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set nocompatible
set enc=utf-8
set fenc=utf-8
set tabstop=4
set softtabstop=4
set shiftwidth=4
set backspace=2
set autoindent
set cindent
syntax on
set t_Co=256
set number
set showmatch
autocmd FileType cpp nnoremap <F9> :w <bar> :! g++ % -std=c++11 -
 02 -Wall && ./a.out<CR>
```

## 1 Java

```
class Scan {
  BufferedReader buffer;
  StringTokenizer tok;
  Scan() {
    buffer = new BufferedReader(new InputStreamReader(System.in))
  boolean hasNext() {
    while(tok == null || !tok.hasMoreElements())
         tok = new StringTokenizer(buffer.readLine());
       } catch (Exception ex) {
         return false;
    return true;
  String next() {
    if(hasNext())
       return tok.nextToken():
    return null;
  String nextLine() {
    if(hasNext())
      return tok.nextToken("\n");
    return null;
  int nextInt() {
    return Integer.parseInt(next());
}
/* Compile: javac %
 * Run: java [Class name] */
import java.util.*;
import java.lang.*;
import java.math.*;
class Main {
    public static void main (String[] args) {
         System.out.print(1);
         System.out.print(2);
System.out.println("Hello World");
         System.out.printf("%.2f", 0.12345);
         Scanner sc = new Scanner(System.in);
System.out.println(sc.nextLine()); //gets()
         System.out.println(sc.next()); //scanf("%s")
         System.out.println(sc.nextInt());
         System.out.println(sc.nextDouble());
         while(sc.hasNext()) { //EOF
              int a = sc.nextInt();
              System.out.println(a);
         }
         int[] a = {1,2,3};
int[][] b = {{1,2},{3,4,5}};
double[] c = new double[90];
         System.out.print(b[0][1]);
         System.out.print(b[1][2]);
         int[] d = {5,2,1,3,4};
         Integer[] e = {6,3,4,1,2};
         Arrays.sort(d);
         Arrays.sort(e, new MyCom());
for(int i=0; i<d.length; i++) {</pre>
              System.out.print(d[i]);
         for(int i=0; i<e.length; i++) {</pre>
              System.out.print(e[i]);
         Set<String> s = new HashSet<String>(); //or TreeSet
         s.add("123");
         s.add("234");
         System.out.println(s);
         System.out.println(s.contains("123"));
         Map<String, Integer> m = new TreeMap<String, Integer>();
m.put("haha", 123);
m.put("hehe", 234);
         System.out.println(m);
         BigInteger b1 = new BigInteger("-1231237182379123712");
         BigInteger b2 = BigInteger.valueOf(234);
         System.out.println(b1.add(b2));
         System.out.println(b1.mod(b2));
```

```
int z = Integer.parseInt("-123");
    System.out.println(z);

    System.out.println(Math.PI);
    System.out.println(Math.sin(1));
}

static class MyCom implements Comparator<Integer> {
    public int compare(Integer i1, Integer i2) {
        return i2 - i1;
    }
}
```

## 2 Math

## 2.1 Math Basic

```
vector<pii> primeFac(int n) {
  vector<pii> ret;
  for(int i=2; n>1; ++i){
    if( n%i != 0 ) continue;
    int e = 0;
    while( n%i == 0 ) ++e , n/=i;
    ret.push_back({i, e});
  return ret;
long long fastPow(long long x, int n, long long m){
  long long ans = 1LL;
  while( n ){
    if( n&1 ) ans = ans * x % m;
x = x*x % m;
    n >>= 1;
  return ans;
long long modInv(long long x, long long p){
  return fastPow(x, p-2, p);
long long modInv_euler(long long x, long long m){
  // must be gcd(x,m)==1
// phi is euler function: O(sqrt(x))
  return fastPow(x, phi(m)-1, m);
long long gt(long long a, long long b) {
  // smallest integer greater than a/b
  long long ret = a/b;
if( ret>0 || a%b==0 ) ++ret;
  return ret;
```

#### 2.2 Euler Function

```
int phi(int n){
   // euler function: in [0,n], # of coprime(i, n)
   vector<pii> fac = primeFac(n);
   int num = 1 , m = 1;
   for(auto &p : fac)
      num *= (p.first-1) , m *= p.first;
   return n/m * num;
}
```

#### 2.3 Extended Euclidean

```
pll recur(long long n, long long m) {
    // solve one integer solution of
    // x*n + y*m = gcd(n,m)
    if( n%m == 0 )
        return {0LL, 1LL};
    pll res = recur(m, n%m);
    pll ret = {res.second, res.first - res.second * (n/m)};
    return ret;
}
```

#### 2.4 China Remain Theorm

```
bool china_solvable(vector<pii> &rule) {
  for(int i=0; i<rule.size(); ++i)
  for(int j=1; j<rule.size(); ++j) {
    int gcd = __gcd(rule[i].second, rule[j].second);
    if( rule[i].first%gcd != rule[j].first%gcd )
      return false;
  }
  return true;</pre>
```

```
Interval of the second of
```

# 2.5 Counting

```
const int MaxNum = 1000004;
const int modNum = 1000000009;
long long fac [MaxNum];
long long facIv[MaxNum];
void initFac(){
  fac[0] = facIv[0] = 1LL;
  for(int i=1; i<MaxNum; ++i) {
  fac [i] = fac[i-1]*i % modNum;</pre>
    facIv[i] = modInv(fac[i], modNum);
  }
long long Cnm(int n, int m){
  if( m==0 || n==m ) return 1LL;
  return fac[n]*facIv[m] % modNum *facIv[n-m] % modNum;
long long nBlock_kColor(int n,int k){
   // n different blocks; k different colors
  // use inclusion-exclusion principle
  long long ans = fastPow(k, n, modNum);
  bool del = true;
  for(int i=k-1; i>0; --i, del=!del){

long long now = Cnm(k, i)*fastPow(i, n, modNum) %modNum;
    if( del ) ans = (ans+modNum-now) % modNum;
    else ans = (ans+now) % modNum;
  return ans;
```

## 2.6 Miller Rabin

```
#include <climits>
typedef unsigned long long int ull;
ull bases[20] = { 2ULL, 3ULL, 5ULL, 7ULL, 11ULL, 13ULL, 17ULL, 19ULL, 23
     ULL,29ULL,31ULL,37ULL };
ull fake_mul(ull n, ull m, ull x);
ull fast_pow(ull n, ull p, ull x);
bool is_prime(ull n)
 if (n < 2ULL) return false;</pre>
  for (int tt = 0; tt < 12; tt++) {
    ull a;
    a = bases[tt] % n;
    if (a == 0 || a == 1 || a == n - 1) {
    int t = 0;
    ull u = n - 1ULL;
    while ((u & 1ULL) == 0ULL) u >>= 1, t++;
    ull x = fast_pow(a, u, n);
                                   // x = a ^ u % n:
    if (x == 1ULL || x == (n - 1)) continue;
for (int i = 0; i < t - 1; i++)
      if (ULLONG_MAX / x < x) {
        x = fake_mul(x, x, n);
      else {
        x = x*x%n;
```

```
if (x == 1) return false;
      if (x == n - 1) break;
    if (x == n - 1) continue;
    return false;
  return true;
}
ull fake_mul(ull n, ull m, ull x)
{
  ull re = 0ULL;
  while (m != 0ÚLL) {
    if ((m & 1ULL) != 0ULL) {
      if (ULLONG_MAX - re < n) {</pre>
        ull temp = ULLONG_MAX%x;
        temp += (n - (ULLONG_MAX - re)) % x;
        re = temp%x;
      }
      else {
        re = (re + n) % x;
      }
    if (ULLONG_MAX - n < n) {
      ull temp = ULLONG_MAX%x;
      temp += (n - (ULLONG_MAX - n)) % x;
      n = temp%x:
    else {
      n = n + n\%x;
    m >>= 1;
  return re;
ull fast_pow(ull n, ull p, ull x)
  ull re = 1ULL;
  while (p != OULL) {
    if ((p & 1ULL) != 0ULL) {
      if (ULLONG_MAX / re < n) {</pre>
        re = fake_mul(n, re, x);
      else {
        re = (re*n) % x;
    if (ULLONG_MAX / n < n) {
      n = fake_mul(n, n, x);
    else {
     n = (n*n) % x;
    p >>= 1;
  return re;
// Below is non-extreme version
ull fake_mul(ull n, ull m, ull x) {
  ull re = OULL;
  n %= x, m %= x;
  while('m') {
    if( m&1ULL )
     \dot{r}e = (re+\dot{n}) \% x;
    n = (n+n) \% x;
    m >>= 1;
  return re;
ull fast_pow(ull n, ull p, ull x) {
  ull re = 1ULL;
  while( p ) {
    if( p&1ULL )
    re = fake_mul(re,n,x);
n = fake_mul(n,n,x);
    p >>= 1;
  return re;
bool is_prime(ull n) {
  static const int bNum = 12;
  static const ull bases[bNum] = {
```

