# BAYES THEOREM

Covid → Home kit / RTPCR

↳ 10% $pop^n$

False positive

" negative

$10\% \; pop^n \to$ +ve → Affected

+ve → Not affected → False +ve

$10\% \; pop^n \to$ -ve → Not affected

-ve → Affected → false -ve

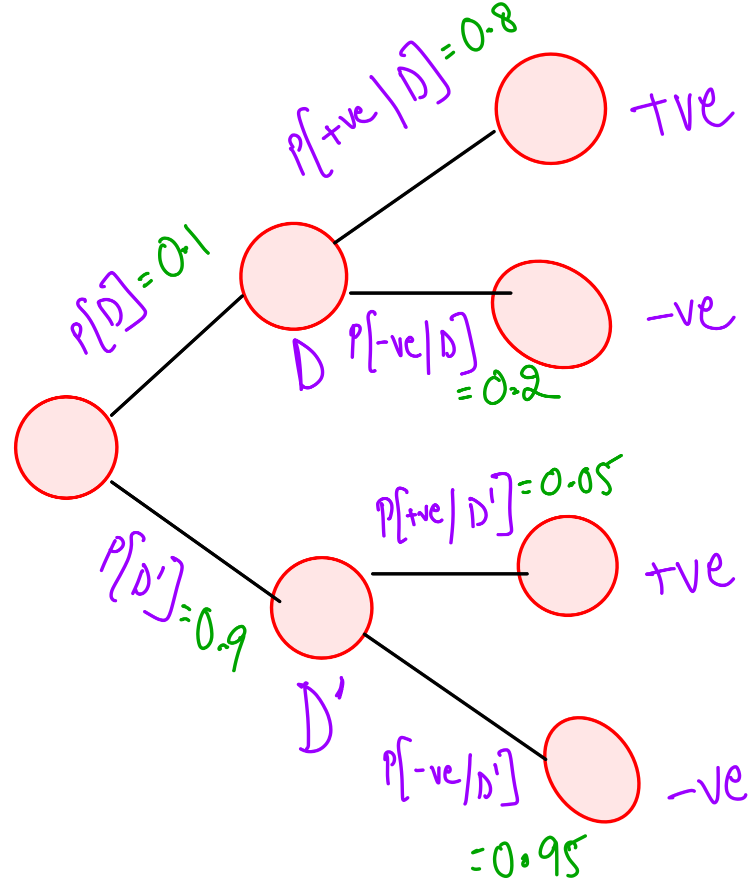
$D \to$ +ve, -ve

$D' \to$ +ve, -ve

1. A disease affects 10% of the population.
Among those who have the disease, 80% get "positive" test result.
Among those who don't have the disease, 5% get "positive" test result.
What is P(+ve | Disease)

43 users have participated

| | | |
|---|---|---|
| A | 0.05 | 19% |
| B | 0.1 | 12% |
| C ✓ | 0.8 | 63% |
| D | 0.9 | 7% |



$$\boxed{P[A/B] = \frac{P[A \cap B]}{P[B]}} \quad \Big\} \text{ Conditional Probability}$$

$$\boxed{\begin{aligned} P[A \cap B] &= P[A/B] \cdot P[B] \\ P[B \cap A] &= P[B/A] \cdot P[A] \end{aligned}} \quad \Big\} \text{ Multiplication Rule}$$

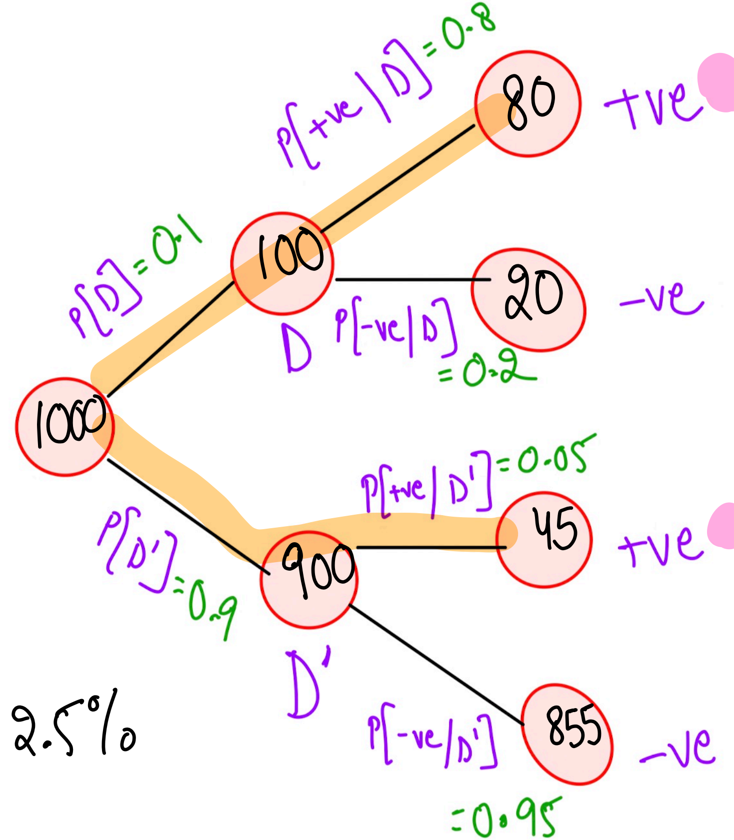
$$P[B/A] \cdot \underline{P[A]} = P[A/B] \cdot P[B]$$

