LA-4

Perceptron

Agenda:

→ Distance b/w lines

→ Loss function

→ Perceptron learning rule

Recap

Vectors

$$\vec{x}, \vec{y} \in \mathbb{R}^d$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \quad \text{or} \quad \vec{x}^T = [x_1 \; x_2 \; \cdots \; x_d]$$

Norm

$$\|\vec{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_d^2} = \sqrt{\sum_{i=1}^{d} x_i^2}$$

Dot Product

inner product

$$\vec{x} \cdot \vec{y} = x^T y = \|x\| \cdot \|y\| \cos\theta = \sum_{i=1}^{d} x_i \cdot y_i$$

Angle $(\vec{v}_1, \vec{v}_2)$

$$\cos\theta = \frac{v_1^T v_2}{\|v_1\| \cdot \|v_2\|}$$

Unit Vector

$$\hat{a} = \frac{\vec{a}}{\|a\|}$$

$Proj_y(x)$

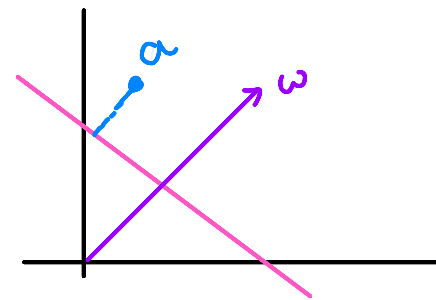
$$\|P\| = x^T \hat{y}$$

$$= \|x\| \cos\theta = \boxed{\frac{x^T y}{\|y\|}}$$

Distance(a, ω)

Point a from line ω

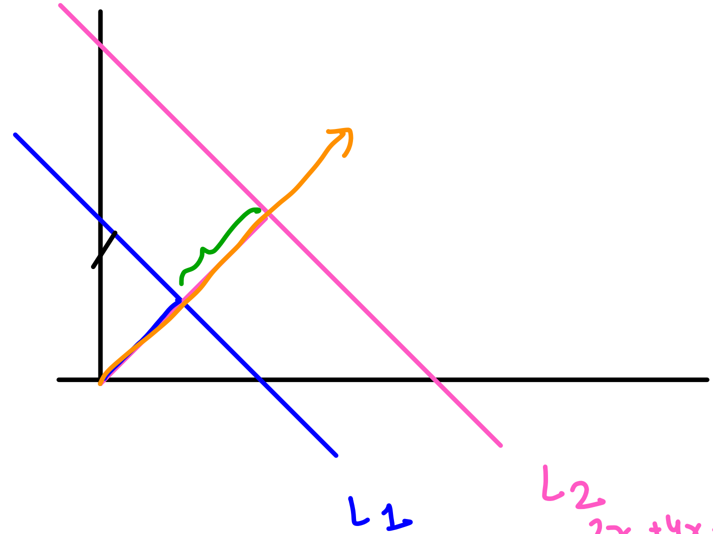
$$d = \frac{\omega^T a + \omega_0}{\|\omega\|}$$



$$= \frac{\omega_1 a_1 + \omega_2 a_2 + \omega_0}{\sqrt{\omega_1^2 + \omega_2^2}}$$

III Dist. b/w 2 lines (Parallel lines)

