Linear Algebra - 2

Agenda :-

- Vectors
- Norms
- Matrix multiplication
- Dot Product
- Angle b/w 2 vectors

Recap

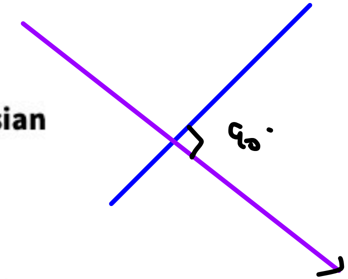
- → Feature
- → Label
- → Datapoint
- → Dataset
- → classifier / ML model

Line Eq.

$$y = m \cdot x + c$$

$$w_1 \cdot x_1 + w_2 \cdot x_2 + w_0 = 0$$

$$m_1 \cdot m_2 = -1$$



Which of the following statements is true about perpendicular lines in a Cartesian coordinate system?

34 users have participated

A	Perpendicular lines have the same slope.	12%
B	Perpendicular lines intersect at a 45-degree angle.	18%
C	Perpendicular lines have slopes that are negative reciprocals of each other.	62%
D	Perpendicular lines never intersect.	9%

End Quiz Now

$$m \times \left(-\frac{1}{m}\right)$$

$$= -1$$

$L_1: 3x - 2y + 6 = 0$

$m = -\frac{w_1}{w_2}$

$l_2: 9x - 6y - 18 = 0$

$3x - 2y + 6 = 0$

$-2y = -3x - 6$

$y = \frac{3}{2}x + 3$

Optimise a classifier

= Finding Best line

optimisation

entire data $\longrightarrow$ $\boxed{\text{magic}}$ $\longrightarrow$ w_1, w_2, w_0 (eqⁿ of a line)

$w_1 = 1$

$w_2 = 1$

$w_0 = -98$

2 features

$\downarrow$

$w_1 x_1 + w_2 x_2 + w_0 = 0$

x_1	x_2	x_3

Y

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

(2D) $w_1 x_1 + w_2 x_2 + w_0 = 0$ → Line

(3D) $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$ → Plane

$\vdots \qquad \vdots \qquad \vdots$

(dD) $w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_d x_d + w_0 = 0$

↳ Hyperplane

Vectors

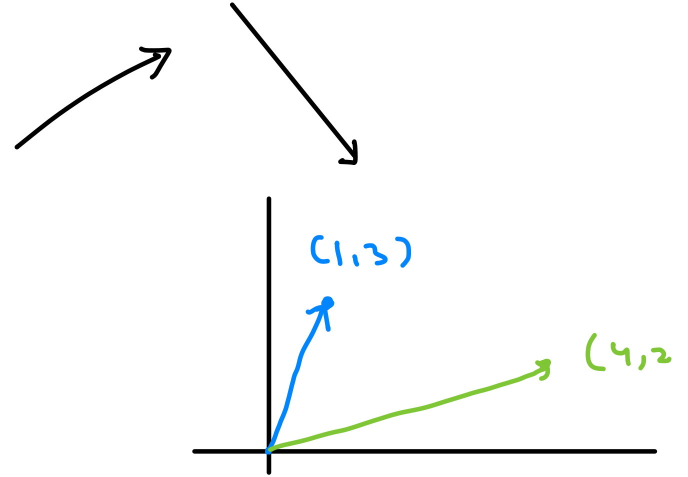
physics → dir^n + magnitude

Maths / C.S → Storing data

C.S → Store info.

applicant Loan $\begin{bmatrix} \text{income} \\ \text{age} \\ \text{Loan} \\ \text{Tenure} \\ \text{credit score} \end{bmatrix}$

Collection of Numbers



$[1, 3]$ $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Vector : $\vec{x}$

