lec 1

Fixed Income Securities

- discount factor: z
- **Compounding Frequency:**
  - $(1+\frac{r_n}{n})^{nt} = (1+r_{year})^t = e^{rt}$
  - $r_n = n(\frac{1}{z^{\frac{1}{nt}}} - 1)$
  - $r = -\frac{ln(z)}{t}$
- Floating Rate Bonds: coupon was determined in last reset date (mostly every half year)
  - spread = 0: price = principal, once pay coupon: price = principal
  - spread = s: price = principal + PV(each spread)

lec 2

Interest Rate Risk

- Price risk: The risk that bond prices change due to interest rate fluctuations
- Reinvestment risk: The risk that interest rate fluctuations change the rate at which bond coupons can be reinvested

duraion

- D = -(dP/p)/dr
  - measures the percentage change in bond price when the level of interest rates changes by 1 percent
  - r is in percentage
  - r: continuously compounded YTM
- when the coupon rate increases, duration falls
  - Lower Average Time of Cash Flow Payments: The higher the coupon, the larger are the intermediate coupons relative to the last one. Thus the average time of payments gets closer to today
  - Lower Sensitivity to Interest Rates: The higher the coupon rate, the larger are cash flows in the near future compared to the more distant future. Cash flows that arrive sooner rather than later are less sensitive to changes in interest rates. Thus, an increase in coupon rate implies an overall lower sensitivity to changes in discount rates
- Duration of a Floating Rate Bond with zero spread
  - equal to the time left to the next coupon payment date
  - If today is the coupon payment/reset date: duration is 0
- Interpreting duration as the average time of future payments only holds for securities with fixed cash flows

Modified Duration

- change r to y (semi-annually compounded YTM)
- MD = D/(1+y/2)
  - y is YTM
  - r = 2ln(1+y/2)
  - dr = 1/(1+y/2)dy
  - into D = -(dP/p)/dr
  - D = -(1+y/2)(dP/p)/dy

dollar duration

- dollar duration = P*D
- dollar duration of portfolio = sum(dollar duration of securities)

Immunization

- choose a portfolio of securities with the same present value and duration of the cash flow commitment to pay

convexity

- C = (d^2P/P)/dr^2
- Convexity states that
  - When interest rates go up, prices do not go down as much as duration alone predicts
  - When interest rates go down, prices go up more than what duration alone predicts
- A common pitfall is to consider convexity as the change in duration, it is not!
  - The duration of a ZCB is its time-to-maturity which is independent of the interest rate. But the convexity of a ZCB is not zero!

Slope and Curvature

- Level: the average yield across all maturities
- Slope: the difference between long- and short-term yields
- Curvature: a combination of short-, medium-, and long-term bond

Factor Duration

- D = sum(w*t*factor)

Lec 3

forward discount factor

- F(t,T1,T2)

Forward rates

- compounding frequency forward rate
  - fn(t,T1,T2)
- continuously compounded forward rate
  - f(t,T1,T2)

Forward curve

- bootstrap methodology
  - exploit the previous relations to extract spot rates (and hence discount factors) from observed forward rates

Forward Rate Agreement (FRA)

- The value of a forward agreement is zero at inception
- $V^{floating}(T_1) = N$
- $V^{FRA}(t) = N * Z(t, T_2) * \Delta * [f_n(0, T_1, T_2) - f_n(t, T_1, T_2)]$
  - fixed leg receiver
  - floating leg payer

Forward Contracts

Interest Rate Swaps

- swap rate
  - $c = n * (\frac{1 - Z(0, T_M)}{\sum_{j=1}^{M} Z(0, T_j)})$
- floating rate is determined at T_(i−1), pay at T_i
- long swap = long floating and short fixed
-
  - Given the swap curve we can determine the implicit discount factors through the **bootstrap methodology**
    - For $i = 1$:

$$Z(t, T_1) = \frac{1}{1 + \frac{c(t, T_1)}{n}}$$

    - While for $i = 2, \ldots, M$:

$$Z(t, T_i) = \frac{1 - \frac{c(t, T_i)}{n} \times \sum_{j=1}^{i-1} Z(t, T_j)}{1 + \frac{c(t, T_i)}{n}}$$