$$d = \frac{w_0^{\text{Pink}}}{\|w\|} - \frac{w_0^{\text{Blue}}}{\|w\|}$$

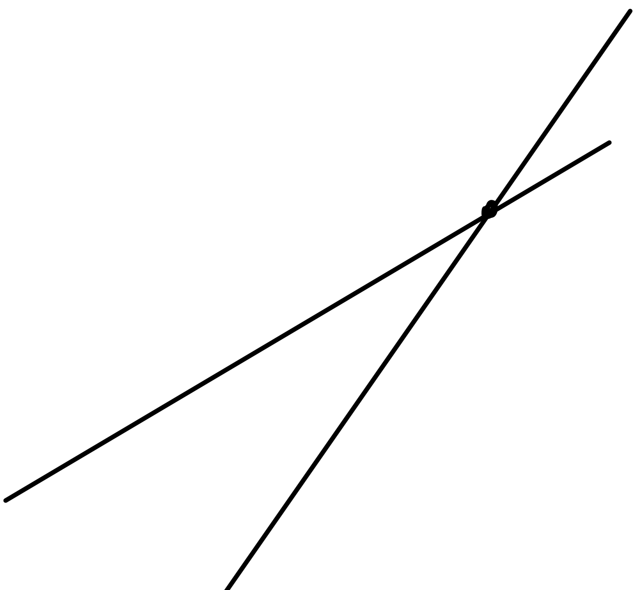


$2x_1 + 4x_2 - 3 = 0$

$\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$2x_1 + 4x - 8 = 0$

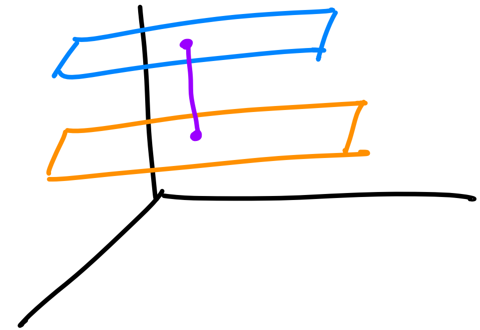
$\begin{bmatrix} 2 \\ 4 \end{bmatrix}$



$$\boxed{w^T x + w_0 = 0}$$

$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ line

$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ plane



$$d = \frac{w_0^{(1)}}{\|w\|} - \frac{w_0^{(2)}}{\|w\|}$$

Q → $w^T x + 5 = 0$, $w^T x + 3 = 0$ dist b/w 2 planes.

$$w = [3, 2, 1]^T$$

$$d = \frac{5}{\sqrt{14}} - \frac{3}{\sqrt{14}} = \frac{2}{\sqrt{14}} = 0.53$$

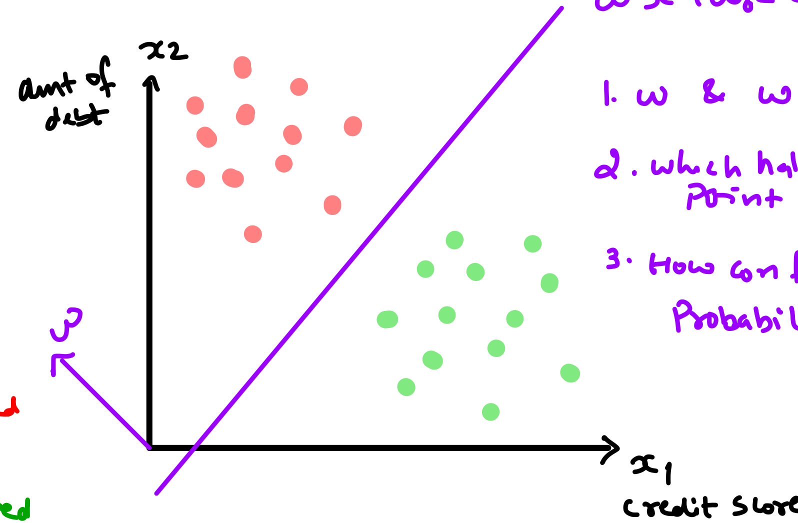
Loan Approval

New applicant:

"x-query" [400, 900]

