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set nocompatible

set enc=utf-8
set fenc=utf-8

set tabstop=4
set softtabstop=4
set shiftwidth=4
set backspace=2
set autoindent
set cindent

syntax on
set t_Co=256
set number
set showmatch
set hls

autocmd FileType cpp noremap <F9> :w <bar> :! g++ % -std
=c++11 -O2 -Wall && ./a.out<CR>

1 Java

```

1 class Scan {
2     BufferedReader buffer;
2     StringTokenizer tok;
2     Scan() {
2         buffer = new BufferedReader(new InputStreamReader(
2             System.in));
2     }
3     boolean hasNext() {
3         while(tok == null || !tok.hasMoreElements())
4             try {
4                 tok = new StringTokenizer(buffer.readLine());
4             } catch (Exception ex) {
5                 return false;
5             }
6         return true;
6     }
6     String next() {
6         if(hasNext())
6             return tok.nextToken();
6         return null;
7     }
7     String nextLine() {
7         if(hasNext())
7             return tok.nextToken("\n");
8         return null;
9     }
9     int nextInt() {
10        return Integer.parseInt(next());
10    }
10 }
10
11 /* Compile: javac %
11 * Run: java [Class name] */
11 import java.util.*;
12 import java.lang.*;
12 import java.math.*;
12
13 class Main {
13     public static void main (String[] args) {
14         System.out.print(1);
14         System.out.print(2);
14         System.out.println("Hello World");
14         System.out.printf("%.2f", 0.12345);
14
15         Scanner sc = new Scanner(System.in);
15         System.out.println(sc.nextLine()); //gets()
15         System.out.println(sc.next()); //scanf("%s")
15         System.out.println(sc.nextInt());
15         System.out.println(sc.nextDouble());
15         while(sc.hasNext()) { //EOF
16             int a = sc.nextInt();
16             System.out.println(a);
16         }
16
16         int[] a = {1,2,3};
16         int[][] b = {{1,2},{3,4,5}};
16         double[] c = new double[90];
16         System.out.print(b[0][1]);
16         System.out.print(b[1][2]);
16
16         int[] d = {5,2,1,3,4};
16         Integer[] e = {6,3,4,1,2};
16         Arrays.sort(d);
16         Arrays.sort(e, new MyCom());
16         for(int i=0; i<d.length; i++) {
16             System.out.print(d[i]);
16         }
16         for(int i=0; i<e.length; i++) {
16             System.out.print(e[i]);
16         }
16
16         Set<String> s = new HashSet<String>(); //or
16         TreeSet

```

```

        s.add("123");
        s.add("234");
        System.out.println(s);
        System.out.println(s.contains("123"));
        Map<String, Integer> m = new TreeMap<String,
Integer>();
        m.put("haha", 123);
        m.put("hehe", 234);
        System.out.println(m);

        BigInteger b1 = new BigInteger("-1231237182379123712");
        BigInteger b2 = BigInteger.valueOf(234);

        System.out.println(b1.add(b2));
        System.out.println(b1.mod(b2));

        int z = Integer.parseInt("-123");
        System.out.println(z);

        System.out.println(Math.PI);
        System.out.println(Math.sin(1));
    }

    static class MyCom implements Comparator<Integer> {
        public int compare(Integer i1, Integer i2) {
            return i2 - i1;
        }
    }
}

```

2 Math

2.1 Math Basic

```

vector<pii> primeFac(int n) {
    vector<pii> ret;
    for(int i=2; n>1; ++i){
        if( n%i != 0 ) continue;
        int e = 0;
        while( n%i == 0 ) ++e , n/=i;
        ret.push_back({i, e});
    }
    return ret;
}

long long fastPow(long long x, int n, long long m){
    long long ans = 1LL;
    while( n ){
        if( n&1 ) ans = ans * x % m;
        x = x*x % m;
        n >>= 1;
    }
    return ans;
}

long long modInv(long long x, long long p){
    return fastPow(x, p-2, p);
}

long long modInv_euler(long long x, long long m){
    // must be gcd(x,m)==1
    // phi is euler function: O(sqrt(x))
    return fastPow(x, phi(m)-1, m);
}

long long gt(long long a, long long b) {
    // smallest integer greater than a/b
    long long ret = a/b;
    if( ret>0 || a%b==0 ) ++ret;
    return ret;
}

```

2.2 Euler Function

```

int phi(int n){
    // euler function: in [0,n], # of coprime(i, n)
    vector<pii> fac = primeFac(n);
    int num = 1 , m = 1;
    for(auto &p : fac)
        num *= (p.first-1) , m *= p.first;
    return n/m * num;
}

```

2.3 Extended Euclidean

```

pll recur(long long n, long long m) {
    // solve one integer solution of
    // x*n + y*m = gcd(n,m)
    if( n%m == 0 )
        return {0LL, 1LL};
    pll res = recur(m, n%m);
    pll ret = {res.second, res.first - res.second * (n/m)};
    return ret;
}

```

2.4 China Remain Theorm

```

bool china_solvable(vector<pii> &rule) {
    for(int i=0; i<rule.size(); ++i)
        for(int j=1; j<rule.size(); ++j) {
            int gcd = __gcd(rule[i].second, rule[j].second);
            if( rule[i].first%gcd != rule[j].first%gcd )
                return false;
        }
    return true;
}

```

```

long long china(const vector<pii> &rule, int nlt=0){
    // solve  $x = a_i \pmod{m_i}$ 
    // rule should solvable
    long long MM = 1LL;
    for(auto &r : rule){
        MM = lcm(MM, r.second);
    }
    long long x = 0LL;
    for(auto &r : rule){
        long long ai = r.first;
        long long mi = r.second;
        long long Mi = MM / r.second;
        long long Mv = modInv_euler(Mi%mi, mi);
        long long tmp = ai*Mi%MM *Mv %MM;
        x = (x+tmp) % MM;
    }
    if( x>=nlt ) return x;
    long long n = ceil((nlt-x)*1.0/MM);
    return x + n*MM;
}

```

2.5 Counting

```

const int MaxNum = 1000004;
const int modNum = 1000000009;
long long fac [MaxNum];
long long facIv[MaxNum];
void initFac(){
    fac[0] = facIv[0] = 1LL;
    for(int i=1; i<MaxNum; ++i) {
        fac[i] = fac[i-1]*i % modNum;
        facIv[i] = modInv(fac[i], modNum);
    }
}
long long Cnm(int n, int m){
    if( m==0 || n==m ) return 1LL;
    return fac[n]*facIv[m] % modNum *facIv[n-m] % modNum;
}
long long nBlock_kColor(int n,int k){
    // n different blocks; k different colors
    // use inclusion-exclusion principle
    long long ans = fastPow(k, n, modNum);
    bool del = true;
    for(int i=k-1; i>0; --i, del=!del){
        long long now = Cnm(k, i)*fastPow(i, n, modNum) % modNum;
        if( del ) ans = (ans+modNum-now) % modNum;
        else ans = (ans+now) % modNum;
    }
    return ans;
}

