PROBABILITY DISTRIBUTIONS
3

- Bernoulli
- Binomial
- Normal
- lognormal

Covered

- Poisson
- Exponential

this Video

Poisson Distribution

- **Where this db^n arises?** Which processes follow Poisson db^n?

X → Measures number of events per unit time

X ↓ RV

e.g.:

No. of visitors to a website per minute → X → Rate: 10 user/min

No. of connections made to a cell tower per hour → X → 100 connect^{n}/hour

- **Conditions** (assumptions)

① Constant rate (λ) - for that time interval

② events must be independent

③ No occurance of simultaneous events → (nano sec gap)

- No. of Patients coming to hospital / day

$X \rightarrow 20, 25, 15, 10 \ldots$ } $\rightarrow$ X is a Random Variable → discrete

↓ # of events in a particular day.

Only one parameter

$\lambda = 25 \Rightarrow X \sim \text{Poisson}(\lambda = 25)$

average/day

what we can compute :

- X is a R V ✓ $X = 0, 1, 2, 3, \ldots$

PMF : $P(X=0)$
$P(X=1)$
$\vdots$
$P(X=20)$

$P(X=k)$

- for $k = 20$

what is the probability that we observe 20 patients in a day.

PMF of Poisson :

$$\boxed{P(X=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}}$$

$\lambda = 25/\text{day}$

$\Rightarrow P(X=20) = ?$

$$P(X=20) = \frac{(25)^{20} \cdot e^{-25}}{20!} = 0.05$$

In python

$\lambda = 25, k = 20$

```
- from scipy.stats import poisson
- poisson.pmf(k=20, mu=25)
```

$= 0.05$

$\Rightarrow P(X \leq 15) = ?$ $\boxed{\text{CDF}}$

$= P(X=0) + P(X=1) + \cdots \cdots + P(X=15)$

- from scipy.stats import poisson
- poisson.cdf(k=20 , mu = 25)

$= 0.185$

Question:

A call center recieves 240 calls / hour. They want to deploy ideal number of employee for answering calls to optimize their cost of hiring. – Cost optimization

(i) What is prob. of getting 5 call in 30 sec?

- hourly rate = 240 calls/hour.

$$60 \text{ min} = 3600 \text{ sec} = 240 \text{ calls}.$$

$$\Rightarrow \quad 1 \text{ sec} = \left(\frac{240}{3600}\right) \text{calls}$$

$$\Rightarrow \quad 30 \text{ sec} = \left(\frac{240}{3600} \times 30\right) \text{calls}$$

- In 30 sec. we get 2 calls on avg. $\longrightarrow$ $\boxed{\lambda = 2}$ for 30 sec.

$$P(X=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

- $P(X=5)$

$$= \frac{(2)^5 \cdot (e)^{-2}}{5!} = 0.036$$

$\sim 3.6\%$

- poisson.pmf(k=5, mu = 2) $\rightarrow$ 0.036

Let's answer some more question

A call center recieves 240 calls / hour.

- hourly rate = 240 calls/hour.

60 min $= 3600$ sec $= 240$ calls.

$\Rightarrow$ 1 sec $= \left(\frac{240}{3600}\right) = \left(\frac{1}{15}\right)$ calls.

Avg. # of Calls./sec

'λ' for 1 second $= (1/15)$

rate parameter

$\Rightarrow$ $\boxed{\lambda_1 = \left(\frac{1}{15}\right)}$

(A) What is the prob. of no calls in 10 sec?