Sign of dist → +ve : class 1 rejected

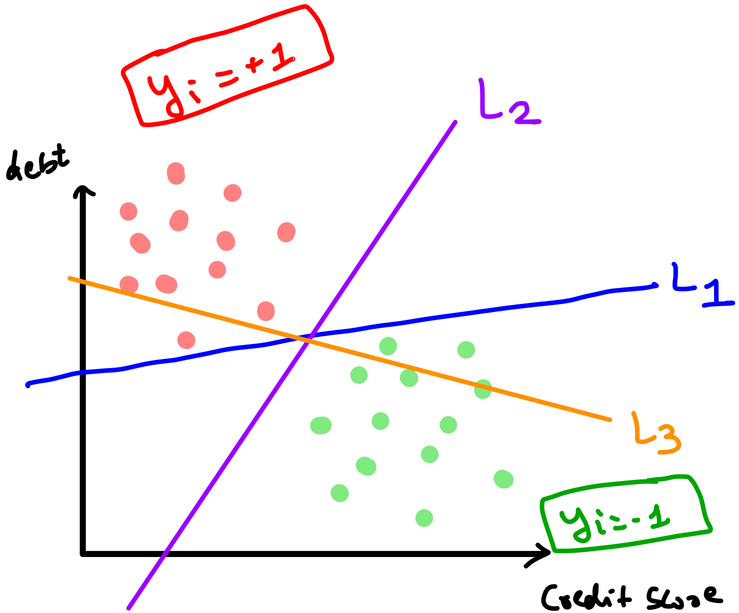
Sign of dist → -ve : class 0 approved



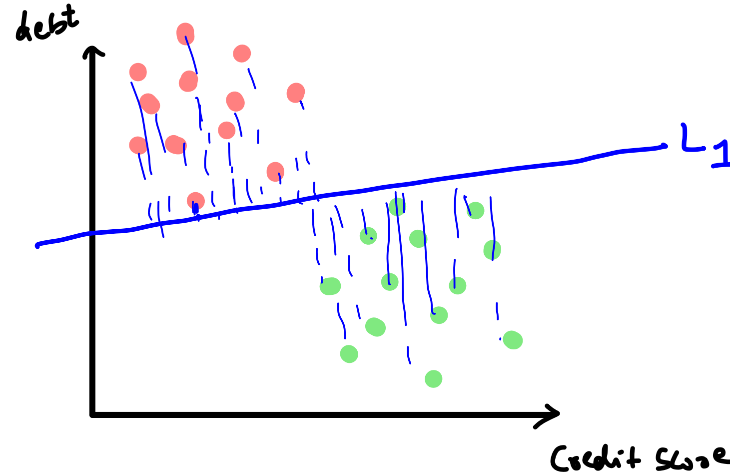
1. w & w_0
2. which half point lies?
3. How confident Probability?

$$x_p \cdot w = \underbrace{\|x\|}_{+ve} \underbrace{\|w\|}_{+ve} \boxed{\cos\theta}$$

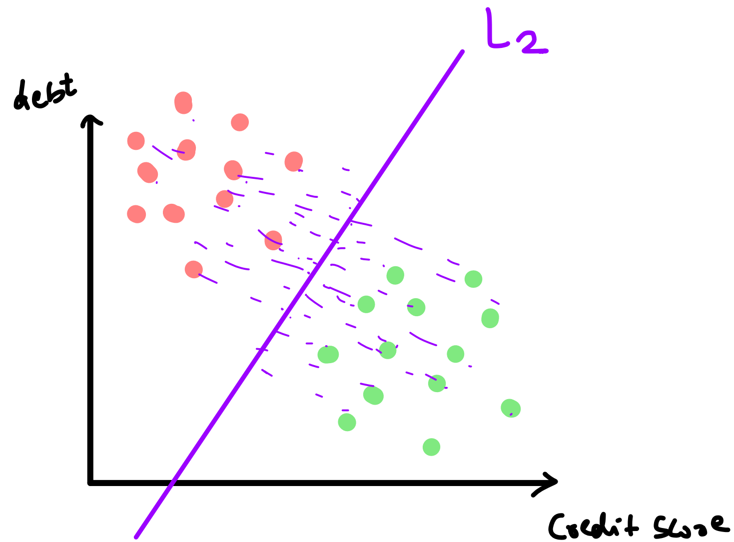
Loss



$y_i = +1$
L_2
debt
L_1
L_3
$y_i = -1$
Credit score



debt
L_1
Credit score



L_2
debt
Credit score

Quantify L2 is Better?

$g(w, w_0) = 18$

$g(w, w_0) = 30$ → avg dist of Points from line

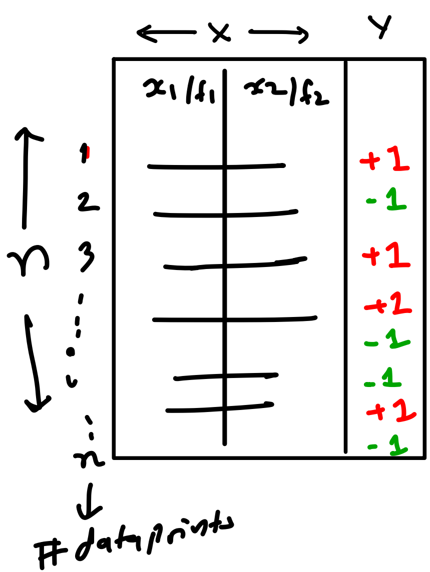
$g(w, w_0) = 7$

If i take dist as it as, it's going to nullify each other:

