Report 8 - Nihal Afsal

PID Control Theory

Note: Homework must be uploaded as a **single pdf file**, not a zip file. If a problem solution requires a video, add it as a hyperlink in the pdf. The hyperlink should open the video file which is stored on your Google Drive. Any problem that requires Python code must show the entire code as well as a description of how the code works. Duplicate code submissions will result in a zero.

1. Consider a first-order vehicle model at rest with a mass of 2000 kg subject to velocity resistive effects of 1500 (i.e. c = 1500). Suddenly a propulsive force of f(t) = 5 * 10^5 N is applied. *(15 pts)*

   a. Find the vehicle's steady state speed and how long it takes to reach that speed

Mass (m) = 2000 kg

resistance (c) = 1500

$f(t) = m \cdot a(t) = f(t) - c \cdot v(t)$

steady state = 0

so

$0 = P(t) - C \cdot a(t)$

$$a(t) = \frac{(5 \cdot 10^5 N)}{1500}$$

Steady speed = 333.33 m/s

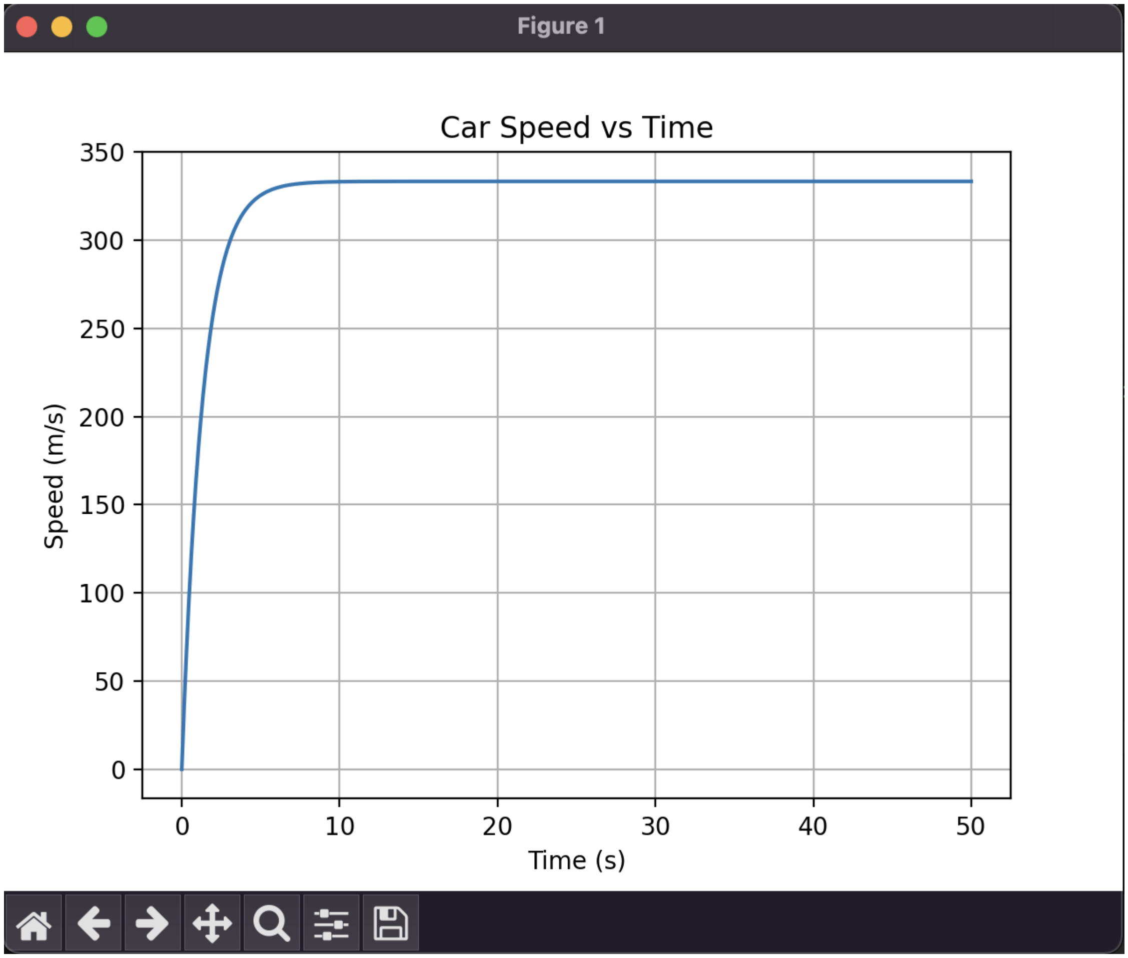
Finding how long it takes to reach that speed

