

Statistics

Combinatorics

Note: Please solve the question and be ready with your answer. I will launch the quiz at 07:08AM

Consider the data, in the following table, recorded over a month with 30 days:

On each day I recorded, whether it was sunny, (S), or not, (NS), and whether my girlfriend's mood was good, G, or not (NG).

$$P(\text{Sunny}) = \frac{10}{30} \quad P(\text{NS}) = \frac{20}{30} \quad P(G) = \frac{15}{30} \quad P(NG) = \frac{15}{30} = 0.5$$

		Whether		
		Sunny	Not Sunny	
	Mood			
	Good	9	6	15
	Not Good	1	14	15
		10	20	30

- If I pick a random day, what's the probability of my girlfriend being in good mood given that the day is sunny? 0.9

$$P(\text{Good mood} | \text{Sunny}) = \frac{9}{10} = 0.9$$

- Is her mood dependent on the whether condition?

Yes

$$P(\text{Good mood} | \text{Sunny}) = P(\text{Good mood})$$

$$0.9 \neq 0.5$$

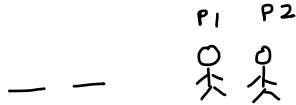
Recap



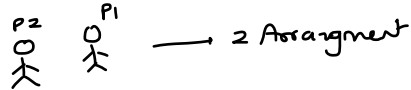
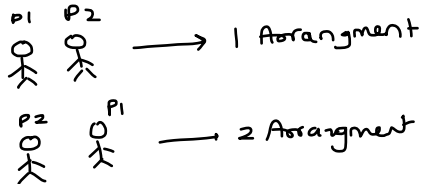
In how many ways you can
Select two balls

$R_1 G_1$ $R_2 G_1$

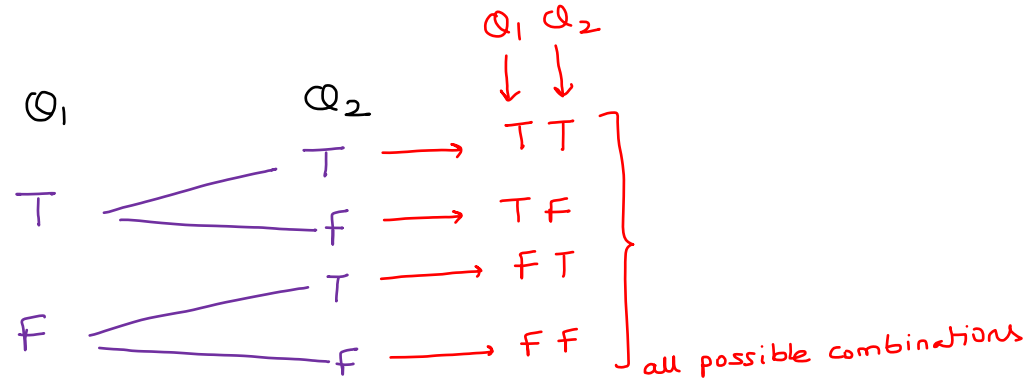
- ① combination \rightarrow Selection
- ② permutation \rightarrow Arrangement



