Hypothesis Testing

Z Test

1. How to write Hypothesis
2. test metric — p-value

How to conduct test ⟶

Hey Girl,
You Must Be p>0.05, Because
I fail To Reject You

Quick Recap Test:

Initial reported value

- A factory manufactures machines that dispenses 80 ml of liquid in a bottle. An employee believes that there is a fault in the production due to which the average amount of fluid is less than 80 ml. Choose Ho and H1

  $H_0: \mu \geq 80ml$
  $H_1: \mu < 80ml$

- A company features car batteries with avg life span of 2 or more years. An engineer believes that this value to be less. Choose Ho and H1

  $H_0: \mu \geq 2$
  $H_1: \mu < 2$

- P value is : prob. of null hypo. to be true

- If the p-value of a test is 0.0001 and the null hypothesis is "The person is not a criminal" what will you conclude?

  H_0: person is innocent
  P-value = 0.0001 $\quad \alpha = 0.05$

  If p is low, null will go $\therefore P < \alpha$ $\therefore$ The prob. of the person being criminal is very very high.

CLT Recap

Suppose we have a sample of 40 people with following parameters

Average height: 65 inches (μ)
Std dev height: 2.5 inches (σ)

Can we come up with the sampling distribution based on the above information?

CLT Recap

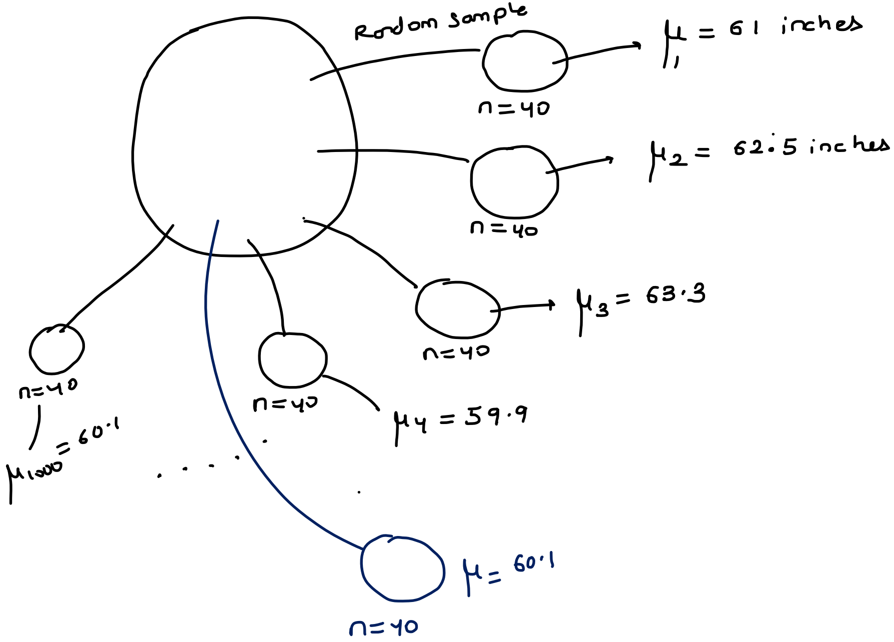
Suppose we have a sample of 40 people with following parameters