$$\boxed{P[B/A] = \frac{P[A/B] \cdot P[B]}{P[A]}} \quad \Big\} \text{ Bayes Theorem.}$$

2. **A disease affects 10% of the population.**
**Among those who have the disease, 80% get "positive" test result**
**Among those who don't have the disease, 5% get "positive" test result.**
**Overall, what percentage of people tested "positive"?**

57 users have participated

| | | |
|---|---|---|
| A | 90 | 7% |
| B | 80 | 21% |
| C | 64 | 4% |
| D | 12.5 | 68% |

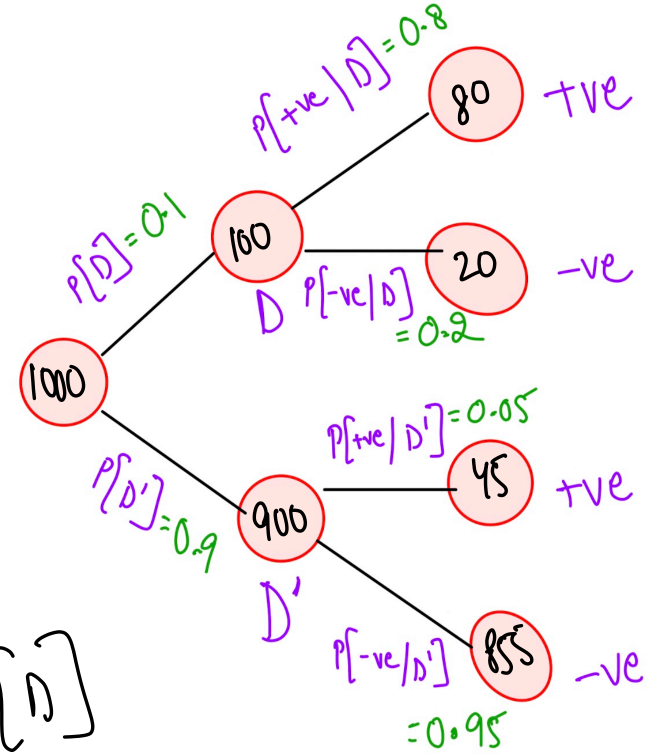



$$P[+ve] = \frac{80 + 45}{1000} = \frac{125}{1000} = 12.5\%$$

**3.** **A disease affects 10% of the population.**
**Among those who have the disease, 80% get "positive" test result.**
**Among those who don't have the disease, 5% get "positive" test result.**
**What is P(+ve ∩ Disease)?**