In how many ways they can
arrange themselves in the two
chairs

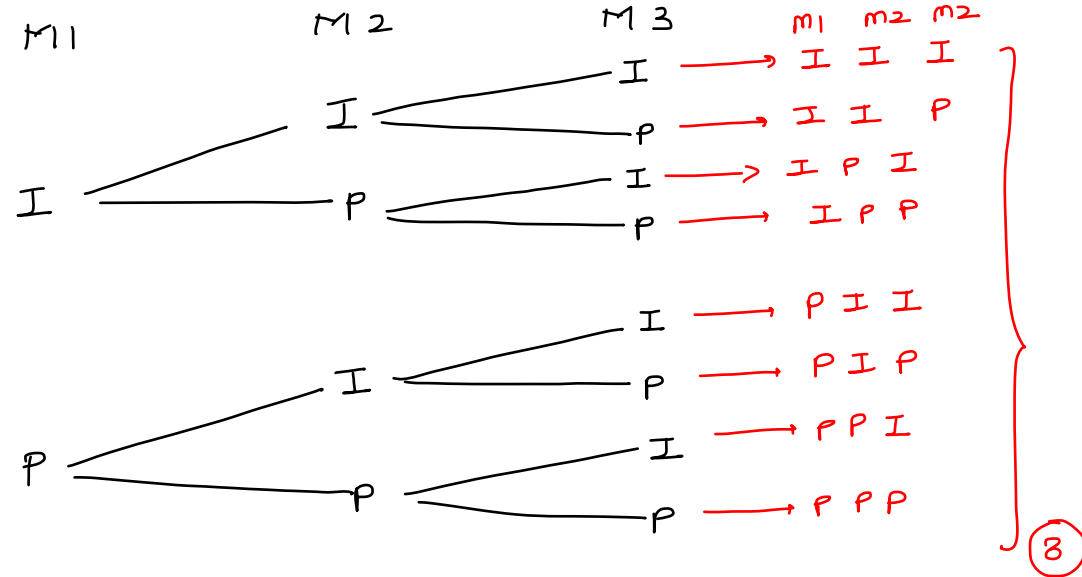


Suppose we have 2 True/False questions. In how many ways can they be solved?



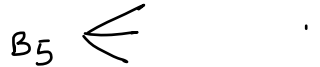
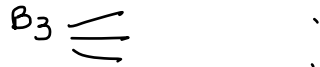
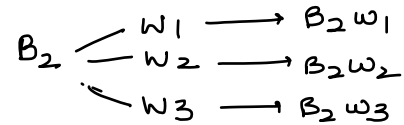
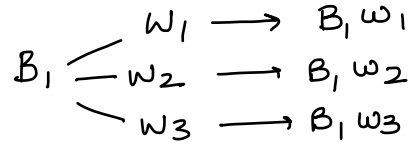
Quiz-1: India and Pakistan play a 3-match series. How many results are possible?
Note that we consider (Ind, Ind, Pak) different from (Ind, Pak, Ind) etc.

Don't answer it in
the chat section



$$\underline{2} \times \underline{2} \times \underline{2} = \textcircled{8}$$

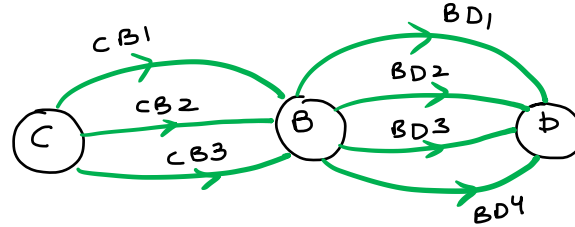
Quiz-2: In a bowl-out, for a specific ball you have to choose a bowler and a wicketkeeper.
Suppose you have 5 bowlers and 3 wicketkeepers. How many ways can you select for a ball?



$$5 \times 3 = 15$$

Quiz-3: There are 3 ways to move from Chennai to Bangalore.
There are 4 ways to move from Bangalore to Delhi.

In how many ways can one reach from Chennai to Delhi via BLR?



$$(3) \times (4) = (12)$$

[CB1 - BD1		CB2 - BD1		CB3 - BD1
CB1 - BD2		CB2 - BD2		CB3 - BD2
CB1 - BD3		CB2 - BD3		CB3 - BD3
CB1 - BD4		CB2 - BD4		CB3 - BD4
(4)		(4)		(4)

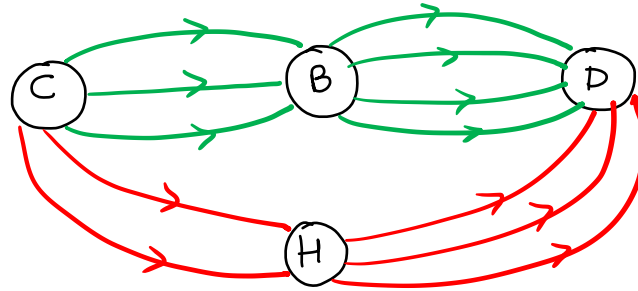
$$(12)$$

Extra: There are 2 ways to move from Chennai to Hyderabad, and 3 ways to move from Hyderabad to Delhi. In how many ways can we move from Chennai to Delhi?

And \rightarrow (x) or \rightarrow (+)

Quiz-4: There are 3 ways to move from Chennai to Bangalore, and 4 ways to move from Bangalore to Delhi.

There are 2 ways to move from Chennai to Hyderabad, and 3 ways to move from Hyderabad to Delhi. In how many ways can we move from Chennai to Delhi?