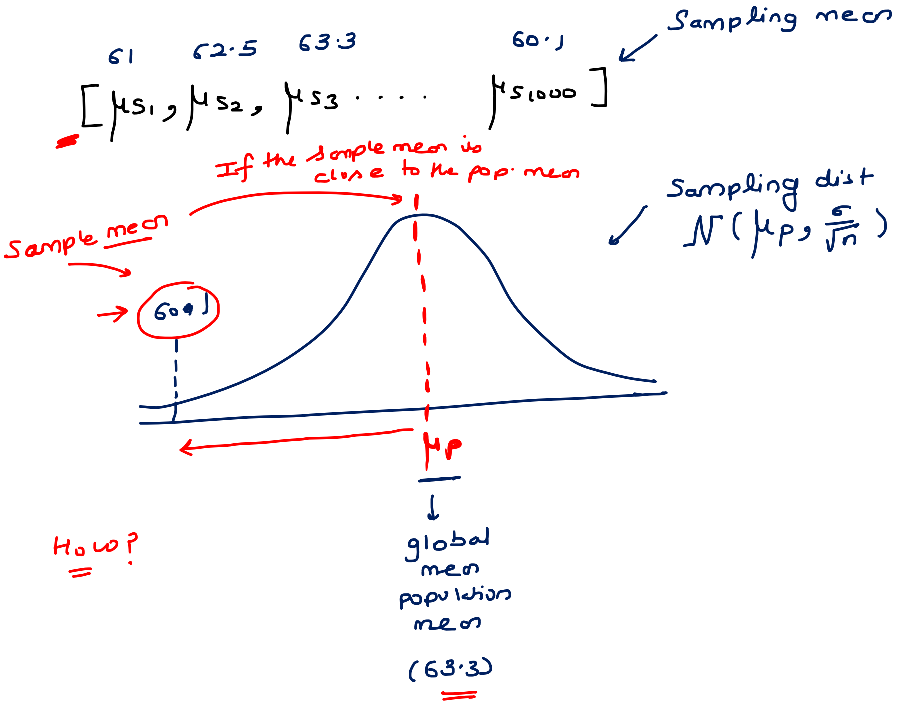
Average height: 65 inches (μ)
Std dev height: 2.5 inches (σ)

Find 90% confidence interval

CLT

Population of all NBA player





Reported value

σ

As per the NBA, the avg height of the NBA player is 66.3 inches and standard deviation of 3.84 inches. The coach at Boston Basketball club is preparing his team and he found that the avg height of the player in his team is 65.9 inches. Can we conclude (with a confidence of 90%) that the avg height in his team is significantly less than the avg height of a NBA registered team? Assume that Boston Basketball team consist of 40 players including fulltime parttime and national players

$\alpha = 10\%$

$\alpha = 0.1$

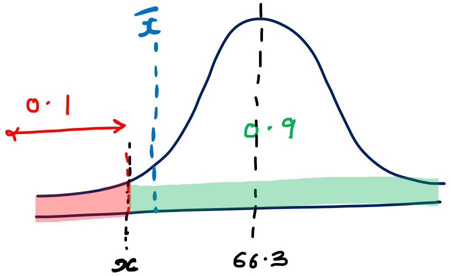
$\bar{x} = 65.9$

Step-1 $\left[\begin{array}{l} H_0: \mu \geq 66.3 \\ H_a: \mu < 66.3 \end{array} \right.$

Step-2 $N\left(66.3, \frac{3.84}{\sqrt{40}}\right)$

Step-3 Type of test

$\bar{x} = 65.9$ inches



norm.ppf(0.1) = −1.28

$LCV = \mu_p - 1.28 \times se$

$= 66.3 - 1.28 \times \left(\frac{3.84}{\sqrt{40}}\right)$

$= 65.5$

critical value method

Lower critical value [65.5]

(we fail to reject null)

Reported value → Sampling dist

↓

critical value

↓

finding where the sample mean lies at?

- If the value fall on rejection side, we reject null
- If the value fall on acceptance side, we fail to reject null

As per the NBA, the avg height of the NBA player is 66.3 inches and standard deviation of 3.84 inches. The coach at Boston Basketball club is preparing his team and he found that the avg height of the player in his team is 65.9 inches. Can we conclude (with a confidence of 90%) that the avg height in his team is significantly less than the avg height of a NBA registered team? Assume that Boston Basketball team consist of 40 players including fulltime parttime and national players

$H_0: \mu \geq 66.3$

$H_a: \mu < 66.3$

Lower tail test

$N\left(66.3, \frac{3.84}{\sqrt{40}}\right)$

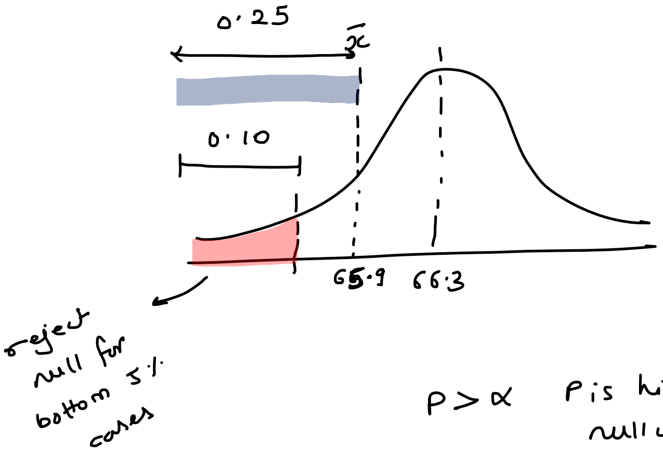
$\bar{x} = 65.9$ lies at bottom 25% cases

