PROJECTILE Type

UNIT 04 (Numerical 13)

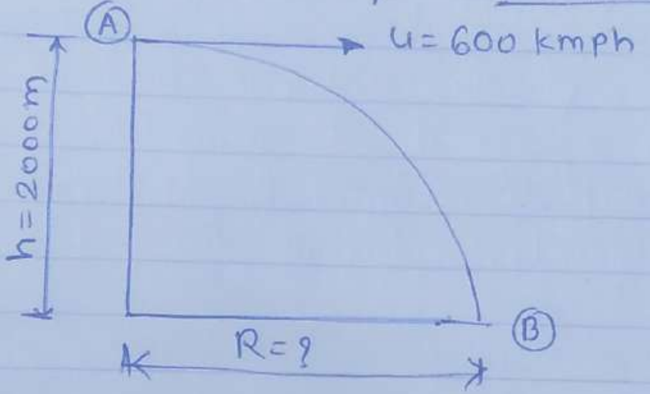
N-4.13 A pilot flying his bomber at height of 2000 m with, a uniform horizontal velocity of 600 kmph wants to strike a target as shown in figure. At what distance from the target, he should release the bomb?

Solu^n →

$h = 2000\ m$

$u = 600\ kmph = \frac{600 \times 1000}{60 \times 60} = 166.17\ m/sec.$

Initial velocity in vertical direction = 0



Gravitational acceleration = $9.81\ m/sec^2$ (downward +ve)

If t is the time of flight, considering vertical motion, we get

$$h = ut + \frac{1}{2} gt^2$$

$$2000 = 0 \times t + \frac{1}{2} \times 9.81 \times t^2$$

$$\therefore \boxed{t = 20.19\ sec}$$

During this period, horizontal distance travelled by the bomb,

$$Range = ut$$

$$R = 166.67 \times 20.19$$

$$\boxed{R = 3365.46\ m}$$

Bomb should be released at 3365.46 m from the target.

UNIT 04 (Numerical 14)

N-4.14 A particle is projected at an angle of $60°$ with the horizontal. The horizontal range of the particle is 5 kilometres. Find :-

i) The velocity of projection

ii) The maximum height attained by the projectile

Solu^n → (i) Given

Angle of projection, $\alpha = 60°$

Horizontal range, $R = 5$ kilometres

$= 5 \times 1000 \Rightarrow 5000$ m.

Let u = velocity of projection

By using equation for horizontal range.

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$\therefore 5000 = \frac{u^2 \times \sin(2\times 60)°}{9.81}$$

$$\therefore u^2 = \frac{5000 \times 9.81}{\sin 120°} = \frac{5000 \times 9.81}{0.866} = 56639.7$$

$$\therefore \boxed{u = 237.99 \text{ m/sec}}$$

ii) Maximum height attained by the projectile

Let h = Maximum height attained by projectile

By using equation of maximum height,

$$h = \frac{u^2 \cdot \sin^2\alpha}{2g}$$

$$h = \frac{237.99^2 \times \sin^2 60}{2 \times 9.81}$$

$$\boxed{h = 2164.98 \text{ m}}$$

UNIT 04 (Numerical - 15)

N-4.15 A particle is projected in air with a velocity 100 m/sec and at an angle of 30° with the horizontal. Find.

i) The horizontal range.

ii) The maximum height by the particle.

iii) The time of flight.

Given,

Solu^n → Velocity of projection, $u = 100$ m/sec

Angle of Projection, $\alpha = 30°$

Let R = Horizontal range = ?

h_{max} = Maximum height attained by particle = ?

