

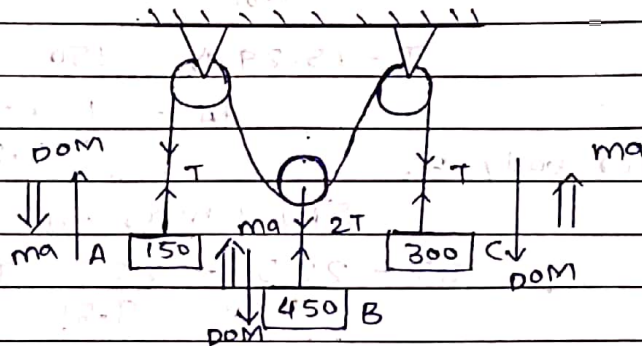
W16

Assignment 4

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1. Determine accn of each block shown in fig.



→ To find Direction of motion:

For, body A is in rest.

$$\sum F_y = 0$$

$$T - 150 = 0$$

$$T = 150 \text{ N}$$

For body B,

$$\sum F_y = 0$$

$$T - 2T - 450 = 0$$

$$T = 225 \text{ N}$$

For body C,

$$\sum F_y = 0$$

$$T - 300 = 0$$

$$T = 300 \text{ N}$$

+ Body A will move in upward direction &

body B & C will move in downward direction

$$T - 150 + T - 2T - 450 = 0$$

To find kinematic relation:

$$v = \frac{dx}{dt} = T$$

$$T_{SA} = 2T_{SB} = T_{SC} = 0$$

$$S_A = 2S_B + S_C$$

$$2S_B = S_A - S_C$$

$$0 = a_A = 2a_B + a_C$$

For body A : $\Sigma F_y - m a_y = 0$

$$T - 150 - \frac{150}{9.81} a_A = 0$$

$$T - 15.29 a_A = 150 \quad (1)$$

$$a_A = \frac{150 - T}{15.29}$$

For body B,

$$\Sigma F_y + m a_y = 0$$

$$2T - 225 + \frac{225}{9.81} a_B = 0$$

$$2T + 22.93 a_B = 225$$

$$a_B = \frac{225 - 2T}{22.93}$$

$$a_B = \frac{225 - 2T}{22.93}$$

For body C,

$$\Sigma F_y + m a_y = 0$$

$$T - 300 + \frac{300}{9.81} a_C = 0$$

$$T - 30.58 a_C = 300$$

$$a_C = \frac{300 - T}{30.58}$$

$$a_A = 2a_B + a_C$$

$$\frac{150 - T}{15.29} = 2 \left(\frac{225 - 2T}{22.93} \right) + \frac{300 - T}{30.58}$$

$$11008 = T$$

$$2(150 - T) = 2(225 - 2T) + 300 - T$$

$$300 - T = 598.5 - 5.32T + 300 - T$$

$$5.32T = 598.5$$

$$T = 112.5 \text{ N}$$

$$(1) \Rightarrow 112.5 - 15.29 a_A = 150$$

$$-137.5 = -15.29 a_A$$

$$a_A = 2.45 \text{ m/s}^2$$

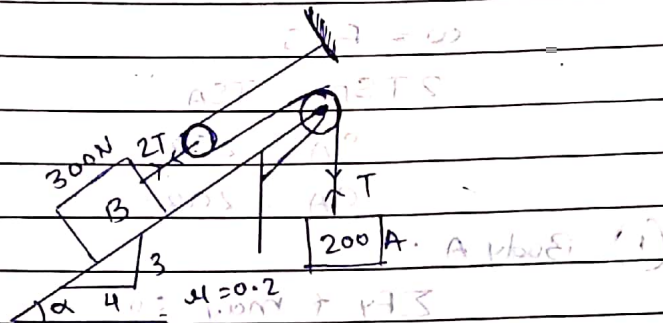
$$(2) \Rightarrow 2(112.5) + 22.93 a_B = 225$$

$$a_B = 0$$

$$(3) \Rightarrow 112.5 - 30.58 a_c = 300$$

$$T = (112.5 - 300) / a_c = 6.13 \text{ m/s}^2$$

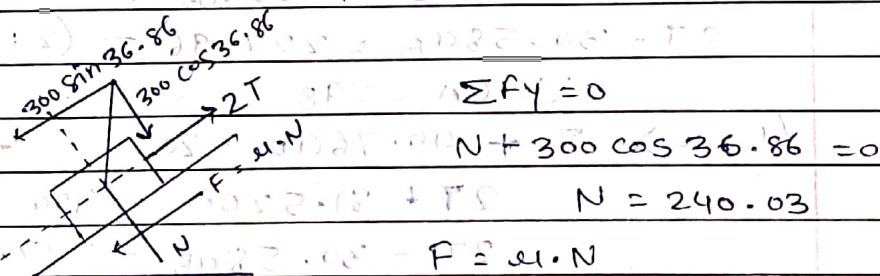
2. For the arrangement shown in fig. compute the accn of body B & tension in cord supporting body A. coefficient of friction is 0.2.



