Contents

1 Math 1
1.1 Math Basic 1
1.2 Euler Function 1
1.3 China Remain Theorm 1
1.4 Math Counting 2
1.5 Miller Rabin 2
1.6 Linear Algebra 2
1.7 FFT 3

2 String 4
2.1 KMP 4
2.2 LPS 4

3 Tree 5
3.1 Tree Min Vertex Cover 5

4 Graph 5
4.1 Bipartite: MaxMatch, MinVerCover, MaxIndSet . 5
4.2 Min Vertex Cover 6

5 Flow 7
5.1 Dinic Maxflow Mincut 7

6 Geometry 8
6.1 Geometry basic 8
6.2 Minimal Enclose Disk 8
6.3 2D Convex Hull 8


```
set nocompatible

set enc=utf-8
set fenc=utf-8

set tabstop=4
set softtabstop=4
set shiftwidth=4
set backspace=2
set autoindent
set cindent

syntax on
set t_Co=256
set number
set showmatch
set hls

autocmd FileType cpp nnoremap <F9> :w <bar> :! g++ % -std
    =c++11 -O2 -Wall && ./a.out<CR>
```

1 Math

1.1 Math Basic

```
vector<pii> primeFac(int n) {
  vector<pii> ret;
  for(int i=2; n>1; ++i){
    if( n%i != 0 ) continue;
    int e = 0;
    while( n%i == 0 ) ++e , n/=i;
    ret.push_back({i, e});
  }
  return ret;
}
long long fastPow(long long x, int n, long long m){
  long long ans = 1LL;
  while( n ){
    if( n&1 ) ans = ans * x % m;
    x = x*x % m;
    n >>= 1;
  }
  return ans;
}
long long modInv(long long x, long long p){
  return fastPow(x, p-2, p);
}
long long modInv_euler(long long x, long long m){
  // must be gcd(x,m)==1
  // phi is euler function: O(sqrt(x))
  return fastPow(x, phi(m)-1, m);
}
```

1.2 Euler Function

```
int phi(int n){
  // euler function: in [0,n], # of coprime(i, n)
  vector<pii> fac = primeFac(n);
  int num = 1 , m = 1;
  for(auto &p : fac)
    num *= (p.first-1) , m *= p.first;
  return n/m * num;
}
```

1.3 China Remain Theorm

```
bool china_solvable(vector<pii> &rule) {
  for(int i=0; i<rule.size(); ++i)
  for(int j=1; j<rule.size(); ++j) {
    int gcd = __gcd(rule[i].second, rule[j].second);
    if( rule[i].first%gcd != rule[j].first%gcd )
      return false;
  }
  return true;
}
long long china(const vector<pii> &rule, int nlt=0){
  // solve x = ai (mod mi)
  // rule should solvable
  long long MM = 1LL;
  for(auto &r : rule)
    MM = lcm(MM, r.second);
  long long x = 0LL;
  for(auto &r : rule){
    long long ai = r.first;
    long long mi = r.second;
    long long Mi = MM / r.second;
    long long Mv = modInv_euler(Mi%mi, mi);
    long long tmp = ai*Mi%MM *Mv %MM;
    x = (x+tmp) % MM;
  }
  if( x>=nlt ) return x;
  long long n = ceil((nlt-x)*1.0/MM);
  return x + n*MM;
}
```

1.4 Math Counting

```
const int MaxNum = 1000004;
const int modNum = 1000000009;
long long fac  [MaxNum];
long long facIv[MaxNum];
void initFac(){
  fac[0] = facIv[0] = 1LL;
  for(int i=1; i<MaxNum; ++i) {
    fac  [i] = fac[i-1]*i % modNum;
    facIv[i] = modInv(fac[i], modNum);
  }
}
long long Cnm(int n, int m){
  if( m==0 || n==m ) return 1LL;
  return fac[n]*facIv[m] % modNum *facIv[n-m] % modNum;
}
long long nBlock_kColor(int n,int k){
  // n different blocks; k different colors
  // use inclusion-exclusion principle
  long long ans = fastPow(k, n, modNum);
  bool del = true;
  for(int i=k-1; i>0; --i, del=!del){
    long long now = Cnm(k, i)*fastPow(i, n, modNum) %
    modNum;
    if( del ) ans = (ans+modNum-now) % modNum;
    else ans = (ans+now) % modNum;
  }
  return ans;
}
```

