EE2703 Assignment 9 - Spectra of Non-Periodic Signals

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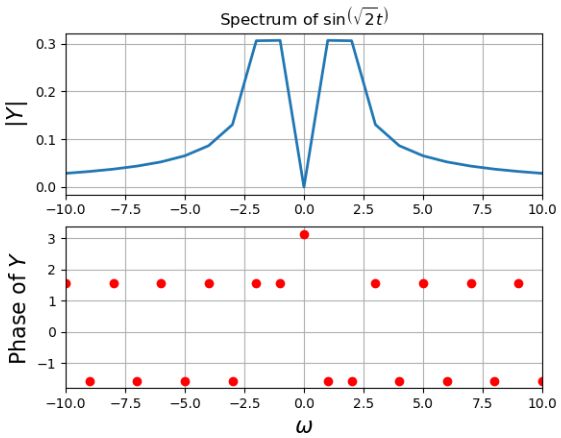
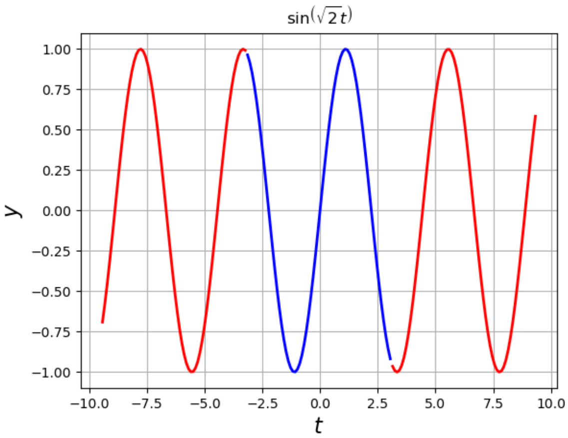
Abstract

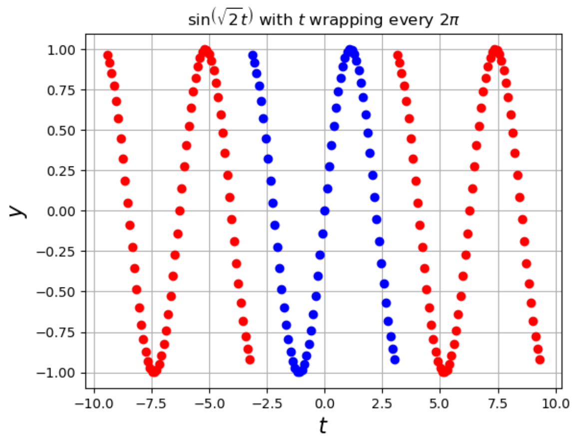
In this assignment, the spectra of non-periodic signals were obtained using DFT. The detailed process and observations from each of the plots is given below.

