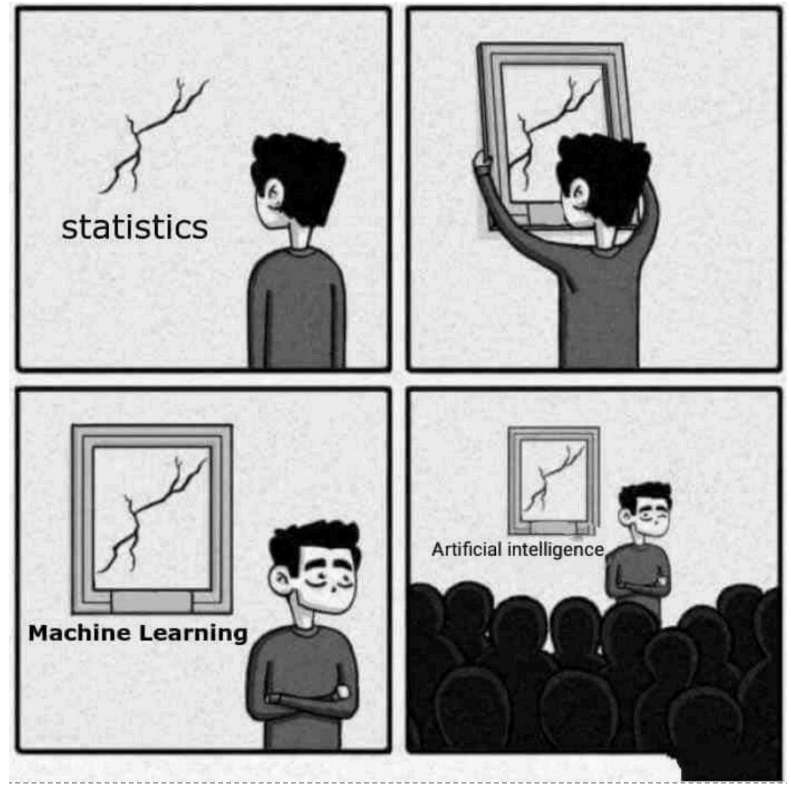
Optimisation - 01



Agenda

→ classifier

→ searching Algo.

→ optimization Problem.

→ functions

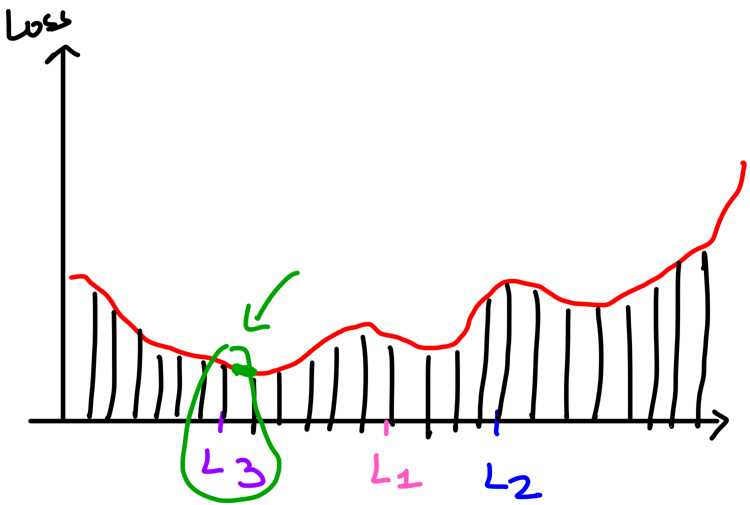
→ Limit

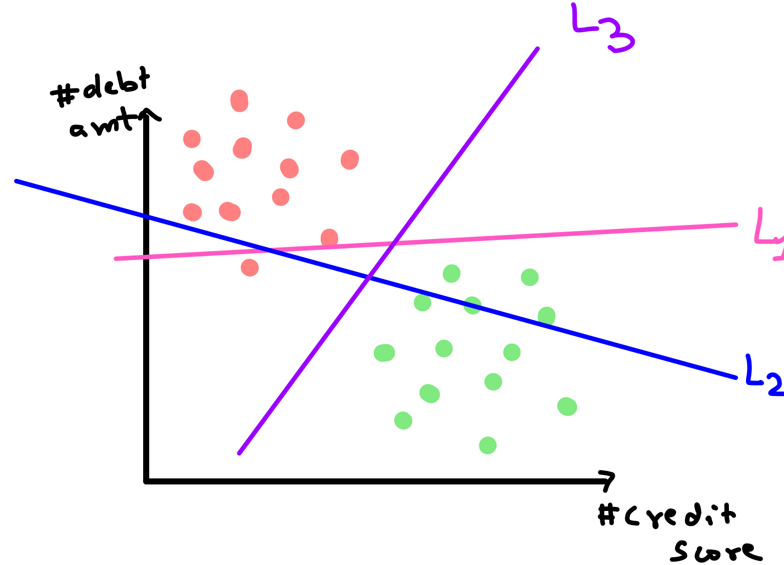
→ Continuity & Differentiation

Classifier

Loss function

$$\mathcal{L}(D, w, w_0) = \mathbb{R}$$





Simple Searching Algo.