— rate parameter λ for 10 seconds

$$\boxed{\lambda_{10} = 10(\lambda_1)} = 10 \cdot \left(\frac{1}{15}\right)$$

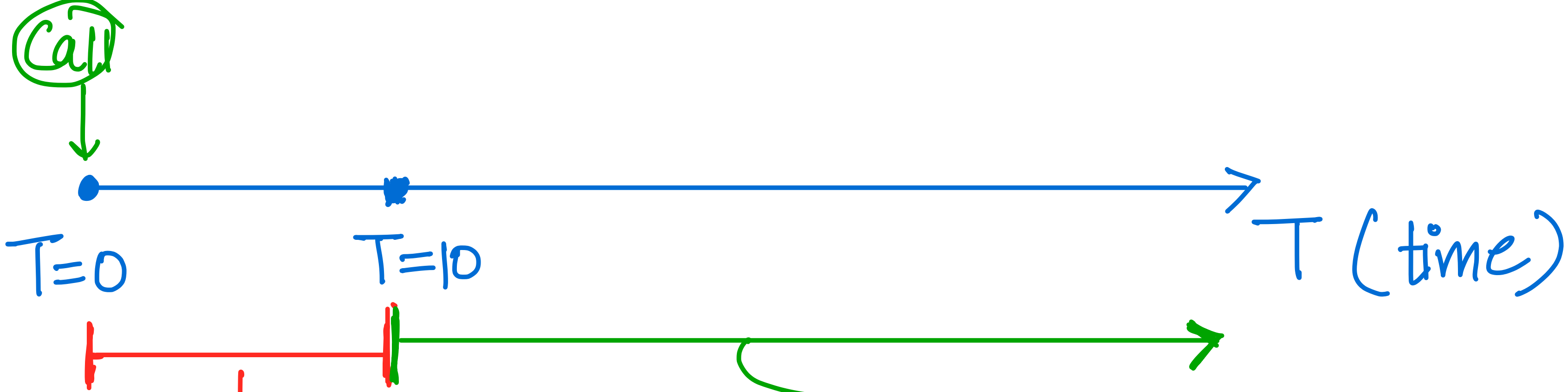
$$P(X=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

$$\Rightarrow P(X=0) = \frac{\left(\frac{10}{15}\right)^0 \cdot e^{-10\cdot(1/15)}}{0!}$$

poisson.pmf(k=0, mu=$\frac{10}{15}$)

$$\boxed{P(X=0) = e^{-10\cdot(1/15)}}$$

(B) What is the prob. of waiting more than 10 sec for next call?



No Call
or
Zero Call in 10 sec

I can expect call any time after T > 10 sec.

$P(X=0)$

$P(T>10)$

These two cases are same

$P(X=0)$ Prob. of zero calls in 10 sec. $\Bigg\}$ $X \sim \text{Poisson}(\lambda_{10})$

– Avg. calls count in fixed time interval.

$P(T>10)$ Prob. of waiting more than 10 sec. for next call $\Bigg\}$ → Random Variable is 'T' time.

• T is waiting time for next call

$\Rightarrow P(X=0) = e^{-10\cdot(1/15)}$

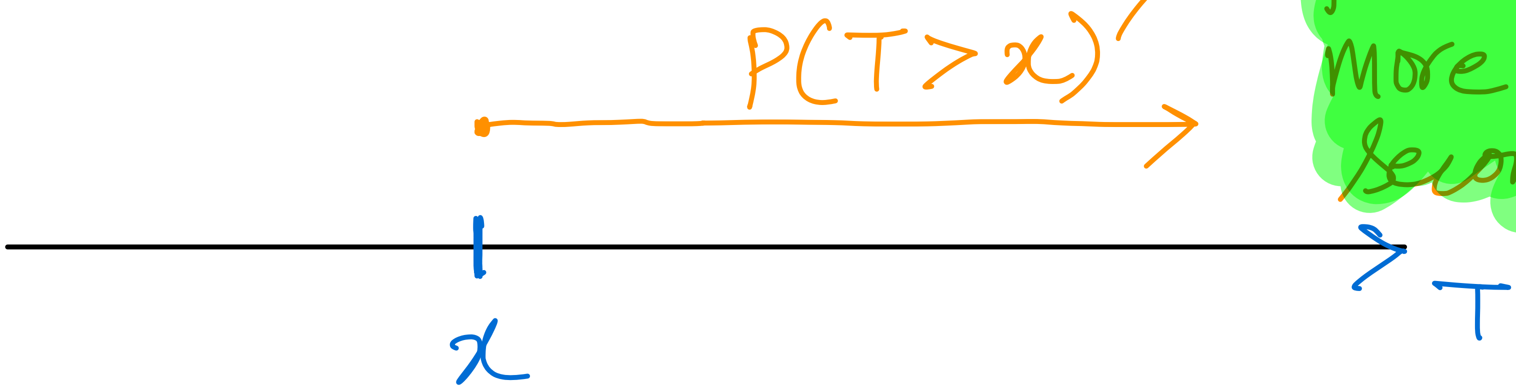
$\Rightarrow P(T>10) = e^{-10\cdot(1/15)}$

$\Bigg\}$ Both Same.

