BAYES THEOREM 2

Independant Events

Coin + Dice

H T   1 to 6

Outcomes: {H 1}, {H 2} . . . . . {H 6}
{T 1}, {T 2} . . . . {T 6}

Sample Space : {H1 . . . . . H6
T1 . . . . . T6}

A = Obtaining Heads = {H1 . . . H6}

B = Obtaining 3 on dice = {H3, T3}

$A \cap B = \{H3\}$

$P[A] = 6/12$

$P[B] = 2/12$

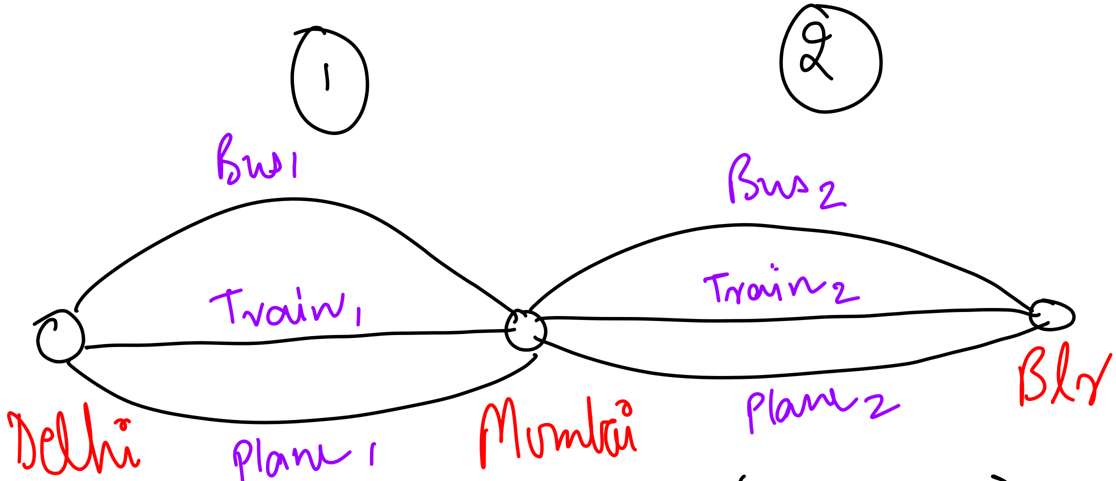
$P[A \cap B] = 1/12$

$$P[A/B] = \frac{P[A \cap B]}{P[B]} = \frac{1/\cancel{12}}{2/\cancel{12}} = 1/2$$

$$P[B/A] = \frac{P[A \cap B]}{P[A]} = \frac{1/\cancel{12}}{6/\cancel{12}} = 1/6$$

$P[A/B] = P[A]$ $\quad$ $P[B/A] = P[B]$

Y $\quad$ F



$\frac{1}{3} \times \frac{1}{3} \Rightarrow \frac{1}{9}$ $\quad$ $T_1 \cap P_2$

$B1 \to B_2$	$T_1 \to B_2$	$P_1 \to B_2$
T_2	T_2	T_2
P_2	P_2	P_2

$\} \to 9$

$$P[A/B] = P[A] \qquad P[B/A] = P[B]$$

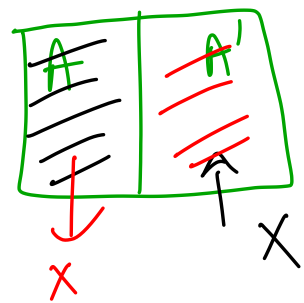
Claim: if A and B are mutually exclusive then A & B are not independant.