Best line

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \; w_0 \qquad \left[w_1^* x_1 + w_2^* x_2 + w_0^* = 0 \right]$$

How to find Best w's ?

"Linear Search" - to find w & w_0 ?

$w_1 \rightarrow [-10, 10]$

$w_2 \rightarrow [-10, 10]$

$w_0 \rightarrow [-10, 10]$

Step-size = 0.1

$[-10, \ldots \ldots -0.1]$ ← 100 →

$[0]$

$[0.1 \ldots \ldots 10]$ ← 100 →

201

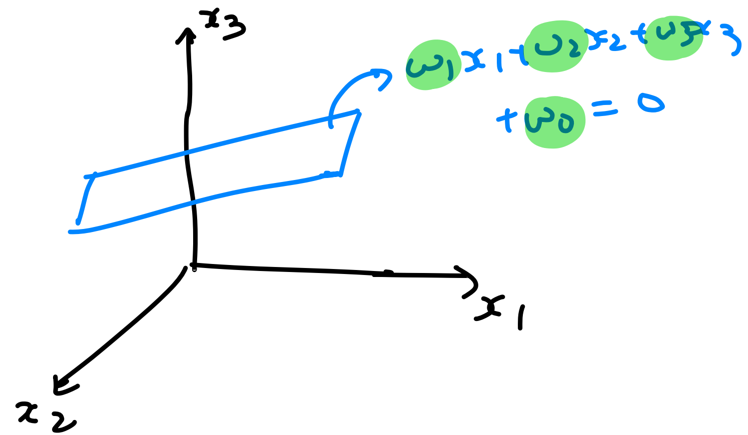
total combination tried = $(201)^3$

C.S $\rightarrow 10^6$ ops/sec

2 features $\rightarrow$ 8 seconds

3 features $\rightarrow$ 27 mins

4 features $\rightarrow$ 91 hours



Solving Optimization Problem

$y = (x-5)^2$

at what value of x does this func. takes min values.

→ Use Gradient Descent to Solve Opt. Problem.

$$f(x) = (x-5)^2$$

$$x^* = \underset{x}{\operatorname{argmin}}\ f(x)$$

$$\boxed{x^* = \underset{x}{\operatorname{argmin}}\ (x-5)^2}$$

→ maxima / minima

→ Calculus in multi-variate

→ Calculus in single-variable

→ derivatives, slope,

→ limit, Continuity, functions

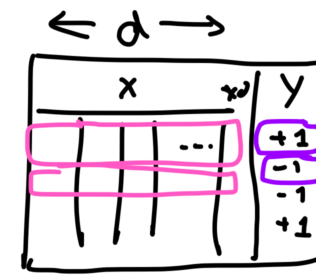
Classification

Given labelled Data

labels

$$D = \left\{ (x^i, y^i)_{i=1}^{N} \; ; \; x^i \in \mathbb{R}^d \; ; \; y^i \in \{-1, +1\} \right\}$$

features



Goal : find a function $f(x)$ s.t.

$$f(x) = \underset{\text{values}}{\text{Predicted}} = w^T x + w_0$$

$y^{(i)}$ = output / label for i^{th} data Point

$\hat{y}^{(i)}$ = Predicted label for i^{th} data Point

$$y^{(i)} \approx \hat{y}^{(i)}$$

Actual label should be similar to Predicted label

Gain function :

$g(D, w, w_0)$ → This is the func i want to optimize

→ measures how good a classifier is performing

$$\boxed{\text{Loss func} = -\text{gain func.}}$$

$$g(D, w, w_0) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{w^T x^i + w_0}{\|w\|} \right) \cdot y^i$$

$$w^*, w_0^* = \underset{w, w_0}{\operatorname{argmax}} \; g(D, w, w_0)$$

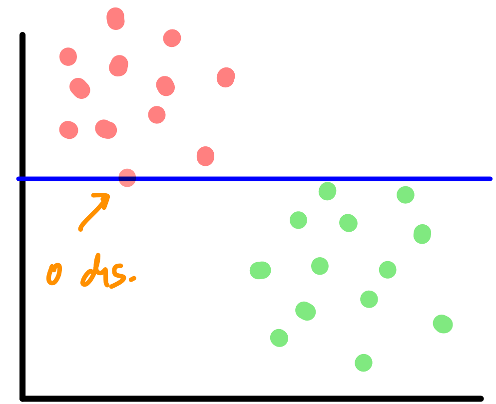
Preffered!!

