

Contents

1 Math	1
1.1 Math Basic	1
1.2 Euler Function	1
1.3 China Remain Theorem	1
1.4 Math Counting	1
1.5 Miller Rabin	1
1.6 Pollard rho	1
1.7 Linear Algebra	1
1.8 FFT	1
2 String	
2.1 KMP	
2.2 LPS	
2.3 AC Automation	
3 Tree	
3.1 Tree Min Vertex Cover	
4 Graph	
4.1 Biconnected Components	
4.2 2 SAT	
4.3 System of Difference Constraints	
4.4 Bipartite: MaxMatch, MinVerCover, MaxIndSet	
4.5 Min Vertex Cover	
5 Flow	
5.1 Dinic Maxflow Mincut	
6 Geometry	
6.1 Geometry basic	
6.2 Minimal Enclose Disk	
6.3 2D Convex Hull	

```
set nocompatible
```

```
set enc=utf-8
set fenc=utf-8
```

```
set tabstop=4
set softtabstop=4
set shiftwidth=4
set backspace=2
set autoindent
set cindent
```

```
syntax on
set t_Co=256
set number
set showmatch
set hls
```

```
autocmd FileType cpp noremap <F9> :w <bar> !! g++ % -std
=c++11 -O2 -Wall && ./a.out<CR>
```

1 Math

1.1 Math Basic

```
1 vector<pii> primeFac(int n) {
2     vector<pii> ret;
3     for(int i=2; n>1; ++i){
4         if( n%i != 0 ) continue;
5         int e = 0;
6         while( n%i == 0 ) ++e , n/=i;
7         ret.push_back({i, e});
8     }
9     return ret;
10 }
11
12 long long fastPow(long long x, int n, long long m){
13     long long ans = 1LL;
14     while( n ){
15         if( n&1 ) ans = ans * x % m;
16         x = x*x % m;
17         n >>= 1;
18     }
19     return ans;
20 }
21
22 long long modInv(long long x, long long p){
23     return fastPow(x, p-2, p);
24 }
25
26 long long modInv_euler(long long x, long long m){
27     // must be gcd(x,m)==1
28     // phi is euler function: O(sqrt(x))
29     return fastPow(x, phi(m)-1, m);
30 }
31 }
```

1.2 Euler Function

```
int phi(int n){
    // euler function: in [0,n], # of coprime(i, n)
    vector<pii> fac = primeFac(n);
    int num = 1 , m = 1;
    for(auto &p : fac)
        num *= (p.first-1) , m *= p.first;
    return n/m * num;
}
```

1.3 China Remain Theorem

```
bool china_solvable(vector<pii> &rule) {
    for(int i=0; i<rule.size(); ++i)
        for(int j=1; j<rule.size(); ++j) {
            int gcd = __gcd(rule[i].second, rule[j].second);
            if( rule[i].first%gcd != rule[j].first%gcd )
                return false;
        }
    return true;
}

long long china(const vector<pii> &rule, int nlt=0){
    // solve x = ai (mod mi)
    // rule should solvable
    long long MM = 1LL;
    for(auto &r : rule)
        MM = lcm(MM, r.second);
    long long x = 0LL;
    for(auto &r : rule){
        long long ai = r.first;
        long long mi = r.second;
        long long Mi = MM / r.second;
        long long Mv = modInv_euler(Mi%mi, mi);
        long long tmp = ai*Mi%MM *Mv %MM;
        x = (x+tmp) % MM;
    }
    if( x>=nlt ) return x;
    long long n = ceil((nlt-x)*1.0/MM);
    return x + n*MM;
}
```

1.4 Math Counting

```

const int MaxNum = 1000004;
const int modNum = 1000000009;
long long fac [MaxNum];
long long facIv[MaxNum];
void initFac(){
    fac[0] = facIv[0] = 1LL;
    for(int i=1; i<MaxNum; ++i) {
        fac [i] = fac[i-1]*i % modNum;
        facIv[i] = modInv(fac[i], modNum);
    }
}
long long Cnm(int n, int m){
    if( m==0 || n==m ) return 1LL;
    return fac[n]*facIv[m] % modNum *facIv[n-m] % modNum;
}
long long nBlock_kColor(int n,int k){
    // n different blocks; k different colors
    // use inclusion-exclusion principle
    long long ans = fastPow(k, n, modNum);
    bool del = true;
    for(int i=k-1; i>0; --i, del=!del){
        long long now = Cnm(k, i)*fastPow(i, n, modNum) %
            modNum;
        if( del ) ans = (ans+modNum-now) % modNum;
        else ans = (ans+now) % modNum;
    }
    return ans;
}