$$A \cap B = \{\}$$

$$P[A/B] = \frac{\boxed{P[A \cap B]}}{P[B]} = 0$$

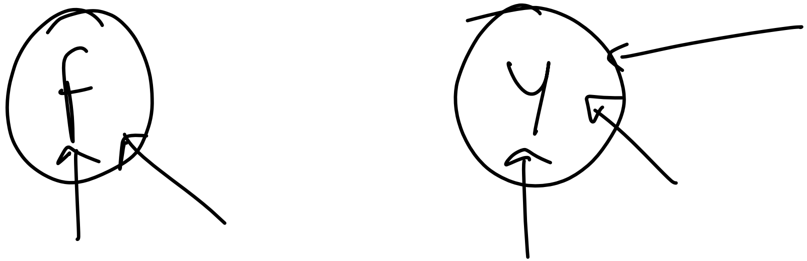
$$= 0$$

$$\underbrace{A \checkmark \qquad B \times}_{\text{Dependant}}$$



$$P[A/A'] =$$

War ↔ Peace



$$P[A \cap B] = P[A] \cdot P[B] \rightarrow \text{Independant Events.}$$

$$P[A/B] = \frac{P[A \cap B]}{P[B]} \Rightarrow \begin{array}{l} P[A \cap B] = P[A/B] \cdot P[B] \\ P[A \cap B] = P[A] \cdot P[B] \end{array}$$

$$P[T_1 \cap P_2] = P[T_1] \cdot P[P_2]$$

$$= 1/3 \cdot 1/3 = 1/9$$

A : obtaining H

B : obtaining T

$A \cap B = \{\}$

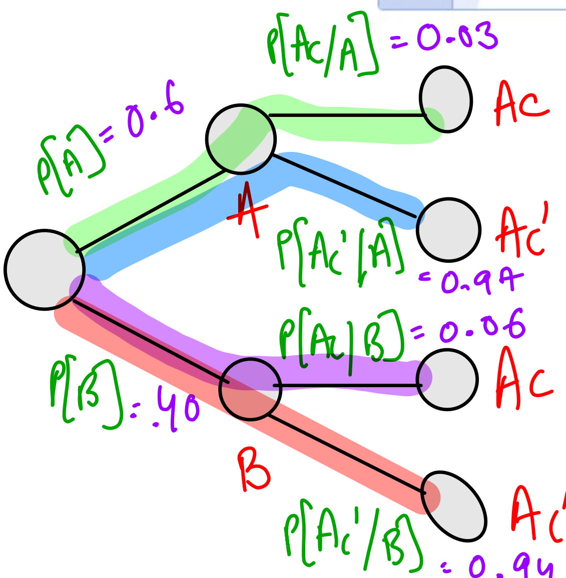
$P[A \cap B]$

$P[A|B] = P[A]$

A city has 2 taxi companies, A and B. A has 60% of the taxis in the city and B has 40%. A's taxis are involved in accidents 3% of the time, B's are involved in 6%. If a taxi is involved in an accident, what is the probability that it belongs to Company B?

43 users have participated

A	0.4	14%
B	0.57	49%
C	0.6	33%
D	0.69	5%



$$P[B/A_c] = \frac{\quad}{\quad + \quad}$$

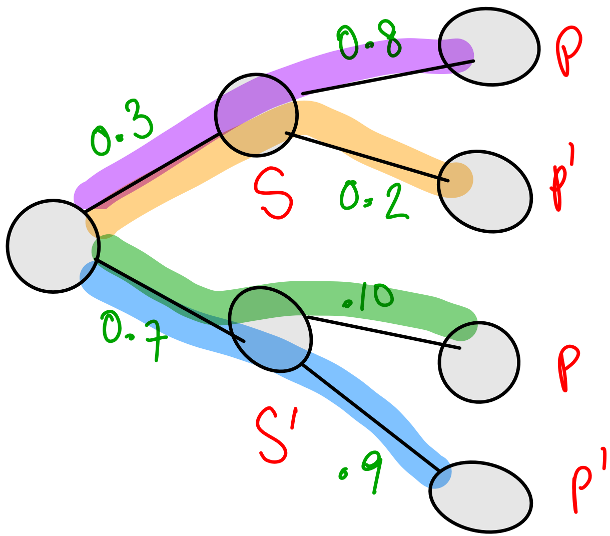
$$= \frac{0.4 \times 0.06}{0.4 \times 0.06 + 0.6 \times 0.03} = \frac{0.024}{0.024 + 0.018} = \frac{24}{42} \approx .57$$

It is known that 30% of emails are spam, and 70% are not spam. The word “purchase” occurs in 80% of spam emails. It also occurs in 10% of non-spam emails. A new mail does not has the word “purchase”. What is the probability that it is spam?

