Solution to Quiz 9

Q1.

Without any restriction, the number of ways that the 11 letters can be arranged is (5+4+2)!/(5!*4!*2!)=6930.

If all of 5 a's are consecutive, we regard them as an integrated whole, then the number of arrangements is (1+4+2)!/(1!*4!*2!)=105.

If all of 4 b's are consecutive, we regard them as an integrated whole, then the number of arrangements is (5+1+2)!/(5!*1!*2!)=168.

If all of 2 c's are consecutive, we regard them as an integrated whole, then the number of arrangements is (5+4+1)!/(5!*4!*1!)=1260.

If all of 5 a's and all of 4 b’s are consecutive, we regard them as two integrated wholes, then the number of arrangements is (1+1+2)!/(1!*1!*2!)=12.

If all of 5 a's and all of 2 c’s are consecutive, we regard them as two integrated wholes, then the number of arrangements is (1+4+1)!/(1!*4!*1!)=30.

If all of 4 b's and all of 2 c’s are consecutive, we regard them as two integrated wholes, then the number of arrangements is (5+1+1)!/(5!*1!*1!)=42.

If all of 5 a's, all of 4 b’s and all of 2 c’s are consecutive, we regard them as three integrated wholes, then the number of arrangements is (1+1+1)!/(1!*1!*1!)=3.

Hence, the number of arrangements that at least all of one kind of same-type letters are consecutive is 105+168+1260-12-30-42+3=1452, then the number of ways that the 11 letters can be arranged so that no same-type letters are all consecutive is 6390-1452=4938.

Q2.

This problem can be easily solved with enumeration method.

Firstly we narrow the value range of x_1, since $x_1+x_2+x_3=28$, $0 \le x_2 \le 12$ and $0 \le x_3 \le 6$, then $x_1 = 28 - x_2 - x_3 \ge 28 - 12 - 6 = 10$, hence $10 \le x_1 \le 20$.

Similarly, we can also obtain $2 \le x_2 \le 12$.

With $0 \le x_3 \le 6$:

As $x_1 = 10$, $x_2 + x_3 = 18$, then $12 \le x_2 = 18 - x_3 \le 18$ whose intersection with $2 \le x_2 \le 12$ is $\{12\}$, the number of solutions in this case is 1.

As $x_1 = 11$, $x_2 + x_3 = 17$, then $11 \le x_2 = 17 - x_3 \le 17$ whose intersection with $2 \le x_2 \le 12$ is $\{11,12\}$, the number of solutions in this case is 2.

As $x_1 = 12$, $x_2 + x_3 = 16$, then $10 \le x_2 = 16 - x_3 \le 16$ whose intersection with $2 \le x_2 \le 12$ is $\{10,11,12\}$, the number of solutions in this case is 3.

As $x_1 = 13$, $x_2 + x_3 = 15$, then $9 \le x_2 = 15 - x_3 \le 15$ whose intersection with $2 \le x_2 \le 12$ is $\{9,10,11,12\}$, the number of solutions in this case is 4.

As $x_1 = 14$, $x_2 + x_3 = 14$, then $8 \le x_2 = 14 - x_3 \le 14$ whose intersection with $2 \le x_2 \le 12$ is $\{8,9,10,11,12\}$, the number of solutions in this case is 5.

As $x_1 = 15$, $x_2 + x_3 = 13$, then $7 \le x_2 = 13 - x_3 \le 13$ whose intersection with $2 \le x_2 \le 12$ is $\{7,8,9,10,11,12\}$, the number of solutions in this case is 6.

As $x_1 = 16$, $x_2 + x_3 = 12$, then $6 \le x_2 = 12 - x_3 \le 12$ whose intersection with $2 \le x_2 \le 12$ is $\{6,7,8,9,10,11,12\}$, the number of solutions in this case is 7.

As $x_1 = 17$, $x_2 + x_3 = 11$, then $5 \le x_2 = 11 - x_3 \le 11$ whose intersection with $2 \le x_2 \le 12$ is $\{5,6,7,8,9,10,11\}$, the number of solutions in this case is 7.

As $x_1 = 18$, $x_2 + x_3 = 10$, then $4 \le x_2 = 10 - x_3 \le 10$ whose intersection with $2 \le x_2 \le 12$ is $\{4,5,6,7,8,9,10\}$, the number of solutions in this case is 7.

As $x_1 = 19$, $x_2 + x_3 = 9$, then $3 \le x_2 = 9 - x_3 \le 9$ whose intersection with $2 \le x_2 \le 12$ is $\{3,4,5,6,7,8,9\}$, the number of solutions in this case is 7.

As $x_1 = 20$, $x_2 + x_3 = 8$, then $2 \le x_2 = 8 - x_3 \le 8$ whose intersection with

$2 \le x_2 \le 12$ is $\{2,3,4,5,6,7,8\}$, the number of solutions in this case is 7.

Then the number of solutions subject to the given restriction is 1+2+3+4+5+6+7+7+7+7+7=56.