```

2.6 Miller Rabin

```

#include <climits>
typedef unsigned long long int ull;

ull bases[20] = { 2ULL, 3ULL,5ULL,7ULL,11ULL,13ULL,17ULL,
    ,19ULL,23ULL,29ULL,31ULL,37ULL };

ull fake_mul(ull n, ull m, ull x);
ull fast_pow(ull n, ull p, ull x);

bool is_prime(ull n)
{
    if (n < 2ULL) return false;

    for (int tt = 0; tt < 12; tt++) {
        ull a;
        a = bases[tt] % n;

        if (a == 0 || a == 1 || a == n - 1) {
            continue;
        }
    }
}

```

```

int t = 0;
ull u = n - 1ULL;
while ((u & 1ULL) == 0ULL) u >>= 1, t++;

ull x = fast_pow(a, u, n); //  $x = a^u \pmod{n}$ 
if (x == 1ULL || x == (n - 1)) continue;
for (int i = 0; i < t - 1; i++)
{
    if (ULLONG_MAX / x < x) {
        x = fake_mul(x, x, n);
    }
    else {
        x = x*x%n;
    }
    if (x == 1) return false;
    if (x == n - 1) break;
}
if (x == n - 1) continue;
return false;
}
return true;
}

```

```

ull fake_mul(ull n, ull m, ull x)
{
    ull re = 0ULL;
    while (m != 0ULL) {
        if ((m & 1ULL) != 0ULL) {
            if (ULLONG_MAX - re < n) {
                ull temp = ULLONG_MAX%x;

                temp += (n - (ULLONG_MAX - re)) % x;
                re = temp%x;
            }
            else {
                re = (re + n) % x;
            }
        }
        m >>= 1;
    }
    return re;
}

```

```

ull fast_pow(ull n, ull p, ull x)
{
    ull re = 1ULL;

    while (p != 0ULL) {
        if ((p & 1ULL) != 0ULL) {
            if (ULLONG_MAX / re < n) {
                re = fake_mul(n, re, x);
            }
            else {
                re = (re*n) % x;
            }
        }
        if (ULLONG_MAX / n < n) {
            n = fake_mul(n, n, x);
        }
        else {
            n = (n*n) % x;
        }
        p >>= 1;
    }
}

```

```

    }
    return re;
}

// Below is non-extreme version
ull fake_mul(ull n, ull m, ull x) {
    ull re = 0ULL;
    n %= x, m %= x;
    while( m ) {
        if( m&1ULL )
            re = (re+n) % x;
        n = (n+n) % x;
        m >>= 1;
    }
    return re;
}

ull fast_pow(ull n, ull p, ull x) {
    ull re = 1ULL;
    while( p ) {
        if( p&1ULL )
            re = fake_mul(re,n,x);
        n = fake_mul(n,n,x);
        p >>= 1;
    }
    return re;
}

bool is_prime(ull n) {
    static const int bNum = 12;
    static const ull bases[bNum] = {
        2ULL,3ULL,5ULL,7ULL,11ULL,13ULL,17ULL,19ULL,23ULL,29
        ULL,31ULL,37ULL
    };
    if( n<=2ULL ) return n==2ULL;
    if( !(n&1ULL) ) return false;

    ull u = n-1;
    while( !(u&1ULL) )
        u >>= 1;
    for(int i=0; i<bNum; i++) {
        if( bases[i]%n == 0 ) continue;
        ull t = u;
        ull a = fast_pow(bases[i], t, n);
        if( a==1 || a==n-1 ) continue;
        while( t!=n-1 && a!=1 && a!=n-1 ) {
            a = fake_mul(a,a,n);
            t <<= 1;
        }
        if( t==n-1 && a==1 ) continue;
        if( a!=n-1 ) return false;
    }
    return true;
}

```

2.7 Pollard rho

```

// need fake_mul, is_prime
ull gcd(ull a, ull b) {
    return (a%b==0)? b : gcd(b, a%b);
}

ull dif(ull a, ull b) {
    return a>b? a-b : b-a;
}

void pollard_rho(ull n, map<ull,int> &facs) {
    while( !(n&1ull) ) {
        // must extract factor 2
        int cnt = 0;
        while( !(n&1ull) )
            ++cnt, n>>=1;
        facs[2] = cnt;
    }
    if( n==1ull ) return;
    if( is_prime(n) ) {
        facs[n]++;
        return;
    }
}

```

```

ull x = rand()%n;
ull y = x;
ull a = rand()%(n-1) + 1;
ull g = 1ull;
while( g==1ull ) {
    x = (fake_mul(x,x,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    if( x==y ) {
        g = n;
        break;
    }
    g = gcd(dif(x,y), n);
}
if( g==n ) // unluck try again
    pollard_rho(n, facs);
else if( g>1ull ) { // luck, found g
    pollard_rho(g, facs);
    pollard_rho(n/g, facs);
}
}

```

2.8 Linear Algebra

```

#ifndef _MATRIX_H_
#define _MATRIX_H_

#include <iostream>
#include <vector>
using namespace std;

template <class T>
class Matrix{
public:
    int rSize, cSize;
    vector< vector<T> > mat;

    Matrix(int r = 0, int c = 0) :rSize(r), cSize(c), mat(
        rSize, vector<T>(cSize)){}
    vector<T>& operator[](int i) {
        return mat[i];
    }
    void print();
};

template <class T>
void Matrix<T>::print() {
    cout << "Matrix elements:" << endl;
    for (int i = 0; i < rSize; i++) {
        cout << "[";
        for (int j = 0; j < cSize; j++) {
            cout << "\t" << mat[i][j];
            if (j != cSize - 1)cout << ",";
        }
        cout << "]" << endl;
    }
}

#endif

#include "Matrix.h"

template <class T>
Matrix<T> matMul(Matrix<T> matA, Matrix<T> matB){
    Matrix<T> matRe(matA.rSize, matB.cSize);

    for (int i = 0; i < matRe.rSize; i++) {
        for (int j = 0; j < matRe.cSize; j++) {
            matRe[i][j] = 0;
            for (int k = 0; k < matA.cSize; k++) {
                matRe[i][j] += matA[i][k] * matB[k][j];
            }
        }
    }
    return matRe;
}

```

```

}
}
return k;
}

```

2.9 FFT

```

#include <complex>
#include <vector>
using namespace std;

const double PI = 3.141592654;
typedef complex<double> Complex;

void _fft(vector<Complex>& buf, vector<Complex>& out,
int st, int step, bool isInv) {

    if (step >= buf.size()) return;

    _fft(out, buf, st, step * 2, isInv);
    _fft(out, buf, st + step, step * 2, isInv);

    int n = buf.size();
    double c = isInv ? 1.0 : -1.0;
    for (int i = 0; i < n; i += 2 * step) {
        Complex t = polar(1.0, c * 2 * PI * i / n) * out[i +
step + st];
        buf[i / 2 + st] = out[i + st] + t;
        buf[(i + n) / 2 + st] = out[i + st] - t;
    }
}

void fft(vector<Complex> &x, bool isInv) {
    int n = x.size(), nxt2 = 0;
    for (int i = 0, mask = 1; i < 31; i++, mask <= 1)
        nxt2 = (n&mask) ? (n != mask) ? 1 << (i + 1) : 1 << i
            : nxt2;
    n = nxt2;
    while (x.size() < n)
        x.push_back(0);

    vector<Complex> out = x;
    _fft(x, out, 0, 1, isInv);
    for (int i = 0; isInv && i < x.size(); i++)
        x[i] /= n;
}