↳ ~~a.~~ Modulus/abs values $|-5| = 5$

↳ ~~b.~~ take Square

↳ c. multibly with y_i. → all dist. are Positive magnitude.



$$x^{(i)} \longrightarrow d^{(i)}$$

$$x^{(i)} \longrightarrow \frac{w^T x^{(i)} + w_0}{\|w\|}$$

$$x^{(i)} \cdot y^{(i)} \rightarrow \frac{w^T x^{(i)} + w_0}{\|w\|} \cdot y^{(i)}$$

For all dataPoint

n total dist. $= \sum_{i=1}^{n} \frac{w^T x^{(i)} + w_0}{\|w\|} \cdot y^{(i)}$

$$\boxed{w^*, w_0^* = \underset{w, w_0}{\operatorname{argmax}} \frac{1}{n} \sum_{i=1}^{n} \frac{w^T x^{(i)} + w_0}{\|w\|} \cdot y^{(i)}}$$

→ i want to find such w & w_0 so that this func. is max.

↳ avg. dis. of points from line is maximized!

" gain function " = maximize gain function.

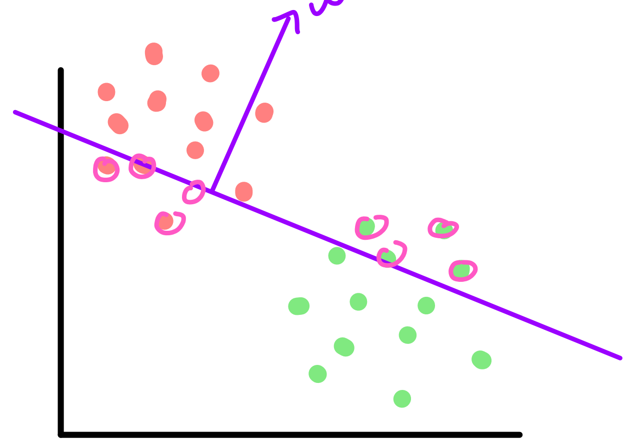
↕ opposite

$$\boxed{\text{Loss func} = -\text{gain function}}$$

minimize !

Perceptron Learning Algorithm

Goal : find Best line
[w, w_0]



Step 1 : Initialize a random w, w_0

close ~ & incorrect classification
[misclassified Points]

Step 2 : Iterate until convergence : → till you have atleast 1 misclassified.

↳ update w & w_0 using : → actual label

$$w_{new} = w_{old} + y^{(i)} \cdot x^{(i)}$$

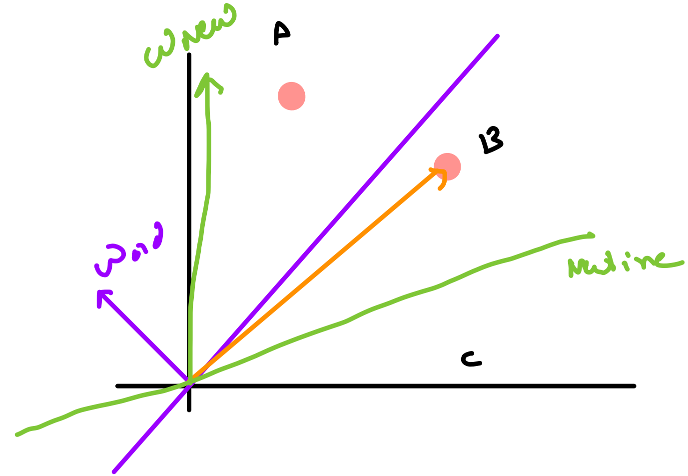
$$w_0 = w_0 + y^{(i)}$$

} only do this for misclassified Points.

