EE2703 Assignment 8 - Digital Fourier Transform

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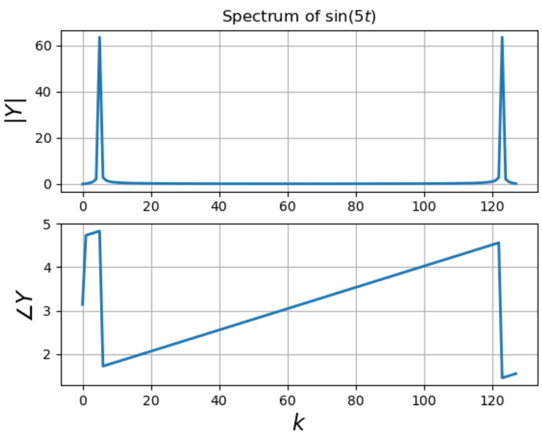
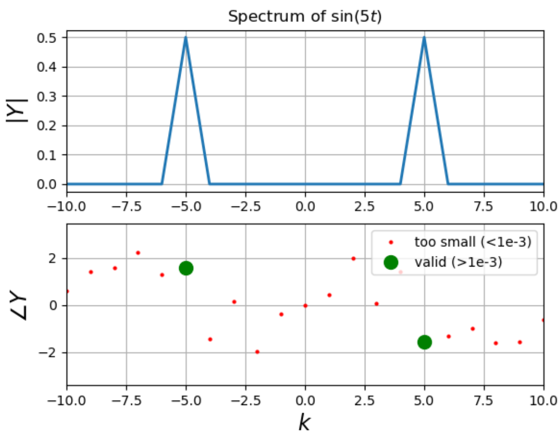
Abstract

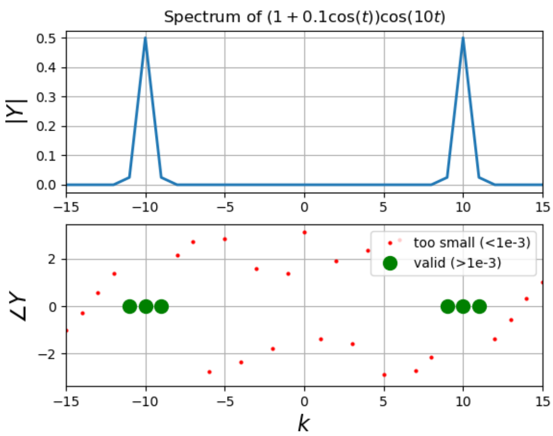
In this assignment, the DFT was explored using FFT functions in Scipy. The detailed process and observations from each of the plots is given below.

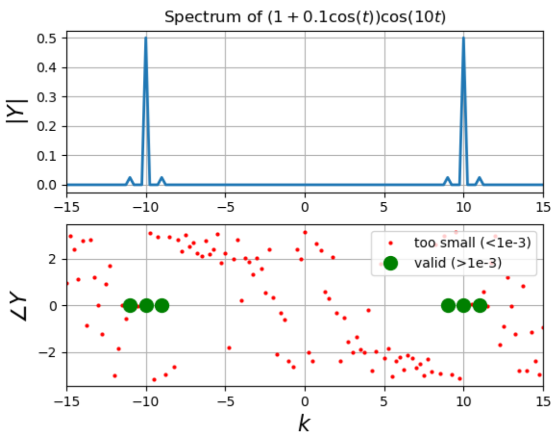
Example Tasks

The example tasks involving sine and amplitude modulation were done, and the following plots were obtained