① $$\ell(D, w, w_0) = \frac{1}{n} \sum_{i=1}^{n} \frac{w^T x^i + w_0}{\|w\|} \cdot y^i$$

Summation

② $$\ell(D, w, w_0) = \frac{1}{n} \prod_{i=1}^{n} \frac{w^T x^i + w_0}{\|w\|} \cdot y^i$$

Product



$$d^{(1)} \times d^{(2)} \times \dots \times \underset{0}{d^{(i)}} \times \dots \times d^{(n)}$$

$= 0$

functions

$y = f(x) = x+5$

$= x^2$

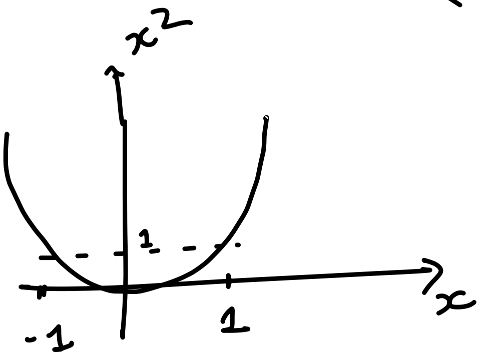
$= (x+2)^3$

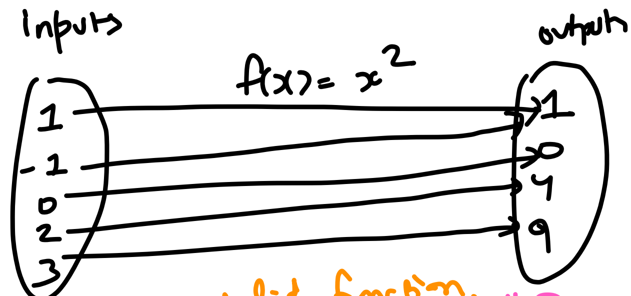
input

$\boxed{f(x) = x^2}$

Name of func

definition of function



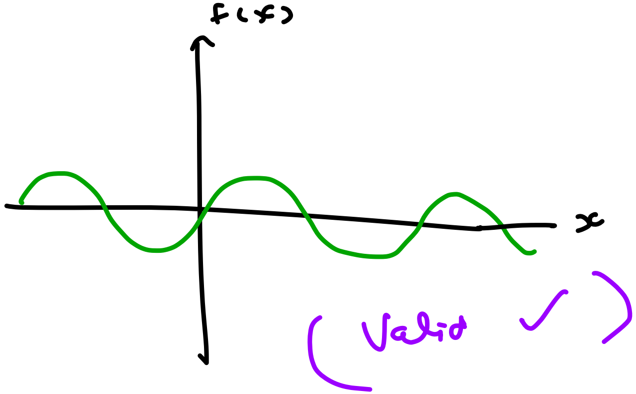


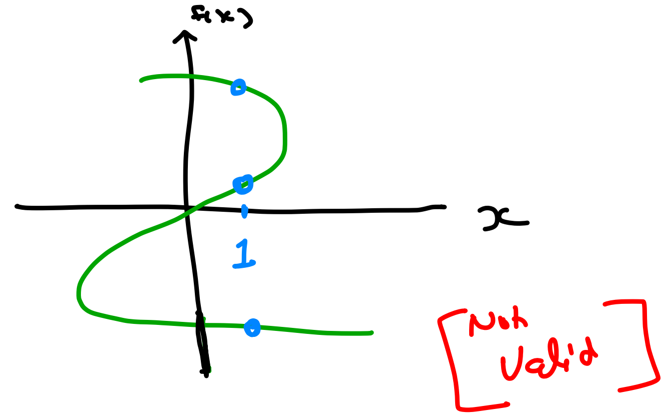
"Domain"

valid function.

"Range"

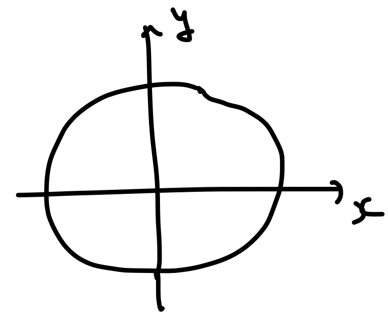
1 input can't have 2 different outputs.