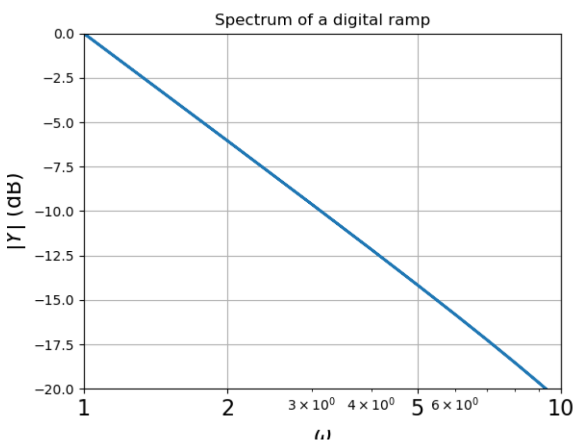
Example Tasks

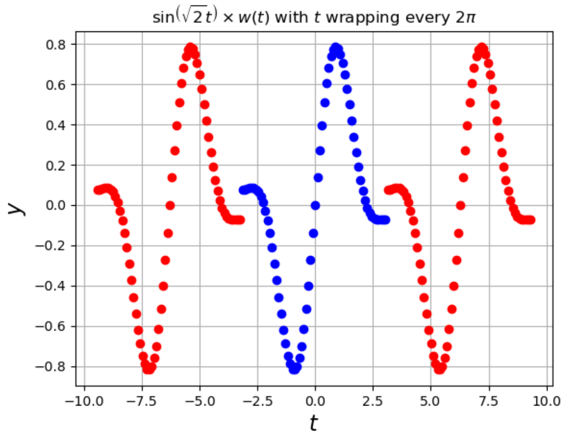
The example tasks showed the effects of Gibbs phenomenon and windowing to mitigate the effects. The following plots were obtained

