Lesson 4 outline

CSPP58001: Numerical Methods
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1 Cubic splines

We take a brief detour to describe another application of matrix computations. A very common problem in data analysis involves interpolation between discretely sampled measurements. If you want to know what the observation should be at a point between your samples, you need to interpolate to get a reasonable guess.

There are many ways to do this. You could come up with a single polynomial that best describes the distribution of points, but polynomial interpolation can have disadvantages with high oscillations.

One popular method is spline interpolation. Instead of using a single polynomial for the entire range, we come up with a piecewise polynomial composed of separate polynomials between each two data points subject to some restrictions.

The simplest form of spline interpolation would be to draw straight lines between the data points. This is known as linear interpolation.

We are going to discuss a popular variant— cubic splines. Say you have $N+1$ observations f_i at $N+1$ different independent variables x_i. Our goal is to find N cubic polynomials $S_0, S_1, \ldots, S_{N-1}$ that satisfy

1. $S_i(x_i) = f_i$ (interpolation) $i \in [0, N-1]$
2. $S_i(x_{i+1}) = S_{i+1}(x_{i+1})$ (continuity) $i \in [0, N-2]$
3. $S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$ (continuity of first derivative) $i \in [0, N-2]$
4. $S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$ (continuity of second derivative) $i \in [0, N-2]$

This gives you $(N+1)+3(N-1) = 4N-2$ independent constraints. For N cubic polynomials of the form

$$S_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i$$

we have $4N$ free variables in the form of the coefficients. We lack 2 constraints. The remaining constraints come from the boundary conditions. The most common boundary conditions are "natural" boundary conditions.

It is clear that with these constraints, the piecewise polynomials coefficients depend on the coefficients of their neighboring polynomials.

You will have to make a cubic spline interpolator on HW3.

2 Understanding stability constraints

See `lesson4.pdf`.