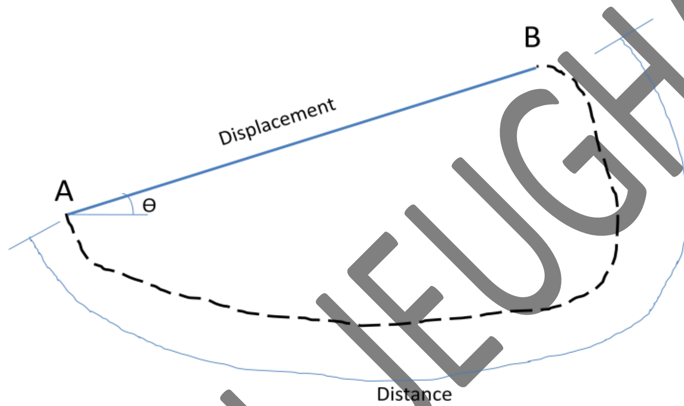
UNIT NO. 4

KINEMATICS

KINEMATICS:- IT is the branch of dynamics which deals with the motion of the body without referencing the force causing motion of the body is called as kinematics.

MOTION:- A body is said to be in motion if it changes its position with respect to reference point.

DISTANCE & DISPLACEMENT:- Let the body moves along the path as shown .



The time taken by the body to move from A to B is "t".

Then the distance moved by the body in time "t" is distance measured along dotted line. Distance is a scalar quantity, since it has only magnitude.

Displacement in this time interval is the linear distance AB which makes angle ϴ with x-axis. Displacement of the body in the time interval may be defined as the linear distance between two positions of the body in the beginning & at the end of the time interval.

It is a vector quantity, since it has magnitude & direction.

SPEED & VELOCITY:- The rate of change of distance with respect of time is called as speed.

The rate of change of displacement with respect to time is called as velocity.

If "S" is the displacement in interval "t", then the average velocity is "V" is given by

$$\frac{S}{t}$$

Velocity of particle at given instant is called as instantaneous velocity.

The relation between speed & velocity is given by $1KMPH = \frac{1x1000}{60x60} = \frac{5}{18} m/s$

ACCELERATION & RETARDATION:- The rate of change of velocity with respect to time is called as acceleration.

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left[\frac{ds}{dt}\right]$$

$$= \frac{d^2s}{dt^2} m/s^2$$

The negative acceleration is called as retardation.

TYPES OF MOTION:- the body may be move in any direction in the space when the motion is in single plane, the motion is called as plain motion.

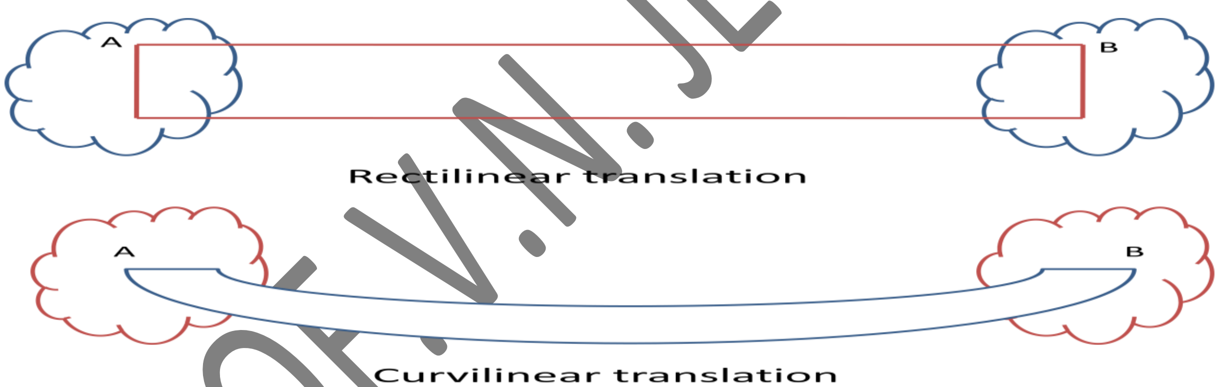
Plain motion may be classified as

1) Translation

2) Rotation

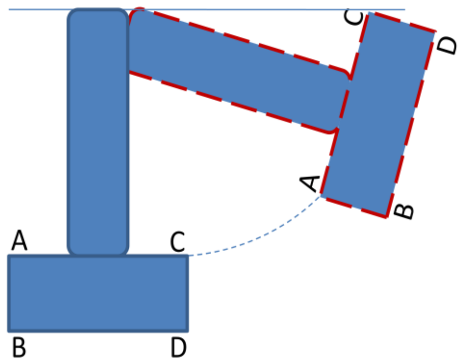
3) General plain motion

1) TRANSLATION:- A motion is said to be translation if a straight line drawn to moving body remains parallel at its original position at any time.



During the translation motion if the path track by the point is a straight line is called as rectilinear translation & if the path is curved one it is called as curvilinear translation.

2) ROTATION:- A motion is said to be rotation if all the particles of rigid body moves in concentric circle.