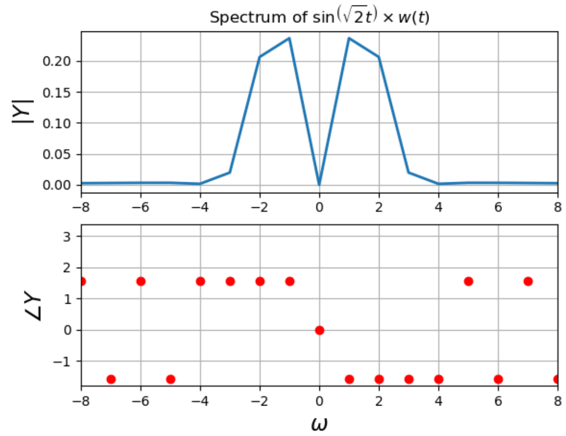


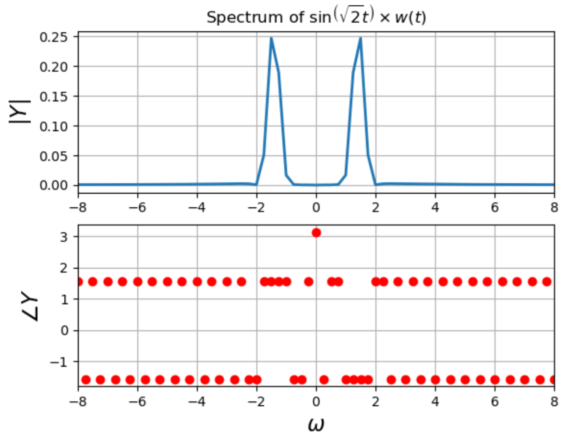










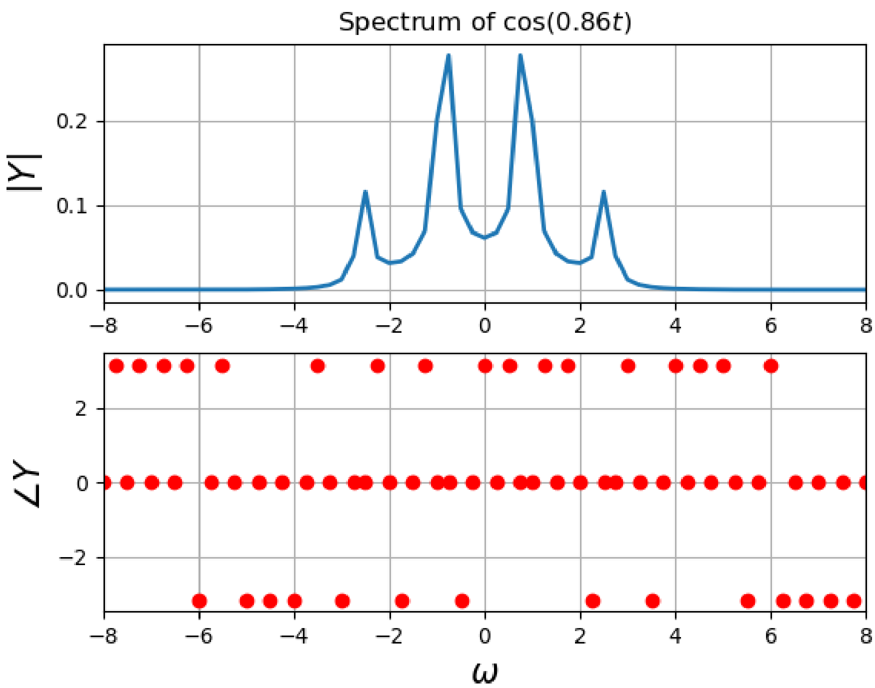


Spectrum of $cos^3(0.86t)$

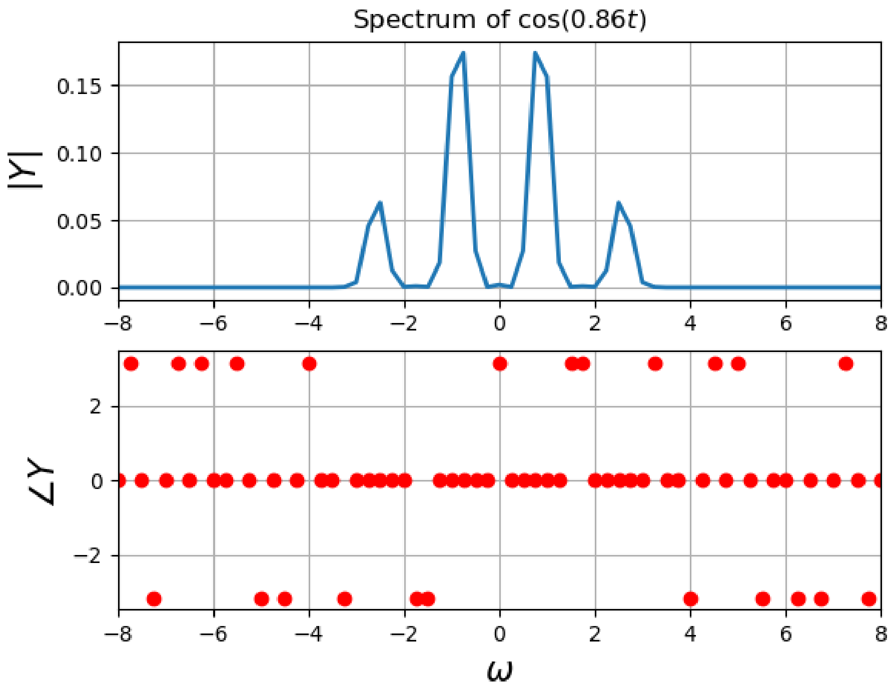
From trigonometric identities, we know that

$$cos^3(0.86t) = \frac{cos(2.58t) + 3cos(0.86t)}{4}$$

The peaks are at $\pm1, \pm3$. The phases are 0, as expected from the complex exponential formula for cosine