Futures

- traded on a regulated exchange

- contract is “standardized”
- marked-to-market daily
- Futures prices (like forward prices) must converge to the price of the underlying security at maturity date

options

- in-the-money
- out-of-the-money
- at-the-money
- put-call parity
    - call - put = long forward

lec 4

Fed, Central Banks

- The Federal Reserve is in charge of conducting U.S. monetary policy
  - The goals of monetary policy are explained in the Federal Reserve Act which states that the Federal Reserve should seek: “to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates”
  - The Federal Reserve has only limited power in affecting prices, employment, and interest rates, and the mechanism by which it can do so is far from perfect
  - The Norwegian equivalent to the Fed is Norges Bank
- Central Banks
  - The central bank determines the short-term level of interest rates
  - By deciding on the money supply in the economy, the first point of the interest rate curve is determined (the interbank overnight rate, in the US: Fed Funds Rate)
  - The central bank changes the money supply by • Printing fiat currency (physical dollar bills) • Setting reserve requirements affecting how much money the banking system creates
  - Longer term interest rates are determined by market participants expectations to the path of short term rates and risk premia
- The Tools of Monetary Policy
  - Open market operations, which are interventions in the market to buy or sell Treasury securities; they are determined by the Federal Open Market Committee (FOMC), and it is by far the main tool of active monetary policy
    - Open market operations have a large impact on this equilibrium rate
      - If the Fed buys Treasury securities in an open market operation from one of the depository institutions, it pays for them by crediting the institution’s account
      - Because the depository institution can then lend these cash balances to other banks at the Fed funds rate, the open market operation increases the total supply of cash balances available for lending
      - This tends to decrease the equilibrium market clearing Fed funds rate
    - More often the Federal Reserve only wishes to counterbalance temporary or seasonal variations in the total supply of reserves so more often the Federal Reserve carries out open market operations through repurchase and reverse repurchase agreements
      - A Repurchase Agreement (Repo) is an agreement to sell some securities to another party and buy them back at a fixed date and for a fixed amount. The price at which the security is bought back is greater than the selling price and the difference implies an interest called Repo Rate
      - A Reverse Repo is the opposite transaction, namely, it is the purchase of the security for cash with the agreement to sell it back to the original owner at a predetermined price, determined, once again, by the Repo Rate
  - Reserve requirements, which are the amount of reserves that depository institutions (banks) are required to have at the Federal Reserve Bank; they are determined by the Board of Governors of the Federal Reserve
  - The Federal discount rate, which is the rate at which the Federal Reserve lends to FDIC-approved depository institutions; it is determined by the directors of the Federal Reserve banks, subject to review by the Board of Governors
- The Fed Funds Rate
  - The Federal funds rate is the rate at which depository institutions can borrow or lend overnight reserves at the Federal Reserve Bank
    - Each depository institution is required to keep reserves with the Federal Reserve
    - Because of its daily operations a depository institution may run a deficit of reserves, and rather than simply replenish them, it may be cheaper to borrow such funds from another depository institution that may be running a surplus
    - The Federal funds rate is the rate at which these transactions occur
  - The Federal funds rate itself then is not decided by the Federal Reserve, but it is an equilibrium level resulting from these transactions

Predicting the Fed Funds Rate

- - Predict the Fed funds rate (r^{FF}) using its current level:

$$r^{FF}(t+1) = a + \beta_1 \times r^{FF}(t) + \varepsilon(t+1)$$

  - The Fed funds rate reacts also to the labor market conditions and the inflation rate, so it appears sensible to try use information from the business cycle (i.e. employment and inflation)

$$r^{FF}(t+1) = a + \beta_2 \times X^{Pay}(t) + \beta_3 \times X^{Inf}(t) + \varepsilon(t+1)$$

    where X^{Pay} is the annual growth of nonfarm payrolls, and X^{Inf} is the annual growth in the CPI Index

  - We can also include all three variables into one model:

$$r^{FF}(t+1) = a + \beta_1 \times r^{FF}(t) + \beta_2 \times X^{Pay}(t) + \beta_3 \times X^{Inf}(t) + \varepsilon(t+1)$$

