Bayes Theorem Application

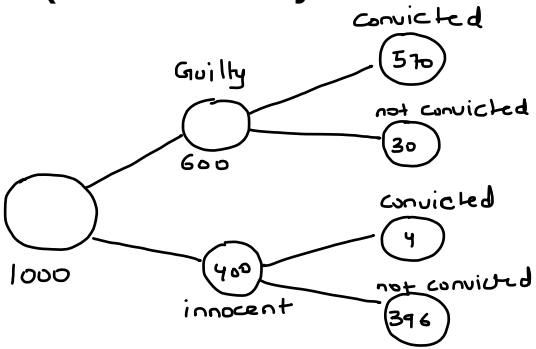
Out of those brought to court, there are 60% which are actually guilty. Of those that are guilty, 95% of them are convicted. But there are 1% of innocent people who get falsely convicted. What is the probability that you are actually innocent given that you are convicted?

$P(\text{Guilty}) = 0.6 \quad P(\text{Innocent}) = 0.4 \quad P(\text{convicted} \mid \text{guilty}) = 0.95 \quad P(\text{not convicted} \mid \text{guilty}) = 0.05$

(Please share your answers privately to me on the chat section.)

$P(\text{convicted} \mid \text{innocent}) = 0.01$

$P(\text{not convicted} \mid \text{innocent}) = 0.99$



	Guilty	Innocent	
convicted	570	4	574
not convicted	30	396	426
	600	400	1000

$$P(\text{innocent} \mid \text{convicted}) = \frac{4}{574}$$

$$= 0.0069$$

$$\approx 0.7\%$$

$$P(\text{innocent} \mid \text{convicted}) = \frac{4}{574}$$

Marginal Prob

$P(\text{convicted}) = \frac{574}{1000}$

$P(\text{not convicted}) = \frac{426}{1000}$

$P(\text{Guilty}) = \frac{600}{1000}$

$P(\text{innocent}) = \frac{400}{1000}$

Joint prob.

$P(\text{convicted} \cap \text{Guilty}) = \frac{570}{1000}$

$P(\text{convicted} \cap \text{Innocent}) = \frac{4}{1000}$

$P(\text{not convicted} \cap \text{Guilty}) = \frac{30}{1000}$

$P(\text{not convicted} \cap \text{Innocent}) = \frac{396}{1000}$

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(column)

A

B (index)	Guilty	Innocent	
convicted	570	4	574
not convicted	30	396	426
	600	400	1000

Joint & marginal table

	Guilty	innocent	
convicted	570/1000	4/1000	574/1000
not convicted	30/1000	396/1000	426/1000
	600/1000	400/1000	1000/1000

Prob. of a person being guilty and convicted

Here we have divided each cell with row total

P(A|B)

	Guilty	innocent	
convicted	570/574	4/574	574/574
not convicted	30/426	396/426	426/426
	600/1000	400/1000	1000/1000

P(Guilty | convicted)

P(Guilty | not convicted)

P(innocent | convicted)

$P(\text{Guilty} \mid \text{convicted}) = \frac{P(\text{Guilty} \cap \text{convicted})}{P(\text{convicted})}$

$P(\text{Guilty} \mid \text{not convicted}) = \frac{P(\text{Guilty} \cap \text{not convicted})}{P(\text{not convicted})}$

P(Innocent | convicted)

P(Innocent | not convicted)

P(B|A)

	Guilty	Innocent	
convicted	570/600	4/400	574/1000
not convicted	30/600	396/400	426/1000

P(convicted | guilty)

- P(A|B) = divide each joint with index total
- P(B|A) = divide each joint with column total