Below is the output with windowing



Estimating Frequency and Phase of $cos(\omega_0 t + \delta)$ with Noise

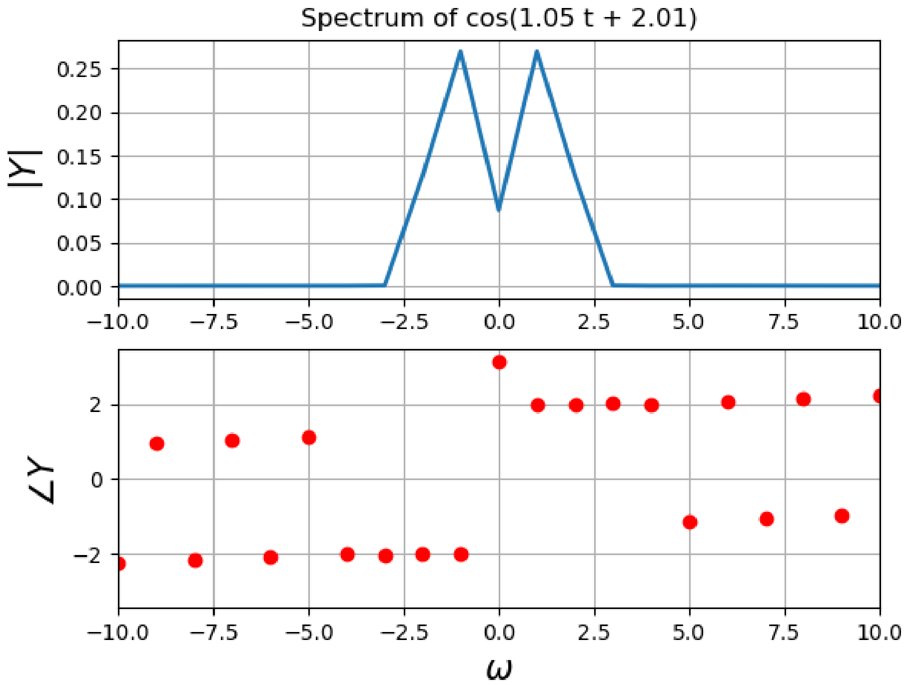
For estimating frequency, we find the ω which equals the average of all ω weighted by their magnitude (similar to finding centre of mass). Since the plot is symmetric about origin, only one half is considered.

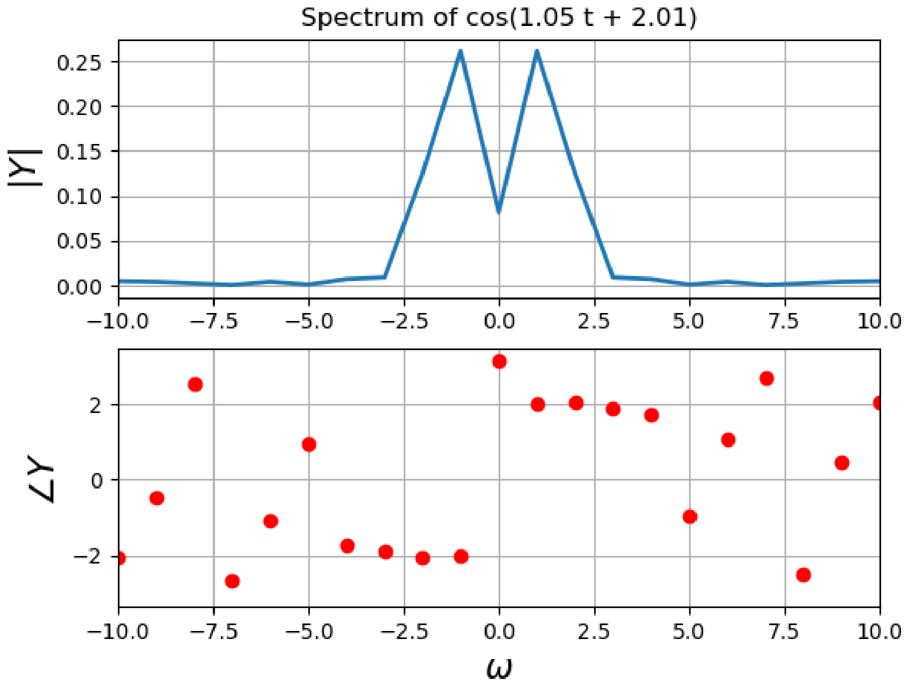
Due to DFT's inaccuracy from windowing and non-integral frequency, the tail of the magnitude plot does not fall fast enough, which caused poor estimates. So only the first 5 frequencies are considered.

For finding phase, since we know the input frequency is between 0.5 and 1.5, and the resolution of our spectrum is 1, the phase at ± 1 will give us δ. Since there can be inaccuracies due to non-integral frequency, we can take average of phase at each frequency weighted by magnitude.

$$cos(\omega_0 t + \delta) = \frac{e^{j(\omega_0 t + \delta)} + e^{-j(\omega_0 t + \delta)}}{2} = \frac{e^{j\delta}}{2} e^{j\omega_0 t} + \frac{e^{-j\delta}}{2} e^{-j\omega_0 t}$$

So $\angle DFT(f, \omega_0) = \delta$, and its negative on the negative frequency side.