Verticle data

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$$

Colm vector

$\longleftarrow$ wide / Horizontal $\longrightarrow$

$$\vec{x} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_d \end{bmatrix}$$

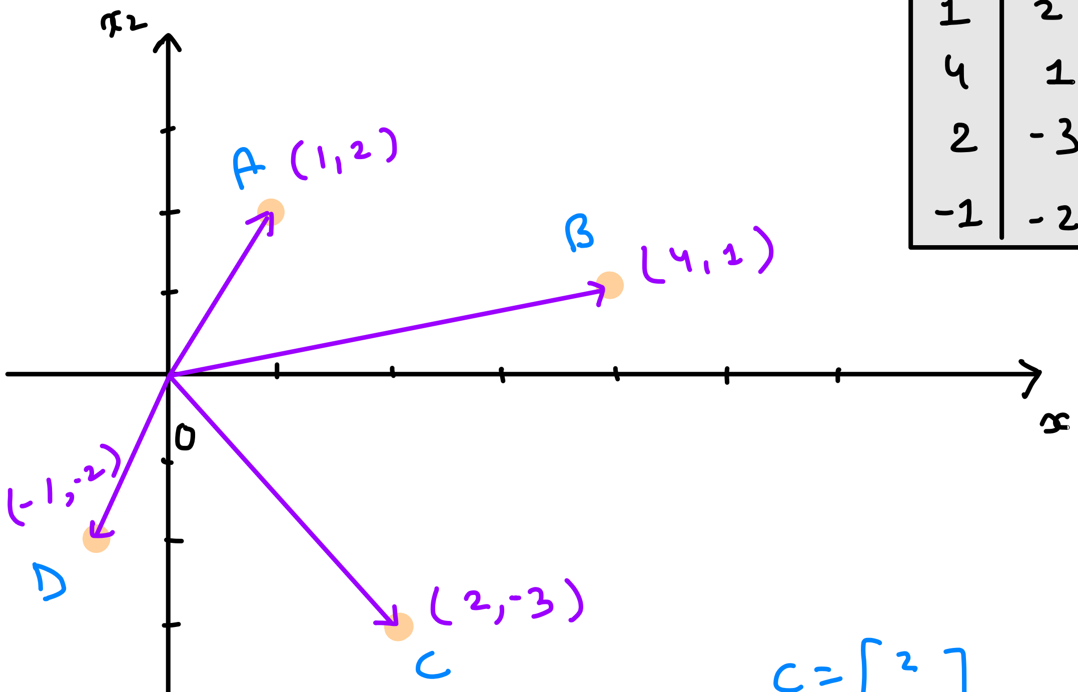
Row vector

$x \in \mathbb{R}^d$ $\rightarrow$ # elements in vector $= d$

$4.5 \in R^1$

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

x_1	x_2
1	2
4	1
2	-3
-1	-2



$B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$C^T = [2, -3]$

$B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$: $len(B) = \sqrt{4^2 + 1^2}$

$\Rightarrow \sqrt{17}$

length of vector

Norm of a vector =

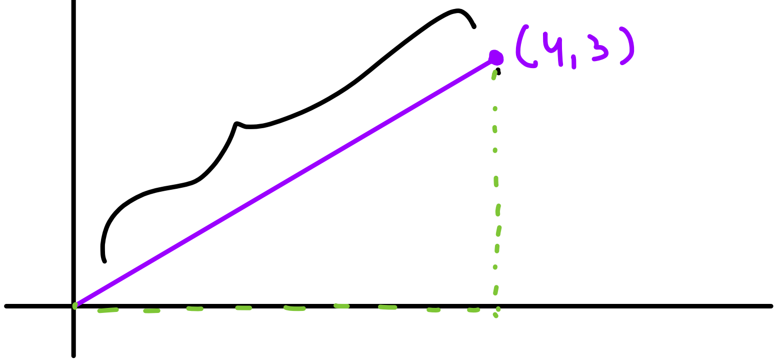
$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} = \|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_d^2}$$