```

2.10 Hash

```

#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
using namespace std;
const int hashRange = 131072;
int hashtable[4][256], shuffleArr[hashRange], ref[
hashRange];
void buildHashTable() {
    memset(ref, -1, sizeof(ref));
    for (int i = 0; i < hashRange; i++)
        shuffleArr[i] = i;
    for (int i = 0; i < 4; i++) {
        random_shuffle(shuffleArr, shuffleArr + hashRange);
        for (int j = 0; j < 256; j++)
            hashtable[i][j] = shuffleArr[j];
    }
}
int myhash(int input) {
    int x[4];
    for (int i = 0; i < 4; i++)
        x[i] = hashtable[i][(input >> (i*8)) & (0xff)];
    int k = x[0] ^ x[1] ^ x[2] ^ x[3];
    if (ref[k] != input)
        for (int i = k; ; i = (i+1)%hashRange)
            if (ref[i] == -1 || ref[i] == input) {
                ref[i] = input;
                return i;
            }
}

```

3 String

3.1 KMP

```
class kmp{
private:
    int prefix[maxLen];
    char pat[maxLen];
public:
    void setPattern(const char *str){
        strcpy(pat, str);
        prefix[0] = -1;
        int i=1, j=0;
        for( ; str[i]!='\0' ; ++i, ++j ){
            if( str[i]==str[j] )
                prefix[i] = prefix[j];
            else
                prefix[i] = j;
            while( j>=0 && str[j]!=str[i] )
                j = prefix[j];
        }
        prefix[i] = j;
    }
    int search(const char *str){
        // return index of str match pattern
        int i=0, j=0;
        for( ; str[i]!='\0' && pat[j]!='\0' ; ++i,++j){
            while( j>=0 && pat[j]!=str[i] )
                j = prefix[j];
        }
        if( pat[j]=='\0' )
            return i-j;
        return -1;
    }
    int countMatched(const char *str){
        // return # of pattern in str
        int cnt = 0;
        int i=0, j=0;
        while( true ){
            if( pat[j]=='\0' ) ++cnt;
            if( str[i]=='\0' ) break;
            while( j>=0 && pat[j]!=str[i] )
                j = prefix[j];
            ++i, ++j;
        }
        return cnt;
    }
};
```

3.2 LPS

```
int lps(const char *str){
    // return len of longest palindrom substring
    static char emptyChar = '@';
    static char tmp[maxLen*2];
    static int lprb[maxLen*2];
    // [i-lprb[i], i+lprb[i]] is the lps when mid is i
    for(int i=0, j=-1; true; ++i){
        if( str[i]=='\0' ){
            tmp[++j] = emptyChar;
            tmp[++j] = '\0';
            break;
        }
        tmp[++j] = emptyChar;
        tmp[++j] = str[i];
    }
    lprb[0] = 0;
    int rightBorder = 0, midId = 0;
    for(int i=1; tmp[i]!='\0'; ++i){
        if( i>rightBorder ){
            rightBorder = i;
            midId = i;
            lprb[i] = 0;
        }
```

```
        int mirId = midId - (i-midId);
        if( i+lprb[mirId] > rightBorder )
            lprb[i] = rightBorder - i;
        else if( i+lprb[mirId] < rightBorder )
            lprb[i] = lprb[mirId];
        else{
            int j=lprb[mirId];
            while( tmp[i+j]!='\0' && i-j>=0 && tmp[i+j]==tmp[i-j] )
                ++j;
            rightBorder = i+j-1;
            midId = i;
            lprb[i] = j-1;
        }
    }
    int ans = 1;
    for(int i=0 ; tmp[i]!='\0' ; ++i)
        if( lprb[i]>ans )
            ans = lprb[i];
    return ans;
}
```

3.3 AC Automation

```
#include <queue>
#include <cstdio>
#include <cstring>
using namespace std;
struct AC_algorithm {
    struct node {
        static const int signNum = 52; //number of kind of character
        node *ch[signNum];
        node *suffix, *dict;
        int index;
        node() {
            memset(ch, 0, sizeof(ch));
            suffix = dict = 0;
            index = -1;
        }
    };
    static const int stringNum = 100010; //number of pattern
    node *root;
    int occur[stringNum]; //string i occur occur[i] times
    int reflect[stringNum]; //string i is the same as string reflect[i];

    AC_algorithm() {
        root = new node();
        memset(occur, 0, sizeof(occur));
        memset(reflect, -1, sizeof(reflect));
    }

    int decode(char c) { //decode char
        return c <= 'Z' ? (c - 'A') : (c - 'a' + 26);
    }

    void insert(char *s, int index) { //add string to trie
        node *p = root;
        for( ; *s; s++) {
            int code = decode(*s);
            if(p->ch[code] == NULL)
                p->ch[code] = new node();
            p = p->ch[code];
        }
        if(p->index == -1)
            p->index = index;
        else
            reflect[index] = p->index;
    }

    void build() { //build machine
        queue<node*> q;
```

```

q.push(root);
while(!q.empty()) {
    node *p = q.front();
    for(int i = 0; i < node::signNum; i++)
        if(p->ch[i]) {
            node *tmp = p->suffix;
            while(tmp && !tmp->ch[i]) tmp = tmp->suffix;
            if(tmp)
                p->ch[i]->suffix = tmp->ch[i];
            else
                p->ch[i]->suffix = root;
            tmp = p->ch[i]->suffix;
            if(tmp->index != -1)
                p->ch[i]->dict = tmp;
            else
                p->ch[i]->dict = tmp->dict;
            q.push(p->ch[i]);
        }
    q.pop();
}

void match(char *s) {          //match patterns with Text
    node *p = root;
    for(; *s; s++) {
        int code = decode(*s);
        while(p && !p->ch[code]) p = p->suffix;
        if(p)
            p = p->ch[code];
        else
            p = root;
        node *tmp = p;
        while(tmp) {
            if(tmp->index != -1)
                occur[tmp->index]++;
            tmp = tmp->dict;
        }
    }
}

~AC_algorithm() {
    queue<node*> q;
    q.push(root);
    while(!q.empty()) {
        node *p = q.front();
        q.pop();
        for(int i = 0; i < node::signNum; i++)
            if(p->ch[i])
                q.push(p->ch[i]);
        delete p;
    }
}
};

```

3.4 Z

```

#include <cstring>
int z[length];
void z_function(char *str) {
    int len = strlen(str), L = 0, R = 1;
    z[0] = len;
    for(int i = 1; i < len; i++)
        if(R <= i || z[i-L] >= R-i) {
            int x = max(R, i);
            while(x < len && str[x] == str[x-i])
                x++;
            z[i] = x-i;
            L = i; R = x;
            //if(i < x) {L = i; R = x;}
        } else
            z[i] = z[i-L];
}
}

