Advance Distribution

Poisson

Quiz: (Read the following questions and be ready with your answers. I will launch the poll at 09:08 PM)

The amount of time, in hours, that the lovestruck Sumit spends daydreaming about his crush Ankita each day is normally distributed with a mean of 3 hours and a standard deviation of 1 hour. A group of Sumit's friends decides to observe him over a period of 32 days to see how lovestruck he really is.

Find the probability that the average amount of time Sumit spends daydreaming about Ankita over these 32 days falls between 2.5 hours and 3.5 hours per day.

A) 0.95
B) 0.99
C) 0.84
D) 0.50

$\mu_p = 3 \text{ hrs}$

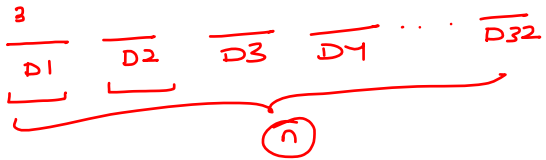
$\sigma = 1 \text{ hr}$

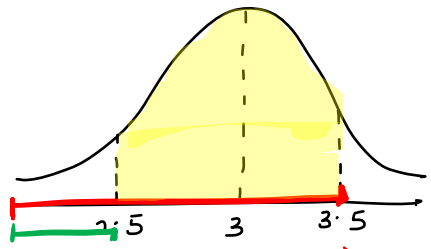
$N\left(\mu_p, \frac{\sigma}{\sqrt{n}}\right) = N\left(3, 1/\sqrt{32}\right)$

norm.cdf(z_2) − norm.cdf(z_1)

$z_1 = \frac{(x-\mu)}{SE} = \frac{(2.5-3)}{1/\sqrt{32}}$

$z_2 = \frac{(3.5-3)}{1/\sqrt{32}}$





z_1

norm.cdf(z_1)

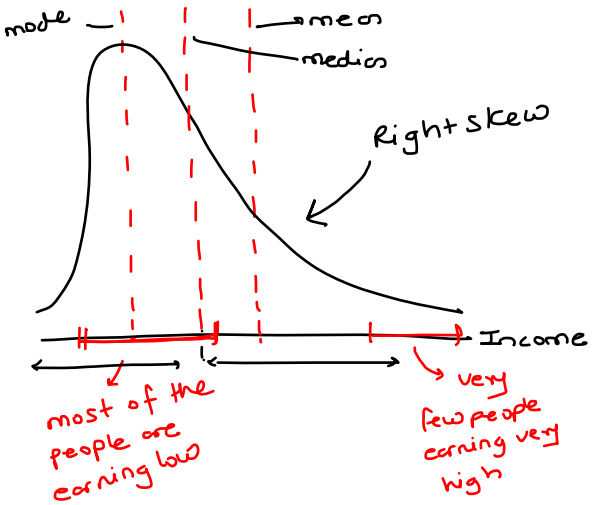
z_2

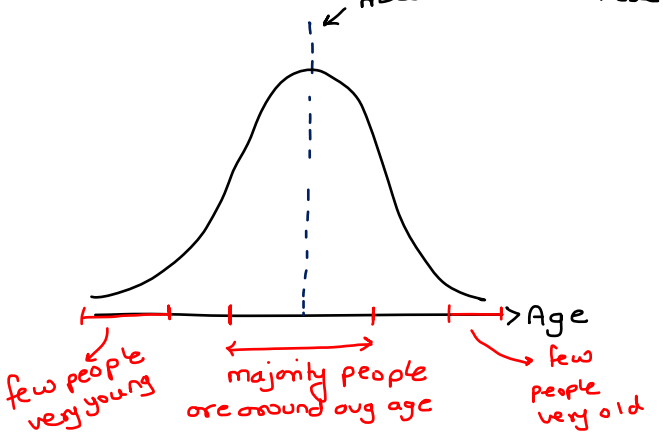
norm.cdf(z_2)