26 users have participated

A ✓	0.08	58%
B	0.09	31%
C	0.1	12%
D	0.11	0%

$$= \frac{0.06}{0.69} = 0.08$$



$$P[S \mid P'] = \frac{\quad}{\quad + \quad}$$

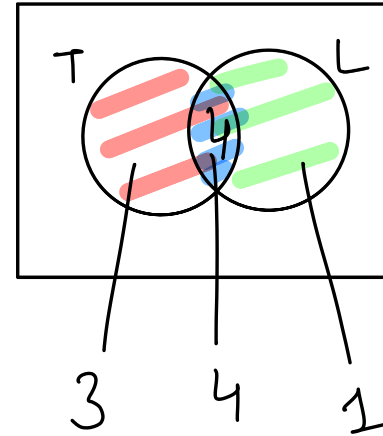
$$= \frac{0.3 \times 0.2}{0.3 \times 0.2 + 0.7 \times 0.9} = \frac{0.06}{0.06 + 0.63}$$

In a city, 7% of people are on Twitter.

5% of people are on LinkedIn.

4% of people are on both LinkedIn and Twitter.

A random LinkedIn user is chosen. What is the probability that he is on Twitter?



42 users have participated

A	0.2	12%
B	0.7	21%
C	0.8	60%
D	0.9	7%

$$P[T/L] = \frac{P[T\cap L]}{P[L]} = \frac{0.04}{0.05} = \frac{4}{5}$$

$$P[T/L] = 4/5 = \underline{0.8}$$

$$P[T] = 0.07$$

$$P[L] = 0.05$$

$$P[T\cap L] = 0.04$$

Independant $= P[T].P[L] = 0.0035$

Extra Info that a person is on L, did it increase /decrease the prob of a person on twitter?

$P[T] = 0.07$ $\qquad$ $7/100$ are on Twitter

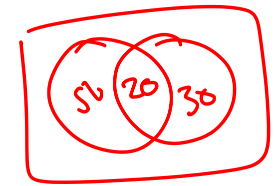
$\underbrace{P[T/L]}$ $= 0.08$

↳ $P[T/L] = P[T]$

A website has noticed the following stats.
Among those who saw the ad, 70% saw it on Youtube, 50% saw it on Amazon, 35% saw it on both.
A random person who saw the ad on Amazon is chosen. What is the probability that he also saw the ad on Youtube?

43 users have participated

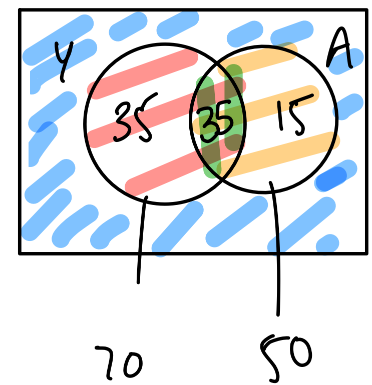
A	0.7	58%
B	0.5	16%
C	0.35	26%



$$P[Y/A] = \frac{35}{50} = 0.7 \quad \Big| \quad P[Y] = 0.7$$

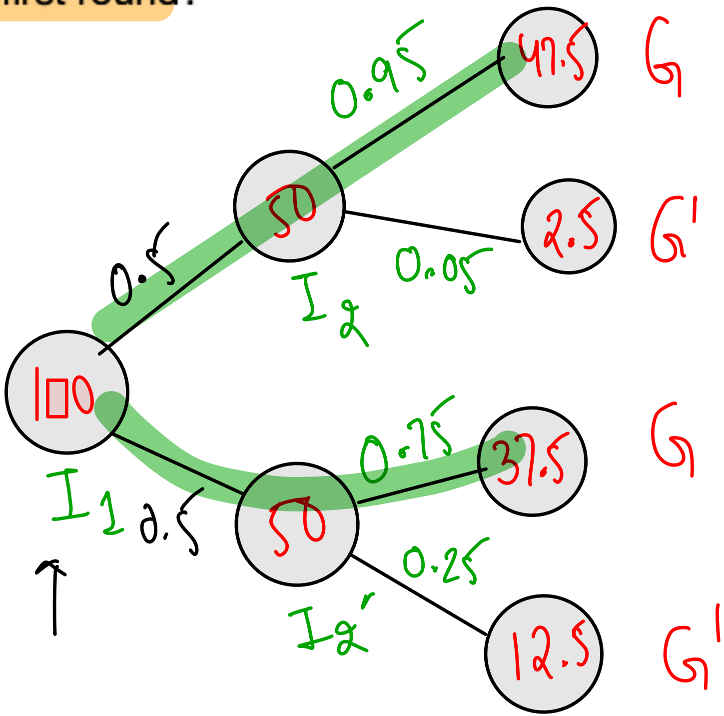
$$P[A/Y] = \frac{35}{70} = 0.5 \quad \Big| \quad P[A] = 0.5$$

