PROBABILITY DISTRIBUTIONS 4

- Bernoulli
- Binomial
- Normal
- lognormal
- Poisson
- Exponential

} Covered

- Geometric
- Interview Questions

} this video

Geometric:

Consider bernoulli's trial → Success (p)
→ failure (1-p)

- Identifies Prob. of Ist Success in 'k' trials

- Tossing a coin
  - heads → success

- What is the Prob. of Ist heads comes in 7th Toss ?
  - Till 6 tosses → failure (i.e Tails) $P(T) = (1-p)$
    7th Toss → Succes (i.e Heads) $P(H) = p$

⇒ Sequence for which I need to find prob.:

T T T T T T H ⇒ $P[X=7] = (1-p)^6 \cdot p$.

- In general,
  Probability of first Success comes in $\underline{R}^{th}$ trial ?

Success prob $= p$

failure prob $= 1-p$

$$P[X=R] = (1-p)^{R-1} \cdot p$$

I am playing a game where the prob of winning a prize is 0.7

What is the probability that I win the prize on the 4th attempt?

$P[X = 4] = (0.3)^3(0.7)$

$P[X = 4] =$ **geom.pmf(k = 4, p = 0.7)** $= 0.018$ **from scipy.stats import geom**

$= 1.8\%$

What is the probability that I don't win in the first two attempts?

$P[X > 2] = 1 - P[X <= 2]$

$P[X > 2] = 1 -$ **geom.cdf(k = 2, p = 0.7)** $= 0.91 \approx 91\%$

Q.) Expected no. of die rolls before we get 1^{st} 6.?

$SS = \{1, 2, 3, 4, 5, 6\}$

$\{6\} \rightarrow$ Succes

$p = P(\text{Succes}) = P(\{6\}) = 1/6 \approx 0.166 \rightarrow p$

$(1-p) \rightarrow$ prob. of failure $\Rightarrow$ S, FS, FFS, FFFS, FFFFS, ...

# of times we roll	fraction of Success we get
1	$(1/6) \approx 0.166$
x	1

$\Rightarrow \frac{1}{x} = \frac{(1/6)}{1} \Rightarrow x = 6$

$x = \frac{1}{p}$

expected # of rolls for 1^{st} Succes.

⇒ Expected # of trials till we get 1st success ⇒ $\boxed{\mu = 1/p}$

What is this process?

– doing trial till first success ?

↓

Geometric db^n

- $p(H) = 0.25 \leftrightarrow 25\%$.

⇒ $E(X) = \frac{1}{p} = \frac{1}{0.25} = 4$

↓

trials

A machine has 5 parts working in parallel, each with a lifetime distributed exponentially have mean of 1000 hours

Q) Calculate the probability that 3 or more parts fail in 1000 hours

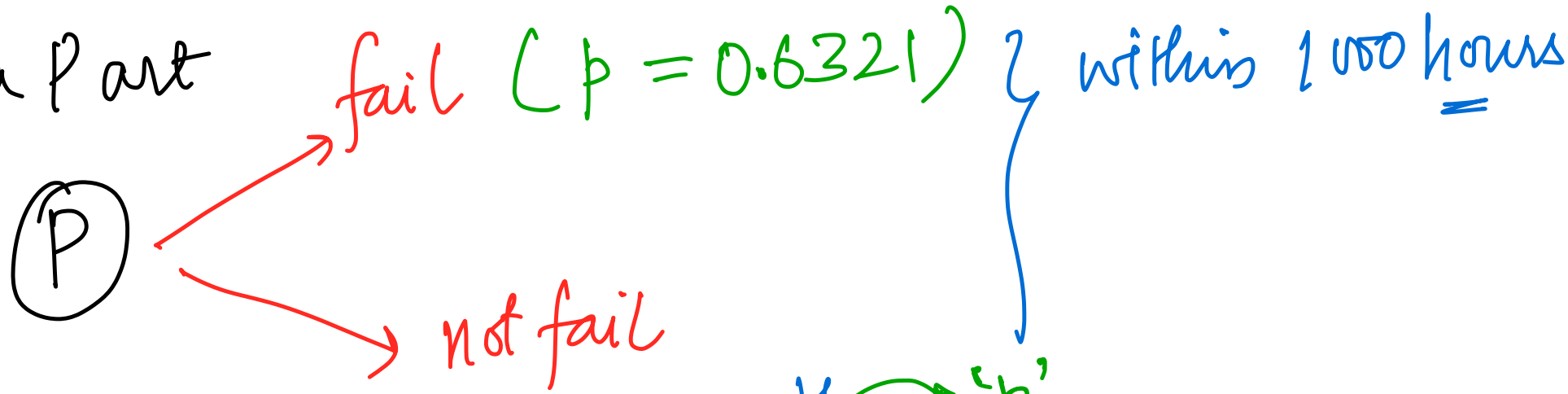
- Let's compute the prob. of failure for single part first.
- For a part;
  life Time ~ Exp dbn.
  Avg life = scale = 1000 → $\lambda = \left(\frac{1}{1000}\right)$
- Prob. that this part fails within 1000 hours

$$P[T \le 1000] = \text{expon.cdf}(x = 1000, \text{scale} = 1000)$$

$p = 0.6321$ failure

Not fail $q = (1-p) \approx 0.3$

for a part



- Consider a part as Coin. H → 'p', T

$P[X = k]$ → PMF

$X \sim \text{Binomial}(n=5, p=0.6321)$

$P[X \geq 3] = 1 - P[X \leq 2]$

$= 1 - \text{binom.cdf}(n=5, k=2, p=0.632)$

$\approx 0.736 \approx 73.6\%$