```

4 Tree

4.1 Tree Min Vertex Cover

```

class TreeMinVertexCover {
private:
    static const int maxNum = 100004;
    vector<int> G[maxNum];
    int in[maxNum];
public:
    bool pick[maxNum];
    int MVC; // min vertex cover
    void init() {
        for(int i=0; i<maxNum; ++i)
            G[i].clear();
        memset(in, 0, sizeof(in));
    }
    void addEdge(int u, int v) {
        G[u].emplace_back(v);
        G[v].emplace_back(u);
        ++in[u];
        ++in[v];
    }
    int treeMinVertexCover() {
        memset(pick, 0, sizeof(pick));
        MVC = 0;
        queue<int> myQ;
        for(int i=1; i<=maxNum; ++i)
            if( in[i]==1 ) myQ.push(i);
        while( myQ.size() ) {
            int nowAt = myQ.front();
            myQ.pop();
            if( in[nowAt]==0 ) continue;
            ++MVC;
            int id;
            for(int i=0; i<G[nowAt].size(); ++i)
                if( in[G[nowAt][i]] ) {
                    id = G[nowAt][i];
                    break;
                }
            for(int i=0; i<G[id].size(); ++i)
                if( in[G[id][i]] ) {
                    --in[G[id][i]];
                    --in[id];
                    if( in[G[id][i]]==1 )
                        myQ.push(G[id][i]);
                }
            }
        return MVC;
    }
};

```

4.2 Treap ordered

```

struct node {
    int v , p , sz ;
    node *l , *r ;
    node() { l = r = NULL ;}
    node(int v_):v(v_),p(rand()),sz(1) {l = r = 0;}
    int size() {
        return this!=NULL ? this->sz : 0 ;
    }
    void maintain() {
        sz = l->size() + r->size() + 1 ;
    }
};

void splite_v(node *t,node* &a,node* &b,int v) {
    if(!t)
        a = b = NULL ;
    else if(v >= t->v) {
        a = t ;
        splite_v(t->r,a->r,b,v) ;
        a->maintain() ;
    } else if(v < t->v) {

```

```

    b = t ;
    splite_v(t->l,a,b->l,v) ;
    b->maintain() ;
}
}
node* merge(node *a,node *b) {
    if(a==NULL || b==NULL)
        return a!=NULL ? a : b;
    if(a->p > b->p) {
        a->r = merge(a->r,b) ;
        a->maintain() ;
        return a ;
    } else if(a->p <= b->p) {
        b->l = merge(a,b->l) ;
        b->maintain() ;
        return b ;
    }
}
int kth(node *t,int k) {
    if(k<=t->l->size())
        return kth(t->l,k) ;
    else if(k>t->l->size()+1)
        return kth(t->r,k-t->l->size()-1) ;
    return t->v ;
}
void release(node *t) {
    if(t) {
        release(t->l) ;
        release(t->r) ;
        delete t ;
    }
}

```

4.3 Treap unordered

```

#include <iostream>
#include <cstdio>
#include <stdlib.h>
#include <cstring>
using namespace std;
const int oo = 1e9 ;
struct node {
    int v , p , sz ;
    int sum , presum , sufsum , maxsum , flag , set ;
    node *l , *r ;
    node(){ }
    node(int v_):p(rand()),sz(1),l(NULL),r(NULL) {
        v = sum = presum = sufsum = maxsum = v_ ;
        flag = 0 ;
        set = oo ;
    }
    int size() { return this ? sz : 0 ; }
    int Sum() { return this ? sum : 0 ; }
    int Presum() { return this ? (!flag ? presum : sufsum) : -oo ; }
    int Sufsum() { return this ? (!flag ? sufsum : presum) : -oo ; }
    int Maxsum() { return this ? maxsum : -oo ; }
    int max(int a,int b) { return a > b ? a : b ; }
    int max(int a,int b,int c) { return max(a,max(b,c)) ; }
    void makesame(int st) {
        if(this) {
            set = st ;
            sum = st*sz ;
            presum = sufsum = maxsum = (st <= 0 ? st : sum) ;
        }
    }
    void pushdown() {
        if(flag) {
            if(l) l->flag = !l->flag ;
            if(r) r->flag = !r->flag ;
            swap(l,r) ;
            swap(presum,sufsum) ;
            flag = 0 ;
        }
    }
}

```

```

    if(set!=oo) {
        v = set ;
        l->makesame(set) ;
        r->makesame(set) ;
        set = oo ;
    }
}
void maintain() {
    presum = max(l->Presum(), l->Sum() + v, l->Sum() + v + r->Presum()) ;
    sufsum = max(r->Sufsum(), r->Sum() + v, r->Sum() + v + l->Sufsum()) ;
    int maxsum1 = max(l->Maxsum(), r->Maxsum(), v) ;
    int maxsum2 = max(l->Sufsum() + v, r->Presum() + v, l->Sufsum() + v + r->Presum()) ;
    maxsum = max(maxsum1, maxsum2) ;
    sum = l->Sum() + r->Sum() + v ;
    sz = 1 + l->size() + r->size() ;
}
} ;
void splite(node *t,node* &a,node* &b,int k) {
    if(t == NULL) {
        a = b = NULL ;
        return ;
    }
    t->pushdown() ;
    if(t->l->size()+1 <= k) {
        a = t ;
        splite(t->r, a->r, b, k-(t->l->size()+1)) ;
        a->maintain() ;
    } else {
        b = t ;
        splite(t->l,a,b->l,k) ;
        b->maintain() ;
    }
}
node* merge(node *a,node *b) {
    if(!a || !b)
        return a ? a : b ;
    if(a->p > b->p) {
        a->pushdown() ;
        a->r = merge(a->r,b) ;
        a->maintain() ;
        return a ;
    } else {
        b->pushdown() ;
        b->l = merge(a,b->l) ;
        b->maintain() ;
        return b ;
    }
}
void Delete(node *t) {
    if(!t) return ;
    Delete(t->l) ;
    Delete(t->r) ;
    delete t ;
}
void INSERT(node* &root) {
    int p , k , v ;
    node *t=0 , *L , *R;
    scanf("%d%d",&p,&k) ;
    for(int i=0 ; i<k ; i++) {
        scanf("%d",&v) ;
        t = merge(t,new node(v)) ;
    }
    splite(root, L, R, p) ;
    root = merge(L, merge(t, R)) ;
}
void DELETE(node* &root) {
    int p , k ;
    scanf("%d%d",&p,&k) ;
    node *L , *M , *R;
    splite(root, L, R, p-1) ;
    splite(R, M, R, k) ;
    Delete(M) ;
}

