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1 Java

class Scan {

```
BufferedReader buffer;
  StringTokenizer tok;
  Scan() {
    buffer = new BufferedReader(new InputStreamReader(
    System.in));
  boolean hasNext() {
    while(tok == null || !tok.hasMoreElements())
      try {
        tok = new StringTokenizer(buffer.readLine());
      } catch (Exception ex) {
        return false;
      }
    return true;
  String next() {
    if(hasNext())
      return tok.nextToken();
    return null;
  String nextLine() {
    if(hasNext())
      return tok.nextToken("\n");
    return null;
  int nextInt() {
    return Integer.parseInt(next());
}
/* Compile: javac %
 * Run: java [Class name] */
import java.util.*;
import java.lang.*;
import java.math.*;
class Main {
    public static void main (String[] args) {
        System.out.print(1);
        System.out.print(2);
        System.out.println("Hello World");
        System.out.printf("%.2f", 0.12345);
        Scanner sc = new Scanner(System.in);
        System.out.println(sc.nextLine()); //gets()
        System.out.println(sc.next()); //scanf("%s")
        System.out.println(sc.nextInt());
        System.out.println(sc.nextDouble());
        while(sc.hasNext()) { //EOF
            int a = sc.nextInt();
            System.out.println(a);
        int[] a = \{1,2,3\};
        int[][] b = {{1,2},{3,4,5}};
        double[] c = new double[90];
        System.out.print(b[0][1]);
        System.out.print(b[1][2]);
        int[] d = {5,2,1,3,4};
        Integer[] e = \{6,3,4,1,2\};
        Arrays.sort(d);
        Arrays.sort(e, new MyCom());
        for(int i=0; i<d.length; i++) {</pre>
            System.out.print(d[i]);
        for(int i=0; i<e.length; i++) {</pre>
            System.out.print(e[i]);
        Set<String> s = new HashSet<String>(); //or
    TreeSet
```

```
s.add("123");
    s.add("234");
    System.out.println(s);
    System.out.println(s.contains("123"));
    Map<String, Integer> m = new TreeMap<String,</pre>
Integer>();
    m.put("haha", 123);
m.put("hehe", 234);
    System.out.println(m);
    BigInteger b1 = new BigInteger("
-1231237182379123712");
    BigInteger b2 = BigInteger.valueOf(234);
    System.out.println(b1.add(b2));
    System.out.println(b1.mod(b2));
    int z = Integer.parseInt("-123");
    System.out.println(z);
    System.out.println(Math.PI);
    System.out.println(Math.sin(1));
}
static class MyCom implements Comparator<Integer> {
    public int compare(Integer i1, Integer i2) {
        return i2 - i1;
}
```

}

2 Math

2.1 Math Basic

```
vector<pii> primeFac(int n) {
  vector<pii> ret;
  for(int i=2; n>1; ++i){
    if( n%i != 0 ) continue;
    int e = 0;
    while( n%i == 0 ) ++e , n/=i;
    ret.push_back({i, e});
  }
  return ret;
long long fastPow(long long x, int n, long long m){
  long long ans = 1LL;
  while( n ){
    if( n\&1 ) ans = ans * x % m;
   x = x*x % m;
   n >>= 1;
  }
  return ans;
long long modInv(long long x, long long p){
  return fastPow(x, p-2, p);
long long modInv_euler(long long x, long long m){
  // must be gcd(x,m)==1
  // phi is euler function: O(sqrt(x))
  return fastPow(x, phi(m)-1, m);
long long gt(long long a, long long b) {
  // smallest integer greater than a/b
  long long ret = a/b;
  if( ret>0 || a%b==0 ) ++ret;
  return ret;
```

2.2 Euler Function

```
int phi(int n){
   // euler function: in [0,n], # of coprime(i, n)
   vector<pri> fac = primeFac(n);
   int num = 1 , m = 1;
   for(auto &p : fac)
      num *= (p.first-1) , m *= p.first;
   return n/m * num;
}
```

2.3 Extended Euclidean

```
pll recur(long long n, long long m) {
    // solve one integer solution of
    // x*n + y*m = gcd(n,m)
    if( n%m == 0 )
        return {0LL, 1LL};
    pll res = recur(m, n%m);
    pll ret = {res.second, res.first - res.second * (n/m)};
    return ret;
}
```

2.4 China Remain Theorm

```
bool china_solvable(vector<pii> &rule) {
  for(int i=0; i<rule.size(); ++i)
  for(int j=1; j<rule.size(); ++j) {
    int gcd = __gcd(rule[i].second, rule[j].second);
    if( rule[i].first%gcd != rule[j].first%gcd )
        return false;
  }
  return true;
}</pre>
```

```
long long china(const vector<pii> &rule, int nlt=0){
 // solve x = ai (mod mi)
  // rule should solvable
 long long MM = 1LL;
 for(auto &r : rule)
   MM = lcm(MM, r.second);
 long long x = 0LL;
  for(auto &r : rule){
    long long ai = r.first;
   long long mi = r.second;
   long long Mi = MM / r.second;
   long long Mv = modInv_euler(Mi%mi, mi);
   long long tmp = ai*Mi%MM *Mv %MM;
   x = (x+tmp) \% MM;
 }
 if( x>=nlt ) return x;
 long long n = ceil((nlt-x)*1.0/MM);
  return x + n*MM;
```

2.5 Counting

```
const int MaxNum = 1000004;
const int modNum = 1000000009;
long long fac [MaxNum];
long long facIv[MaxNum];
void initFac(){
  fac[0] = facIv[0] = 1LL;
  for(int i=1; i<MaxNum; ++i) {</pre>
    fac [i] = fac[i-1]*i % modNum;
    facIv[i] = modInv(fac[i], modNum);
 }
}
long long Cnm(int n, int m){
 if( m==0 || n==m ) return 1LL;
  return fac[n]*facIv[m] % modNum *facIv[n-m] % modNum;
long long nBlock_kColor(int n,int k){
  // n different blocks; k different colors
  // use inclusion-exclusion principle
  long long ans = fastPow(k, n, modNum);
 bool del = true;
  for(int i=k-1; i>0; --i, del=!del){
    long long now = Cnm(k, i)*fastPow(i, n, modNum) %
    modNum:
    if( del ) ans = (ans+modNum-now) % modNum;
    else ans = (ans+now) % modNum;
 }
 return ans;
```