Theories on the term structure

- The Expectations Hypothesis
  - $$\begin{aligned} r(t,T) = & \left[\frac{1}{\tau} * r(t,t+1) + \frac{(\tau-1)}{\tau} * E_t[r(t+1,T)]\right] && \text{(Expected future yield)} \\ & + \frac{\lambda}{\tau} && \text{(Risk premium)} \\ & - \frac{(\tau-1)^2}{2\tau} Var_t[r(t+1,T)] && \text{(Convexity)} \end{aligned}$$
  - The third term is a convexity adjustment due to the convex relationship between yields and prices
  - The expectations hypothesis assumes that:
  - $$\lambda = (\tau-1)^2 * \frac{V_t[r(t+1,T)]}{2}$$
  - Finally
  - $$E_t[r(t+1,t+\tau) - r(t,t+\tau)] = \frac{1}{(\tau-1)}[r(t,t+\tau) - r(t,t+1)]$$
  - the expected change in yield (LHS) is directly related to the slope of the term structure (RHS) i.e. the Expectations Hypothesis

Conclusion

- The statement that a positively sloped term structure implies that market participants expect higher future yields is largely rejected by the data
- In fact, quite the opposite is true: A positively sloped term structure predicts lower future yields, because it is related to a risk premium that market participants require to hold long-term bonds
- This result is also consistent with the fact that the forward spread predicts returns on long-term bonds
- In short, returns on zero coupon bonds are predictable by using some predicting factors and this is due to a variation in risk premia, rather than variation in expectation of future yields

Inflation Risk

- Inflation risk refers to the loss of purchasing power of the dollar. All assets that pay fixed amount of dollars in the future are subject to inflation risk
- Treasury Inflation Protected Securities (TIPS)

- If the CPI increases, then the principal amount increases proportionally
  - The Treasury publishes index ratios, which are simply given by the change in the CPI index between the issuance of the TIPS and the reference CPI reading
  - The reference CPI reading is not the current CPI, however, but the average of the CPI value at the beginning of the month of the coupon payment and the CPI value at the beginning of the previous month
  - Both the coupon payment and the principal amount increases with the CPI

lec 5

Binomial Model

- i is the period number
- t is the time in years
- j is the future state/nod
- Replicating portfolios
  - - Solving for N_2:

      $$N_2 = \frac{V_{1,u} - V_{1,d}}{P_{1,u}(2) - P_{1,d}(2)}$$

    - Given N_2, the solution for N_1 is:

      $$N_1 = [V_{1,u} - N_2 \times P_{1,u}(2)] \;/\; 100$$

    - We then get:

      $$V_0 = \Pi_0 = N_1 \times P_0(1) + N_2 \times P_0(2)$$

    - This method provides a **first recipe** for pricing derivatives
- risk premium embedded in the price of longer term bonds
- market price of interest rate risk
  - $$\frac{e^{-r_0\Delta} \times E[P_1(2)] - P_0(2)}{P_{1,u}(2) - P_{1,d}(2)} = \frac{e^{-r_0\Delta} \times E[V_1] - V_0}{V_{1,u} - V_{1,d}}$$
  - So we have that, all interest rate securities on a binomial tress have **the same ratio** between risk premium and risk:

    $$\frac{Risk\ premium}{Risk} = \frac{e^{-r_0\Delta} \times E[V_1] - V_0}{V_{1,u} - V_{1,d}} = \boldsymbol{\lambda_0}$$

    where $\boldsymbol{\lambda_0}$ **is common across all** interest rate securities and is called the market price of (interest rate) risk
- Another Pricing Formula
  -

- If we know λ_0 at time $i = 0$, we can compute the price of any security as:
$$V_0 = e^{-r_0 \Delta} \times E[V_1] - \lambda_0 \times (V_{1,u} - V_{1,d})$$

- How can we compute λ_0? Using information on the bond with maturity $i = 2$
$$\lambda_0 = \frac{e^{-r_0 \Delta} \times E[P_1(2)] - P_0(2)}{P_{1,u}(2) - P_{1,d}(2)}$$

- This method provides a **second recipe** for pricing derivatives

- Risk Neutral Probability
  - $$p^* = \frac{e^{r0 \times \Delta} P_0(2) - P_{1,d}(2)}{P_{1,u}(2) - P_{1,d}(2)}$$
- Risk Neutral Expected Interest Rates and Forward Rates
  - Forward rates are not equal to the market expectation of future interest rates
  - If today we observe high forward rates we should think about two possibilities:
    1. Market participants expect higher future interest rates
    2. They are strongly averse to risk, and thus the price of long term bonds is low today
  - The forward rate is not even equal to the risk neutral future interest rate, although they are quite close
    - Recall that risk neutral pricing is based on the notion of dynamic replication, which involves trading in securities
    - Interest rates are related to securities prices through a convex relation
    - Thus the divergence between rates, is because there is a convexity adjustment missing to make both interest rates the same

lec 6

Two-Step Binomial Tree

- tool.xlsx lec 6

match term structure

- we can find p* by choosing a value for it such that observed price = model price
- tool.xlsx exercise 6c