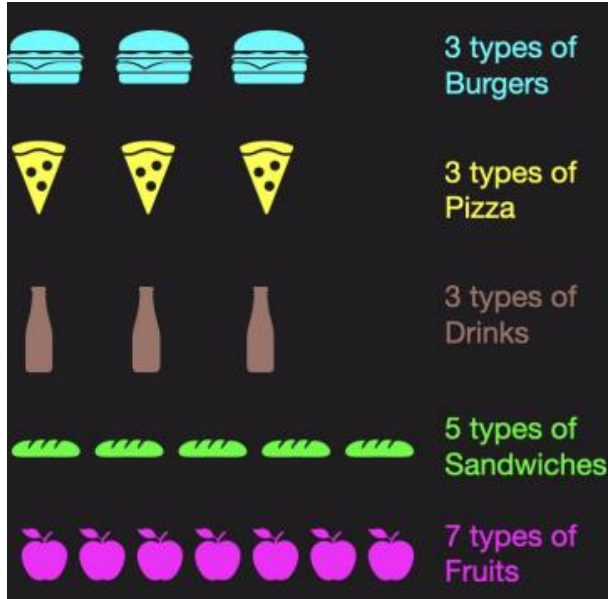


$$\text{Route-1 : } C B D = 3 \times 4 = 12 \text{ or}$$

$$\text{Route-2 : } C H D = 2 \times 3 = 6$$

$$\underline{18}$$

Quiz-5: A fast food outlet has the following types of items in their menu.



$$\begin{array}{r} \text{Combo-1} \\ 3 \quad 5 = 15 \end{array}$$

$$\begin{array}{r} \text{or} \\ \text{Combo-2} \\ 7 \quad 3 = 21 \end{array}$$

$$\begin{array}{r} \text{or} \\ \text{Combo-3} \\ 3 = 3 \\ \hline 39 \end{array}$$

B1S1	B2S1	B3S1
B1S2	B2S2	B3S2
B1S3	B2S3	B3S3
B1S4	B2S4	B3S4
B1S5	B2S5	B3S5

You can choose one of the following combos:

- 1 Buger and 1 Sandwhich
- 1 Fruit and 1 Drink
- 1 Pizza

How many such combos can you make ?

Quiz-6: What is the number of ways of ARRANGING three characters A, B, C?

$$\frac{3}{1} \times \frac{2}{2} \times \frac{1}{3} = 3 \times 2 \times 1 = 3! = 6$$

→ A B C

Select 3 characters → 1 (A B C)

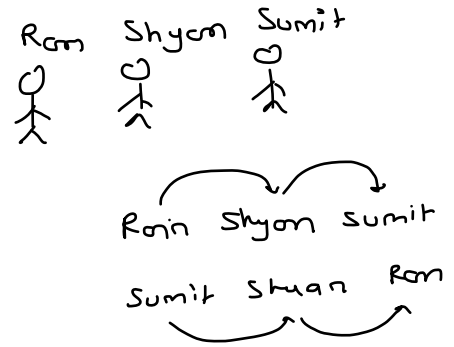
→ A B C

How many arrangement are possible

arrange 3 characters

→ 6

A B C
A C B
B A C
B C A
C A B
C B A



Q-7: In how many ways can the letters of the word "COMPUTE" be arranged such that the vowels always come together?

C M P T

E O U

3!

1 2 3 4 5

5!

$$5! \times 3! = \underline{\underline{720}}$$

Quiz-8: Given 5 different characters, in how many ways can we arrange them in 2 places?

A B C D E

$${}^n P_r = \frac{n!}{(n-r)!}$$

n = # of objects

r = # of positions

$$\underline{5} \quad \underline{4} = 20$$

- In how many ways we can arrange 5 ch. in 5 position

$$5! = 5 \times 4 \times 3 \times 2 \times 1 \quad \leftarrow$$

- In how many ways we can arrange 5 ch. in 4 position

$$\frac{5!}{1!} = 5 \times 4 \times 3 \times 2 \quad \leftarrow$$

- 5 ch. in 3 position

$$\frac{5!}{2!} = 5 \times 4 \times 3 \quad \leftarrow$$

Quiz:9 There are 4 players P1, P2, P3, and P4 who can play in the top-order batting positions of 1, 2, and 3. How many arrangements of top-order can we make from 3 of these 4 players, keeping in mind the order in which these batsmen come?

$$n = \text{objects} = 4 \quad (P_1, P_2, P_3, P_4)$$
$$r = \text{position} = 3 \quad (—, —, —)$$

$$\frac{4}{\quad} \frac{3}{\quad} \frac{2}{\quad} = \textcircled{24}$$

$${}^n P_r = \frac{n!}{(n-r)!} = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4 \times 3 \times 2 \times 1}{1} = \textcircled{24}$$

P1 = Sachin P3 = Rohit
P2 = David P4 = Virat

Combinations

Select 3 players from group of 4

There 4 players who are good in top order batting. Top order positions are No1, No2 and No3.

The names of 4 players are Sachin, Sehwag, Rohit and Kohli.

So how many top order line-ups you can make with these batsman.