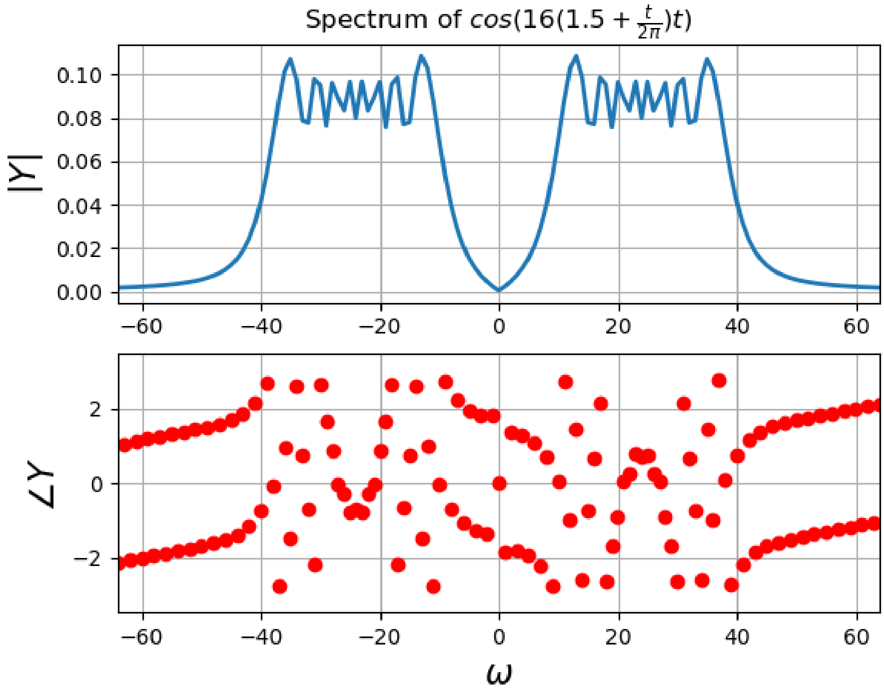




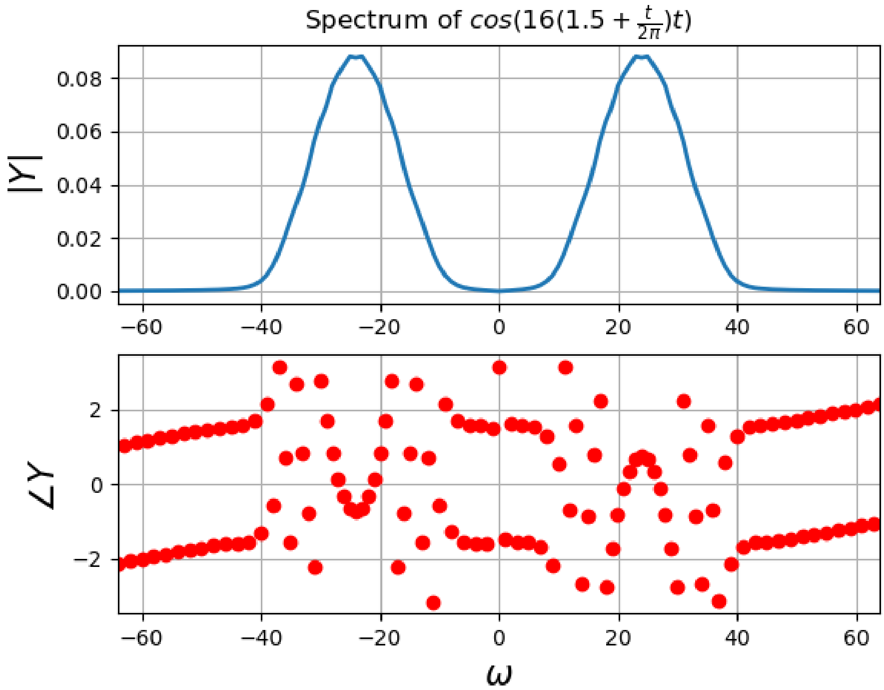
Actual w0, d = [1.05322321] [2.00524995]
Estimated w0 (clean, noisy): 1.0805567090426142 1.1726716462801463
Estimated d (clean, noisy): 2.0020734363356922 2.009562085973709