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{65.9 - 66.3}{3.84/\sqrt{40}}$$

$$= -0.65$$

norm.cdf$(-0.65) = 0.25$

= p-value



$p > \alpha$ p is high null will fly

we fail to reject null

Reported and proved

According to a report, the average package of an IIT Kanpur student (Master's) is $35,500 with a standard deviation of $4,200. (Note: This data was collected based on a survey that the college conducted on 140 students after the end of the placement season on campus). Sumit, a student in the IIT Kanpur M.Sc. program, was offered a package of $34,325 and he believes that the average packages as reported by the official report are wrong, and that the average is different from $35,500. Can we test this at a significance level of 0.10?

Step-1 $H_0 : \mu = 35500$

$H_a : \mu \neq 35500$

Step-2 $\mathcal{N}\left(35500, \frac{4200}{\sqrt{140}}\right)$

$\alpha = 0.10$

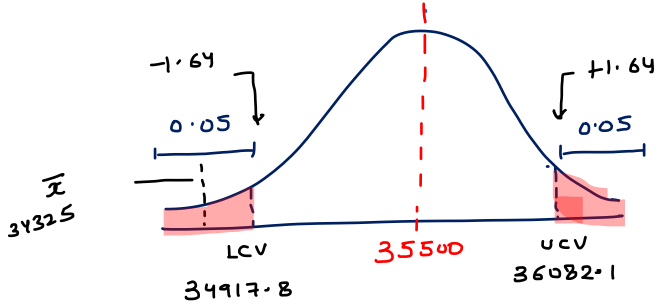
$\frac{\alpha}{2} = 0.05$

Step-3 Two tailed test

norm.ppf(0.05) =

$LCV = 35500 - 1.64\left(\frac{4200}{\sqrt{140}}\right)$

$UCV = 35500 + 1.64\left(\frac{4200}{\sqrt{140}}\right)$



Ha

> Right →

< Left ←

≠ Both / Two

Since 34325 ($\bar{x}$) is in rejection region we reject null

→ The reported value is not 35500

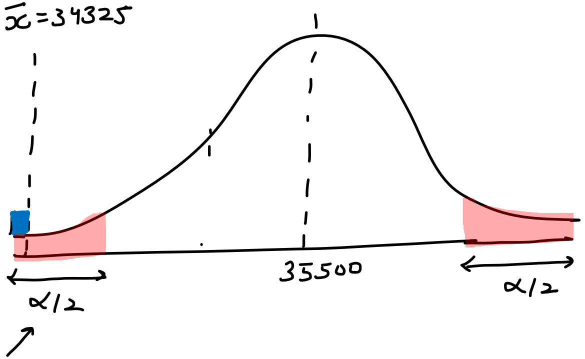
According to a report, the average package of an IIT Kanpur student (Master's) is \$35,500 with a standard deviation of \$4,200. (Note: This data was collected based on a survey that the college conducted on 140 students after the end of the placement season on campus). Sumit, a student in the IIT Kanpur M.Sc. program, was offered a package of \$34,325 and he believes that the average packages as reported by the official report are wrong, and that the average is different from \$35,500. Can we test this at a significance level of 0.10?

$\alpha = 0.10$

$H_0: \mu = 35500$

$H_a: \mu \neq 35500$

$\mathcal{N}\left(35500, \frac{4200}{\sqrt{140}}\right)$



$$z = \frac{34325 - 35500}{4200/\sqrt{140}}$$

$z = -3.31$

$p = \text{norm.cdf}(z)$

$p = 0.0004 \times 2$ → Because two tail

$p\text{-value} = 0.0008$

$\alpha = 0.10$

P is low null will go

we reject null

You want to rent an unfurnished one-bedroom apartment in NYC. According to the Housenice.com, a website that lists apartments for rent, the average rent in NYC is $1000 with a standard deviation of $200. You collected data on 40 such apartments listed on the website, the average rent of your sample was $1100?