2.6 Miller Rabin

```
#include <climits>
typedef unsigned long long int ull;

ull bases[20] = { 2ULL, 3ULL,5ULL,7ULL,11ULL,13ULL,17ULL
    ,19ULL,23ULL,29ULL,31ULL,37ULL };

ull fake_mul(ull n, ull m, ull x);

ull fast_pow(ull n, ull p, ull x);

bool is_prime(ull n)
{
    if (n < 2ULL) return false;

    for (int tt = 0; tt < 12; tt++) {
        ull a;
        a = bases[tt] % n;

        if (a == 0 || a == 1 || a == n - 1) {
            continue;
        }
}</pre>
```

```
int t = 0;
    ull u = n - 1ULL;
    while ((u & 1ULL) == 0ULL) u >>= 1, t++;
    ull x = fast_pow(a, u, n); // x = a ^ u % n;
    if (x == 1ULL \mid \mid x == (n - 1)) continue;
    for (int i = 0; i < t - 1; i++)
      if (ULLONG_MAX / x < x) {</pre>
        x = fake_mul(x, x, n);
      }
      else {
        x = x*x%n;
      if (x == 1) return false;
      if (x == n - 1) break;
    }
    if (x == n - 1) continue;
    return false;
  return true;
ull fake_mul(ull n, ull m, ull x)
{
  ull re = 0ULL;
  while (m != OULL) {
    if ((m & 1ULL) != 0ULL) {
      if (ULLONG_MAX - re < n) {</pre>
        ull temp = ULLONG_MAX%x;
        temp += (n - (ULLONG_MAX - re)) % x;
        re = temp%x;
      }
      else {
        re = (re + n) % x;
    }
    if (ULLONG_MAX - n < n) {</pre>
      ull temp = ULLONG_MAX%x;
      temp += (n - (ULLONG_MAX - n)) \% x;
      n = temp%x;
    }
    else {
      n = n + n\%x;
    m >>= 1;
  }
  return re;
ull fast_pow(ull n, ull p, ull x)
{
  ull re = 1ULL;
  while (p != OULL) {
    if ((p & 1ULL) != 0ULL) {
      if (ULLONG_MAX / re < n) {</pre>
        re = fake_mul(n, re, x);
      else {
        re = (re*n) % x;
    }
    if (ULLONG_MAX / n < n) {</pre>
      n = fake_mul(n, n, x);
    else {
      n = (n*n) % x;
    p >>= 1;
```

3

```
return re;
// Below is non-extreme version
ull fake_mul(ull n, ull m, ull x) {
  ull re = 0ULL;
  n \% = x, m \% = x;
  while( m ) {
    if( m&1ULL )
      re = (re+n) % x;
    n = (n+n) \% x;
    m >>= 1;
  }
  return re;
ull fast_pow(ull n, ull p, ull x) {
  ull re = 1ULL;
  while( p ) {
    if( p&1ULL )
     re = fake_mul(re,n,x);
    n = fake_mul(n,n,x);
    p >>= 1;
  }
  return re;
bool is_prime(ull n) {
  static const int bNum = 12;
  static const ull bases[bNum] = {
    2ULL,3ULL,5ULL,7ULL,11ULL,13ULL,17ULL,19ULL,23ULL,29
    ULL,31ULL,37ULL
  if( n<=2ULL ) return n==2ULL;</pre>
  if( !(n&1ULL) ) return false;
  ull u = n-1;
  while( !(u&1ULL) )
    u >>= 1;
  for(int i=0; i<bNum; i++) {</pre>
    if( bases[i]%n == 0 ) continue;
    ull t = u;
    ull a = fast_pow(bases[i], t, n);
    if( a==1 || a==n-1 ) continue;
    while( t!=n-1 && a!=1 && a!=n-1 ) {
      a = fake_mul(a,a,n);
      t <<= 1;
    if( t==n-1 && a==1 ) continue;
    if( a!=n-1 ) return false;
  return true;
2.7
        Pollard rho
```

```
// need fack_mul, is_prime
ull gcd(ull a, ull b) {
  return (a%b==0)? b : gcd(b, a%b);
ull dif(ull a, ull b) {
  return a>b? a-b : b-a;
void pollard_rho(ull n, map<ull,int> &facs) {
  while( !(n&1ull) ) {
    // must extract factor 2
    int cnt = 0;
    while( !(n&1ull) )
     ++cnt, n>>=1;
    facs[2] = cnt;
  if( n==1ull ) return;
  if( is_prime(n) ) {
    facs[n]++;
    return;
  }
```

```
ull x = rand()%n;
  ull y = x;
  ull a = rand()\%(n-1) + 1;
  ull g = 1ull;
  while( g==1ull ) {
    x = (fake_mul(x,x,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    if( x==y ) {
      g = n;
      break;
    }
    g = gcd(dif(x,y), n);
  }
  if( g==n ) // unluck try again
    pollard_rho(n, facs);
  else if( g>1ull ) { // luck, found g
    pollard_rho(g, facs);
    pollard_rho(n/g, facs);
 }
}
```

2.8 Linear Algebra

```
#ifndef _MATRIX_H_
#define _MATRIX_H_
#include <iostream>
#include <vector>
using namespace std;
template <class T>
class Matrix{
public:
  int rSize, cSize;
  vector< vector<T> > mat;
  Matrix(int r = 0, int c = 0) :rSize(r), cSize(c), mat(
    rSize, vector<T>(cSize)){}
  vector<T>& operator[](int i) {
    return mat[i];
  void print();
template <class T>
void Matrix<T>::print() {
  cout << "Matrix elements:" << endl;</pre>
  for (int i = 0; i < rSize; i++) {</pre>
    cout << "[";
    for (int j = 0; j < cSize; j++) {</pre>
      cout << "\t" << mat[i][j];</pre>
      if (j != cSize - 1)cout << ",";</pre>
    cout << " ]" << endl;
  }
}
#endif
#include "Matrix.h"
template <class T>
Matrix<T> matMul(Matrix<T> matA, Matrix<T> matB){
  Matrix<T> matRe(matA.rSize, matB.cSize);
  for (int i = 0; i < matRe.rSize; i++) {</pre>
    for (int j = 0; j < matRe.cSize; j++) {</pre>
      matRe[i][j] = 0;
      for (int k = 0; k < matA.cSize; k++) {</pre>
        matRe[i][j] += matA[i][k] * matB[k][j];
    }
  return matRe;
```

5

```
return k;
        FFT
2.9
#include <complex>
#include <vector>
using namespace std;
const double PI = 3.141592654;
typedef complex<double> Complex;
void _fft(vector<Complex>& buf, vector<Complex>& out,
  int st, int step, bool isInv) {
  if (step >= buf.size()) return;
  _fft(out, buf, st, step * 2, isInv);
  _fft(out, buf, st + step, step * 2, isInv);
  int n = buf.size();
  double c = isInv ? 1.0 : -1.0;
  for (int i = 0; i < n; i += 2 * step) {
    Complex t = polar(1.0, c * 2 * PI * i / n) * out[i +
    step + st];
    buf[i / 2 + st] = out[i + st] + t;
    buf[(i + n) / 2 + st] = out[i + st] - t;
 }
void fft(vector<Complex> &x, bool isInv) {
  int n = x.size(), nxt2 = 0;
  for (int i = 0, mask = 1; i < 31; i++, mask <<= 1)
    nxt2 = (n\&mask) ? (n != mask) ? 1 << (i + 1) : 1 << i
     : nxt2;
  n = nxt2;
  while (x.size() < n)</pre>
    x.push_back(0);
  vector<Complex> out = x;
  _fft(x, out, 0, 1, isInv);
  for (int i = 0; isInv && i < x.size(); i++)</pre>
    x[i] /= n;
2.10
          Hash
#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
using namespace std;
const int hashRange = 131072;
int hashtable[4][256], shuffleArr[hashRange], ref[
    hashRange];
void buildHashTable() {
  memset(ref, -1, sizeof(ref));
  for(int i = 0; i < hashRange; i++)</pre>
    shuffleArr[i] = i;
  for(int i = 0; i < 4; i++) {
    random_shuffle(shuffleArr, shuffleArr + hashRange);
    for(int j = 0; j < 256; j++)
      hashtable[i][j] = shuffleArr[j];
 }
int myhash(int input) {
  int x[4];
  for(int i = 0; i < 4; i++)
    x[i] = hashtable[i][(input >> (i*8)) & (0xff)];
  int k = x[0] ^ x[1] ^ x[2] ^ x[3];
  if(ref[k] != input)
    for(int i = k; ; i = (i+1)%hashRange)
      if(ref[i] == -1 || ref[i] == input) {
        ref[i] = input;
```

return i;