Spectrum of Chirp Signal

The spectrum is obtained over the entire sample, during which the frequency of the wave increases from 16 to 32, and hence the peaks are rather broad. The distributions are centred at ±24. The spread width is 24 (different from the expected value of 16) and there are sharper ripples on the edges, but this is corrected after windowing.
The phase doesn't have any particular pattern, apart from being an odd function.

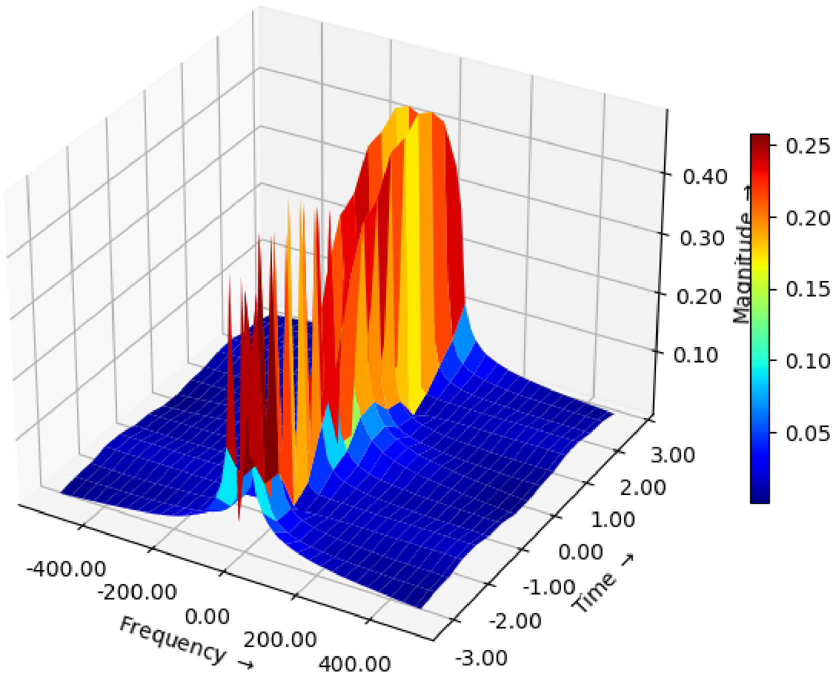


Below is the output with windowing

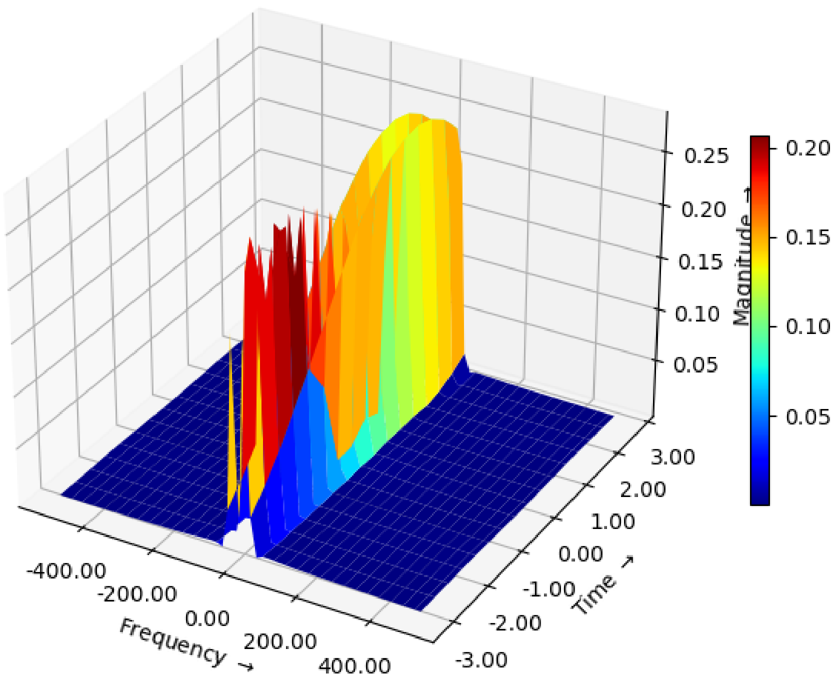


Spectrogram – Time-Frequency Plot of Chirp