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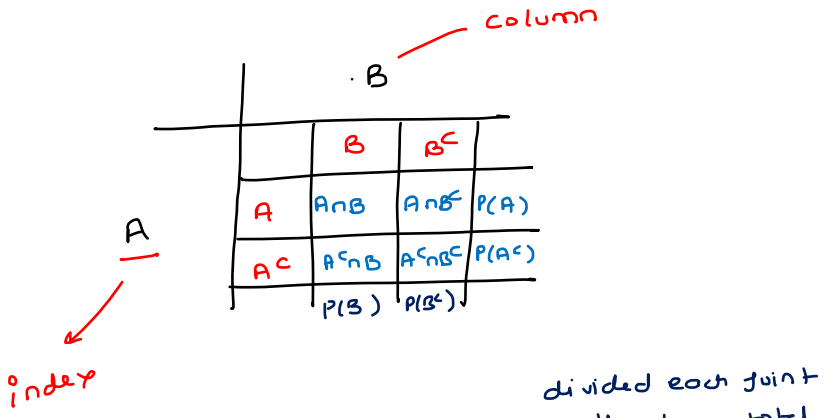
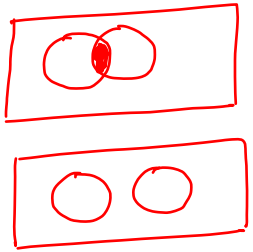


divided each joint with column total

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

what so ever is given to us forms the denominator

Summary of formulas



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ → Event A and B are not mutually exclusive

$P(A \cup B) = P(A) + P(B)$ → event A and B are mutually exclusive

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$ Baye's theorem

Independent cond.

$$\begin{cases} P(A|B) = P(A) \\ P(A \cap B) = P(A) \cdot P(B) \end{cases}$$

Dependent cond.

$$\begin{cases} P(A|B) = \frac{P(A \cap B)}{P(B)} \\ P(A \cap B) = P(A|B) \cdot P(B) \end{cases}$$

Independent & Dependent Events

Dependent Event

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{——— (1)}$$

$P(\text{Rain}) = 0.5$

$P(\text{not Rain}) = 0.5$

Independent Event

$$\left[P(A|B) = P(A) \right]$$

This Eq. means that the additional info. B is not impacting the prior prob. of A

$P(\text{Rain} \mid \text{Heavy clouds and no sun}) = 0.79 \; [> P(\text{Rain})]$

$P(\text{Rain} \mid \text{No clouds and Bright sun}) = 0.30 \; [< P(\text{Rain})]$

these info were impacting the prior

$P(\text{Rain} \mid \text{I had poha in breakfast}) = P(\text{Rain})$

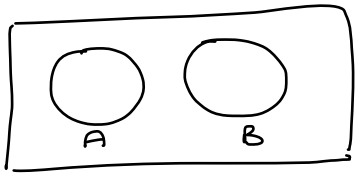
This info is not impacting the prob. of Rain

$$\left[P(A \cap B) = P(A|B) \cdot P(B) \right]$$

multiplication rule when two events are dependent

$$\left[P(A \cap B) = P(A) \cdot P(B) \right]$$

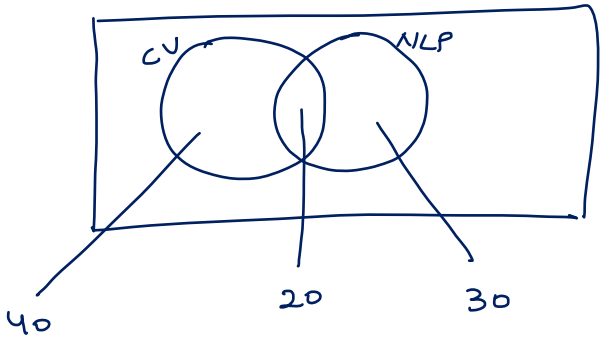
multiplication rule when two events are independent



Two events are mutually exp. then they are definently Dependent

Quiz-1. Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language processing (NLP). Also, it is seen that 20 have taken both CV and NLP.

Given that a person has taken NLP, what is the probability that he has also taken CV?



$P(CV \mid NLP) = \frac{20}{50} = 2/5$

Are these two events independent?