1.5 Miller Rabin

```
#include <climits>
typedef unsigned long long int ull;

ull bases[20] = { 2ULL, 3ULL,5ULL,7ULL,11ULL,13ULL,17ULL
    ,19ULL,23ULL,29ULL,31ULL,37ULL };

ull fake_mul(ull n, ull m, ull x);
ull fast_pow(ull n, ull p, ull x);

bool is_prime(ull n)
{
  if (n < 2ULL) return false;

  for (int tt = 0; tt < 12; tt++) {
    ull a;
    a = bases[tt] % n;

    if (a == 0 || a == 1 || a == n - 1) {
      continue;
    }

    int t = 0;
    ull u = n - 1ULL;
    while ((u & 1ULL) == 0ULL) u >>= 1, t++;

    ull x = fast_pow(a, u, n);   // x = a ^ u % n;
    if (x == 1ULL || x == (n - 1)) continue;
    for (int i = 0; i < t - 1; i++)
    {
      if (ULLONG_MAX / x < x) {
        x = fake_mul(x, x, n);
      }
      else {
        x = x*x%n;
      }
      if (x == 1) return false;
      if (x == n - 1) break;
    }
    if (x == n - 1) continue;
    return false;
  }
  return true;
}

ull fake_mul(ull n, ull m, ull x)
{
  ull re = 0ULL;
  while (m != 0ULL) {
    if ((m & 1ULL) != 0ULL) {
      if (ULLONG_MAX - re < n) {
        ull temp = ULLONG_MAX%x;

        temp += (n - (ULLONG_MAX - re)) % x;
        re = temp%x;
      }
      else {
        re = (re + n) % x;
      }
    }

    if (ULLONG_MAX - n < n) {
      ull temp = ULLONG_MAX%x;

      temp += (n - (ULLONG_MAX - n)) % x;
      n = temp%x;
    }
    else {
      n = n + n%x;
    }

    m >>= 1;
  }
  return re;
}

ull fast_pow(ull n, ull p, ull x)
{
  ull re = 1ULL;

  while (p != 0ULL) {
    if ((p & 1ULL) != 0ULL) {
      if (ULLONG_MAX / re < n) {
        re = fake_mul(n, re, x);
      }
      else {
        re = (re*n) % x;
      }
    }

    if (ULLONG_MAX / n < n) {
      n = fake_mul(n, n, x);
    }
    else {
      n = (n*n) % x;
    }
    p >>= 1;
  }
  return re;
}
```

1.6 Linear Algebra

```
#ifndef _MATRIX_H_
#define _MATRIX_H_

#include <iostream>
#include <vector>
using namespace std;

template <class T>
class Matrix{
public:
  int rSize, cSize;
  vector< vector<T> > mat;

  Matrix(int r, int c) :rSize(r), cSize(c), mat(rSize,
    vector<T>(cSize)){}
```

```
  vector<T>& operator[](int i) {
    return mat[i];
  }
  void print();
};

template <class T>
void Matrix<T>::print() {
  cout << "Matrix elements:" << endl;
  for (int i = 0; i < rSize; i++) {
    cout << "[";
    for (int j = 0; j < cSize; j++) {
      cout << "\t" << mat[i][j];
      if (j != cSize - 1)cout << ",";
    }
    cout << " ]" << endl;
  }
}

#endif
```

```
#include "Matrix.h"

template <class T>
Matrix<T> matMul(Matrix<T> matA, Matrix<T> matB){
  Matrix<T> matRe(matA.rSize, matB.cSize);

  for (int i = 0; i < matRe.rSize; i++) {
    for (int j = 0; j < matRe.cSize;; j++) {
      matRe[i][j] = 0;
      for (int k = 0; k < matA.cSize; k++) {
        matRe[i][j] += matA[i][k] * matB[k][j];
      }
    }
  }
  return matRe;
}
```

