Z - Test Contd.

In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9. } population info $\mu = 65$, $\sigma = 9$

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving. In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5

sample size

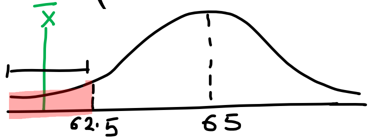
observed sample mean $\bar{X}$

- Investigate, at the 5% level of significance, the teachers' suspicion. (Find critical value and p-value with the conclusion)

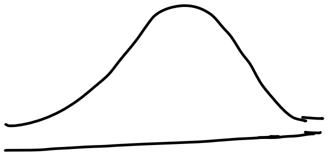
$H_0: \mu \geq 65$
$H_a: \mu < 65$

Lower tail test

$N\left(65, \frac{9}{\sqrt{35}}\right)$



- Explain, in the context of this question, the meaning of a Type I error.



$\alpha = 0.05$

$z = \frac{(\bar{x} - \mu)}{se}$

$z = \frac{61.5 - 65}{se}$

$z =$

$P = 0.0107$

$P < \alpha$ null will go

we reject null

$z = \text{norm.ppf}(0.05)$

$= -1.64$

$LCV = 65 - 1.64 \times \frac{9}{\sqrt{35}}$

$=$

In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9.

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving. In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5

- Explain, in the context of this question, the meaning of a Type I error.

Type-I : Reject null when null was actually True

Type-II : we fail to Reject null was null was actually False

$H_0: \mu \geq 65$

$H_a: \mu < 65$

$\alpha = 0.05$

1 → Teacher concluded that their students are underachieving when they were not

2 → Teacher concluded that their students are not underachieving when they were

new alpha $\alpha = 0.10$

One Sample Z-Test

[We compared the sample avg with the known/proven population avg]

$\bar{x} \rightleftharpoons \mu$

Sample mean (observed mean)

population mean (alredy proven value)

Ha   (a) → pop. mean

$\mu = a$

$\mu > a$

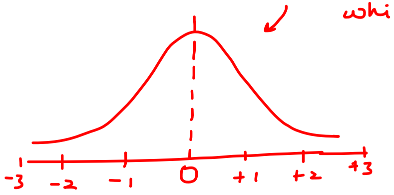
$\mu < a$

1. formulated Hypothesis
2. which tailed test
3. Decided the distribution → Z-Dist
4. Calculate Z-value

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

5. find p-value
6. conclusion using p-value

Sampling dist which follows a normal dist



Two Sample Z-Test

we compare the average across two independent groups

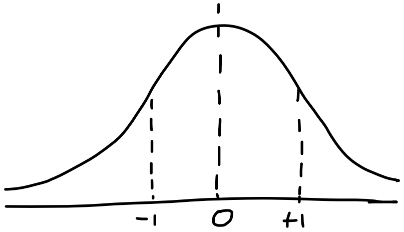
Ha:

1. $\mu_m \neq \mu_F$
2. $\mu_m > \mu_F$
3. $\mu_m < \mu_F$

males income → μ_m → avg male income

females income → μ_F → avg female income

$\mu_m \rightleftharpoons \mu_F$



1. formulate Hypo.
2. which tailed test
3. Decide Dist → Z-Dist.
4. compute Z-value

$$\left[Z = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right]$$

5. Compute p-value
6. conclusion using p-value

One Sample Z-Test
Two indep. Sample Z-Test } → average

One Sample Z-proportion Test

we compare the sample proportion with the population prop.

Example : Report says 30% student in college, smoke you are suspicious about the reported value and you surveyed 130 student in your college to find 40 of them smoking. can we conclude that the report is wrong and the prop. is actually less at a sig. of 5%.

