SUMMER-2017

UNIT 1

Q.1 a) Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$. **(7)**

b) Solve $(D^3 - D^2 - 6D)\, y = x^2 + a^2 \sin x$. **(6)**

Q.2 a) Solve: $(D^2 - D - 6)\, y = e^x \cosh 2x$. **(6)**

b) Solve by the method of variation parameters

$(D^2 + 4)\, y = 4 \sec^2 2x$. **(7)**

UNIT 2

Q.3 a) Using convolution theorem find

$L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$. **(5)**

b) Find Laplace transform of the periodic function

$$f(t) = \frac{t}{a}, \qquad 0 < t < a$$

$$= \frac{1}{a}(2a - t), \qquad a < t < 2a$$

and $f(t) = f(t + 2a)$. **(5)**

c) Find f(t) if $\bar{f}(s) = \log\left(\frac{s+b}{s+a}\right)$. **(4)**

Q.4 a) Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$. **(4)**

b) Find $L\left\{e^{-4t}\int_0^t \frac{\sin 3t}{t} dt\right\}$. **(5)**

c) Solve by using Laplace transform

$\frac{d^2y}{dt^2} + y = 6\cos 2t$ with $y = 3$, $\frac{dy}{dt} = 1$ at $t = 0$. **(5)**

UNIT 3

Q.5 a) i. $y_{n+2} - 3y_{n+1} + 2y_n = n^2 + 2_{n-1}$. **(4)**

ii. $y_{n+2} + 2y_{n+1} + y_n = n$. **(4)**

b) Find inverse z-transform of $\dfrac{2z(z^2-1)}{(z^2+1)^2}$ by direct division method. **(5)**

Q.6 a) i. $y_{n+2} - 7y_{n+1} + 10y_n = 12(3)^n + 4^n$. **(4)**

ii. $y_{n+2} - 2y_{n+1} + y_n = 3n + 2$. **(4)**

b) Solve by Z transform

$$y_{n+1} + \frac{1}{4}y_n = \left(\frac{1}{4}\right)^n, \; n \geq 0, \; y_0 = 0.$$ **(5)**

UNIT 4

Q.7 a) Solve the following partial differential equations:

i. $\sqrt{p} + \sqrt{q} = 1$. **ii.** $p^2 + q^2 = z$. **(4)**

b) Find the Fourier sine transform of following function:

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$ **(6)**

Q.8 a) Solve the following partial differential equations:

i. $yzp + xzq + zxy = 0$, **ii.** $p \cos(x + y) + q \sin(x + y) = z$. **(8)**

b) Express f(x) as Fourier sine integral

where $f(x) = \begin{cases} 1, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$

and hence evaluate $\displaystyle\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda$. **(6)**

UNIT 5

Q.9 a) Determine analytic function whose real part is

$E^{2x}(x \cos 2y - y \sin 2y)$. **(6)**

b) Find the bilinear transformation which maps the points $z = 1, -i, -1$ onto the points $w = i, 0, -i$. **(7)**

Q.10 a) Expand the function cos z in a Taylor's series about the point $z = \pi/4$. **(6)**

b) Prove that: $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(7)**

UNIT 6

Q.11 a) Find the directional derivative of $f(x, y, z) = x^2 y^2 z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$. **(6)**

b) Find the divergence and curl of the vector

$\overline{V} = xyzi + 3x^2yj + (xz^2 - y^2z)k$. **(7)**

Q.12 a) Find the tangential and normal components of acceleration at any time t, of a particle whose position (x, y) at any time t is given by

$x = \cos t + t \sin t$, $y = \sin t - t \cos t$. **(6)**

b) Evaluate $\int_S \overline{F}.\hat{n}ds$, where $\overline{F} = 2x^2yi - y^2j + 4xz^2k$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $x = 2$, $y = 0$, $z = 0$. **(7)**