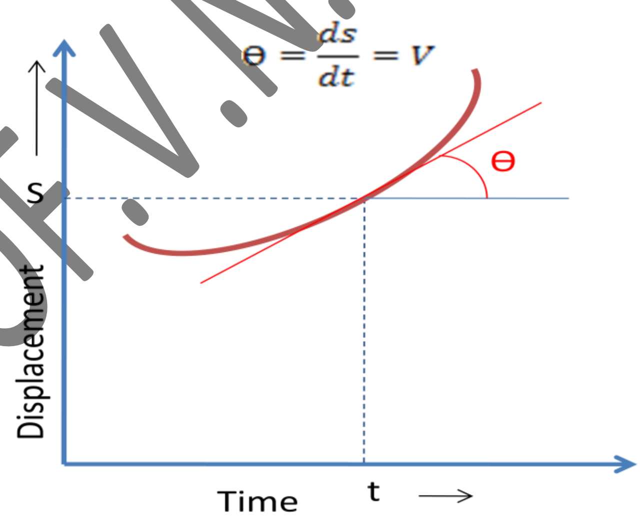


3) GENERAL PLAIN MOTION:- The general plane motion is combination of translation & rotation.

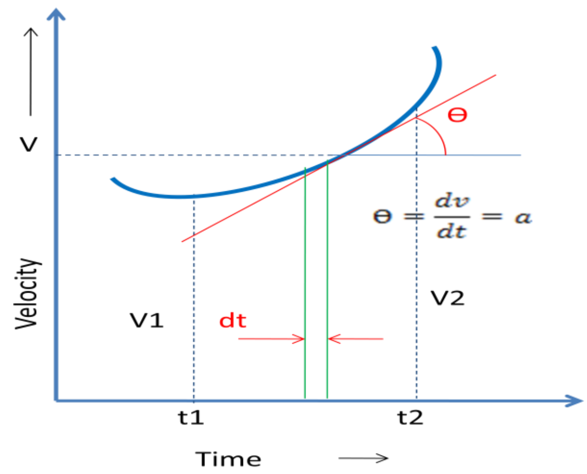
Ex:- 1) Wheels of moving vehicles.

MOTION CURVES:- Motion curves are the graphical representation of distance, velocity & acceleration with respect to time.

1) DISPLACEMENT –TIME CURVE:- (S-t curve):- s-t curve is a curve with time as abscissa & displacement as ordinate. At any instant of time "t", velocity is given by $V = \frac{ds}{dt}$ if the body is having non-uniform motion. Its displacement at various times interval may be observed on s-t curve plotted. Velocity at any time may be found out from the slope of s-t curve.



2) VELOCITY –TIME CURVE:- (v-t curve):- in v-t curve, the abscissa represents time & the ordinate velocity of motion. Acceleration "a" is given by slope of v-t curve i.e. $a = \frac{dv}{dt}$



Thus the acceleration at any if the slope of v-t curve at that point.

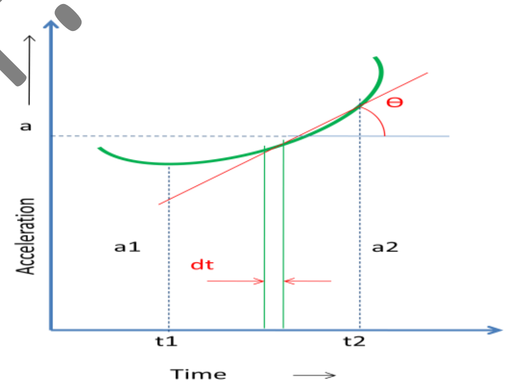
Thus in v-t curve,1) Slope of curve represents acceleratiom2) Area under the curve represents displacement.

3) ACCELERATION- TIME CURVE:- (a-t curve):- If the body is moving with varying acceleration, its motion can be studied by drawing a curve with time as abscissa & acceleration as ordinate, such a curve is called as acceleration-time curve. $a = \frac{dv}{dt}$

$$dv = a\,dt$$

$$v = \int a\,dt$$

The area under curve represents velocity.



MOTION WITH UNIFORM VELOCITY:-

Consider the motion of body with uniform acceleration "a"

