Hypothesis Testing

T Test

H_0 : [$\mu \geq 100$] → population mean is greater than or equals to 100.

H_a : [$\mu < 100$] → we want to prove that pop. mean is less than 100

population

Note: we test and conclude things that are proven and reported

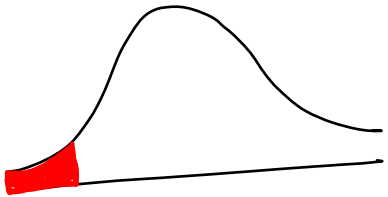
Mr. Null Hypothesis

Quick Recap Test:

One Sample z-Test

(Sample mean) 46.45 ⇌ (population mean) 48

1. Suppose the life expectancy of "two candy" bars has a population that is normally distributed with a standard deviation of 3. Tomorrow you sample 41 "two candy" bars from this population and obtain a mean life expectancy of 46.45 and a standard deviation of 3.5814. Using an alpha value of α = 0.01, is this observed mean significantly less than an expected life expectancy of 48?



$\alpha = 0.01$

$H_0: \mu \geq 48$

$H_a: \mu < 48$ ←

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{46.45 - 48}{3/\sqrt{41}}$$

$$= -3.3$$

$P = normcdf(z)$

$= 0.0004$

$P < \alpha$ P is low, null will go

we reject null hypo.

conclusion - Life expentency is less than 48

Quick Recap Test:

1. A company wanted to compare the performance of its call center employees in two different centers located in two different parts of the country – Hyderabad, and Bengaluru, in terms of the number of tickets resolved in a day (hypothetically speaking). The company randomly selected 30 employees from the call center in Hyderabad and 30 employees from the call center in Bengaluru. The following data was collected: Hyderabad: $\bar{x}1 = 750$, $\sigma 1 = 20$ | Bengaluru: $\bar{x}2 = 780$, $\sigma 2 = 25$ The company wants to determine if the performance of the employees in Hyderabad less than from the performance of the employees in the Bengaluru center.

$\alpha = 0.05$

H

$\bar{x}_1 = 750$

$\sigma_1 = 20$

B

$\bar{x}_2 = 780$

$\sigma_2 = 25$

compare avg of two independent groups

"2 sample z-Test"

$H_0: \mu_H \geq \mu_B$

$H_a: \mu_H < \mu_B$

$$z = \frac{\mu_H - \mu_B}{\sqrt{\frac{\sigma_H^2}{n_1} + \frac{\sigma_B^2}{n_2}}}$$

$P = 0.0000$

$\alpha = 0.05$ $\quad P < \alpha$

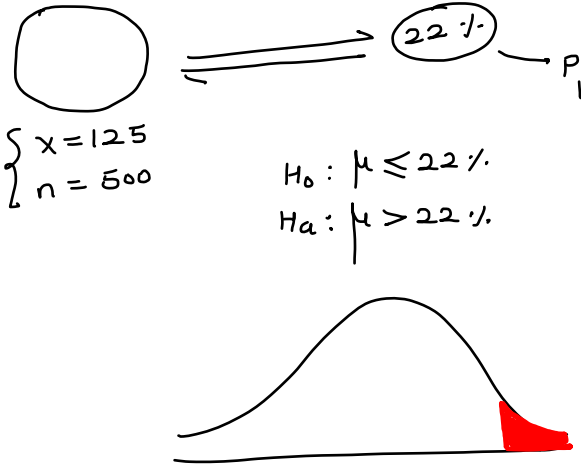
P is low null will go

yes, the Hyd. team is lower performance

Quick Recap Test:

1. A researcher who is studying the effects of income levels on breastfeeding of infants hypothesizes that countries where the income level is lower have a higher rate of infant breastfeeding than higher income countries. It is known that in Germany, considered a high-income country by the World Bank, 22% of all babies are breastfeed. In Tajikistan, considered a low-income country by the World Bank, researchers found that in a random sample of 500 new mothers that 125 were breastfeeding their infant. At the 5% level of significance, does this show that low-income countries have a higher incident of breastfeeding?

"one sample z-proportion test"



$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

p = population prop.
$\hat{p}$ = sample prop.
(1-p) = q
n = sample size

p-value = 0.052

$p > \alpha$ p is high, null will fly

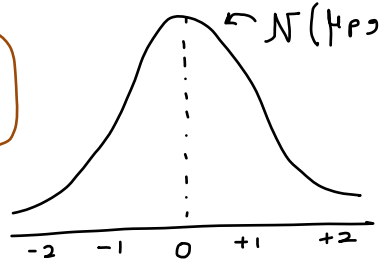
we fail to reject null

When Z-Test Fails?

