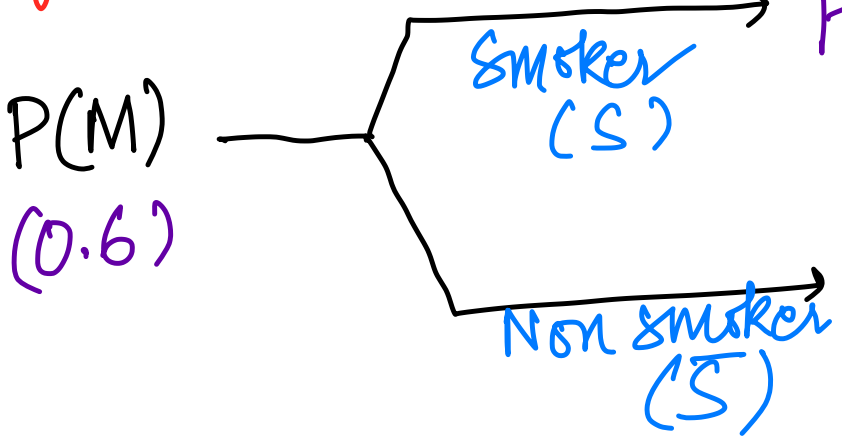
Bayes's Theorem :

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B) \cdot P(B)$

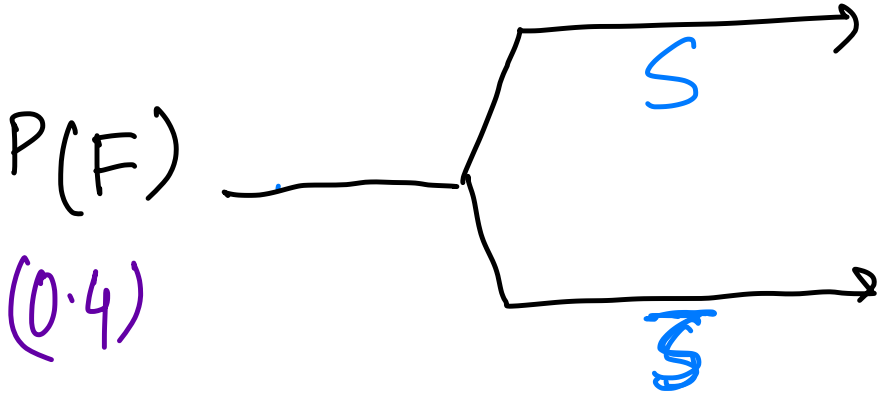
Male => 60 % , P(M) = 0.6
Female => 40%, P(F) = 0.4
5 % males are smoker and 2% females are smoker.

Marginal | Conditional | joint Probabilities



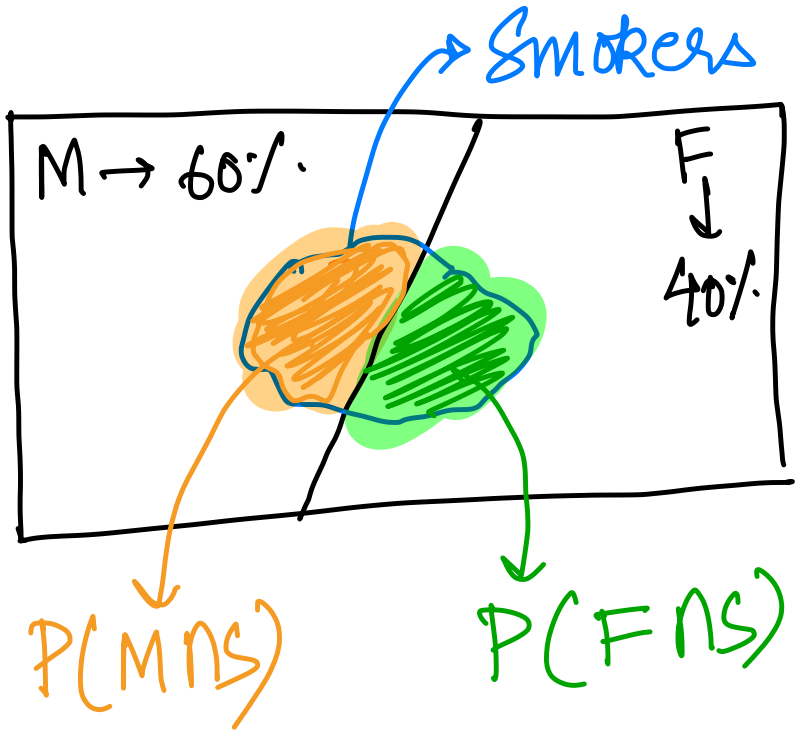
$P(S|M) = 0.05 \rightarrow P(M \cap S) = 0.05 \times 0.6 = 0.03$ ✓

$P(\bar{S}|M) = 0.95 \rightarrow P(M \cap \bar{S}) = 0.95 \times 0.6 = 0.57$



$P(S|F) = 0.02 \rightarrow P(F \cap S) = 0.02 \times 0.4 = 0.008$ ✓

$P(\bar{S}|F) = 0.98 \rightarrow P(F \cap \bar{S}) = 0.98 \times 0.4 = 0.392$



(i) find the total Prob. of Smoker
$= P(S)$

$\Rightarrow P(S) =$ ● + ●

$$P(S) = P(M \cap S) + P(F \cap S)$$

Total Prob. of being a Smoker.

$P(S) = 0.03 + 0.008$

$P(S) = 0.038$

(ii) A random person is found to be a Smoker.

(a) Prob. that this person is a Male.

(b) Prob. that this person is a female

(a) $P(M|S) = \frac{P(M \cap S)}{P(S)} = \frac{0.03}{0.038} \approx 0.789$

$\approx 79\%$

i.e. If you see a random person is Smoking.

↳ $P(\text{Male}) \approx 79\%$.

(b) $P(F|S) = \frac{P(F \cap S)}{P(S)} = \frac{0.008}{0.038} \approx 21\%$

i.e. If you see a random person is Smoking.

↳ $P(\text{Female}) \approx 21\%$.

(iii) $P(M \mid \overline{S}) = \dfrac{P(M \cap \overline{S})}{P(\overline{S})}$

(iv) $P(F \mid \overline{S}) = \dfrac{P(F \cap \overline{S})}{P(\overline{S})}$