ADVANCED DISTRIBUTIONS-1

RV

- Discrete
  - pmf
  - cdf
  - Geometric
  - Binomial
- Continuous
  - pdf
  - cdf
  - Normal / Gaussian

ppf

POISSON

"Count of occurrences in a given time/space interval"

DISCRETE

CONTINUOUS

Football game

Avg # of goals per 90 mins = 2.5

What is the prob. of having 1 goal in the last 30 mins?

Rate = 2.5 G / 90 mins

2.5 G → 90 mins

1.25 G → 45 mins

Customer going to a store

Avg # customer / day = 100

P[10 Customers will arrive in the next 1 hour]

Rate: 100 C / day

100 C → 1 day

1 day ≈ 8 hours

100 C / 8 ⇒ 12.5

Support Team

100 calls/hour

P[50 calls in 2 hours]

Rate. 100 C → 1 hour

200 C → 2 hours

Hospital Fortis

Avg # accident/day = 5

P[5 patients will arrive tomorrow]

Rate : 5 A / day

5 A → 1 day

25 A → 5 days

Rate: Avg $\underbrace{\text{no. of occurences}}_{\text{DISCRETE}}$ in a given $\underbrace{\text{time/space interval}}_{\text{Continuous}}$.

Notation

λ — mathematical

mu — Coding

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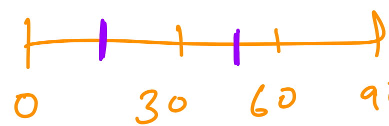
1. Counting: Random Variable that we have chosen must be no. of occurences in a given time/space interval.
2. Independance: Occurences are independant
3. No simultaneous occurences



4. Rate is independant of actual occurence.

BINOMIAL $\longrightarrow$ POISSON

Lim $x \to 0$

& $y \to \infty$

$$^{n}C_{k}\,(p)^{k}\,(1-p)^{n-k}$$

$$\boxed{\text{POISSON} : \frac{\lambda^{k} e^{-\lambda}}{k!}}$$

λ = rate

k = value

Count of occurrence

A city sees 3 accidents per day on average.
Find the probability that there will be 5 accidents tomorrow?

Rate : 3 A / Day $\Rightarrow \lambda$ $k = 5$

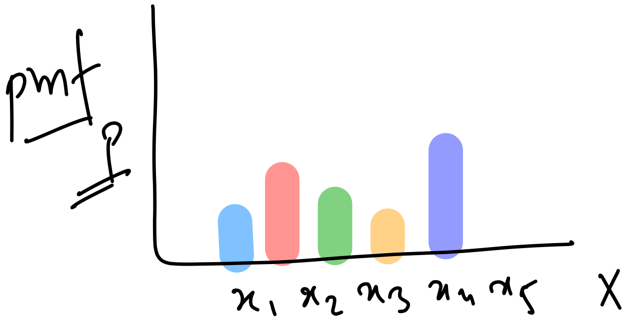
"X" : "No of Accidents in a day"

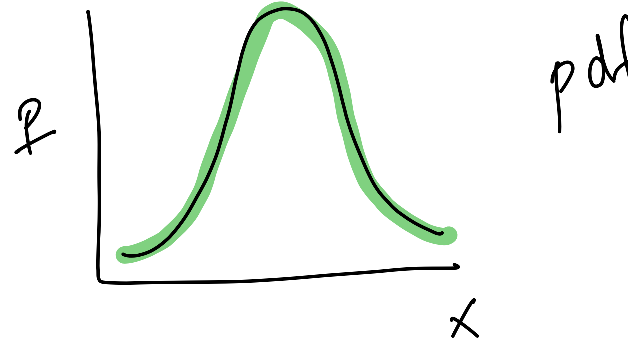
$$P[X=5] = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{3^5 e^{-3}}{5!} = 0.1008$$

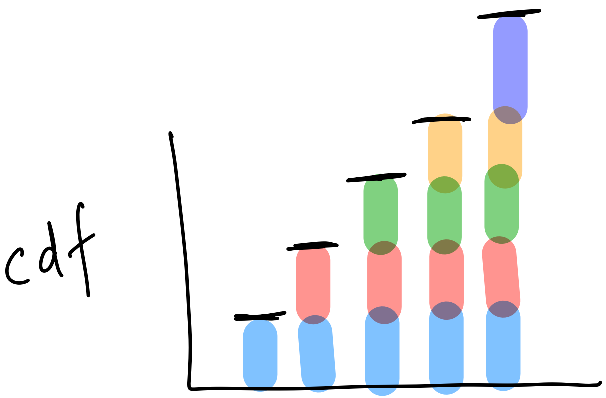
$P[X=5]$ = poisson.pmf$(k=5, mu=3)$ = $\underline{0.1008}$