$$w_{new} = w_{old} + B$$

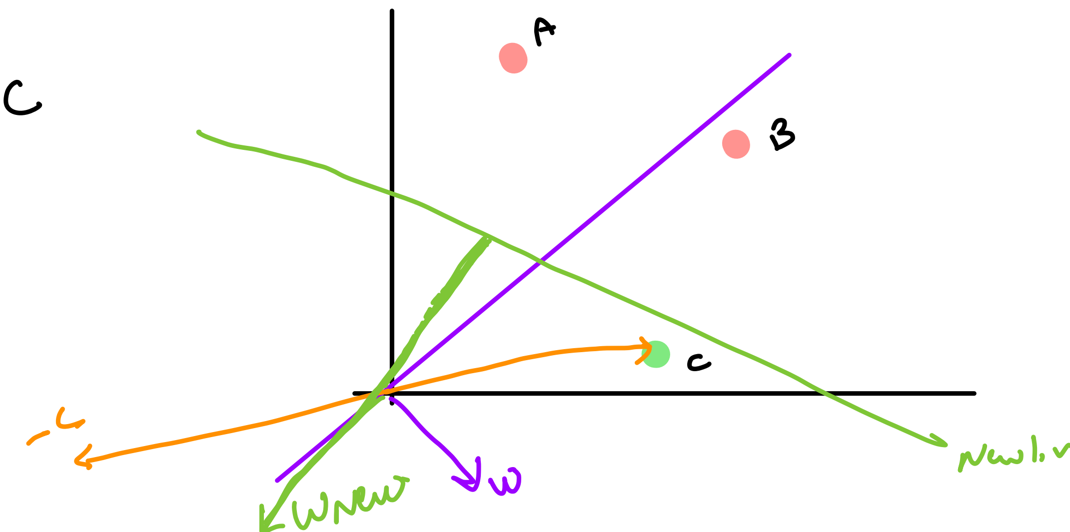
for $\boxed{y^{(i)} = +1}$ misclassification

$$w_{new} = w_{old} + B$$



for $\boxed{y^{(i)} = -1}$ misclassification

$$w_{new} = w_{old} - C$$



$$w_{old} = \begin{bmatrix} -8.8 \\ 6.9 \end{bmatrix} \qquad P = \begin{bmatrix} 24 \\ 43 \end{bmatrix}$$

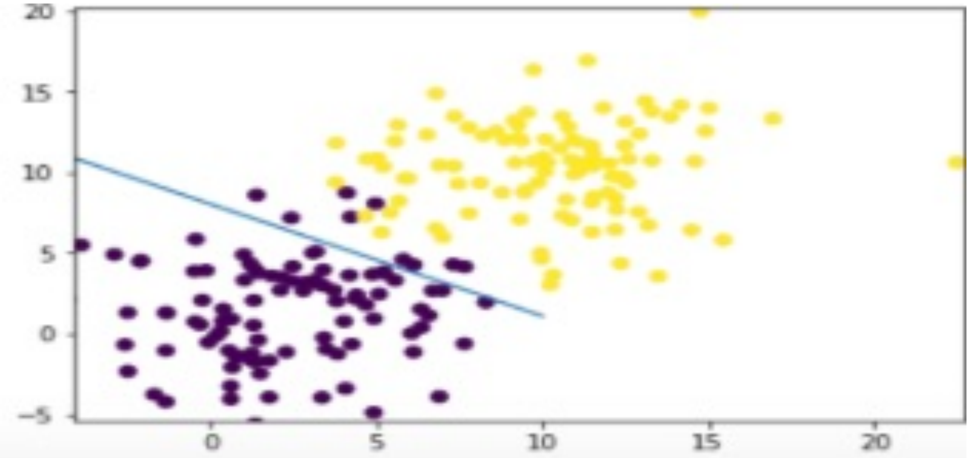
$y^{(i)} = -1$ misclassif

$$w_{new} = w_{old} - P$$

$$w_{new} = \begin{bmatrix} -8.8 - 24 \\ 6.9 - 43 \end{bmatrix}$$

$$\begin{bmatrix} -32.8 \\ -36.1 \end{bmatrix}$$

$w_0 =$



Rejected (y = 1)

Prediction → Approved (y = − 1)

$$w = w_{old} + y^{(i)} + x$$

$$w = w_{old} + x^{(i)}$$

$$w = w_{old} + (y^{i})\, x^{(i)}$$

$$w = w_{old} - x^{(i)}$$