Q: Is the following example a random experiment? A manufacturing unit has a lathe machine that is operating in poor condition. The mechanic has informed the owners of the machine that the machine might stop working after 5 hours of continuous operation.

Yes because neither the owner nor the mechanic is certain about the exact time, when the machine is going to stop.

Q: What's the sample space of the above experiment?

**Send your answer privately to me on the chat section. We will discuss at 07:05 AM**

at infinite point of time

$\{0 - \infty\}$ → continuous sample space

immediately

1

1:05

5 min

5 sec

# Recap

| | Sachin century: Yes | Sachin century: No | |
|---|---|---|---|
| India: Yes | 20 | 30 | 50 |
| India: No | 50 | 16 | 66 |
| | 70 | 46 | 116 |

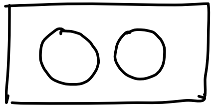
Joints

20/116

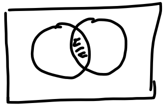
P(S-y ∩ I-y)

marginals

P(S-y) = 70/116



If A B are mutually exclusive (No common event) - $P(A \cup B) = P(A) + P(B)$



If A B are not mutually exc. $P(A \cup B) = \underline{P(A)} + P(B) - \underline{P(A \cap B)}$

Random Experiment

outcome

Sample space

Events

mutually exclusive

collectively exhaustive

$$P(x) = \frac{\#\text{ of outcomes in Event A}}{\text{Total no. of outcomes in the sample space}}$$

Random exp.

Q: Toss of two coins

$\Omega = \{HH, HT, TH, TT\}$

A: Both coin lands with same outcome

B: Both coin lands with different outcome

A: {HH, TT}

B: {HT, TH}

$A \cup B$ : {HH, TT, HT, TH} = $\Omega$

collectively exhaustive

$A \cap B$ : { } = $\phi$

mutually exclusive

## Conditional Probability

What's the probability that it will rain today?

$P(\text{Rain}) = 0.5$

$P(\text{not Rain}) = 0.5$

[No sun and Heavy Black clouds]

If you know this information then what's the prob. that it will rain today?

Additional info is impacting your prior prob.

[Clear sky and bright sun]

given

conditional prob

$P(\text{Rain} \mid \text{heavy clouds and no sun}) > P(\text{Rain})$

$P(\text{Rain} \mid \underbrace{\text{no clouds and bright sun}}_{\text{Additional info}}) < P(\text{Rain})$

It is known that –

60% people use Swiggy,

50% use Zomato,

20% people use both.

- when no info. is provided, we focus on the entire sample space
- with the additional info, the sample space will get reduced to that info.

(Atual)

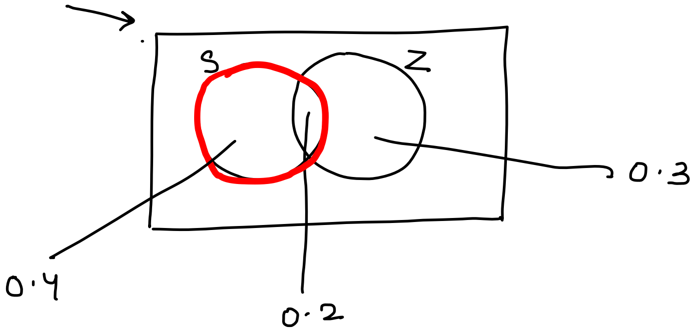
Among those who use Swiggy, what fraction also use Zomato?

$P(\text{Swiggy}) = \frac{60}{100} = 0.6$

$P(\text{zomato}) = \frac{50}{100} = 0.5$

$P(\text{Swiggy and zomato}) = \frac{20}{100} = 0.2$

{ given will always be in the denominator }




Additional Info: Atual is already a swiggy user

$P\left(\text{Atual is using Zomato} \mid \text{Atual is already using swiggy}\right) = \frac{20}{60}$

$= 1/3$

P(

$\frac{20}{60}$

$P(Z|S) = \frac{P(Z \cap S)}{P(S)} = \frac{0.2}{0.6} = 2/6$

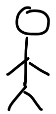
$=$

It is known that –

80% people like cappuccino,
40% people like espresso, and
30% like both.