Let, u = Initial velocity

v= Final velocity

t= Time taken for change in velocity u to v

We know, acceleration is defined as rate of change of velocity

$$a = \frac{v - u}{t}$$

$$v = u + at \rightarrow 1)$$

Displacement "s" is given by

$$s = \text{Avg. velocity x time}$$

$$s = \frac{v + u}{2} \text{ x } t$$

Substituting the value from equation 1

$$s = \frac{u + (u + at)}{2} \text{ x } t$$

$$s = \frac{2u + at}{2} \text{ x } t$$

$$s = ut + \frac{1}{2} a t^2 \rightarrow 2)$$

From equation 1

$$t = \frac{v - u}{a}$$

Substituting this value in equation 2

$$s = \frac{u + v}{2} \text{ x } \frac{v - u}{a}$$

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as \rightarrow 3)$$

Thus equations of motion of a moving body with constant acceleration are

$$v = u + at$$

$$s = ut + \frac{1}{2}\, a\, t^2$$

$$v^2 - u^2 = 2as$$

From definition of acceleration

$$\frac{dv}{dt} = a$$

$$dv = a\, dt$$

Since "a" is constant

$$v = at + c \rightarrow a)$$

When t=0

Final velocity = Initial velocity

$$v = u$$

Substituting this value in equation a

$$c = u$$

$$v = u + at \rightarrow i)$$

From the definition of velocity

$$\frac{ds}{dt} = v$$

$$\frac{ds}{dt} = u + at \text{ from eq. i}$$

$$ds = (u + at)dt$$

$$s = ut + \frac{1}{2}at^2 + c1$$

To find out c1,t=0, S=0

$$s = ut + \frac{1}{2}at^2 \rightarrow ii)$$

From definition of acceleration,

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} x \frac{ds}{dt}$$

$$a = \frac{dv}{ds} xv \qquad \because \frac{ds}{dt} = v$$

$$a\, ds = v\, dv$$

Integrating

$$a \int ds = \int_u^v v\, dv$$

$$a\, s = \left[\frac{v^2}{2}\right]_u^v$$

$$a\, s = \frac{v^2}{2} - \frac{u^2}{2}$$

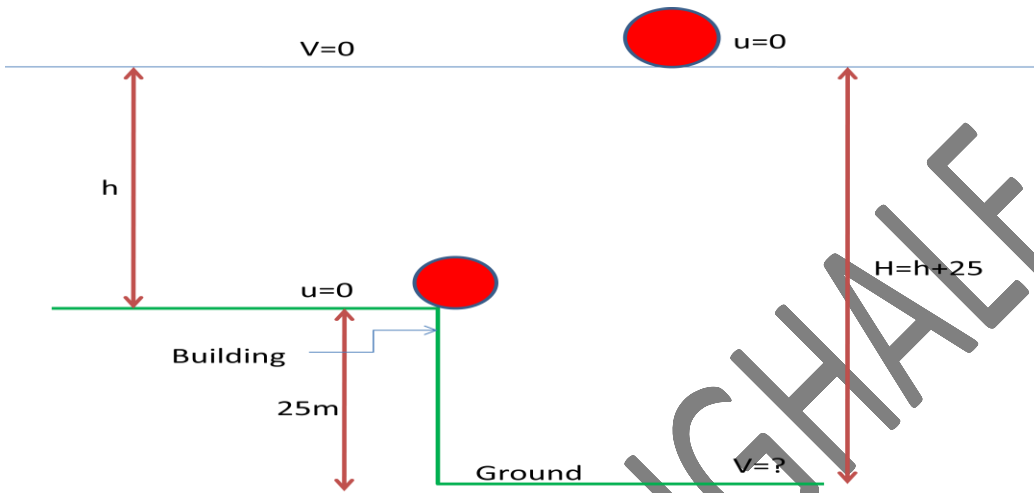
$$a\, s = \frac{v^2 - u^2}{2}$$

$$2as = v^2 - u^2$$

$$v^2 - u^2 = 2as \rightarrow iii)$$

Numerical:-

1) A small ball is thrown vertically upward from the top of building 25m above the ground with an initial velocity of 18m/s. Find a) In what time it will reach up to the maximum height, b) How high above the building will the ball rise, c) Compute the velocity with which it will strike the ground & the total time it is in motion.



For upward motion of ball

$$u = 18m/s$$

$$v = 0$$

$$s = h\ m$$

$$g = a = -9.81m/s^2$$

Using equation of motion

$$v = u + at_1$$

$$0 = 18 + (-9.81)t_1$$

$$t_1 = \frac{18}{9.81}$$

$$t_1 = 1.83\ sec$$

Using equation of motion

$$s = ut_1 + \frac{1}{2}at^2$$

$$h = 18x1.83 + \frac{1}{2}(-9.81)x1.83^2$$

$$h = 16.53 \text{ m}$$

Total height of ball from ground

$$H = h + 25$$

$$= 16.53 + 25$$

$$H = 41.53 \text{ m}$$

For downward motion

$$u = 0$$

$$v = ?$$

$$s = H = 41.53\text{m}$$

$$g = a = 9.81\text{m/s}^2$$

Using equation of motion

$$v^2 - u^2 = 2as$$

$$v^2 - 0^2 = 2x9.81x41.53$$

$$v = \sqrt{2x9.81x41.53}$$

$$v = 28.53 \text{ m/s}$$

Using equation of motion

$$v = u + at_2$$

$$28.53 = 0 + 9.81t_2$$

$$t_2 = \frac{28.53}{9.81}$$

$$t_2 = 2.91 \text{ sec}$$

The total time for which the ball is in motion

$$T = t_1 + t_2$$

$$T = 1.83 + 2.91$$

$$T = 4.74 \text{ sec}$$

2) A motorist travelling at 80km/Hr when he observed a traffic signal 200m ahead of him turn red. The traffic light is time to stay red for 10 sec. if the motorist wants to pass the signal without stopping just as it turns green. Determine,1} The required uniform de-acceleration of motor, 2} The speed of motor as it passes the signal.

Solution:-

Given:-

$$u = 80\text{Km/h}$$

$$u = 80\text{x}\frac{5}{18} = 22.22 \text{ m/s}$$

$$S = 200 \text{ m}$$

$$t = 10 \text{ sec}$$

$$a = ?$$

$$V = ?$$

Using equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$200 = 22.22\text{x}10 + \frac{1}{2}a\text{x}10^2$$

$$a = \frac{200 - 22.22\text{x}10}{50}$$

$$a = -0.45\text{m/s}^2$$

As the acceleration is negative, it is retardation/de-acceleration of motor.