```
2ULL, 3ULL, 5ULL, 7ULL, 11ULL, 13ULL, 17ULL, 19ULL, 23ULL, 29ULL, 31ULL
if( n<=2ULL ) return n==2ULL;
if( !(n&1ULL) ) return false;</pre>
ull u = n-1;
while( !(u&1ULL) )
  u >>= 1;
for(int i=0; i<bNum; i++) {</pre>
  if( bases[i]%n == 0 ) continue;
  ull t = u;
  ull a = fast_pow(bases[i], t, n);
  if( a==1 || a==n-1 ) continue;
  while( t!=n-1 && a!=1 && a!=n-1 ) {
    a = fake_mul(a,a,n);
    t <<= 1;
  if( t==n-1 && a==1 ) continue;
  if( a!=n-1 ) return false;
return true:
```

## 2.7 Pollard rho

```
// need fack_mul, is_prime
ull gcd(ull a, ull b) {
  return (a%b==0)? b : gcd(b, a%b);
ull dif(ull a, ull b) {
  return a>b? a-b : b-a;
void pollard_rho(ull n, map<ull,int> &facs) {
  while( !(n&1ull) ) {
    // must extract factor 2
    int cnt = 0;
    while( !(n&lull) )
    ++cnt, n>>=1;
facs[2] = cnt;
  if( n==1ull ) return;
  if( is_prime(n) ) {
    facs[n]++;
    return;
  ull x = rand()%n;
  ull y = x;
  ull a = rand()\%(n-1) + 1;
  ull g = 1ull;
  while( g==1ull ) {
    x = (fake_mul(x,x,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    y = (fake_mul(y,y,n) + a) %n;
    if( x==y ) {
      g = n;
      break;
    g = gcd(dif(x,y), n);
  if( g==n ) // unluck try again
  pollard_rho(n, facs);
else if( g>lull ) { // luck, found g
  pollard_rho(g, facs);
    pollard_rho(n/g, facs);
```

## 2.8 Linear Algebra

```
#ifndef _MATRIX_H_
#define _MATRIX_H_
#include <iostream>
#include <vector>
using namespace std;

template <class T>
class Matrix{
public:
    int rSize, cSize;
    vector< vector<T> > mat;

Matrix(int r = 0, int c = 0) :rSize(r), cSize(c), mat(rSize, vector<T>(cSize)){}
    vector<T> & operator[](int i) {
        return mat[i];
    }
}
```

```
void print();
template <class T>
                                                                                  }
void Matrix<T>::print() {
  cout << "Matrix elements:" << endl;</pre>
  for (int i = 0; i < rSize; i++) {</pre>
                                                                                  int x[4];
    cout << "[";
    for (int j = 0; j < cSize; j++) {
  cout << "\t" << mat[i][j];</pre>
       if (j != cSize - 1)cout << ",";
    cout << " ]" << endl;
#endif
                                                                                  return k;
#include "Matrix.h"
                                                                                2.11
template <class T>
Matrix<T> matMul(Matrix<T> matA, Matrix<T> matB){
  Matrix<T> matRe(matA.rSize, matB.cSize);
                                                                                Requirement:
  for (int i = 0; i < matRe.rSize; i++) {</pre>
    for (int j = 0; j < matRe.cSize; j++) {
  matRe[i][j] = 0;</pre>
                                                                                     same
       for (int k = 0; k < matA.cSize; k++) +</pre>
         matRe[i][j] += matA[i][k] * matB[k][j];
                                                                                Solution:
    }
  return matRe;
2.9
         FFT
const double PI = acos(-1.0);
typedef complex<double> Complex;
void _fft(const Complex *in, Complex *out, const vector<Complex>
     &pls, int step, int size, double dir) {
                                                                                3.1 KMP
    if(size < 1) return;</pre>
    if(size == 1) {
         out[0] = in[0];
                                                                                class kmp{
         return;
                                                                                  private:
    _fft(in, out, pls, step * 2, size / 2, dir);
_fft(in + step, out + size / 2, pls, step * 2, size / 2, dir)
                                                                                  public:
    for(int i = 0; i < size / 2; ++i) {</pre>
         Complex even = out[i];
         Complex odd = out[i + size / 2];
         out[i] = even + pls[i * step] * odd;
out[i + size / 2] = even + pls[(i + size / 2) * step] *
// x.size() must be 2<sup>m</sup>
vector<Complex> fft(const vector<Complex> &x, bool isInv) {
    int n = x.size();
    double dir = isInv ? -1. : 1.;
    vector<Complex> pls(n);
    for(int i=0; i<n; ++i)
         pls[i] = polar(1.0, dir * 2 * PI * i / n);
    vector<Complex> out = x;
    fft(&x[0], &out[0], pls, 1, n, dir);
for(int i = 0; isInv && i < x.size(); i++)
   out[i] /= n;</pre>
    return out;
2.10 Hash
#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
using namespace std;
const int hashRange = 131072;
int hashtable[4][256], shuffleArr[hashRange], ref[hashRange];
void buildHashTable() {
  memset(ref, -1, sizeof(ref));
for(int i = 0; i < hashRange; i++)</pre>
```

shuffleArr[i] = i; for(int i = 0; i < 4; i++) {

```
random_shuffle(shuffleArr, shuffleArr + hashRange);
     for(int j = 0; j < 256; j++)
  hashtable[i][j] = shuffleArr[j];</pre>
int myhash(int input) {
  for(int i = 0; i < 4; i++)
    x[i] = hashtable[i][(input >> (i*8)) & (0xff)];
  int k = x[0] ^ x[1] ^ x[2] ^ x[3];
  if(ref[k] != input)
     for(int i = k; ; i = (i+1)%hashRange)
  if(ref[i] == -1 || ref[i] == input) {
         ref[i] = input;
          return i;
```

#### NIM

```
- Two player take turns
- In the same state, the action both players can take are the
- Game must end in finite turn
- Lose if one player can not move in his/her turn
- Using SG value to represent a game
- SG value of multiple game are the xor of all of them
- SG value = 0 iff first move lose. Vice versa.
Find SG value of each game:
- 0 when no move can be taken in the game
- Find the smallest non negative integer that is not belong to
    "All the SG value after one move"
```

# String

```
int prefix[maxLen];
    char pat[maxLen];
    void setPattern(const char *str){
      strcpy(pat, str);
       prefix[0] = -1;
       int i=1 , j=0;
       for(; str[i]!='\0'; ++i , ++j ){
    if( str[i]==str[j] )
           prefix[i] = prefix[j];
        prefix[i] = j;
while( j>=0 && str[j]!=str[i] )
           j = prefix[j];
      prefix[i] = j;
    int search(const char *str){
       // return index of str match pattern
       int i=0 , j=0;
       for( ; str[i]!='\0' && pat[j]!='\0' ; ++i,++j){
         while( j>=0 && pat[j]!=str[i] )
           j = prefix[j];
       if( pat[j]=='\0' )
        return i-j;
      return -1;
     int countMatched(const char *str){
      // return # of pattern in str
       int cnt = 0;
       int i=0 , j=0;
      while( true ){
        if( pat[j]=='\0' ) ++cnt;
if( str[i]=='\0' ) break;
         while( j>=0 && pat[j]!=str[i] )
           j = prefix[j];
         ++i , ++j;
      return cnt;
};
```

#### 3.2 LPS

```
int lps(const char *str){
  // return len of longest palindrom substring
  static char emptyChar = '@';
  static char tmp[maxLen*2];
  static chait comp[maxLen*2];
static int lprb[maxLen*2];
// [i-lprb[i], i+lprb[i]] is the lps when mid is i
for(int i=0, j=-1; true; ++i){
   if( str[i]=='\0' ){
        tmp[++j] = emptyChar;
        tmp[++j] = '\0';
        break;
     tmp[++j] = emptyChar;
     tmp[++j] = str[i];
  lprb[0] = 0;
  int rightBorder = 0 , midId = 0;
for(int i=1; tmp[i]!='\0'; ++i){
   if( i>rightBorder ){
        rightBorder = i;
        midId = i;
        lprb[i] = 0;
     int mirId = midId - (i-midId);
if( i+lprb[mirId] > rightBorder )
     lprb[i] = rightBorder - i;
else if( i+lprb[mirId] < rightBorder )</pre>
        lprb[i] = lprb[mirId];
     else{
        int j=lprb[mirId];
while( tmp[i+j]!='\0' && i-j>=0 && tmp[i+j]==tmp[i-j])
          ++j;
        rightBorder = i+j-1;
        midId = i;
        lprb[i] = j-1;
    }
  int ans = 1;
for(int i=0; tmp[i]!='\0'; ++i)
     if( lprb[i]>ans )
        ans = lprb[i];
  return ans;
```

## 3.3 AC Automation