1.7 FFT

```
#include <complex>
#include <vector>
#include <cmath>

const double PI = 3.141592654;
typedef complex<double> Complex;

void _fft(vector<Complex>& buf, vector<Complex>& out,
  int st, int step, bool isInv) {

  if (step >= buf.size()) return;

  _fft(out, buf, st, step * 2, isInv);
  _fft(out, buf, st + step, step * 2, isInv);

  int n = buf.size();
  double c = isInv ? 1.0 : -1.0;
  for (int i = 0; i < n; i += 2 * step) {
    Complex t = polar(1.0, c * 2 * PI * i / n) * out[i +
    step + st];
    buf[i / 2 + st] = out[i + st] + t;
    buf[(i + n) / 2 + st] = out[i + st] - t;
  }
}

void fft(vector<Complex> &x, bool isInv) {
  int n = x.size(), nxt2 = 0;
  for (int i = 0, mask = 1; i < 31; i++, mask <<= 1)
    nxt2 = (n&mask) ? (n != mask) ? 1 << (i + 1) : 1 << i
     : nxt2;
  n = nxt2;
  while (x.size() < n)
    x.push_back(0);

  vector<Complex> out = x;
  _fft(x, out, 0, 1, isInv);
  for (int i = 0; isInv && i < x.size(); i++)
    x[i] /= n;
}
```

2 String

2.1 KMP

```
class kmp{
  private:
    int  prefix[maxLen];
    char pat[maxLen];
  public:
    void setPattern(const char *str){
      strcpy(pat, str);
      prefix[0] = -1;
      int i=1 , j=0;
      for( ; str[i]!='\0' ; ++i , ++j ){
        if( str[i]==str[j] )
          prefix[i] = prefix[j];
        else
          prefix[i] = j;
        while( j>=0 && str[j]!=str[i] )
          j = prefix[j];
      }
      prefix[i] = j;
    }
    int search(const char *str){
      // return index of str match pattern
      int i=0 , j=0;
      for( ; str[i]!='\0' && pat[j]!='\0' ; ++i,++j){
        while( j>=0 && pat[j]!=str[i] )
          j = prefix[j];
      }
      if( pat[j]=='\0' )
        return i-j;
      return -1;
    }
    int countMatched(const char *str){
      // return # of pattern in str
      int cnt = 0;
      int i=0 , j=0;
      while( true ){
        if( pat[j]=='\0' ) ++cnt;
        if( str[i]=='\0' ) break;
        while( j>=0 && pat[j]!=str[i] )
          j = prefix[j];
        ++i , ++j;
      }
      return cnt;
    }
};
```

2.2 LPS

```
int lps(const char *str){
  // return len of longest palindrom substring
  static char emptyChar = '@';
  static char tmp[maxLen*2];
  static int lprb[maxLen*2];
  // [i-lprb[i], i+lprb[i]] is the lps when mid is i
  for(int i=0, j=-1; true; ++i){
    if( str[i]=='\0' ){
      tmp[++j] = emptyChar;
      tmp[++j] = '\0';
      break;
    }
    tmp[++j] = emptyChar;
    tmp[++j] = str[i];
  }
  lprb[0] = 0;
  int rightBorder = 0 , midId = 0;
  for(int i=1; tmp[i]!='\0'; ++i){
    if( i>rightBorder ){
      rightBorder = i;
      midId = i;
      lprb[i] = 0;
    }
    int mirId = midId - (i-midId);
    if( i+lprb[mirId] > rightBorder )
      lprb[i] = rightBorder - i;
    else if( i+lprb[mirId] < rightBorder )
      lprb[i] = lprb[mirId];
    else{
      int j=lprb[mirId];
      while( tmp[i+j]!='\0' && i-j>=0 && tmp[i+j]==tmp[i-
      j])
        ++j;
      rightBorder = i+j-1;
      midId = i;
      lprb[i] = j-1;
    }
  }
  int ans = 1;
  for(int i=0 ; tmp[i]!='\0' ; ++i)
    if( lprb[i]>ans )
      ans = lprb[i];
  return ans;
}
```