Using equation of motion

$$v = u + at$$

$$v = 22.22 + 0.45x10$$

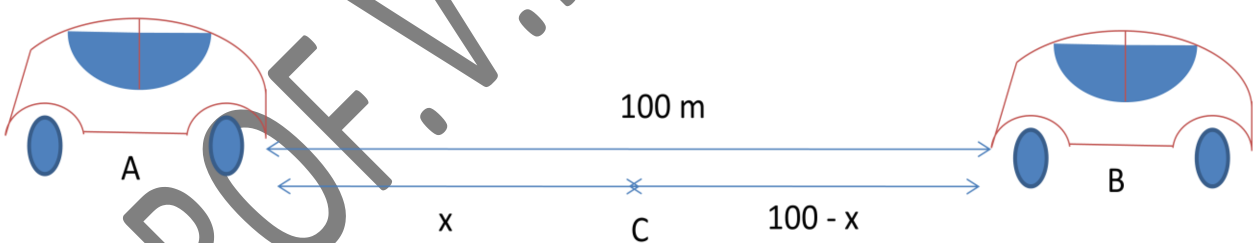
$$v = 17.72 \text{ m/s}$$

$$v = 17.72x\frac{18}{5}$$

$$v = 63.79 \text{ Kmph}$$

The de-acceleration of motor should be0.45m/s^2 & the speed of motor when it passes the signal should be 63.79Km/h

3) Two cars are travelling towards each other on a single lane road at the velocity of 12m/s & 9m/s respectively. When 100m apart both the drivers realize the situation & apply their breaks, they succeed in stopping simultaneously at just short of collusion. Assuming constant de-acceleration for each case, determine1} Time required for cars to stop, 2} The distance travelled by each car before slowing down.



Let,

A = position of first car.

B = Position of second car

When drivers realize the situation & apply breaks, let they meet at point C.

Let distance travelled by car A be x meter in t sec.

So distance travelled by car B is 100-x meter in t sec.

Considering the motion of 1st car

$$u = 12 m/s$$

$$V = 0$$

$$S = x\ m$$

$$\text{Let } a_1 \text{ be the de} - \text{acceleration of first car}$$

Using equation of motion

$$v = u + at$$

$$0 = 12 + a_1 t$$

$$a_1 = -\frac{12}{t} \quad \rightarrow 1)$$

Using equation of motion

$$v^2 - u^2 = 2a_1 s$$

$$0 - 12^2 = 2\ x - \frac{12}{t} x\ X$$

$$0 = 144 - \frac{24}{t} x\ X$$

$$x = 6t \quad \rightarrow 2)$$

Considering the motion of car B

$$\text{Let } a_2 \text{ be the de} - \text{acceleration of second car}$$

$$u = 9\ m/s$$

$$V = 0$$

$$S = 100 - x\ m$$

Using equation of motion

$$v = u + at$$

$$0 = 9 + a_2 t$$

$$a_2 = -\frac{9}{t} \qquad \rightarrow 3)$$

Using equation of motion

$$v^2 - u^2 = 2a_2 s$$

$$0 - 9^2 = 2 \text{ x} - \frac{9}{t} \text{x } 100 - x$$

$$0 = 81 - \frac{18}{t} \text{x } 100 - x$$

$$100 - x = 4.5t \qquad \rightarrow 4)$$

Adding equation 2 & 4

$$(100 - x) + x = 4.5t + 6t$$

$$100 = 10.5t$$

$$t = \frac{100}{10.5}$$

$$t = 9.52 \text{ sec}$$

putting this value in eq. 2

$$x = 6t \qquad \rightarrow 2)$$

$$x = 6 \text{ x } 9.52$$

$$x = 57.14 \text{ m}$$

So, distance travelled by car A before stopping will be 57.14m

Distance travelled by car B before stopping = 100-57.14 = 42.86m

From eq. 1

$$a_1 = -\frac{12}{t} \qquad \rightarrow 1)$$

$$a_1 = -\frac{12}{9.52}$$

$$a_1 = -1.26\,m/s^2$$

From eq. 3

$$a_2 = -\frac{9}{t} \qquad \rightarrow 3)$$

$$a_2 = -\frac{9}{9.52}$$

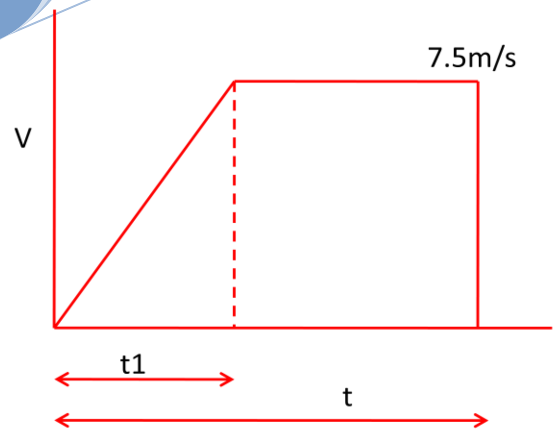
$$a_2 = -0.94\,m/s^2$$

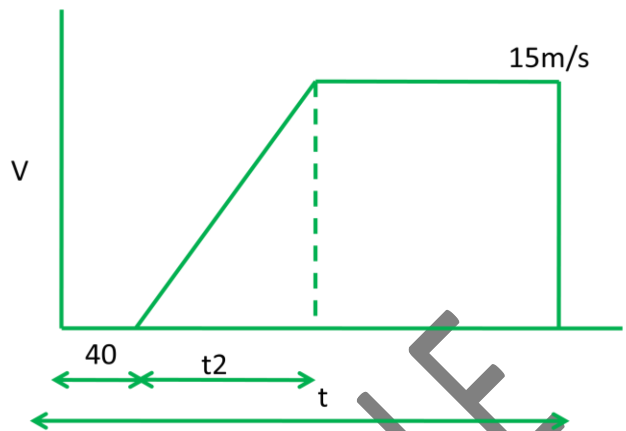
The time required for cars to stop is 9.52 sec,

De-acceleration of car A & B are $1.26m/s^2$& $0.94m/s^2$ respectively,

The distance travelled by car A & car B before stopping are 57.14m & 42.86m respectively.