Keep in mind here order does not matter.

of combinations
= 4

$$= \frac{24}{6}$$

$$= (4)$$

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2

$$\frac{\text{\# of arrangement}}{\text{\# of arrangement with each combination}} = \text{\# of combinations}$$

$${}^nC_r = \frac{{}^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

In a group of 6 boys and 4 girls, four children are to be selected.

$$= {}^{10}C_4 = \frac{10!}{4!6!}$$

$$n = \# \text{ of objects} = 10 \quad \swarrow$$

$$\text{Select 4 children from 6 boys} = {}^6C_4$$

$$\text{Select 4 children from 4 girls} = {}^4C_4$$

Select 4 children (3 and 1) from 6 boys and 4 girls

↑ girls ↑ boy

$$\downarrow$$
$$[{}^4C_3 \times {}^6C_1]$$
$$=$$

In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

$$\begin{array}{lcl} 1 \text{ Boy} & 3 \text{ girls} & \longrightarrow 6C_1 \times 4C_3 \\ & \text{or} & + \\ 2 \text{ Boy} & 2 \text{ girls} & \longrightarrow 6C_2 + 4C_2 \\ & \text{or} & + \\ 3 \text{ Boy} & 1 \text{ girl} & \longrightarrow 6C_3 + 4C_1 \\ & \text{or} & + \\ 4 \text{ Boy} & 0 \text{ girls} & \longrightarrow 6C_4 + 4C_0 \end{array} \left. \vphantom{\begin{array}{l} 1 \text{ Boy} \\ 2 \text{ Boy} \\ 3 \text{ Boy} \\ 4 \text{ Boy} \end{array}} \right\} \longrightarrow 209$$

=

Quiz-10: A Maruti Showroom has 3 colours in their “Baleno” model and 3 colours in the “Swift” model.

In how many ways can they place it such that Baleno and Swift are kept in alternate slots?

Select 3 people to
sit in 3 chair

①

Arrangement

$$\frac{3}{c_1}$$

$$\frac{2}{c_2}$$

$$\frac{1}{c_3}$$

P1



P2



P3



$$3! = 3 \times 2 \times 1 = 6$$

In how many ways we
may arrange them in 3
chairs



_____ 1 Arrangement



_____ 2 Arrangement



_____ 3 Arrangement

Astrologer \rightarrow "ADN"