3 String

3.1 KMP

```
class kmp{
  private:
    int prefix[maxLen];
    char pat[maxLen];
  public:
    void setPattern(const char *str){
      strcpy(pat, str);
      prefix[0] = -1;
      int i=1 , j=0;
      for( ; str[i]!='\0' ; ++i , ++j ){
        if( str[i]==str[j] )
          prefix[i] = prefix[j];
        else
          prefix[i] = j;
        while( j>=0 && str[j]!=str[i] )
          j = prefix[j];
      }
      prefix[i] = j;
    int search(const char *str){
      // return index of str match pattern
      int i=0 , j=0;
      for( ; str[i]!='\0' && pat[j]!='\0'; ++i,++j){
        while( j>=0 && pat[j]!=str[i] )
          j = prefix[j];
      if( pat[j]=='\0' )
       return i-j;
      return -1;
    int countMatched(const char *str){
      // return # of pattern in str
      int cnt = 0;
      int i=0 , j=0;
      while( true ){
        if( pat[j]=='\0' ) ++cnt;
        if( str[i]=='\0' ) break;
        while( j>=0 && pat[j]!=str[i] )
          j = prefix[j];
        ++i , ++j;
      }
      return cnt;
};
```

3.2 LPS

```
int lps(const char *str){
  // return len of longest palindrom substring
  static char emptyChar = '@';
  static char tmp[maxLen*2];
  static int lprb[maxLen*2];
  // [i-lprb[i], i+lprb[i]] is the lps when mid is i
  for(int i=0, j=-1; true; ++i){
  if( str[i]=='\0' ){
       tmp[++j] = emptyChar;
       tmp[++j] = '\setminus 0';
      break;
    tmp[++j] = emptyChar;
    tmp[++j] = str[i];
  lprb[0] = 0;
  int rightBorder = 0 , midId = 0;
for(int i=1; tmp[i]!='\0'; ++i){
    if( i>rightBorder ){
       rightBorder = i;
       midId = i;
       lprb[i] = 0;
    }
```

```
int mirId = midId - (i-midId);
  if( i+lprb[mirId] > rightBorder )
    lprb[i] = rightBorder - i;
  else if( i+lprb[mirId] < rightBorder )</pre>
    lprb[i] = lprb[mirId];
  else{
    int j=lprb[mirId];
    while( tmp[i+j]!='\0' && i-j>=0 && tmp[i+j]==tmp[i-
    rightBorder = i+j-1;
    midId = i;
    lprb[i] = j-1;
 }
}
int ans = 1;
for(int i=0 ; tmp[i]!='\0' ; ++i)
  if( lprb[i]>ans )
    ans = lprb[i];
return ans;
```

3.3 AC Automation

```
#include <queue>
#include <cstdio>
#include <cstring>
using namespace std;
struct AC_algorithm {
  struct node {
    static const int signNum = 52; //number of kind of
    character
    node *ch[signNum];
    node *suffix, *dict;
    int index;
    node() {
      memset(ch, 0, sizeof(ch));
      suffix = dict = 0;
      index = -1;
    }
  };
  static const int stringNum = 100010;//number of pattern
  node *root:
  int occur[stringNum];
                              //string i occur occur[i]
    times
  int reflect[stringNum];
                                //string i is the same as
     string reflect[i];
  AC_algorithm() {
    root = new node();
    memset(occur, 0, sizeof(occur));
    memset(reflect, -1, sizeof(reflect));
  int decode(char c) {
                              //decode char
    return c <= 'Z' ? (c - 'A') : (c - 'a' + 26);
  void insert(char *s,int index) { //add string to trie
    node *p = root;
    for(; *s; s++) {
      int code = decode(*s);
      if(p->ch[code] == NULL)
        p->ch[code] = new node();
      p = p->ch[code];
    if(p->index == -1)
      p->index = index;
    else
      reflect[index] = p->index;
  void build() {
                             //build machine
    queue<node*> q;
```

```
q.push(root);
    while(!q.empty()) {
      node *p = q.front();
      for(int i = 0; i < node::signNum; i++)</pre>
        if(p->ch[i]) {
          node *tmp = p->suffix;
          while(tmp && !tmp->ch[i]) tmp = tmp->suffix;
          if(tmp)
            p->ch[i]->suffix = tmp->ch[i];
            p->ch[i]->suffix = root;
          tmp = p->ch[i]->suffix;
          if(tmp->index != -1)
            p->ch[i]->dict = tmp;
          else
            p->ch[i]->dict = tmp->dict;
          q.push(p->ch[i]);
      q.pop();
    }
  }
  void match(char *s) {
                               //match patterns with Text
    node *p = root;
    for(; *s; s++) {
      int code = decode(*s);
      while(p && !p->ch[code]) p = p->suffix;
      if(p)
        p = p->ch[code];
      else
        p = root;
      node *tmp = p;
      while(tmp) {
        if(tmp->index != -1)
          occur[tmp->index]++;
        tmp = tmp->dict;
      }
   }
  }
  ~AC_algorithm() {
    queue<node*> q;
    q.push(root);
    while(!q.empty()) {
      node *p = q.front();
      q.pop();
      for(int i = 0; i < node::signNum; i++)</pre>
        if(p->ch[i])
          q.push(p->ch[i]);
      delete p;
    }
 }
};
```

3.4 Z

```
#include <cstring>
int z[length];
void z_function(char *str) {
  int len = strlen(str), L = 0, R = 1;
  z[0] = len;
  for(int i = 1; i <len; i++)</pre>
    if(R \le i \mid \mid z[i-L] >= R-i) {
      int x = max(R, i);
      while(x < len && str[x] == str[x-i])</pre>
        x++;
      z[i] = x-i;
      L = i; R = x;
      //if(i < x) \{L = i; R = x;\}
    } else
      z[i] = z[i-L];
}
```

4 Tree

4.1 Tree Min Vertex Cover

```
class TreeMinVertexCover {
private:
  static const int maxNum = 100004;
  vector<int> G[maxNum];
  int in[maxNum];
public:
  bool pick[maxNum];
  int MVC; // min vertext cover
  void init() {
    for(int i=0; i<maxNum; ++i)</pre>
      G[i].clear();
    memset(in, 0, sizeof(in));
  void addEdge(int u, int v) {
    G[u].emplace_back(v);
    G[v].emplace_back(u);
    ++in[u];
    ++in[v];
  int treeMinVertexCover() {
    memset(pick, 0, sizeof(pick));
    MVC = 0;
    queue<int> myQ;
    for(int i=1; i<=maxNum; ++i)</pre>
      if( in[i]==1 ) myQ.push(i);
    while( myQ.size() ) {
      int nowAt = myQ.front();
      myQ.pop();
      if( in[nowAt]==0 ) continue;
      ++MVC;
      int id;
      for(int i=0; i<G[nowAt].size(); ++i)</pre>
        if( in[G[nowAt][i]] ) {
          id = G[nowAt][i];
          break;
      for(int i=0; i<G[id].size(); ++i)</pre>
        if( in[G[id][i]] ) {
          --in[G[id][i]];
          --in[id];
          if( in[G[id][i]]==1 )
            myQ.push(G[id][i]);
    return MVC;
  }
};
```