3 Tree

3.1 Tree Min Vertex Cover

```
class TreeMinVertexCover {
private:
  static const int maxNum = 100004;
  vector<int> G[maxNum];
  int in[maxNum];
public:
  bool pick[maxNum];
  int MVC; // min vertext cover
  void init() {
    for(int i=0; i<maxNum; ++i)
      G[i].clear();
    memset(in, 0, sizeof(in));
  }
  void addEdge(int u, int v) {
    G[u].emplace_back(v);
    G[v].emplace_back(u);
    ++in[u];
    ++in[v];
  }
  int treeMinVertexCover() {
    memset(pick, 0, sizeof(pick));
    MVC = 0;
    queue<int> myQ;
    for(int i=1; i<=maxNum; ++i)
      if( in[i]==1 ) myQ.push(i);
    while( myQ.size() ) {
      int nowAt = myQ.front();
      myQ.pop();
      if( in[nowAt]==0 ) continue;
      ++MVC;
      int id;
      for(int i=0; i<G[nowAt].size(); ++i)
        if( in[G[nowAt][i]] ) {
          id = G[nowAt][i];
          break;
        }
      for(int i=0; i<G[id].size(); ++i)
        if( in[G[id][i]] ) {
          --in[G[id][i]];
          --in[id];
          if( in[G[id][i]]==1 )
            myQ.push(G[id][i]);
        }
    }
    return MVC;
  }
};
```

4 Graph

4.1 Bipartite: MaxMatch, MinVerCover, MaxIndSet

```
class Bipartite {
private:
  static const int MaxNum = 1004;
  vector<int> g[MaxNum];
  bool visited [MaxNum];

  bool bipart(int nowAt, int nowSide) {
    visited[nowAt] = true;
    side[nowAt] = nowSide;
    for(auto &id : g[nowAt])
      if( !visited[id] )
        bipart(id , !nowSide);
      else if( side[id]==nowSide )
        return false;
    return true;
  }
  bool maxMatch(int nowAt) {
    visited[nowAt] = true;
    for(auto &id : g[nowAt])
      if( cp[id]==-1
        || (!visited[cp[id]] && maxMatch(cp[id])) ){
        cp[id]    = nowAt;
        cp[nowAt] = id;
        return true;
      }
    return false;
  }
  void minVertexCover(int nowAt) {
    MVC[nowAt] = 1;
    for(auto &id : g[nowAt])
      if( !MVC[id] ) {
        MVC[id] = 1;
        minVertexCover(cp[id]);
      }
  }
  void maxIndependentSet(int nowAt) {
    MIS[nowAt] = 1;
    for(auto &id : g[nowAt])
      if( !MIS[cp[id]] )
        maxIndependentSet(cp[id]);
  }
public:
  int  matchNum;     // max match num
  int  cp  [MaxNum]; // id and cp[id] is couple
  bool side[MaxNum]; // left/right side
  bool MVC [MaxNum]; // min vertex cover
  bool MIS [MaxNum]; // max indepent set
  void addEdge(int u, int v) {
    g[u].emplace_back(v);
    g[v].emplace_back(u);
  }
  void init() {
    for(int i=0; i<MaxNum; ++i)
      g[i].clear();
  }
  bool countAll() {
    // if graph is not bipartite return false

    // bipartite
    memset(side, 0, sizeof(side));
    memset(visited , 0 , sizeof(visited));
    for(int i=0 ; i<MaxNum ; ++i)
      if( !visited[i] && !bipart(i, 0) )
        return false;


    // maximum match
    // O(VE), this code can be more optimized
    // alternative: dinic O(V^0.5*E)
```

```
    matchNum = 0;
    memset(cp , -1 , sizeof(cp));
    for(int i=0 ; i<MaxNum ; ++i){
      if( cp[i]!=-1 ) continue;
      memset(visited , 0 , sizeof(visited));
      if( maxMatch(i) )
        ++matchNum;
    }

    // min vertex cover
    memset(MVC, 0, sizeof(MVC));
    for(int i=0; i<MaxNum; ++i)
      if( side[i]==1 && cp[i]==-1 )
        minVertexCover(i);
    for(int i=0; i<MaxNum; ++i)
      if( side[i]==1 )
        MVC[i] = !MVC[i];

    // max independent set
    memset(MIS, 0, sizeof(MIS));
    for(int i=0; i<MaxNum; ++i)
      if( cp[i]==-1 )
        maxIndependentSet(i);
    for(int i=0; i<MaxNum; ++i)
      if( side[i]==1 && cp[i]!=-1
        && !MIS[i] && !MIS[cp[i]] )
        MIS[i] = 1;

    return true;
  }
};
```

