Topic 7: The Capital Asset Pricing Model (CAPM)

Overview

- Implications of Portfolio Theory

- The CAPM

- The Security Market Line and the Capital Market Line

- Estimating Beta and testing the CAPM

- Stock Performance Evaluation

- CAPM, Arbitrage and the APT

Implications of Portfolio Theory

● We have seen that all investors hold combinations of the tangency portfolio with borrowing or lending at the risk-free rate.

-Very risk-averse people put a high weight on the risk-free asset.

-Less risk-averse people place more weight on the tangency portfolio.

● As a group, investors put some money in the risk-free asset and some money in the tangency portfolio.

This is <u>total demand</u> for assets.

Implications of Portfolio Theory (Cont'd)

● Let the total supply of risky asset j equal $P_j s_j$, where P_j is the price of one share and s_j is the number of shares.

The total supply of all risky assets equals

$$\sum_{j=1}^{N} P_j s_j$$

● This big portfolio of all risky assets is called the market portfolio, or more precisely, the market portfolio of all risky assets.

The weight of asset k in the market portfolio is

$$w_k = \frac{P_k s_k}{\sum_{j=1}^{N} P_j s_j}$$

Implications of Portfolio Theory (Cont'd)

● Market equilibrium means total demand=total supply. Since all investors hold the same portfolio of risky assets, this must be the market portfolio.

Key implication: In equilibrium, the tangency portfolio is the market portfolio.

● Example: If the market portfolio has 1% GM, then the tangency portfolio has 1% GM.

If the tangency portfolio has less than 1% GM, then demand for GM < supply of GM. Thus, the price of GM will fall, reducing GM's share of total market.

What happens to expected returns when GM's price falls?

CAPM

- Since the market portfolio is the tangency portfolio, we know from Topic 6 that the reward-to-risk ratio is the same for all risky assets in the market portfolio:

$$\frac{E(R_j) - R_f}{2Cov(R_j, R_m)}$$

where R_m=return on the market portfolio.

Since the market portfolio itself is a risky asset, the reward-to-risk ratio for the market equals the reward-to-risk ratio for all individual assets. So,

$$\frac{E(R_j) - R_f}{2Cov(R_j, R_m)} = \frac{E(R_m) - R_f}{2V(R_m)} \Rightarrow$$

$$E(R_j) - R_f = \frac{Cov(R_j, R_m)}{V(R_m)}\left(E(R_m) - R_f\right)$$

This is the CAPM!

CAPM (Cont'd)

- The Beta of asset *j* is defined as follows:

$$\beta_j = \frac{Cov(R_j, R_m)}{V(R_m)}$$

Using the definition of beta,

$$E(R_j) - R_f = \beta_j \left(E(R_m) - R_f \right)$$

- So, CAPM says that the expected excess return on asset *j* is determined by its beta (relative to the market) and the expected excess return on the market portfolio.

- Key insight of CAPM: Beta is the relevant measure of risk for an asset.

-What matters: how much does an asset contribute to the volatility of the market portfolio.

-What does not matter: how volatile is a stock on its own.

CAPM (Cont'd)

Intuition: Beta measures the sensitivity of a stock's return to returns in the market.

- If beta is high, asset is very sensitive to market movements.

What stocks have high betas?

- If beta is low, asset is not sensitive to market movements.

What stocks have low betas?

- If beta is zero, asset is uncorrelated with the market.

- If beta is negative, asset is negatively correlated with the market.

What is the expected return of negative beta stocks?

- What is the beta for the "typical" stock?

CAPM (Cont'd)

To derive CAPM, we made some assumptions:

- There is a riskless asset and people can borrow or lend at this rate.

- Investors have mean-variance preferences.

- All assets are traded.

- Investors have a 1-period horizon.

- There are no transactions costs.

- Investors have identical information and beliefs.

Many of these can be relaxed.

CAPM (Cont'd)

Uses of CAPM.

- Portfolio evaluation

How well has a portfolio or stock performed?

- Valuation of stocks (Topic 8)

What is the right stock price?

- Capital budgeting (Topic 10)

How can you evaluate a risky project?

CAPM gives the rate to discount expected cash flows.

The SML and the CML

● The Capital Market Line (CML) plots the relationship between the expected return and standard deviation for frontier portfolios:

$$E(R) = R_f + \frac{E(R_m) - R_f}{\sigma(R_m)} \sigma(R)$$

The CML holds only for frontier portfolios!

Key point about CML: The market portfolio lies on the CML.

● Recall Fact # 1: Riskier indices carry a higher average return than safer ones.

This follows because well-diversified portfolio (i.e. indices) lie on (or near) the CML.

The SML and the CML (Cont'd)

● The Security Market Line (SML) plots the relationship between expected return and beta:

$$E(R_j) = R_f + \beta_j \left(E(R_m) - R_f \right)$$

This relationship holds for all assets.