4) Two trains A &B leaves the same station in parallel lines. Train A starts with a uniform acceleration of $0.15m/s^2$ & attain a speed of 27Kmph. When the stream is reduced to keep the speed constant trains B lease 40sec later with uniform acceleration of $0.3m/s^2$ to attain a maximum speed of 54Kmph. When & where will train B overtake train A. Use motion curve?





Motion curve for train A

Motion curve for train B

Constant speed of train A= 27Kmph

$$= 27\text{x}\frac{5}{18}$$

$$= 7.5 \text{ m/s}$$

Constant speed of train B= 54Kmph

$$= 54\text{x}\frac{5}{18}$$

$$= 15 \text{ m/s}$$

Let, Train B overtake train A after ‘t’ second at a distance ‘s’ from station.

Considering motion of train A.

Let, t1 be time taken to attain uniform velocity.

Using equation of motion.

$$v = u + at$$

$$v = u + at1$$

$$7.5 = o + 0.15t1$$

$$t1 = \frac{7.5}{0.15}$$

$$t1 = 50 \text{ sec}$$

Distance travelled by train A in ‘t’ sec

$$S = \text{Area of triangle} + \text{Area of rectrangle}$$

$$S = \frac{1}{2}x7.5xt1 + (t - t1)x7.5$$

$$S = \frac{1}{2}x7.5x50 + (t - 50)x7.5$$

$$S = 7.5t - 187.5 \rightarrow 1)$$

Considering motion of train B.

Let, t2 be time taken to attain uniform velocity.

Using equation of motion.

$$v = u + at$$

$$v = u + at2$$

$$15 = o + 0.3t2$$

$$t2 = \frac{15}{0.3}$$

$$t2 = 50 \text{ sec}$$

$$S = \text{Area of triangle} + \text{Area of rectrangle}$$

$$S = \frac{1}{2}x15xt2 + (t - 40 - t2)x15$$

$$S = \frac{1}{2}x15x50 + (t - 40 - 50)x15$$

$$S = 15t - 975 \rightarrow 2)$$

From eq. 1& 2

$$7.5t - 187.5 = 15t - 975$$

$$7.5t - 15t = 187.5 - 975$$

$$-7.5t = -787.5$$

$$t = \frac{-787.5}{-7.5}$$

$$t = 105 \text{ sec}$$

Put this value in eq.2

$$S = 15t - 975 \rightarrow 2)$$

$$S = 15x105 - 975$$

$$S = 600 \text{ m}$$

The train B overtakes train A in 105 sec at a distance 600m from station.

MOTION WITH VARYING ACCELERATION:-

A vehicle is normally not accelerate uniformly, initially it starts with zero acceleration, then the rate of acceleration is increased & when the desired speed is nearing the rate of acceleration is reduced by the time desired is pickup, acceleration is brought to zero.

Thus there are situations with varying acceleration. If the variation of acceleration or velocity or displacement with respect to time is known. Such a problem can be solve by using differential equation.

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$a = v.\frac{dv}{dt}$$

1) A motion of a particle moving in a straight line is given by the expression $s = t^3 - 3t^2 + 2t + 5$ where 's' is displacement in meter & 't' is time in second.

Determine: 1) Maximum & minimum velocity & corresponding displacement.

2) Velocity & acceleration after 4 second.

3) Time at which velocity is zero.

Solution:-

$$\text{Given eq. } s = t^3 - 3t^2 + 2t + 5 \rightarrow 1)$$

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2 \rightarrow 2)$$

$$a = \frac{d^2s}{dt^2} = 6t - 6 \rightarrow 3)$$

To find velocity & acceleration after 4 second

Using equation 2&3

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2 \rightarrow 2)$$

$$v = 3x4^2 - 6x4 + 2$$

$$v = 48 - 24 + 2$$

$$v = 26 \text{ m/s}$$

$$a = 6t - 6 \rightarrow 3)$$

$$a = 6x4 - 6$$

$$a = 24 - 6$$

$$a = 18\ \ m/s^2$$

The velocity is maximum or minimum when acceleration is zero (a=0)

From equation 3

$$a = \frac{d^2s}{dt^2} = \ 6t - 6 \rightarrow 3)$$

$$a = 6t - 6$$

$$0 = 6t - 6$$

$$6t = 6$$

$$t = 1\ \text{sec}$$

Minimum velocity of partial at $t = 1$

Using equation 2

$$v = 3t^2 - 6t + 2 \rightarrow 2)$$

$$v = 3t^2 - 6t + 2$$

$$v = 3x1^2 - 6x1 + 2$$

$$v = 3 - 6 + 2$$

$$v = -1m\ /s$$

Let, at time 't' velocity is zero

Using equation 2

$$v = 3t^2 - 6t + 2 \rightarrow 2)$$

$$0 = 3t^2 - 6t + 2$$

$$a = 3, b = -6, c = 2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4x3x2}}{2x3}$$

$$t = 1.57 \text{or } t = 0.42 \text{ sec}$$

The particle will attain the maximum velocity when t = 1.57sec & the maximum velocity at that time is given by