4.2 Treap ordered

```
struct node {
  int v , p , sz ;
  node *l , *r ;
  node() \{ l = r = NULL ; \}
  node(int v_{-}):v(v_{-}),p(rand()),sz(1) \{l = r = 0;\}
  int size() {
    return this!=NULL ? this->sz : 0 ;
  void maintain() {
    sz = l->size() + r->size() + 1;
 }
};
void splite_v(node *t,node* &a,node* &b,int v) {
 if(!t)
   a = b = NULL;
  else if(v >= t -> v) {
    a = t ;
    splite_v(t->r,a->r,b,v) ;
    a->maintain();
  } else if(v < t->v) {
```

if(set!=oo) {

v = set;

set = oo ;

void maintain() {

if(t == NULL) {
 a = b = NULL;

t->pushdown();

a->maintain();

b->maintain();

if(t->l->size()+1 <= k) {

splite(t->l,a,b->l,k) ;

return ;

} else {
 b = t;

}

+ r->Presum());

+ l->Sufsum());

->Sufsum() + v + r->Presum());

maxsum = max(maxsum1, maxsum2);
sum = l->Sum() + r->Sum() + v;

sz = 1 + l->size() + r->size() ;

void splite(node *t,node* &a,node* &b,int k) {

splite(t->r, a->r, b, k-(t->l->size()+1));

}

}

} ;

l->makesame(set) ;
r->makesame(set) ;

presum = $max(l\rightarrow Presum(), l\rightarrow Sum() + v, l\rightarrow Sum() + v$

sufsum = max(r->Sufsum(), r->Sum() + v, r->Sum() + v

int maxsum1 = max(l->Maxsum(), r->Maxsum(), v);
int maxsum2 = max(l->Sufsum() + v, r->Presum() + v, l

```
b = t;
    splite_v(t->l,a,b->l,v) ;
    b->maintain();
}
node* merge(node *a,node *b) {
  if(a==NULL || b==NULL)
    return a!=NULL ? a : b;
  if(a->p > b->p) {
    a->r = merge(a->r,b);
    a->maintain();
    return a ;
  } else if(a->p <= b->p) {
    b->l = merge(a,b->l);
    b->maintain();
    return b;
 }
int kth(node *t,int k) {
  if(k<=t->l->size())
    return kth(t->l,k) ;
  else if(k>t->l->size()+1)
    return kth(t->r,k-t->l->size()-1) ;
  return t->v;
}
void release(node *t) {
  if(t) {
    release(t->l);
    release(t->r) ;
    delete t ;
 }
```

4.3 Treap unordered

```
}
#include <iostream>
                                                             node* merge(node *a,node *b) {
#include <cstdio>
                                                               if(!a || !b)
#include <stdlib.h>
                                                                  return a ? a : b ;
#include <cstring>
                                                               if(a->p > b->p) {
using namespace std;
                                                                  a->pushdown();
const int oo = 1e9 ;
                                                                  a->r = merge(a->r,b);
struct node {
                                                                 a->maintain();
  int v , p , sz ;
                                                                 return a ;
  int sum , presum , sufsum , maxsum , flag , set ;
                                                               } else {
 node *l , *r ;
node(){ }
                                                                  b->pushdown();
                                                                 b->l = merge(a,b->l) ;
 node(int v_):p(rand()),sz(1),l(NULL),r(NULL) {
                                                                 b->maintain();
    v = sum = presum = sufsum = maxsum = v_;
                                                                  return b;
    flag = 0;
                                                               }
    set = oo ;
                                                             }
                                                             void Delete(node *t) {
 int size() { return this ? sz : 0 ; }
                                                               if(!t) return ;
 int Sum() { return this ? sum : 0 ; }
                                                               Delete(t->l);
  int Presum() { return this ? (!flag ? presum : sufsum)
                                                               Delete(t->r);
    : -00; }
                                                               delete t ;
  int Sufsum() { return this ? (!flag ? sufsum : presum)
    : -00;}
                                                             void INSERT(node* &root) {
 int Maxsum() { return this ? maxsum : -oo ; }
                                                               int p , k , v ;
                                                               node *t=0 , *L, *R;
  int max(int a,int b) { return a > b ? a : b ; }
  int max(int a,int b,int c) { return max(a,max(b,c)) ; }
                                                               scanf("%d%d",&p,&k)
 void makesame(int st) {
                                                               for(int i=0 ; i<k ; i++) {</pre>
    if(this) {
                                                                  scanf("%d",&v);
      set = st;
                                                                  t = merge(t, new node(v));
      sum = st*sz;
      presum = sufsum = maxsum = (st <= 0 ? st : sum) ;</pre>
                                                               splite(root, L, R, p) ;
                                                               root = merge(L, merge(t, R)) ;
 }
 void pushdown() {
                                                             void DELETE(node* &root) {
    if(flag) {
                                                               int p , k ;
scanf("%d%d",&p,&k) ;
      if(l) l->flag = !l->flag ;
      if(r) r->flag = !r->flag ;
                                                               node *L, *M, *R;
      swap(l,r);
                                                               splite(root, L, R, p-1) ;
      swap(presum, sufsum) ;
                                                               splite(R, M, R, k);
                                                               Delete(M);
      flag = 0;
```

```
root = merge(L, R) ;
void MAKE_SAME(node* &root) {
  int p , k , l ;
 scanf("%d%d%d",&p,&k,&l);
 node *L, *M, *R;
 splite(root, L, R, p-1) ;
  splite(R, M, R, k) ;
 M->makesame(l);
 root = merge(L, merge(M, R));
void REVERSE(node* &root) {
 int p , k ;
scanf("%d%d",&p,&k) ;
 node *L, *M, *R;
 splite(root, L, R, p-1) ;
 splite(R, M, R, k);
 M->flag = !M->flag ;
 root = merge(L, merge(M, R));
int GET_SUM(node* &root) {
 int p , k , v ;
scanf("%d%d",&p,&k) ;
 node *L, *M, *R;
 splite(root, L, R, p-1);
 splite(R, M, R, k) ;
 v = M \rightarrow Sum();
 root = merge(L, merge(M, R));
 return v ;
int MAX_SUM(node* &root) {
 return root->Maxsum();
int main () {
 int n, m;
 srand(860514) ;
 while(scanf("%d%d",&n,&m)==2) {
    node *root=0 ;
    for(int i=0,v ; i<n ; i++) {</pre>
      scanf("%d",&v);
      root = merge(root, new node(v)) ;
    while(m--) {
      char s[10] ;
      scanf("%s",s);
      if(strcmp(s,"INSERT")==0) {
        INSERT(root) ;
        continue;
      if(strcmp(s,"DELETE")==0) {
        DELETE(root) ;
        continue;
      if(strcmp(s,"MAKE-SAME")==0) {
        MAKE_SAME(root) ;
        continue;
      if(strcmp(s,"REVERSE")==0) {
        REVERSE(root) ;
        continue;
      if(strcmp(s,"GET-SUM")==0) {
        printf("%d\n",GET_SUM(root));
        continue ;
      if(strcmp(s,"MAX-SUM")==0) {
        printf("%d\n",MAX_SUM(root));
        continue;
      }
    }
 }
}
```

```
#include <iostream>
#include <cstdio>
#include <vector>
#include <string.h>
#define N 50000
using namespace std;
vector<int> G[N];
int ind[N], f[N], d[N], head[N], son[N], pn, n;
int dfs(int u,int parent) {
  int i, v, x, w = 0, sum = 1;
  f[u] = parent;
  son[u] = -1;
  for(i = 0; i < G[u].size(); i++) {</pre>
    v = G[u][i];
    if(v != parent) {
      d[v] = d[u] + 1;
      x = dfs(v,u);
      sum += x;
      if(x > w) {
        son[u] = v;
        w = x;
    }
  }
  return sum;
void chain(int u) {
  int i , v;
  ind[u] = pn++;
  if(f[u] == -1 || son[f[u]] != u)
   head[u] = u;
  else
    head[u] = head[f[u]];
  if(son[u]!=-1)
    chain(son[u]);
  for(i=0; i<G[u].size(); i++) {</pre>
    v = G[u][i];
    if(v != f[u] && v != son[u])
      chain(v);
}
void modify(int u, int v, ...) {
  while(head[u] != head[v])
    if(d[head[u]] > d[head[v]]) {
      //do something in rang(ind[head[u]], ind[u]);
      u = f[head[u]];
    } else {
      //do something in rang(ind[head[v]], ind[v]);
      v = f[head[v]];
  if(d[u] > d[v])
    //do something in rang(ind[head[u]], ind[u]);
  else
    //do something in rang(ind[head[v]], ind[v]);
int getLca(int u, int v) {
  while(head[u] != head[v])
    if(d[head[u]] > d[head[v]])
      u = f[head[u]];
    else
      v = f[head[v]];
  if(d[u] > d[v])
    return v;
    return u;
void build() {
  pn = 1;
  d[0] = 0; //set depth of root to 0
  dfs(0,-1);
  chain(0); //relabel node
```