Sample proportion (observed prop.) → $\hat{p}$

$\frac{40}{130} = 0.3076$

$\rightleftharpoons$

p → population prop. (Reported)

0.30

$$\boxed{Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}}$$

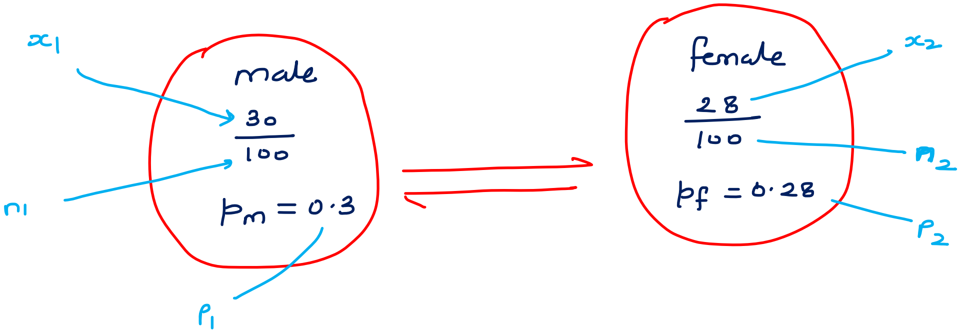
p → population prop.

$\hat{p}$ → sample prop.

n → sample size

Two independent sample z-proportion Test

We compare proportion between two independent sample/groups



metric = proporation

$$Z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2}$$

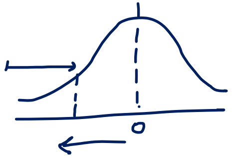
x_1 = no. of favourable respondents

n_1 = total of respondents in sample 1

x_2 = no. of favourable respondents in sample-2

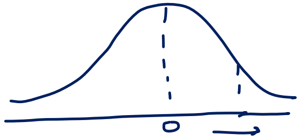
n_2 = total no. of respondents in sample-2 one tail test

$Z = -ve$ (Lef tail)



p-value = norm.cdf(z)

$Z = +ve$ (Right tail)

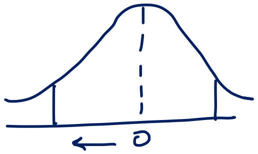


p-value = 1 − norm.cdf(z)

<u>Two tailed test</u>

&

$Z = -ve$



p-value = (norm.cdf(z)) × 2

$Z = +ve$

p-value = [1 − norm.cdf(z)] × 2

A corporation offers five different health plans to its employees.Each year the company offers them an opportunity to switch plans.In 2021, 17% of the employees switched from one plan to another.Recent attention has increased focus on the health care plans within the company. The corporation thinks that due to this attention, more employees will switch plans this year.In a random sample of 150 employees, we find that 30 employees plan to switch plans this year.Is the proportion of employees who plan to switch higher than in 2021?Run the test at a 5% level of significance.

$H_0: P \leq 0.17$

$H_a: P > 0.17$

Quiz: A fast-food restaurant claims that 80% of their customers prefer their new burger over the old one. In a random sample of 100 customers, 85 said they preferred the new burger. What is the null and alternative hypothesis?

Quiz: A researcher is studying the satisfaction level of customers after implementing a new customer service system. They collected survey responses from 250 customers and found that 65 of them were dissatisfied with the new system. The researcher wants to test the null hypothesis that no more than 30% of customers are dissatisfied with the new system. Use the p-value technique to test the claim with a significance level of $\alpha = 0.05$.

Quiz: A company introduces a new feature in its mobile App that allows users to subscribe to a premium service. They want to evaluate if the introduction of this feature has led to an increase in the number of premium users. They collect data from two different time periods: before the feature was introduced (Group A) and after the feature was introduced (Group B). Which test should you use to determine if the new feature has significantly increased in the number of premium users?

Quiz: A shoe manufacturer claims that their new running shoes make people run faster. To test this claim, they select two groups: Group A wears the new shoes, and Group B wears the old ones. After a 4-week trial, you find that Group A improved their running speed by 15%, while Group B improved by only 10%. Which test should you use to determine if the new shoes are more effective?

Consider number of male doctors in military and civilian hospitals. Test the claim there is a *smaller* proportion of male doctors in military than in civilian life at $\alpha = 0.05$.

	military (1)	civilian (2)
male doctors	358	6786
total doctors	407	7363

$H_0: p_m \geq p_c$

$H_a: p_m < p_c$