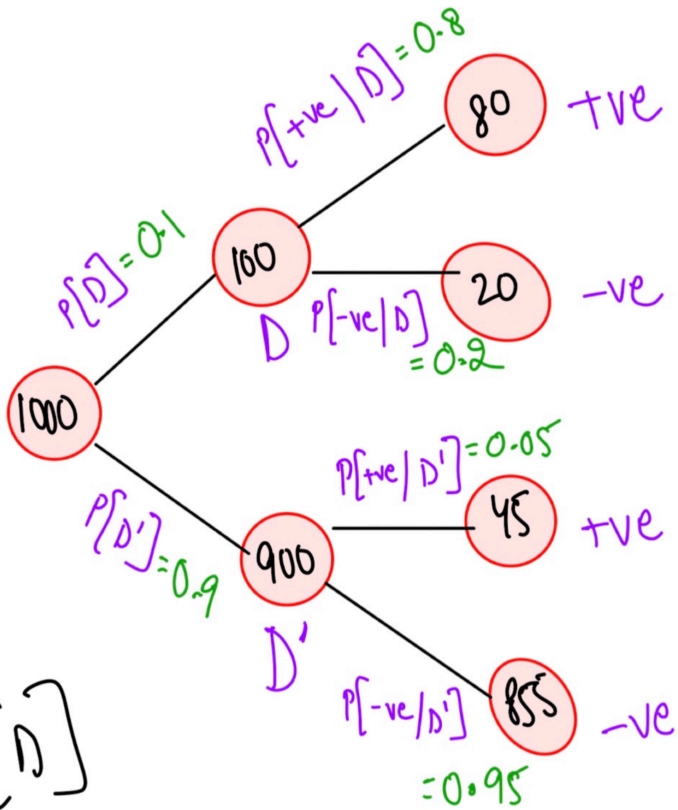
51 users have participated

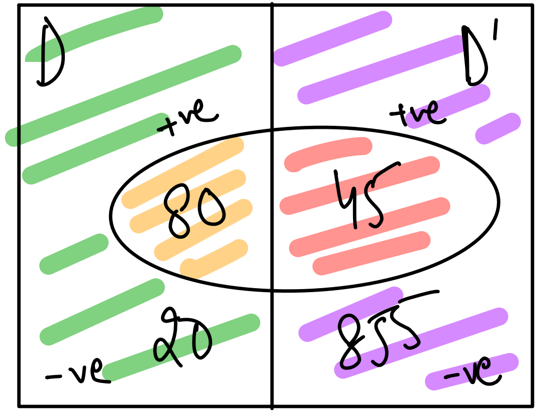
| | | |
|---|---|---|
| A | 0.8 | 39% |
| B | 0.64 | 2% |
| C | 0.1 | 12% |
| D | 0.08 | 47% |




$$P[+ve \cap \text{Disease}] = P[+ve / D] \cdot P[D]$$
$$= 0.8 \times 0.1 = 0.08$$
$$= \frac{80}{1000} = 0.08$$

1000

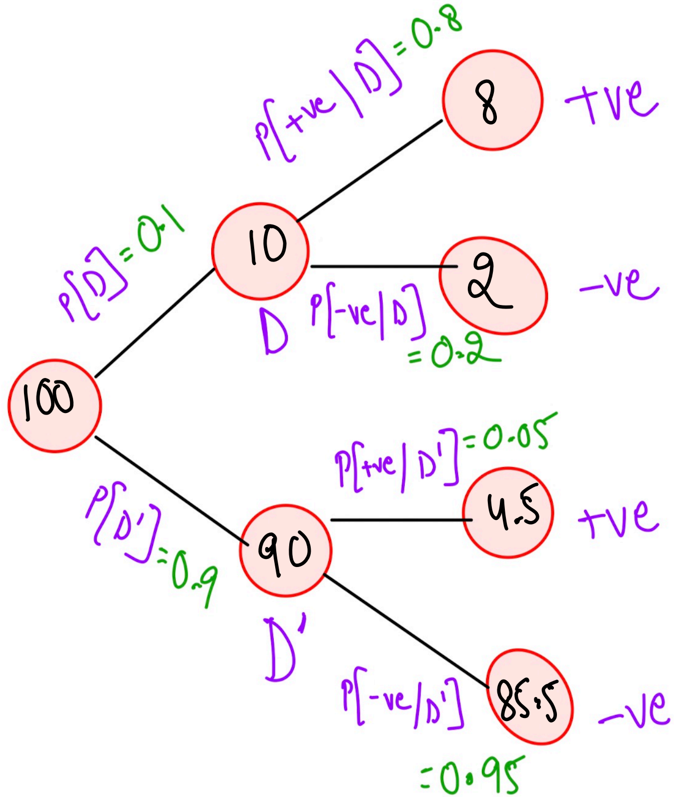



$$P[\underline{+ve} \cap \underline{\text{Disease}}] = \frac{80}{1000}$$

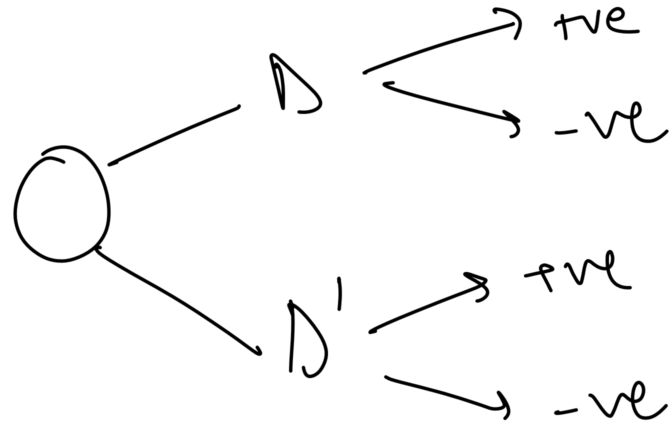
$$= \underline{0.08}$$

P[D]




$$P[+ve \cap \text{Disease}] = 8/100$$

**3.** **A disease affects 10% of the population.**
**Among those who have the disease, 80% get “positive” test result.**
**Among those who don’t have the disease, 5% get “positive” test result.**
**What is P(+ve ∩ Disease)?**