```



```

    root = merge(L, R) ;
}
void MAKE_SAME(node* &root) {
    int p , k , l ;
    scanf("%d%d%d",&p,&k,&l) ;
    node *L, *M, *R ;
    splite(root, L, R, p-1) ;
    splite(R, M, R, k) ;
    M->makesame(l) ;
    root = merge(L, merge(M, R)) ;
}
void REVERSE(node* &root) {
    int p , k ;
    scanf("%d%d",&p,&k) ;
    node *L, *M, *R ;
    splite(root, L, R, p-1) ;
    splite(R, M, R, k) ;
    M->flag = !M->flag ;
    root = merge(L, merge(M, R)) ;
}
int GET_SUM(node* &root) {
    int p , k , v ;
    scanf("%d%d",&p,&k) ;
    node *L, *M, *R ;
    splite(root, L, R, p-1) ;
    splite(R, M, R, k) ;
    v = M->Sum() ;
    root = merge(L, merge(M, R)) ;
    return v ;
}
int MAX_SUM(node* &root) {
    return root->Maxsum() ;
}
int main () {
    int n, m ;
    srand(860514) ;
    while(scanf("%d%d",&n,&m)==2) {
        node *root=0 ;
        for(int i=0,v ; i<n ; i++) {
            scanf("%d",&v) ;
            root = merge(root,new node(v)) ;
        }
        while(m-->0) {
            char s[10] ;
            scanf("%s",s) ;
            if(strcmp(s,"INSERT")==0) {
                INSERT(root) ;
                continue ;
            }
            if(strcmp(s,"DELETE")==0) {
                DELETE(root) ;
                continue ;
            }
            if(strcmp(s,"MAKE-SAME")==0) {
                MAKE_SAME(root) ;
                continue ;
            }
            if(strcmp(s,"REVERSE")==0) {
                REVERSE(root) ;
                continue ;
            }
            if(strcmp(s,"GET-SUM")==0) {
                printf("%d\n",GET_SUM(root)) ;
                continue ;
            }
            if(strcmp(s,"MAX-SUM")==0) {
                printf("%d\n",MAX_SUM(root)) ;
                continue ;
            }
        }
    }
}

```

4.4 Treap unordered

```

#include <iostream>
#include <cstdio>
#include <vector>
#include <string.h>
#define N 50000
using namespace std;

vector<int> G[N];
int ind[N], f[N], d[N], head[N], son[N], pn, n;

int dfs(int u,int parent) {
    int i, v, x, w = 0, sum = 1;
    f[u] = parent;
    son[u] = -1;
    for(i = 0; i < G[u].size(); i++) {
        v = G[u][i];
        if(v != parent) {
            d[v] = d[u] + 1;
            x = dfs(v,u);
            sum += x;
            if(x > w) {
                son[u] = v;
                w = x;
            }
        }
    }
    return sum;
}

void chain(int u) {
    int i, v;
    ind[u] = pn++;
    if(f[u] == -1 || son[f[u]] != u)
        head[u] = u;
    else
        head[u] = head[f[u]];
    if(son[u] != -1)
        chain(son[u]);
    for(i=0; i<G[u].size(); i++) {
        v = G[u][i] ;
        if(v != f[u] && v != son[u])
            chain(v);
    }
}

void modify(int u, int v, ...) {
    while(head[u] != head[v])
        if(d[head[u]] > d[head[v]]) {
            //do something in rang(ind[head[u]], ind[u]);
            u = f[head[u]];
        } else {
            //do something in rang(ind[head[v]], ind[v]);
            v = f[head[v]];
        }
    if(d[u] > d[v])
        //do something in rang(ind[head[u]], ind[u]);
    else
        //do something in rang(ind[head[v]], ind[v]);
}

int getLca(int u, int v) {
    while(head[u] != head[v])
        if(d[head[u]] > d[head[v]])
            u = f[head[u]];
        else
            v = f[head[v]];
    if(d[u] > d[v])
        return v;
    else
        return u;
}

void build() {
    pn = 1;
    d[0] = 0; //set depth of root to 0
    dfs(0,-1);
    chain(0); //relabel node
}

```

5 Graph

5.1 Biconnected Components

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <stack>
#include <vector>
using namespace std;
//Biconnected Components
struct BCC {
    static const int maxNum = 1010;
    vector<int> G[maxNum], bccGroup[maxNum];
    int Node;
    int bcc_cnt;
    int timeStamp;
    int low[maxNum];
    int visit[maxNum];
    int bcc[maxNum];
    bool is_ap[maxNum];
    stack< pair<int,int> > S;
    BCC(int Node) {
        for(int i = 0; i < maxNum; i++) {
            G[i].clear();
            bccGroup[i].clear();
            low[i] = visit[i] = bcc[i] = -1;
            is_ap[i] = false;
        }
        this->Node = Node;
        bcc_cnt = 0;
    }
    void DFS(int u,int parent) {
        int children = 0;
        low[u] = visit[u] = timeStamp++;
        for(int i = 0; i < G[u].size(); i++) {
            int v = G[u][i];
            if(visit[v] == -1) {
                S.push(make_pair(u, v));
                children++;
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if(low[v] >= visit[u]) {
                    is_ap[u] = true;
                    pair<int,int> e;
                    do {
                        e = S.top();
                        if(bcc[e.first] != bcc_cnt) {
                            bccGroup[bcc_cnt].push_back(e.first);
                            bcc[e.first] = bcc_cnt;
                        }
                        if(bcc[e.second] != bcc_cnt) {
                            bccGroup[bcc_cnt].push_back(e.second);
                            bcc[e.second] = bcc_cnt;
                        }
                        S.pop();
                    }while(e.first!=u || e.second!=v);
                    bcc_cnt++;
                }
            } else if(v != parent) {
                S.push(make_pair(u, v));
                low[u] = min(low[u], visit[v]);
            }
        }
        if(u == parent) // u is root
            is_ap[u] = (children >= 2);
    }
    void articulation_vertex() {
        timeStamp = 0;
        for(int i = 0; i < Node; i++)
            if(low[i] == -1)
                DFS(i, i);
    }
};
```

5.2 Strong Connected Components

```
class SCC {
private:
    static const int maxN = 10004;
    vector<int> G[maxN];
    vector<int> invG[maxN];
    vector<int> stk;
    bool visited[maxN];
    void dfs_1(int nowAt) {
        visited[nowAt] = true;
        for(auto v : G[nowAt])
            if( !visited[v] )
                dfs_1(v);
        stk.emplace_back(nowAt);
    }
    void dfs_2(int nowAt, const int id) {
        sccID[nowAt] = id;
        for(auto v : invG[nowAt])
            if( sccID[v]==-1 )
                dfs_2(v, id);
    }
public:
    int sccNum;
    int sccID[maxN];

    void init() {
        for(int i=0; i<maxN; ++i) {
            G [i].clear();
            invG[i].clear();
        }
    }
    void addEdge(int u, int v) {
        G [u].emplace_back(v);
        invG[v].emplace_back(u);
    }
    vector<vector<int>> findAllSCC(int base, int n) {
        memset(visited, 0, sizeof(visited));
        stk.clear();
        for(int i=base; i<=n; ++i)
            if( !visited[i] )
                dfs_1(i);

        sccNum = 0;
        memset(sccID, -1, sizeof(sccID));
        for(int i=stk.size()-1; i>=0; --i)
            if( sccID[stk[i]]==-1 ) {
                dfs_2(stk[i], sccNum);
                ++sccNum;
            }

        // returned zero base scc dag
        vector<vector<int>> sccDAG(sccNum);
        vector<unordered_set<int>> have(sccNum);
        for(int u=base; u<=n; ++u) {
            int sccU = sccID[u];
            for(auto v : G[u]) {
                int sccV = sccID[v];
                if( sccU==sccV ) continue;
                if( have[sccU].find(sccV) == have[sccU].
                    end() ) {
                    have [sccU].insert (sccV);
                    sccDAG[sccU].emplace_back(sccV);
                }
            }
        }
        return sccDAG;
    }
};
```

5.3 2 SAT