```
#include <queue>
#include <cstdio>
#include <cstring>
using namespace std;
struct AC_algorithm {
  struct node {
    static const int signNum = 52; //number of kind of character
    node *ch[signNum];
    node *suffix, *dict;
    int index;
    node() {
       memset(ch, 0, sizeof(ch));
       suffix = dict = 0;
       index = -1;
    }
  };
  static const int stringNum = 100010;//number of pattern
  node *root;
  int occur[stringNum];
                                   //string i occur occur[i] times
  int reflect[stringNum];
                                     //string i is the same as string
     reflect[i];
  AC_algorithm() {
    root = new node();
    memset(occur, 0, sizeof(occur));
memset(reflect, -1, sizeof(reflect));
  int decode(char c) {      //decode char
    return c <= 'Z' ? (c - 'A') : (c - 'a' + 26);</pre>
  void insert(char *s,int index) { //add string to trie
    node *p = root;
    for(; *s; s++) {
       int code = decode(*s);
       if(p->ch[code] == NULL)
  p->ch[code] = new node();
         = p->ch[code];
```

```
if(p->index == -1)
      p->index = index;
    else
       reflect[index] = p->index;
  void build() {
                                //build machine
    queue<node*> q;
    q.push(root);
    while(!q.empty()) {
       node *p = q.front();
for(int i = 0; i < node::signNum; i++)</pre>
         if(p->ch[i]) {
           node *tmp = p->suffix;
while(tmp && !tmp->ch[i]) tmp = tmp->suffix;
           if(tmp)
             p->ch[i]->suffix = tmp->ch[i];
           else
             p->ch[i]->suffix = root;
           tmp = p->ch[i]->suffix;
           if(tmp->index != -1)
             p->ch[i]->dict = tmp;
           else
             p->ch[i]->dict = tmp->dict;
           q.push(p->ch[i]);
      q.pop();
    }
  void match(char *s) {
                                  //match patterns with Text
    node *p = root;
for(; *s; s++) {
       int code = decode(*s);
       while(p && !p->ch[code]) p = p->suffix;
       if(p)
         p = p->ch[code];
       else
        p = root;
       node *tmp = p;
       while(tmp) {
         if(tmp->index != -1)
           occur[tmp->index]++;
         tmp = tmp->dict;
      }
    }
  }
  ~AC_algorithm() {
    queue<node*> q;
    q.push(root);
    while(!q.empty()) {
       node *p = q.front();
       q.pop();
for(int i = 0; i < node::signNum; i++)</pre>
         if(p->ch[i])
           q.push(p->ch[i]);
      delete p;
  }
};
        Ζ
3.4
#include <cstring>
int z[length];
void z_function(char *str) {
  int len = strlen(str), L = 0, R = 1;
  z[0] = len;
for(int i = 1; i <len; i++)
    if(R \le i \mid \mid z[i-L] >= R-i) {
       int x = max(R, i);
       while(x < len && str[x] == str[x-i])</pre>
        x++;
       z[i] = x-i;
      L = i; R = x;
//if(i < x) {L = i; R = x;}
    } else
      z[i] = z[i-L];
```

## 3.5 Suffix Array

```
vector<int> buildSuffixArray(const vector<int> &str, int
    endOfString=-1) {
    // sa: i -> start position of str
    // O(n*lgn*lgn). probably faster than O(n*lgn) version
    int len = str.size();
```

```
vector<int> sa(len+1), rank(len+1);
                                                                                      vector<int> sa(len), rank(len);
     for(int i=0; i<len; ++i) rank[sa[i] = i] = str[i];</pre>
                                                                                      for(int i=0; i<len; ++i) rank[sa[i] = i] = src[i];</pre>
     rank[sa.back() = len] = endOfString;
                                                                                      for(int ll=1, cnt=0; cnt!=len; ll<<=1, cnt=rank[sa.back()]+1)</pre>
     for(int ll=1, cnt=0; cnt!=len; ll<<=1, cnt=rank[sa.back()]) {
  auto cmp = [&](const int l, const int r) {</pre>
                                                                                           auto cmp = [&](const int l, const int r) {
              if( rank[l]!=rank[r] ) return rank[l] < rank[r];</pre>
                                                                                               if( rank[l]!=rank[r] ) return rank[l] < rank[r];</pre>
              int lv = (l+ll < len) ? rank[l+ll] : 0;</pre>
                                                                                               return rank[(l+ll)%len] < rank[(r+ll)%len];</pre>
              int rv = (r+ll < len) ? rank[r+ll] : 0;</pre>
              return lv < rv;</pre>
                                                                                           sort(sa.begin(), sa.end(), cmp);
                                                                                           vector<int> tmp = rank;
                                                                                           tmp[sa[0]] = 0;
         sort(sa.begin(), sa.end(), cmp);
         vector<int> tmp = rank;
                                                                                           for(int i=1; i<sa.size(); ++i)</pre>
         tmp[sa[0]] = 0;
                                                                                               tmp[sa[i]] = tmp[sa[i-1]] + cmp(sa[i-1], sa[i]);
         for(int i=1; i < sa.size(); ++i)</pre>
              tmp[sa[i]] = tmp[sa[i-1]] + cmp(sa[i-1], sa[i]);
         rank = tmp;
                                                                                      vector<int> rst(len);
     }
                                                                                      for(int i=0; i<len; ++i) rst[i] = src[(sa[i]+len-1)%len];</pre>
     return sa;
                                                                                      return rst;
}
vector<int> buildLcp(const vector<int> &str, const vector<int> &
                                                                                 vector<int> btwDecode(const vector<int> &rst) {
                                                                                     int len = rst.size();
vector<pair<int,int> > pre(len);
for(int i=0; i<len; ++i) pre[i] = {rst[i], i};</pre>
     // lcp: longest common prefix for sa[i-1] sa[i]
     int len = sa.size();
    roctor = Sa.Size();
vector<int> lcp(len, 0), idx(len);
for(int i=0; i<len; ++i) idx[sa[i]] = i;
for(int i=0, l=0; i<len; ++i) {
    if( idx[i] == 0 ) {
}</pre>
                                                                                      sort(pre.begin(), pre.end());
                                                                                     vector<int> table(len);
for(int i=0; i<len; ++i) table[pre[i].second] = i;
vector<int> src(len);
for(int i=rst.size()-1, idx=0; i>=0; --i, idx=table[idx])
              l = 0:
                                                                                          src[i] = rst[table[idx]];
              continue:
                                                                                      return src;
         int j = sa[idx[i]-1];
         while( i+l<len && j+l<len && str[i+l]==str[j+l] )</pre>
                                                                                       Tree
         lcp[idx[i]] = l;
         l -= l>0;
                                                                                 4.1 Tree Min Vertex Cover
     return lcp;
}
                                                                                 class TreeMinVertexCover {
pair<int, int> longestRepeatedSubstring(const vector<int> &sa,
                                                                                 private:
      const vector<int> &lcp, int k) {
                                                                                   static const int maxNum = 100004;
     // longest repeated substring who occurs at least k times
                                                                                   vector<int> G[maxNum];
     // return <longest length, start position>
                                                                                   int in[maxNum];
     pair<int,int> ret = {-1, -1};
                                                                                 public:
     if( k<=1 ) {
                                                                                   bool pick[maxNum];
         for(int i=0; i<sa.size(); ++i)</pre>
                                                                                   int MVC; // min vertext cover
              if( sa[i]==0 ) return {sa.size()-1, i};
                                                                                   void init() {
                                                                                      for(int i=0; i<maxNum; ++i)</pre>
     if( lcp.size() < k ) return ret;</pre>
                                                                                        G[i].clear();
     deque<pair<int,int>> dq;
                                                                                     memset(in, 0, sizeof(in));
     auto maintain = [&](pair<int,int> v) {
         while( dq.size() && dq.front().second <= v.second-(k-1) )</pre>
                                                                                   void addEdge(int u, int v) {
              dq.pop_front();
                                                                                     G[u].emplace_back(v);
         while( dq.size() && dq.back() >= v )
                                                                                     G[v].emplace_back(u);
              dq.pop_back();
                                                                                      ++in[u];
         dq.push_back(v);
                                                                                      ++in[v];
     for(int i=0; i<k-2; ++i) maintain({lcp[i], i});</pre>
    for(int i=k-2; i<lcp.size(); ++i) {
    maintain({lcp[i], i});</pre>
                                                                                   int treeMinVertexCover() {
                                                                                      memset(pick, 0, sizeof(pick));
         ret = max(ret, dq.front());
                                                                                      queue<int> myQ;
                                                                                      for(int i=1; i<=maxNum; ++i)</pre>
     return ret;
                                                                                        if( in[i]==1 ) myQ.push(i);
                                                                                      while( myQ.size() )
                                                                                        int nowAt = myQ.front();
pair<int, int> lcs(const vector<int> &s1, const vector<int> &s2,
                                                                                        myQ.pop();
      int e1, int e2) {
                                                                                        if( in[nowAt] == 0 ) continue;
     // Longest Common "Substring" in O(n*lgn*lgn) n = s1+s2
                                                                                        ++MVC;
     vector<int> arr(s1.size()+s2.size()+2);
     int idx = -1;
                                                                                        for(int i=0; i<G[nowAt].size(); ++i)</pre>
    for(int i=0; i<s1.size(); ++i) arr[++idx] = s1[i];
arr[++idx] = e1;
                                                                                          if( in[G[nowAt][i]] ) {
                                                                                             id = G[nowAt][i];
    for(int i=0; i<s2.size(); ++i) arr[++idx] = s2[i];
arr[++idx] = e2;
vector<int> sa = buildSuffixArray(arr);
vector<int> lcp = buildLcp(arr, sa);
                                                                                             break;
                                                                                        for(int i=0; i<G[id].size(); ++i)</pre>
    for(int i=1; i<lcp.size(); ++i)
   if( (sa[i]<=sl.size()) ^ (sa[i-1]<=sl.size()) )</pre>
                                                                                           if( in[G[id][i]] ) {
                                                                                             --in[G[id][i]];
                                                                                             --in[id];
                                                                                             if( in[G[id][i]]==1 )
              ret = max(ret, {lcp[i], sa[i]});
                                                                                               myQ.push(G[id][i]);
     return ret;
                                                                                          }
                                                                                      return MVC;
3.6 BWT
                                                                                   }
```

#### 3.0 DWI