● SML says that for individual stocks (e.g. assets *not* on the Portfolio Frontier), expected return depends on its contribution to variance of the market portfolio.

This explains Fact #2: The relationship between volatility and risk for individual stocks is much less apparent.

Estimating Beta and Testing CAPM

We can always write excess returns as follows:

$$R_j - R_f = \alpha_j + \beta_j (R_m - R_f) + \varepsilon_j$$

So, excess returns are the sum of three things:

● A constant (α_j)

● The <u>systematic part of the return</u> ($\beta_j(R_m\text{-}R_f)$). This is the part of the return that is perfectly correlated with the excess return on the market.

● The <u>idiosyncratic part of the return</u> (ε_j). This is the part of the return that is uncorrelated with the excess return on the market.

That is:

$$E(\varepsilon_j) = 0$$
$$Cov(R_m, \varepsilon_j) = 0$$

● What does CAPM imply about α_j?

Estimating Beta and Testing CAPM (Cont'd)

To compute beta, estimate the following with a linear regression:

$$R_j - R_f = \alpha_j + \beta_j (R_m - R_f) + \varepsilon_j$$

This regression model is called the market model.

● We do not know α_j and β_j, but we can estimate them from historical data on R_j, R_m and R_f.

$$b_j = \frac{\overline{Cov(R_j - R_f, R_m - R_f)}}{s^2(R_m - R_f)}$$

$$a_j = (\overline{R_j} - \overline{R_f}) - b_j(\overline{R_m} - \overline{R_f})$$

● In practice, use excess returns in the market model. (Does not matter in theory.)

Estimating Beta and Testing CAPM (Cont'd)

● The market model allows you to decompose the total return variability as follows

$$R_j - R_f = \alpha_j + \beta_j (R_m - R_f) + \varepsilon_j \Rightarrow$$

$$V(R_j) = \beta_j^2 V(R_m) + V(\varepsilon_j)$$

● The variance decomposition allows us to define idiosyncratic and systematic risk, as follows

$$IdiosyncraticRisk = \sigma(\varepsilon_j)$$
$$SystematicRisk = \beta_j \sigma(R_m)$$

● These can be estimated from the market model regression as

$$IdiosyncraticRisk = s(e_j)$$
$$SystematicRisk = b_j s(R_m - R_f)$$

Estimating Beta and Testing CAPM (Cont'd)

Here are some estimates of Beta using monthly data (1967-1998):

	Beta	Sample Std. Dev. of Return	Sample Mean Excess Return	CAPM Expected Excess Return
Duke Pow.	0.54	5.43%	0.42%	0.29%
Exxon	0.73	5.03%	0.79	0.39
IBM	0.85	6.74%	0.33	0.45
GM	0.90	6.87%	0.35	0.48
Coke	0.91	6.38%	1.03	0.48
Goodyear	1.10	9.17%	0.60	0.58
Boeing	1.29	9.87%	1.06	0.68
Disney	1.36	9.04%	1.39	0.72
United	1.47	12.02%	0.78	0.78
"Market"	1.00	4.62%	0.53	0.53

Estimating Beta and Testing CAPM (Cont'd)

Problems testing CAPM...

- The return on the "market" is not observable.

Best proxy: Value-Weighted Return on NYSE/Amex/NASDAQ stocks

Weight on asset *k*:

$$w_k = \frac{P_k s_k}{\sum_{j=1}^{N} P_j s_j}$$

where N is the number of stocks trading on NYSE/Amex/NASDAQ.

-What about international stocks?
-What about non-traded assets?
-What about bonds?

Estimating Beta and Testing CAPM (Cont'd)

More problems testing CAPM...

- Betas and expected excess return on market may change over time.

Is IBM the same today as it was in the 1970s?

- We do not observe true beta, only an estimate.

- Unanswered questions:

What determines risk-free rate?

What determines expected excess return on market?

Estimating Beta and Testing CAPM (Cont'd)

- Bottom line on CAPM

CAPM Says

(1) Expected Excess Return increase linearly with Beta

-True

(2) With a slope equal to market risk premium

-False. Too flat.

(3) And nothing else matters...

-False. Size and Book-to-Market (15.433).

Stock Performance Evaluation

● Suppose we estimate the market model regression for asset j:

$$R_j - R_f = \alpha_j + \beta_j (R_m - R_f) + \varepsilon_j$$

CAPM says α_j=0, but an individual stock may do better or worse than its expected return over any given period of time.

Thus, the estimate of α_j over a period (a_j) of time represents how well (or badly) a stock performed relative to CAPM expectations.

CAPM says

$$\alpha_j = E(R_j - R_f) - \beta_j E(R_m - R_f) = 0$$

But, in any sample...

$$a_j = (\overline{R_j} - \overline{R_f}) - b_j (\overline{R_m} - \underline{\overline{R_f}})$$

...can be greater or less than zero.

● a_j is called Jensen's Alpha for stock *j*.