Among the people who like cappuccino, what fraction of people like espresso?

manjot

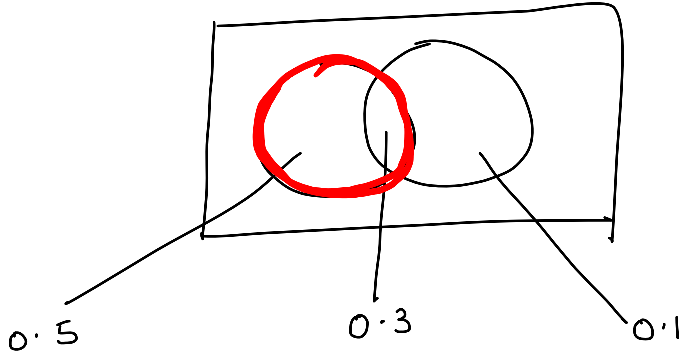


$P(\text{manjot} - \text{capp}) = \frac{80}{100}$

$P(\text{manjot} - \text{espresso}) = \frac{40}{100}$

$P(\text{manjot} - \text{cap and esp}) = \frac{30}{100}$

Add info: manjot alredy loves capp




$\overline{\phantom{00}}$
$80$

$P(\text{Espresso} \mid \text{Cappuccino}) = \frac{30}{80}$

$= 3/8$

$=$

$$P(E|C) = \frac{P(E \cap C)}{P(C)} = \frac{0.3}{0.8} = 3/8$$

# Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = P(B|A) \cdot P(A)$$

$$P(A \cap B) = \overbrace{P(A|B) \cdot P(B)}$$

It is known that-
30% of emails are spam, and 70% are not spam.
The word "purchase" occurs in 80% of spam emails.
It also occurs in 10% of non-spam emails.
Overall, in what percentage of emails would we see the word "purchase"?

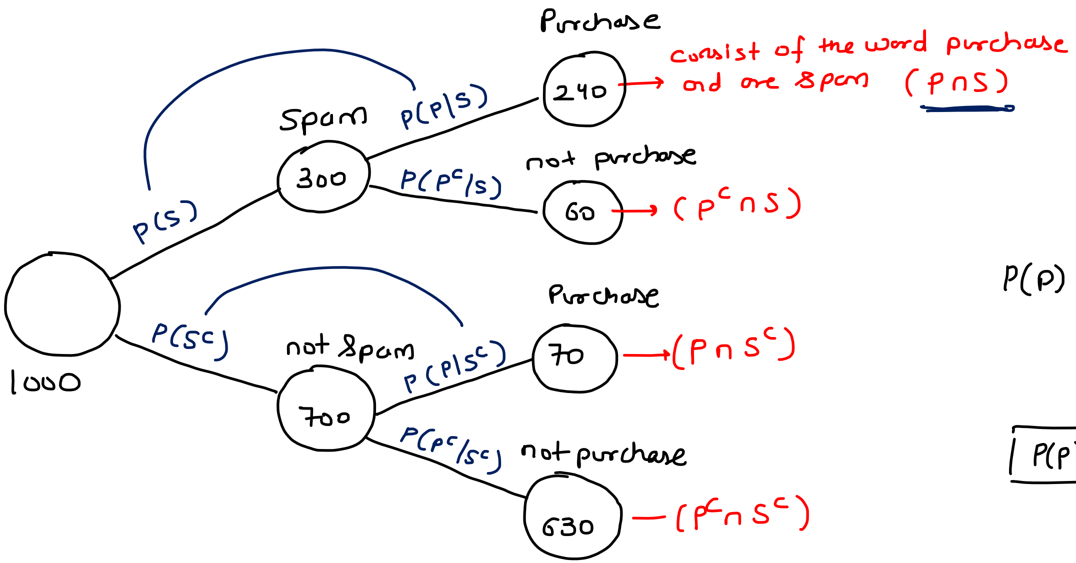
Tree dig.

$P(Spam) = 0.3$

$P(not\ spam) = 0.7$

$P(purchase \mid spam) = 0.8$

$P(purchase \mid not\ spam) = 0.1$




$$P(purchase) = \frac{240+70}{1000} = \frac{310}{1000} = 31\%$$

$$P(P) = P(P\cap S) + P(P\cap S^c)$$

$$= \frac{240}{1000} + \frac{70}{100}$$

$$\boxed{P(P) = 0.31}$$

$$P(P\cap S) = P(P\mid S)\cdot P(S)$$

$$P(P\cap S^c) = P(P\mid S^c)\cdot P(S^c)$$

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Spam
purchase

Spam

purchase

| | S | $S^c$ | |
|---|---|---|---|
| P | 240 | 70 | 310 |
| $P^c$ | 60 | 630 | 690 |
| | 300 | 700 | 1000 |

$P(\text{Spam} \mid \text{purchase}) =$

$$\frac{P(\text{Spam} \cap \text{Purchase})}{P(\text{purchase})}$$

$$= \frac{240}{310}$$

$$P(S^c \mid P^c) = \frac{630}{690}$$

Marginals prob.