T = Time of flight = ?

i) By equation of Horizontal range

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{100^2 \times \sin(2\times 30)^\circ}{9.81}$$

$$\therefore \boxed{R = 882.77 \text{ m}}$$

(ii) By using equation for maximum height.

$$h_{max} = \frac{u^2 \sin^2\alpha}{2g} = \frac{100^2 \times \sin^2 30}{2 \times 9.81}$$

$$\therefore \boxed{h_{max} = 127.42 \text{ m}}$$

(iii) By using equation for time of flight.

$$T = \frac{2u\sin\alpha}{g} = \frac{2\times 100 \times \sin 30}{9.81}$$

$$\therefore \boxed{T = 10.19 \text{ sec}}$$

UNIT 04 (Numerical 16)

N-4.16 A particle is projected at such an angle with the horizontal that the horizontal range is four times the greatest height attained by the particle. Find the angle of projection.

Solu^n Given

Horizontal range = 4 times the greatest height

i.e. $\boxed{R = 4 \times h_{max}}$

where R = Horizontal range

h_{max} = Maximum height obtained

Let α = Angle of projection

By using equation for horizontal range.

$$\boxed{R = \frac{u^2 \cdot \sin 2\alpha}{g}} \quad \text{——— } (1)$$

By using equation for maximum height

$$\boxed{h_{max} = \frac{u^2 \cdot \sin^2\alpha}{2g}} \quad \text{——— } (2)$$

From eq^n (1) & (2) and given condition $R = 4 \times h_{max}$ we get

$$R = 4 \times h_{max}$$

$$\left(\frac{u^2 \cdot \sin 2\alpha}{g}\right) = 4\left(\frac{u^2 \cdot \sin^2\alpha}{2g}\right)$$

$$\sin 2\alpha = 2 \cdot \sin^2\alpha$$

$$2\sin\alpha \cdot \cos\alpha = 2 \sin^2\alpha$$

$$\cos\alpha = \sin\alpha$$

$$\therefore \frac{\sin\alpha}{\cos\alpha} = 1$$

$$\therefore \tan\alpha = 1$$

$$\therefore \alpha = \tan^{-1}(1)$$

$$\therefore \boxed{\alpha = 45^\circ}$$

UNIT 04 (Numerical 17)

M-4.17 A body is projected at an angle such that it's horizontal range is 3 times the maximum height. Find the angle of projection.

Solu^n →

Let 'u' be the velocity of projection and 'α' be the angle of projection. then

$$\boxed{\text{Maximum height reached} = \frac{u^2 \sin^2\alpha}{2g}}$$

and

$$\boxed{\text{Range} = \frac{u^2 \sin 2\alpha}{g}}$$

By using given condition.

$$R = 3 \times h_{max}$$

$$\left(\frac{u^2 \sin 2\alpha}{g}\right) = 3\left(\frac{u^2 . \sin^2\alpha}{2g}\right)$$

$$\sin 2\alpha = \frac{3}{2} . \sin^2\alpha$$

$$2 . \sin\alpha . \cos\alpha = \frac{3}{2} . \sin^2\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{4}{3}$$

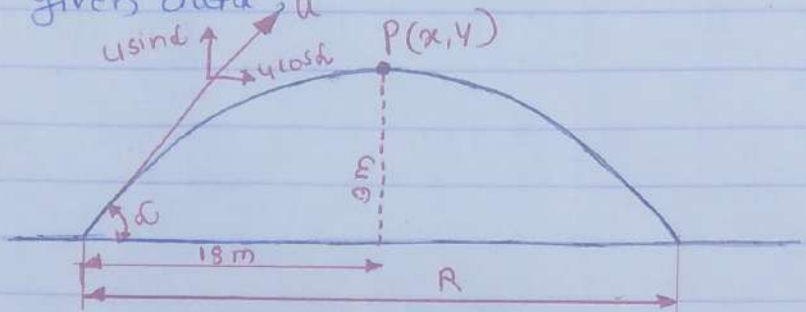
$$\tan\alpha = \frac{4}{3}$$

$$\alpha = \tan^{-1}(4/3)$$

$$\boxed{\alpha = 53.13^{\circ}}$$

N-4.18 The horizontal component of the velocity of a projectile is twice it's initial vertical component. Find ~~average~~ the range on the horizontal plane, if the projectile passes through a point 18 m horizontally and 3m vertically above the point of projection.