```
class TwoSAT {
private:
```

```

static const int maxN = 100004;
static const int size = 2*maxN + 4;
bool pick[size];
vector<int> G [size];

int id(int i, int T) { return (i<<1) + T; }
int alter(int i) { return i^1; }
bool dfsTry(int nowAt, vector<int> &stk) {
    if( pick[alter(nowAt)] )
        return false;
    stk.emplace_back(nowAt);
    pick[nowAt] = true;
    for(auto v : G[nowAt]) {
        if( !pick[v] && !dfsTry(v, stk) )
            return false;
    }
    return true;
}
public:
void init() {
    memset(pick, 0, sizeof(pick));
    for(int i=0; i<size; ++i)
        G[i].clear();
}
void addClause(bool TA, int A, bool TB, int B) {
    // Add clause (TA + TB)
    // When TA not true, TB must true. vise versa.
    G[id(A, !TA)].emplace_back(id(B, TB));
    G[id(B, !TB)].emplace_back(id(A, TA));
}
bool solve() {
    // O(n) solve
    memset(pick, 0, sizeof(pick));
    for(int i=0; i<maxN; ++i) {
        if( pick[id(i, 0)] || pick[id(i, 1)] )
            continue;
        vector<int> stk;
        if( dfsTry(id(i, 0), stk) )
            continue;
        for(auto v : stk)
            pick[v] = false;
        if( !dfsTry(id(i, 1), stk) )
            return false;
    }
    return true;
}
bool T(int i) {
    // should solve() first
    return pick[id(i, 1)];
}
};

```

5.4 System of Difference Constraints

```

class System_of_DifConstrain {
private:
    static const int maxN = 504;
    static const int maxM = 3004;
    struct Edge {
        int s, t;
        long long cost;
    };
    Edge es[maxM];
    int eSize;
public:
    bool solvable;
    long long x[maxN]; // one solution
    void init() {
        eSize = -1;
    }
    void addConstrain(int xI, int xJ, long long c) {
        // add xi - xj <= c
        es[++eSize] = {xJ, xI, c};
    }
}

```

```

}
bool solve(int n=maxN) {
    // n is max # of node of CC
    memset(x, 0, sizeof(x));
    for(int i=0; i<n; ++i)
        for(int j=0; j<=eSize; ++j)
            if( x[es[j].s] + es[j].cost < x[es[j].t] )
                x[es[j].t] = x[es[j].s] + es[j].cost;
    for(int j=0; j<=eSize; ++j)
        if( x[es[j].s] + es[j].cost < x[es[j].t] )
            return solvable = false;
    return solvable = true;
}
};

```

5.5 Bipartite: MaxMatch, Min-VerCover, MaxIndSet

```

class Bipartite {
private:
    static const int MaxNum = 1004;
    vector<int> g[MaxNum];
    bool visited [MaxNum];

    bool bipart(int nowAt, int nowSide) {
        visited[nowAt] = true;
        side[nowAt] = nowSide;
        for(auto &id : g[nowAt])
            if( !visited[id] )
                bipart(id, !nowSide);
            else if( side[id]==nowSide )
                return false;
        return true;
    }
    bool maxMatch(int nowAt) {
        visited[nowAt] = true;
        for(auto &id : g[nowAt])
            if( cp[id]==-1
                || (!visited[cp[id]] && maxMatch(cp[id])) ) {
                cp[id] = nowAt;
                cp[nowAt] = id;
                return true;
            }
        return false;
    }
    void minVertexCover(int nowAt) {
        MVC[nowAt] = 1;
        for(auto &id : g[nowAt])
            if( !MVC[id] ) {
                MVC[id] = 1;
                minVertexCover(cp[id]);
            }
    }
    void maxIndependentSet(int nowAt) {
        MIS[nowAt] = 1;
        for(auto &id : g[nowAt])
            if( !MIS[cp[id]] )
                maxIndependentSet(cp[id]);
    }
public:
    int matchNum; // max match num
    int cp [MaxNum]; // id and cp[id] is couple
    bool side[MaxNum]; // left/right side
    bool MVC [MaxNum]; // min vertex cover
    bool MIS [MaxNum]; // max indepent set
    void addEdge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
    void init() {
        for(int i=0; i<MaxNum; ++i)
            g[i].clear();
    }
    bool countAll() {

```

```

// if graph is not bipartite return false

// bipartite
memset(side, 0, sizeof(side));
memset(visited, 0, sizeof(visited));
for(int i=0; i<MaxNum; ++i)
    if( !visited[i] && !bipart(i, 0) )
        return false;

// maximum match
// O(VE), this code can be more optimized
// alternative: dinic O(V^0.5*E)
matchNum = 0;
memset(cp, -1, sizeof(cp));
for(int i=0; i<MaxNum; ++i){
    if( cp[i]!=-1 ) continue;
    memset(visited, 0, sizeof(visited));
    if( maxMatch(i) )
        ++matchNum;
}

// min vertex cover
memset(MVC, 0, sizeof(MVC));
for(int i=0; i<MaxNum; ++i)
    if( side[i]==1 && cp[i]==-1 )
        minVertexCover(i);
for(int i=0; i<MaxNum; ++i)
    if( side[i]==1 )
        MVC[i] = !MVC[i];

// max independent set
memset(MIS, 0, sizeof(MIS));
for(int i=0; i<MaxNum; ++i)
    if( cp[i]==-1 )
        maxIndependentSet(i);
for(int i=0; i<MaxNum; ++i)
    if( side[i]==1 && cp[i]!=-1
        && !MIS[i] && !MIS[cp[i]] )
        MIS[i] = 1;

return true;
}
};

```

5.6 Bipartite: KM

```

#include <cstring>
#include <iostream>
using namespace std;
struct KM {
    static const int N = 105, big_value = 1000000000;
    int G[N][N], visx[N], visy[N];
    int n, labelx[N], labely[N], matchx[N], matchy[N];

    KM(int n_):n(n_) {} ;

    bool DFS(int x) {
        visx[x] = true;
        int y;
        for(y=0; y<n; y++){
            if(!visy[y] && labelx[x]+labely[y]==G[x][y]) {
                visy[y] = true;
                if(matchy[y]==-1 || DFS(matchy[y])) {
                    matchx[x] = y;
                    matchy[y] = x;
                    return true;
                }
            }
        }
        return false;
    }

    int max_match() { //Maximum Weight Perfect Bipartite
        Matching
        memset(labelx,0,sizeof(labelx));

```

```

        memset(labely,0,sizeof(labely));
        memset(matchx,-1,sizeof(matchx));
        memset(matchy,-1,sizeof(matchy));
        int i, x, y;
        for(x=0; x<n; x++){
            for(y=0; y<n; y++){
                labelx[x] = max(labelx[x],G[x][y]);
            }
        }
        while(true) {
            memset(visx,0,sizeof(visx));
            memset(visy,0,sizeof(visy));
            if(DFS(0)) break;
            int d=big_value;
            for(x=0; x<n; x++){
                if(visx[x])
                    for(y=0; y<n; y++){
                        if(!visy[y])
                            d = min(d,labelx[x]+labely[y]-G[x][y]);
                    }
            }
            if(d==big_value) return -1; //faile to exist
            perfect matching
            for(int j=0; j<n; j++){
                if(visx[j]) labelx[j] -= d;
                if(visy[j]) labely[j] += d;
            }
        }
        int total=0;
        for(i=0; i<n; i++) //must be perfect!!!
            total += G[i][matchx[i]];
        return total;
    }
};

```

5.7 Min Vertex Cover