$$v = 3x1.57^2 - 6x1.57 + 2$$

$$v = 3.04 \text{ m/s}$$

2) The velocity of particle moving in varying acceleration is given by $V = t^3 - t^2 - 2t + 2$, the particle is found to be at a distance of 4m from station A after 2sec. Determine 1) Acceleration & displacement after 4sec, 2) Maximum or minimum acceleration.
Solution:-

$$\text{Given eq. } V = t^3 - t^2 - 2t + 2 \rightarrow 1)$$

$$a = \frac{dv}{dt} = 3t^2 - 2t - 2 \rightarrow 2)$$

Acceleration after 4 sec (t=4sec)

$$a = 3t^2 - 2t - 2 \rightarrow 2)$$

$$a = 3x4^2 - 2x4 - 2$$

$$a = 38 \text{ m/s}^2$$

Equation for displacement

$$s = \int t^3 - t^2 - 2t + 2 \, dt$$

$$s = \frac{t^4}{4} - \frac{t^3}{3} - \frac{2t^2}{2} + 2t + c1$$

When s=4m & t=2sec

$$4 = \frac{2^4}{4} - \frac{2^3}{3} - \frac{2x2^2}{2} + 2x2 + c1$$

$$c1 = 2.66$$

$$s = \frac{t^4}{4} - \frac{t^3}{3} - \frac{2t^2}{2} + 2t + 2.66$$

Displacement after 4 sec

$$s = \frac{4^4}{4} - \frac{4^3}{3} - \frac{2x4^2}{2} + 2x4 + 2.66$$

$$s = 37.32 \text{ m}$$

Acceleration will be minimum or maximum when $\frac{da}{dt} = 6t - 2$

When a=0

$$0 = 6t - 2$$

$$t = \frac{1}{3}\text{sec}$$

Acceleration will be minimum or maximum when t=1/3 sec

$$a = 3t^2 - 2t - 2 \rightarrow 2)$$

$$a = 3x\left[\frac{1}{3}\right]^2 - 2x\left[\frac{1}{3}\right] - 2$$

$$a = -2.33 \text{ m/s}^2$$

3) A body moves along a straight line & its acceleration which varying with the time is given by $a = 2 - 3t$, 5 seconds after start of observation, its velocity is found to be 20m/s, 10second after start, the body is at 85m from origin, determine i) Acceleration, velocity& distance from origin, ii) Time in which velocity become zero, iii) Describe the motion diagrammatically.

Solution:-

$$\text{Given equation } a = 2 - 3t$$

$$a = \frac{dv}{dt} = 2 - 3t$$

$$v = \int 2 - 3t \, dt$$

$$v = 2t - \frac{3t^2}{2} + c1 \rightarrow 1)$$

When $t = 5$ sec & $v = 20m/s$

$$20 = 2x5 - \frac{3x5^2}{2} + c1$$

$$c1 = 47.5$$

$$v = 2t - \frac{3t^2}{2} + 47.5 \rightarrow 2)$$

$$v = \frac{ds}{dt} = 2t - \frac{3t^2}{2} + 47.5$$

$$s = \int 2t - \frac{3t^2}{2} + 47.5$$

$$s = \frac{2t^2}{2} - \frac{3}{2}x\frac{t^3}{3} + 47.5 + c2$$

$$s = t^2 - \frac{t^3}{2} + 47.5t + c2 \rightarrow 3)$$

When $t = 10$ sec & S=85 m

$$85 = 10^2 - \frac{10^3}{2} + 47.5x10 + c2$$

$$c2 = 10$$

$$s = t^2 - \frac{t^3}{2} + 47.5t + 10 \rightarrow 4)$$

Acceleration, When $t = 0$

$$a = 2 - 3t$$

$$a = 2 - 3x0$$

$$a = 2\ m/s^2$$

Velocity, When $t = 0$

$$v = 2t - \frac{3t^2}{2} + 47.5$$

$$v = 2x0 - \frac{3x0^2}{2} + 47.5$$

$$v = 47.5 \text{ m/s}$$

The time at which velocity become zero,

$$v = 2t - \frac{3t^2}{2} + 47.5$$

$$0 = 2t - \frac{3t^2}{2} + 47.5$$

$$a = 2, b = 3/2 \ \& \ c = 47.5$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(3/2) \pm \sqrt{(3/2)^2 - 4x2x47.5}}{2x2}$$