Z-Distribution $\leftarrow N(\mu_p, \frac{\sigma}{\sqrt{n}})$

$$z = \frac{(x-\mu)}{\sigma/\sqrt{n}}$$

Standard Error



Standard normal dist

Z-value
p-value → norm.cdf →

Z-Test

① n (sample size) > 30
② pop. std. deviation is known

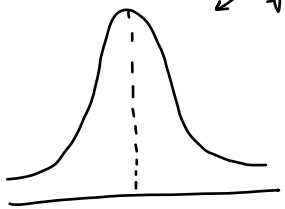
as per statistics this no. represents sufficiently large sample size

(If pop. std. dev is unknown, we may use sample std dev as pop. std. dev. but the sample size has to be significantly greater than 30)

T-Test

① n (sample size) < 30
② pop. std. deviation is unknown

T-Distribution $\leftarrow N(\mu_p, \frac{s}{\sqrt{n}})$



$$T = \frac{(x-\mu)}{s/\sqrt{n}}$$

Sample Standard Error

① when sample size n < 30 we use T-Test (T-Dist)

② But when n > 30, T-Dist becomes Z-Dist and hence we can use either of them to conduct the test

n < 30 — T-Test

n > 30 — T-Test / Z-Test = Same as

at n = 30

T-Dist ⟶ Z-Dist

as n approaches to 30

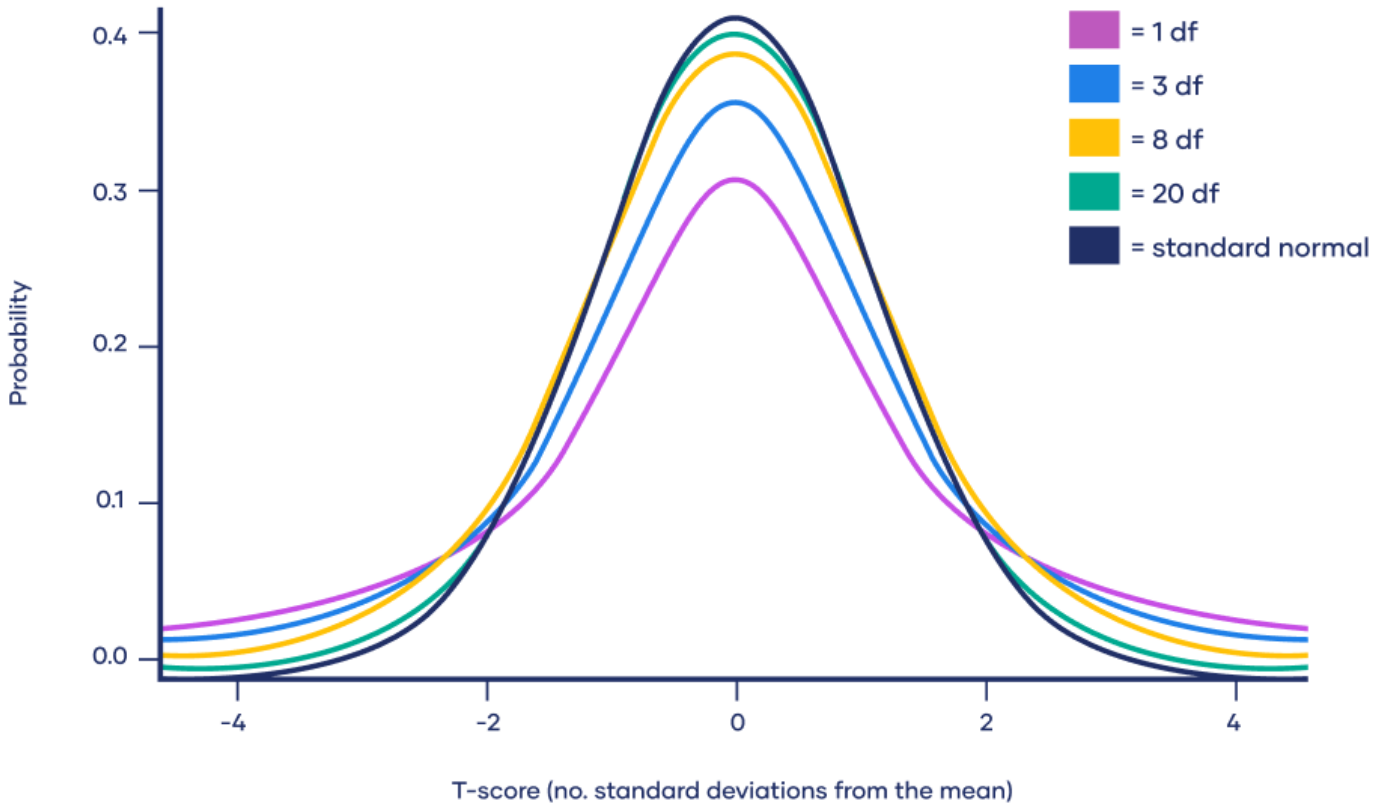
T-Dist ⟶ Z-Dist.

s = Sample std. devi

$$\text{Sample std. dev} = \sum_{i=1}^{n} \frac{(x-\bar{x})^2}{(n-1)}$$

dof

The variability depends only on n-1 sample


0.4
0.3
0.2
0.1
0.0
Probability
-4
-2
0
2
4
T-score (no. standard deviations from the mean)
= 1 df
= 3 df
= 8 df
= 20 df
= standard normal

Scribbr

Z-Test : one sample Z-Test

- Two ind. sample Z-Test
- one sample Z-prop Test
- Two ind. sample Z-prop. Test

[ttest_1samp]

① one sample T-Test

we compare the sample mean (observed) with population mean (reported)

A researcher claims that his pill can improve the IQ level among high school students. The avg IQ level of thigh school students was known to be 100.