addn info.



Y & A are independant

50 % of the people who gave the first interview were called for 2nd interview.
95% of the people who got invited for the second round felt that they had a good first round.
75% of the people who did not get invited for 2nd round also felt that they had a good first round.
Given that the person felt good about the first round, what is the probability that he cleared the first round?



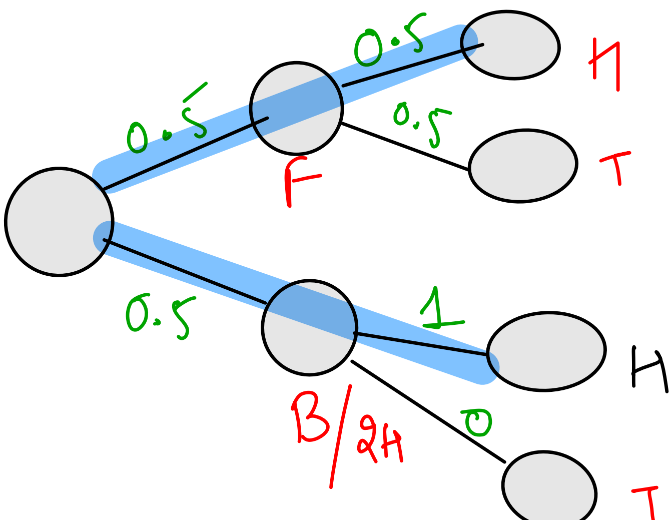
$$P[I_2/G] = \frac{47.5}{47.5+37.5}$$

$$= \underline{0.558}$$

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin?

40 users have participated

A	1/2	43%
B	1/3	40%
C	1/5	7%
D	1	10%



$$P[F/H] = \frac{\cancel{0.5}\times 0.5}{\cancel{0.5}\times 0.5 + \cancel{0.5}\times 1}$$

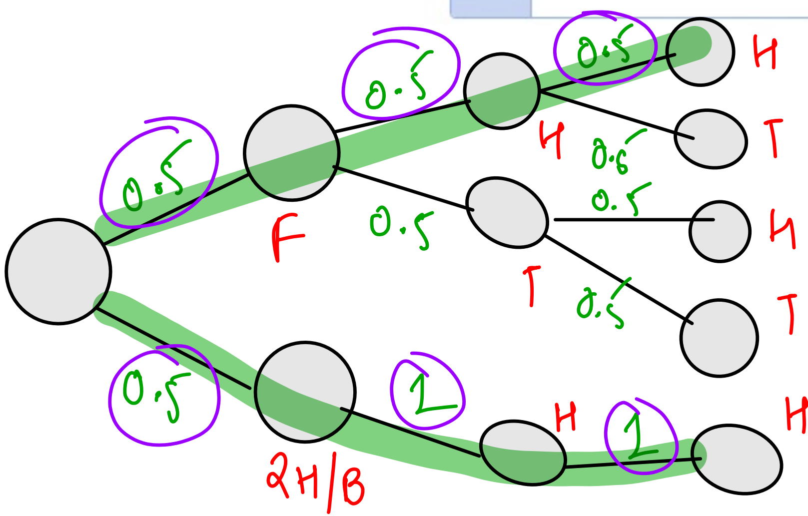
$$= \frac{0.5}{1.5} = \frac{1}{3}$$

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it twice. It shows heads both the times. What is the probability that it is the fair coin? → 3 times