$$t = 6.33 \text{ sec}$$

Displacement, When $t = 6.33$ sec

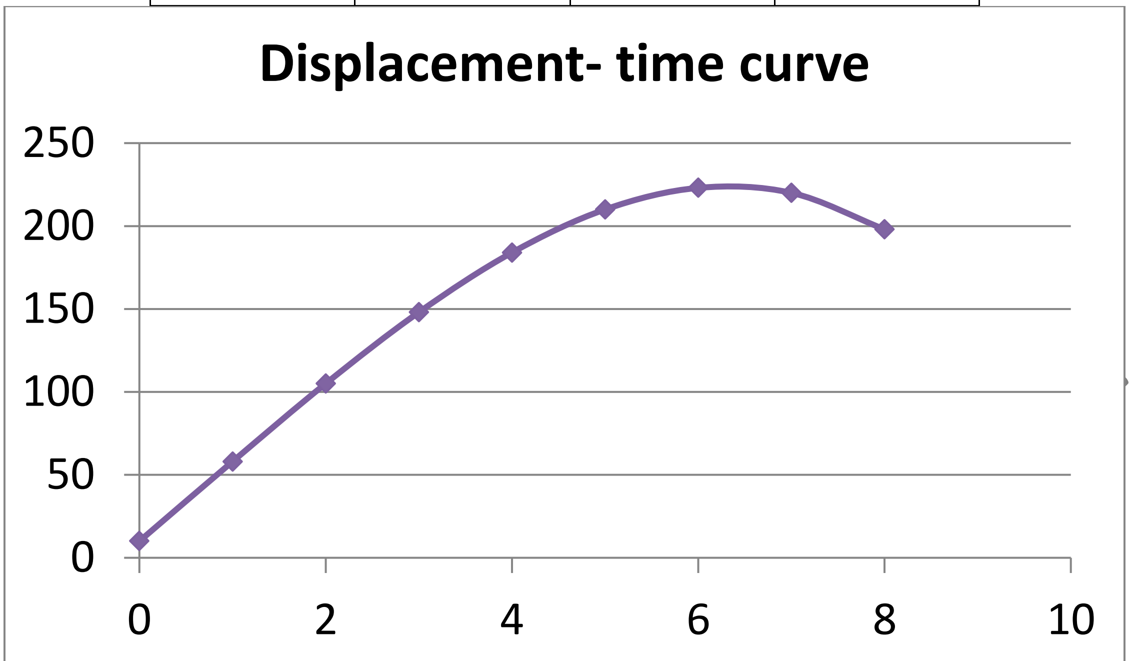
$$s = t^2 - \frac{t^3}{2} + 47.5t + 10$$

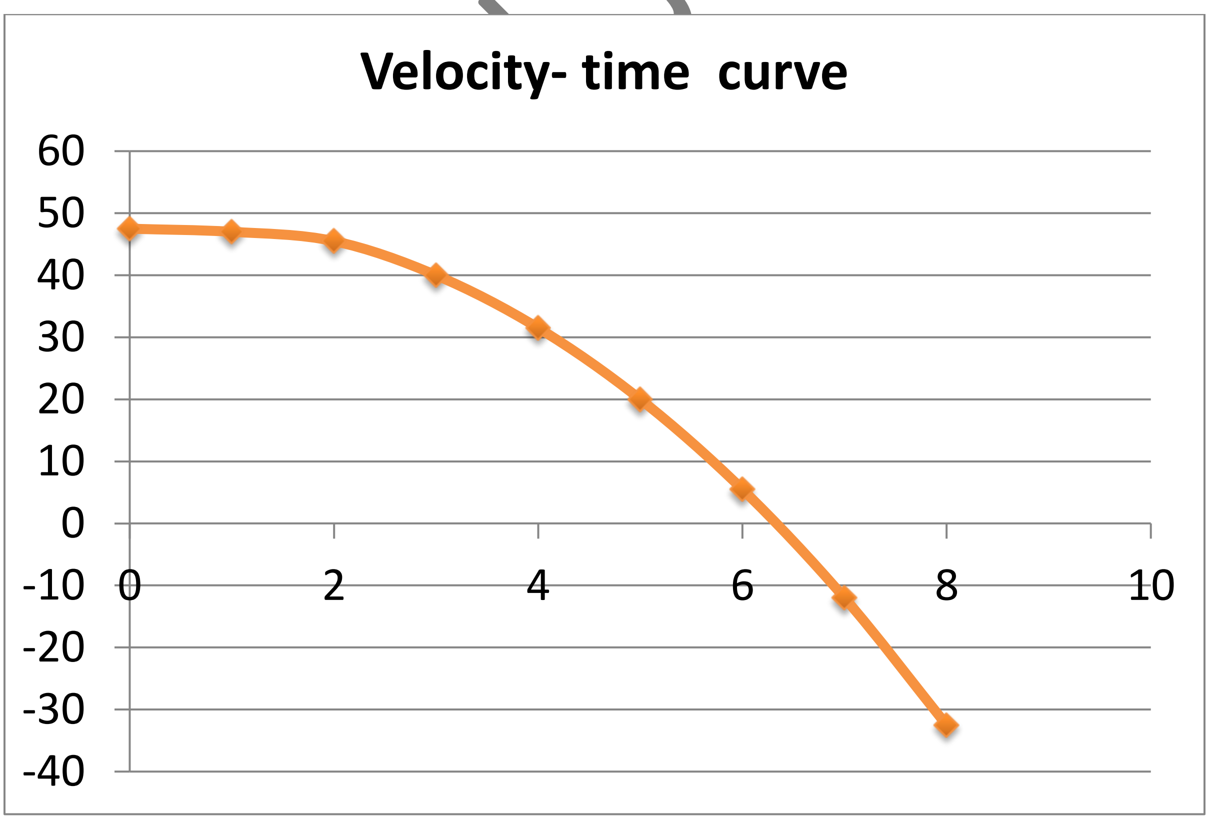
$$s = 6.33^2 - \frac{6.33^3}{2} + 47.5x6.33 + 10$$