```

1.5 Miller Rabin

```

#include <climits>
typedef unsigned long long int ull;

ull bases[20] = { 2ULL, 3ULL, 5ULL, 7ULL, 11ULL, 13ULL, 17ULL,
    , 19ULL, 23ULL, 29ULL, 31ULL, 37ULL };

ull fake_mul(ull n, ull m, ull x);
ull fast_pow(ull n, ull p, ull x);

bool is_prime(ull n)
{
    if (n < 2ULL) return false;

    for (int tt = 0; tt < 12; tt++) {
        ull a;
        a = bases[tt] % n;

        if (a == 0 || a == 1 || a == n - 1) {
            continue;
        }

        int t = 0;
        ull u = n - 1ULL;
        while ((u & 1ULL) == 0ULL) u >>= 1, t++;

        ull x = fast_pow(a, u, n); // x = a ^ u % n;
        if (x == 1ULL || x == (n - 1)) continue;
        for (int i = 0; i < t - 1; i++)
        {
            if (ULLONG_MAX / x < x) {
                x = fake_mul(x, x, n);
            }
            else {
                x = x*x%n;
            }
            if (x == 1) return false;
            if (x == n - 1) break;
        }
        if (x == n - 1) continue;
        return false;
    }
    return true;
}

```

```

}

ull fake_mul(ull n, ull m, ull x)
{
    ull re = 0ULL;
    while (m != 0ULL) {
        if ((m & 1ULL) != 0ULL) {
            if (ULLONG_MAX - re < n) {
                ull temp = ULLONG_MAX%x;

                temp += (n - (ULLONG_MAX - re)) % x;
                re = temp%x;
            }
            else {
                re = (re + n) % x;
            }
        }

        if (ULLONG_MAX - n < n) {
            ull temp = ULLONG_MAX%x;

            temp += (n - (ULLONG_MAX - n)) % x;
            n = temp%x;
        }
        else {
            n = n + n%x;
        }

        m >>= 1;
    }
    return re;
}

```

```

ull fast_pow(ull n, ull p, ull x)
{
    ull re = 1ULL;

    while (p != 0ULL) {
        if ((p & 1ULL) != 0ULL) {
            if (ULLONG_MAX / re < n) {
                re = fake_mul(n, re, x);
            }
            else {
                re = (re*n) % x;
            }
        }

        if (ULLONG_MAX / n < n) {
            n = fake_mul(n, n, x);
        }
        else {
            n = (n*n) % x;
        }
        p >>= 1;
    }
    return re;
}

```

```

// Below is non-extreme version
ull fake_mul(ull n, ull m, ull x) {
    ull re = 0ULL;
    n %= x, m %= x;
    while( m ) {
        if( m&1ULL )
            re = (re+n) % x;
        n = (n+n) % x;
        m >>= 1;
    }
    return re;
}

ull fast_pow(ull n, ull p, ull x) {
    ull re = 1ULL;
    while( p ) {
        if( p&1ULL )
            re = fake_mul(re,n,x);
    }
}

```

```

    n = fake_mul(n,n,x);
    p >>= 1;
}
return re;
}
bool is_prime(ull n) {
    static const int bNum = 12;
    static const ull bases[bNum] = {
        2ULL,3ULL,5ULL,7ULL,11ULL,13ULL,17ULL,19ULL,23ULL,29
        ULL,31ULL,37ULL
    };
    if( n<=2ULL ) return n==2ULL;
    if( !(n&1ULL) ) return false;

    ull u = n-1;
    while( !(u&1ULL) )
        u >>= 1;
    for(int i=0; i<bNum; i++) {
        if( bases[i]%n == 0 ) continue;
        ull t = u;
        ull a = fast_pow(bases[i], t, n);
        if( a==1 || a==n-1 ) continue;
        while( t!=n-1 && a!=1 && a!=n-1 ) {
            a = fake_mul(a,a,n);
            t <<= 1;
        }
        if( t==n-1 && a==1 ) continue;
        if( a!=n-1 ) return false;
    }
    return true;
}

```

1.6 Pollard rho

```

// need fack_mul, is_prime
ull gcd(ull a, ull b) {
    return (a%b==0)? b : gcd(b, a%b);
}
ull dif(ull a, ull b) {
    return a>b? a-b : b-a;
}
void pollard_rho(ull n, map<ull,int> &facs) {
    while( !(n&1ull) ) {
        // must extract factor 2
        int cnt = 0;
        while( !(n&1ull) )
            ++cnt, n>>=1;
        facs[2] = cnt;
    }
    if( n==1ull ) return;
    if( is_prime(n) ) {
        facs[n]++;
        return;
    }
    ull x = rand()%n;
    ull y = x;
    ull a = rand()%(n-1) + 1;
    ull g = 1ull;
    while( g==1ull ) {
        x = (fake_mul(x,x,n) + a) %n;
        y = (fake_mul(y,y,n) + a) %n;
        y = (fake_mul(y,y,n) + a) %n;
        if( x==y ) {
            g = n;
            break;
        }
        g = gcd(dif(x,y), n);
    }
    if( g==n ) // unluck try again
        pollard_rho(n, facs);
    else if( g>1ull ) { // luck, found g
        pollard_rho(g, facs);
        pollard_rho(n/g, facs);
    }
}

