☺Differential Equation

1. **Separable differential equations**
   $y' = f\ (t) \cdot g(y)$
   $$\int \frac{1}{g(y)} dy = \int \frac{1}{f(t)} dt$$
2. **Linear differential equations**
   $y' + a(t)y = b(t)$
   $\mathrm{u} = \mathrm{e}^{\int a(t)dt}$
   $uy' + a(t)uy\ =\ b(t)u$
   $(uy)' =\ b(t)u$
   $$uy\ = \int b(t)udt$$
   $$y\ = \frac{1}{u}\int b(t)udt$$
   $y' + ay = b \Rightarrow y = \frac{b}{a} + Ce^{-at}$
3. **Exact differential equations**
   $p(t,y)y' + q(t,y) = 0$
   $\begin{cases} \mathrm{h}'_{\mathrm{y}} = p(t,y) \\ h'_t = q(t,y) \end{cases}$
   $h(t,y) = C$
4. **Superposition principle**
   $y_1\ for\ y'' + ay' + by = f_1(t)\ and\ y_2\ for\ y'' + ay' + by = f_2(t)$
   $Then\ c_1y_1 + c_2y_2\ for\ y'' + ay' + by = c_1f_1(t) + c_2f_2(t)$
   Homogeneous case
   $y'' + ay' + by = 0$
   $$\mathrm{r} = \frac{\mathrm{a} \pm \sqrt{a^2 - 4b}}{-2}$$
   a. $a^2 - 4b\ > 0$
   $$y\ = C_1e^{r_1t} + C_2e^{r_2t}$$
   b. $a^2 - 4b = 0$
   $$y\ = (C_1{+}C_2t)e^{rt}$$
   c. $a^2 - 4b < 0$
   $$y\ =\ e^{-\frac{a}{2}t}(C_1\cos\beta t + C_2\ \sin\beta t)\ where\ \beta = \frac{\sqrt{4b - a^2}}{2}$$
   Inhomogeneous case
   $y'' + ay' + by = f(t)$
   We may guess a solution $y = y_p$, often assume y has the same form as $f(t), f'(t), f''(t)$
   Then we have $\mathbf{y = y_h + y_p}$
5. **Superposition principle for first order linear differential equations**
   $y' + ay = b(t)$

$y = Ce^{-at} + y_p$

6. **Systems of differential equations**

$\underline{\boldsymbol{y}'} = \boldsymbol{A}\underline{\boldsymbol{y}}$

$\underline{\boldsymbol{y}} = \mathrm{C}_1\underline{\mathbf{v_1}}\mathrm{e}^{\lambda_1 \mathrm{t}} + \mathrm{C}_2\underline{\mathbf{v_2}}\mathrm{e}^{\lambda_2 \mathrm{t}} + \cdots + \mathrm{C}_\mathrm{n}\underline{\mathbf{v_n}}\mathrm{e}^{\lambda_\mathrm{n} \mathrm{t}}$

7. **Equilibrium states, stable, globally asymptotically stable**
   - **a.** $\mathrm{y}' = 0 \Rightarrow \mathrm{y_e}$
   - **b.** $F'(y_e) < 0 \Rightarrow stable$
   - **c.** $F' < 0 \Rightarrow$globally asymptotically stable