Lesson 3 outline

CSPP58001: Numerical Methods
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1 Returning to the heat equation...

In the beginning of this lecture, we introduce another scheme for modeling the heat equation— the Backwards Euler (BECS) a.k.a. backwards difference method. Refer to `lesson2.pdf` for the mathematics for the 1D system. The take away point is that this method is implicit instead of explicit. That is to say, you have to solve a linear system to get your answer because the temperature at any time t is written as a function of the temperature at $t + 1$. The PDF also introduces the Crank-Nicholson scheme, which combines features of backwards and forwards difference methods.

1.1 Going 2D

We write the analytic heat equation in the form of a 2D problem:

$$\frac{\partial T}{\partial t} = \alpha[\frac{\partial^2 T}{\partial x \partial x} + \frac{\partial^2 T}{\partial y \partial y}] = \nabla^2 T$$

where the ∇^2 is referred to as the Laplacian operator.

Now, using FTCS scheme, we have a larger explicit formula

$$T_{i,j}^{n+1} = T_{i,j}^{n} + \frac{\alpha \delta t}{\delta x^2}[T_{i+1,j}^{n} + T_{i-1,j}^{n} + T_{i,j+1}^{n} + T_{i,j-1}^{n} - 4T_{i,j}^{n}]$$

For BECS,

$$T_{i,j}^{n+1} = T_{i,j}^{n} + \frac{\alpha \delta t}{\delta x^2}[T_{i+1,j}^{n+1} + T_{i-1,j}^{n+1} + T_{i,j+1}^{n+1} + T_{i,j-1}^{n+1} - 4T_{i,j}^{n+1}]$$

The majority of this lecture consisted of prototyping these schemes in MATLAB. The codes are available on the Dropbox.