Solu^n → As per given data



Let 'u' be the initial velocity.
'α' be the angle of projection.
vertical component of velocity = u.sinα
Horizontal component of velocity = u.cosα

As per condition

$$u\cos\alpha = 2 \times u\sin\alpha$$

$$\therefore \frac{\sin\alpha}{\cos\alpha} = \frac{1}{2}$$

$$\therefore \tan\alpha = \frac{1}{2}$$

$$\therefore \boxed{\alpha = 26.565^\circ}$$

It's given that $x = 18\,m$, $y = 3\,m$

$\therefore$ Using the equation of trajectory

$$y = x\tan\alpha - \frac{1}{2} \cdot \frac{gx^2}{u^2\cos^2\alpha}$$

$$3 = 18 \times \frac{1}{2} - \frac{1}{2} \times \frac{9.81 \times 18^2}{u^2 \cdot \cos^2 26.565^\circ}$$

$$\therefore u^2 = \frac{9.81 \times 18^2}{6 \times 2 \times \cos^2 26.565^\circ}$$

$$\therefore \boxed{u = 18.196\, m/sec}$$

$\therefore$ Range on the horizontal plane $= \dfrac{u^2 \sin 2\alpha}{g}$

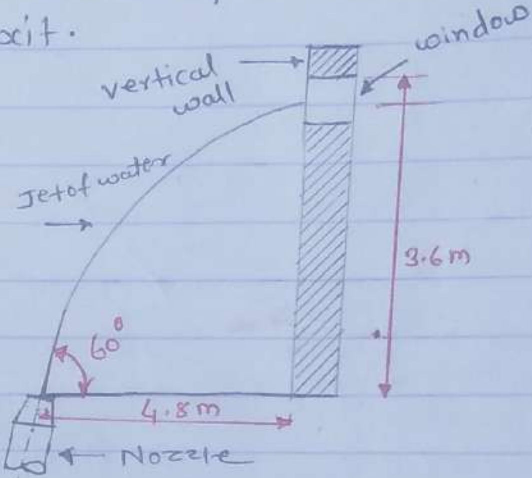
$$= \frac{18.196^2 \sin(2 \times 26.265^\circ)}{9.81}$$

$$\boxed{R = 27\ m}$$

UNIT 04 (Numerical 19)

N-4.19 A fireman holding a nozzle at a horizontal distance of 4.8 m from a vertical wall, wishes to send a jet of water through a small window in the wall located 3.6 m vertically above the nozzle. If the inclination of the jet with the horizontal is 60° at the nozzle, calculate the required velocity of the jet at the nozzle exit.

solu^n →



Horizontal distance, $x = 4.8$ m
vertical distance, $y = 3.6$ m
Angle of projection, $\alpha = 60°$

Let u = required velocity of projection

The equation of path travelled by projectile is given by

$$y = x \cdot \tan\alpha - \frac{g x^2}{2u^2 \cos^2\alpha}$$

$$3.6 = 4.8 \times \tan 60° - \frac{9.81 \times 4.8^2}{2 \times u^2 \times \cos^2 60°}$$

$$3.6 = 4.8 \times 1.732 - \frac{9.81 \times 4.8^2}{2 \times u^2 \times (0.5)^2}$$

