Bayes theorem - 2

Agenda
- → Recap of last Class
- → Independent Events
- → Practical Question
- → Real world Scenario

⇒ Recap →

⇒ Conditional Probability →

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Coin Tosses →

a) Probability of getting a Head → $\frac{1}{2}$

b) I get a Head → what is prob of getting tails $= 0$

Roll a dice

a) Probabi of each number → $\frac{1}{6}$

b) I know that the number >3 →

what is the prob → 1, 2, 3 → 0

4, 5, 6 → $\frac{1}{3}$

$\Rightarrow$ Given a condition $\rightarrow$ limited no. of possible events / outcomes

$$\underline{P(n>3)} = \frac{1}{2} = \frac{3}{6} \Rightarrow 3 \text{ outcomes}$$

$\Rightarrow$ What are chance of getting $n=4 \Rightarrow \frac{1}{3}$

$$P(A|B) = \frac{\text{Scenarios where A and B are true}}{\text{Scenario where B is true}}$$

$$= \frac{A \cap B}{B} = \frac{A\cap B/ss}{B/ss}$$

$\underline{B}$ $\underline{n>3} \Rightarrow 3$ scenario

$\underline{A}$ $n=2 \Rightarrow$

$$P(n=2 \mid n>3) = \frac{0}{3} = 0$$

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

Conditional prob

$\Rightarrow \quad P(A \cap B) = P(A|B) \times P(B)$ $\Rightarrow$ Multiplication Rule

$\Rightarrow \quad P(A \cap B) = P(B \cap A)$

$\Rightarrow P(A|B) \times P(B) = P(B|A) \times P(A)$

$$\boxed{P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}}$$

Bayes Theorem

$\Rightarrow$ Law of Total probability $\rightarrow$

$\Rightarrow$

College (1000) $\xrightarrow{400}$ Mumbai $\rightarrow$ 30% are in DS

College (1000) $\xrightarrow{300}$ Bangalore $\rightarrow$ 20% are in DS

College (1000) $\xrightarrow{300}$ Delhi $\rightarrow$ 35% are in DS

Tell me what total % are in DS $\rightarrow$

$(0.3)\,400 + (0.2)\,300 + (0.35)\,300$

$120 + 60 + 105 = \underline{\underline{285}}$

Overall people in DS = People in DS from Mum + in DS fro Del + in DS from Bangalor

$$DS = M \cap DS + D \cap DS + B \cap DS$$

$$\boxed{P(DS) = P(M \cap DS) + P(D \cap DS) + P(B \cap DS)}$$

⇒ Independent Events →

Event A doesnot impact Event B

⇓

Independent event

→ A and B has nothing in Common

⇓

Mutually Exclusive/ Disjoint

⇒ Coin Toss → H | T

2 Coins → H | T   H | T

C1   C2

2 coins $\Rightarrow \{HH, TH, HT, TT\}$

event A = Getting 2 Heads $= \{HH\}$

event B = Getting 2 Tails $= \{TT\}$

event C = Getting 1 Head $= \{HT, TH\}$

event D = Getting atleast 1 Head $= \{HT, TH, HH\}$

$\Rightarrow$ A & B are disjoint

B & C disjoint

A & D are non exclusive

$A \cap D \neq \{\}$ $\Rightarrow$ non mutually exclusive

$\Rightarrow$ Coin and Dice $\Rightarrow$
(H/T) (1, 2, 3, 4, 5, 6)

SS $\Rightarrow$ H1 H2 H3 H4 H5 H6

T1 T2 T3 T6

A $\rightarrow$ getting Head $\Rightarrow \dfrac{6}{12}$

P(A)

$\underline{\underline{B}} \Rightarrow$ getting a 3 $\Rightarrow \frac{2}{12}$ $\quad P(\underline{B})$

$\Rightarrow$ $\underline{\underline{Q}}$ Getting a Head and 3 $\Rightarrow 1/12$

$$P(A \cap B) = 1/12$$

$\underline{\underline{Q}}$ Prob of getting heads given 3 has occured on the dice $\rightarrow$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{1/12}{2/12} = 1/2$$

A $\rightarrow$ getting Head

B $\rightarrow$ getting 3

$$P(A) = 1/2$$

$$\underline{\underline{P(A|B) = P(A)}} \Rightarrow \underline{\underline{\text{Independent events}}}$$

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$ If this holds

then A & B are independent

⇒ Mutually exclusive events are always dependent!