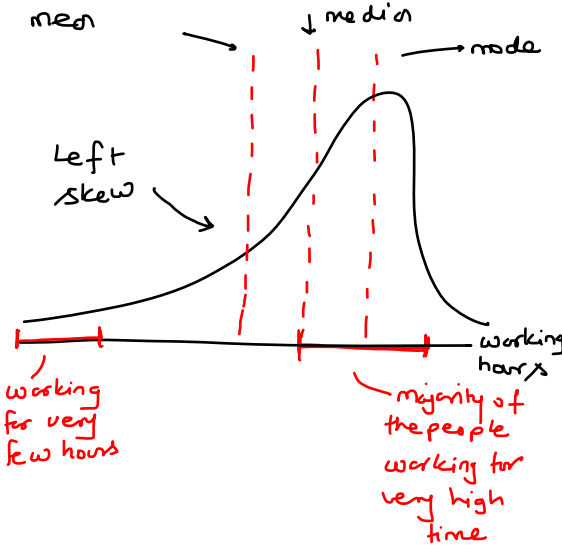
Recap

- Log-normal distribution
- poisson distribution
- poisson & Binomial

=







Log normal dist.

{ A right skew dist. which can be transformed to normal dist. when $\log_{10}$ is taken }

mode = most occurance

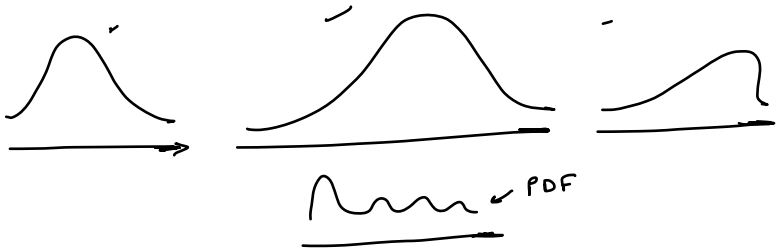
median = middle value such that (50% data below this point & 50% above)

mean = since it is impacted by the outliers, it will shift towards the outlier

Continuous random

PDF

Normal dist



Income
duration
height
weight
working hrs
Age

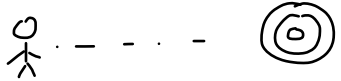
} continuous R.V

discrete Random variable

Pmf

Binomial Dist

# of success in n-trails



n = 10 (trails)

x = 3 (no. of success)

Conditions or Assumption

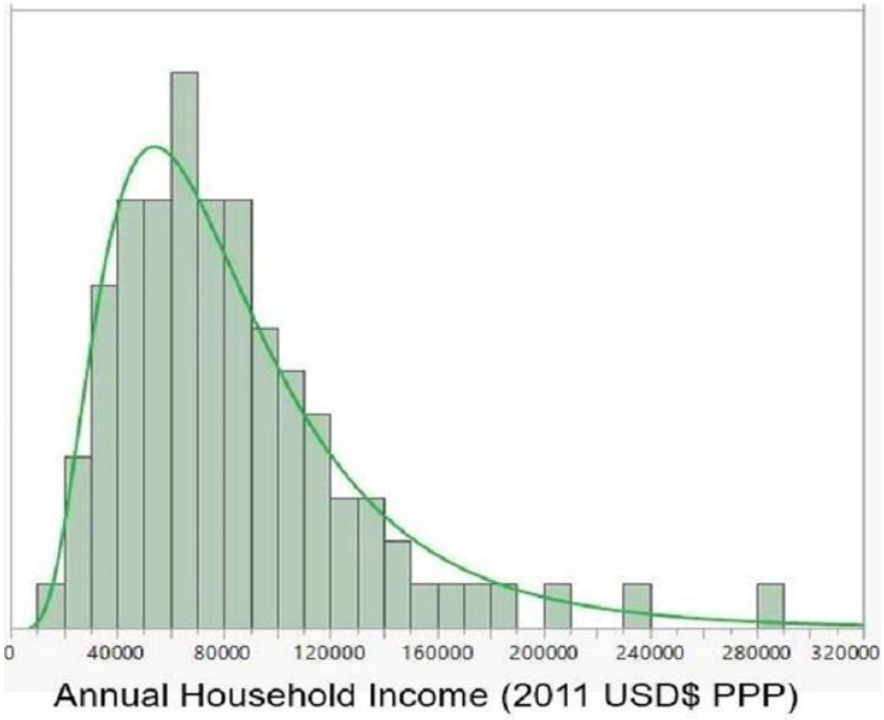
1. outcome is binary
2. P of success is constant across each trail
3. ~~prob~~ n-trails