4.4 Treap unordered

Graph 5

Biconnected Components 5.1

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <stack>
#include <vector>
using namespace std;
//Biconnected Components
struct BCC {
 static const int maxNum = 1010;
 vector<int> G[maxNum], bccGroup[maxNum];
 int Node;
  int bcc_cnt;
  int timeStamp;
  int low[maxNum];
  int visit[maxNum];
  int bcc[maxNum];
 bool is_ap[maxNum];
  stack< pair<int,int> > S;
 BCC(int Node) {
    for(int i = 0; i < maxNum; i++) {</pre>
      G[i].clear();
      bccGroup[i].clear();
      low[i] = visit[i] = bcc[i] = -1;
      is_ap[i] = false;
    this->Node = Node;
    bcc_cnt = 0;
 void DFS(int u,int parent) {
    int children = 0;
    low[u] = visit[u] = timeStamp++;
    for(int i = 0; i < G[u].size(); i++) {</pre>
      int v = G[u][i];
      if(visit[v] == -1) {
        S.push(make_pair(u, v));
        children++;
        DFS(v, u);
low[u] = min(low[u], low[v]);
        if(low[v] >= visit[u]) {
          is_ap[u] = true;
          pair<int,int> e;
          do {
            e = S.top();
            if(bcc[e.first] != bcc_cnt) {
              bccGroup[bcc_cnt].push_back(e.first);
              bcc[e.first] = bcc_cnt;
            if(bcc[e.second] != bcc_cnt) {
              bccGroup[bcc_cnt].push_back(e.second);
              bcc[e.second] = bcc_cnt;
          }while(e.first!=u || e.second!=v);
          bcc_cnt++;
      } else if(v != parent) {
        S.push(make_pair(u, v));
        low[u] = min(low[u], visit[v]);
    if(u == parent) // u is root
      is_ap[u] = (children >= 2);
 void articulation_vertex() {
    timeStamp = 0;
    for(int i = 0; i < Node; i++)</pre>
      if(low[i] == -1)
        DFS(i, i);
};
```

Strong Connected Components

```
class SCC {
private:
    static const int maxN = 10004;
    vector<int> G[maxN];
    vector<int> invG[maxN];
    vector<int> stk;
    bool visited[maxN];
    void dfs_1(int nowAt) {
        visited[nowAt] = true;
        for(auto v : G[nowAt])
            if( !visited[v] )
                dfs_1(v);
        stk.emplace_back(nowAt);
    }
    void dfs_2(int nowAt, const int id) {
        sccID[nowAt] = id;
        for(auto v : invG[nowAt])
            if( sccID[v]==-1 )
                dfs_2(v, id);
public:
    int sccNum;
    int sccID[maxN];
    void init() {
        for(int i=0; i<maxN; ++i) {</pre>
            G [i].clear();
            invG[i].clear();
    void addEdge(int u, int v) {
        G [u].emplace_back(v);
        invG[v].emplace_back(u);
    vector<vector<int>> findAllSCC(int base, int n) {
        memset(visited, 0, sizeof(visited));
        stk.clear();
        for(int i=base; i<=n; ++i)</pre>
            if( !visited[i] )
                dfs_1(i);
        sccNum = 0;
        memset(sccID, -1, sizeof(sccID));
        for(int i=stk.size()-1; i>=0; --i)
            if( sccID[stk[i]]==-1 ) {
                dfs_2(stk[i], sccNum);
                 ++sccNum;
            }
        // returned zero base scc dag
        vector<vector<int>> sccDAG(sccNum);
        vector<unordered_set<int>> have(sccNum);
        for(int u=base; u<=n; ++u) {</pre>
            int sccU = sccID[u];
            for(auto v : G[u]) {
                 int sccV = sccID[v];
                 if( sccU==sccV ) continue;
                 if( have[sccU].find(sccV) == have[sccU].
    end() ) {
                     have [sccU].insert
                                               (sccV):
                     sccDAG[sccU].emplace_back(sccV);
                 }
            }
        }
        return sccDAG;
};
```

2 SAT

```
class TwoSAT {
private:
```

```
static const int maxN = 100004;
 static const int size = 2*maxN + 4;
 bool pick[size];
 vector<int> G [size];
 int id(int i, int T) { return (i<<1) + T; }
  int alter(int i) { return i^1; }
 bool dfsTry(int nowAt, vector<int> &stk) {
    if( pick[alter(nowAt)] )
      return false;
    stk.emplace_back(nowAt);
    pick[nowAt] = true;
    for(auto v : G[nowAt]) {
     if( !pick[v] && !dfsTry(v, stk) )
        return false;
    return true;
 }
public:
 void init() {
    memset(pick, 0, sizeof(pick));
    for(int i=0; i<size; ++i)</pre>
     G[i].clear();
  void addClause(bool TA, int A, bool TB, int B) {
    // Add clause (TA + TB)
    // When TA not true, TB must true. vise versa.
    G[id(A, !TA)].emplace_back(id(B, TB));
   G[id(B, !TB)].emplace_back(id(A, TA));
 bool solve() {
    // O(n) solve
    memset(pick, 0, sizeof(pick));
    for(int i=0; i<maxN; ++i) {</pre>
      if( pick[id(i, 0)] || pick[id(i, 1)] )
        continue;
      vector<int> stk;
      if( dfsTry(id(i, 0), stk) )
        continue;
      for(auto v : stk)
       pick[v] = false;
      if( !dfsTry(id(i, 1), stk) )
       return false;
    return true;
 }
 bool T(int i) {
    // should solve() first
    return pick[id(i, 1)];
};
```

5.4 System of Difference Constraints