$A \cap B = \{\}$

$P(A \cap B) = 0$

$\Rightarrow A \cap B = \{\}$

⇒ I know that A is true ←

↓

I also know that B is false

if $P(A \cap B) = 0 \Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \neq P(A)$

↳ dependent events

Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language processing (NLP).
Also, it is seen that 20 have taken both CV and NLP.
Given that a person has taken NLP, what is the probability that he has also taken CV?

$P(CV) = 0.6$

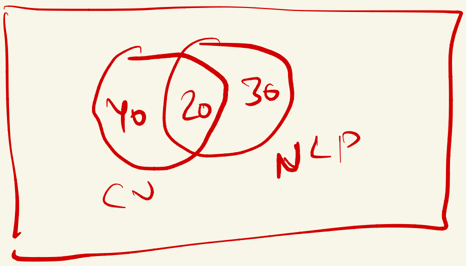
$P(CV \cap NLP) = 0.2$

$P(NLP) = 0.5$

$$P(CV \mid NLP) = \frac{P(CV \cap NLP)}{P(NLP)}$$

$$= \frac{0.2}{0.5} = \frac{2}{5}$$

Approach -2



$$= \frac{20}{50}$$

Non Exclusive

$CV \cap NLP \neq 0$

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

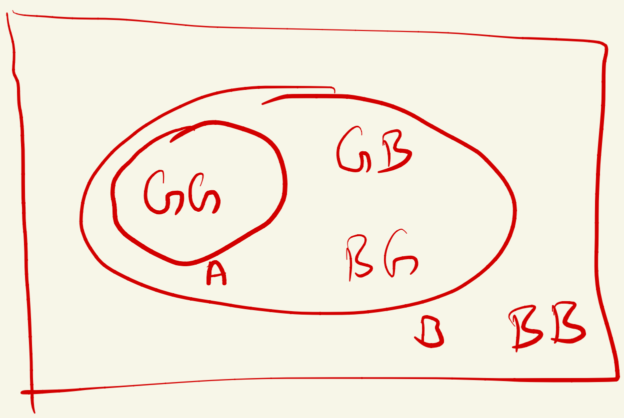
A family has 2 children, at least one of them is a girl.
What is the probability that both are girls?

$\Rightarrow$ SS $\Rightarrow$ {GG, GB, BG, BB}

A $\Rightarrow$ Both are girl $\Rightarrow$ $P(A) = 1/4$

B $\Rightarrow$ atleast 1 is girl $\Rightarrow$ $P(B) = 3/4$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = 1/3$$



$\Rightarrow \frac{1}{3}$

A = 2 girls $\Rightarrow$ GG

B = atleast 1 girl $\Rightarrow$ GG, GB, BG

$A \cap B$ $\Rightarrow$ GG $\Rightarrow$ 1 case

$P(A \cap B) = \frac{1}{\text{size of SS}} = \frac{1}{4} =$

2 children

C1 → B, G

C2 → B, G

G B → first child is Girl

B G → 2nd —"—

In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD

$\Rightarrow \quad P(F) = 0.3 \qquad P(M) = 0.7$

A) $P(PHD \mid F) = 0.6 \qquad P(PHD \mid M) = 0.4$

~~B) $P(PHD \cap F) = 0.6$~~

Q. Prob that a randomly choosen person is female and have a pHD ??

~~A $P(PHD \mid F)$~~ $\qquad$ B $P(PHD \cap F)$

$$P(PHD \cap F) = P(PHD \mid F)\, P(F)$$

$$= (0.6)(0.3) = 0.18$$

Q random choice → male & PHD holder

$P(PHD \cap M) = P(PHD|M)\,P(M)$

$= (0.4)(0.7)$

$= 0.28$

Q random person → has a PHD ??