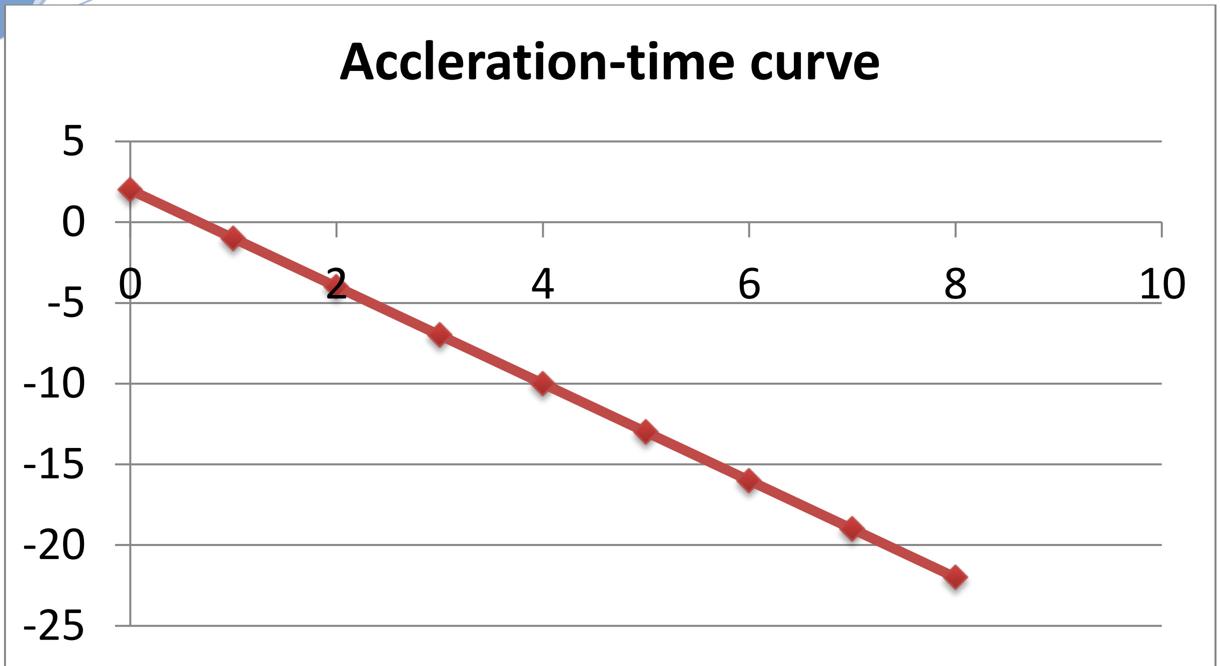
$$s = 223.92 \text{ m}$$

When t =	Displacement	Velocity	Acceleration
0	10	47.5	2
1	58	47	-1
2	105	45.5	-4
3	148	40	-7
4	184	31.5	-10
5	210	20	-13
6	223	5.5	-16
7	220	-12	-19

8	198	-32.5	-22


Displacement- time curve
250
200
150
100
50
0
0
2
4
6
8
10


Velocity- time curve
60
50
40
30
20
10
0
-10
-20
-30
-40
0
2
4
6
8
10



PROJECTILES

In the previous part, we consider the motion of particle along a straight line but the particle actually move along a curved path, if it is freely projected in the air in the direction other than vertical.

These freely projecting particles which are having combined effect of a vertical and horizontal motion are called as projectile.

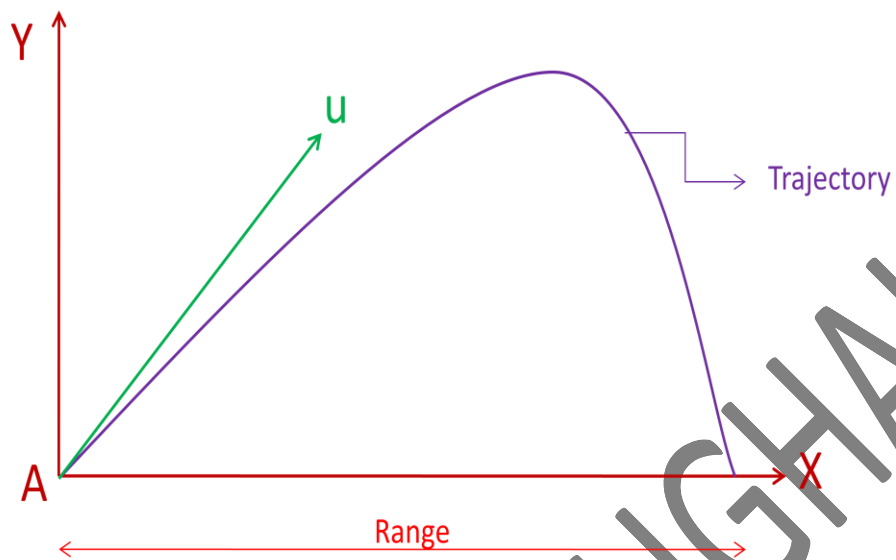
The motion of projectile has a vertical component & horizontal component. The vertical component of motion is subjected to gravitational acceleration/ retardation while horizontal component remains constant, if are resistance is neglected.

The motion of projectile can be analyze independently in a vertical & horizontal direction & the total is the combine effect of these two.

DEFINITIONS:-

1) VELOCITY OF PROJECTILE:- (u m/s)

The velocity with which the particle is projected is called as velocity of particle.



2) ANGLE OF PROJECTION:- (α)

The angle between direction of projection & horizontal direction is called as angle of projection.

3) TRAJECTORY:-

The path tracked by projectile is called as its trajectory.

HORIZONTAL RANGE :- (R)