Stock Performance Evaluation (Cont'd)

● Suppose you want to evaluate well-diversified portfolios, like mutual funds.

Best fund is the one closest to the Portfolio Frontier.

The Tangency Portfolio (Market) has highest Sharpe Ratio:

$$SharpeRatio = \frac{E(R_m) - R_f}{\sigma(R_m)}$$

● So, a good mutual fund is one that has a high Sharpe Ratio estimated with historical data.

$$MeasuredSharpeRatio = \frac{\overline{R_p} - \overline{R_f}}{s(R_p)}$$

where R_p is the return on a portfolio (e.g. a mutual fund).

CAPM, Arbitrage and the APT

● We have derived CAPM as an equilibrium model based on supply and demand.

(Key assumption: People are risk averse and want to hold lowest variance portfolio for a given amount of expected return...)

What if people care about things other than variance?

● We can get an approximate version of the CAPM for <u>any preferences</u> using an arbitrage argument.

Only approximate since we are making weaker assumptions about preferences.

CAPM, Arbitrage and the APT (Cont'd)

- Market model again:

$$R_j - R_f = \alpha_j + \beta_j (R_m - R_f) + \varepsilon_j$$

We can derive CAPM for any asset (or portfolio) that has no idiosyncratic risk (think mutual fund) using an arbitrage argument.

Imagine such a portfolio (P). Its return is given by

$$R_p - R_f = \alpha_p + \beta_p (R_m - R_f)$$

Now, build a second portfolio by combining a position in the market portfolio and the risk-free asset, with weight β_p on the market and 1-β_p on the risk-free asset.

This portfolio's return (R_{rep})is

$$R_{rep} = (1 - \beta_p) R_f + \beta_p R_m \Rightarrow$$
$$R_{rep} - R_f = \beta_p (R_m - R_f)$$

- To avoid an arbitrage opportunity, α_p must be zero! That's CAPM.

CAPM, Arbitrage and the APT (Cont'd)

● The Arbitrage Pricing Theory (APT) is a generalization of this argument.

● Consider a Multi-factor return model

$$R_j = \alpha_j + \sum_{k=1}^{K} b_{j,k} f_k + \varepsilon_j$$

Now, all we say is that returns are generated by K aggregate factors (systematic factors) plus an idiosyncratic factor.

-f_k is the kth aggregate risk factor, these have 0 expected value.
Analogue to R_m-R_f in CAPM.

-$b_{j,k}$ is the "factor loading" for asset *j* on factor *k*.
Analogue to Beta in CAPM.

-ε_j is the idiosyncratic part of the return.

CAPM, Arbitrage and the APT (Cont'd)

● Assume that there exist *k* factor portfolios, each of which depends on just one return factor (f_k):

$$R_{fk} = E(R_{fk}) + f_k$$

● Then, APT says that for any well-diversified portfolio P, the following is true

$$E(R_p) - R_f = \sum_{k=1}^{K} b_{p,k}\left(E(R_{fk}) - R_f\right)$$

● This generalizes CAPM. The expected excess return on factor portfolios...

$$E(R_{fk}) - R_f$$

...enters just like the expected excess return on the market does in the CAPM.

CAPM, Arbitrage and the APT (Cont'd)

- The proof is the same as with 1 factor.

Example: Imagine 2 factors.

$$R_p = \alpha_p + b_{p,1} f_1 + b_{p,2} f_2$$

What is the replicating portfolio?

What is the return on the replicating portfolio?

CAPM, Arbitrage and the APT (Cont'd)

What is the difference between the return on the portfolio (R_p) and the return on the replicating portfolio?

What does α_p equal (to avoid arbitrage)?

CAPM, Arbitrage and the APT (Cont'd)

- What if there is some idiosyncratic risk?

For most assets:

$$E(R_j) - R_f \approx \sum_{k=1}^{K} b_{j,k}\left(E(R_{fk}) - R_f\right)$$

If this did not hold, you can create a portfolio from the individual stocks and use the same arguments as before...

CAPM, Arbitrage and the APT (Cont'd)

● What the heck are these factors?

Theory says nothing...

● To apply to real life, we include factors that seem sensible.

That is, systematic economic factors that generate variation in stock returns.

● Examples

-Interest Rate Factor: Change in T-bill Rate

-Term Structure Factor: Return on LT bond – T-bill

-Exchange Rate Risk: Change in exchange rate

-Production Risk: Change in forecasted GDP.

-Inflation Risk: Change in forecasted inflation.

-Market Risk: Return on market portfolio.

CAPM, Arbitrage and the APT (Cont'd)

CAPM vs. APT

Advantages of APT

- Requires weak set of assumptions.
- Don't need to measure the market portfolio.

Advantages of CAPM

- We know what the relevant risk factor is.
- Results are exact. In the APT, results are approximate.

In practice, for example in capital budgeting, may be prudent to use both CAPM and APT.