```

struct MinVertexCover {
private:
    static const int MaxNum = 54;
    vector<int> G[MaxNum];
    int in[MaxNum];

    int undo(vector<int> &record) {
        for(int i=0; i<record.size(); ++i)
            ++in[record[i]];
        record.clear();
    }

    int delNode(int u, vector<int> &record) {
        for(int i=0; i<G[u].size(); ++i)
            if( in[G[u][i]] ) {
                --in[G[u][i]];
                --in[u];
                record.push_back(G[u][i]);
                record.push_back(u);
            }
    }

    int cnt(int from, int *visited, bool type) {
        if( visited[from] ) return 0;
        if( type==1 ) visited[from] = 1;
        for(int i=0; i<G[from].size(); ++i)
            if( in[G[from][i]] && !visited[G[from][i]] )
                return type+cnt(G[from][i], visited, !type);
        return type;
    }

    int cnt(int *visited) {
        int ret = 0;
        for(int i=0; i<MaxNum; ++i)
            if( in[i]==1 && !visited[i] )
                ret += cnt(i, visited, 0);
        for(int i=0; i<MaxNum; ++i)
            if( in[i]==2 && !visited[i] )
                ret += cnt(i, visited, 0);
        return ret;
    }

public:
    int MVCPick[MaxNum];
    int MVC; // min vertex cover

```

```

void init() {
    for(int i=0; i<MaxNum; ++i)
        G[i].clear();
    memset(in, 0, sizeof(in));
}
void addEdge(int u, int v) {
    G[u].push_back(v);
    G[v].push_back(u);
    ++in[u];
    ++in[v];
}
void minVertexCover(int nowMVC=0, const int *lastPick=
    NULL) {
    //  $O(n^2 * 1.38^n)$ 
    int nowPick[MaxNum] = {};
    if( nowMVC==0 ) {
        MVC = MaxNum;
        memset(MVCPick, 0, sizeof(MVCPick));
    }
    else memcpy(nowPick, lastPick, sizeof(nowPick));

    int maxid = 0;
    for(int i=0; i<MaxNum; ++i)
        if( in[i]>in[maxid] )
            maxid = i;
    if( in[maxid]<=2 ) {
        nowMVC += cnt(nowPick);
        if( nowMVC<MVC ) {
            MVC = nowMVC;
            memcpy(MVCPick, nowPick, sizeof(nowPick));
        }
        return;
    }

    vector<int> record;

    delNode(maxid, record);
    nowPick[maxid] = 1;
    minVertexCover(nowMVC+1, nowPick);
    nowPick[maxid] = 0;
    undo(record);

    int cnt = 0;
    for(int i=0; i<G[maxid].size(); ++i)
        if( in[G[maxid][i]] ) {
            ++cnt;
            delNode(G[maxid][i], record);
            nowPick[G[maxid][i]] = 1;
        }
    minVertexCover(nowMVC+cnt, nowPick);
    undo(record);
}
};

```

Define **hot** position: the position is **any** applicant's first choice

Solve: whether there is a popular matching

- (1) Each applicant build an edge with his first choice and his first choice except **all hot** position (if not **exist**, build a dummy position)
- (3) Return whether **all** applicant can have a position

Solve: maximum popular matching

- (1) If popular matching not **exist** return No
- (2) Use the result above
- (3) Remove **all** dummy position
- (4) Find augmenting **path** for **all** unmatched applicant
- (5) For **all** unmatched **hot** position, rob an applicant (all its applicant choice non **hot** position)
- (6) **return** the matching

5.8 Kirchhoff's theorem

D = Degree matrix (diagonal entries are node's degree)

A = Adjacency matrix ($0/1$ matrix)

Q = Laplacian matrix = $D - A$

Q^* = deleting **any** row and **any** column from Q

Number of spanning tree = $\det(Q^*)$

5.9 Popular matching

N applicant M position
each applicant have his own preference list (not required containing **all** position)

Target: **find** a matching s.t. not existing another matching have **more** applicant prefer. this matching called popular matching

A applicant u prefer another matching **which** imply that this matching doesn't have u but the other does or the other matching give u the position that he **more** prefer

7 Geometry

7.1 Geometry basic

```
struct Point{
    double x,y;
    Point(double xi=0.0,double yi=0.0){
        x = xi , y = yi;
    }
    Point operator - (const Point &r)const{
        return Point(x-r.x , y-r.y);
    }
};
typedef Point Vector;

double angle(const Vector &v,const Vector &u){
    // return rad [0, pi] of two vector
    return acos( dot(v,u)/len(v)/len(u) );
}
Vector rotate(const Vector &v,double rad){
    return Vector(
        v.x*cos(rad) - v.y*sin(rad),
        v.x*sin(rad) + v.y*cos(rad)
    );
}
double pointSegLen(const Point &A,const Point &B,const
    Point &Q){
    if(A==B) return len(Q-A);
    if( dot(B-A , Q-A)<0 ) return len(Q-A);
    if( dot(B-A , Q-B)>0 ) return len(Q-B);
    return fabs( cross(B-A , Q-A) ) / len(B-A);
}
bool pointOnSeg(const Point &A,const Point &B,const Point
    &Q){
    return fabs( len(Q-B)+len(Q-A)-len(A-B) ) < 1e-9;
}

struct Line{
    Point P0;
    Vector v;
    Line(const Point &pi,const Vector &vi):p0(pi) , v(vi)
    {}
};

double pointLineLen(const Line &L,const Point &Q){
    return fabs( cross(L.v , Q-L.P0) ) / len(L.v);
}
Point projectToLine(const Line &L,const Point &Q){
    double t = dot(Q-L.P0 , L.v) / dot(L.v , L.v);
    return L.P0 + L.v * t;
}

Point innerCircle(point &p1, point &p2, point &p3){
    // p1,p2,p3 should not on same line
    double a1 = (-2*p1.x + 2*p2.x);
    double b1 = (-2*p1.y + 2*p2.y);
    double c1 = (p2.x*p2.x + p2.y*p2.y - p1.x*p1.x - p1.y*
        p1.y);
    double a2 = (-2*p1.x + 2*p3.x);
    double b2 = (-2*p1.y + 2*p3.y);
    double c2 = (p3.x*p3.x + p3.y*p3.y - p1.x*p1.x - p1.y*
        p1.y);
    double cx = (c1*b2-c2*b1) / (a1*b2-a2*b1);
    double cy = (a1*c2-a2*c1) / (a1*b2-a2*b1);
    return Point(cx, cy);
}
Point outerCircle(point &p1, point &p2, point &p3) {
    // p1,p2,p3 should not on same line
    double x1 = (p1.x+p2.x)/2.0;
    double y1 = (p1.y+p2.y)/2.0;
    double x2 = (p2.x+p3.x)/2.0;
    double y2 = (p2.y+p3.y)/2.0;
    double vx = p2.x-p1.x;
    double vy = p2.y-p1.y;
    double ux = p3.x-p2.x;
```