Spot Rate Duratio

- spot rate duration measures the percentage sensitivity of an interest rate security price to the interest rate
  - $$D = -\frac{1}{V} \times \frac{dV}{dr}$$
- In a binomial tree we compute it by
  - $$\frac{dV}{dr} \approx \frac{V_{1,u} - V_{1,d}}{r_{1,u} - r_{1,d}}$$

A Structured 5-Year Bond

- The same quantity computed for the standard 5-year zero coupon bond yields D5 = 4.62 (here D = 2.08), the structured bond is less risky than the 5-year zero coupon bond
- Intuition: Bond prices decline when interest rate increase. The structured bond provides protection against this scenario, as it increases the final payout when the interest rate increases
- To evaluate the long-term payoff distribution we must use the true probabilities
- The result is very different depending on which probability we use
- Risk neutral probabilities make the expectation of future interest rates higher. This structured security pays more in high-interest rate states so it is overly optimistic

lec 7

Ho-Lee Model

- $r_{i+1} = r_i + \theta_i * \Delta + -\sigma * \sqrt{\Delta}$
- p = 0.5
- σ consistent with the empirical interest rate volatility
- use binominal tree to calibrate θ_i
- see "tool.xlsx" "holee model"

Simple BDT model

- not allow for negative interest rates
- $z = ln(r)$
- $z_{i+1} = z_i + \theta_i * \Delta + -\sigma * \sqrt{\Delta}$
- p = 0.5
- calibrate θ_i
- use the volatility σ of the log interest rate
- see "tool.xlsx" "simple BDT model"

Valuation of options, swaps, and swaptions

- see "tool.xlsx" "exercise 7a"
- receiver swaption/swap: receive the fixed rate
- empirical volatility = std of Δr or $\Delta ln(r)$
- The Ho-Lee model appears to overprice short term caps, and underprice long term caps, while the Simple BDT model in this case always underprices

implied flat volatility

- in the interest rate model that exactly price

full BDT Model

- σ varying
- tree is no longer recombining
- calibrate z and σ to match bond price and cap price

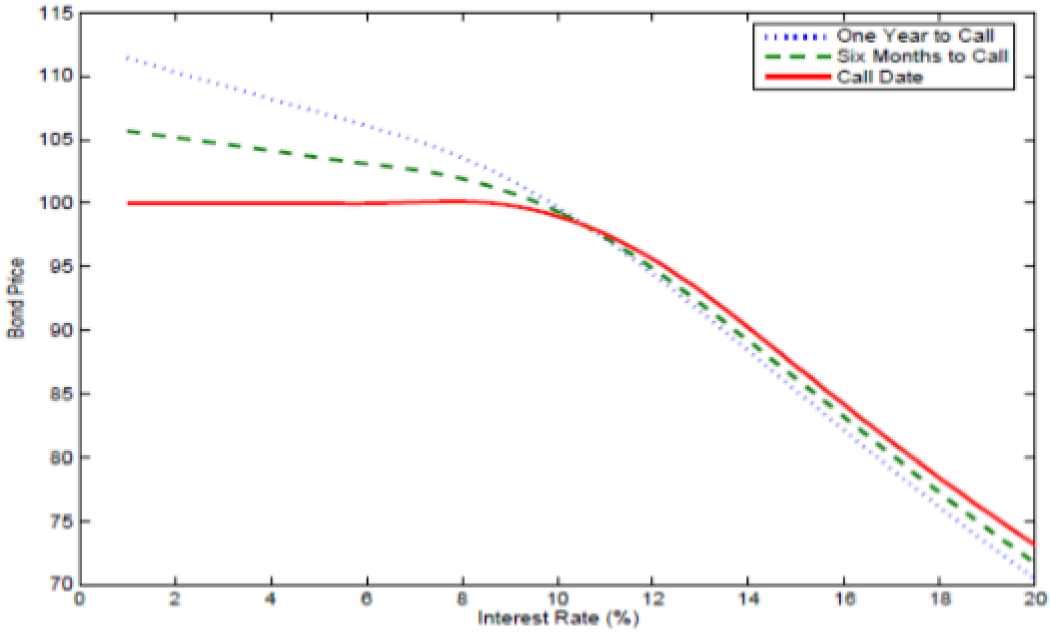
Valuation of Futures

- expected profit should be zero
- Eurodollar futures contract: quote as 100 - futures rate in percentage
- Eurodollar futures LIBOR rate = N * quarterly compounding rate
  - future rate = 4 * (exp(r/4) - 1)

