EE2703 Assignment 6L - Laplace

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Abstract

In this assignment, linear time-invariant (LTI) systems were analysed with Laplace Transforms using Scipy
The detailed process and observations from each of the plots is given below.

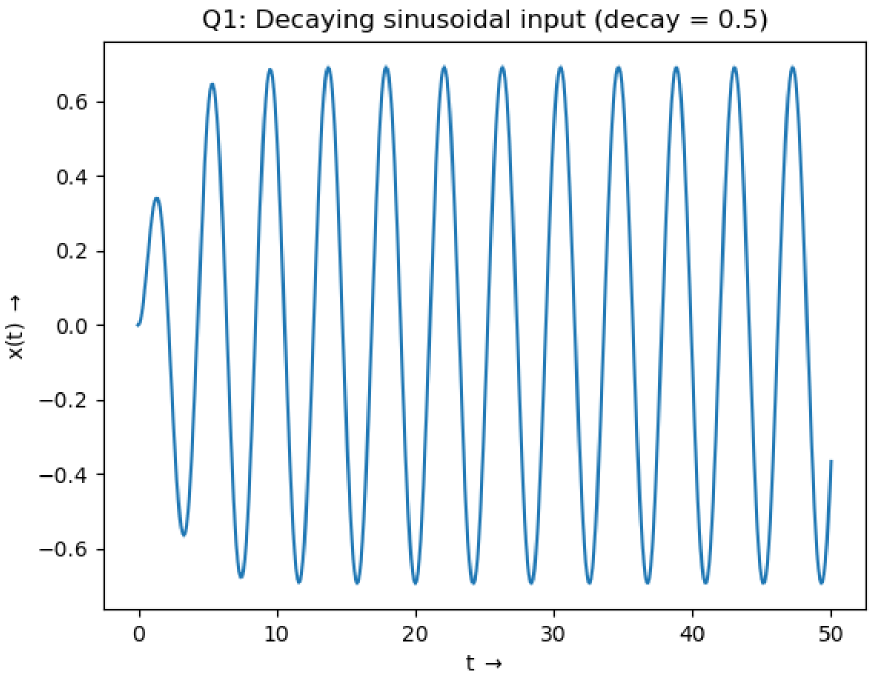
Decaying Sinusoids on Spring System (Q1,Q2)

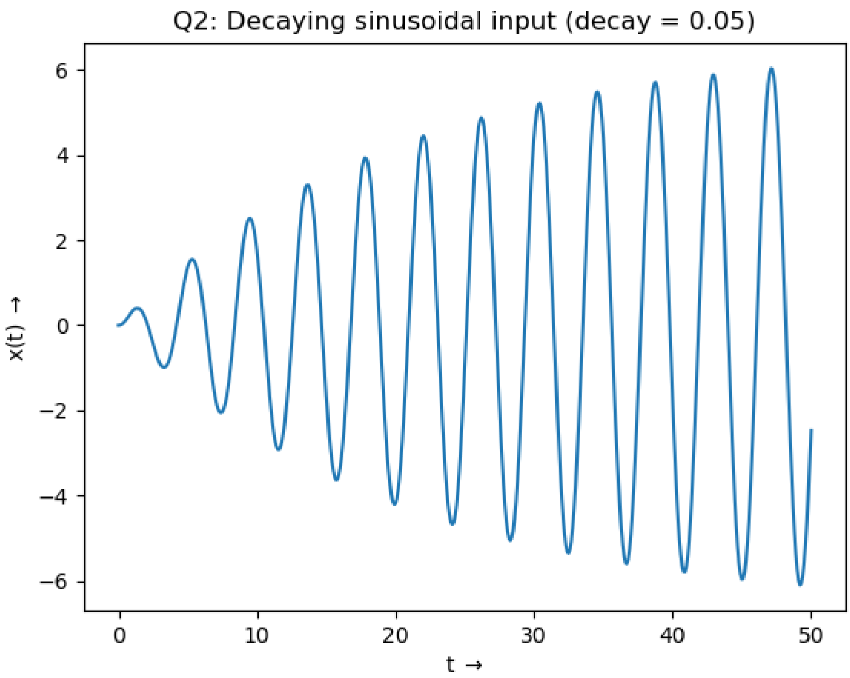
First, an exponentially decaying sinusoidal force is fed as input to a spring system obeying a 2nd order differential equation. On doing the Laplace transform of this equation, we get