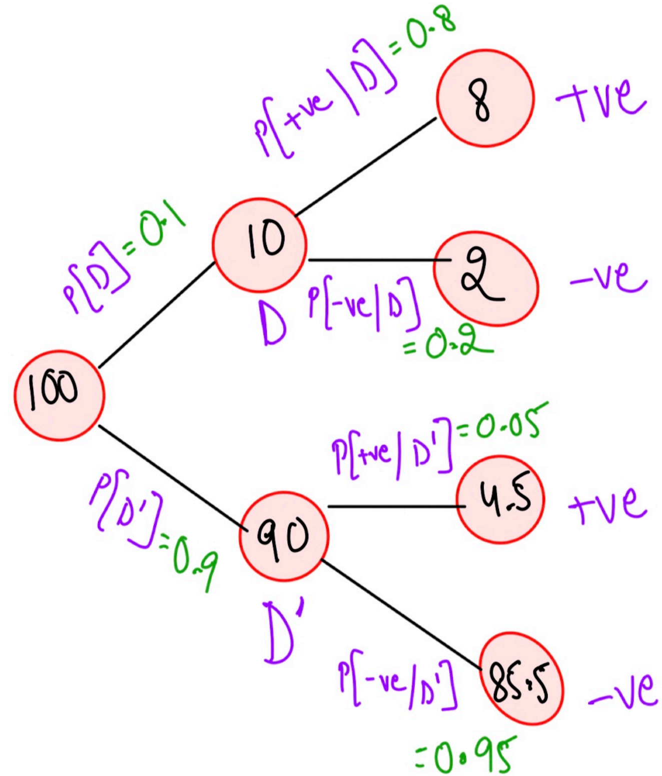
# Quiz time!

Quiz Ended!

**4.** **A disease affects 10% of the population.**
**Among those who have the disease, 80% get "positive" test result.**
**Among those who don't have the disease, 5% get "positive" test result.**
**What is P(+ve ∩ No Disease)**

54 users have participated

- **A** 0.1 — 4%
- **B** 0.06 — 7%
- **C** 0.0450 — 87% ✓
- **D** 0.012 — 2%



**Suppose that you are tested +ve.**
**What is the prob that you have the disease?**

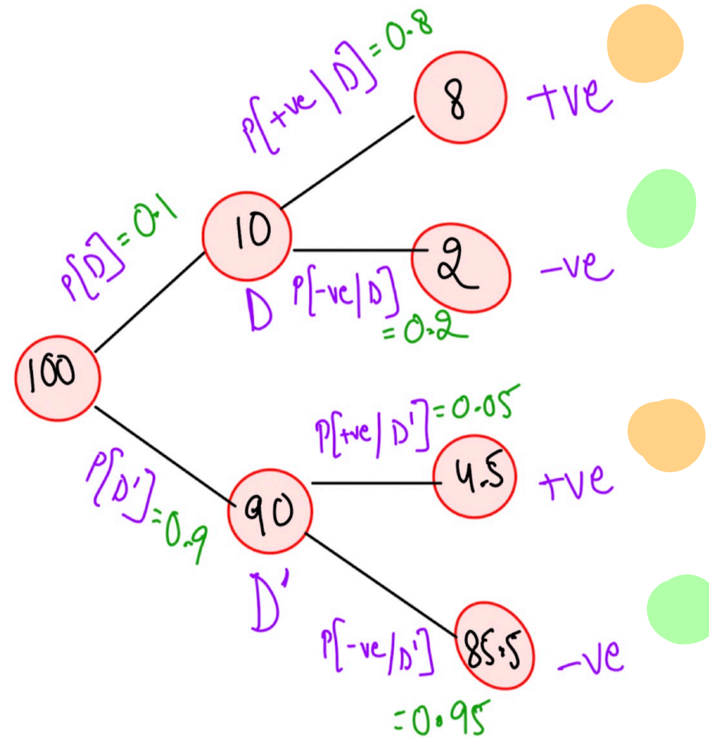
a) $P[D]$

b) $P[+ve \mid D]$

c) $P[D \mid +ve]$

d) $P[+ve]$

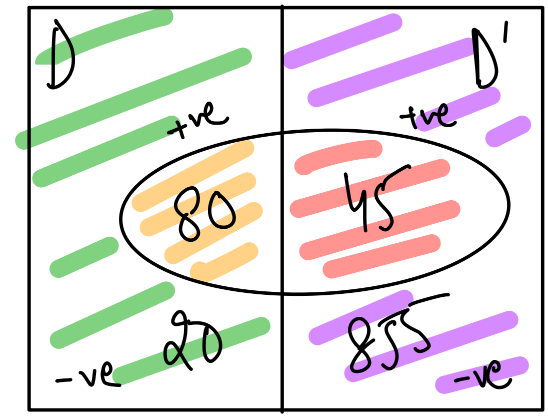
$$P[D \mid \underbrace{+ve}_{\text{given}}]$$