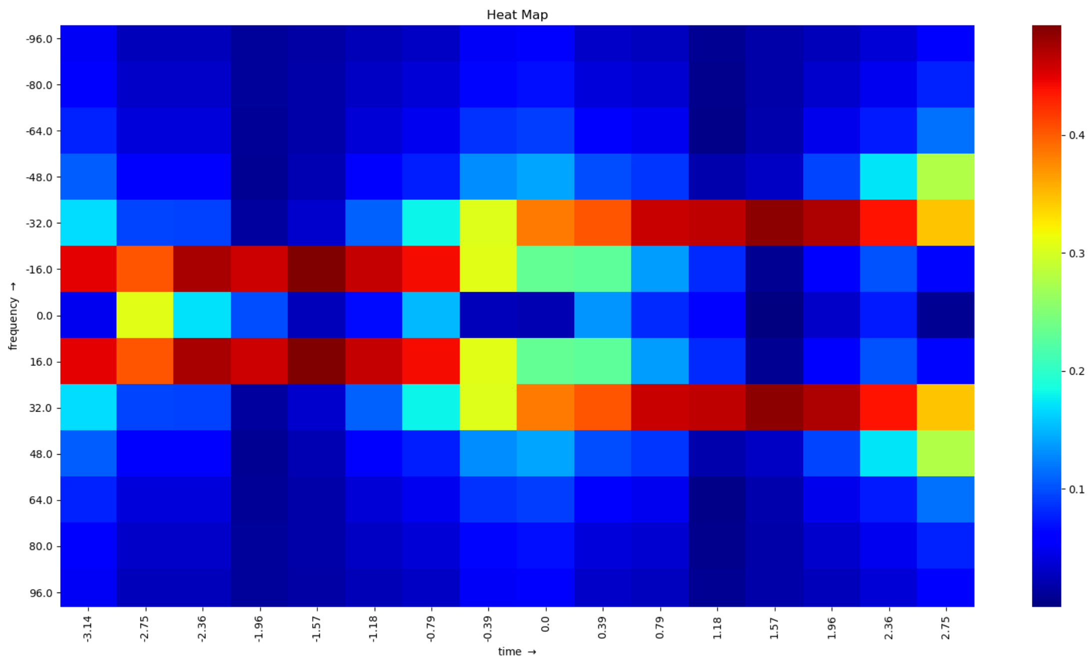
By splitting the long sample into chunks and performing DFT on each, we get a time-frequency plot, which shows what frequencies are present in a particular time window. This shows the dominant frequency components in each moment of time. The two 'walls' diverge slightly, indicating that the frequency of the cosine is increasing.
There is rise and fall in the values since the plot can accomodate only a resolution of 16 in the frequency axis, which means when the frequency is at 24, it equally spreads between 16 and 32, making it look like half the value.