```
vector<int> btwEncode(const vector<int> &src) {
   // 0(n*lgn*lgn). probably faster than 0(n*lgn) version
   int len = src.size():
```

#### 4.2 Treap ordered

};

```
struct node {
  int v , p , sz ;
node *l , *r ;
node() { l = r = NULL ;}
  node(int v_{-}):v(v_{-}),p(rand()),sz(1) \{l = r = 0;\}
  int size() {
    return this!=NULL ? this->sz : 0 ;
  void maintain() {
    sz = l \rightarrow size() + r \rightarrow size() + 1;
void splite_v(node *t,node* &a,node* &b,int v) {
  if(!t)
    a = b = NULL;
  else if(v >= t -> v) {
    a = t;
    splite_v(t->r,a->r,b,v) ;
    a->maintain()
  } else if(v < t \rightarrow v) {
    b = t;
    splite_v(t->l,a,b->l,v) ;
    b->maintain();
}
node* merge(node *a,node *b) {
  if(a==NULL || b==NULL)
return a!=NULL ? a : b;
  if(a->p > b->p) {
    a->r = merge(a->r,b) ;
     a->maintain() ;
     return a ;
  } else if(a->p <= b->p) {
  b->l = merge(a,b->l) ;
     b->maintain();
    return b;
  }
int kth(node *t,int k) {
  if(k<=t->l->size())
    return kth(t->l,k) ;
  else if(k>t->l->size()+1)
    return kth(t->r,k-t->l->size()-1) ;
  return t->v ;
void release(node *t) {
  if(t) {
    release(t->l);
     release(t->r);
     delete t :
```

## 4.3 Treap unordered

#include <iostream>

```
#include <cstdio>
#include <stdlib.h>
#include <cstring>
using namespace std;
const int oo = 1e9 ;
struct node {
  int v , p , sz ;
  int sum , presum , sufsum , maxsum , flag , set ;
  node *l , *r ;
node(){ }
  node(int v_):p(rand()),sz(1),l(NULL),r(NULL) {
    v = sum = presum = sufsum = maxsum = v_ ;
    flag = 0 ;
   set = oo ;
  int size() { return this ? sz : 0 ; }
  int Sum() { return this ? sum : 0 ; }
  int Presum() { return this ? (!flag ? presum : sufsum) : -oo ;
  int Sufsum() { return this ? (!flag ? sufsum : presum) : -oo ;
  int Maxsum() { return this ? maxsum : -oo ;
  int max(int a, int b) { return a > b ? a : b ;
  int max(int a,int b,int c) { return max(a,max(b,c)) ; }
  void makesame(int st) {
   if(this) {
      set = st ;
      sum = st*sz;
      presum = sufsum = maxsum = (st <= 0 ? st : sum) ;</pre>
   }
  void pushdown() {
   if(flag) {
```

```
swap(presum, sufsum) ;
       flag = 0;
     if(set!=oo) {
       l->makesame(set) ;
       r->makesame(set);
       set = oo ;
  void maintain() {
    presum = max(l\rightarrow Presum(), l\rightarrow Sum() + v, l\rightarrow Sum() + v + r\rightarrow
     Presum());
     sufsum = max(r->Sufsum(), r->Sum() + v, r->Sum() + v + l->
     Sufsum());
int maxsum1 = max(l->Maxsum(), r->Maxsum(), v);
int maxsum2 = max(l->Sufsum() + v, r->Presum() + v, l->Sufsum
      () + v + r \rightarrow Presum());
    maxsum = max(maxsum1, maxsum2);
    sum = l -> Sum() + r -> Sum() + v
    sz = 1 + l->size() + r->size() ;
  }
void splite(node *t,node* &a,node* &b,int k) {
  if(t == NULL) {
    a = b = NULL;
    return ;
  t->pushdown();
  if(t->l->size()+1 <= k) {
     splite(t->r, a->r, b, k-(t->l->size()+1));
    a->maintain();
  } else {
    b = t ;
     splite(t->l,a,b->l,k) ;
    b->maintain();
  }
node* merge(node *a,node *b) {
  if(!a || !b)
    return a ? a : b ;
  if(a->p > b->p) {
     a->pushdown();
     a \rightarrow r = merge(a \rightarrow r, b);
     a->maintain();
     return a ;
  } else {
    b->pushdown();
     b->l = merge(a,b->l) ;
    b->maintain();
    return b;
void Delete(node *t) {
  if(!t) return ;
  Delete(t->l) ;
  Delete(t->r)
  delete t ;
void INSERT(node* &root) {
  int p , k , v ;
node *t=0 , *L, *R;
scanf("%d%d",&p,&k) ;
for(int i=0 ; i<k ; i++) {
    scanf("%d",&v) ;
}</pre>
    t = merge(t, new node(v));
  splite(root, L, R, p);
  root = merge(L, merge(t, R)) ;
void DELETE(node* &root) {
  int p , k ;
scanf("%d%d",&p,&k) ;
  node *L, *M, *R;
splite(root, L, R, p-1);
  splite(R, M, R, k);
Delete(M);
  root = merge(L, R) ;
void MAKE_SAME(node* &root) {
  int p , k , l ;
scanf("%d%d%d",&p,&k,&l) ;
  node *L, *M, *R;
splite(root, L, R, p-1);
splite(R, M, R, k);
  M->makesame(l) ;
```

if(l) l->flag = !l->flag ;

if(r) r->flag = !r->flag ;

swap(l,r);

```
root = merge(L, merge(M, R)) ;
void REVERSE(node* &root) {
  int p , k ;
scanf("%d%d",&p,&k) ;
  node *L, *M, *R;
  splite(root, L, R, p-1);
splite(R, M, R, k);
M->flag = !M->flag;
  root = merge(L, merge(M, R));
int GET_SUM(node* &root) {
  int p , k , v ;
scanf("%d%d",&p,&k) ;
node *L, *M, *R ;
splite(root, L, R, p-1) ;
splite(R, M, R, k) ;
  v = M -> Sum();
  root = merge(L, merge(M, R)) ;
  return v ;
int MAX_SUM(node* &root) {
  return root->Maxsum();
int main () {
  int n, m
  srand(860514);
while(scanf("%d%d",&n,&m)==2) {
    node *root=0 ;
    for(int i=0,v ; i<n ; i++) {
  scanf("%d",&v) ;</pre>
       root = merge(root, new node(v)) ;
    while(m--) {
       char s[10] ;
scanf("%s",s) ;
       if(strcmp(s,"INSERT")==0) {
         INSERT(root) ;
         continue;
       if(strcmp(s,"DELETE")==0) {
         DELETE(root) ;
         continue;
       if(strcmp(s,"MAKE-SAME")==0) {
         MAKE_SAME(root) ;
          continue;
       if(strcmp(s,"REVERSE")==0) {
         REVERSE(root) ;
         continue ;
       if(strcmp(s,"GET-SUM")==0) {
         printf("%d\n",GET_SUM(root));
       if(strcmp(s,"MAX-SUM")==0) {
         printf("%d\n",MAX_SUM(root));
         continue:
 }
```

## 4.4 Tree Heavy Light Decomposition

```
#include <iostream>
#include <cstdio>
#include <vector>
#include <string.h>
#define N 50000
using namespace std;
vector<int> G[N];
int ind[N], f[N], d[N], head[N], son[N], pn, n;
int dfs(int u,int parent) {
  int i, v, x, w = 0, sum = 1;
f[u] = parent;
  son[u] = -1;
  for(i = 0; i < G[u].size(); i++) {</pre>
    v = G[u][i];
    if(v != parent) {
      d[v] = d[u] + 1;
      x = dfs(v,u);
      sum += x;
      if(x > w) {
        son[u] = v;
        w = x:
```