$$X(s) = \frac{F(s)}{s^2 + 2.25} = \frac{s + 0.5}{(s^2 + 2.25)((s + 0.5)^2 + 2.25)}$$

x(t) is obtained through the sp.impulse() function, which takes a transfer function and computes impulse response. Using impulse response works since convolving x(t) with $\delta(t)$ gives x(t) itself, and convolution in time domain is multiplication in frequency domain, where $L\{\delta(t)\} <-> 1$. So X(s) can be converted to x(t) using sp.impulse().

Two such inputs were given, with different decay factors. The graphs are given below –





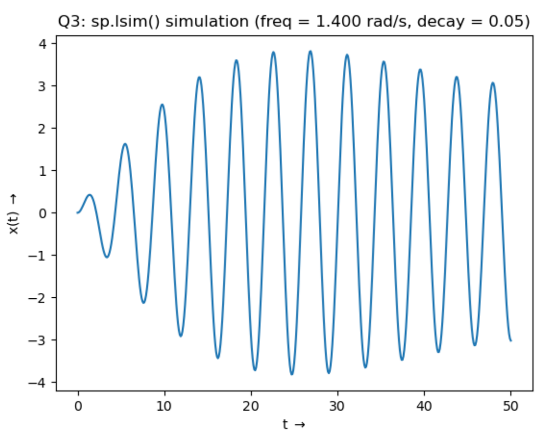
We notice that the smaller decay factor takes longer time to settle to sinusoidal steady state. This is because a smaller decay factor means less decay rate, so the force acts on the system for a longer time. Since it is at resonant frequency (the natural frequency of the system), the force always adds energy to the system. So while the force is active, the amplitude of the sinusoid only increases. This also explains why the final amplitude is higher for low decay factor, since the force is active for more time.

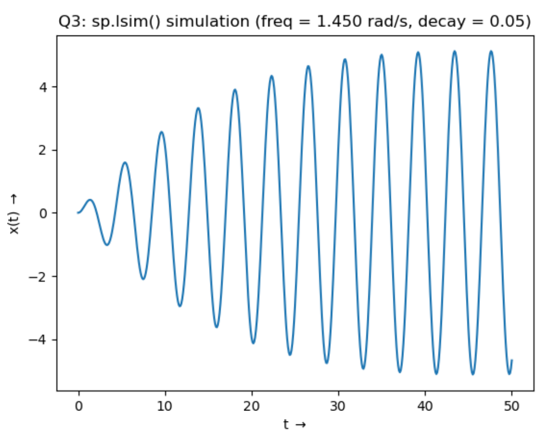
LTI System Formulation of Spring System (Q3)

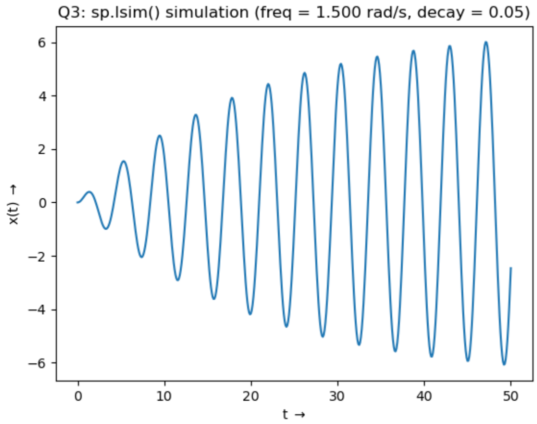
We find the transfer function of the system as