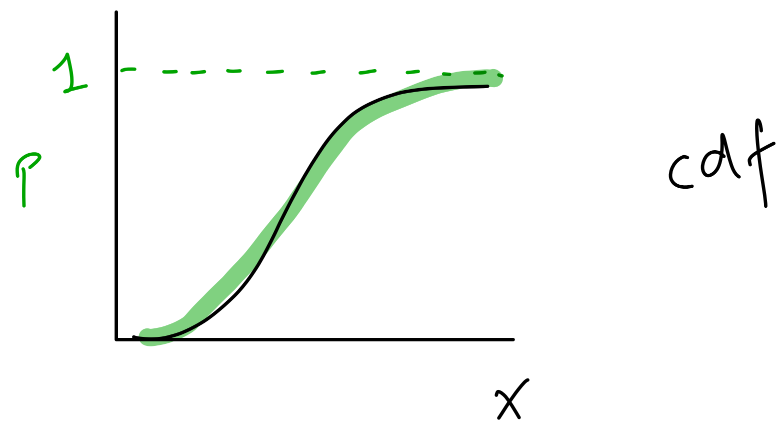
DISCRETE

Continuous









Let " X" be the number of typos in a page in a printed book, with mean 3 typos per page.
What is probability that a randomly selected page has atmost 1 typo?

Rate. 3 Typos / page λ , mu

"X" : " No. of typos in a page"

$$P[X \leq 1] = P[X=0] + P[X=1] = \frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!} = 0.199$$

$$P[X \leq 1] = \text{poisson.pmf}(k=0, mu=3) + \text{poisson.pmf}(k=0, mu=3) = 0.199$$

$$P[X \leq 1] = \text{poisson.cdf}(k=1, mu=3) = 0.199$$

A shop is open for 8 hours. The average number of customers is 74. Assume Poisson distributed.

Rate = 74 C / 8 hour

(a) What is average or expected number of customers in 2 hours?
(b) What is the probability that in 2 hours, there will be 15 customers?
(c) What is the probability that in 2 hours, there will be at least 7 customers?

(a) $\lambda_{8hr} = 74$

74 C $\rightarrow$ 8 hours

74/4 C $\rightarrow$ 8/4 hours

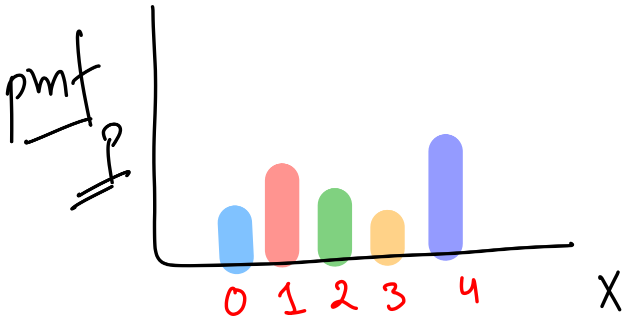
$\boxed{18.5 \text{ C} \rightarrow 2 \text{ hours.}}$

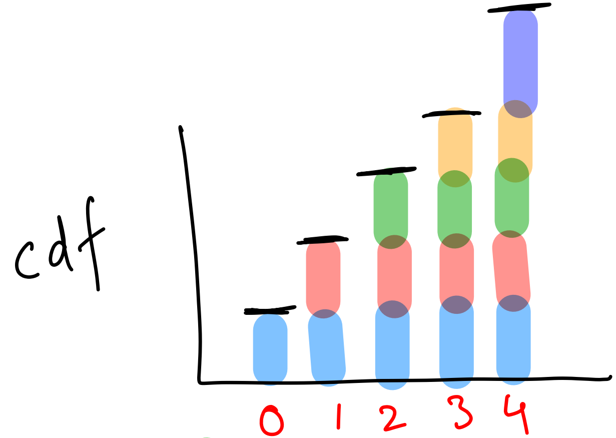
(b) $\lambda_{2hour} = 18.5$ $P[X = 15]$ = poisson.pmf(k = 15, mu = 18.5)