```
class System_of_DifConstrain {
private:
 static const int maxN = 504;
  static const int maxM = 3004;
 struct Edge {
    int s, t;
    long long cost;
 Edge es[maxM];
 int eSize;
public:
 bool solvable;
  long long x[maxN]; // one solution
 void init() {
    eSize = -1;
 void addConstrain(int xI, int xJ, long long c) {
    // add xi - xj <= c
    es[++eSize] = {xJ, xI, c};
```

```
}
bool solve(int n=maxN) {
    // n is max # of node of CC
    memset(x, 0, sizeof(x));
    for(int i=0; i<n; ++i)
        for(int j=0; j<=eSize; ++j)
            if( x[es[j].s] + es[j].cost < x[es[j].t] )
            x[es[j].t] = x[es[j].s] + es[j].cost;
    for(int j=0; j<=eSize; ++j)
        if( x[es[j].s] + es[j].cost < x[es[j].t] )
            return solvable = false;
    return solvable = true;
}
</pre>
```

5.5 Bipartite: MaxMatch, Min-VerCover, MaxIndSet

```
class Bipartite {
private:
  static const int MaxNum = 1004;
  vector<int> g[MaxNum];
  bool visited [MaxNum];
  bool bipart(int nowAt, int nowSide) {
    visited[nowAt] = true;
    side[nowAt] = nowSide;
    for(auto &id : g[nowAt])
      if( !visited[id] )
        bipart(id , !nowSide);
      else if( side[id] == nowSide )
        return false;
    return true;
  bool maxMatch(int nowAt) {
    visited[nowAt] = true;
    for(auto &id : g[nowAt])
      if( cp[id] == -1
        || (!visited[cp[id]] && maxMatch(cp[id])) ){
        cp[id]
                  = nowAt;
        cp[nowAt] = id;
        return true;
      7
    return false;
  void minVertexCover(int nowAt) {
    MVC[nowAt] = 1;
    for(auto &id : g[nowAt])
      if( !MVC[id] ) {
        MVC[id] = 1;
        minVertexCover(cp[id]);
  void maxIndependentSet(int nowAt) {
    MIS[nowAt] = 1;
    for(auto &id : g[nowAt])
      if( !MIS[cp[id]] )
        maxIndependentSet(cp[id]);
public:
  int matchNum;
                      // max match num
  int cp [MaxNum]; // id and cp[id] is couple
  bool side[MaxNum]; // left/right side
bool MVC [MaxNum]; // min vertex cover
  bool MIS [MaxNum]; // max indepent set
  void addEdge(int u, int v) {
    g[u].emplace_back(v);
    g[v].emplace_back(u);
  }
  void init() {
    for(int i=0; i<MaxNum; ++i)</pre>
      g[i].clear();
  bool countAll() {
```

```
// if graph is not bipartite return false
    // bipartite
    memset(side, 0, sizeof(side));
    memset(visited , 0 , sizeof(visited));
    for(int i=0 ; i<MaxNum ; ++i)</pre>
      if( !visited[i] && !bipart(i, 0) )
        return false;
    // maximum match
    // O(VE), this code can be more optimized
    // alternative: dinic O(V^0.5*E)
    matchNum = 0;
    memset(cp , -1 , sizeof(cp));
    for(int i=0 ; i<MaxNum ; ++i){</pre>
      if( cp[i]!=-1 ) continue;
      memset(visited , 0 , sizeof(visited));
      if( maxMatch(i) )
        ++matchNum;
    // min vertex cover
    memset(MVC, 0, sizeof(MVC));
    for(int i=0; i<MaxNum; ++i)</pre>
      if( side[i]==1 && cp[i]==-1 )
        minVertexCover(i);
    for(int i=0; i<MaxNum; ++i)</pre>
      if( side[i] == 1 )
        MVC[i] = !MVC[i];
    // max independent set
    memset(MIS, 0, sizeof(MIS));
    for(int i=0; i<MaxNum; ++i)</pre>
      if( cp[i]==-1 )
        maxIndependentSet(i);
    for(int i=0; i<MaxNum; ++i)</pre>
      if( side[i]==1 && cp[i]!=-1
        && !MIS[i] && !MIS[cp[i]] )
        MIS[i] = 1;
    return true:
  }
};
```

5.6 Bipartite: KM

```
#include <cstring>
#include <iostream>
using namespace std;
struct KM {
 static const int N = 105, big_value = 1000000000;
  int G[N][N] , visx[N] , visy[N];
  int n , labelx[N] , labely[N] , matchx[N] , matchy[N] ;
 KM(int n_):n(n_) {};
 bool DFS(int x) {
    visx[x] = true ;
    for(y=0 ; y<n ; y++)</pre>
      if(!visy[y] && labelx[x]+labely[y]==G[x][y]) {
        visy[y] = true ;
        if(matchy[y] == -1 || DFS(matchy[y])) {
          matchx[x] = y;
          matchy[y] = x;
          return true ;
        }
    return false;
 int max_match() {
                     //Maximum Weight Perfect Bipartite
    memset(labelx,0,sizeof(labelx));
```

```
memset(labely,0,sizeof(labely)) ;
    memset(matchx,-1,sizeof(matchx)) ;
    memset(matchy,-1,sizeof(matchy));
    int i , x , y
    for(x=0 ; x<n ; x++)</pre>
      for(y=0; y<n; y++)
        labelx[x] = max(labelx[x],G[x][y]);
    for(i=0 ; i<n ; i++)</pre>
      while(true) {
        memset(visx,0,sizeof(visx));
        memset(visy,0,sizeof(visy));
        if(DFS(i)) break ;
        int d=big_value ;
        for(x=0 ; x<n ; x++) if(visx[x])</pre>
           for(y=0 ; y<n ; y++) if(!visy[y])</pre>
            d = min(d,labelx[x]+labely[y]-G[x][y]);
        if(d==big_value) return -1; //faile to exist
    perfect matching
        for(int j=0 ; j<n ; j++) {</pre>
          if(visx[j]) labelx[j] -= d ;
          if(visy[j]) labely[j] += d ;
      }
    int total=0 ;
    for(i=0 ; i<n ; i++)</pre>
                                    //must be perfect!!!
      total += G[i][matchx[i]] ;
    return total ;
};
```

5.7 Min Vertex Cover

```
struct MinVertexCover {
private:
  static const int MaxNum = 54;
  vector<int> G[MaxNum];
  int in[MaxNum];
  int undo(vector<int> &record) {
    for(int i=0; i<record.size(); ++i)</pre>
      ++in[record[i]];
    record.clear();
  int delNode(int u, vector<int> &record) {
    for(int i=0; i<G[u].size(); ++i)</pre>
      if( in[G[u][i]] ) {
        --in[G[u][i]];
        --in[u];
        record.push_back(G[u][i]);
        record.push_back(u);
  int cnt(int from, int *visited, bool type) {
    if( visited[from] ) return 0;
    if( type==1 ) visited[from] = 1;
    for(int i=0; i<G[from].size(); ++i)</pre>
    if( in[G[from][i]] && !visited[G[from][i]] )
      return type+cnt(G[from][i], visited, !type);
    return type;
  int cnt(int *visited) {
    int ret = 0;
    for(int i=0; i<MaxNum; ++i)</pre>
      if( in[i] == 1 && !visited[i] )
        ret += cnt(i, visited, 0);
    for(int i=0; i<MaxNum; ++i)</pre>
      if( in[i] == 2 && !visited[i] )
        ret += cnt(i, visited, 0);
    return ret;
public:
  int MVCPick[MaxNum];
  int MVC; // min vertex cover
```