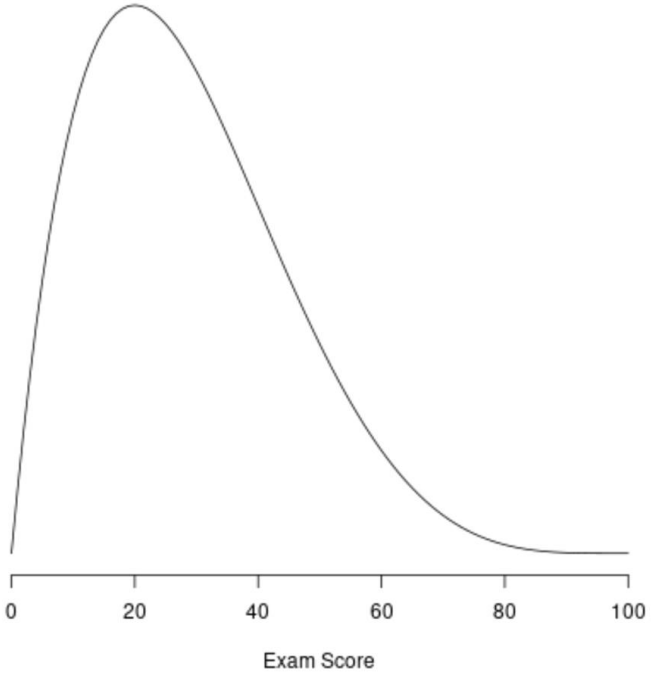
X = # of success

{0, 1, 2, 3, 4, . . . 10}

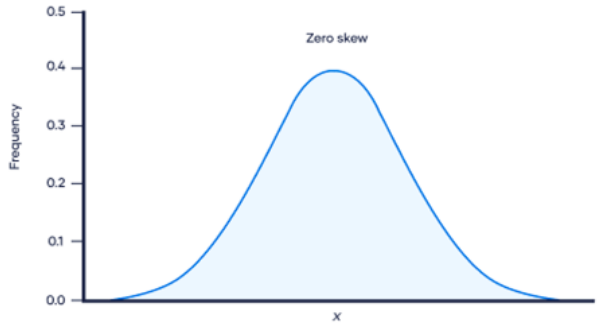
Pmf

Skewed Distribution

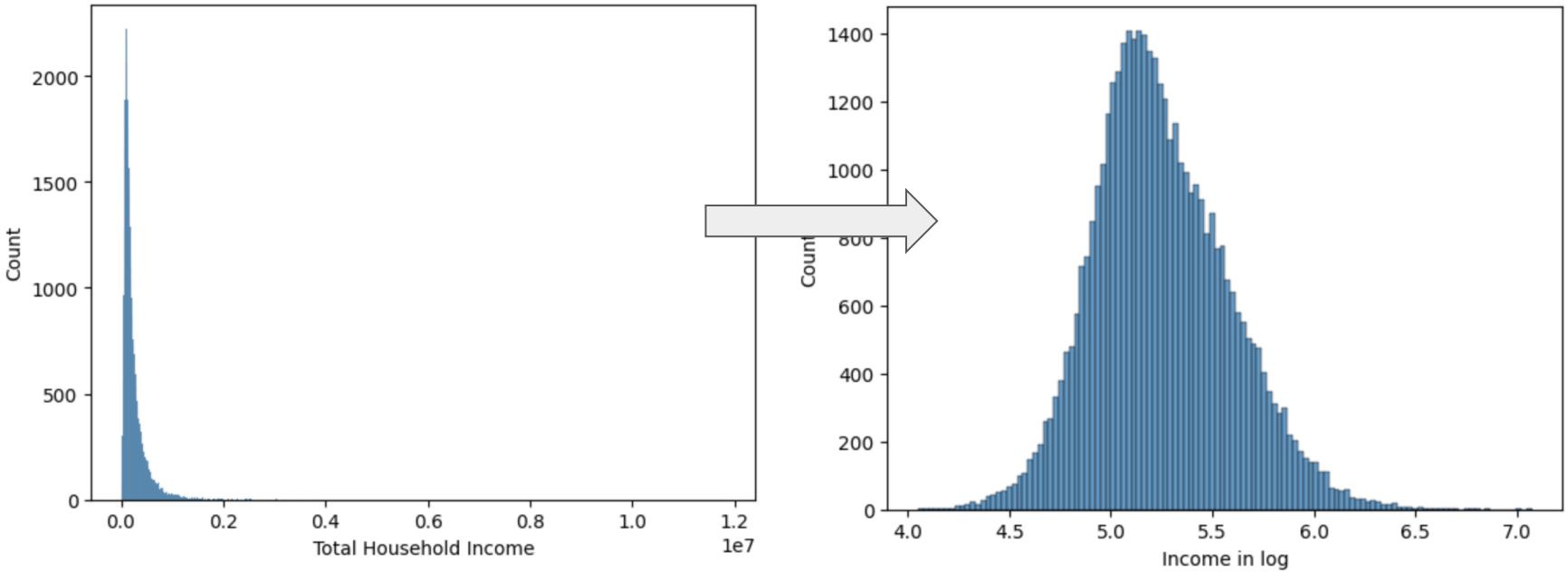




Skewed Distribution



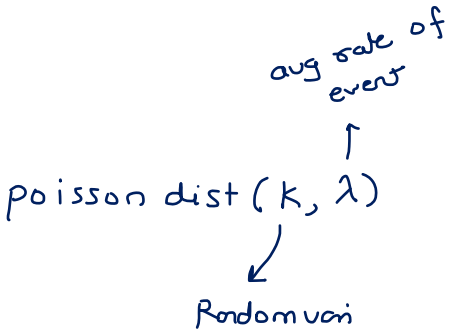
Skewed Distribution



Poisson

x = Discrete

A Poisson distribution is a discrete probability distribution. It gives the probability of an event happening a certain number of times (k) within a given interval of time or space. The Poisson distribution has only one parameter, λ (lambda), which is the mean number of events.



Q: I am a shop owner and based on historical knowledge I know on an avg 3 people visits per hr.
what's the prob that 5 people may visit in next 1 hr.

Rate λ (lambda)

λ is the Poisson rate parameter that indicates the expected value of the average number of events in the fixed time interval.

Examples

- Customer care center receives 100 calls per hour.
- In a cafe, the customer arrives at a mean rate of 2 per min.
- A research has analysed the data of past 3 football world cups to find that the average number of goals in a 90 mins game is 2.5

$\lambda = 100$ calls / hr

$\lambda = 2$ customers / min

$\lambda = 2.5$ goals / game

$\lambda = 2.5$ goals / 90 mins

λ per sec

100 calls ⟶ 60 mins

60 min ⟶ 100

1 min ⟶ $\frac{100}{60}$

1 sec ⟶ $\frac{100}{3600}$

$\lambda = \frac{100}{3600}$ calls / sec

n

There are 80 students in a kindergarten class. Each one of them has 0.015 probability of forgetting their lunch on any given day.

What is the average or expected number of students who forgot lunch in the class?

100 students → 1.5

1 students → $\frac{1.5}{100}$

100 → 1.5

80 → $\left\{\frac{1.5}{100} \times 80\right\}$

→ $0.015 \times n$

$\lambda = 1.2$ students / day

$\lambda = n \times p$

If 3% of electronic units manufactured by a company are defective and the company is able to produce 200 bulbs a day(in a shift of 8 hrs), find the average number of defective bulbs that will be produced in 4 hrs.