$\alpha = 0.05$ (5% cases we will reject null)

Is the information posted by Housenice.com wrong?

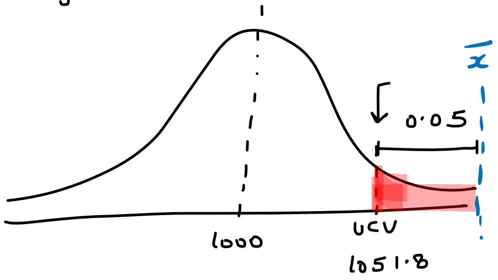
either greater then 1000 or less than → $H_0: \mu = 1000$, $H_a: \mu \neq 1000$

→ avg rent not equal to 100

→ $H_0: \mu \leq 1000$, $H_a: \mu > 1000$ → prove avg rent is greater than 1000

$\sim N\left(1000, \frac{200}{\sqrt{40}}\right)$

Right tail



$\text{norm.ppf}(0.95) = 1.64$

$UCV = 1000 + 1.64\left(\frac{200}{\sqrt{40}}\right)$

$= 1000$

we reject null

Conclude → wrong value reported

You want to rent an unfurnished one-bedroom apartment in NYC. According to the Housenice.com, a website that lists apartments for rent, the average rent in NYC is $1000 with a standard deviation of $200. You collected data on 40 such apartments listed on the website, the average rent of your sample was $1100?

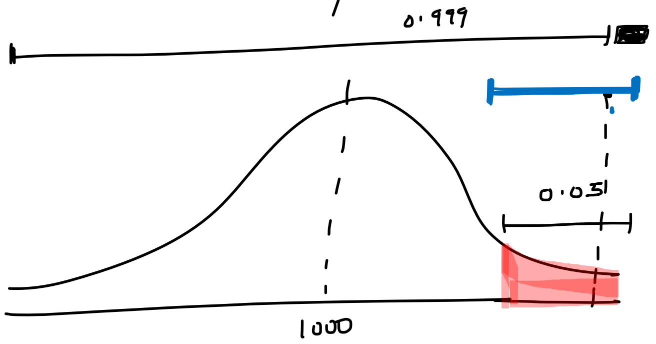
$\alpha = 0.05$

Is the information posted by Housenice.com wrong?

$H_0: \mu \leq 1000$

$H_a: \mu > 1000$

$N\left(1000, \frac{200}{\sqrt{40}}\right)$



$$z = \frac{1100 - 1000}{200 / \sqrt{40}}$$

$z = 3.16$

norm.cdf(3.16) =

p-value = 1 − norm.cdf(3.16)

= 0.0007

$\alpha = 0.05$

p is low, null will go

we reject null

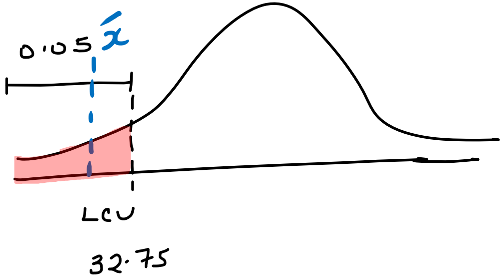
The "RKumar Home Cleaning" service claims that they can maintain your home at an average cost of \$35 per month per home. You are suspicious about their claim and believe that they charge less than what they claim. You carefully monitored their next 35 services and collected the data. You found the average cost to be \$32.50 with a standard deviation of \$8.10. Can you reject the null hypothesis at a significance level of 5%?