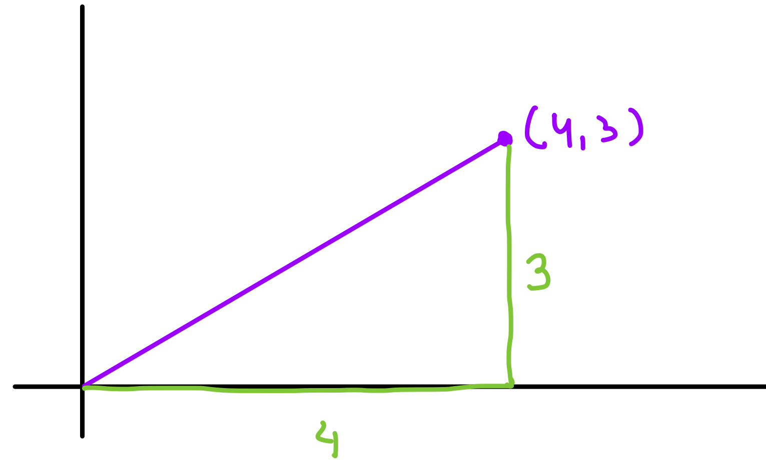
L2 Norm

$$\|\vec{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_d^2}$$

L1 Norm

$$\|\vec{x}\|_1 = |x_1| + |x_2| + |x_3| + \cdots + |x_d|$$





L_2 Norm $= \sqrt{4^2+3^2}$

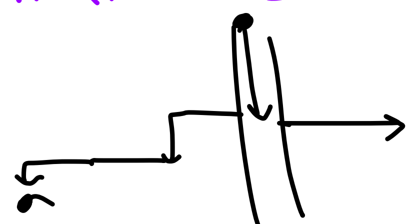
$= 5$ units

Euclidean dist

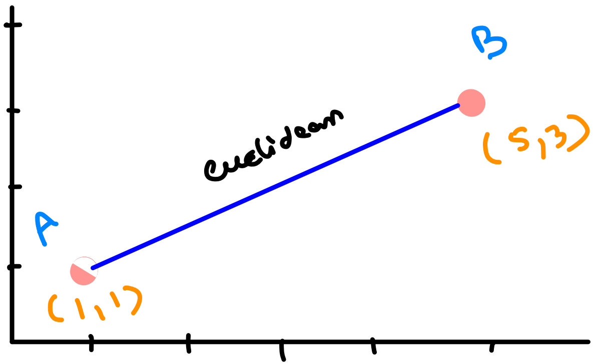
L_1 Norm $= 4+3$

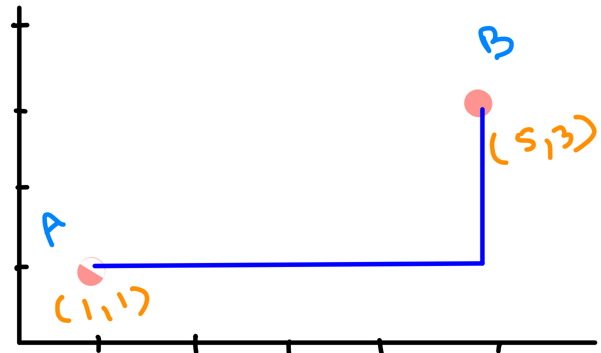
$= 7$

Manhattan distance



$A = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$





$$L_2 \text{ Dist}(A,B) \begin{bmatrix} 5-1 \\ 3-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

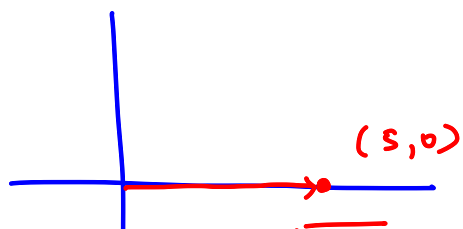
$$\sqrt{4^2 + 2^2}$$

$$= \sqrt{20}$$

$$L_1 \text{ Dist}(A,B) = \begin{bmatrix} 5-1 \\ 3-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= |4| + |2|$$

$$= 6$$



$L_2 = \sqrt{5^2 + 0^2} = 5 \qquad L_1 = 5$

Transpose

col^m ~~vector~~

$$x = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \qquad x^T = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$w^T = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$$

$$w^T \cdot x$$

$$\Rightarrow \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3$$

Dot Product = Matrix Multiplication

$$\mathrm{dot}(w, x) \qquad \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Downarrow$$

$$\mathrm{dot}(w^T, x) \qquad \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{(2 \times 3)}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{(3 \times 2)}$$

