# PROBABILITY AND STATISTICS

## CRASH COURSE

## Lecture #2

### Agenda:

* Conditional Probability

* Important Probability Questions

# Conditional Probability:

Throwing 2 die, $D_1$ and $D_2$ and noting sum of $D_1$ & $D_2$.

- $P(D_1 = 2) = \dfrac{6}{36}$

- $P(D_1 + D_2 \leq 5) = \dfrac{10}{36}$

- $P[(D_1 = 2) \cap (D_1 + D_2 \leq 5)] = \dfrac{3}{36}$

- $P[(D_1 = 2) \mid (D_1 + D_2 \leq 5)] = \dfrac{3}{10}$ ✓

$\boxed{D_1 + D_2}$

$D_2$

|       | 1 | 2 | 3 | 4 | 5 | 6 |
|-------|---|---|---|---|---|---|
| **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| **6** | 7 | 8 | 9 | 10 | 11 | 12 |

$D_1$

⟶ Conditional Prob.

$$\Rightarrow P[(D_1 = 2) \mid (D_1 + D_2 \leq 5)]$$
$$= \frac{P[(D_1 = 2) \cap (D_1 + D_2) \leq 5]}{P(D_1 + D_2 \leq 5)}$$
$$= \frac{3/36}{10/36} = \frac{3}{10} \checkmark$$

$$\Rightarrow \boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

- Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \checkmark$$

- $P(A \cap B) = P(A|B) \cdot P(B) \checkmark \longrightarrow$ Multiplication Rule
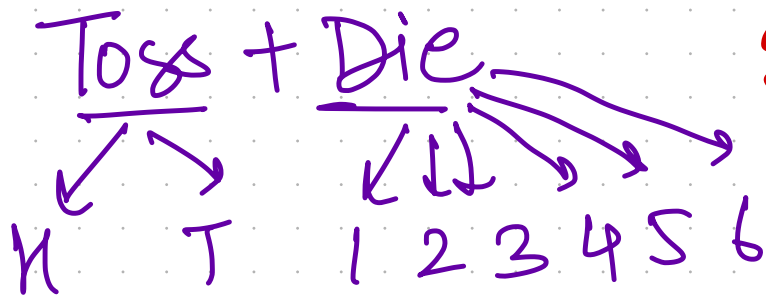
Joint probability.

Conditional Prob.

Marginal Prob.

✓ Simple Prob.

- Two events:
  A & B

$$P(A|B) = P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \} \checkmark$$

Independent Events.

$$\underline{Toss} + \underline{Die}$$

H   T   1 2 3 4 5 6

$$SS = \left\{ \begin{array}{c} (H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6) \\ (T,1)\ (T,2)\ (T,3)\ (T,4)\ (T,5)\ (T,6) \end{array} \right\}$$

$$\Rightarrow A \rightarrow \text{head on toss} \} \text{ Events.}$$
$$\quad B \rightarrow 3 \text{ on die } \}$$

- $P(A) = 6/12 = 1/2$
- $P(B) = 2/12 = 1/6$

- $P(A \cap B) = 1/12$

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{1/12}{1/6} = \frac{6}{12} = \frac{1}{2}$$

- $P(A|B) = P(A)$ ✓

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

· If A y B are independent. $\rightarrow P(A|B) = P(A)$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)} \ ✷$$

# Questions :

1. A group of 5 boys and 10' girls is lined up in random order — that is, each of the 15! permutations are assumed to be equally likely.

a. What is the probability that the person in the 4th position is a boy?

b. What about the person in the 12th position?

c. What is the probability that a particular boy is in the 3rd position?

Soln

5 Boys    10 Girls    }    Total 15 people $\rightarrow$ $= 15!$

a)

b)

$1^{st}$    $2^{nd}$    $3^{rd}$    $4^{th}$    - - - - - - - - - - . $15^{th}$ posn

$\downarrow$

Boy

• $5C_1$ ways = 5 ways.

then possible arrangements are

$= 5C_1 * 14!$

• Total Arrangement = 15 !

$\Rightarrow$ Prob $= \dfrac{5C_1 \times 14!}{15!} = \dfrac{5 \times \cancel{14!}}{15 \times \cancel{14!}} = \boxed{\dfrac{1}{3}}$

(C) $3^{rd}$ pos$^n$ → Shivam

$\underline{\quad}$ $\underline{\quad}$ ⊘ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ $\}$ 14 pos$^n$ left.

↑
Shivam.

Total possible assignment = $\underline{14!}$

$$\text{Prob.} = \frac{14!}{15!} = \frac{14!}{15 \times 14!} = \boxed{\frac{1}{15}} \checkmark$$