PROBABILITY DISTRIBUTIONS

- Bernoulli
- Binomial

} distributions

- Normal
- Poisson
- Exponential
- Geometric
- log Normal

Bernoulli

$$X \begin{cases} \to \text{Success } (1) \\ \to \text{failure } (0) \end{cases}$$

$$X = \begin{cases} 1 \; ; \; p \\ 0 \; ; \; 1-p \end{cases}$$

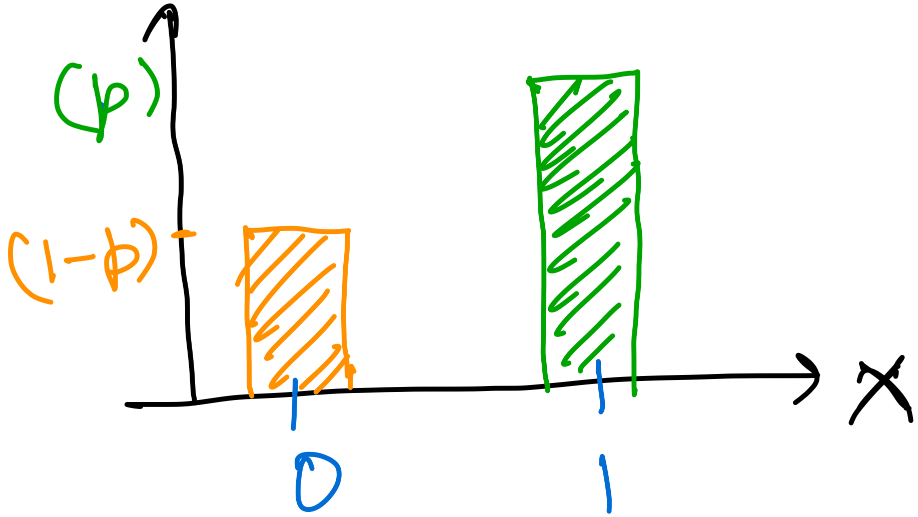
- Coin toss

$$\text{Coin} \begin{cases} \to H \to \text{success } `p' \\ \to T \to \text{failure } `1-p' \end{cases}$$

$$\begin{cases} X = \{0, 1\} \\ P(X=0) = 1-p \\ P(X=1) = p \end{cases}$$

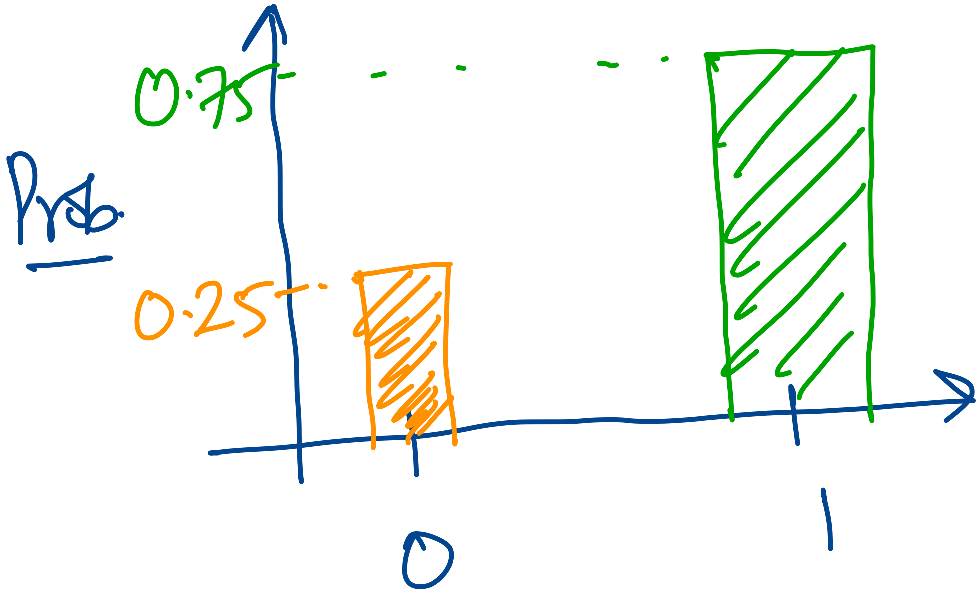
$X \sim$ Bernoulli db^n

$X \sim \text{bernoulli}(p)$ → prob. of success



- Toss a coin → Bernoulli Trial
  - → 100 Times
    - H → 75
    - T → 25

$\Rightarrow$ Prob. of Success = 'p' $= \frac{75}{100} = 0.75$



$P(X=0) = 0.25$

$P(X=1) = 0.75$

$P(\cancel{X=0.5})$

Discrete dbⁿ

Qns) $'p' = 0.8 \rightarrow$ survival

$'1-p' = 0.2 \rightarrow$ Die

% pop. 1 mill. $\rightarrow$ 1 person = 0.8

$1{,}000{,}000 = 1{,}000{,}000 \times 0.8$

$= 800{,}000$

② Binomial dbⁿ

Toss → S 'p'