**If you are tested positive then you belong to ( 8 + 4.5 )% people. among these individuals how many do actually have the disease?**

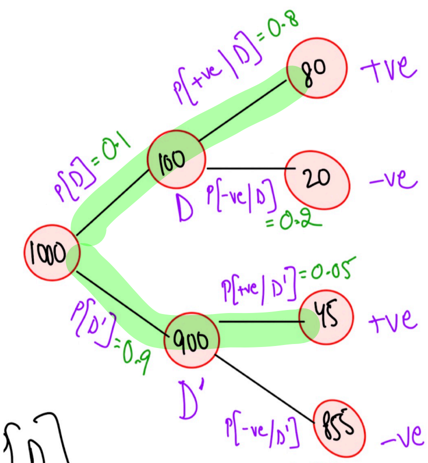
$$P[D / +ve] \quad \frac{8}{8+4.5} = \frac{8}{12.5} = 0.64$$

$$P\left[D/\underset{\sim\sim\sim}{+ve}\right] = \frac{80}{80+45} = 0.64$$




$$P[B|A] = \frac{P[A/B] \cdot P[B]}{P[A]}$$

$$P[D/+ve] = \frac{P[+ve/D] \cdot P[D]}{P[+ve]}$$



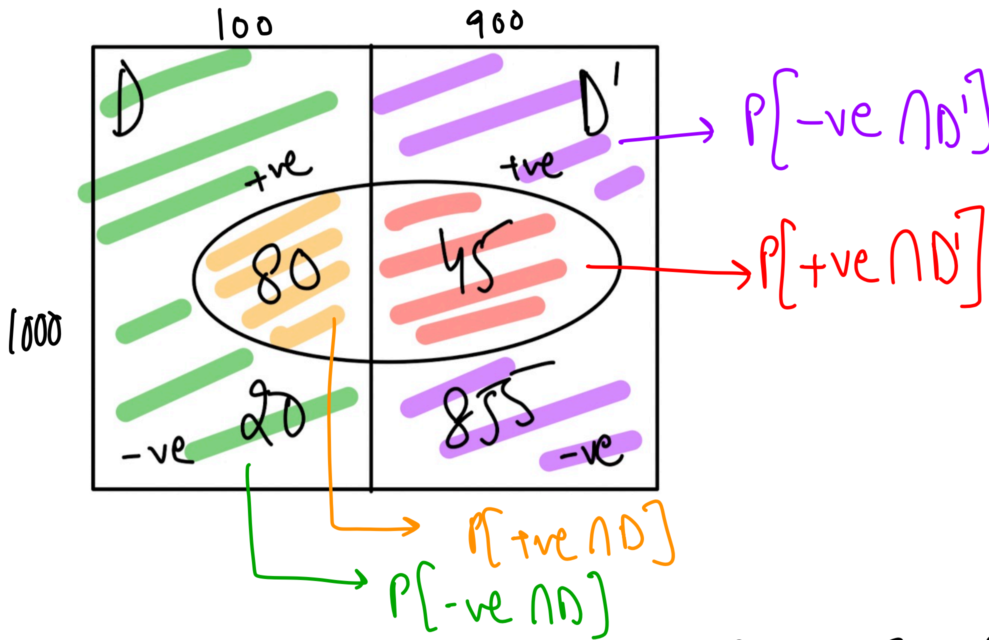

$[D]$

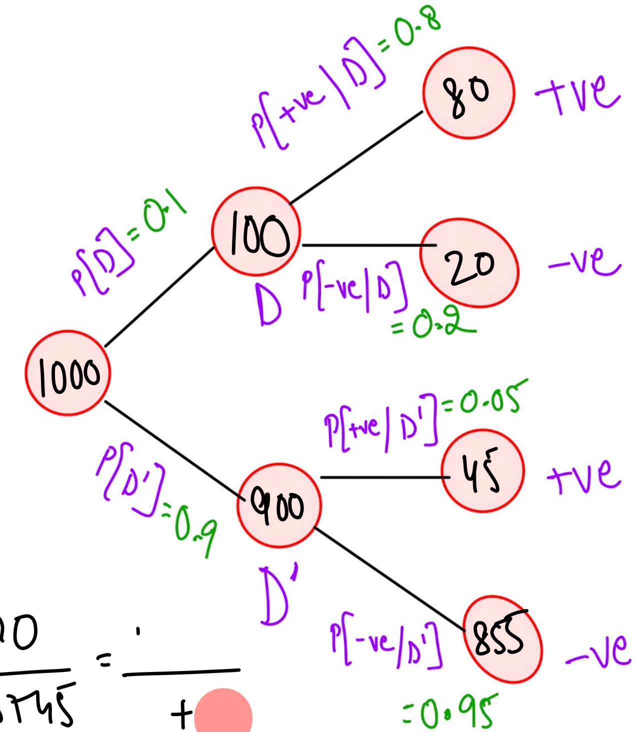
$$P[+ve] = P[+ve \cap D] + P[+ve \cap D']$$

$$= P[+ve/D] \cdot P[D] + P[+ve/D'] \cdot P[D']$$

$$P[D|+ve] = \frac{P[+ve/D] \cdot P[D]}{P[+ve/D] \cdot P[D] + P[+ve/D'] \, P[D']} = \frac{80}{80+45}$$