4.2 Min Vertex Cover

```
struct MinVertexCover {
private:
  static const int MaxNum = 54;
  vector<int> G[MaxNum];
  int in[MaxNum];

  int undo(vector<int> &record) {
    for(int i=0; i<record.size(); ++i)
      ++in[record[i]];
    record.clear();
  }
  int delNode(int u, vector<int> &record) {
    for(int i=0; i<G[u].size(); ++i)
      if( in[G[u][i]] ) {
        --in[G[u][i]];
        --in[u];
        record.push_back(G[u][i]);
        record.push_back(u);
      }
  }

  int cnt(int from, int *visited, bool type) {
    if( visited[from] ) return 0;
    if( type==1 ) visited[from] = 1;
    for(int i=0; i<G[from].size(); ++i)
    if( in[G[from][i]] && !visited[G[from][i]] )
      return type+cnt(G[from][i], visited, !type);
    return type;
  }
  int cnt(int *visited) {
    int ret = 0;
    for(int i=0; i<MaxNum; ++i)
      if( in[i]==1 && !visited[i] )
        ret += cnt(i, visited, 0);
    for(int i=0; i<MaxNum; ++i)
      if( in[i]==2 && !visited[i] )
        ret += cnt(i, visited, 0);
    return ret;
  }

public:
  int MVCPick[MaxNum];
  int MVC; // min vertex cover
  void init() {
    for(int i=0; i<MaxNum; ++i)
      G[i].clear();
    memset(in, 0, sizeof(in));
  }
  void addEdge(int u, int v) {
    G[u].push_back(v);
    G[v].push_back(u);
    ++in[u];
    ++in[v];
  }
  void minVertexCover(int nowMVC=0, const int *lastPick=
    NULL) {
    // O(n^2 * 1.38^n)
    int nowPick[MaxNum] = {};
    if( nowMVC==0 ) {
      MVC = MaxNum;
      memset(MVCPick, 0, sizeof(MVCPick));
    }
    else memcpy(nowPick, lastPick, sizeof(nowPick));

    int maxid = 0;
    for(int i=0; i<MaxNum; ++i)
      if( in[i]>in[maxid] )
        maxid = i;
    if( in[maxid]<=2 ) {
      nowMVC += cnt(nowPick);
      if( nowMVC<MVC ) {
        MVC = nowMVC;
        memcpy(MVCPick, nowPick, sizeof(nowPick));
      }
      return;
    }

    vector<int> record;

    delNode(maxid, record);
    nowPick[maxid] = 1;
    minVertexCover(nowMVC+1, nowPick);
    nowPick[maxid] = 0;
    undo(record);

    int cnt = 0;
    for(int i=0; i<G[maxid].size(); ++i)
      if( in[G[maxid][i]] ) {
        ++cnt;
        delNode(G[maxid][i], record);
        nowPick[G[maxid][i]] = 1;
      }
    minVertexCover(nowMVC+cnt, nowPick);
    undo(record);
  }
};
```