Toss → F '1-p'

Toss a coin 10 times,

↳ # of heads.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 → X is R.V.

n is # of trials = 10

$X \sim \text{Binomial}(n, p)$

p is prob. of success

→ Coin is fair; 'p' = 0.5

$P(X=0)$

$P(X=1)$ → prob. of exactly 1 heads

$\vdots$

$P(X=10)$

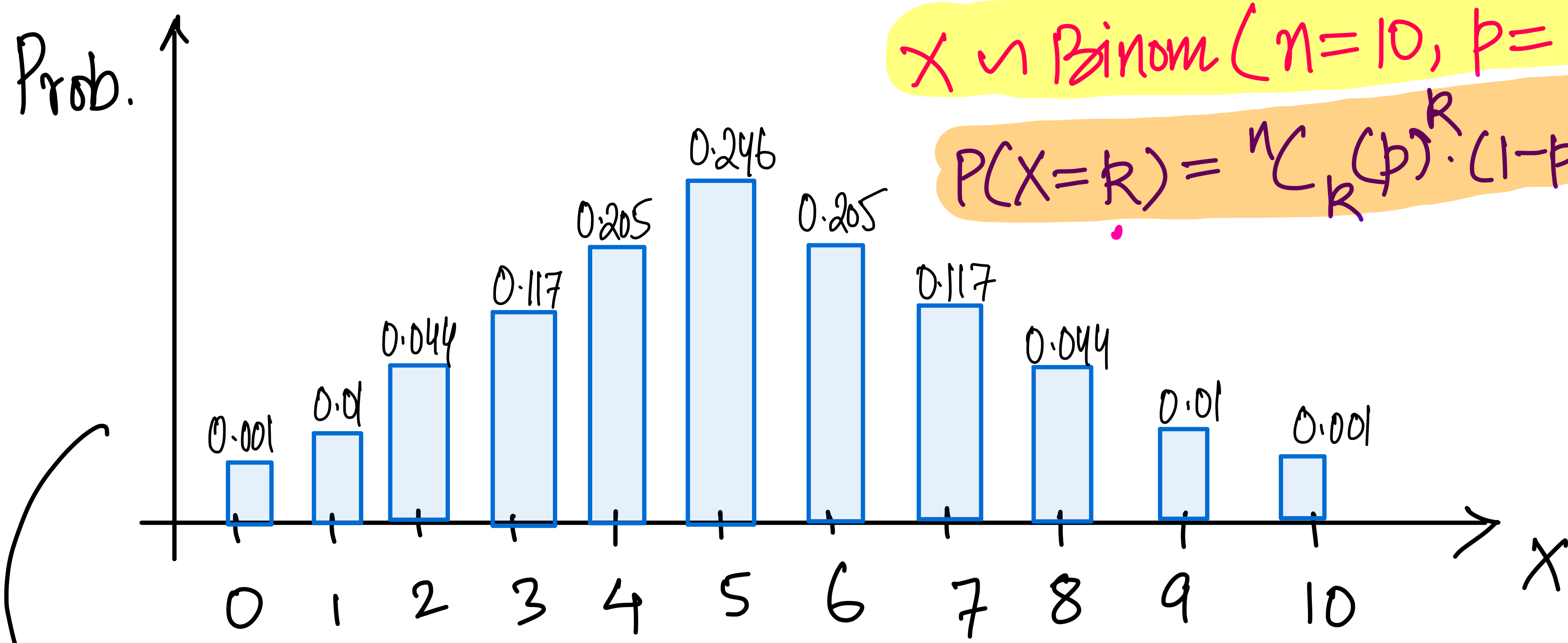
$\left.\right\}$ $P(X=k)$

$$\Rightarrow \boxed{P(X=k) = {}^nC_k (p)^k \cdot (1-p)^{n-k}}, \quad X \sim \text{binom}(n=10,\ p=0.5)$$