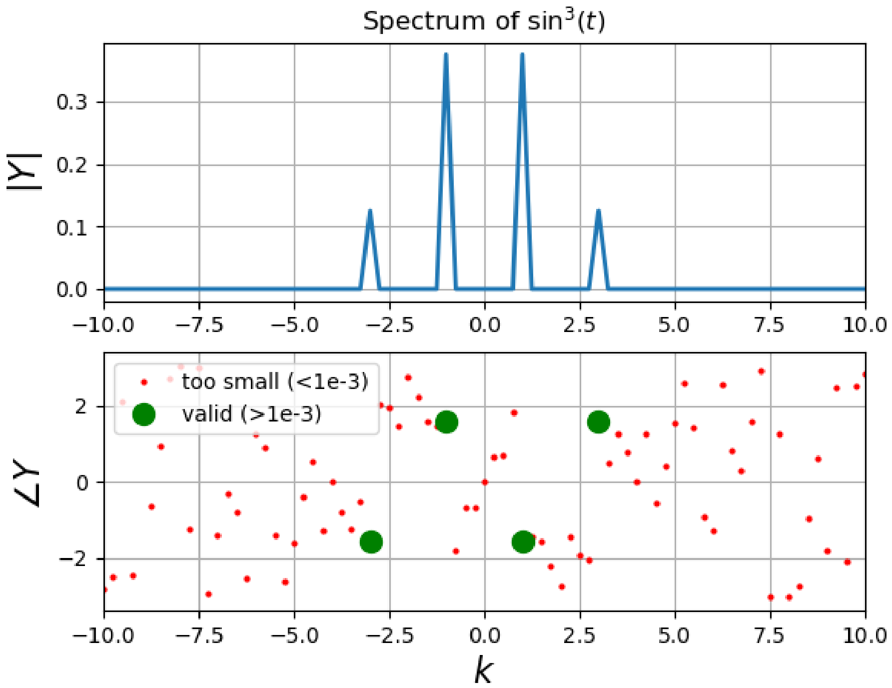


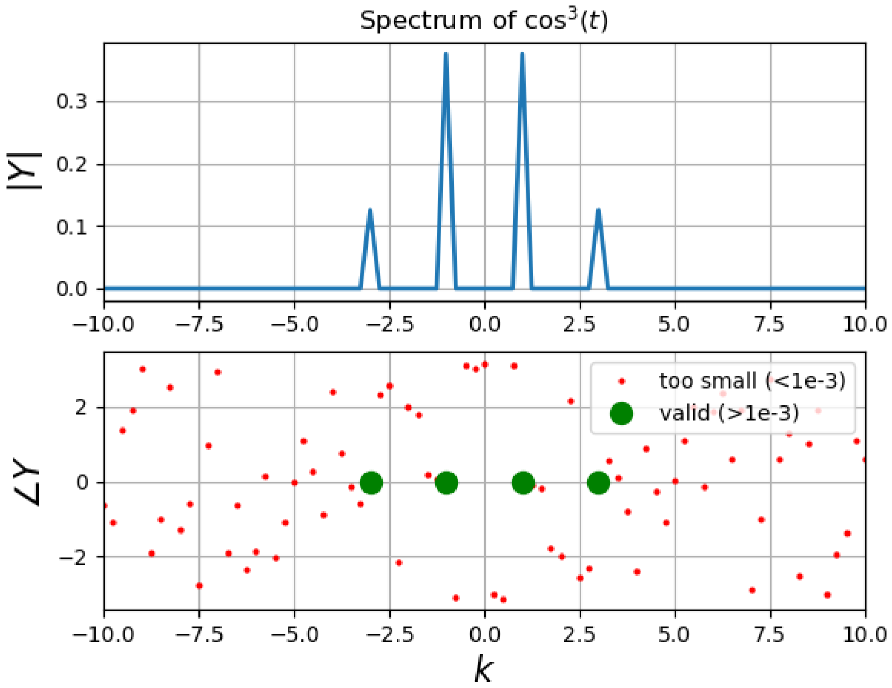
Spectra of $sin^3(t)$ and $cos^3(t)$

From trigonometric identities, we know that

$$sin^3(t) = \frac{3sin(t) - sin(3t)}{4} \qquad cos^3(t) = \frac{cos(3t) + 3cos(t)}{4}$$

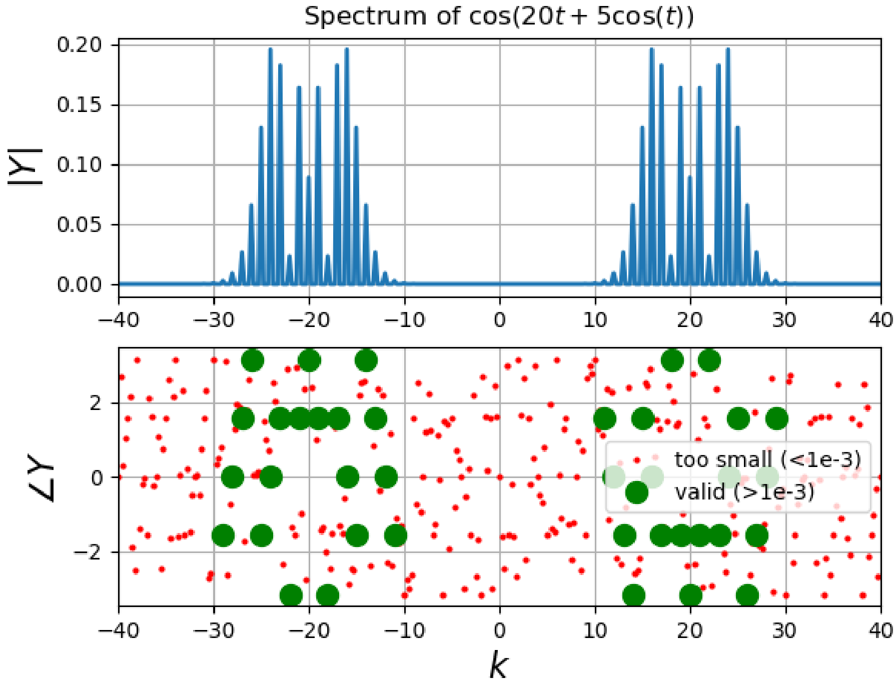
So both spectra have peaks at $\pm 1, \pm 3$. The phases are 0 for cosine and $\pm\frac{\pi}{2}$ for sine, as expected from the exponential formulae for sine and cosine





