ADVANCED DISTRIBUTIONS-2

Suppose we receive 3 support tickets every 20 days.
(a) What is average or expected no.of tickets in 1 day?
(b) What is the probability that there will not be more than 1 ticket in a day?

(a) $3T \rightarrow 20$ days

$xT \rightarrow 1$ day

$\frac{3}{20}$ ← λ_1

(b) $P[X \leq 1] =$ poisson.cdf$\left(k=1, mu=\frac{3}{20}\right)$

$= 0.9898$

(c) $P[X \leq 1]_{\lambda_{30}}$

$\lambda \rightarrow 3T \rightarrow 20$ days

$x \rightarrow 30$ day

$\lambda_{30} = \frac{3 \times 30}{20} = 4.5$

There are 80 students in a kinder garten class.
Each of them has a 0.015 probability of forgetting their lunch on any given day.
(a) What is the average or expected no.of students who forgot lunch in the class?
(b)What is probability that exactly 3 of the will forget their lunch today?

$\lambda = n \times p$

(a)

$1 \text{ S} \longrightarrow \boxed{0.015}\ p$

$80 \text{ S} \longrightarrow x$

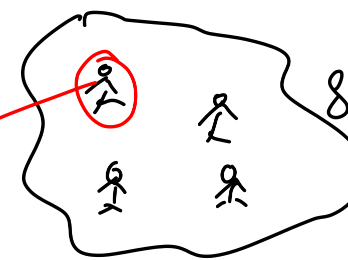
n

$1.2 \text{ S} / 1 \text{ day}$

(b) $P[X=3] =$ poisson.pmf(k = 3, mu = 1.2)

$= 0.086$

0.015



80

There are 80 students in a kinder garten class.
Each of them has a 0.015 probability of forgetting their lunch on any given day.
(a) What is the average or expected no.of students who forgot lunch in the class?
(b)What is probability that exactly 3 of the will forget their lunch today?

$P[X=k] = {}^{n}C_k (p)^k (1-p)^{n-k}$

$n = 80$
$p = 0.015$
$k = 3$

$P(s) = 0.015$
Success → forgetting lunch
$P(f) = 1 - 0.015$

$P[X=3] = {}^{80}C_3 (0.015)^3 (1-0.015)^{77}$
$= \text{binom.pmf}(k=3, p=0.015, n=80)$
$= \underline{0.086}$

$${}^{n}C_{k}\,(p)^{k}\,(1-p)^{n-k}$$

$$\frac{\lambda^{k} e^{-\lambda}}{k!}$$

S. forgetting lunch

$P(S) = 0.015$

$n = 80$

$k = 3$

$\lambda = n \times p$

λ = Rate

k = Count of occurrences

Binomial : Counting $\boxed{k}$ no. of success in $\boxed{n}$ trials

Poisson : Count no. of occurences in a given time/space interval.

1 success $\rightarrow$ probability (0.015)

n successes $\rightarrow$ n $\rightarrow$ $\boxed{np}$ — λ

We can approximate binomial distributions using poisson distribution

1. $n \geq 30$
2. $p \leq 0.05$

$\rightarrow \lambda = n \times p$

$$P[X=k] = \frac{e^{-\lambda} \lambda^{k}}{k!}$$

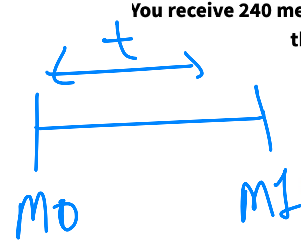
You receive 240 messages per hour on average - assume Poisson distributed.

Q1) What is the average time to wait between two messages?

$240\ M \rightarrow 3600$ secs

$1\ M \rightarrow \frac{3600}{240}$

$\lambda_{15} = 1$



You receive 240 messages per hour on average - assume Poisson distributed. What is the average time to wait between two messages?

31 users have participated

A	1s	10%
B	15s ✓	58%
C	30s	32%
D	45s	0%

Q2) What is the average number of messages per second ?