$$P(X=2) = {}^{10}C_2 (0.5)^2 (1-0.5)^8 = 0.044$$

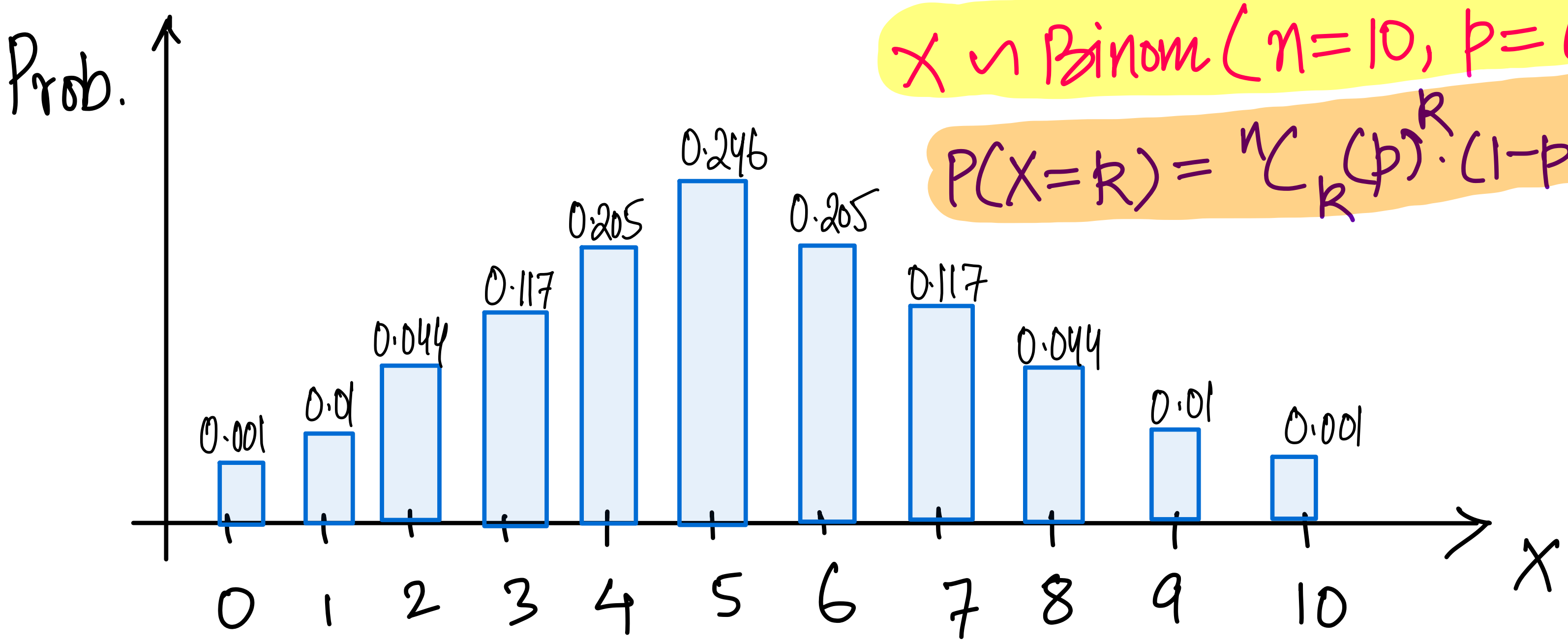
$$X \sim \text{Binom}(n=10,\ p=0.5)$$

$$P(X=k) = {}^{n}C_{k}\,(p)^{k}\cdot(1-p)^{n-k}$$



PMF → Prob. Mass Function

Discrete distribution



$X \sim Binom(n=10, p=0.5)$

$P(X=k) = {}^{n}C_{k} (p)^{k} \cdot (1-p)^{n-k}$ ✓

- $P(X=4) = {}^{10}C_4 (0.5)^4 \cdot (1-0.5)^6$

  $=$ binom.pmf(n=10, k=4 , p =0.5)

- from scipy.stats import binom

- $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

  $=$ binom.cdf(n=10 , k =3 , p=0.5)

cdf → Cummulative distribution function.

Airline Overbooking

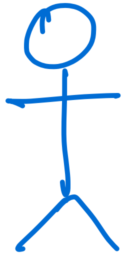
Five percent of the people making reservations on a flight will not show up.
Suppose the airline sells 52 tickets for a flight that can hold only 50 passengers.

What is the probability that there will be a seat available for every passenger who shows up?

Soln.

5% → not show up.

95% → show up.



→ 0.95 (show up)

→ 0.05 (not show up)

→ sold 52 tickets → $\boxed{n=52}$

X → # of people show up.

0 1 2 3 4 - - - . to . . 52

Capacity = 50

⇒ $P(X \leq 50)$ → Prob. of # of people showing up ≤ 50 ✓

$$\boxed{X \sim \text{Binom}(n = 52,\ p = 0.95)}\ *$$

- $P(X \leq 50) = \text{binom.cdf}(n = 52,\ k = 50,\ p = 0.95)$

$$\boxed{P(X \leq 50) = 0.74} \rightarrow \text{Ans}$$

74% chances