```

1.7 Linear Algebra

```

#ifndef _MATRIX_H_
#define _MATRIX_H_

#include <iostream>
#include <vector>
using namespace std;

template <class T>
class Matrix{
public:
    int rSize, cSize;
    vector< vector<T> > mat;

    Matrix(int r, int c) :rSize(r), cSize(c), mat(rSize,
        vector<T>(cSize)){}
    vector<T>& operator[](int i) {
        return mat[i];
    }
    void print();
};

template <class T>
void Matrix<T>::print() {
    cout << "Matrix elements:" << endl;
    for (int i = 0; i < rSize; i++) {
        cout << "[";
        for (int j = 0; j < cSize; j++) {
            cout << "\t" << mat[i][j];
            if (j != cSize - 1)cout << ",";
        }
        cout << "]" << endl;
    }
}

#endif

#include "Matrix.h"

template <class T>
Matrix<T> matMul(Matrix<T> matA, Matrix<T> matB){
    Matrix<T> matRe(matA.rSize, matB.cSize);

    for (int i = 0; i < matRe.rSize; i++) {
        for (int j = 0; j < matRe.cSize; j++) {
            matRe[i][j] = 0;
            for (int k = 0; k < matA.cSize; k++) {
                matRe[i][j] += matA[i][k] * matB[k][j];
            }
        }
    }
    return matRe;
}

```

1.8 FFT

```

#include <complex>
#include <vector>
using namespace std;

const double PI = 3.141592654;
typedef complex<double> Complex;

void _fft(vector<Complex>& buf, vector<Complex>& out,
    int st, int step, bool isInv) {

    if (step >= buf.size()) return;

    _fft(out, buf, st, step * 2, isInv);
    _fft(out, buf, st + step, step * 2, isInv);

    int n = buf.size();
    double c = isInv ? 1.0 : -1.0;
}

```

```

for (int i = 0; i < n; i += 2 * step) {
    Complex t = polar(1.0, c * 2 * PI * i / n) * out[i +
step + st];
    buf[i / 2 + st] = out[i + st] + t;
    buf[(i + n) / 2 + st] = out[i + st] - t;
}
}

void fft(vector<Complex> &x, bool isInv) {
    int n = x.size(), nxt2 = 0;
    for (int i = 0, mask = 1; i < 31; i++, mask <= 1)
        nxt2 = (n&mask) ? (n != mask) ? 1 << (i + 1) : 1 << i
            : nxt2;
    n = nxt2;
    while (x.size() < n)
        x.push_back(0);

    vector<Complex> out = x;
    _fft(x, out, 0, 1, isInv);
    for (int i = 0; isInv && i < x.size(); i++)
        x[i] /= n;
}

```

2 String

2.1 KMP

```

class kmp{
private:
    int prefix[maxLen];
    char pat[maxLen];
public:
    void setPattern(const char *str){
        strcpy(pat, str);
        prefix[0] = -1;
        int i=1, j=0;
        for( ; str[i]!='\0' ; ++i, ++j){
            if( str[i]==str[j] )
                prefix[i] = prefix[j];
            else
                prefix[i] = j;
            while( j>=0 && str[j]!=str[i] )
                j = prefix[j];
        }
        prefix[i] = j;
    }
    int search(const char *str){
        // return index of str match pattern
        int i=0, j=0;
        for( ; str[i]!='\0' && pat[j]!='\0' ; ++i, ++j){
            while( j>=0 && pat[j]!=str[i] )
                j = prefix[j];
        }
        if( pat[j]=='\0' )
            return i-j;
        return -1;
    }
    int countMatched(const char *str){
        // return # of pattern in str
        int cnt = 0;
        int i=0, j=0;
        while( true ){
            if( pat[j]=='\0' ) ++cnt;
            if( str[i]=='\0' ) break;
            while( j>=0 && pat[j]!=str[i] )
                j = prefix[j];
            ++i, ++j;
        }
        return cnt;
    }
};

```

2.2 LPS

```

int lps(const char *str){
    // return len of longest palindrom substring
    static char emptyChar = '@';
    static char tmp[maxLen*2];
    static int lprb[maxLen*2];
    // [i-lprb[i], i+lprb[i]] is the lps when mid is i
    for(int i=0, j=-1; true; ++i){
        if( str[i]!='\0' ){
            tmp[++j] = emptyChar;
            tmp[++j] = '\0';
            break;
        }
        tmp[++j] = emptyChar;
        tmp[++j] = str[i];
    }
    lprb[0] = 0;
    int rightBorder = 0, midId = 0;
    for(int i=1; tmp[i]!='\0'; ++i){
        if( i>rightBorder ){
            rightBorder = i;
            midId = i;
            lprb[i] = 0;
        }
    }
}