How many words we may create from ADN

$$\frac{3}{1} \frac{2}{2} \frac{1}{3} = 3!$$

ADN
AND
DAN
DNA
NAD
NDA

}
6



condition →

make shiva & shakti
sit next to
each other



- ① Select 4 people from
Ram, Shyam, Shiva, Shakti
= ① combination



How many ways we can
make them sit

$$\rightarrow \text{Ram Shyam (shiv + shakti)} = 3! = ⑥$$

$$\rightarrow \text{shiva and shakti} = 2! = ②$$

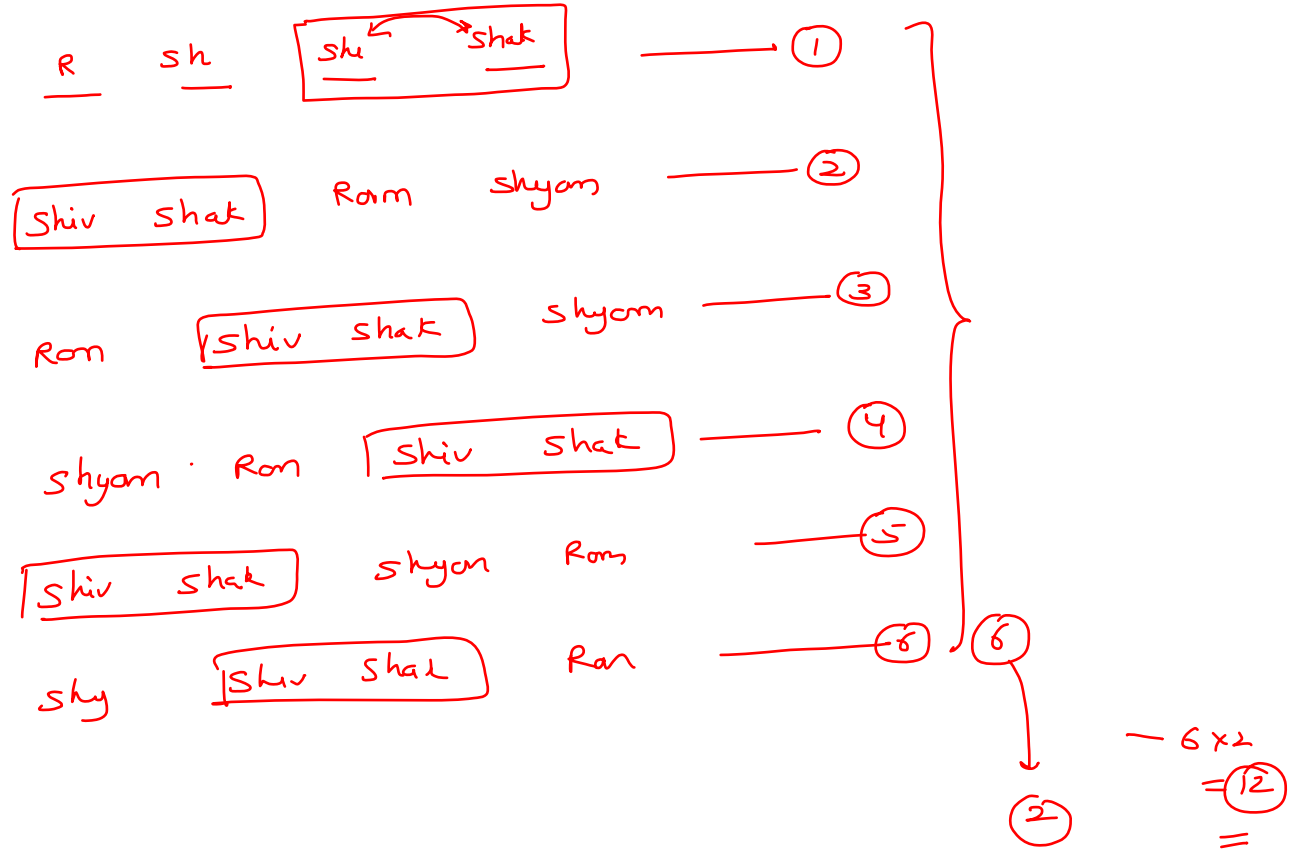
$$> 6 \times 2 = ⑫$$

- ② Arrange Ram, Shyam, Shiva, Shakti
in 4 chairs = ⑫ permutation

R shy shiva shakti — ①
Ram Shyam Shakti Shiva — ②

$$\boxed{3! \times 2!}$$

<u>shiva</u>	<u>shakti</u>
<u>shakti</u>	<u>shiva</u>



A B C

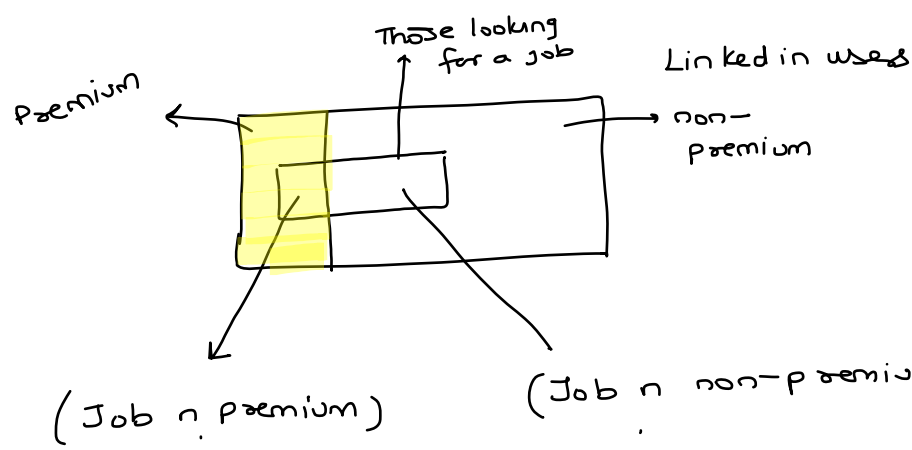
$$\underline{3} \quad \underline{2} = 6$$

- AB - BA - BC
- AC - CA - CB

arrange 2 characters out
of three

city	gender	Buy	
		Yes	No
A	M	12	
A	F		
B	M		
B	F		

of people who belong to city A and gender male and Bought the product = 12



$$\begin{aligned}
 \text{marginal} \quad p(\text{Job}) &= \overbrace{p(\text{Job n premium})}^{\text{Joint}} + \overbrace{p(\text{Job n non-premium})}^{\text{Joint}} \\
 p(\text{Job}) &= p(\text{Job} | \text{premium}) \cdot p(\text{premium}) + p(\text{Job} | \text{non-premium}) \cdot p(\text{non-premium})
 \end{aligned}$$

Job
p
n

	4	2
m	0	0 = 0
f		