```
}
  return sum;
void chain(int u) {
  int i , v;
ind[u] = pn++;
  if(f[u] == -1' \mid \mid son[f[u]]  != u)
    head[u] = u;
    head[u] = head[f[u]];
  if(son[u]!=-1)
    chain(son[u]);
  for(i=0; i<G[u].size(); i++) {
  v = G[u][i] ;</pre>
    if(v != f[u] && v != son[u])
      chain(v);
  }
void modify(int u, int v, ...) {
  while(head[u] != head[v])
    if(d[head[u]] > d[head[v]]) {
       //do something in rang(ind[head[u]], ind[u]);
      u = f[head[u]];
    } else {
       //do something in rang(ind[head[v]], ind[v]);
      v = f[head[v]];
  if(d[u] > d[v])
    //do something in rang(ind[head[u]], ind[u]);
    //do something in rang(ind[head[v]], ind[v]);
int getLca(int u, int v) {
  while(head[u] != head[v])
    if(d[head[u]] > d[head[v]])
      u = f[head[u]];
    else
      v = f[head[v]];
  if(d[u] > d[v])
    return v;
  else
    return u;
void build() {
  pn = 1;
d[0] = 0; //set depth of root to 0
  dfs(0,-1);
  chain(0); //relabel node
```

# 5 Graph

## 5.1 Biconnected Components

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <stack>
#include <vector>
using namespace std;
//Biconnected Components
struct BCC {
  static const int maxNum = 1010;
  vector<int> G[maxNum], bccGroup[maxNum];
  int Node;
  int bcc_cnt;
  int timeStamp;
  int low[maxNum];
  int visit[maxNum];
  int bcc[maxNum];
  bool is_ap[maxNum];
  stack< pair<int,int> > S;
  BCC(int Node) {
    for(int i = 0; i < maxNum; i++) {</pre>
      G[i].clear();
      bccGroup[i].clear();
      low[i] = visit[i] = bcc[i] = -1;
      is_ap[i] = false;
    this->Node = Node;
    bcc_cnt = 0;
  void DFS(int u,int parent) {
    int children = 0;
    low[u] = visit[u] = timeStamp++;
    for(int i = 0; i < G[u].size(); i++) {</pre>
```

```
int v = G[u][i];
       if(visit[v] == -1) {
         S.push(make_pair(u, v));
         children++;
         DFS(v, u);
         low[u] = min(low[u], low[v]);
         if(low[v] >= visit[u]) {
  is_ap[u] = true;
           pair<int,int> e;
           do {
   e = S.top();
              if(bcc[e.first] != bcc_cnt) {
                bccGroup[bcc_cnt].push_back(e.first);
bcc[e.first] = bcc_cnt;
              if(bcc[e.second] != bcc_cnt) {
                bccGroup[bcc_cnt].push_back(e.second);
                bcc[e.second] = bcc_cnt;
           S.pop();
}while(e.first!=u || e.second!=v);
           bcc_cnt++;
      } else if(v != parent) {
         S.push(make_pair(u, v));
low[u] = min(low[u], visit[v]);
    if(u == parent) // u is root
       is_ap[u] = (children >= 2);
  void articulation_vertex() {
    timeStamp = 0;
    for(int i = 0; i < Node; i++)
       i\dot{f}(low[i] = -1)
         DFS(i, i);
  }
};
```

## 5.2 Strong Connected Components

```
class SCC {
private:
     static const int maxN = 10004;
     vector<int> G[maxN];
vector<int> invG[maxN];
     vector<int> stk;
     bool visited[maxN];
     void dfs_1(int nowAt) {
  visited[nowAt] = true;
  for(auto v : G[nowAt])
    if( !visited[v] )
          dfs_1(v);
stk.emplace_back(nowAt);
     void dfs_2(int nowAt, const int id) {
          sccID[nowAt] = id;
for(auto v : invG[nowAt])
    if( sccID[v]==-1 )
                     dfs_2(v, id);
public:
     int sccNum;
     int sccID[maxN];
     void init() {
          for(int i=0; i<maxN; ++i) {
    G [i].clear();</pre>
                invG[i].clear();
     void addEdge(int u, int v) {
          G [u].emplace_back(v);
           invG[v].emplace_back(u);
     vector<vector<int>> findAllSCC(int base, int n) {
          memset(visited, 0, sizeof(visited));
           stk.clear();
           for(int i=base; i<=n; ++i)</pre>
                if( !visitéd[i] )
                     dfs_1(i);
          memset(sccID, -1, sizeof(sccID));
          for(int i=stk.size()-1; i>=0; --i)
    if( sccID[stk[i]]==-1 ) {
                     dfs_2(stk[i], sccNum);
                      ++sccNum;
                }
```

```
// returned zero base scc dag
vector<vector<int>> sccDAG(sccNum);
vector<unordered_set<int>> have(sccNum);
for(int u=base; u<=n; ++u) {
    int sccU = sccID[u];
    for(auto v : G[u]) {
        int sccV = sccID[v];
        if( sccU=sccV ) continue;
        if( have[sccU].find(sccV) == have[sccU].end() ) {
            have [sccU].insert (sccV);
            sccDAG[sccU].emplace_back(sccV);
        }
    }
}
return sccDAG;
}</pre>
```

## 5.3 Strong Connected Components Short

```
const int N = 100000 + 10;
int visit[N], low[N], instack[N], time_stamp = 0;
int sccID[N], sccNum;
int numOfScc[N];
vector<int> G[N];
vector<int> stack;
void scc_dfs(int u) {
    visit[u] = low[u] = ++time_stamp;
    stack.push_back(u);
     instack[u] = true;
     for (int i = 0; i < G[u].size(); i++) {
          int v = G[u][i];
         if (!visit[v])
              scc_dfs(v);
          if (instack[v])
             low[u] = min(low[u], low[v]);
    }
     if (visit[u] == low[u]) {
         int v;
         do {
              v = stack.back();
              stack.pop_back();
instack[v] = false;
              sccID[v] = sccNum;
              numOfScc[sccNum]++;
         } while (v != u);
         sccNum++;
    }
}
// 0 base
void tarjan(int n) {
    memset(visit, 0, sizeof(visit));
     memset(numOfScc, 0, sizeof(numOfScc));
memset(instack, 0, sizeof(instack));
      time_stamp = 0;
      sccNum = 0;
for (int i = 0; i < n; ++i)
         if (!visit[i])
              scc_dfs(i);
```

#### 5.4 2 SAT

```
class TwoSAT {
private:
   int V;
   vector<bool> pick;
   vector< vector<int> > G;

int id(int i, int T) { return (i<<1) + T; }
   int alter(int i) { return i^1; }

bool dfsTry(int nowAt, vector<int> &stk) {
    if( pick[alter(nowAt)] )
       return false;
    stk.emplace_back(nowAt);
    pick[nowAt] = true;
   for(auto v : G[nowAt]) {
       if( !pick[v] && !dfsTry(v, stk) )
            return false;
    }
    return true;
}
```

```
public:
  void init(int varNum) {
    V = varNum;
    pick = vector<bool>(V*2 + 4, false);
    G = vector< vector<int> >(V*2 + 4);
  void addClause(bool TA, int A, bool TB, int B) {
    // Add clause (TA + TB)
    G[id(A, !TA)].emplace_back(id(B, TB));
    G[id(B, !TB)].emplace_back(id(A, TA));
  void imply(bool TA, int A, bool TB, int B) {
    addClause(!TA, A, TB, B);
  void preset(bool TA, int A) {
    pick[id(A, TA)] = true;
  bool solve() {
  vector<int> stk;
    for(int i=0; i<V; ++i) {
  if( pick[id(i, true)] && !dfsTry(id(i, true), stk) )</pre>
       return false;
if( pick[id(i, false)] && !dfsTry(id(i, false), stk) )
         return false;
       stk.clear();
    for(int i=0; i<V; ++i) {
  if( pick[id(i, 0)] || pick[id(i, 1)] )</pre>
         continue
       stk.clear()
       if( dfsTry(id(i, 0), stk) )
         continue;
       for(auto v : stk)
  pick[v] = false;
       if( !dfsTry(id(i, 1), stk) )
         return false;
    return true;
  bool T(int i) {
    // should solved first
    return pick[id(i, 1)];
};
```

# 5.5 k-coloring

```
#include <cstdio>
#include <set>
#include <cstring>
#include <vector>
#include <cassert>
using namespace std;
const int k = 4;
vector<int> G[104];
int lable[104];
int available[104];
bool notok[104][4];
void update_available(int u, int val, set<pair<int,int>> &nds) {
   if( nds.find({available[u], u}) != nds.end() ) {
         nds.erase({available[u], u});
         nds.insert({val, u});
    available[u] = val;
}
bool kcolor(set<pair<int,int>> &nds) {
    if( nds.size() == 0 )
         return true;
    int nowAt = nds.begin()->second;
    nds.erase(nds.begin());
    for(int i=0; i<k; ++i)
         if( notok[nowAt][i] )
            continue;
         bool gg = false;
         lable[nowAt] = i;
         vector<int> affect;
         for(auto u : G[nowAt])
             if( !notok[u][i] )
                  if( available[u]-1 < 1 ) {</pre>
                      gg = true;
break;
                  update_available(u, available[u]-1, nds);
```

notok[u][i] = true;

```
affect.push_back(u);
            if( !gg && kcolor(nds) )
                 return true;
            lable[nowAt] = -1;
            for(auto u : affect) {
                 update_available(u, available[u]+1, nds);
                 notok[u][i] = false;
      nds.insert({available[nowAt], nowAt});
     return false;
int main() {
      int u, v;
      scanf("%d", &N);
     while( scanf("%d-%d", &u, &v) != EOF ) {
    G[u].push_back(v);
           G[v].push_back(u);
     memset(lable, -1, sizeof(lable));
memset(notok, 0, sizeof(notok));
set<pair<int,int>> nodes;
for(int i=1; i<=N; ++i) {
    available[i] = k;
}</pre>
           nodes.insert({available[i], i});
      assert(kcolor(nodes));
      assert(nodes.size() == 0);
for(int i=1; i<=N; ++i)
    printf("%d %d\n", i, lable[i]+1);</pre>
      return 0;
```