$P(\text{Spam}) = 300/1000$

$P(\text{not spam}) = 700/1000$

$P(\text{Purchase}) = 310/1000$

$P(\text{not purchase}) = 690/1000$

Joint prob.

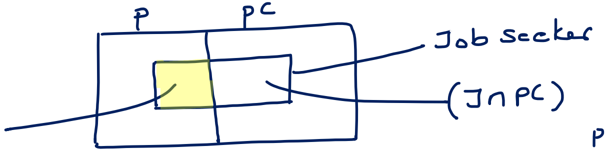
$P(\text{spam and purchase}) = 240/1000$

$P(S^c \cap P) = 70/1000$

$P(S \cap P^c) = 60/1000$

$P(S^c \cap P^c) = 630/1000$

Quiz-1




$J = (J \cap P) + (J \cap P^c)$

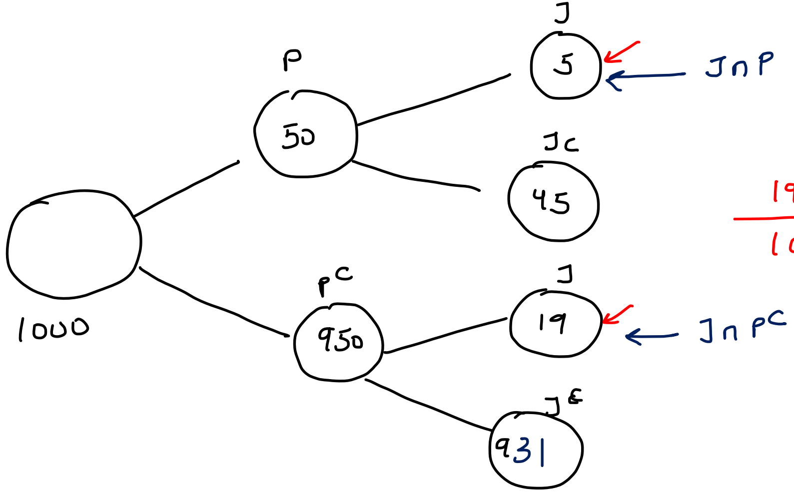
$P(J) = P(J \cap P) + P(J \cap P^c)$

$= P(J|P) \cdot P(P) + P(J|P^c) \cdot P(P^c)$

It is known that –

5% of all LinkedIn users are premium users 10% of premium users are actively seeking new job opportunities.

Only 2% of non-premium users are actively seeking new job opportunities. Overall, what percentage of people are actively seeking new job opportunities




$\frac{19+5}{100} = \frac{24}{1000}$

Premium LinkedIn users

Job seekers

| | P | $P^c$ | |
|---|---|---|---|
| J | 5 | 19 | 24 |
| $J^c$ | 45 | 931 | 976 |
| | 50 | 950 | 1000 |

$P(\text{Job}) = \frac{24}{1000} = 0.24\%$

$\boxed{P(J) = P(J \cap P) + P(J \cap P^c)}$

$= P(J|P) \cdot P(P) + P(J|P^c) \cdot P(P^c)$

$= [0.1 \times 0.05 + 0.02 \times 0.95]$

Quiz-2:

$P(\text{Disease}) = 0.1$

$P(+ve \mid \text{Disease}) = 0.8$

$P(+ve \mid \text{not Disease}) = 0.05$

A disease affects 10% of the population. Among those who have the disease, 80% get “positive” test result Among those who don’t have the disease, 5% get “positive” test result. What is P(+ve | Disease) ?