$$3.6 = 8.3136 - \frac{452.0448}{u^2}$$

$$\frac{452.0448}{u^2} = 8.3136 - 3.6 \Rightarrow 4.7136$$

$$\therefore u^2 = \frac{452.0448}{4.7136}$$

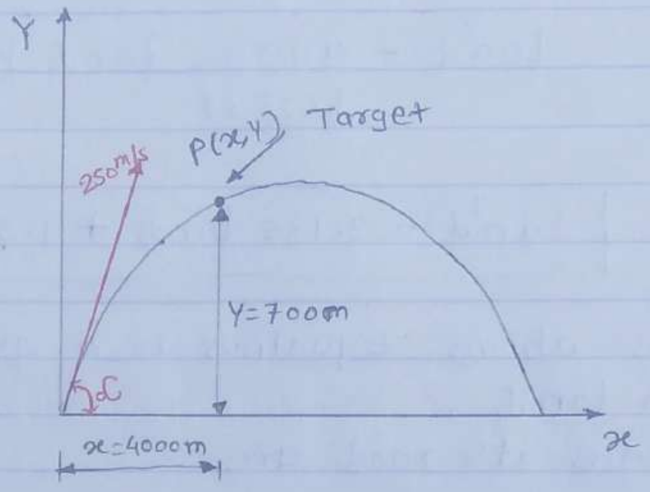
$$u^2 = 95.9$$

$$\boxed{u = 9.78 \text{ m/sec}}$$

UNIT 04 (Numerical 20)

N-4.20 A projectile is fired with an initial velocity of 250 m/sec at a target located at a horizontal distance of 4 km and vertical distance of 700m above the gun. Determine the value of firing angle to hit the target. Neglect air resistance.

Solun



Given

Initial velocity, $u = 250$ m/sec

Horizontal distance, $x = 4$ km $= 4000$ m

Vertical distance, $y = 700$ m

Let α = angle of firing

The equation of the path travelled by a projectile is given by :-

$$\boxed{y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}}$$

$$700 = 4000 \times \tan\alpha - \frac{9.81 \times 4000^2}{2 \times 250^2 \times \cos^2\alpha}$$

$$700 = 4000\tan\alpha - \frac{1255.68}{\cos^2\alpha}$$

$$700 = 4000\tan\alpha - 1255.68 \cdot \sec^2\alpha$$

$$700 = 4000\tan\alpha - 1255.68\,(1 + \tan^2\alpha)$$

$$700 = 4000\tan\alpha - 1255.68 - 1255.68\tan^2\alpha$$

$$\therefore\ 1255.68\tan^2\alpha - 4000\tan\alpha + 1255.68 + 700 = 0$$

$$\therefore\ 1255.68\tan^2\alpha - 4000\tan\alpha + 1955.68 = 0$$

$$\tan^2\alpha - \frac{4000}{1255.68}\tan\alpha + \frac{1955.68}{1255.68} = 0$$

$$\therefore\ \boxed{\tan^2\alpha - 3.185\tan\alpha + 1.557 = 0}$$

The above equation is a quadratic equation in $\tan\alpha$,
Hence it's root are

$$\tan\alpha = \frac{3.185 \pm \sqrt{3.185^2 - 4\times 1.557}}{2}$$

$$\tan\alpha = \frac{3.185 \pm 1.979}{2}$$

$$\tan\alpha = 2.582 \ \& \ 0.603$$

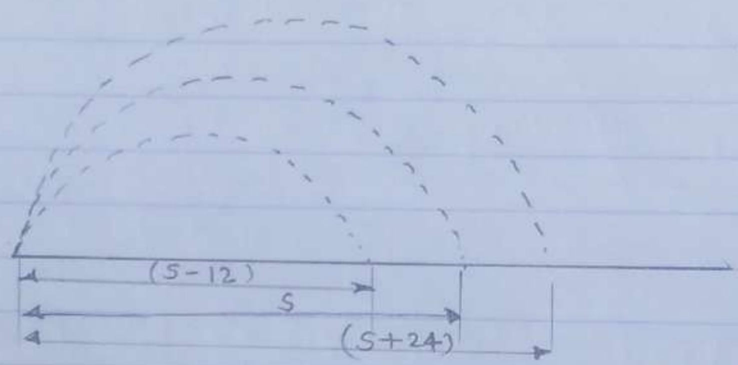
$$\therefore\ \alpha = \tan^{-1}(2.582)\ \&\ \tan^{-1}(0.603)$$

$$\therefore\ \boxed{\alpha = 68.82^\circ}\ \&\ \boxed{\alpha = 31.08^\circ}$$

UNIT 04 (Numerical 21)

N.4.21 A projectile is aimed at a target on the horizontal plane and falls 12 m short when the angle of projection is 15°, while it overshoots by 24 m when the angle is 45°. Find the angle of projection to hit the target.