$$= 64\%$$




$$P[+ve/D] = 80/100 = 0.8$$

$$P[+ve/D'] = 45/900 = 0.05$$

$$P[-ve/D] = 20/100 = 0.2$$

$$P[-ve/D'] = 855/900 = 0.95$$

$$P[D/+ve] = \frac{80}{80+45} = \frac{\;}{\;+\;}$$

$$P[D/-ve] = \frac{20}{20+855} = \frac{\;}{\;+\;}$$

$$P[D'/+ve] = \frac{45}{80+45} = \frac{\;}{\;+\;}$$

$$P[D'/-ve] = \frac{855}{855+20} = \frac{\;}{\;+\;}$$

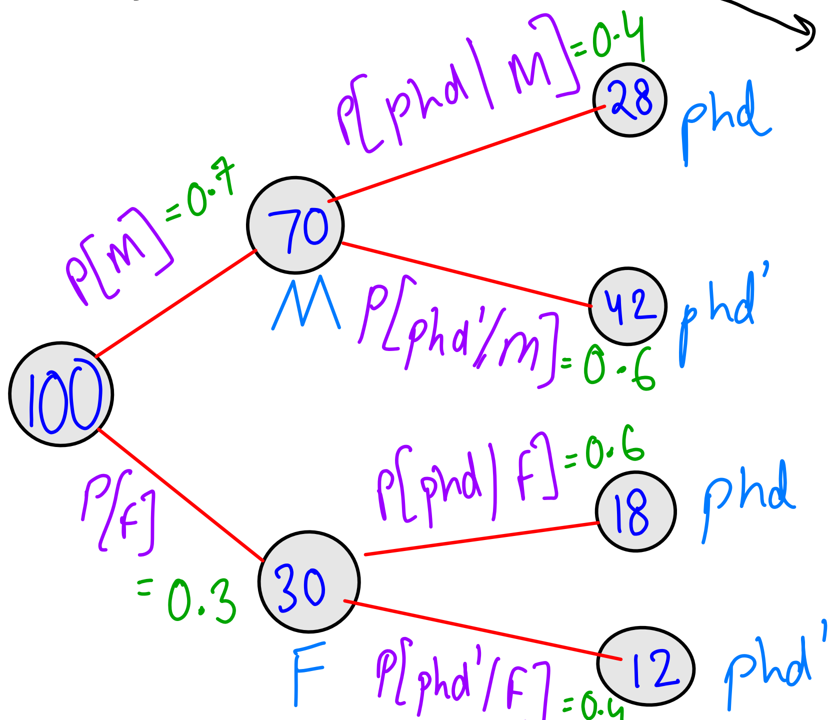
In a university, 30% of the faculty members are Female. Of the Female faculty members, 60% have PhD. Of the Male faculty members, 40% have PhD. What is the Probability that

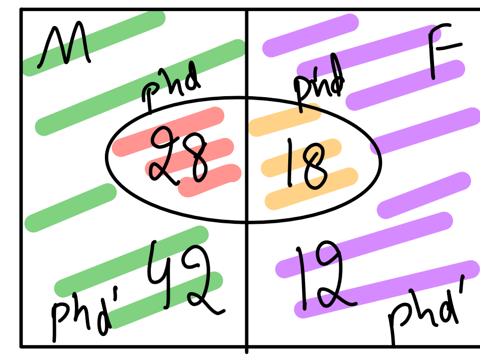
(a) randomly chosen faculty member is Female and has a PhD? $P[F \cap Phd]$