$240\ M \rightarrow 3600$ secs

$x \rightarrow 1$ secs

$\lambda_1 = 0.066 \quad x = \frac{240}{3600} = 0.066$

You receive 240 messages per hour on average - assume Poisson distributed. What is the average number of messages per second?

53 users have participated

A ✓	0.06	89%
B	0.6	8%
C	6	0%
D	60	4%

Q3) What is the probability of having no messages in 10 seconds?

240 M $\rightarrow$ 3600 secs

x M $\rightarrow$ 10 secs

$\lambda_{10} = 0.66$

$$P[X=0] = \frac{e^{-0.66}(0.66)^0}{0!} = e^{-0.66} \rightarrow e^{-\lambda_{10}} = e^{-10\lambda_1}$$

If the poisson rate is 0.067 messages per second,
What is probability of getting no messages in the next 10 seconds?

42 users have participated

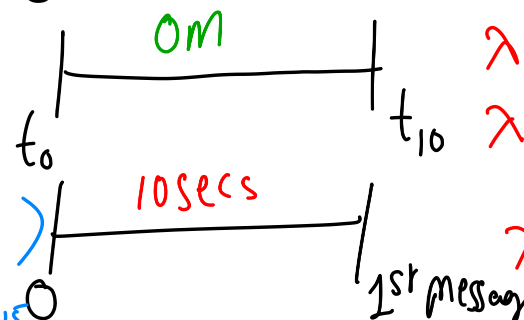
A	0.28	5%
B	0.31	17%
C	0.37	21%
D	0.51	57%

Q4) What is the probability of waiting for more than 10 seconds for the next message?

P[waiting time > 10secs for next occurrence]

$P[T > 10] = e^{-10\lambda_1}$

$P[T>10] = 1 - P[T \leq 10] = 1 - $ expon.cdf($x=10$, scale=15)



$\lambda_1 = 0.066$

$\lambda_{10} = 10 \times 0.066$
$= 10 \times \lambda_1$

$\lambda_k = k \times \lambda_1$

Q5) What is the probability of of waiting less than or equal to 10 seconds for next message?

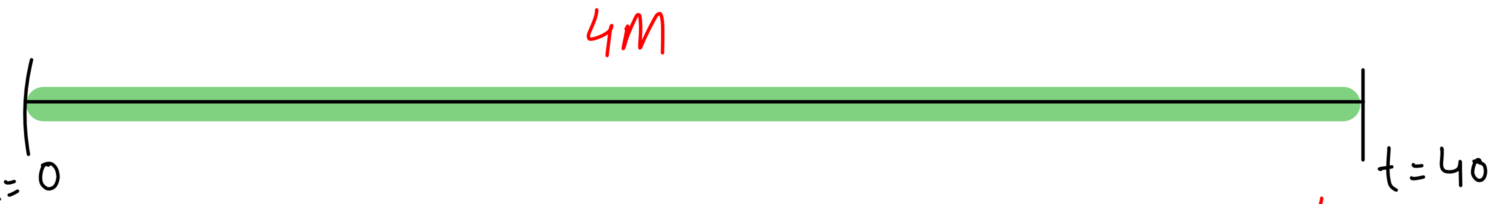
Scale $= 1/\lambda_1$

$P[X \leq 10] = $ expon.cdf($x=10$, scale=15) $= 0.4865$

$$P[X > k] = e^{-k\lambda} = 1 - \text{expon.cdf}(x = k, \text{scale} = \tfrac{1}{\lambda})$$

$$P[X \leq k] = 1 - e^{-k\lambda} = \text{expon.cdf}(x = k, \text{scale} = \tfrac{1}{\lambda})$$

poisson



poisson dist

λ : Avg no. of occurences in a given time/space interval

λ : $\frac{4}{40}$ M/secs. = $1/10$

4m → 40 secs

Exponential dist

Scale. Avg time b/w 2 occurences.

Scale = $\frac{40}{4} = 10$

$\boxed{\text{Scale} = \frac{1}{\lambda}}$

You are working as a data engineer who has to resolve any bugs/ failures of machine learning models in predictions
The time taken to debug is exponentially distributed with mean of 5 minutes.

scale = 5 ~~scale = 1/5~~

Q1) Find the probability of debugging in 4 to 5 mins?