$\frac{3}{100} = 0.03$

$\lambda = (0.03 \times 200) = 6$

$\lambda = 6$ defective bulbs / day

$\lambda = 6$ defective bulbs / 8 hrs

$\lambda = 3$ defective bulbs / 4 hrs

A city sees 3 accidents per day on average. $\lambda = 3$ acc / day

X = no of accidents per day (0,1,2,3…..) Random variable

Find the probability that there will be 5 accidents tomorrow

$k = 5$ accident $\quad \lambda = 3$ acc / day

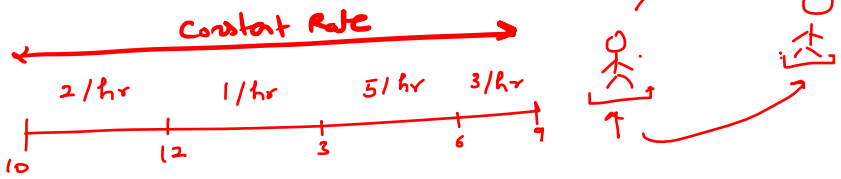
$$P(x=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

at most 3 accidents $\quad$ poisson.cdf (k = 3, mu = 3)

Assumptions Of Poisson Distribution

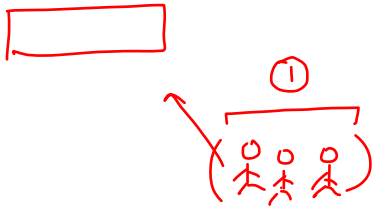
1. **Counting:** (x = Random var. should be discrete)

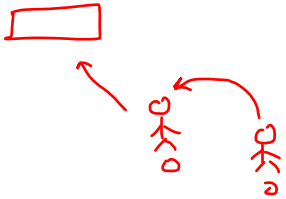
2. **Independence:** one observation should not be influenced by other



3. **Rate:** Is assumed to be constant (in given interval)

4. **No Simultaneous Events:**





Quiz-1: It is known that a certain website makes 10 sales per hour.
In a given hour, what is the probability that the site makes exactly 8 sales?

$\lambda = 10$ sales / hr

$k = 8$

poisson.pmf (k = 8, mu = 10)

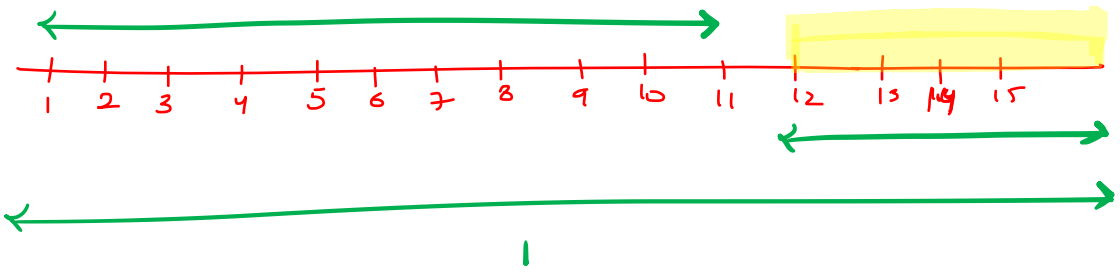
Quiz-2: It is known that a certain hospital experience 4 births per hour. In a given hour, what is the probability that 4 or less births occur?

$\lambda = 4$ births / hr

$P(K \leq 4) = \text{poisson.cdf}(K = 4, \lambda = 4)$

0.6288

Quiz-3: An e-commerce website experiences an average of 10 credit card transactions per day.
What is the probability that there will be at least 12 credit card transactions in a given day?



1 − poissoncdf (k = 11, mu = 10)

A shop is open for 8 hours. The average number of customers is 74 - assume Poisson distributed.

What is the probability that in 2 hours, there will be at most 15 customers?

20000 1000 0.07 0.1

Quiz: There are 80 students in a kindergarten class. Each one of them has 0.015 probability of forgetting their lunch on any given day.

n = 80

What is the probability that exactly 3 of them will forget their lunch today?

1. Is the outcome binary = yes
2. $P(x) = 0.015$
3. $n = 80$

$n = 80$, $p = 0.015$, $x = 3$

Success = p student forgetting lunch

binom.pmf (x = 3, p = 0.015, n = 80) ≈ 0.086

poisson = $\lambda = np = 1.2$/day

poisson.pmf (k = 3, mu = 1.2) ≈ 0.086

- Poisson will approximate to binomial

1. n → very large → Sufficiently larg.
2. p → very small
3. λ → small value