lec 8

American options

- callable bonds
  - 1.5-year, 3% coupon bond, that is callable at par (100) starting on i = 1
    1. bond (long)
    2. An American call option (short)
  - The Negative Convexity of Callable Bonds
    - **Figure 12.1** The Negative Convexity in Callable Bonds



- Option Adjusted Spread (OAS)
  - There are numerous reasons why the OAS of a bond may be different from zero (such as liquidity)
  - The higher the option adjusted spread (OAS), the more underpriced is the bond compared to the model, and thus the higher should be the incentive to purchase the bond
  - market < model, market r > model rate, OAS positive

American Swaptions

Monte Carlo

- Monte Carlo is very useful for pricing securities with path-dependent payoffs (asian option)
- (the MC here is a weak one)

Asian Interest Rate Options

- (...)
- How Many Simulations are Enough
  - $standarderror = std(price)/\sqrt{N}$
  - Confidence Interval
  - unfortunatly, the question was ignored

lec 9

Securitization

- Specific Purpose Vehicle (SPV) is created which formally buys the assets and raises the necessary capital from the investors

fixed-rate mortgages

- r: monthly rate
- A: discount factor
- C: coupon monthly (fixed)
- L: total mortgage
- $C = \frac{L}{\sum_{i=1}^{30*12} A^i}$
- interest paid at t: $I_t = r/12 * L_t$
- principal paid at t: $L_t^{paid} = C - I_t$
- principal at t+1: $L_{t+1} = L_t - L_t^{paid}$
-

$$L_t = \sum_{i=1}^{n} \frac{C}{\left(1+\frac{\bar{r}_{12}^{m}}{12}\right)^{i}} = C \times A \times \frac{1-A^{n}}{1-A}$$

is:

$$P(t) = \sum_{i=1}^{n} \frac{C}{\left(1+r_{12}^{m}(t,T_i)/12\right)^{i}}$$

Default Risk and Prepayment Risk

- Default risk refers to the probability that an unusually large number of homeowners' defaults on their mortgage payments
- Prepayment risk refers to the probability of receiving cash flows too early compared to the expected life of the mortgage
  - Prepayments bring down the rate of return for a bank
  - (when interest rate decrease) has to re-invests in a low interest environment
  - Seasonality: Summers are characterized by large prepayments, as this is the period in which people move from one place to another for various reasons
  - Age of the mortgage pool: Young mortgages are characterized by large interest rate payments and low principal, by paying early homeowners can save the interest payments
  - Family circumstances: Default, disasters, or sale of the house
  - Housing prices: If the property value of a house declines, it is more difficult to refinance, and thus prepayments tend to decline
  - Burnout effect: Mortgage pools heavily refinanced in the past then to be insensitive to interest rates
- Interest rate risk: both price risk and re-investment risk

MBS

- Prepayment speed

    - p: a constant probability that the mortgage will be prepaid after the next coupon
    - Conditional Prepayment Rate (CPR): CPR = 1 – (1 – p)12
    - PSA Experience
        - CPR = 0.2% of the principal is paid in the first month
        - CPR increases by 0.2% in each of the following 30 months
        - CPR levels off at 6% thereafter until maturity
    - The prepayment speed is typically measured as a percentage of this scenario (e.g. 150% PSA)
- Pass-Through Securities
    - the simplest type of MBS
    - Agency MBS are essentially default risk free
    - If we think of these securities having zero or small default risk, then we can use the Treasury discount curve to discount these cash flows