38 users have participated

A	1/2	21%
B	1/3	18%
C	1/5 ✓	61%
D	1	0%

✓



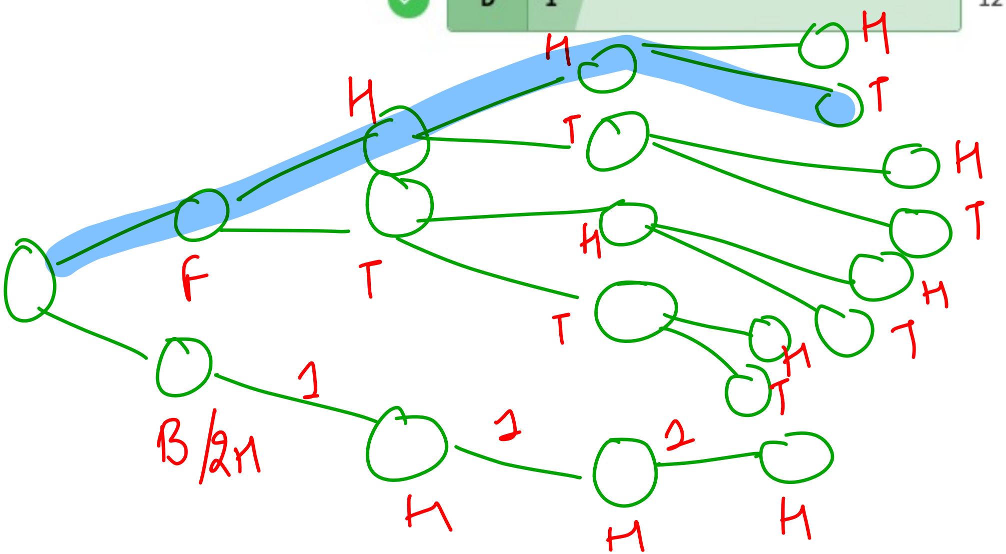
$$P[F/HH] = \frac{0.5 \times 0.5 \times 0.5}{(0.5)^3 + 1 \times 1 + 0.5}$$

$$= \frac{0.25}{1.25} = \frac{1}{5} \checkmark$$

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it three times. He gets {HHT}. What is the probability that it is the fair coin?

26 users have participated

A	1/2	19%
B	1/3	23%
C	1/5	46%
D	1	12%



$P[F/HHT] = 1$

The probabilities that "A" and "B" will tell the truth are 2/3 and 4/5 respectively. While acting as a witness in the court what is the probability that i) they agree with each other ii) they contradict each other.

$P\left[W_1/W_2\right] = P[W_1]$

$P\left[W_2/W_1\right] = P[W_2]$

T T F F

$\frac{2}{3} \times \frac{4}{5} + \frac{1}{3} \times \frac{1}{5}$

$\frac{8}{15} + \frac{1}{15} = \frac{9}{15} \frac{3}{5} = 0.6$

Agu

$0.5 \times \frac{2}{3} + 0.5 \times \frac{4}{5}$

$\frac{10 + 12}{15} = \frac{22}{1}$

T F   F T

$\frac{2}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{4}{5}$

$\frac{2}{15} + \frac{4}{15} = \frac{\cancel{6}\,2}{\cancel{15}\,5} = 0.4 = 40\%$

Disagree

$1 = 0.6 + 0.4$

$$
\begin{aligned}
P[\text{Agree}] &= P[W_1(T) \cap W_2(T)] + P[W_1(F) \cap W_2(F)] \\
&= P[W_1(T) / W_2(T)] \cdot P[W_2(T)] \\
&= P[W_1(T)] \cdot P[W_2(T)] + P[W_1(F)] \cdot P[W_2(F)] \\
&= \frac{2}{3} \cdot \frac{4}{5} + \frac{1}{3} \cdot \frac{1}{5} \Rightarrow \frac{8}{15} + \frac{1}{15} = \frac{9}{15} = 0.6
\end{aligned}
$$