$P(CV \mid NLP) = P(CV)$

$\frac{20}{50} \neq \frac{60}{100}$

$P(CV \cap NLP) = P(CV) \times P(NLP)$

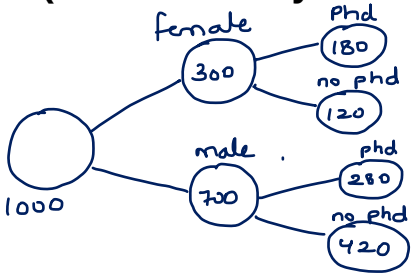
$\frac{20}{100} \neq \frac{60}{100} \times \frac{50}{100}$

Dependent

In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD

1 - What is the probability that a randomly chosen faculty member is a female and has PHD?
2 - What is the probability that a randomly chosen faculty member is a male and has PHD?
3 - What is the probability that a randomly chosen faculty member has a PHD?
4 - What is the probability that a randomly chosen PHD holder is female?

(Please share your answers privately to me on the chat section.)



	phd	not phd	
male	280	420	700
Female	180	120	300
	460	540	1000

$P(\text{female and phd}) = \frac{180}{1000}$ 0.18

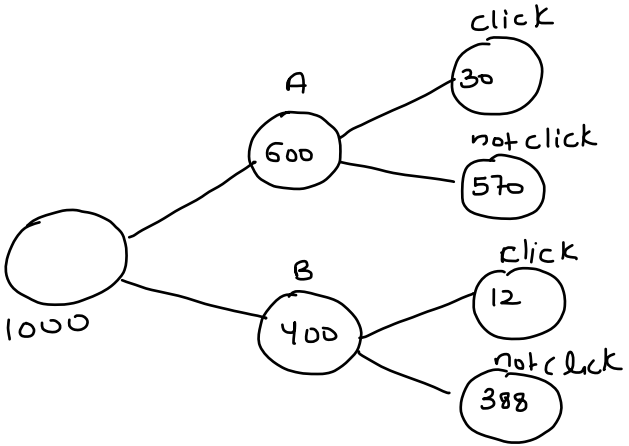
$P(\text{phd}) = \frac{460}{1000}$ 0.46

$P(\text{male and phd}) = \frac{280}{1000}$ 0.28

$P(\text{female} \mid \text{phd}) = \frac{180}{460}$ 0.39

Quiz-2. An website shows two types of ads:
60% of the visitors see Type A ads, and 40% visitors see Type B ads.
The click-through rate for A is 5%, and for B is 3%.

A visitor to the website does not click the ad. What is the probability that he saw Type A ad?



	A	B	
click	30	12	42
not click	570	388	958
	600	400	1000

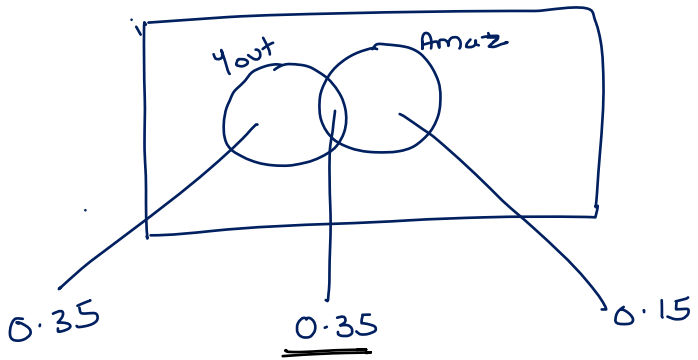
$$P(A \mid \text{not click}) = \frac{570}{958} = 0.59$$

$$P(A \mid \text{not click}) = \frac{570}{570+388}$$

$$= \frac{570}{958} = 0.59$$

Quiz-3. A website has noticed the following stats.
Among those who saw the ad, 70% saw it on Youtube, 50% saw it on Amazon, 35% saw it on both.
A random person who saw the ad on Amazon is chosen. What is the probability that he also saw the ad on Youtube?

Independent or Dependent



Independent But not mutually exclusive

P(youtube ∩ Amazon)

= P(youtube) × P(Amazon)

= 0.7 × 0.5

= 0.35

$$P(\text{Youtube} \mid \text{Amazon}) = \frac{0.35}{.50} = 0.7$$

$P(y|A) = P(y)$

$0.7 = 0.7$] independent