Duration and convexity

- The effective duration of a MBS is given by the formula:

$$D \approx \frac{1}{P}\frac{P(+x\_bps)-P(-x\_bps)}{2\times x\_bps}$$

- This duration is much smaller than the duration that we obtained when we neglected the impact on the change in PSA due to changes in interest rates
- issing the impact of interest rate variation on the speed of prepayment may grossly overstate the sensitivity of the pass-through security to changes in interest rates, and thus the performance of any duration-based hedging activity
- An important characteristic of MBS is negative convexity
    - Fixed income security prices typically increase when interest rates fall
    - Negative convexity, instead, states that if interest rate decline makes the rate of prepayment increase, then the price of the security does not increase as much
- The effective convexity of a MBS is given by the formula:

$$C \approx \frac{1}{P}\frac{P(+x\_bps)+P(-x\_bps)-2\times P}{(x\_bps)^2}$$

- Suppose that as the interest rate declines, the PSA increases, while if the interest rate increases, the PSA declines

Collateralized Mortgage Obligations (CMO)

- more complex structures than pass through securities
- offers different levels of prepayment risk
- appeal to different investors' clienteles, which may increase the liquidity of these securities
- divide the total principal into smaller groups called "Tranche A", "Tranche B", "Tranche C", and so on
- Each tranche receives:
    - A fixed coupon rate payment, in percentage of the tranche principal
    - Sequentially all the principal payments, scheduled or prepaid, up to the point at which the whole principal of the tranche is paid out
- Sometimes a "Tranche Z" is added, which receives no cash flows (like a zero coupon), but the coupon is accrued over time to its principal
    - Once all other classes have been retired, the principal payments go to Tranche Z
- CMO planned amortization class (PAC)
    - Also structured in tranches which receive prepayments according to a fixed schedule
- Interest only and principal only strips
    - Interest only receives all interest payments from the underlying mortgages
    - Principal only receives all scheduled and unscheduled principal from the underlying mortgages

lec 10

Modeling Credit Risk

Corporate Bond Markets

- interest rate risk and credit risk
- Credit risk refers to the possibility of default by one of the counterparties in a financial transaction
- Credit rating agencies define default as any missed or delayed disbursement of contractual obligations, bankruptcy, or distressed exchanges
-

	Moody's	Standard & Poor's	Safety
Investment grade	Aaa	AAA	The strongest rating; ability to repay interest and principal is very strong.
	Aa	AA	Very strong likelihood that interest and principal will be repaid.
	A	A	Strong ability to repay, but some vulnerability to changes in circumstances.
	Baa	BBB	Adequate capacity to repay; more vulnerability to changes in economic circumstances.
Speculative grade (Junk bonds)	Ba	BB	Considerable uncertainty about ability to repay.
	B	B	Likelihood of interest and principal payments over sustained periods is questionable.
	Caa	CCC	Bonds that may already be in default or in danger of imminent default.
	Ca	CC	
	C	C	Little prospect for interest or principal on the debt ever to be repaid.

- workouts
  - can lead to exchange offers in which old debt contracts are replaced by new debt contracts which may be less valuable to the creditors
- Default and workouts are referred to as credit events

Structural Credit Risk Models

- option pricing theory
- The BS model is based on the following assumptions
  - Stock prices follows an Ito process
  - No restrictions on short-selling
  - No transactions costs nor taxes
  - No dividend payment on the underlying security
  - No arbitrage opportunities
  - Trading is continuous and the risk-free rate is constant
- The Black-Scholes (1973) Model: P13
- The basic Merton (1974) model: P23
  - B_t: market value of debt at time t
  - D: par value of debt, maturity date is time T
  - S_t: market value of equity
  - V_t: market value of the firm's assets
  - S_T = max(V_T-D, 0)
  - B_T = min(D, V_T) = D - max(V_T-D, 0)
  -

The value of equity is

$$S_0 = c^{BS}(V_0, D, T, \sigma, r)$$
$$= V_0 \times N(d_1) - e^{-rT} \times D \times N(d_2)$$

where

$$d_1 = \frac{ln(V_0/D) + \left(r + \frac{1}{2} \times \sigma^2\right) \times T}{\sigma \times \sqrt{T}}$$
$$d_2 = d_1 - \sigma \times \sqrt{T}$$

NB: σ is the **asset volatility** and NOT the stock volatility!