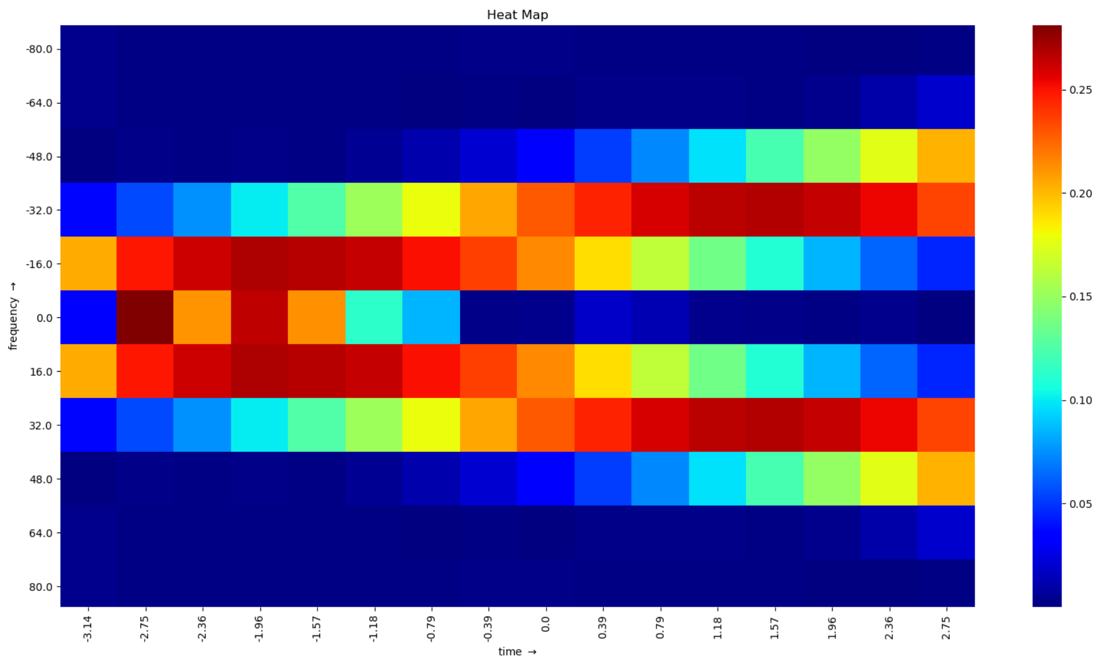
Below is the output with windowing



Also, a more familiar (and more readable) heatmap – the spectrogram – was plotted



Below is the output with windowing



Conclusion

This assignment explored the process of finding spectra of non-periodic or non-integral frequency signals using DFT implemented in Scipy. Briefly, the following results were obtained

- Q1 (Examples): The example tasks showed the effects of Gibbs phenomenon and windowing to mitigate the effects. The plots obtained are the same as the question paper
- Q2 ($cos^3(0.86t)$ spectrum): After windowing, the peaks are sharper, and match the expected cosine spectrum from the trigonometric formula. Windowing also eliminates the magnitude spread in frequencies between the peaks
- Q3, Q4 ($cos(\omega_0 t + \delta)$ Estimation): The frequency was estimated using the average of frequencies weighted by their magnitudes. The phase was estimated using average of phase weighted by the magnitude at that frequency. The error in frequency and phase for randomly generated values did not cross around 10% usually. This estimation method works with similar performance in the presence of Added White Gaussian Noise
- Q5 (Chirp DFT): The Chirp DFT contains ripples showing the frequencies that were sampled by the DFT as its frequency increased. They are centred around the middle frequency of the chirp (at t=0). This is similar to a rect function, and the side lobes at the edge have higher ripple than the middle. For applications needing lower energy in the side lobes, windowing is used to remove the ripples and shows two broad peaks, free of high side lobes.
- Q6 (Time-Frequency Plot): The 3D surface plot shows the evolution of magnitude over time and frequency. There 2 'walls' indicating the present frequencies of the cosine. The frequency increases with time from 16 to 48. Windowing makes the non dominant frequencies close to 0, but the peaks become slightly broader, as shown by the yellowness in the windowed surface plot.
  A 2D spectrogram was also plotted, which makes this data easier to visualise.