## 5.6 System of Difference Constraints

```
class System_of_DifConstrain {
private:
  static const int maxN = 504;
  static const int maxM = 3004;
  struct Edge {
    int s. t:
    long long cost;
  Edge es[maxM]:
  int eSize;
public:
  bool solvable;
  long long x[maxN]; // one solution
void init() {
    eSize = -1;
  void addConstrain(int xI, int xJ, long long c) {
    // add xi - xj <= c
es[++eSize] = {xJ, xI, c};
  bool solve(int n=maxN) {
    // n is max # of node of CC
    memset(x, 0, sizeof(x));
for(int i=0; i<n; ++i)</pre>
    for(int j=0; j<=eSize; ++j)
  if( x[es[j].s] + es[j].cost < x[es[j].t] )</pre>
         x[es[j].t] = x[es[j].s] + es[j].cost;
     for(int j=0; j<=eSize; ++j)</pre>
       if( x[es[j].s] + es[j].cost < x[es[j].t] )</pre>
         return solvable = false;
     return solvable = true;
  }
};
```

# 5.7 Bipartite: MaxMatch, MinVer-Cover, MaxIndSet

```
class Bipartite {
private:
    static const int MaxNum = 1004;
    vector<int> g[MaxNum];
    bool visited [MaxNum];

bool bipart(int nowAt, int nowSide) {
    visited[nowAt] = true;
    side[nowAt] = nowSide;
    for(auto &id : g[nowAt])
        if( !visited[id] )
        bipart(id , !nowSide);
```

```
else if( side[id]==nowSide )
            return false;
      return true;
   bool maxMatch(int nowAt) {
      visited[nowAt] = true;
      for(auto &id : g[nowAt])
         if( cp[id]==-1
            || (!visited[cp[id]] && maxMatch(cp[id])) ){
            cp[id] = nowAt;
cp[nowAt] = id;
            return true;
      return false;
   void minVertexCover(int nowAt) {
      MVC[nowAt] = 1;
      for(auto &id : g[nowAt])
  if( !MVC[id] ) {
    MVC[id] = 1;
            minVertexCover(cp[id]);
   void maxIndependentSet(int nowAt) {
      MIS[nowAt] = 1;
      for(auto &id : g[nowAt])
  if( !MIS[cp[id]] )
            maxIndependentSet(cp[id]);
  int matchNum;  // max match num
int cp [MaxNum]; // id and cp[id] is couple
bool side[MaxNum]; // left/right side
bool MVC [MaxNum]; // min vertex cover
bool MIS [MaxNum]; // max indepent set
void addEdge(int u, int v) {
   g[u].emplace_back(v);
   g[v].emplace_back(v);
public:
      g[v].emplace_back(u);
   void init() {
  for(int i=0; i<MaxNum; ++i)</pre>
         g[i].clear();
   bool countAll() {
      // if graph is not bipartite return false
      memset(side, 0, sizeof(side));
      memset(visited , 0 , sizeof(visited));
for(int i=0 ; i<MaxNum ; ++i)
  if( !visited[i] && !bipart(i, 0) )</pre>
            return false;
      // maximum match
      // O(VE), this code can be more optimized
      // alternative: dinic O(V^0.5*E)
      matchNum = 0;
      memset(cp , -1 , sizeof(cp));
for(int i=0 ; i<MaxNum ; ++i){
  if( cp[i]!=-1 ) continue;</pre>
         memset(visited , 0 , sizeof(visited));
if( maxMatch(i) )
             ÷+matchNum;
      // min vertex cover
      memset(MVC, 0, sizeof(MVC));
      for(int i=0; i<MaxNum; ++i)
if( side[i]==1 && cp[i]==-1 )
            minVertexCover(i);
      for(int i=0; i<MaxNum; ++i)
  if( side[i]==1 )</pre>
            MVC[i] = !MVC[i];
      // max independent set
      memset(MIS, 0, sizeof(MIS));
for(int i=0; i<MaxNum; ++i)
   if( cp[i]==-1 )</pre>
            maxIndependentSet(i);
      for(int i=0; i<MaxNum; ++i)</pre>
         if( side[i]==1 && cp[i]!=-1 && !MIS[i] && !MIS[cp[i]] )
            MIS[i] = 1;
      return true:
};
```

## 5.8 Bipartite: KM

```
#include <cstring>
#include <iostream>
using namespace std;
struct KM {
  static const int N = 105;
   int visx[N], visy[N];
  int nx, ny, matchx[N], matchy[N];
int labelx[N], labely[N], G[N][N];
  bool labeled[N];
  int max_match_value;
  bool isPerfect;
  KM(int nx_,int ny_):nx(nx_),ny(ny_) {};
  bool DFS(int x) {
     visx[x] = true ;
     int y ;
     for(int y = 0; y < ny ; y++)
  if(hasEdge(x, y) && !visy[y] && labelx[x] + labely[y] == G[</pre>
      x][y] ) {
          visy[y] = true ;
if(matchy[y]==-1 || DFS(matchy[y])) {
             matchx[x] = y ;
             matchy[y] = x;
            return true ;
     return false;
  bool max_match() {
                             //Maximum Weight Perfect Bipartite
      Matching
     isPerfect = true;
     for(int y = 0; y < ny; y++)</pre>
       labely[y] = 0;
    memset(labeled, 0, sizeof(labeled));
memset(matchx,-1,sizeof(matchx));
memset(matchy,-1,sizeof(matchy));
     for(int x = 0; x < nx; x++)
for(int y = 0; y < ny; y++)
          if(hasEdge(x, y))
  if(!labeled[x]) {
               labelx[x] = G[x][y];
labeled[x] = true;
             } else
labelx[x] = max(labelx[x], G[x][y]);
     for(int i = 0; i < nx ; i++)
       while(true) {
          memset(visx, 0, sizeof(visx));
          memset(visy, 0, sizeof(visy));
          if(DFS(i)) break ;
          int d;
bool flag = false;
for(int x = 0; x < nx; x++) if(visx[x])</pre>
             for(int y = 0; y < ny; y++) if(!visy[y])</pre>
               if(hasEdge(x, y))
                  if(!flag)
                    d = labelx[x] + labely[y] - G[x][y];
                    flag = true;
                  } else
                    d = min(d, labelx[x] + labely[y] - G[x][y]);
          if(!flag) {
             isPerfect = false;
             break:
          for(int j = 0; j < nx; j++)
  if(visx[j]) labelx[j] -= d;
for(int j = 0; j < ny; j++)
  if(visy[j]) labely[j] += d;</pre>
     int total = 0;
     for(int x = 0; x < nx; x++)
if(matchx[x] != -1)
                                                    //must be perfect!!!
          total += G[x][matchx[x]];
     max_match_value = total;
     return isPerfect;
  bool hasEdge(int u,int v) {
    //TODO
  }
* change edge data type
 * negative edge for min_match
 * initialize G[][]
 * call km.max_match()
 * check km.isPerfect
```

```
* check km.max_match_value
*/
```

## 5.9 Min Vertex Cover

```
struct MinVertexCover {
private:
  static const int MaxNum = 54;
  vector<int> G[MaxNum];
  int in[MaxNum];
  int undo(vector<int> &record) {
    for(int i=0; i<record.size(); ++i)</pre>
      ++in[record[i]];
    record.clear();
  int delNode(int u, vector<int> &record) {
    for(int i=0; i<G[u].size(); ++i)
  if( in[G[u][i]] ) {</pre>
        --in[G[u][i]];
        --in[u];
        record.push_back(G[u][i]);
        record.push_back(u);
  }
  int cnt(int from, int *visited, bool type) {
    if( visited[from] ) return 0;
    if( type==1 ) visited[from] = 1;
for(int i=0; i<G[from].size(); ++i)</pre>
    if( in[G[from][i]] && !visited[G[from][i]] )
      return type+cnt(G[from][i], visited, !type);
    return type:
  int cnt(int *visited) {
    int ret = 0;
    for(int i=0; i<MaxNum; ++i)
  if( in[i]==1 && !visited[i] )</pre>
        ret += cnt(i, visited, 0);
    for(int i=0; i < MaxNum; ++i)</pre>
      if( in[i]==2 && !visited[i] )
        ret += cnt(i, visited, 0);
    return ret;
  }
public:
  int MVCPick[MaxNum];
  int MVC; // min vertex cover
  void init() {
    for(int i=0; i<MaxNum; ++i)</pre>
      G[i].clear();
    memset(in, 0, sizeof(in));
  void addEdge(int u, int v) {
    G[u].push_back(v);
    G[v].push_back(u);
    ++in[u];
    ++in[v];
  void minVertexCover(int nowMVC=0, const int *lastPick=NULL) {
    // 0(n<sup>2</sup> * 1.38<sup>n</sup>)
    int nowPick[MaxNum] = {};
    if( nowMVC==0 ) {
      MVC = MaxNum:
      memset(MVCPick, 0, sizeof(MVCPick));
    else memcpy(nowPick, lastPick, sizeof(nowPick));
    int maxid = 0;
    for(int i=0; i<MaxNum; ++i)</pre>
      if( in[i]>in[maxid] )
        maxid = i;
    if( in[maxid] <= 2 ) {</pre>
      nowMVC += cnt(nowPick);
       if( nowMVC<MVC ) {</pre>
        MVC = nowMVC;
        memcpy(MVCPick, nowPick, sizeof(nowPick));
      return;
    vector<int> record;
    delNode(maxid, record);
    nowPick[maxid] = 1;
    minVertexCover(nowMVC+1, nowPick);
    nowPick[maxid] = 0;
    undo(record);
```

```
int cnt = 0;
  for(int i=0; i<G[maxid].size(); ++i)
    if( in[G[maxid][i]] ) {
        ++cnt;
        delNode(G[maxid][i], record);
        nowPick[G[maxid][i]] = 1;
    }
  minVertexCover(nowMVC+cnt, nowPick);
  undo(record);
}
</pre>
```