```

```

int mirId = midId - (i-midId);
if( i+lprb[mirId] > rightBorder )
    lprb[i] = rightBorder - i;
else if( i+lprb[mirId] < rightBorder )
    lprb[i] = lprb[mirId];
else{
    int j=lprb[mirId];
    while( tmp[i+j]!='\0' && i-j>=0 && tmp[i+j]==tmp[i-j] )
        ++j;
    rightBorder = i+j-1;
    midId = i;
    lprb[i] = j-1;
}
}
int ans = 1;
for(int i=0 ; tmp[i]!='\0' ; ++i)
    if( lprb[i]>ans )
        ans = lprb[i];
return ans;
}

```

2.3 AC Automation

```

#include <queue>
#include <cstdio>
#include <cstring>
using namespace std;
struct AC_algorithm {
    struct node {
        static const int signNum = 52; //number of kind of character
        node *ch[signNum];
        node *suffix, *dict;
        int index;
        node() {
            memset(ch, 0, sizeof(ch));
            suffix = dict = 0;
            index = -1;
        }
    };
};

static const int stringNum = 100010; //number of pattern
node *root;
int occur[stringNum]; //string i occur occur[i] times
int reflect[stringNum]; //string i is the same as string reflect[i];

AC_algorithm() {
    root = new node();
    memset(occur, 0, sizeof(occur));
    memset(reflect, -1, sizeof(reflect));
}

int decode(char c) { //decode char
    return c <= 'Z' ? (c - 'A') : (c - 'a' + 26);
}

void insert(char *s,int index) { //add string to trie
    node *p = root;
    for(; *s; s++) {
        int code = decode(*s);
        if(p->ch[code] == NULL)
            p->ch[code] = new node();
        p = p->ch[code];
    }
    if(p->index == -1)
        p->index = index;
    else
        reflect[index] = p->index;
}

void build() { //build machine
    queue<node*> q;

```

```

q.push(root);
while(!q.empty()) {
    node *p = q.front();
    for(int i = 0; i < node::signNum; i++)
        if(p->ch[i]) {
            node *tmp = p->suffix;
            while(tmp && !tmp->ch[i]) tmp = tmp->suffix;
            if(tmp)
                p->ch[i]->suffix = tmp->ch[i];
            else
                p->ch[i]->suffix = root;
            tmp = p->ch[i]->suffix;
            if(tmp->index != -1)
                p->ch[i]->dict = tmp;
            else
                p->ch[i]->dict = tmp->dict;
            q.push(p->ch[i]);
        }
    q.pop();
}
}

```

```

void match(char *s) { //match patterns with Text
    node *p = root;
    for(; *s; s++) {
        int code = decode(*s);
        while(p && !p->ch[code]) p = p->suffix;
        if(p)
            p = p->ch[code];
        else
            p = root;
        node *tmp = p;
        while(tmp) {
            if(tmp->index != -1)
                occur[tmp->index]++;
            tmp = tmp->dict;
        }
    }
}

~AC_algorithm() {
    queue<node*> q;
    q.push(root);
    while(!q.empty()) {
        node *p = q.front();
        q.pop();
        for(int i = 0; i < node::signNum; i++)
            if(p->ch[i])
                q.push(p->ch[i]);
        delete p;
    }
}
};

```

3 Tree

3.1 Tree Min Vertex Cover

```
class TreeMinVertexCover {
private:
    static const int maxNum = 100004;
    vector<int> G[maxNum];
    int in[maxNum];
public:
    bool pick[maxNum];
    int MVC; // min vertex cover
    void init() {
        for(int i=0; i<maxNum; ++i)
            G[i].clear();
        memset(in, 0, sizeof(in));
    }
    void addEdge(int u, int v) {
        G[u].emplace_back(v);
        G[v].emplace_back(u);
        ++in[u];
        ++in[v];
    }
    int treeMinVertexCover() {
        memset(pick, 0, sizeof(pick));
        MVC = 0;
        queue<int> myQ;
        for(int i=1; i<=maxNum; ++i)
            if( in[i]==1 ) myQ.push(i);
        while( myQ.size() ) {
            int nowAt = myQ.front();
            myQ.pop();
            if( in[nowAt]==0 ) continue;
            ++MVC;
            int id;
            for(int i=0; i<G[nowAt].size(); ++i)
                if( in[G[nowAt][i]] ) {
                    id = G[nowAt][i];
                    break;
                }
            for(int i=0; i<G[id].size(); ++i)
                if( in[G[id][i]] ) {
                    --in[G[id][i]];
                    --in[id];
                    if( in[G[id][i]]==1 )
                        myQ.push(G[id][i]);
                }
        }
        return MVC;
    }
};
```