He took a sample 15 high school students and asked them to take the pill.

After a month all students took IQ test as the results are as follows

X [105, 101, 99, ... 109] → avg(x)

Sample mean

avg(x) ⇌ 100

T-Test [ttest_ind]

② Two ind. sample T-Test

we compare avg of two independent groups

It is said that the avg height of girls is less than that of boys in high school.

To test the claim 20 students participated and their heights were reported

g = [ ] → avg(g)

b = [ ] → avg(b)

avg(g) ⇌ avg(b)

[ttest_rel]

③ paired sample T-Test

we compare the avg of a group in two different time

A researcher has developed a new medicine that can reduce the weight of overweighted people.

5 volunteers participated

	weight before medicine	weight after medicine
V1		
V2		
V3		
V4		
V5		

avg (weight) before medicine

avg (weight) after medicine

Quiz-1: What is the null hypothesis in a one-sample t-test?

Quiz-2: What is the formula for the t-statistic in a one-sample t-test?

Quiz-3: Which type of t-test is used to compare the means of two independent groups?

Quiz-4: In a two-sample independent t-test, what is the alternative hypothesis if you're interested in determining if the means of the two groups are different (two-tailed test)?

Example - 01

Population IQ average = 100

One researcher claims that his pill will improve IQ

[110, 105, 98, 102, 99, 104, 115, 95]

Establish result at 99% confidence (0.01 significance "alpha")

Example- 02 (IQ Test Two Groups)

Example- 03 Sachin (Batting pattern in first and second innings)

Example- 04 Sachin (Victory Vs Defeat)

Example- 05 (Drug recovery)

Example- 06 (Aerofit)

Example- 06 (Aerofit)

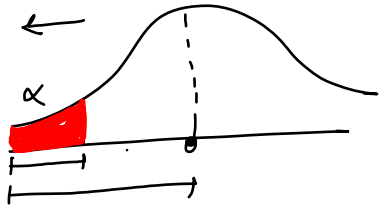
$H_a: \mu < 100$

$z = -ve$

Lower tail test

$z = -ve$

p-value = norm.cdf(z)



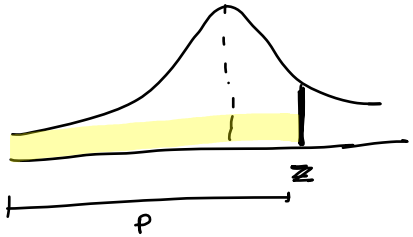
fail to reject H_0,

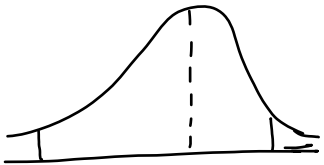
z

Upper tail test

$z \rightarrow +ve$

p-value = 1 − norm.cdf(z)





two sided

z − tve → [1 − norm.cdf(z)] × 2

z − ve → [norm.cdf(z)] × 2

Quiz: Test the hypothesis that eating dry fruits make a student smarter. A random sample of 12 students is taken and they were provided with dry fruits for a year and then they were given an IQ Test.

[16, 111, 101, 120, 99, 94, 106, 115, 107, 101, 110, 92]

Test if these students are having IQ level more than the normal IQ level of 100 at significance level of 0.01

Quiz: **A pharmaceutical company is testing a new drug designed to lower blood pressure. They conduct a study involving two groups: Group A receives the new drug, While Group B receives a placebo treatment. The blood pressure drop in both these groups, after treatment is recorded as:**

group_a = [8, 7, 6, 5, 6]

group_b = [6, 5, 5, 4, 3]

The company wants to determine if there is a significant difference in the mean blood pressure reduction between the two groups after one month of treatment.

Which of the following Python code snippets correctly performs a two-sample t-test at a significance level of 0.05?

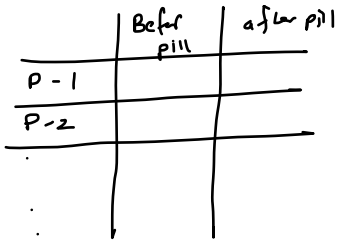
1 Samp T-Test ✗

Two Sample T-Test →

paired - ✗

ttest_ind(a, b)

=



Ha : group a ≠ group b

Two-sided