Solun → Let 's' be the distance of the target from the point of projection and 'u' be the velocity of projection as shown in fig. ~~Range of projection is gi~~



∴ Range of projection is given by expression.

$$\text{Range}, R = \frac{u^2 \sin 2\alpha}{g}$$

Applying it to first case (i.e. $R = (S-12)$)

$$\therefore S-12 = \frac{u^2 \sin(2\times 15^\circ)}{g}$$

$$S-12 = \frac{u^2}{g} \times \frac{1}{2} = \frac{1}{2}\frac{u^2}{g}$$

$$\therefore \boxed{2(S-12) = \frac{u^2}{g}} \quad \text{— (1)}$$

from second case $(S+24)$

$$\therefore (S+24) = \frac{u^2 \sin 2\alpha}{g}$$

$$(S+24) = \frac{u^2}{g} \times \sin(2\times 45^\circ)$$

$$\therefore \boxed{(S+24) = \frac{u^2}{g}} \quad \text{— (2)}$$

from eqn (1) & (2)

$$(S+24) = 2(S-12)$$

$$\therefore \boxed{S = 48 \text{ m}}$$

Let the correct angle of projection be 'α' then

$$R = \frac{u^2}{g} \sin 2\alpha$$

$$\therefore 48 = \frac{u^2}{g} \times \sin 2\alpha$$

but $\frac{u^2}{g} = S+24 = 48+24$

$$\boxed{\frac{u^2}{g} = 72 \text{ m}}$$ put in

$$\therefore 48 = 72 \times \sin 2\alpha$$

$$\therefore 2\alpha = \sin^{-1}\left(\frac{48}{72}\right)$$

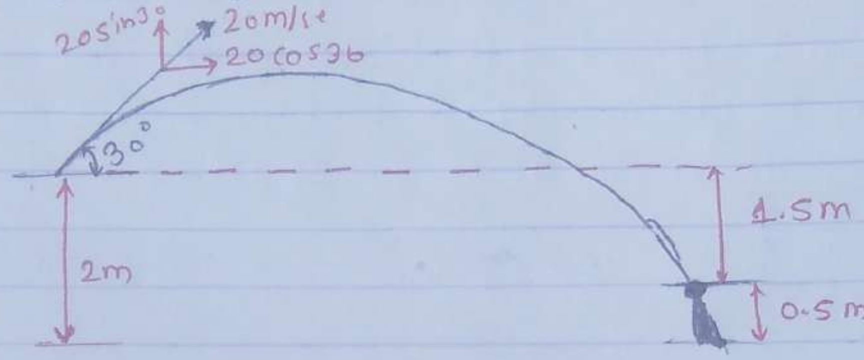
$$2\alpha = 41.81$$

$$\boxed{\alpha = \frac{41.81}{2} = 20.91^\circ}$$

UNIT 04 (Numerical 22)

N-4.22 A cricket ball thrown by a fielder from a height of 2m, at an angle of 30° to the horizontal, with an initial velocity of 20 m/sec, hits the wickets at a height of 0.5 m from the ground. how far was the fielder from the wickets ?

Solu^n



Solu^n

Initial velocity $u = 20$ m/sec
Angle of projection, $\alpha = 30°$

$$y_0 = -(2.0 - 0.5) = -1.5 m$$

∴ Time of flight t is given by the expression.

$$y = (u \sin\alpha) t - \frac{1}{2} g t^2$$

$$-1.5 = (20 \sin 30) t - \frac{1}{2} \times 9.81 \times t^2$$

$$t^2 - 2.0387 t - 0.3058 = 0$$

$$\therefore \boxed{t = 2.179 \text{ sec}}$$

The distance of the fielder from the wickets

$$= \text{Range} = u \cos\alpha \times t$$

$$= 20 \times \cos 30 \times 2.179$$

$$\therefore \boxed{\text{Range} = 37.742 \text{ m}}$$

∴ The distance between fielder & wickets is 37.742 m.