$P[4 < T < 5]$ = expon.cdf(x=5, scale=5) - expon.cdf(x=4, scale=5)

= 0.0814

Q2) Find the probability of needing more than 6 minutes to debug?

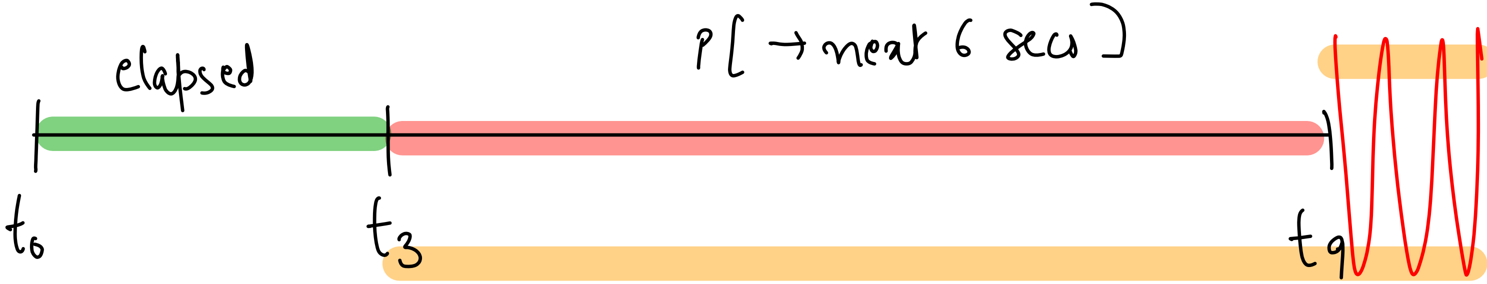
$P[T > 6] = 1 - P[T \le 6]$ = 1 - expon.cdf(x=6, scale=5)

= 0.3011

Q3) Given that you have already spent 3 minutes, what is the probability of needing more than 9 minutes?

$$P[T > 9 \mid T > 3] = \frac{P[(T>9) \cap (T>3)]}{P[T>3]}$$

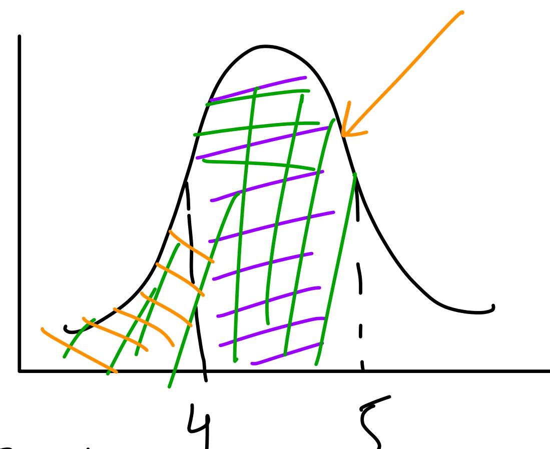
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



$$P[T>9 \mid T>3] = \frac{P[T>9]}{P[T>3]} = \frac{e^{-9\lambda}}{e^{-3\lambda}} = e^{-6\lambda}$$

$$= \frac{1-P[T\le 9]}{1-P[T\le 3]}$$

$$= \frac{1-\text{expon.cdf}(x=9, \text{scale}=5)}{1-\text{expon.cdf}(x=3, \text{scale}=5)} = 0.3011$$



"Exponential Distributions are memory less"

A call centre gets 3.5 calls per hour.