$$4\lambda = 4\left(\frac{m}{c}\right)$$

$$= 4\left(\frac{2000}{1500}\right)$$

$$= 5.33 \text{ seconds}$$

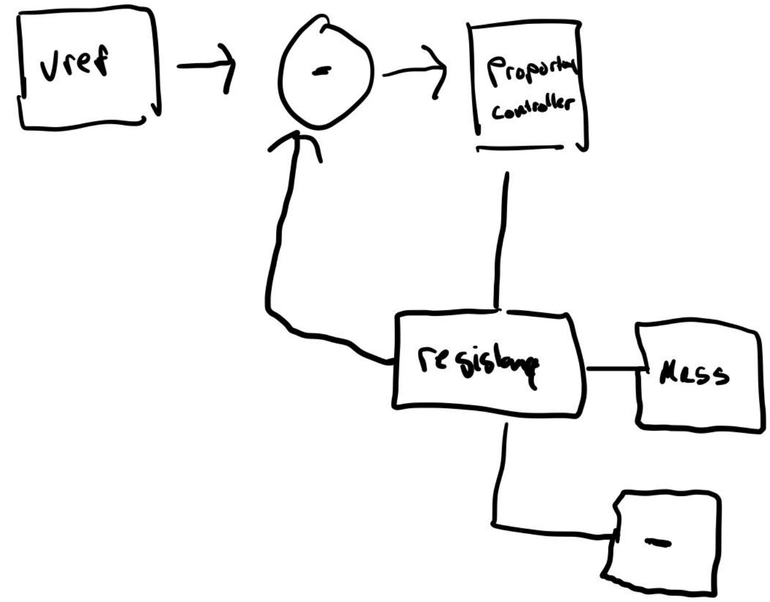
b. Find the vehicle's speed as a function of time and plot it in numerically in Python (i.e. use the NumPy package and the linspace command rather than SymPy)



2. A second-order vehicle model has a mass of 2000 kg, velocity resistive effects of 1500, and position resistive effects of 500. If its initial position is 50 and its initial velocity is 10 m/sec, plot the free response numerically using Python. *(15 pts)*

3. We are applying proportional control to a first-order vehicle model with a mass of 2000 kg subject to velocity resistive effects of 1500. *(20 pts)*

   a. Create a custom the block diagram for this controller (hint: we are controlling the velocity variable, the plant input variable can be force or torque)



b. Find the smallest value of the gain K_p required so that the steady state velocity offset error will be no greater than 0.2 if the input velocity is a unit-step input. Evaluate the resulting time constant and steady state response due to a unit step disturbance.