4 Graph

4.1 Biconnected Components

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <stack>
#include <vector>
using namespace std;
//Biconnected Components
struct BCC {
    static const int maxNum = 1010;
    vector<int> G[maxNum], bccGroup[maxNum];
    int Node;
    int bcc_cnt;
    int timeStamp;
    int low[maxNum];
    int visit[maxNum];
    int bcc[maxNum];
    bool is_ap[maxNum];
    stack< pair<int,int> > S;
    BCC(int Node) {
        for(int i = 0; i < maxNum; i++) {
            G[i].clear();
            bccGroup[i].clear();
            low[i] = visit[i] = bcc[i] = -1;
            is_ap[i] = false;
        }
        this->Node = Node;
        bcc_cnt = 0;
    }
    void DFS(int u,int parent) {
        int children = 0;
        low[u] = visit[u] = timeStamp++;
        for(int i = 0; i < G[u].size(); i++) {
            int v = G[u][i];
            if(visit[v] == -1) {
                S.push(make_pair(u, v));
                children++;
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if(low[v] >= visit[u]) {
                    is_ap[u] = true;
                    pair<int,int> e;
                    do {
                        e = S.top();
                        if(bcc[e.first] != bcc_cnt) {
                            bccGroup[bcc_cnt].push_back(e.first);
                            bcc[e.first] = bcc_cnt;
                        }
                        if(bcc[e.second] != bcc_cnt) {
                            bccGroup[bcc_cnt].push_back(e.second);
                            bcc[e.second] = bcc_cnt;
                        }
                    } while(S.pop());
                }while(e.first!=u || e.second!=v);
                bcc_cnt++;
            }
            else if(v != parent) {
                S.push(make_pair(u, v));
                low[u] = min(low[u], visit[v]);
            }
        }
        if(u == parent) // u is root
            is_ap[u] = (children >= 2);
    }
    void articulation_vertex() {
        timeStamp = 0;
        for(int i = 0; i < Node; i++)
            if(low[i] == -1)
                DFS(i, i);
    }
};
```

4.2 2 SAT

```
class TwoSAT {
private:
    static const int maxN = 100004;
    static const int size = 2*maxN + 4;
    bool pick[size];
    vector<int> G [size];

    int id(int i, int T) { return (i<<1) + T; }
    int alter(int i) { return i^1; }
    bool dfsTry(int nowAt, vector<int> &stk) {
        if( pick[alter(nowAt)] )
            return false;
        stk.emplace_back(nowAt);
        pick[nowAt] = true;
        for(auto v : G[nowAt]) {
            if( !pick[v] && !dfsTry(v, stk) )
                return false;
        }
        return true;
    }
public:
    void init() {
        memset(pick, 0, sizeof(pick));
        for(int i=0; i<size; ++i)
            G[i].clear();
    }
    void addClause(bool TA, int A, bool TB, int B) {
        // Add clause (TA + TB)
        // When TA not true, TB must true. vise versa.
        G[id(A, !TA)].emplace_back(id(B, TB));
        G[id(B, !TB)].emplace_back(id(A, TA));
    }
    bool solve() {
        // O(n) solve
        memset(pick, 0, sizeof(pick));
        for(int i=0; i<maxN; ++i) {
            if( pick[id(i, 0)] || pick[id(i, 1)] )
                continue;
            vector<int> stk;
            if( dfsTry(id(i, 0), stk) )
                continue;
            for(auto v : stk)
                pick[v] = false;
            if( !dfsTry(id(i, 1), stk) )
                return false;
        }
        return true;
    }
    bool T(int i) {
        // should solve() first
        return pick[id(i, 1)];
    }
};
```

4.3 System of Difference Constraints

```
class System_of_DifConstrain {
private:
    static const int maxN = 504;
    static const int maxM = 3004;
    struct Edge {
        int s, t;
        long long cost;
    };
    Edge es[maxM];
    int eSize;
public:
    bool solvable;
    long long x[maxN]; // one solution
    void init() {
        eSize = -1;
    }
```

```

    }
    void addConstrain(int xI, int xJ, long long c) {
        // add xi - xj <= c
        es[++eSize] = {xJ, xI, c};
    }
    bool solve(int n=maxN) {
        // n is max # of node of CC
        memset(x, 0, sizeof(x));
        for(int i=0; i<n; ++i)
            for(int j=0; j<=eSize; ++j)
                if( x[es[j].s] + es[j].cost < x[es[j].t] )
                    x[es[j].t] = x[es[j].s] + es[j].cost;
        for(int j=0; j<=eSize; ++j)
            if( x[es[j].s] + es[j].cost < x[es[j].t] )
                return solvable = false;
        return solvable = true;
    }
};
```

4.4 Bipartite: MaxMatch, Min-VerCover, MaxIndSet