```
double uy = p3.y-p2.y;
double A = vx*x1 + vy*y1;
double B = ux*x2 + uy*y2;
double cx = (uy*A - vy*B) / (uy*vx - ux*vy);
double cy = (ux*A - vx*B) / (ux*vy - uy*vx);
return Point(cx, cy);
}
```

7.2 Minimal Enclose Disk

```
struct Circle {
    Point c;
    double R2; // square of radius
    Circle() {}
    Circle(const Point &p1, const Point &p2) {
        c.x = (p1.x+p2.x)/2.0;
        c.y = (p1.y+p2.y)/2.0;
        R2 = dot(p1-p2, p1-p2)/4.0;
    }
    Circle(const Point &p1, const Point &p2, const Point &
        p3) {
        // p1, p2, p3 should not on same line
        c = outerCircle(p1, p2, p3);
        double dx = p1.x - c.x;
        double dy = p1.y - c.y;
        R2 = dx*dx + dy*dy;
    }
    bool contain(const Point &p) const {
        double dx = c.x - p.x;
        double dy = c.y - p.y;
        return fdif(dx*dx + dy*dy - R2)<=0;
    }
};

Circle minEncloseDisk(vector<Point> &ps) {
    // Find minimal circal enclose all point
    // worst case O(n^3), expected O(n)
    Circle D;
    if( ps.size()==0 ) return D;
    if( ps.size()==1 ) {
        D.c = ps[0];
        D.R2 = 0.0;
        return D;
    }

    random_shuffle(ps.begin(), ps.end());
    D = Circle(ps[0], ps[1]);
    for(int i=2; i<ps.size(); ++i) {
        if( D.contain(ps[i]) )
            continue;
        D = Circle(ps[i], ps[0]);
        for(int j=1; j<i; ++j) {
            if( D.contain(ps[j]) )
                continue;
            D = Circle(ps[i], ps[j]);
            for(int k=0; k<j; ++k) {
                if( D.contain(ps[k]) )
                    continue;
                D = Circle(ps[i], ps[j], ps[k]);
            }
        }
    }
}
```

7.3 2D Convex Hull

```
bool turnLeft(const Vector &v1, const Vector &v2) {
    return fdif(cross(v1, v2)) > 0LL;
}
vector<Point> convexHull(vector<Point> &ps) {
    // return convex hull without redundant point
    sort(ps.begin(), ps.end());

    vector<Point> up;
```



```

for(int i=0; i<ps.size(); ++i) {
    while( up.size()>1
        && !turnLeft(up.back()-up[up.size()-2],
            ps[i]-up.back()) )
        up.pop_back();
    up.emplace_back(ps[i]);
}

vector<Point> btn;
for(int i=ps.size()-1; i>=0; --i) {
    while( btn.size()>1
        && !turnLeft(btn.back()-btn[btn.size()-2],
            ps[i]-btn.back()) )
        btn.pop_back();
    btn.emplace_back(ps[i]);
}

vector<Point> res(up);
res.insert(res.end(), btn.begin()+1, btn.end());
res.pop_back();
return res;
}

```

7.4 Closest Point

```

#include <iostream>
#include <cstdio>
#include <algorithm>
#include <cstring>
#include <cmath>
#define N 10010
#define INFINITY__ 1e10
using namespace std;
int diff(double f) {
    if(fabs(f) < 1e-9)
        return 0;
    return f < 0 ? -1 : 1;
}
struct point {
    double x, y;
    bool operator < (const point &p) const {
        return diff(x - p.x) < 0;
    }
};
point pt[N], tmp[N];
double dis(point a, point b) {
    return sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y));
}
double closestpoint(int L, int R) {
    if(R - L + 1 == 1) return INFINITY__;
    int M = (L+R)/2;
    double middle = pt[M].x;
    double Ldis = closestpoint(L, M);
    double Rdis = closestpoint(M+1, R);
    double radi = min(Ldis, Rdis);
    int cntpt = 0;
    merge(pt+L, pt+M+1, pt+M+1, pt+R+1, tmp+L, [](point a,
        point b) {
        return diff(a.y - b.y) < 0;
    });
    copy(tmp + L, tmp + R + 1, pt + L);
    for(int i = L; i <= R; i++)
        if(diff(fabs(pt[i].x - middle) - radi) < 0)
            tmp[cntpt++] = pt[i];
    for(int i = 0; i < cntpt; i++)
        for(int j = 1; i + j < cntpt && j < 8; j++)
            radi = min(radi, dis(tmp[i], tmp[i+j]));
    return radi;
}

```

7.5 Min Max Triangle

```

pair<double, double> findMinMaxTri(vector<Point> &ps) {

```

```

    static const double PI = acos(-1.0);
    struct Seg {
        double rad; // [0.5pi, 1.5pi]
        int s1, s2;
    };

    const int n = ps.size();
    sort(ps.begin(), ps.end(), [](const Point &l, const
        Point &r) {
        if( fdif(l.x - r.x) == 0 )
            return l.y > r.y;
        return l.x < r.x;
    });
    vector<int> id(n+4);
    for(int i=0; i<n; ++i)
        id[i] = i;

    // sort all pair of point
    vector<Seg> segs;
    for(int i=0; i<n; ++i)
        for(int j=i+1; j<n; ++j) {
            double m = atan2(ps[j].y-ps[i].y, ps[j].x-ps[i].x) +
                PI;
            segs.push_back({m, i, j});
        }
    sort(segs.begin(), segs.end(), [](const Seg &l, const
        Seg &r) {
        return fdif(l.rad - r.rad) < 0;
    });

    // find min max triangle
    pair<double, double> ret;
    ret.first = ret.second = fabs(cross(ps[0], ps[1], ps
        [2]));
    for(auto seg : segs) {
        swap(ps[id[seg.s1]], ps[id[seg.s2]]);
        swap(id[seg.s1], id[seg.s2]);

        const Point &p1 = ps[id[seg.s1]];
        const Point &p2 = ps[id[seg.s2]];
        int id1 = min(id[seg.s1], id[seg.s2]);
        int id2 = max(id[seg.s1], id[seg.s2]);

        // find min triangle
        if( id1-1 >= 0 ) {
            double a = fabs(cross(p1, p2, ps[id1-1]));
            if( a < ret.first )
                ret.first = a;
        }
        if( id2+1 < n ) {
            double a = fabs(cross(p1, p2, ps[id2+1]));
            if( a < ret.first )
                ret.first = a;
        }

        // find max triangle
        if( id1 != 0 ) {
            double a = fabs(cross(p1, p2, ps[0]));
            if( a > ret.second )
                ret.second = a;
        }
        if( id2 != n-1 ) {
            double a = fabs(cross(p1, p2, ps[n-1]));
            if( a > ret.second )
                ret.second = a;
        }
    }
    ret.first /= 2.0;
    ret.second /= 2.0;
    return ret;
}

```