$P(PHD) = P(PHD \cap F) + P(PHD \cap M)$

$= 0.18 + 0.28$

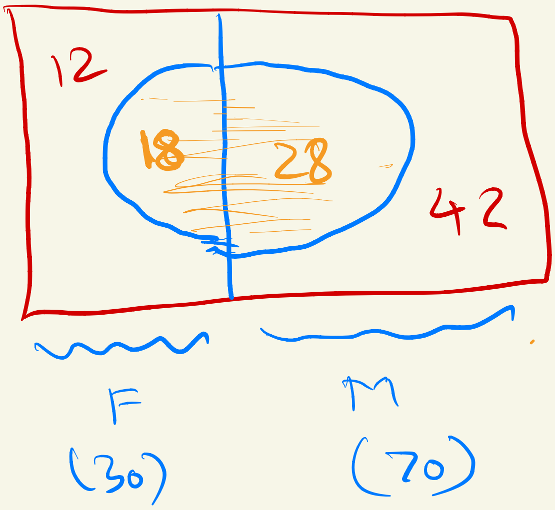
$= \underline{\underline{0.46}}$

Q a random PHD holder is selected ⇒ Prob that they are female.

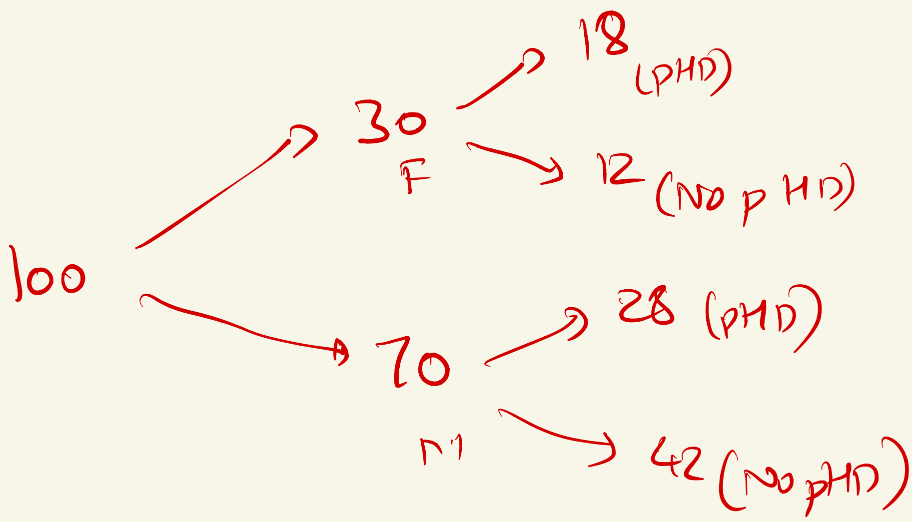
$$P(F|PHD) = \frac{P(PHD|F)\,P(F)}{P(PHD)}$$

$$= \frac{(0.6)(0.3)}{0.46} = \frac{18}{46}$$

Approach 2



Approach 3



1) $F \cap pHD$

$= \frac{18}{100}$

2) $M \cap pHD$

$= \frac{28}{100}$

3) $F \mid pHD$

$= \frac{18}{46}$

An website shows two types of ads:
60% of the visitors see Type A ads, and 40% visitors see Type B ads. The click-through rate for A is 5%, and for B is 3%.
A visitor to the website does not click the ad.
What is the probability that he saw Type A ad?

Approach 1

1000 → 600 A → 570
600 A → 30 click
1000 → 400 B → 388
400 B → 12 click

Given that No click ⇒ $570 + 388 = 958$

$$\Rightarrow \frac{570}{958} = 0.59$$

Approach 2

$P(A) = 0.6$ $\qquad$ $P(B) = 0.4$

$P(\text{click} \mid A) = 0.05$ $\qquad$ $P(\text{click} \mid B) = 0.03$

$P(NC \mid A) = 0.95$ $\qquad$ $P(NC \mid B) = 0.97$

** $P(A \mid NC) = ???$

$$P(A|NC) = \frac{P(A \cap NC)}{P(NC)} \qquad P(A|NC) = \frac{P(NC|A)\,P(A)}{P(NC)}$$