```
class Bipartite {
private:
    static const int MaxNum = 1004;
    vector<int> g[MaxNum];
    bool visited[MaxNum];

    bool bipart(int nowAt, int nowSide) {
        visited[nowAt] = true;
        side[nowAt] = nowSide;
        for(auto &id : g[nowAt])
            if( !visited[id] )
                bipart(id, !nowSide);
            else if( side[id]==nowSide )
                return false;
        return true;
    }
    bool maxMatch(int nowAt) {
        visited[nowAt] = true;
        for(auto &id : g[nowAt])
            if( cp[id]==-1
                || (!visited[cp[id]] && maxMatch(cp[id])) ) {
                cp[id] = nowAt;
                cp[nowAt] = id;
                return true;
            }
        return false;
    }
    void minVertexCover(int nowAt) {
        MVC[nowAt] = 1;
        for(auto &id : g[nowAt])
            if( !MVC[id] ) {
                MVC[id] = 1;
                minVertexCover(cp[id]);
            }
    }
    void maxIndependentSet(int nowAt) {
        MIS[nowAt] = 1;
        for(auto &id : g[nowAt])
            if( !MIS[cp[id]] )
                maxIndependentSet(cp[id]);
    }
public:
    int matchNum; // max match num
    int cp [MaxNum]; // id and cp[id] is couple
    bool side[MaxNum]; // left/right side
    bool MVC [MaxNum]; // min vertex cover
    bool MIS [MaxNum]; // max indepent set
    void addEdge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
    void init() {
```

```

    for(int i=0; i<MaxNum; ++i)
        g[i].clear();
}
bool countAll() {
    // if graph is not bipartite return false

    // bipartite
    memset(side, 0, sizeof(side));
    memset(visited, 0, sizeof(visited));
    for(int i=0; i<MaxNum; ++i)
        if( !visited[i] && !bipart(i, 0) )
            return false;

    // maximum match
    // O(VE), this code can be more optimized
    // alternative: dinic O(V^0.5*E)
    matchNum = 0;
    memset(cp, -1, sizeof(cp));
    for(int i=0; i<MaxNum; ++i){
        if( cp[i]!=-1 ) continue;
        memset(visited, 0, sizeof(visited));
        if( maxMatch(i) )
            ++matchNum;
    }

    // min vertex cover
    memset(MVC, 0, sizeof(MVC));
    for(int i=0; i<MaxNum; ++i)
        if( side[i]==1 && cp[i]==-1 )
            minVertexCover(i);
    for(int i=0; i<MaxNum; ++i)
        if( side[i]==1 )
            MVC[i] = !MVC[i];

    // max independent set
    memset(MIS, 0, sizeof(MIS));
    for(int i=0; i<MaxNum; ++i)
        if( cp[i]==-1 )
            maxIndependentSet(i);
    for(int i=0; i<MaxNum; ++i)
        if( side[i]==1 && cp[i]!=-1
            && !MIS[i] && !MIS[cp[i]] )
            MIS[i] = 1;

    return true;
}
};

```

4.5 Min Vertex Cover

```

struct MinVertexCover {
private:
    static const int MaxNum = 54;
    vector<int> G[MaxNum];
    int in[MaxNum];

    int undo(vector<int> &record) {
        for(int i=0; i<record.size(); ++i)
            ++in[record[i]];
        record.clear();
    }

    int delNode(int u, vector<int> &record) {
        for(int i=0; i<G[u].size(); ++i)
            if( in[G[u][i]] ) {
                --in[G[u][i]];
                --in[u];
                record.push_back(G[u][i]);
                record.push_back(u);
            }
    }

    int cnt(int from, int *visited, bool type) {
        if( visited[from] ) return 0;
        if( type==1 ) visited[from] = 1;

```

```

        for(int i=0; i<G[from].size(); ++i)
            if( in[G[from][i]] && !visited[G[from][i]] )
                return type+cnt(G[from][i], visited, !type);
        return type;
    }

    int cnt(int *visited) {
        int ret = 0;
        for(int i=0; i<MaxNum; ++i)
            if( in[i]==1 && !visited[i] )
                ret += cnt(i, visited, 0);
        for(int i=0; i<MaxNum; ++i)
            if( in[i]==2 && !visited[i] )
                ret += cnt(i, visited, 0);
        return ret;
    }

public:
    int MVCPick[MaxNum];
    int MVC; // min vertex cover
    void init() {
        for(int i=0; i<MaxNum; ++i)
            G[i].clear();
        memset(in, 0, sizeof(in));
    }

    void addEdge(int u, int v) {
        G[u].push_back(v);
        G[v].push_back(u);
        ++in[u];
        ++in[v];
    }

    void minVertexCover(int nowMVC=0, const int *lastPick=
        NULL) {
        // O(n^2 * 1.38^n)
        int nowPick[MaxNum] = {};
        if( nowMVC==0 ) {
            MVC = MaxNum;
            memset(MVCPick, 0, sizeof(MVCPick));
        }
        else memcpy(nowPick, lastPick, sizeof(nowPick));