- The value of debt is

$$B_0 = D \times e^{-rT} - p^{BS}(V_0, D, T, \sigma, r)$$

We can use the put-call parity to obtain the put option value

$$p^{BS}(V_0, D, T, \sigma, r) = c^{BS}(V_0, D, T, \sigma, r) + D \times e^{-rT} - V_0$$

Note also that

$$B_0 = D \times e^{-rT} - p^{BS}(V_0, D, T, \sigma, r)$$
$$= D \times e^{-rT} - (c^{BS}(V_0, D, T, \sigma, r) + D \times e^{-rT} - V_0)$$
$$= V_0 - c^{BS}(V_0, D, T, \sigma, r)$$

- the value of the bond is:
  -

Bond Price	Intuition
Increasing in V_0	The (short) put option value decreases when the underlying asset value increases
Increasing in D	The call option (S_0) is less valuable when the strike price increases. Hence, from $B_0 = V_0 - S_0$ the bond price increases
Decreasing in r	The call option value increases because the PV of the strike price is lower. Hence, $B_0 \downarrow$
Decreasing in T	The call option value increases, hence $B_0 \downarrow$
Decreasing in σ	The call option value increases

- The Term Structure of Credit Spreads
  - P28
- Crosbie and Bohn (2003) describes how to implement the Merton model

Reduced-Form Models

```
- Reduced-form models directly model the probability of default and the recovery rates
- default event is modeled as a random event
- hazard rate: $\lambda$
- default probability: $\lambda\Delta t$
- survival probability: $1-\lambda\Delta t$
```

- credit spread in the continuous limit is given by
  - $y - r = \lambda(1 - R)$
  - R: recovery rate

Credit Default Swap (CDS)

- The protection buyer pays a quarterly or semi-annual CDS premium to the protection seller until whichever comes first: expiration or default
- In case of default the protection seller pays a compensation to the protection buyer
- Why is the CDS Market Popular?
  - An easy way of trading credit
  - Often used in structured products
  - Illiquid secondary markets in corporate bonds may make it difficult to take on credit risk
  - Shorting a corporate bond may be even harder (i.e. to short credit risk). Repo can be expensive or even non-existent whereas selling a CDS contract is much simpler

Valuation of CDS

- in a frictionless market we should have that:
  - $Bond\ yield = CDS\ premium + Risk\ free\ rate$
- Computing the CDS Premium: P63 - P68

lec 11

Continuous time models

- reasons
  - Realism
  - Simplicity
  - Analytical formulas
- Ho-Lee model
  - $dr_t = \theta_t dt + \sigma dX_t$
- BM
- Stochastic Processes
  - drift term, predictable
  - diffusion term, unpredictable
- Vasicek model
  - $dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$
- Driftless Ho-Lee Model

Valuation with Deterministic Interest Rate

- The no-arbitrage restriction allows us to obtain the solution:

  $$Z(r_t, t; T) = e^{A(t;T) - B(t;T) r_t}$$

  where

  $$B(t;T) = \frac{1}{\gamma}\left(1 - e^{-\gamma(T-t)}\right)$$
  $$A(t;T) = \bar{r}\left(B(t;T) - (T-t)\right)$$

  We will not derive these equations but note that
  - $A(t;T)$ and $B(t;T)$ are only functions of time and converge to zero as t approaches T
  - Since $B(t;T) > 0$, a higher r_t implies a lower bond price
  - If $r_t = \bar{r}$, then the price reverts to the standard formula $Z(r_t, t; T) = e^{-\bar{r}(T-t)}$
  - If $r_t > \bar{r}$, the price is lower the lower γ is, the opposite is true for $r_t < \bar{r}$

Valuation with stochastic interest rate

Course Overview

- Bond valuation
  - The relation between discount factors and interest rates
  - Valuation of zero-coupon, fixed-coupon, and floating rate bonds
- Risk management
  - Duration, convexity, and factor duration
  - Portfolio immunization
- Fixed income derivatives
  - Valuation of FRAs, forwards/futures, and swaps
  - Options on interest rates, swaps, and bonds
- Monetary policy

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- How and why the central bank conducts monetary policy
- The expectations hypothesis
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- Term structure models in discrete time
  - The binomial model: Ho-Lee and BDT models
  - Calibration to market prices and risk-neutral trees
  - Valuation of fixed income securities with fixed and/or stochastic cash flows
- Monte Carlo simulations
  - How to simulate future interest rates in the binomial model
  - How to value securities using MC simulations
- Residential mortgage backed securities
  - The institutional setup of the RMBS market
  - The risk and return in RMBS
- Credit risk modeling and CDS
  - Structural and reduced-form models
  - CDS contracts
- Introduction to term structure models in continuous time
  - Stochastic processes and Ito’s Lemma
  - The Ho-Lee and Vasicek models