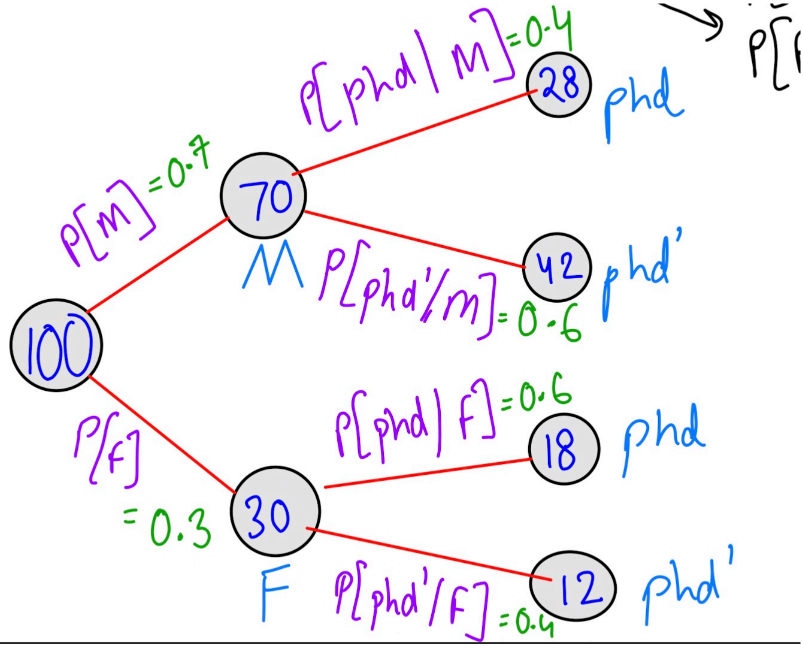
(b) randomly chosen faculty member is Male and has a PhD? $\rightarrow P[M \cap Phd]$

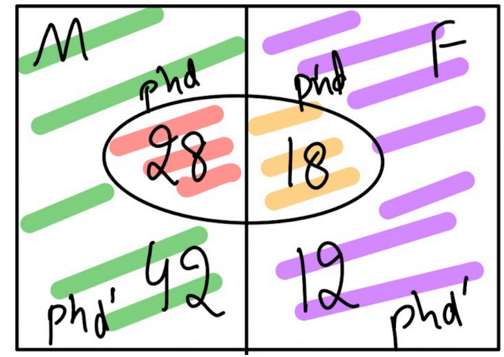
(c) randomly chosen faculty member has a PhD? $\rightarrow P[Phd]$

(d) randomly chosen PhD holder is Female? $\rightarrow P[F / Phd]$






$P[M] = 0.7$

$P[F] = 0.3$

$P[phd/F] = 0.6$

$P[phd/M] = 0.4$

$P[phd'/F] = 0.4$

$P[phd'/M] = 0.6$

$$P[phd \cap F] = 0.18 = P[phd/F] \cdot P[F]$$
$$= 0.6 \times 0.3$$
$$= 0.18$$

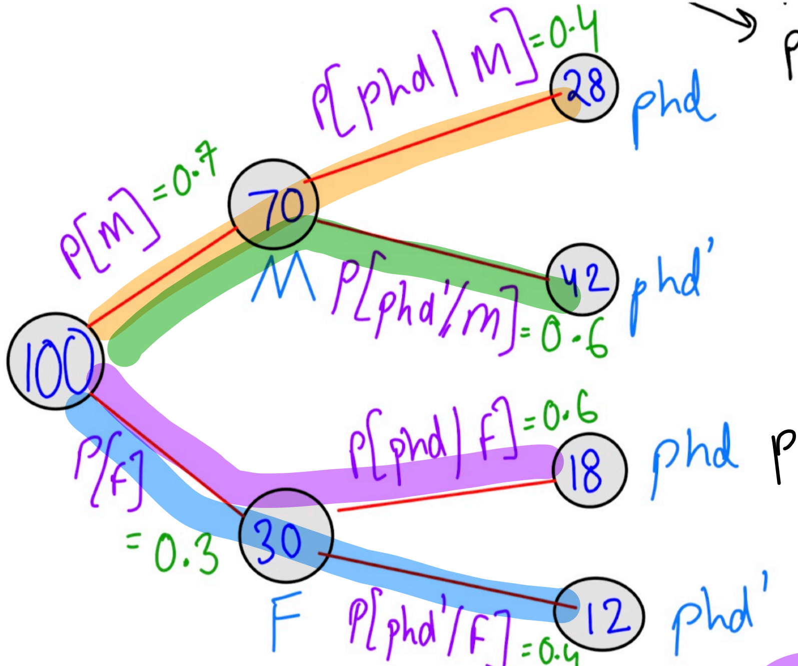
$$P[phd \cap M] = 0.28 = P[phd/M] \cdot P[M]$$
$$= 0.7 \times 0.4$$
$$= 0.28$$