```
void init() {
    for(int i=0; i<MaxNum; ++i)</pre>
      G[i].clear();
    memset(in, 0, sizeof(in));
  void addEdge(int u, int v) {
    G[u].push_back(v);
    G[v].push_back(u);
    ++in[u];
    ++in[v];
  void minVertexCover(int nowMVC=0, const int *lastPick=
    NULL) {
    // 0(n^2 * 1.38^n)
    int nowPick[MaxNum] = {};
    if( nowMVC==0 ) {
      MVC = MaxNum;
      memset(MVCPick, 0, sizeof(MVCPick));
    else memcpy(nowPick, lastPick, sizeof(nowPick));
    int maxid = 0;
    for(int i=0; i<MaxNum; ++i)</pre>
      if( in[i]>in[maxid] )
        maxid = i;
    if( in[maxid] <= 2 ) {</pre>
      nowMVC += cnt(nowPick);
      if( nowMVC<MVC ) {</pre>
        MVC = nowMVC;
        memcpy(MVCPick, nowPick, sizeof(nowPick));
      }
      return;
    }
    vector<int> record;
    delNode(maxid, record);
    nowPick[maxid] = 1;
    minVertexCover(nowMVC+1, nowPick);
    nowPick[maxid] = 0;
    undo(record);
    int cnt = 0;
    for(int i=0; i<G[maxid].size(); ++i)</pre>
      if( in[G[maxid][i]] ) {
        ++cnt;
        delNode(G[maxid][i], record);
        nowPick[G[maxid][i]] = 1;
    minVertexCover(nowMVC+cnt, nowPick);
    undo(record);
};
```

6 Flow

6.1 Dinic Maxflow Mincut

```
class Dinic{
private:
    typedef pair<int,int> pii;
    static const int maxN = 504;
    static const int infF = 1023456789;
    int cap [maxN][maxN];
    int pipe[maxN][maxN];
    vector<int> g[maxN];
    int level[maxN], visited[maxN];
    bool bfsLabeling(int s, int t){
        memset(level , 0 , sizeof(level));
        queue<int> myQ;
        myQ.push( s );
        level[s] = 1;
        while( !myQ.empty() ){
            int nowAt = myQ.front();
            myQ.pop();
            for(int i=0;i<g[nowAt].size();++i)</pre>
                 if( !level[g[nowAt][i]] && pipe[nowAt][g[
    nowAt][i]] ){
                     level[g[nowAt][i]] = level[nowAt] +
    1;
                     myQ.push( g[nowAt][i] );
                }
        }
        return level[t];
    int dfsFindRoute(int nowAt, int t, int maxC) {
        visited[nowAt] = true;
        if( nowAt==t ){
            maxFlow += maxC;
            return maxC;
        for(int i=0; i<g[nowAt].size(); ++i) {</pre>
            int next = g[nowAt][i];
            if( visited[next] ) continue;
            if( level[next] != level[nowAt]+1 ) continue;
            if( !pipe[nowAt][next] ) continue;
            int nowOut = dfsFindRoute(next ,t ,min(maxC ,
     pipe[nowAt][next]));
            if( nowOut==0 )
                continue;
            pipe[nowAt][next] -= nowOut;
            pipe[next][nowAt] += nowOut;
            return nowOut;
        }
        return 0;
    void dfsFindMinCut(int nowAt) {
        sside[nowAt] = 1;
        for(auto v : g[nowAt])
            if( !sside[v] && pipe[nowAt][v] )
                dfsFindMinCut(v);
    }
public:
    int maxFlow;
    bool sside[maxN];
    vector<pii> minCut;
    void init() {
        memset(cap , 0, sizeof(cap));
        memset(pipe , 0, sizeof(pipe));
        memset(sside, 0, sizeof(sside));
        for(int i=0;i<maxN;++i)</pre>
            g[i].clear();
        maxFlow = 0;
        minCut.clear();
    void addEdge(int u, int v, int c) {
```

```
if( u==v ) return;
        if( !cap[u][v] && !cap[v][u] ) {
            g[u].emplace_back(v);
            g[v].emplace_back(u);
        cap[u][v] += c;
    void coculAll(int s, int t) {
        memcpy(pipe, cap, sizeof(pipe));
        // max flow
        while( bfsLabeling(s,t) ) {
            memset(visited, 0, sizeof(visited));
            while( dfsFindRoute(s,t,infF) )
        }
        // min cut
        dfsFindMinCut(s);
        for(int u=0; u<maxN; ++u)</pre>
        if( sside[u] )
            for(auto v : g[u])
            if( !sside[v] )
                minCut.push_back({u, v});
};
```

7 Geometry

7.1 Geometry basic

```
struct Point{
  double x,y;
  Point(double xi=0.0,double yi=0.0){
    x = xi , y = yi;
  Point operator - (const Point &r)const{
    return Point(x-r.x , y-r.y);
  }
};
typedef Point Vector;
double angle(const Vector &v,const Vector &u){
  // return rad [0, pi] of two vector
  return acos( dot(v,u)/len(v)/len(u) );
Vector rotate(const Vector &v,double rad){
  return Vector(
   v.x*cos(rad) - v.y*sin(rad),
    v.x*sin(rad) + v.y*cos(rad)
  );
}
double pointSegLen(const Point &A,const Point &B,const
    Point &Q){
  if(A==B) return len(Q-A);
  if( dot(B-A , Q-A)<0 ) return len(Q-A);</pre>
  if( dot(B-A , Q-B)>0 ) return len(Q-B);
  return fabs( cross(B-A , Q-A) ) / len(B-A);
bool pointOnSeg(const Point &A,const Point &B,const Point
     &Q){
  return fabs( len(Q-B)+len(Q-A)-len(A-B) ) < 1e-9;</pre>
struct Line{
  Point P0;
  Vector v;
  Line(const Point &pi,const Vector &vi):p0(pi) , v(vi)
double pointLineLen(const Line &L,const Point &Q){
  return fabs( cross(L.v , Q-L.P0) ) / len(L.v);
Point projectToLine(const Line &L,const Point &Q){
  double t = dot(Q-L.P0 , L.v) / dot(L.v , L.v);
  return L.P0 + L.v * t;
Point innerCircle(point &p1, point &p2, point &p3){
  // p1,p2,p3 should not on same line
  double a1 = (-2*p1.x + 2*p2.x);
  double b1 = (-2*p1.y + 2*p2.y);
  double c1 = (p2.x*p2.x + p2.y*p2.y - p1.x*p1.x - p1.y*
    p1.y);
  double a2 = (-2*p1.x + 2*p3.x);
double b2 = (-2*p1.y + 2*p3.y);
  double c2 = (p3.x*p3.x + p3.y*p3.y - p1.x*p1.x - p1.y*
    p1.y);
  double cx = (c1*b2-c2*b1) / (a1*b2-a2*b1);
  double cy = (a1*c2-a2*c1) / (a1*b2-a2*b1);
  return Point(cx, cy);
Point outerCircle(point &p1, point &p2, point &p3) {
  // p1,p2,p3 should not on same line
  double x1 = (p1.x+p2.x)/2.0;
  double y1 = (p1.y+p2.y)/2.0;
  double x2 = (p2.x+p3.x)/2.0;
  double y2 = (p2.y+p3.y)/2.0;
  double vx = p2.x-p1.x;
  double vy = p2.y-p1.y;
  double ux = p3.x-p2.x;
```