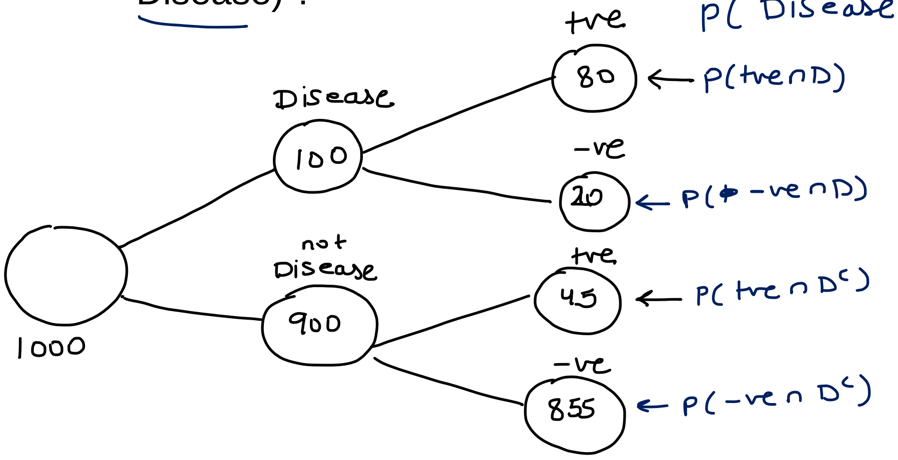
$P(\text{Disease} \mid +ve) = ?$

↓

denominator

$$\frac{80}{80+45}$$

$P(+ve \mid D)$




$$P(D \mid +ve) = \frac{P(D \cap +ve)}{P(+ve)}$$

$$= \frac{80}{80+45}$$

$$\frac{125}{1000}$$

Disease

| Positive | D | $D^c$ | |
|---|---|---|---|
| +ve | 80 | 45 | 125 |
| -ve | 20 | 855 | 875 |
| | 100 | 900 | 1000 |

$$P(D \mid +ve) = \frac{80}{125}$$

$$P(D \cap +ve) = \frac{80}{1000} = 0.08$$

Quiz-3:

A disease affects 10% of the population. Among those who have the disease, 80% get “positive” test result. Among those who don’t have the disease, 5% get “positive” test result. What is P(+ve ∩ Disease)?

A disease affects 10% of the population. Among those who have the disease, 80% get “positive” test result. Among those who don’t have the disease, 5% get “positive” test result.

Now, if a person tests +ve, what is the prob that they actually have the disease?

Q: A ecommerce website shows two type of Ad $< \begin{matrix} A \\ B \end{matrix}$

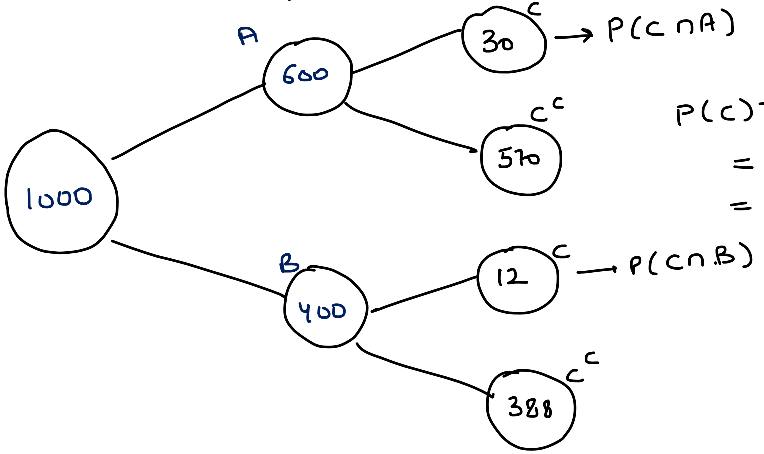
60% of visitor see type-A

40% of visitor see type-B

5% of those whoo see type-A clicks on the Ad $\longrightarrow P(\text{click} \mid A)$

3% of those who see type-B clicks on the Ad $— P(\text{click} \mid B)$

$P(\text{click}) = ?$




$$P(c) = P(c \cap A) + P(c \cap B)$$
$$= P(c \mid A) \cdot P(A) + P(c \mid B) \cdot P(B)$$
$$= 0.05 \times 0.6 + 0.03 \times 0.4$$
$$= 0.042$$

Ad-type

| | A | B | |
|---|---|---|---|
| C | 30 | 12 | 42 |
| $C^c$ | 570 | 388 | 958 |
| | 600 | 400 | 1000 |

click

$$P(\text{click}) = \frac{42}{1000}$$

# Bayes theorm

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \text{---} \; (1)$$

$$P(A\cap B) = P(B\cap A)$$

$$P(B\cap A) = P(B|A)\cdot P(A) \quad \text{---} \; (2)$$

likelihood

Prior

$$\boxed{P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)}}$$

Posterior prob.

marginal

Bayes theorm

connects A|B with B|A