        int maxid = 0;
        for(int i=0; i<MaxNum; ++i)
            if( in[i]>in[maxid] )
                maxid = i;
        if( in[maxid]<=2 ) {
            nowMVC += cnt(nowPick);
            if( nowMVC<MVC ) {
                MVC = nowMVC;
                memcpy(MVCPick, nowPick, sizeof(nowPick));
            }
            return;
        }

        vector<int> record;

        delNode(maxid, record);
        nowPick[maxid] = 1;
        minVertexCover(nowMVC+1, nowPick);
        nowPick[maxid] = 0;
        undo(record);

        int cnt = 0;
        for(int i=0; i<G[maxid].size(); ++i)
            if( in[G[maxid][i]] ) {
                ++cnt;
                delNode(G[maxid][i], record);
                nowPick[G[maxid][i]] = 1;
            }
        minVertexCover(nowMVC+cnt, nowPick);
        undo(record);
    }
};

```


5 Flow

5.1 Dinic Maxflow Mincut

```

class Dinic{
private:
    static const int maxN = 104;
    static const int infF = 1023456789;
    int cap [maxN][maxN];
    int pipe[maxN][maxN];
    vector<int> g[maxN];
    bool sside[maxN];

    int level[maxN];
    bool bfsLabeling(int s, int t){
        memset(level, 0, sizeof(level));
        queue<int> myQ;
        myQ.push( s );
        level[s] = 1;
        while( !myQ.empty() ){
            int nowAt = myQ.front();
            myQ.pop();
            for(int i=0; i<g[nowAt].size(); ++i)
                if( !level[g[nowAt][i]] && pipe[nowAt][g[nowAt][i]]
                ){
                    level[g[nowAt][i]] = level[nowAt] + 1;
                    myQ.push( g[nowAt][i] );
                }
        }
        return level[t];
    }
    int dfsFindRoute(int nowAt, int t, int maxC) {
        if( nowAt==t ){
            maxFlow += maxC;
            return maxC;
        }
        for(int i=0; i<g[nowAt].size(); ++i) {
            int next = g[nowAt][i];
            if( level[next] != level[nowAt]+1 ) continue;
            if( !pipe[nowAt][next] ) continue;
            int nowOut = dfsFindRoute(next, t, min(maxC, pipe[
nowAt][next]));
            if( nowOut==0 )
                continue;
            pipe[nowAt][next] -= nowOut;
            pipe[next][nowAt] += nowOut;
            return nowOut;
        }
        return 0;
    }
    void dfsFindMinCut(int nowAt) {
        sside[nowAt] = 1;
        for(auto v : g[nowAt])
            if( !sside[v] && pipe[nowAt][v] )
                dfsFindMinCut(v);
    }

public:
    int maxFlow;
    vector<pii> minCut;

    void init(){
        memset(cap, 0, sizeof(cap));
        memset(pipe, 0, sizeof(pipe));
        memset(sside, 0, sizeof(sside));
        for(int i=0; i<maxN; ++i)
            g[i].clear();
        maxFlow = 0;
        minCut.clear();
    }
    void addEdge(int u, int v, int c) {
        if( u==v ) return;
        if( cap[u][v]==0 )
            g[u].emplace_back(v);
        cap[u][v] += c;
    }
}

void coculAll(int s, int t) {
    memcpy(pipe, cap, sizeof(pipe));

    // max flow
    while( bfsLabeling(s,t) )
        while( dfsFindRoute(s,t,infF) )
            ;

    // min cut
    dfsFindMinCut(s);
    for(int u=0; u<maxN; ++u)
        if( sside[u] )
            for(auto v : g[u])
                if( !sside[v] )
                    minCut.push_back({u, v});
}

```

6 Geometry

6.1 Geometry basic

```

struct Point{
    double x,y;
    Point(double xi=0.0,double yi=0.0){
        x = xi , y = yi;
    }
    Point operator - (const Point &r)const{
        return Point(x-r.x , y-r.y);
    }
};
typedef Point Vector;

double angle(const Vector &v,const Vector &u){
    // return rad [0, pi] of two vector
    return acos( dot(v,u)/len(v)/len(u) );
}
Vector rotate(const Vector &v,double rad){
    return Vector(
        v.x*cos(rad) - v.y*sin(rad),
        v.x*sin(rad) + v.y*cos(rad)
    );
}
double pointSegLen(const Point &A,const Point &B,const
    Point &Q){
    if(A==B) return len(Q-A);
    if( dot(B-A , Q-A)<0 ) return len(Q-A);
    if( dot(B-A , Q-B)>0 ) return len(Q-B);
    return fabs( cross(B-A , Q-A) ) / len(B-A);
}
bool pointOnSeg(const Point &A,const Point &B,const Point
    &Q){
    return fabs( len(Q-B)+len(Q-A)-len(A-B) ) < 1e-9;
}

struct Line{
    Point P0;
    Vector v;
    Line(const Point &pi,const Vector &vi):p0(pi) , v(vi)
    {}
};

double pointLineLen(const Line &L,const Point &Q){
    return fabs( cross(L.v , Q-L.P0) ) / len(L.v);
}
Point projectToLine(const Line &L,const Point &Q){
    double t = dot(Q-L.P0 , L.v) / dot(L.v , L.v);
    return L.P0 + L.v * t;
}