```
double uy = p3.y-p2.y;
double A = vx*x1 + vy*y1;
double B = ux*x2 + uy*y2;
double cx = (uy*A - vy*B) / (uy*vx - ux*vy);
double cy = (ux*A - vx*B) / (ux*vy - uy*vx);
return Point(cx, cy);
}
```

7.2 Minimal Enclose Disk

```
struct Circle {
  Point c;
  double R2; // square of radius
  Circle() {}
  Circle(const Point &p1, const Point &p2) {
    c.x = (p1.x+p2.x)/2.0;
    c.y = (p1.y+p2.y)/2.0;
    R2 = dot(p1-p2, p1-p2)/4.0;
  Circle(const Point &p1, const Point &p2, const Point &
    p3) {
    // p1, p2, p3 should not on same line
    c = outerCircle(p1, p2, p3);
    double dx = p1.x - c.x;
    double dy = p1.y - c.y;
    R2 = dx*dx + dy*dy;
  bool contain(const Point &p) const {
    double dx = c.x - p.x;
    double dy = c.y - p.y;
    return fdif(dx*dx + dy*dy - R2)<=0;</pre>
};
Circle minEncloseDisk(vector<Point> &ps) {
  // Find minimal circal enclose all point
  // worst case O(n^3), expected O(n)
  if( ps.size()==0 ) return D;
  if( ps.size()==1 ) {
    D.c = ps[0];
    D.R2 = 0.0;
    return D;
  }
  random_shuffle(ps.begin(), ps.end());
  D = Circle(ps[0], ps[1]);
  for(int i=2; i<ps.size(); ++i) {</pre>
    if( D.contain(ps[i]) )
      continue;
    D = Circle(ps[i], ps[0]);
    for(int j=1; j<i; ++j) {</pre>
      if( D.contain(ps[j]) )
        continue;
      D = Circle(ps[i], ps[j]);
      for(int k=0; k<j; ++k) {</pre>
        if( D.contain(ps[k]) )
          continue;
        D = Circle(ps[i], ps[j], ps[k]);
    }
 }
```

7.3 2D Convex Hull

```
bool turnLeft(const Vector &v1, const Vector &v2) {
  return fdif(cross(v1, v2)) > 0LL;
}
vector<Point> convexHull(vector<Point> &ps) {
  // return convex hull without redundant point
  sort(ps.begin(), ps.end());
  vector<Point> up;
```

```
for(int i=0; i<ps.size(); ++i) {</pre>
 while( up.size()>1
   && !turnLeft(up.back()-up[up.size()-2],
      ps[i]-up.back()) )
   up.pop_back();
 up.emplace_back(ps[i]);
vector<Point> btn;
for(int i=ps.size()-1; i>=0; --i) {
 while( btn.size()>1
   && !turnLeft(btn.back()-btn[btn.size()-2],
      ps[i]-btn.back()) )
   btn.pop_back();
  btn.emplace_back(ps[i]);
vector<Point> res(up);
res.insert(res.end(), btn.begin()+1, btn.end());
res.pop_back();
return res;
```

7.4 Closest Point

```
#include <iostream>
#include <cstdio>
#include <algorithm>
#include <cstring>
#include <cmath>
#define N 10010
#define INFINITY__ 1e10
using namespace std;
int diff(double f) {
  if(fabs(f) < 1e-9)
    return 0;
  return f < 0 ? -1 : 1;
struct point {
  double x, y;
  bool operator < (const point &p) const {</pre>
    return diff(x - p.x) < 0;
};
point pt[N], tmp[N];
double dis(point a,point b) {
  return sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y
    - b.y);
double closestpoint(int L,int R) {
  if(R - L + 1 == 1) return INFINITY__;
  int M = (L+R)/2;
  double middle = pt[M].x;
  double Ldis = closestpoint(L, M);
  double Rdis = closestpoint(M+1, R);
  double radi = min(Ldis, Rdis);
  int cntpt = 0;
  merge(pt+L, pt+M+1, pt+M+1, pt+R+1, tmp+L, [](point a,
    point b) {
    return diff(a.y - b.y) < 0;</pre>
  });
  copy(tmp + L, tmp + R + 1, pt + L);
  for(int i = L; i <= R; i++)
    if(diff(fabs(pt[i].x - middle) - radi) < 0)</pre>
      tmp[cntpt++] = pt[i];
  for(int i = 0; i < cntpt; i++)</pre>
    for(int j = 1; i + j < cntpt && j < 8; j++)
      radi = min(radi, dis(tmp[i], tmp[i+j]));
  return radi:
}
```

7.5 Min Max Triangle

```
pair<double, double> findMinMaxTri(vector<Point> &ps) {
```

```
static const double PI = acos(-1.0);
struct Seg {
  double rad; // [0.5pi, 1.5pi]
  int s1, s2;
};
const int n = ps.size();
sort(ps.begin(), ps.end(), [](const Point &l, const
  Point &r) {
  if(fdif(l.x - r.x) == 0)
    return l.y > r.y;
  return l.x < r.x;</pre>
});
vector<int> id(n+4);
for(int i=0; i<n; ++i)</pre>
  id[i] = i;
// sort all pair of point
vector<Seg> segs;
for(int i=0; i<n; ++i)</pre>
for(int j=i+1; j<n; ++j) {</pre>
  double m = atan2(ps[j].y-ps[i].y, ps[j].x-ps[i].x) +
  segs.push_back({m,i,j});
}
sort(segs.begin(), segs.end(), [](const Seg &l, const
  return fdif(l.rad - r.rad) < 0;</pre>
});
// find min max triangle
pair<double, double> ret;
ret.first = ret.second = fabs(cross(ps[0], ps[1], ps
  [2]));
for(auto seg : segs) {
  swap(ps[id[seg.s1]], ps[id[seg.s2]]);
  swap(id[seg.s1], id[seg.s2]);
  const Point &p1 = ps[id[seg.s1]];
  const Point &p2 = ps[id[seg.s2]];
  int id1 = min(id[seg.s1], id[seg.s2]);
int id2 = max(id[seg.s1], id[seg.s2]);
  // find min triangle
  if( id1-1 >= 0 ) {
    double a = fabs(cross(p1, p2, ps[id1-1]));
    if( a < ret.first )</pre>
      ret.first = a;
  if( id2+1 < n ) {
    double a = fabs(cross(p1, p2, ps[id2+1]));
    if( a < ret.first )</pre>
      ret.first = a;
  // fin max triangle
  if( id1 != 0 ) {
    double a = fabs(cross(p1, p2, ps[0]));
    if( a > ret.second )
      ret.second = a;
  if( id2 != n-1 ) {
    double a = fabs(cross(p1, p2, ps[n-1]));
    if( a > ret.second )
      ret.second = a;
  }
}
ret.first /= 2.0;
ret.second /= 2.0;
return ret;
```