## 5.10 Max Independent Set

Pick those vertice which are not picked by min vertex cover.

## 5.11 Max Clique

```
Create a new graph G'(V, E') in which E' = \{(u, v) \mid \text{if } (u, v) \text{ not in } E\}
Max clique in G === Max independent set in G'
```

#### 5.12 Kirchhoff's theorem

```
D = Degree matrix (diagonal entries are node's degree) A = Adjacency matrix (01 matrix) Q = Laplacian matrix = D - A Q* = deleting any row and any column from Q Number of spanning tree = det(Q*)
```

## 5.13 Popular matching

```
N applicant M position
Each applicant have his own perference list (not required
     containing all position)
Target: find a matching s.t. not existing another matching have
     more applicant prefer. This matching called popular matching
A applicant u prefer another matching which imply that
    "This matching doesn't have u but the other does"
    "The other matching give u the position that he more prefer"
Define hot position: the position is any applicant's first choice
Solve: Whethere there is a popular matching
(1) Each applicant build and edge with "his first choice" and "
     his first choice except all hot position" (if not exist,
     build a dummy position)
(3) Return whether all applicant can have a position
Solve: Maximum popular matching
(1) If popular matching not exist return No
(2) Use the result above
(3) Remove all dummy position
(4) Find augmenting path for all unmatch applicant(5) For all unmatch hot position, rob an applicant(all it's
    applicant choice non hot position)
(6) return the matching
```

#### 5.14 Blossom Algorithm

```
#include <cstdio>
#include <queue>
#include <cstring>
using namespace std;
struct BlossomAlgorithm {
         // number of vertex, zero base
         static const int N = 205;
         int match[N];
         int d[N];
         queue<int> q;
         deque<int> path[N];
         void label_one_side(int x, int y, int bi) {
    for(int i = bi+1; i < path[x].size(); i++) {</pre>
                   int z = path[x][i];
                   if (d[z] == 1) {
                       path[z] = path[y];
                       path[z].insert(path[z].end(), path[x].rbegin
     (), path[x].rend()-i);
d[z] = 0;
                       q.push(z);
```

```
}
        }
        bool BFS(int r) {
    for (int i = 0; i < n; ++i)</pre>
                path[i].clear();
            path[r].push_back(r);
            memset(d, -1, sizeof(d));
            d[r] = 0;
            while(!q.empty())
                q.pop();
            q.push(r);
            while (!q.empty()) {
                int x = q.front();
                if (match[y] == -1) {
    for (int i = 0; i + 1 < path[x].
     size(); i += 2) {
                                     match[path[x][i]] = path[x][i
     +1];
                                     match[path[x][i+1]] = path[x]
    ][i];
                                 match[x] = y; match[y] = x;
                                 return true;
                             else {
                                 int z = match[y];
                                 path[z] = path[x];
                                 path[z].push_back(y);
                                 path[z].push_back(z);
                                 d[y] = 1; d[z] = 0;
                                 q.push(z);
                         else if (d[y] == 0) {
    bi++;
                             bi--;
                             label_one_side(x, y, bi);
                             label_one_side(y, x, bi);
            return false;
    public:
        int max_match;
        int n;
        bool map[N][N];
        void matchAll() {
            memset(match, -1, sizeof(match));
            max_match = 0;
            for (int i = 0; i < n; i++)
    if (match[i] == -1)
                    if (BFS(i))
                        max_match++;
                    else
                        match[i] = i;
        }
        void init() {
            memset(map, false, sizeof(map));
        void addedge(int u,int v) {
  map[u][v] = map[v][u] = true;
} solver;
    Step:
        solver.init();
        solver.n = n;
        loop:
            solver.addedge(i, j);
```

## 6 Flow

## 6.1 Dinic Maxflow Mincut

```
class Dinic{
private:
     typedef pair<int,int> pii;
     static const int maxN = 504;
static const int infF = 1023456789;
     int cap [maxN][maxN];
     int pipe[maxN][maxN];
     vector<int> g[maxN];
     int level[maxN], visited[maxN];
     bool bfsLabeling(int s, int t){
          memset(level , 0 , sizeof(level));
queue<int> myQ;
           myQ.push( s );
           level[s] = 1;
           while( !myQ.empty() ){
                int nowAt = myQ.front();
                myQ.pop();
                for(int i=0;i<g[nowAt].size();++i)</pre>
                     if( !level[g[nowAt][i]] && pipe[nowAt][g[nowAt][i
      ]]){
                           level[g[nowAt][i]] = level[nowAt] + 1;
                           myQ.push( g[nowAt][i] );
           return level[t];
      int dfsFindRoute(int nowAt, int t, int maxC) {
           visited[nowAt] = true;
           if( nowAt==t ){
                maxFlow += maxC;
                return maxC;
           for(int i=0; i<g[nowAt].size(); ++i) {</pre>
                int log; InglineMat].size(), ..., {
   int next = g[nowAt][i];
   if( visited[next] ) continue;
   if( level[next] != level[nowAt]+1 ) continue;
   if( !pipe[nowAt][next] ) continue;
   int nowOut = dfsFindRoute(next ,t ,min(maxC , pipe[
      nowAt][next]));
                if( nowOut==0 )
                     continue;
                pipe[nowAt][next] -= nowOut;
pipe[next][nowAt] += nowOut;
                return nowOut;
          }
           return 0;
     void dfsFindMinCut(int nowAt) {
          sside[nowAt] = 1;
for(auto v : g[nowAt])
    if( !sside[v] && pipe[nowAt][v] )
                     dfsFindMinCut(v);
     }
public:
     int maxFlow;
     bool sside[maxN];
     vector<pii> minCut;
     void init() {
          memset(cap , 0, sizeof(cap));
memset(pipe , 0, sizeof(pipe));
memset(sside, 0, sizeof(sside));
           for(int i=0;i<maxN;++i)</pre>
                g[i].clear();
           maxFlow = 0;
          minCut.clear();
     void addEdge(int u, int v, int c) {
          if( u==v ) return;
if( !cap[u][v] && !cap[v][u] ) {
                g[u].emplace_back(v);
                g[v].emplace_back(u);
           cap[u][v] += c;
     void coculAll(int s, int t) {
          memcpy(pipe, cap, sizeof(pipe));
           // max flow
          while( bfsLabeling(s,t) ) {
    memset(visited, 0, sizeof(visited));
                while( dfsFindRoute(s,t,infF) )
```

```
}

// min cut
dfsFindMinCut(s);
for(int u=0; u<maxN; ++u)
if( sside[u] )
    for(auto v : g[u])
    if( !sside[v] )
        minCut.push_back({u, v});
};</pre>
```

