SUMMER-2012

UNIT 1

Q.1 a) Solve: $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}.\sec^2 x(1+2\tan x)$ **(7)**

b) Solve: $(x^2D^2-3xD+1)y=\frac{\sin(\log x)}{x}$ **(6)**

Q.2 a) Solve: $(D^2 - 2D + 2)\, y = e^x \tan x + 3x$ **(7)**

b) Use method of variation of parameter to solve

$(D^2 - 1)\, y = e^{-x}.\sin(e^{-x}) + \cos(e^{-x})$ **(6)**

UNIT 2

Q.3 a) Find Laplace transform of $t(3 \sin 2t - 2 \cos 2t)$ **(4)**

b) Find $L^{-1}\left\{\frac{1}{S^2(S-3)}\right\}$ **(4)**

c) Express f(t) in terms of unit step function and hence find Laplace transform

$$f(t) = \begin{cases} t^2; & 0 < t < 1 \\ 4t; & t > 1 \end{cases}$$ **(6)**

Q.4 a) Show that:

$$L^{-1}\left\{\frac{1}{S}.\cos\frac{1}{S}\right\}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\frac{t^6}{(6!)^2}+\ldots.$$ **(4)**

b) Evaluate $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}dt$ **(4)**

c) Solve the differential equation by using Laplace transform. $\frac{d^2x}{dt}+x=6\cos 2t$, with x = 3, $\frac{dx}{dt}=1$ at t = 0 **(6)**

UNIT 3

Q.5 a) Solve the difference equation:

i. $u_{n+2} - 16u_n = \cos\frac{n}{2}$, **ii.** $(E^2 + E + 1)\, u_n = n^2 + n + 1$ **(4)**

b) Find the inverse z-transform of $\frac{3z^2 + 2z + 1}{z^2 + 3z + 2}$ **(5)**

Q.6 a) Solve the difference equation

$U_{n+2} - 2u_{n+1} + 6u_n = 4$ **(4)**

b) Find z-transform of sin (3n + 5) **(4)**

c) Solve by z-transform $y_{n+2} + 5y_{n+1} + 6y_n = 6^n + 3n$ **(5)**

UNIT 4

Q.7 a) Solve the following partial differential equations.

$$\int_0^\infty \frac{\cos wx}{k^2 + w^2}\, dw = \frac{\pi e^{-kx}}{2k}$$ **(4)**

b) Solve the following partial differential equations.

i. $x^2p + y^2q = (x + y)\, z$, **ii.** $(pq - p - q)(z - px - qy) = pq$. **(4)**

Q.8 a) Find the Fourier transform of

$$f(x) = 1 - x^2; \quad |x| < 1$$
$$= 0; \quad |x| > 1$$

Hence evaluate $\int_0^\infty \left\{\frac{\sin x - x\cos x}{x^3}\right\}.\cos\frac{x}{2}.dx$ **(6)**

b) Solve the following partial differential equations.

i. $q(p^2z + q^2) = 4$, **ii.** $(2x - y)\frac{\partial z}{\partial x} + (x + z)\frac{\partial z}{\partial y} + (2x + y) = 0$ **(4)**

UNIT 5

Q.9 a) Find analytic function f(z) = u + iv such that

$v(r, \theta) = r^2 \cos 2\theta - r \cos + 2$ **(6)**

b) Find the bilinear transformation which maps the points z = 1, i – 1 into the points w = 0, 1, ∞. **(7)**

Q.10 a) Find the analytic function

$f(z) = u + iv$ if $\frac{u}{v} = \cot y$ **(6)**

b) Find the Laurent's expansion of

$f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < |z + 1| < 3$ **(7)**

UNIT 6

Q.11 a) A particle P moves along a plane with constant angular velocity w about 'O'. If the rate of increase of acceleration is parallel to PO prove that, $\frac{d^2r}{dt^2} = \frac{1}{3}rw^2$ **(7)**

b) Find the directional derivative of $\phi = 4.e^{2x-y+z}$ at point (1, 1, –1) in the direction towards the point (–3, 5, 6). **(6)**

Q.12 a) Prove that $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi.\nabla\psi + \psi\nabla^2\phi$. **(3)**

b) $\nabla^2\left[\nabla\left(\bar{r}/r^2\right)\right] = \frac{2}{r^4}$ **(3)**

c) Show that

$\bar{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is a conservative vector field and find a function ϕ such, that $\bar{F} = \nabla\phi$. Also find the work done in moving the particle form

(0, 1, –1) to $\left(\frac{\pi}{2}, -1, 2\right)$ **(7)**