= 0.7188

(c) $\lambda_{2hour} = 18.5$ $P[X \geq 7]$ = 1 - poisson.cdf(k = 6, mu = 18.5)

= 0.992





poisson.pmf(k = 2, mu = x) = $\;_2 \rightarrow P[X = 2]$

poisson.cdf(k = 0, mu = x) = $\;^{0}$ $\quad P[X \leq 0]$

poisson.cdf(k = 1, mu = x) = $\;^{0} + \;^{1}$ $\quad P[X \leq 1]$

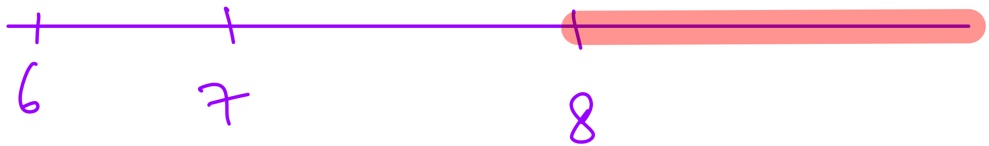
poisson.cdf(k = 2, mu = x) = $\;_0 + \;_1 + \;_2$ $\quad P[X \leq 2]$

$P[X \ge 2] = 1 - P[X \le 1]$

$= 1 - \text{poisson.cdf}(k = 1, mu = x)$

$\boxed{0, 1}, \boxed{2, 3, 4}$

$P[X > 7]$

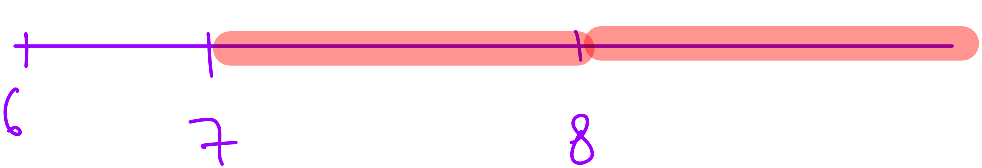


Discrete

$P[X \ge 7]$

Discrete

$P[X > 7]$



Continuous

$P[X \ge 7]$

Continuous

You receive 240 messages per hours on average. Assume Poisson Distributed.

(a) What is the average or expected number of messages in 30 seconds?

(b) What is the probability of 1 message arriving over a 30 seconds time interval?

(c) What is the probability that there will be no message 15 seconds?

(d) What is the probability that there are 3 messages in 20 seconds?

$\lambda = 240 m/hr$

(a) 240 m → 1 hour

240 m → 3600 sec ⇒ $\frac{240 m}{120}$ → $\frac{3600 \text{ sec}}{120}$

2 m → 30 sec.

$\lambda_{1 hour} = 240$

$\lambda_{30 secs} = 2$

(b) P[1 message in 30 Secs]

$\lambda_{30 secs} = 2$, $P[X = 1]$

poisson.pmf(k=1, mu=2) =

You receive 240 messages per hour on average - assume Poisson distributed. What is the probability of one message arriving over a 30 second time interval?

42 users have participated

A	0.18	14%
B	0.27	64%
C	0.39	10%
D	0.56	12%

(c) 240M → 3600 secs

$$\frac{\cancel{240}\ \cancel{24}}{\cancel{3600}\ \cancel{360}\ \cancel{15}} \times \cancel{15} = 1 \qquad \times 15$$

$\lambda_{15\ secs} = 1$

$P[X=0]$

poisson.pmf(k=0, mu=1) = 0.36

You receive 240 messages per hour on average - assume Poisson distributed. What is the probability that there are no messages in 15 seconds?

43 users have participated

A	0.27	7%
B ✓	0.36	77%
C	0.45	7%
D	0.54	9%

(d) $\lambda_{20\ secs}$

240 M → 3600 secs

$$\frac{2\ \cancel{240}}{3\ \cancel{3600}} \times 2\cancel{0} \qquad 1.33$$

$\lambda_{20\ secs} = 1.33$ or $4/3$

$P[X=3]$ = poisson.pmf(k=3, mu=4/3)
= 0.10413