Point innerCircle(point &p1, point &p2, point &p3){
    // p1,p2,p3 should not on same line
    double a1 = (-2*p1.x + 2*p2.x);
    double b1 = (-2*p1.y + 2*p2.y);
    double c1 = (p2.x*p2.x + p2.y*p2.y - p1.x*p1.x - p1.y*
        p1.y);
    double a2 = (-2*p1.x + 2*p3.x);
    double b2 = (-2*p1.y + 2*p3.y);
    double c2 = (p3.x*p3.x + p3.y*p3.y - p1.x*p1.x - p1.y*
        p1.y);
    double cx = (c1*b2-c2*b1) / (a1*b2-a2*b1);
    double cy = (a1*c2-a2*c1) / (a1*b2-a2*b1);
    return Point(cx, cy);
}
Point outerCircle(point &p1, point &p2, point &p3) {
    // p1,p2,p3 should not on same line
    double x1 = (p1.x+p2.x)/2.0;
    double y1 = (p1.y+p2.y)/2.0;
    double x2 = (p2.x+p3.x)/2.0;
    double y2 = (p2.y+p3.y)/2.0;
    double vx = p2.x-p1.x;
    double vy = p2.y-p1.y;
    double ux = p3.x-p2.x;

```

```

    double uy = p3.y-p2.y;
    double A = vx*x1 + vy*y1;
    double B = ux*x2 + uy*y2;
    double cx = (uy*A - vy*B) / (uy*vx - ux*vy);
    double cy = (ux*A - vx*B) / (ux*vy - uy*vx);
    return Point(cx, cy);
}

```

6.2 Minimal Enclose Disk

```

struct Circle {
    Point c;
    double R2; // square of radius
    Circle() {}
    Circle(const Point &p1, const Point &p2) {
        c.x = (p1.x+p2.x)/2.0;
        c.y = (p1.y+p2.y)/2.0;
        R2 = dot(p1-p2, p1-p2)/4.0;
    }
    Circle(const Point &p1, const Point &p2, const Point &
        p3) {
        // p1, p2, p3 should not on same line
        c = outerCircle(p1, p2, p3);
        double dx = p1.x - c.x;
        double dy = p1.y - c.y;
        R2 = dx*dx + dy*dy;
    }
    bool contain(const Point &p) const {
        double dx = c.x - p.x;
        double dy = c.y - p.y;
        return fdif(dx*dx + dy*dy - R2)<=0;
    }
};

Circle minEncloseDisk(vector<Point> &ps) {
    // Find minimal circal enclose all point
    // worst case O(n^3), expected O(n)
    Circle D;
    if( ps.size()==0 ) return D;
    if( ps.size()==1 ) {
        D.c = ps[0];
        D.R2 = 0.0;
        return D;
    }

    random_shuffle(ps.begin(), ps.end());
    D = Circle(ps[0], ps[1]);
    for(int i=2; i<ps.size(); ++i) {
        if( D.contain(ps[i]) )
            continue;
        D = Circle(ps[i], ps[0]);
        for(int j=1; j<i; ++j) {
            if( D.contain(ps[j]) )
                continue;
            D = Circle(ps[i], ps[j]);
            for(int k=0; k<j; ++k) {
                if( D.contain(ps[k]) )
                    continue;
                D = Circle(ps[i], ps[j], ps[k]);
            }
        }
    }
}

```

6.3 2D Convex Hull

```

bool turnLeft(const Vector &v1, const Vector &v2) {
    return fdif(cross(v1, v2)) > 0LL;
}
vector<Point> convexHull(vector<Point> &ps) {
    // return convex hull without redundant point
    sort(ps.begin(), ps.end());

    vector<Point> up;

```

```
for(int i=0; i<ps.size(); ++i) {
    while( up.size()>1
        && !turnLeft(up.back()-up[up.size()-2],
            ps[i]-up.back()) )
        up.pop_back();
    up.emplace_back(ps[i]);
}

vector<Point> btn;
for(int i=ps.size()-1; i>=0; --i) {
    while( btn.size()>1
        && !turnLeft(btn.back()-btn[btn.size()-2],
            ps[i]-btn.back()) )
        btn.pop_back();
    btn.emplace_back(ps[i]);
}

vector<Point> res(up);
res.insert(res.end(), btn.begin()+1, btn.end());
res.pop_back();
return res;
}
```