1 Kurtosis optimization

Two statistically independent sources with $\langle \mathrm{s}_i \mathrm{s}_j \rangle = \delta_{ij}$
Extraction from observations (one source)

$$\begin{aligned} \widehat{\mathrm{s}} &= \underline{\mathbf{z}}^T \underline{\mathbf{s}} \\ &= \mathrm{z}_1 \mathrm{s}_1 + \mathrm{z}_2 \mathrm{s}_2 \end{aligned} \tag{1}$$

optimization problem

$$\begin{aligned} &\mathrm{kurt}(\widehat{\mathrm{s}}) \stackrel{!}{=} \max_{\underline{\mathbf{z}}} \\ &\mathrm{z}_1^2 + \mathrm{z}_2^2 \stackrel{!}{=} 1 \end{aligned} \tag{2}$$

method of lagrange multiplies

$$\mathrm{kurt}(\widehat{\mathrm{s}}) = \mathrm{z}_1^4 \underbrace{\mathrm{kurt}(\mathrm{s}_1)}_{=a} + \mathrm{z}_2^2 \underbrace{\mathrm{kurt}(\mathrm{s}_2)}_{=b} \tag{3}$$

let $a, b > 0$ (special case)

$$a\mathrm{z}_1^4 + b\mathrm{z}_2^4 - \lambda\left(\mathrm{z}_1^2 + \mathrm{z}_2^2\right) \stackrel{!}{=} \text{exts.} \tag{4}$$

$$\begin{aligned} 4a\mathrm{z}_1^3 - 2\lambda \mathrm{z}_1 = 0 \quad &\rightsquigarrow \mathrm{z}_1\left(4a\mathrm{z}_1^2 - 2\lambda\right) = 0 \quad \rightsquigarrow \mathrm{z}_1 = \pm\tfrac{\lambda}{2a}, 0 \\ 4b\mathrm{z}_2^3 - 2\lambda \mathrm{z}_2 = 0 \quad &\rightsquigarrow \mathrm{z}_2\left(4a\mathrm{z}_2^2 - 2\lambda\right) = 0 \quad \rightsquigarrow \mathrm{z}_2 = \pm\tfrac{\lambda}{2b}, 0 \end{aligned} \tag{5}$$

matrix of second derivatives

$$\left| \begin{pmatrix} 12a\mathrm{z}_1^2 - 2\lambda & 0 \\ 0 & 12b\mathrm{z}_2^2 - 2\lambda \end{pmatrix} \right| = \left(12a\mathrm{z}_1^2 - 2\lambda\right)\left(12b\mathrm{z}_2^2 - 2\lambda\right) \tag{6}$$

solutions

z_1	z_2	λ	1%1	$a\mathrm{z}_1^4 + b\mathrm{z}_2^4$
0	± 1	$2b$	$-32b^2$	b
± 1	0	$2a$	$-32a^2$	a
$\pm\frac{b}{\sqrt{a^2+b^2}}$	$\pm\frac{a}{\sqrt{a^2+b^2}}$	$\frac{2ab}{\sqrt{a^2+b^2}}$	depending on values for a,b	at most local optima: $\frac{ab}{(a^2+b^2)^2}(a^3+b^3)$