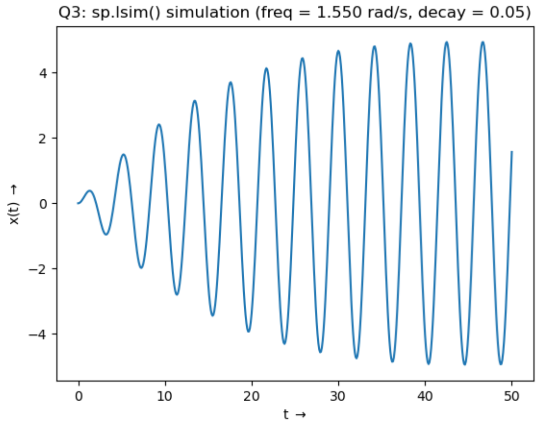
$$H(s) = \frac{1}{s^2 + 2.25}$$

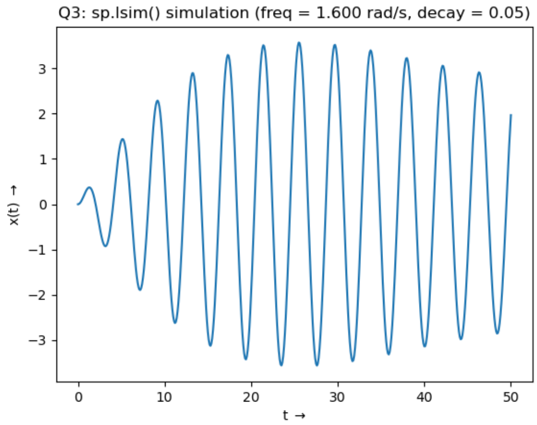
Using sp.lsim(), inputs of different decaying sinusoid frequencies were passed to the system, and outputs were found and plotted below











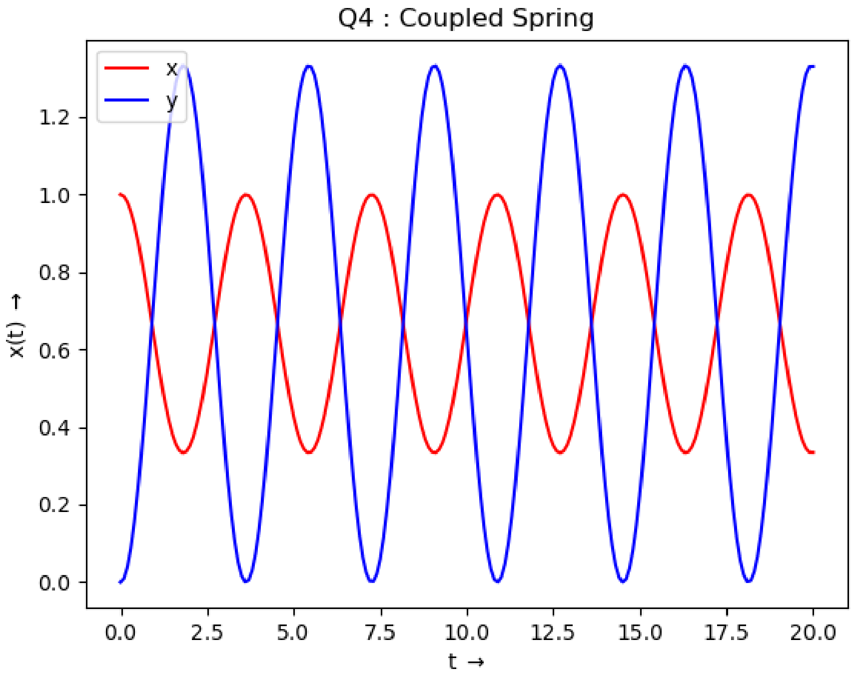
At both lower and higher frequencies, the envelope of the sinusoid reaches a peak, then falls. This is because the input frequency does not match the natural frequency. So instead of the resonant case where the force always increased energy, in this case the force will in fact become out of phase with the natural response of the system, hence reducing the energy, which is seen by the dip in amplitude.
As the frequencies come closer to the natural frequency, the dipping reduces.