5 Flow

5.1 Dinic Maxflow Mincut

```
class Dinic{
private:
  static const int maxN = 104;
  int pipe[maxN][maxN];
  vector<int> g[maxN];

  int level[maxN];
  bool bfsLabeling(int s, int t){
    memset(level , 0 , sizeof(level));
    queue<int> myQ;
    myQ.push( s );
    level[s] = 1;
    while( !myQ.empty() ){
      int nowAt = myQ.front();
      myQ.pop();
      for(int i=0;i<g[nowAt].size();++i)
        if( !level[g[nowAt][i]] && pipe[nowAt][g[nowAt][i
    ]] ){
          level[g[nowAt][i]] = level[nowAt] + 1;
          myQ.push( g[nowAt][i] );
        }
    }
    return level[t];
  }
  int dfsFindRoute(int nowAt, int t, int maxC) {
    if( nowAt==t ){
      maxFlow += maxC;
      return maxC;
    }
    for(int i=0; i<g[nowAt].size(); ++i) {
      int next = g[nowAt][i];
      if( level[next] != level[nowAt]+1 ) continue;
      if( !pipe[nowAt][next] ) continue;
      int nowOut = dfsFindRoute(next ,t ,min(maxC , pipe[
    nowAt][next]));
      if( nowOut==0 )
        continue;
      pipe[nowAt][next] -= nowOut;
      pipe[next][nowAt] += nowOut;
      return nowOut;
    }
    return 0;
  }

public:
  int maxFlow;
  vector<pii> minCut;

  void init(){
    memset(pipe , 0 , sizeof(pipe));
    for(int i=0;i<maxN;++i)
      g[i].clear();
    maxFlow = 0;
    minCut.clear();
  }
  void addEdge(int u, int v, int c) {
    if( u==v ) return;
    if( pipe[u][v]==0 ) {
      g[u].emplace_back(v);
      g[v].emplace_back(u);
    }
    pipe[u][v] += c;
    pipe[v][u] += c;
  }
  void coculAll(int s, int t) {
    // max flow
    while( bfsLabeling(s,t) )
      while( dfsFindRoute(s,t,1023456789) )
        ;

    // min cut
    for(int i=0;i<maxN;++i) {
      if( level[i]!=0 )
        continue;
      for(int j=0; j<g[i].size(); ++j)
        if( level[g[i][j]]==0 )
          minCut.push_back({i, g[i][j]});
    }
  }
};
```

6 Geometry

6.1 Geometry basic

```
struct Point{
  double x,y;
  Point(double xi=0.0,double yi=0.0){
    x = xi , y = yi;
  }
  Point operator - (const Point &r)const{
    return Point(x-r.x , y-r.y);
  }
};
typedef Point Vector;

double angle(const Vector &v,const Vector &u){
  // return rad [0, pi] of two vector
  return acos( dot(v,u)/len(v)/len(u) );
}
Vector rotate(const Vector &v,double rad){
  return Vector(
    v.x*cos(rad) - v.y*sin(rad),
    v.x*sin(rad) + v.y*cos(rad)
  );
}
double pointSegLen(const Point &A,const Point &B,const
    Point &Q){
  if(A==B) return len(Q-A);
  if( dot(B-A , Q-A)<0 ) return len(Q-A);
  if( dot(B-A , Q-B)>0 ) return len(Q-B);
  return fabs( cross(B-A , Q-A) ) / len(B-A);
}
bool pointOnSeg(const Point &A,const Point &B,const Point
     &Q){
  return fabs( len(Q-B)+len(Q-A)-len(A-B) ) < 1e-9;
}

struct Line{
  Point P0;
  Vector v;
  Line(const Point &pi,const Vector &vi):p0(pi) , v(vi)
    {}
};

double pointLineLen(const Line &L,const Point &Q){
  return fabs( cross(L.v , Q-L.P0) ) / len(L.v);
}
Point projectToLine(const Line &L,const Point &Q){
  double t = dot(Q-L.P0 , L.v) / dot(L.v , L.v);
  return L.P0 + L.v * t;
}

Point innerCircle(point &p1, point &p2, point &p3){
  // p1,p2,p3 should not on same line
  double a1 = (-2*p1.x + 2*p2.x);
  double b1 = (-2*p1.y + 2*p2.y);
  double c1 = (p2.x*p2.x + p2.y*p2.y - p1.x*p1.x - p1.y*
    p1.y);
  double a2 = (-2*p1.x + 2*p3.x);
  double b2 = (-2*p1.y + 2*p3.y);
  double c2 = (p3.x*p3.x + p3.y*p3.y - p1.x*p1.x - p1.y*
    p1.y);
  double cx = (c1*b2-c2*b1) / (a1*b2-a2*b1);
  double cy = (a1*c2-a2*c1) / (a1*b2-a2*b1);
  return Point(cx, cy);
}
Point outerCircle(point &p1, point &p2, point &p3) {
  // p1,p2,p3 should not on same line
  double x1 = (p1.x+p2.x)/2.0;
  double y1 = (p1.y+p2.y)/2.0;
  double x2 = (p2.x+p3.x)/2.0;
  double y2 = (p2.y+p3.y)/2.0;
  double vx = p2.x-p1.x;
  double vy = p2.y-p1.y;
  double ux = p3.x-p2.x;
  double uy = p3.y-p2.y;
  double A  = vx*x1 + vy*y1;
  double B  = ux*x2 + uy*y2;
  double cx = (uy*A - vy*B) / (uy*vx - ux*vy);
  double cy = (ux*A - vx*B) / (ux*vy - uy*vx);
  return Point(cx, cy);
}
```