Frequency Modulation

A cosine term inside the argument of a cosine causes varying frequency over time. This can be used to send information on the low frequency cosine, which shows up as changing frequency of the overall wave. This is called frequency modulation.
The function given is $cos(20t + 5cos(t))$. So the centres of the spectra are at ± 20. Also, the phase is such that the spectrum coefficients are either purely real or purely imaginary.

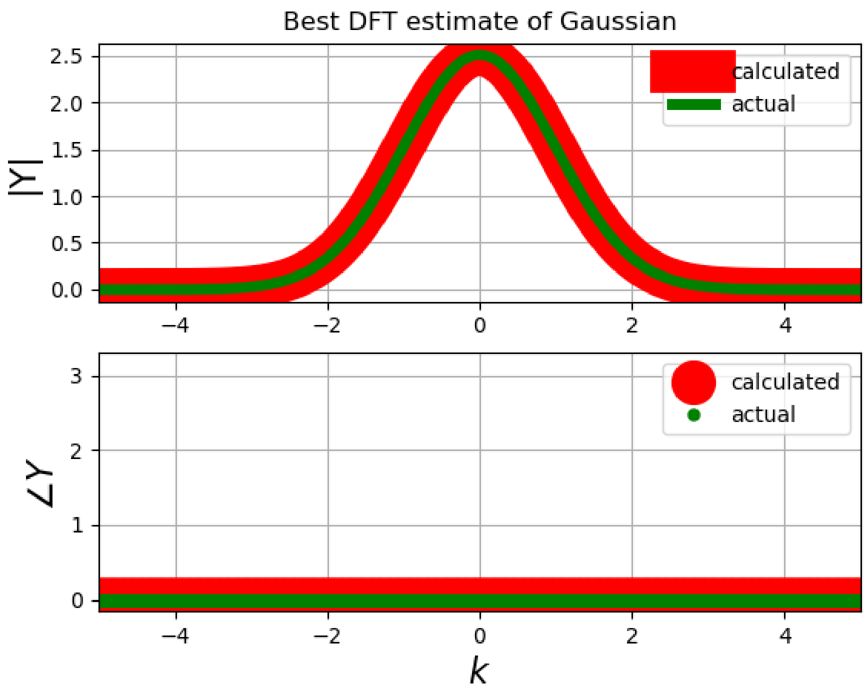


Spectrum of Gaussian

The analog FT of a Gaussian is given by

$$F\{e^{-\frac{t^2}{2}}\} = \sqrt{2\pi}e^{-\frac{\omega^2}{2}}$$

On increasing the time window of DFT, we are capturing more frequencies of the actual function (i.e., Gaussian is not bandlimited). We can iterate over time windows till we get an error less than the tolerance of 1e-6. Each time the window size is doubled, so is the number of samples to keep the resolution constant. Using this, we get -2π to 2π as the ideal time window. The error here is 2.768e-15.



Conclusion

This assignment explored the DFT using FFT functions from Scipy. Briefly, the following results were obtained

- Q1 (Examples): The examples showed that the output of fft() needs to be shifted to centre at 0, and needs amplitude scaling. It also showed the importance of resolution in frequency to get accurate coefficients
- Q2 ($sin^3(t)$ and $cos^3(t)$): The coefficients are as expected, both in magnitude and phase, from the trigonometric formulae relating the cube of sine to sines with 3 times the frequency. There are lot of stray points with considerable phase, but very less magnitude. This is due to the finite duration sequence that we supplied
- Q3 (Frequency Modulation): The spectrum is centred around 2 points – ± 20. This is because the centre frequency of the modulation is 20. There is a spread of magnitude over frequencies ranging from ± 10 to ± 30. This is due to the cosine inside another cosine's argument. The spread width of 10 to 30 can be derived by applying cos(a+b) formula to get cos(5cosx), whose expansion is given by the Jacobi-Anger Identity.

$$cos(5cos(x)) = J_0(5) + \sum_{n=1}^{\infty}(-1)^n J_{2n}(5)cos(2nx)$$

- Q4 (Gaussian): An iterative loop was run to check different time windows to find the most accurate DFT to approximate the analog FT. From the internet, the actual analog FT was used to determine the error. The window obtained is -4π to 4π, and the number of samples used is 1024. The error starts increasing from here, very gradually, since the number of samples are doubling, and the datatype may not hold the exact value for the coefficients