# 7 Geometry

## 7.1 Geometry basic

```
struct Point{
    double x,y;
    Point(double xi=0.0,double yi=0.0){
         x = xi, y = yi;
    Point operator - (const Point &r)const{
         return Point(x-r.x , y-r.y);
typedef Point Vector;
double angle(const Vector &v,const Vector &u){
    // return rad [0, pi] of two vector
    return acos( dot(v,u)/len(v)/len(u) );
Vector rotate(const Vector &v,double rad){
    return Vector(
         v.x*cos(rad) - v.y*sin(rad),
         v.x*sin(rad) + v.y*cos(rad)
double pointSegLen(const Point &A,const Point &B,const Point &Q){
    if(A==B) return len(Q-A);
    if( dot(B-A , Q-A)<0 ) return len(Q-A);
if( dot(B-A , Q-B)>0 ) return len(Q-B);
return fabs( cross(B-A , Q-A) ) / len(B-A);
bool pointOnSeg(const Point &A,const Point &B,const Point &Q){
    return fabs( len(Q-B)+len(Q-A)-len(A-B) ) < 1e-9;</pre>
bool pointInPoly(const vector<Point> &poly, const Point &p) {
    // Poly should be counter clockwise. O(logN)
    int r_bound = poly.size();
    while( poly[0] == poly[r_bound-1] )
         r_bound--;
    int l = 1, r = r_bound;
    const Point target = p - poly[0];
    while( l < r ) {
   int c = (l + r) / 2;</pre>
         if( fdif(cross(poly[c] - poly[0], target)) <= 0 )</pre>
             r = c;
         else
             l = c + 1;
    if( l == 1 || l == r_bound ) return false;
    )>0);
}
struct Line{
    Line(const Point &pi,const Vector &vi):p0(pi) , v(vi) {}
double pointLineLen(const Line &L,const Point &Q){
    return fabs( cross(L.v , Q-L.PO) ) / len(L.v);
Point projectToLine(const Line &L,const Point &Q){
    double t = dot(Q-L.P0 , L.v) / dot(L.v , L.v);
    return L.PO + L.v * t;
Point innerCircle(point &p1, point &p2, point &p3){
    // p1,p2,p3 should not on same line
    double a1 = (-2*p1.x + 2*p2.x);
double b1 = (-2*p1.y + 2*p2.y);
    double c1 = (p2.x*p2.x + p2.y*p2.y - p1.x*p1.x - p1.y*p1.y);
```

```
double a2 = (-2*p1.x + 2*p3.x);
    double b2 = (-2*p1.y + 2*p3.y);
    double c2 = (p3.x*p3.x + p3.y*p3.y - p1.x*p1.x - p1.y*p1.y);
    double cx = (c1*b2-c2*b1) / (a1*b2-a2*b1);
double cy = (a1*c2-a2*c1) / (a1*b2-a2*b1);
    return Point(cx, cy);
Point outerCircle(point &p1, point &p2, point &p3) {
     // p1,p2,p3 should not on same line
    double x1 = (p1.x+p2.x)/2.0;
    double y1 = (p1.y+p2.y)/2.0;
    double x2 = (p2.x+p3.x)/2.0;
    double y2 = (p2.y+p3.y)/2.0;
    double vx = p2.x-p1.x;
    double vy = p2.y-p1.y;
    double ux = p3.x-p2.x;
    double uy = p3.y-p2.y;
    double A = vx*x1 + vy*y1;
    double B = ux*x2 + uy*y2;
    double cx = (uy*A - vy*B) / (uy*vx - ux*vy);
double cy = (ux*A - vx*B) / (ux*vy - uy*vx);
    return Point(cx, cy);
```

## 7.2 Minimal Enclose Disk

```
struct Circle {
  Point c;
  double R2:
               // square of radius
  Circle() {}
  Circle(const Point &p1, const Point &p2) {
    c.x = (p1.x+p2.x)/2.0;

c.y = (p1.y+p2.y)/2.0;
    R2 = dot(p1-p2, p1-p2)/4.0;
  Circle(const Point &p1, const Point &p2, const Point &p3) {
   // p1, p2, p3 should not on same line
     c = outerCircle(p1, p2, p3);
    double dx = p1.x - c.x;
double dy = p1.y - c.y;
    R2 = dx*dx + dy*dy;
  bool contain(const Point &p) const {
    double dx = c.x - p.x;
    double dy = c.y - p.y;
    return fdif(dx*dx + dy*dy - R2)<=0;
};
Circle minEncloseDisk(vector<Point> &ps) {
  // Find minimal circal enclose all point
  // worst case O(n^3), expected O(n)
  Circle D;
  if( ps.size()==0 ) return D;
  if( ps.size()==1 ) {
    D.c = ps[0];
    D.R2 = 0.0;
    return D:
  random_shuffle(ps.begin(), ps.end());
  D = Circle(ps[0], ps[1]);
  for(int i=2; i<ps.size(); ++i) {
  if( D.contain(ps[i]) )</pre>
       continue:
    D = Circle(ps[i], ps[0]);
    for(int j=1; j<i; ++j)</pre>
       if( D.contain(ps[j]) )
         continue:
       D = Circle(ps[i], ps[j]);
       for(int k=0; k<j; ++k) {</pre>
         if( D.contain(ps[k]) )
           continue:
         D = Circle(ps[i], ps[j], ps[k]);
      }
    }
 }
}
```

#### 7.3 2D Convex Hull

```
bool turnLeft(const Vector &v1, const Vector &v2) {
  return fdif(cross(v1, v2)) > 0LL;
}
vector<Point> convexHull(vector<Point> &ps) {
  // return convex hull without redundant point
  sort(ps.begin(), ps.end());
```

```
vector<Point> up;
for(int i=0; i<ps.size(); ++i) {</pre>
  while( up.size()>1
    && !turnLeft(up.back()-up[up.size()-2],
     ps[i]-up.back()) )
    up.pop_back();
  up.emplace_back(ps[i]);
vector<Point> btn;
for(int i=ps.size()-1; i>=0; --i) {
  while( btn.size()>1
    && !turnLeft(btn.back()-btn[btn.size()-2],
      ps[i]-btn.back()) )
    btn.pop_back();
  btn.emplace_back(ps[i]);
vector<Point> res(up);
res.insert(res.end(), btn.begin()+1, btn.end());
res.pop_back();
return res;
```

## 7.4 Closest Point

```
#include <iostream>
#include <cstdio>
#include <algorithm>
#include <cstring>
#include <cmath>
#define N 10010
#define INFINITY_.
using namespace std;
int diff(double f) {
  if(fabs(f) < 1e-9)
    return 0;
  return f < 0 ? -1 : 1;
struct point {
  double x, y;
  bool operator < (const point &p) const {</pre>
    return diff(x - p.x) < 0;
  }
point pt[N], tmp[N];
double dis(point a,point b) {
  return sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y));
double closestpoint(int L,int R) {
  if(R - L + 1 == 1) return INFINITY__;
  int M = (L+R)/2;
  double middle = pt[M].x;
  double Ldis = closestpoint(L, M);
  double Rdis = closestpoint(M+1, R);
  double radi = min(Ldis, Rdis);
  int cntpt = 0;
  merge(pt+L, pt+M+1, pt+M+1, pt+R+1, tmp+L, [](point a,point b)
    return diff(a.y - b.y) < 0;
  });
  copy(tmp + L, tmp + R + 1, pt + L);
  for(int i = L; i <= R; i++)
    if(diff(fabs(pt[i].x - middle) - radi) < 0)</pre>
  tmp[cntpt++] = pt[i];
for(int i = 0; i < cntpt; i++)
  for(int j = 1; i + j < cntpt && j < 8; j++)</pre>
      radi = min(radi, dis(tmp[i], tmp[i+j]));
  return radi:
```

# 7.5 Min Max Triangle

```
pair<double,double> findMinMaxTri(vector<Point> &ps) {
   static const double PI = acos(-1.0);
   struct Seg {
      double rad; // [0.5pi, 1.5pi]
      int s1, s2;
   };

   const int n = ps.size();
   sort(ps.begin(), ps.end(), [](const Point &l, const Point &r) {
      if( fdif(l.x - r.x) == 0 )
        return l.y > r.y;
      return l.x < r.x;
   });
   vector<int> id(n+4);
   for(int i=0; i<n; ++i)</pre>
```

```
id[i] = i;
// sort all pair of point
vector<Seg> segs;
for(int i=0; i<n; ++i)</pre>
for(int j=i+1; j<n; ++j) {</pre>
  double m = atan2(ps[j].y-ps[i].y, ps[j].x-ps[i].x) + PI;
  segs.push_back({m,i,j});
sort(segs.begin(), segs.end(), [](const Seg &l, const Seg &r) {
  return fdif(l.rad - r.rad) < 0;</pre>
// find min max triangle
pair<double, double> ret;
ret.first = ret.second = fabs(cross(ps[0], ps[1], ps[2]));
for(auto seg : segs) {
  swap(ps[id[seg.s1]], ps[id[seg.s2]]);
  swap(id[seg.s1], id[seg.s2]);
  const Point &p1 = ps[id[seg.s1]];
  const Point &p2 = ps[id[seg.s2]];
  int id1 = min(id[seg.s1], id[seg.s2]);
int id2 = max(id[seg.s1], id[seg.s2]);
   // find min triangle
  if( id1-1 >= 0 )
     double a = fabs(cross(p1, p2, ps[id1-1]));
     if( a < ret.first )</pre>
       ret.first = a:
  if( id2+1 < n ) {
   double a = fabs(cross(p1, p2, ps[id2+1]));</pre>
     if( a < ret.first )</pre>
       ret.first = a;
  // fin max triangle
  if( id1 != 0 ) {
  double a = fabs(cross(p1, p2, ps[0]));
     if( a > ret.second )
       ret.second = a;
  if( id2 != n-1 ) {
   double a = fabs(cross(p1, p2, ps[n-1]));
     if( a > ret.second )
       ret.second = a;
ret.first /= 2.0;
ret.second /= 2.0;
return ret;
```