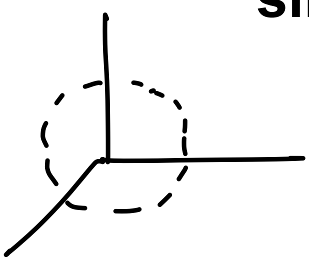


function:

- for a single value of x it should have only a single value of y.



$$\boxed{x^2+y^2=r^2}$$



$$z=f(x,y)=(x^2+y^2)$$

"2 inputs"

$$f(x) = \log x$$

Domain = $(0, \infty)$ $[0, \infty)$

excluded included

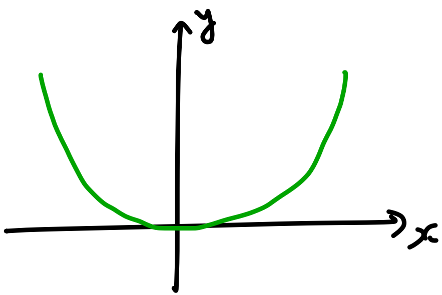
0.0000000001

Range = $(-\infty, \infty)$

III Limits

functions → Continous functions

functions → Non-Continous functions

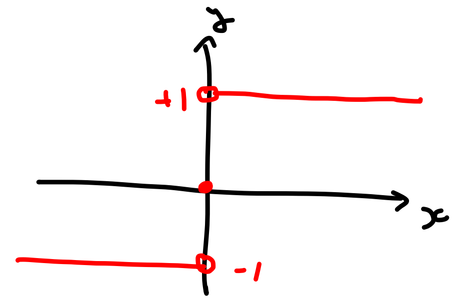


$f(x) = x^2$

Continous function

drawn without lifting pen

$$y = f(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0 \end{cases}$$



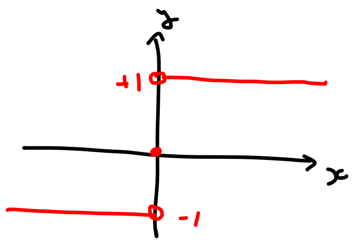
Discontinous function

Limit

↳ Approaching a Number.

1. Right Hand Limit
2. Left Hand Limit

$$y = f(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0 \end{cases}$$



Discontinuous function

RHL

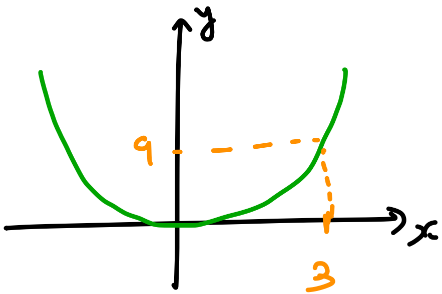
$$\lim_{x \to 0^+} f(x) = 1$$

$x = 0.1, 0.01, 0.001, 0.00001$

LHL

$$\lim_{x \to 0^-} f(x) = -1$$

$x = -0.1, -0.001, -0.000001$



$$f(x) = x^2$$

RHL

$$\lim_{x \to 3^+} f(x) = 9$$

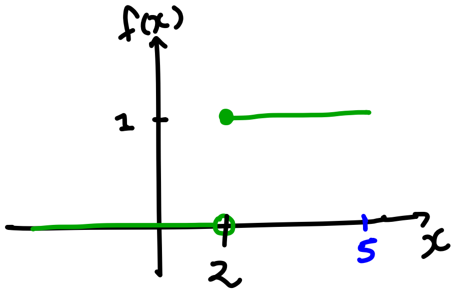
LHL

$$\lim_{x \to 3^-} f(x) = 9$$

function is continous at Point a if :

$$LHL = RHL = f(x)_{x=a}$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$



$$f(x) = \begin{cases} 1 \; ; & x \geq 2 \\ 0 \; : & x < 2 \end{cases}$$

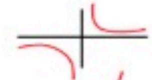
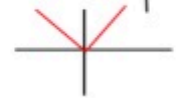
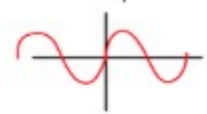
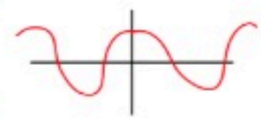
[discontinous @ $x = 2$]

let's take $a = 5$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5)$$

$$1 = 1 = 1$$

continous function → are continuous everywhere

Functions	Domain	Range	Continuous	Plot
① $y = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	continuous	
② $y = \frac{1}{x}$	$(-\infty, \infty)$	$(-\infty, \infty)$	discontinuous	
③ $y = e^x$	$(-\infty, \infty)$	$(0, \infty)$	continuous	
④ $y = \lvert x \rvert$	$(-\infty, \infty)$	$(0, \infty)$	continuous	
⑤ $y = \ln(x)$	$(0, \infty)$	$(-\infty, \infty)$	continuous	
⑥ $y = \frac{1}{1+e^{-x}}$	$(-\infty, \infty)$	$(0, 1)$	continuous	
⑦ $y = \sin\theta$	$(-\infty, \infty)$	$(-1, 1)$	continuous	
⑧ $y = \cos\theta$	$(-\infty, \infty)$	$[-1, 1]$	continuous	
⑨ $y = \tan\theta$	$(-\infty, \infty)$	$(-\infty, \infty)$	discontinuous