$A \cdot B = C$

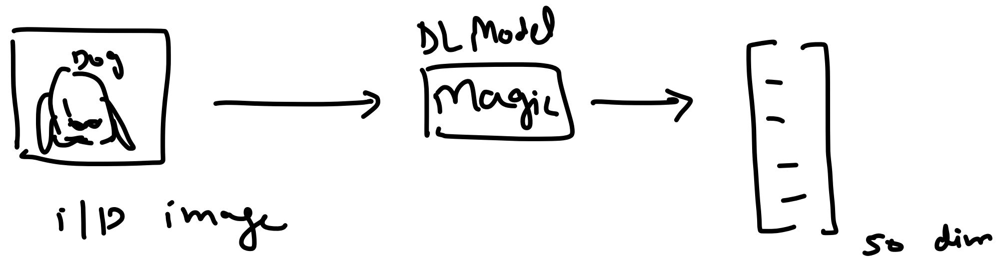
$(m, n) \quad (n, c) \quad (m, c)$

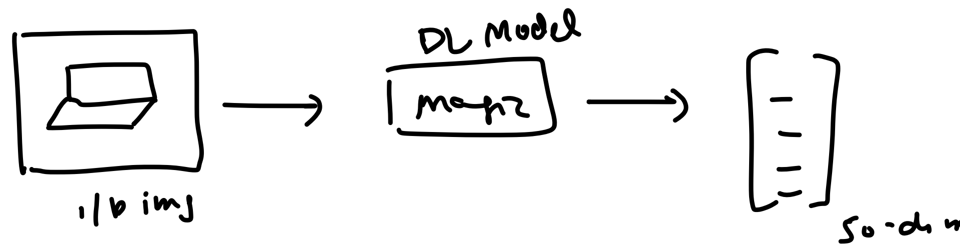
cols in Ist = # rows in IInd

Image Similarity

Dog Cat

Macbook Phone





Dot Product

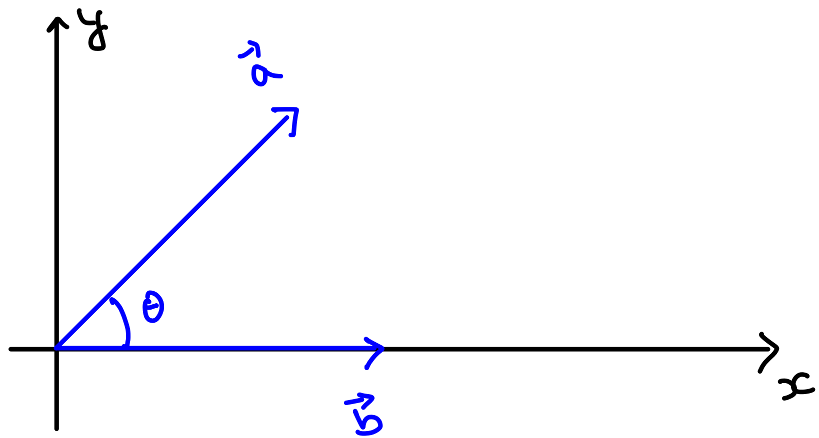
→ Calculate similarity x & y

$$\text{dot prod}(V_{dog}, V_{cat}) = \uparrow \qquad V_{dog} = \begin{bmatrix} 22 \\ 0.1 \\ 9 \\ \vdots \end{bmatrix}$$

$$\text{dot prod}(V_{dog}, V_{macbook}) = \downarrow \qquad V_{cat} = \begin{bmatrix} 15 \\ 0.05 \\ 28 \end{bmatrix}$$

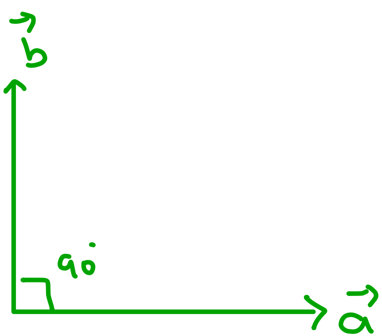
Angle b/w 2 Vectors

$\vec{a}.\vec{b} = a^T b$



$$\boxed{\cos\theta = \frac{a^T b}{\|a\|\,\|b\|}}$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\|a\|\,\|b\|} = a^T b$$



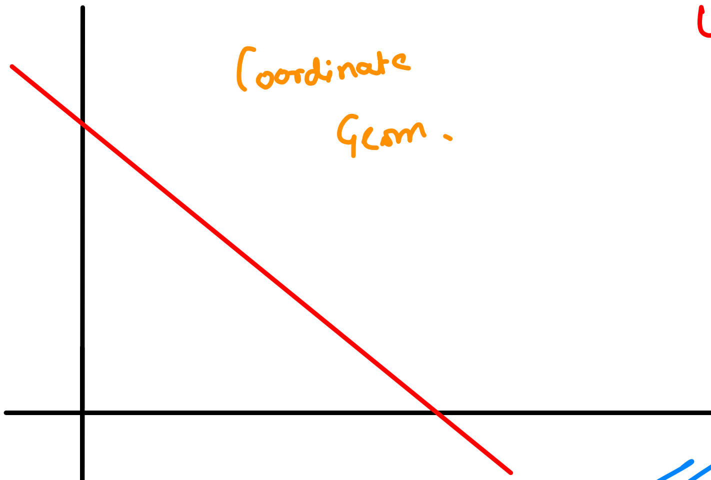
(dot Prod)

$$\underbrace{a.b}_{(-\infty,+\infty)} = \|a\|\,\|b\|\underbrace{\cos\theta}_{(-1,+1)}$$

$$= \|a\|.\|b\|\,.\,\underbrace{\cos 90^\circ}_{0}$$

$$\boxed{\vec{a}.\vec{b} = 0} \quad = \boxed{a^T b = 0}$$

Coordinate Geom & Linear Algebra



Coordinate Geom.