$H_0: \mu \geq 35$

$H_a: \mu < 35$

$N\left(35, \frac{8.10}{\sqrt{35}}\right)$ S.E → sampling dist

Lower tail test



norm.ppf(0.05) = −1.64

$LCU = 35 - 1.64 \times \left(\frac{8.10}{\sqrt{35}}\right)$

$= 32.75$

we reject null → RK home service charge less than what they claim

$\alpha = 0.01$ (I will reject null only for 1% cases)

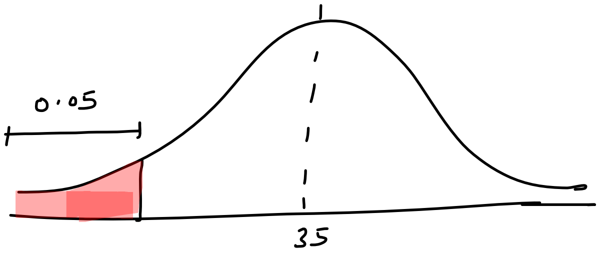
The "RKumar Home Cleaning" service claims that they can maintain your home at an average cost of $35 per month per home. You are suspicious about their claim and believe that they charge less than what they claim. You carefully monitored their next 35 services and collected the data. You found the average cost to be $32.50 with a standard deviation of $8.10. Can you reject the null hypothesis at a significance level of 5%?

$H_0: \mu \geq 35$

$H_a: \mu < 35$

$\mathcal{N}\left(35, \frac{8.10}{\sqrt{35}}\right)$

$\boxed{\alpha = 0.01}$

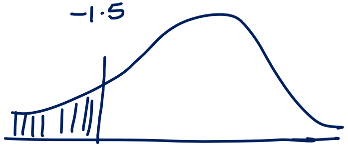


$Z = \frac{32.50 - 35}{8.10/\sqrt{35}}$

$Z = -1.8$

$P = \text{norm.cdf}(-1.8)$

$= 0.033$



norm.cdf(-1.5)

=

$\alpha = 0.05$

P is low, null will go we Reject null

P-Value Method

Case Study:

Suppose there is a Retail Store Chain that sells Shampoo bottles
This chain has 2000 stores across India.

The parameters for weekly sales of the shampoo bottle were reported as:

mean: 1800
std dev: 100
This was calculated by analyzing a lot of historical data

As a Manager / Owner / Data Scientist, you want to increase these sales, to generate more revenue.

What are the techniques at your disposal?: Hire a marketing team

But there is an important factor to consider. These marketing teams/firms are not cheap, and would add a significant cost.

It stands to reason that you would not straightaway handover all 2000 stores to them.

You would want an assurance that their work actually does impact the sales, and generate enough revenue that it is feasible to hire them.

Case Study:

You decide to do this experiment with 2 competing marketing firms

Firm A

- Worked on 50 stores
- Sold an average 1850 bottles of shampoo

Firm B

- Worked on 5 stores
- Sold an average 1900 bottles of shampoo

Power of test

Type-I : Reject null, when null is actually True (α)

Type-II : failed to Reject null, when null was actually False (β)

→ prob. of commiting type-2 error

Result from test

[Power of test $= 1 - \beta$] prob. of not commiting type-II error

Ho : person is not cancerous

Ha : person is cancerous

β

very critical case ← [Type-II : Concluded that person is not cancerous when the patient was ~~actu~~ actually cancerous

Type-I : Concluded that person is cancerous when the patient was not cancerous } → can be resolved with more tests conducted

α

α = prob. of rejecting null Hypo. $\longrightarrow$ $\alpha = 0.5$

Type-I : Reject null, when null is actually true

conclusion from test

$\alpha \uparrow$ Type-I $\uparrow$

Type-II $\downarrow$

$\alpha \downarrow$ Type-I $\downarrow$

Type-II $\uparrow$

	0	1
0	✓	H L
1	L H	✓

Quiz: A coffee shop claims that their coffee cups contain, on average, at least 12 ounces of coffee. A random sample of 36 coffee cups showed an average of 11.8 ounces with a standard deviation of 1.5 ounces. Conduct a Z-test to determine if the coffee shop's claim is supported. What is the p-value?

Quiz: A fitness App claims that its users walk an average of 8,000 steps per day. A random sample of 30 users showed an average of 7,600 steps per day with a standard deviation of 1,200 steps. Conduct a left-tailed Z-test at a 5% significance level to determine if the App's claim is supported. What is the p-value?