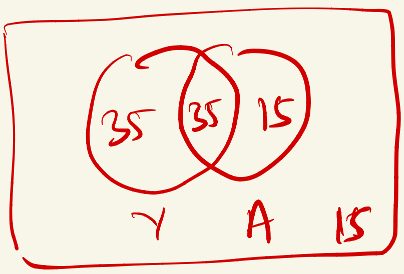
$$P(A|NC) = \frac{P(NC|A)\,P(A)}{P(NC)}$$

$$= \frac{(0.95)(0.6)}{(0.95)(0.6) + (0.97)(0.4)} = 0.59$$

A website has noticed the following stats.
Among those who saw the ad, 70% saw it on Youtube, 50% saw it on Amazon, 35% saw it on both.
A random person who saw the ad on Amazon is chosen, What is the probability that he also saw the ad on Youtube?

⇒

A1 1000



$$= \frac{35}{50} = 0.7$$

⇒

A2

$P(Y) = 0.7 \quad P(A) = 0.5 \quad P(A \cap Y) = 0.35$

$P(Y|A) = ??$

$$P(Y|A) = \frac{P(A \cap Y)}{P(A)} = \frac{0.35}{0.5}$$

$$= 0.7$$

$P(Y) = P(Y|A) = 0.7$

independent

⇒ Kerala

1) Annual Rainfall > Threshold ⇒ Flood happen

2) Compare against median rainfall

3) Maybe only certain months are important

Doubts

⇒ $A \cap B$ ⇒

Tossing 2 Coins → One by one
↳ Simultaneous

A → getting Head on C_1

B → —— " —— C_2

→ SS → {HH, HT, TH, TT}

→ 2 coins at same time
→ 2 coins one after another
→ 1 coin 2 times

⇒ experiment

⇒ Event A → getting Heads on Coin 1 ⇒ {HH, HT}

B → getting Heads on Coin 2 → {HH, TH}

$A \cap B$ ⇒ Getting Head on C1 & on Coin 2
⇒ {HH} ⇒ 1/4

Condition ⇒ Getting Head on C1 given you Know C2 Has a Head →

$$\frac{\{HH\}}{\{HH, TH\}} = \frac{1}{2} =$$

⇒ Mutually Exclusive → Nothing in Common

A → Getting 2 Heads ⇒ {HH}

B ⇒ Getting 2 Tails ⇒ {~~T~~ T}

⇒ $A \cap B = \{\ \}$ ⇒ Mutually exclusive

⇒ If $P(A|B) = P(A)$ ⇒ then they are independent

$\left(P(A|B) = 0\right) \neq \left(P(A) = \frac{1}{4}\right)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 0$ → A & B are dependent

→ All mutually exclusive events are dependent

⇒ All Mutually exclusive are dependent ✓

⇒ All dependent are mutually exclusive ✗

Q

	Reality: Yes	Reality: No	
Test Results: +ve	119	24	143
Test Results: -ve	3	154	157
	122	178	300

a) person tested +ve ~~and~~ given that he or she actually used mari

$$P(+ve \mid yes) = \frac{119}{122}$$

b) person had mari give that they test +ve

$$P(yes \mid +ve) = \frac{119}{143}$$

Q Rolling a dice

A → even number = $\{2, 4, 6\}$

B → value > 4 = $\{5, 6\}$

$A \cap B = \{6\}$ Non mutually exclusive

they can be independent

$\Rightarrow$ Mutually exclusive $\rightarrow$ always dependent

Q $P(A) = 1/2 = \frac{3}{6}$

$P(A|B) = \frac{\{6\}}{\{5,6\}} = 1/2$

$P(A|B) = P(A)$

$\hookrightarrow$ independent

$C \Rightarrow$ outcome $> 3 \Rightarrow \{4,5,6\}$

$P(A \cap C) = \{4,6\}$

$P(A|C) = \frac{2}{3} \neq P(A) = 1/2$

$\Rightarrow$ dependent

CA

Collab Link https://colab.research.google.com/drive/18IIOm9qVnzgypH2ZZAMIOOs2dwbQQ83f?usp=sharing