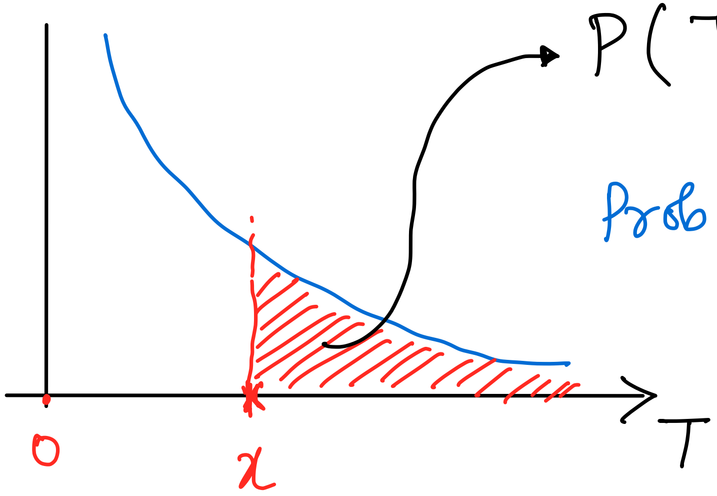
$\lambda_1 = \left(\frac{1}{15}\right) \longrightarrow$ rate/sec or $\left(\frac{1}{15}\right)$ calls/sec.

$\Rightarrow P(T > 10) = e^{-10 \cdot \lambda_1}$

$\Rightarrow \boxed{P(T > x) = e^{-x \cdot \lambda_1}}$



Prob. of waiting more then 'x' seconds for next call.



$P(T > x) = e^{-x \cdot \lambda_1}$

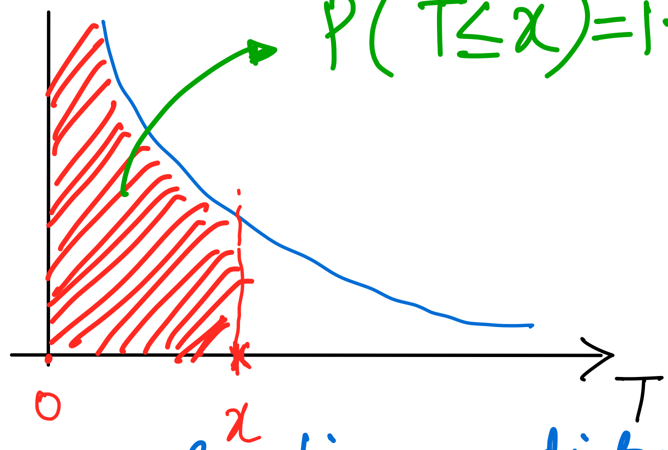
Prob. of waiting more than 'x' seconds for next call

$T \sim$ exponential distribution

Ⓒ What is Prob. of waiting less than 10 sec. for next call

$\Rightarrow$ If $x = 10$ $\Rightarrow$ $P(T > 10) = e^{-10\lambda_1}$

$\Rightarrow P(T \leq 10) = 1 - e^{-10\lambda_1}$



$P(T \leq x) = 1 - e^{-x \cdot \lambda_1}$

$\therefore$ $\boxed{P(T \leq x) = 1 - e^{-x\lambda_1}}$

CDF of Exponential distribution

$\longrightarrow$ Contineous distribution

Random Var. $T \sim$ Exponential distribution

$$P(T \le 10) = 1 - e^{-10\cdot\lambda_1}$$

x

$$= 1 - e^{-10\cdot(1/15)}$$

$\lambda_1 = \left(\frac{1}{15}\right)$ calls per sec.

$= \frac{1}{\lambda} \rightarrow$ scale $\rightarrow$ 15 sec per call

<u>In Python</u>

```
from scipy.stats import expon
expon.cdf(x = 10, scale = 15)
```

Scale parameter

$$\underbrace{\text{Scale}}_{\text{avg. time for next call}} = \frac{1}{\lambda}$$

0.486

i.e. 48.6% chances We reciere next call within 10 seconds

$\lambda_1 = \left(\frac{1}{15}\right)$ calls/sec.

- Avg. Time of getting a call?

5 calls in 10 sec. → 1 call every 2 second

$\therefore$ Avg. time $= 2$ sec.

here, 240 calls in 3600 sec.

$\therefore$ $\underbrace{\text{Avg time}}_{\text{Scale}} = \left(\frac{3600}{240}\right)$ sec. $= 15$ sec/call

(mean time)

$\therefore$ $\boxed{\text{Scale} = \frac{1}{\lambda}}$