SUMMER-2018

UNIT 1

Q.1 a) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ **(6)**

b) Solve by the method of variation parameters

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$ **(7)**

Q.2 a) Solve $(D^2 + 3D + 2)\, y = 4 \cos^2 x$ **(6)**

b) Solve $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$ **(7)**

UNIT 2

Q.3 a) Find Laplace transform of $\frac{\cos bt - \cos at}{t}$. Hence Evaluate $\int_0^\infty \frac{\cos bt - \cos at}{t} dt$. **(5)**

b) Using convolution theorem. Find $L^{-1}\left\{\frac{1}{(s+2)(s^2+16)}\right\}$ **(4)**

c) Find Laplace transform of period function $f(t) = \begin{cases} \frac{t}{a} & 0 < t < a \\ \frac{1}{a}(2a-t) & a < t < 2a \end{cases}$ and f(t) and f(t) = f (t + 2a) **(5)**

Q.4 a) Find $\int_0^\infty \frac{\cos bt - \cos at}{t} dt$ **(4)**

b) Express the following function in terms of Haviside's unit step function and hence find its Laplace transform, if $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$ **(5)**

c) Solve $(D + 1)\, y = t^2 e^{-t}$ by using Laplace transform method, provided $y(0) = 3$ **(5)**

UNIT 3

Q.5 a) Solve the difference equation

$$y_{n+2} - 16y_n = \cos\left(\frac{n}{2}\right)$$ **(5)**

b) Prove that, $z^{-1}\left\{\frac{z}{z-a}\right\} = a^k$. **(4)**

c) Find z-transform of sin (3k + 5) **(4)**

Q.6 a) Solve the difference equations:

i) $y_{n+2} - 7y_{n+1} + 10y_n = 12e^{3n} + 4$

ii) $y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$ **(8)**

b) Solve by z-transform method. $y_{n+2} - 5y_{n+1} + 6y_n = 6^n$, if $y(0) = y(1) = 0$ **(5)**

UNIT 4

Q.7 a) Find the Fourier transform of: $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$ **(5)**

b) Solve the following partial differential equations.

i. $p^2 - pq = 1 - z^2$ **ii.** $p^2 - q^2 = x - y$ **(8)**

Q.8 a) Express $f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 0 & x > \pi \end{cases}$ As a Fourier sine integral and hence evaluate: $\int_0^{\infty} \frac{1 - \cos \pi\lambda}{\lambda} \sin \lambda x \, d\lambda$ **(6)**

b) Solve the following partial differential equations:

i. $yzp + xzq + 2xy = 0$ **ii.** $p^2 + q^2 = \frac{3a^3}{z^2}$ **(8)**

UNIT 5

Q.9 a) Find the analytic function

$f(z) = u + iv$, if $\frac{u}{v} = \cot y$ **(5)**

b) Expand the function $\frac{e^{2z}}{(z-1)^3}$ about the point z = 1 **(4)**

c) If f(z) is analytic then show that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$ **(4)**

Q.10 a) If f(z) = u + iv is analytic function and (u – v) = (x – y) $(x^2 + 4xy + y^2)$; find f(z) in terms of z. **(6)**

b) Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region

i. $|z| < 1$, **ii.** $1 < |z| < 2$ **(7)**

UNIT 6

Q.11 a) Find the directional derivative of $\phi = xyz$ in the direction normal to the surface $x^2y + yz^2 = 3$ at the point P(1, 1, 1). **(7)**

b) Compute the divergence and curl of vector:

$\overline{F} = \nabla(yz + zx + xy)$ **(7)**

Q.12 a) Show that $\overline{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational. Hence find a scalar function ϕ such that $\overline{F} = \nabla\phi$. **(7)**

b) Evaluate the line integral $\int_C \overline{F} \cdot d\overline{r}$ where c is the circle $x^2 + y^2 = 1$ in xy-plane and $\overline{F} = (2x^2 - y^2)i + (x^2 + y^2)j$. **(7)**