Q1) Calculate the probability that the next call will arrive at least 30 minutes after the previous call.

$$P[T > 30] = 1 - P[T \leq 30]$$

$$= 1 - \text{expon.cdf}(x = 30, \text{scale} = \quad)$$

3.5 Calls in 1 hour

3.5 C → 60 mins

x C → 30 mins

1 C → $\frac{60}{3.5}$ mins → Scale

$$1 - \text{expon}\left(x = 30, \text{scale} = \frac{60}{3.5}\right)$$

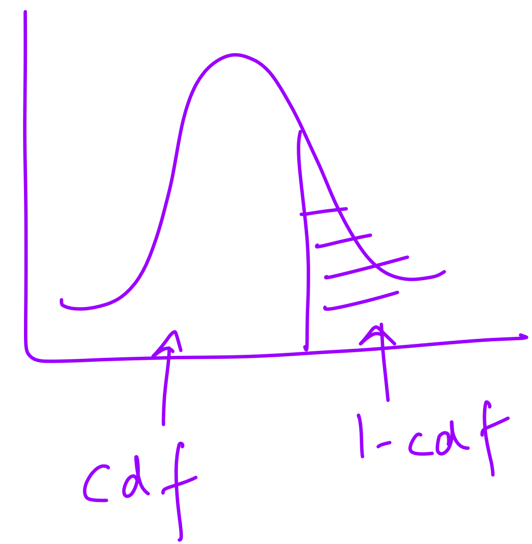
3.5 Calls in 1hour

1 Call → 1/3.5 hours.

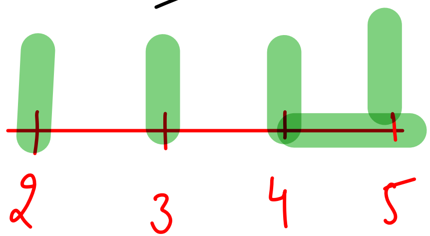
$$1 - \text{expon}\left(x = 0.5, \text{scale} = \frac{1}{3.5}\right)$$

$P[X \leq k] = \text{cdf}$

$P[X > k] = 1 - \text{cdf}$



Discrete



$P[X = k]$ = poisson.pmf(k=k, mu=λ)

$P[X < k]$ = poisson.cdf(k=k-1, mu=λ)

$P[X \leq k]$ = poisson.cdf(k=k, mu=λ)

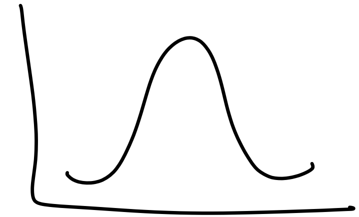
$P[X > k]$ = 1 - poisson.cdf(k=k, mu=λ)

$P[X > 4]$

$P[X \geq k]$ = 1 - poisson.cdf(k=k-1, mu=λ)

$P[X \geq 4]$

Continuous RV



$P[X = k]$ ✗

$P[X \le k]$ = expon.cdf(x=k, scale=scale)

$P[X < k]$ = expon.cdf(x=k, scale=scale)

$P[X > k]$ = 1 - expon.cdf(x=k, scale=scale)

$P[X \ge k]$ = 1 - expon.cdf(x=k, scale=scale)

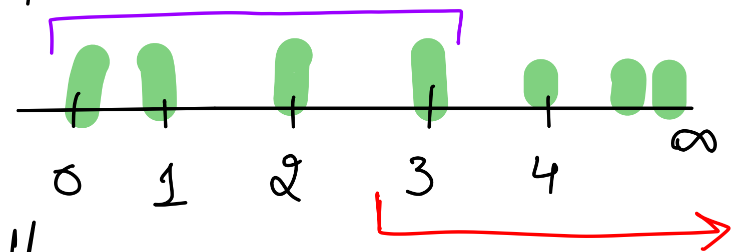
$P[a < X < b]$ = expon.cdf(x=b, scale=scale) - expon.cdf(x=a, scale=scale)

$P[a \le X \le b]$ = '' ''

Atmost

atmost 3 Calls

$$P[X \le 3] = P[X=0] + P[X=1] + P[X=2] + P[X=3]$$



Atleast

atleast 3 Calls

$$P[X \ge 3] = P[X=3] + P[X=4] + P[X=5] \ldots\ldots$$

$$P[X \ge 3] = 1 - P[X \le 2]$$

1 - dist.cdf(2, ...)

Exactly	$P[X = 3]$	
Between	$P[a < X < b]$	$P[a \leq X \leq b]$
Greater	$P[X > k]$	$P[X > k]$
less	$P[X < k]$	$P[X < k]$