Welcome to,

Probability And Statistics

Crash Course

Outcomes:

Tossing a coin: $\{H\}, \{T\}$

Die: $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$

Sample Space

Coin: $\{H, T\}$

Die: $\{1, 2, 3, 4, 5, 6\}$

Probability

Coin toss

$SS = \{H, T\} \rightarrow$ 2 elements.

$E_1 \rightarrow$ getting a head $\Rightarrow E_1 = \{H\} \rightarrow$ 1 element.

$$P(E_1) = \frac{\#\text{ of Elements in } E_1}{\#\text{ of Elements in } SS} \Rightarrow \boxed{P(E_1) = \frac{1}{2}} = 0.5$$

Event ss of die = {1, 2, 3, 4, 5, 6}

E_1: getting all odd #'s : {1, 3, 5} $\Rightarrow P(E_1) = 3/6$

E_2: ——— even #'s : {2, 4, 6} $\Rightarrow P(E_2) = 3/6$

E_3: getting all perfect square: {1, 4} $\Rightarrow P(E_3) = 2/6$

Event → Subset of ss

D_1 & D_2 : two die.

$D_1 \rightarrow 1$ to 6

$D_2 \rightarrow 1$ to 6

D_1 \ D_2	1	2	3	4	5	6
1	(1,1)	(1,2)	- - -			(1,6)
2	⋮	⋱				⋮
3	⋮		⋱ (3,3)			⋮
4	⋮			⋱		⋮
5	⋮				⋱	⋮
6	(6,1)	- - - - - - - - -				(6,6)

$6 \times 6 = 36$ elements in my Sample space.

C_1 & C_2 : tossing 2 coins together

C_1	C_2
H	H
H	T
T	H
T	T

Sample space = {HH, HT, TH, TT}

Event

E_1 : Atleast one heads = {HH, HT, TH}

↳ $P(E_1) = 3/4$

E_2 : Both same = {HH, TT}

↳ $P(E_2) = 2/4$

Betting game. Die → SS = {1, 2, 3, 4, 5, 6}

P_1 : {1, 3, 5} = E_1

P_2 : {1, 5, 6} = E_2

① Win together

$E_1 \cap E_2 = \{1, 5\}$ → Intersection of Events.

② Anyone Wins

$\{1, 3, 5, 6\} = E_1 \cup E_2$ → Union of Events

SET OPERATIONS *

P_1 : $\{1, 3, 5\} = E_1$

P_2 : $\{2, 4, 6\} = E_2$

disjoint set

① $E_1 \cap E_2 = \text{null} = \{\} \Rightarrow '\phi'$

(Mutually Exclusive Events)*

$P_1 : \{1,2\} \rightarrow E_1$

$P_2 : \{3,4,5,6\} \rightarrow E_2$

$SS = \{1,2,3,4,5,6\}$

$E_1 + E_2 = SS$

- E_2 is complement of E_1 and vice versa.

Complement

$$E_2 = E_1^c$$

$$E_1 = E_2^c$$

Betting game. Die → SS = {1, 2, 3, 4, 5, 6}

P_1 : {1, 3, 5} = E_1

P_2 : {1, 5, 6} = E_2

① Win together

$E_1 \cap E_2 = \{1, 5\}$ → Intersection of Events.

$P(E_1) = \frac{2}{6} = \frac{1}{3}$

② Anyone Wins

{1, 3, 5, 6} = $E_1 \cup E_2$ → Union of Events

$P(E_2) = 4/6 = \frac{2}{3}$

SET OPERATIONS *

Die: $SS = \{1,2,3,4,5,6\}$.

$P_1 = \{1,2,3\} = E_1$

$P(E_1) = \frac{1}{2}$.

$P_2 = \{1,4,5\} = E_2$

$P(E_2) = \frac{1}{2}$.

(i) $(E_1 \cap E_2) = \{1\} \Rightarrow P(E_1 \cap E_2) = 1/6$ ✓

(ii) $(E_1 \cup E_2) = \{1,2,3,4,5\} \Rightarrow P(E_1 \cup E_2) = 5/6$ ✓

$\Rightarrow P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$5/6 \neq \frac{1}{2} + \frac{1}{2}$

$\frac{5}{6} \neq 1$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow \frac{5}{6} = \frac{1}{2} + \frac{1}{2} - \frac{1}{6}$

$= 1 - \frac{1}{6} = \frac{5}{6}$

Addition Rule

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$ *