Coupled Spring (Q4)

On taking Laplace transform of the given equations and solving with the given initial conditions, we get

$$X(s) = \frac{s^2+2}{s^3+3s} \qquad Y(s) = \frac{2}{s^3+3s}$$

Using sp.impulse(), we get x(t) and y(t)



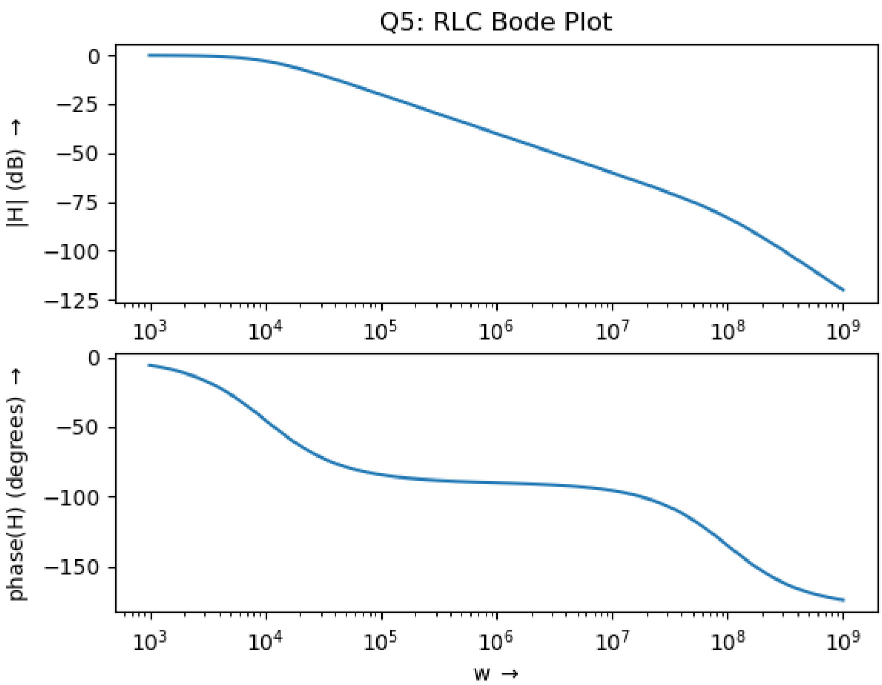
x(t) and y(t) are exactly 180 degrees out of phase. This is because $y = \frac{d^2x}{dt^2} + x$, and second derivative of sinusoid is a scaled and 180 degrees phase shifted version of original sinusoid.

RLC Response (Q5,Q6)