$$P[F/phd] = \frac{P[phd/F] \cdot P[F]}{P[phd]}$$

$$P[phd] = P[phd \cap M] + P[phd \cap F]$$
$$= 0.18 + 0.28 = 0.46$$
$$\Rightarrow P[phd/M].P[M] + P[phd|F].P[F]$$

$$P[F/phd] = \frac{0.6 \times 0.3}{0.46} = \frac{0.18}{0.46}$$

$$P[F/phd] = \frac{P[phd/F].P[F]}{P[phd|F].P[F] + P[phd/M].P[M]} = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.7} = \frac{0.18}{0.46}$$




$$P[M/phd] = \frac{\quad}{\quad + \quad} = \frac{28}{28+18}$$

$$P[M/phd] = \frac{P[M]\cdot P[phd|M]}{P[M]\cdot P[phd|M] + P[F]\cdot P[phd|F]}$$

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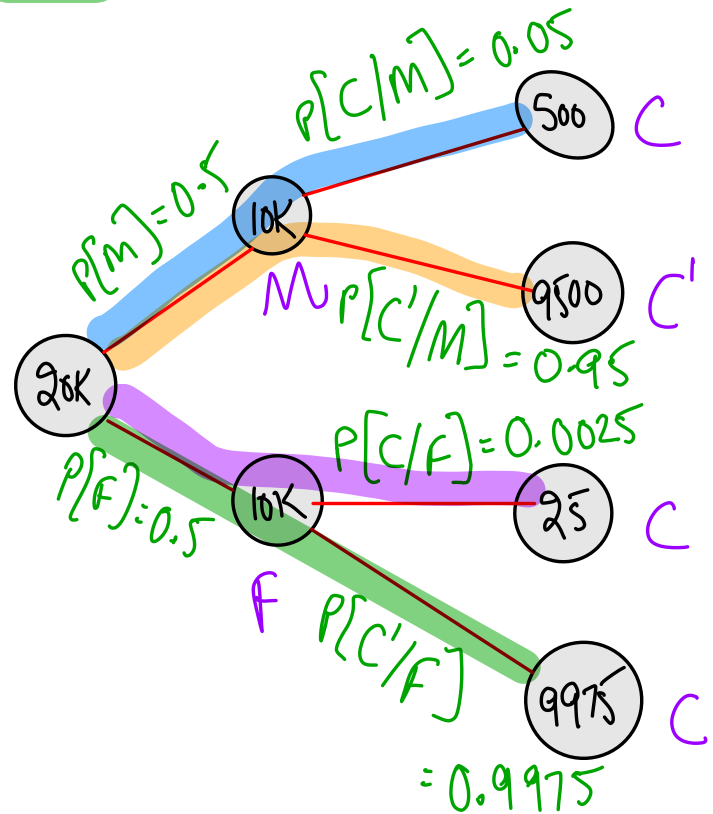
$$P[F/phd] = \frac{\quad}{\quad + \quad} = \frac{18}{18+28} = \frac{P[F]\cdot P[phd|F]}{P[F]\cdot P[phd|F] + P[M]\cdot P[phd|M]}$$

$$P[F/phd'] = \frac{\quad}{\quad + \quad}$$

$$P[M/phd'] = \frac{\quad}{\quad + \quad}$$

**Suppose 5 % Men and 0.25% Women are color blind. A Randomly color blind person is chosen. What is the prob that this person is male? Assume same number of Men and Women.**




$$P[M/C] = \frac{\quad}{\quad + \quad}$$

$$= \frac{\cancel{0.5} \times 0.05}{\cancel{0.5} \times 0.05 + \cancel{0.5} \times 0.0025}$$

$$= \frac{0.05}{0.05 + 0.0025} = \frac{0.0500}{0.0525}$$

$$= \frac{500}{525} = 95.23\% = 0.9523$$

$$P[F/C] = \frac{\quad}{\quad + \quad} \qquad \Bigg| \qquad P[F/C'] = \frac{\quad}{\quad + \quad}$$

$$P[M/C'] = \frac{\quad}{\quad + \quad}$$