6.2 Minimal Enclose Disk

```
struct Circle {
  Point c;
  double R2; // square of radius
  Circle() {}
  Circle(const Point &p1, const Point &p2) {
    c.x = (p1.x+p2.x)/2.0;
    c.y = (p1.y+p2.y)/2.0;
    R2 = dot(p1-p2, p1-p2)/4.0;
  }
  Circle(const Point &p1, const Point &p2, const Point &
    p3) {
    // p1, p2, p3 should not on same line
    c = outerCircle(p1, p2, p3);
    double dx = p1.x - c.x;
    double dy = p1.y - c.y;
    R2 = dx*dx + dy*dy;
  }
  bool contain(const Point &p) const {
    double dx = c.x - p.x;
    double dy = c.y - p.y;
    return fdif(dx*dx + dy*dy - R2)<=0;
  }
};

Circle minEncloseDisk(vector<Point> &ps) {
  // Find minimal circal enclose all point
  // worst case O(n^3), expected O(n)
  Circle D;
  if( ps.size()==0 ) return D;
  if( ps.size()==1 ) {
    D.c  = ps[0];
    D.R2 = 0.0;
    return D;
  }

  random_shuffle(ps.begin(), ps.end());
  D = Circle(ps[0], ps[1]);
  for(int i=2; i<ps.size(); ++i) {
    if( D.contain(ps[i]) )
      continue;
    D = Circle(ps[i], ps[0]);
    for(int j=1; j<i; ++j) {
      if( D.contain(ps[j]) )
        continue;
      D = Circle(ps[i], ps[j]);
      for(int k=0; k<j; ++k) {
        if( D.contain(ps[k]) )
          continue;
        D = Circle(ps[i], ps[j], ps[k]);
      }
    }
  }
}
```

6.3 2D Convex Hull

```
bool turnLeft(const Vector &v1, const Vector &v2) {
  return fdif(cross(v1, v2)) > 0LL;
}
vector<Point> convexHull(vector<Point> &ps) {
  // return convex hull without redundant point
  sort(ps.begin(), ps.end());

  vector<Point> up;
```

```
  for(int i=0; i<ps.size(); ++i) {
    while( up.size()>1
      && !turnLeft(up.back()-up[up.size()-2],
        ps[i]-up.back()) )
      up.pop_back();
    up.emplace_back(ps[i]);
  }

  vector<Point> btn;
  for(int i=ps.size()-1; i>=0; --i) {
    while( btn.size()>1
      && !turnLeft(btn.back()-btn[btn.size()-2],
        ps[i]-btn.back()) )
      btn.pop_back();
    btn.emplace_back(ps[i]);
  }

  vector<Point> res(up);
  res.insert(res.end(), btn.begin()+1, btn.end());
  res.pop_back();
  return res;
}
```