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

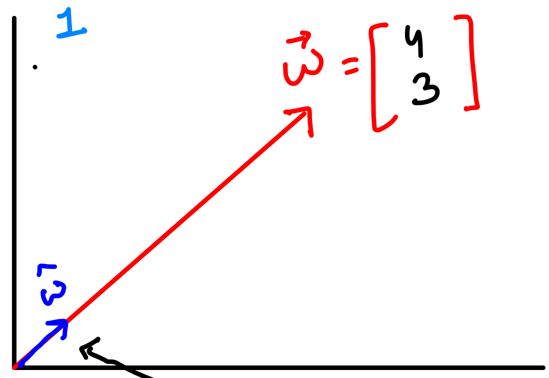
$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_d \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

real no.

$$\boxed{w^T x + w_0 = 0}$$

$$w_1 x_1 + w_2 x_2 + \cdots + w_d x_d + w_0 = 0$$

Unit Vector



$$\vec{w} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\|\hat{w}\| = 1$$

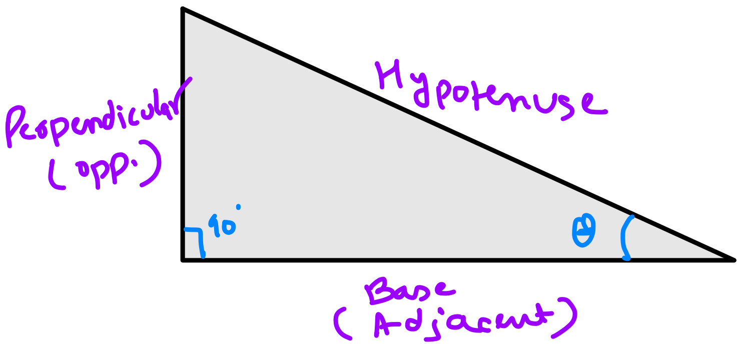
$$\|w\| = \sqrt{4^2+3^2}$$

$$\Rightarrow 5$$

$$\boxed{\hat{w} = \frac{\vec{w}}{\|w\|}}$$ (unit vector)

$$\hat{w} = \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$$

$$\|\hat{w}\| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16+9}{25}} = 1$$



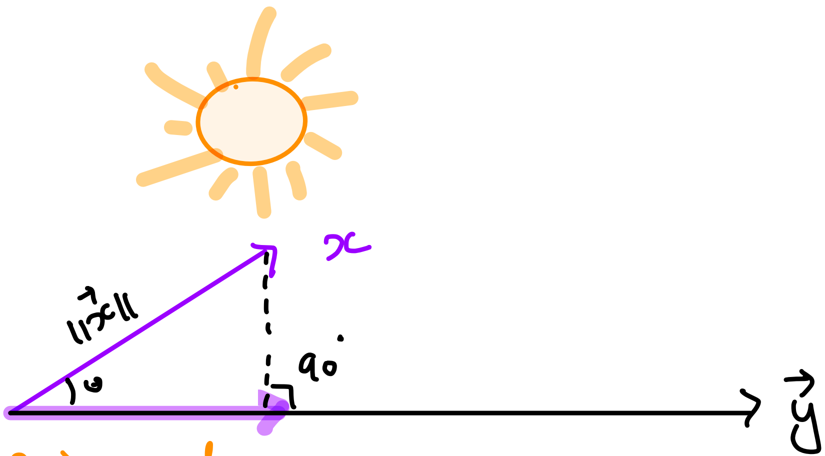
$$\sin(\theta) = \frac{\text{Perpendicular}}{\text{Hypotenous}}$$

$$\cos(\theta) = \frac{\text{Base}}{\text{Hypo}}$$

$$\tan(\theta) = \frac{\text{Perpendicular}}{\text{Base}}$$

Projection of a Vector → [PCA]

$$\vec{x} \cdot \vec{y} = x^T y$$



Projection of $\vec{x}$ onto $\vec{y}$ (shadow)

$$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{\|x\| \, \|y\|}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypo}} = \frac{\text{Proj}}{\|x\|}$$

$$\frac{\text{Proj}}{\cancel{\|x\|}} = \frac{\vec{x} \cdot \vec{y}}{\cancel{\|x\|} \, \|y\|}$$

$$\boxed{\text{Proj}_y x = \frac{\vec{x} \cdot \vec{y}}{\|y\|}}$$

$$\boxed{\text{Proj}_y \vec{x} = \vec{x} \, \hat{y}}$$

$$\mathrm{Proj}_y x = \frac{\|x\| \cancel{\|y\|} \cos\theta}{\cancel{\|y\|}}$$

$$\boxed{\mathrm{Proj}_y x = \frac{x^T y}{\|y\|}}$$

$$\boxed{\mathrm{Proj}_y x = \|x\| \cos\theta}$$