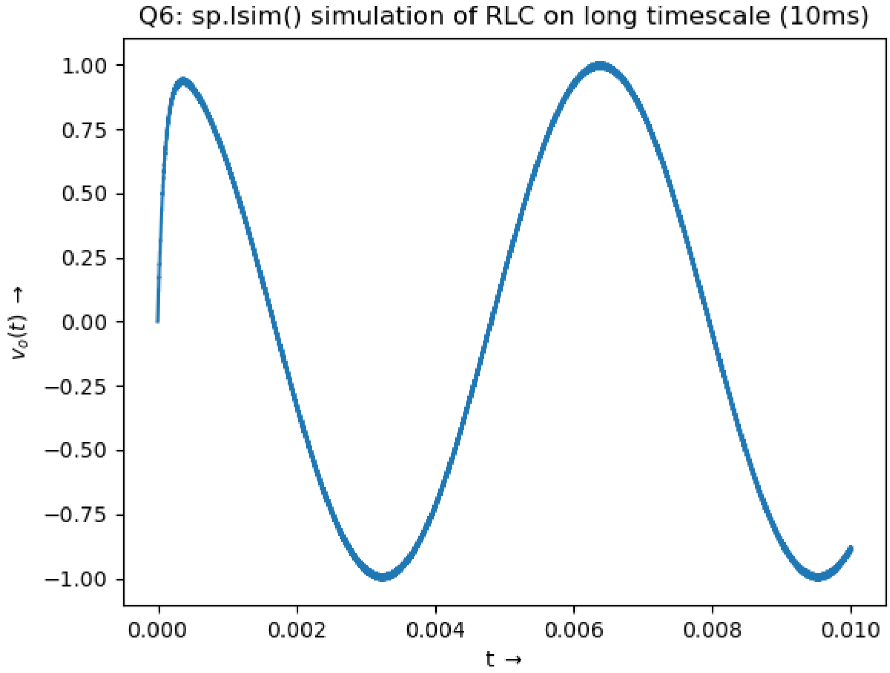
First, the transfer function was found

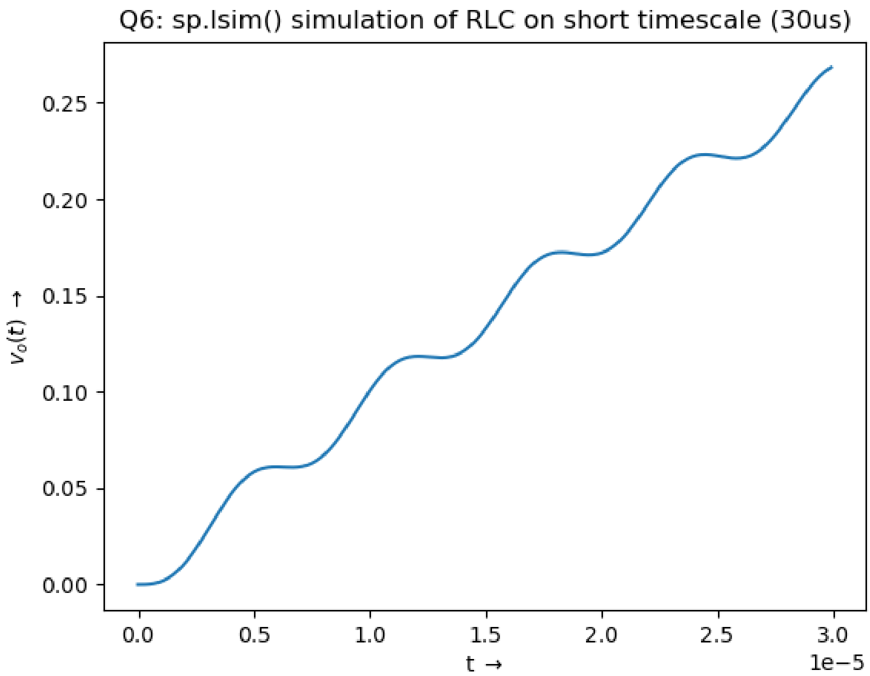
$$H(s) = \frac{1}{10^{-12}s^2 + 10^{-4}s + 1}$$

The magnitude and phase Bode plots were found



On factorising the denominator of H(s), we get 2 poles, approximately at $w = 10^8, 10^4$. This is reflected in the Bode plots as the slope of magnitude becomes more negative around these points. Since the frequency terms in the denominator control the phase, higher jw in denominator causes phase to decrease while passing a pole.
Then for Q6, the given voltage is applied as input on the system. The response is observed on the millisecond and microsecond timescales to notice different behaviours.





The lower frequency sinusoid has mostly been preserved while the higher frequency sinusoid has very small effect. This is possibly due to the magnitude response at 1e3 being 0dB, while being around -50dB at 1e6 (between 100 to 1000 times attenuated).

Conclusion

This assignment explored the analysis of LTI systems using Scipy in Python. Briefly, the following results were obtained

- Q1,Q2: A lower decay factor causes larger final amplitude and slower settling time, if the input sinusoid matches the natural frequency of the system.
- Q3: The force goes out of phase with the natural response at some point of time if the two frequencies don't match. Once this happens, the force reduces energy and the amplitude dips. Moving the forcing frequency closer to natural frequency reduces the dipping effect.
- Q4: After solving the differential equations using Laplace transform and plotting, we notice that both x and y are sinusoidal. The denominator is of the form $s(s^2 + 3)$ which can be separated using partial fractions, which will finally give a step function and sinusoid. Also, y and x are out of phase since y is formed from the second derivative of s.
- Q5,Q6: From the magnitude response, we see that the high frequency is attenuated while the low frequency passes. This is seen in the rippling of the small timescale plot where the high frequency ripple amplitude is small. The large timescale plot looks sinusoidal, but the first peak occurs a bit early. This is because of transients in the initial few time periods.