$$m \cdot \left(\frac{dv}{dt}\right) = F - c \cdot v$$

$F = K\_p \cdot e$ → force

e, is error between reference/velocity

$e = v\_ref - v$

steady state = $0 = K\_p \cdot e - c \cdot v_s$

$$= K\_p = 1500 \cdot \frac{(1-0.2)}{0.2}$$

$kp = 6000$

Therefore using s value as s approaches 0

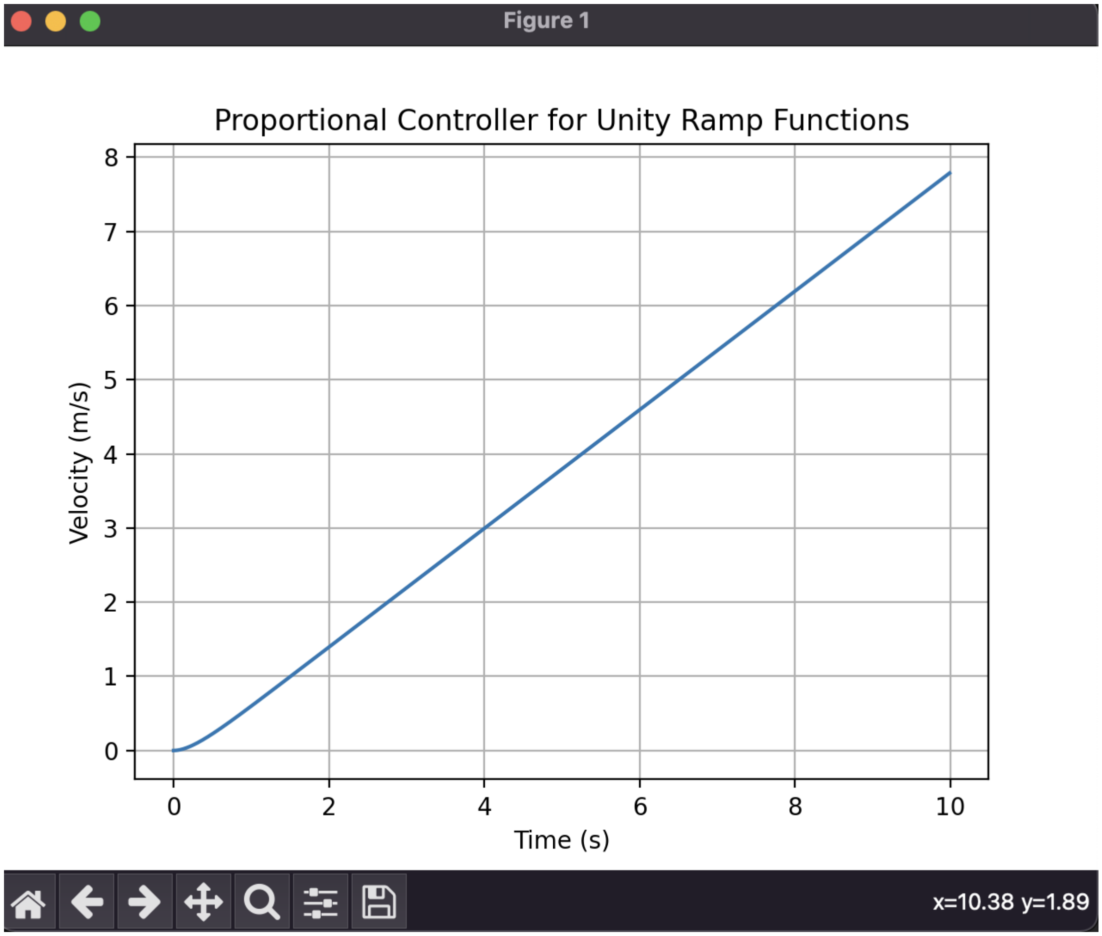
$S = k\_p / (c_c + k\_p)$

$s = 6000 / 7500$

$s = 0.8$

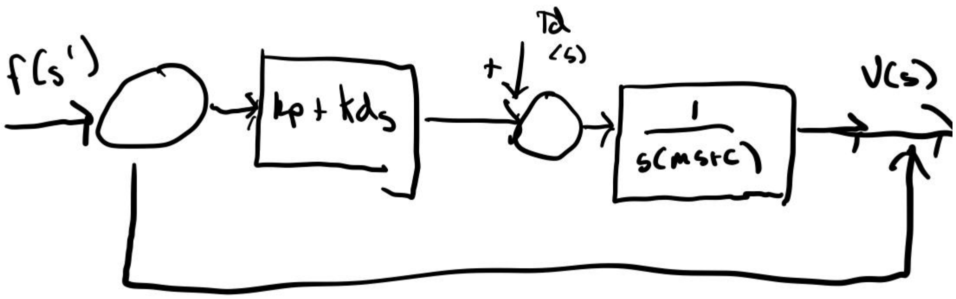
Steady state = 0.8

c. Suppose both the command input and the disturbance are unity ramp functions. Evaluate the response of the proportional controller from part b by making a plot in Python.



4. Now we are applying proportional and derivative control to a second-order vehicle model with a mass of 2000 kg subject to velocity resistive effects of 1500. *(20 pts)*

   a. Create a custom the block diagram for this controller (hint: we are controlling the velocity variable, the plant input variable can be force or torque)



Controller → $k_p + k_d s$

plant → $\frac{1}{s(ms+c)}$

$F(s')$ → force

b. If we seek a time constant of 1 and a damping coefficient of 0.707, what are the controller values?

Finding damping coefficient and natural frequency

$\lambda = (c + Kd)/2 \cdot m \cdot ct)$

$Kd = 2 \cdot \lambda \cdot m \cdot ct - C$

$= 2 \cdot 0.707 \cdot 2000 \cdot 1 - 1500$

$= 1314$

$ct = 1$

therefore

$Kp = 2000 \cdot 1^2$

Controller values $Kp = 2000$ and

$Kd = 1314$

c. What is the command transfer function and the disturbance transfer function for this system?

Command transfer function

Input reference = output

therefore

$$CT = \frac{(K_p + K_d \cdot s)}{M \cdot s^2 + (c + K_d) \cdot s + K_p}$$

Disturbance transfer

set contol output $(K_p + K_d \cdot s)$

$$DT = \frac{1}{(M \cdot s^2 + (c + K_d) \cdot s (K_p)}$$

d. If the command input is a unit ramp, what is the steady state error?

Steady state

- use command transfer function

$$E(s) = \frac{\frac{1}{s^2}(kp + kd \cdot s)}{(m \cdot s^2 + (c + kd) \cdot s + kp) \cdot 1}$$

error = 0