$$\tan \alpha = \frac{3}{4}$$

$$\alpha = 36.86^\circ$$

Case I : Body A is in rest position :



$$\Sigma F_y = 0$$

$$N + 300 \cos 36.86 = 0$$

$$N = 240.03$$

$$F = \mu \cdot N$$

$$F = 0.2 \times 240.03$$

$$F = 48.006 \text{ N}$$

Body B moves up the plane then the forces responsible for that

$$2T = 2 \times 200.5 = 400$$

$$2T = 400 \text{ N}$$

Now, force responsible for down the plane

$$= 300 \sin(36.86) + F$$

$$= 300 \sin(36.86) + 48.006$$

$$= 227.96 \text{ N}$$

$$\approx 228 \text{ N}$$

$$228 < 400$$

\therefore Body B moves up the plane & body A will move downward.

Kinematic relation:

$$w = F \times S$$

$$2TS_B = TS_A$$

$$S_A = 2S_B$$

$$a_A = 2a_B$$

(1) Body A:

$$\Sigma F_y + m a_y = 0$$

$$T - 200 + \frac{200}{9.81} a_A = 0$$

$$T + 20.38 a_A = 200 \quad \text{--- (1)}$$

(2) Body B:

$$\Sigma F_x - m a_x = 0$$

$$-300 \sin(36.86) + 2T - 48.006$$

$$- 30.58 a_B = 0$$

$$2T - 30.58 a_B = 227.96 \quad \text{--- (2)}$$

$$\therefore a_A = 2a_B$$

$$(1) \Rightarrow T + 40.76 a_B = 200 \quad \text{--- x(2)}$$

$$2T + 81.52 a_B = 400$$

$$2T - 30.58 a_B = 227.96$$

$$112.1 a_B = 172.04$$

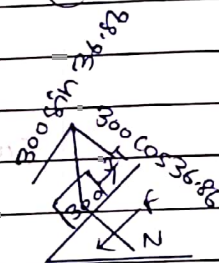
$$a_B = 1.53 \text{ m/s}^2$$

$$(2) \Rightarrow 2T - 30.58 \times 1.53 = 227.96$$

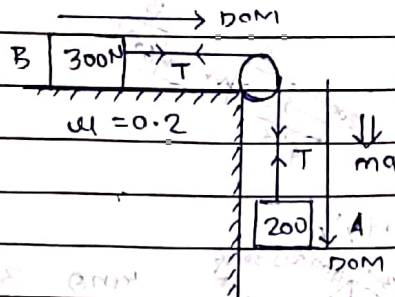
$$T = 137.37 \text{ N}$$

$$a_A = 2 \times 1.53$$

$$a_A = 3.06 \text{ m/s}^2$$



3. Determine the accelⁿ of the bodies shown in fig if the coefficient of friction is 0.2 at all contact surface.



for body A

$$\sum F_y = 0$$

$$T - 200 - 200a = 0$$

$$T = 200 + 200a$$

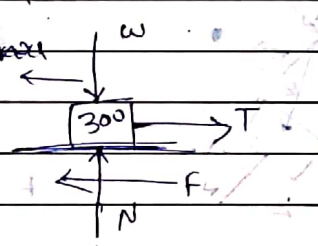
$$\sum F_y = 0$$

$$T - 200 - 200a = 0$$

$$T - 200 - 200a = 0$$

$$T - 20.38a = 200 \quad \text{--- (1)}$$

for Body B



$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$N - 300 = 0$$

$$T - F_f = 0$$

$$N = 300$$

$$T = 0.2 \times 300$$

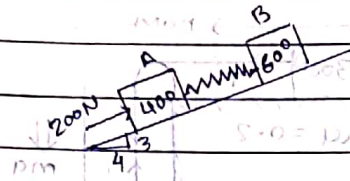
$$T = 60 \text{ N}$$

$$T - 300a - 60 = 0$$

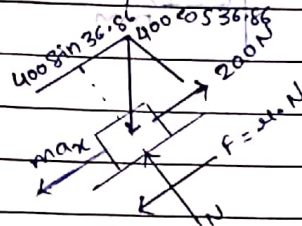
$$T - 30.58a = 60 \quad \text{--- (2)}$$

$$T = 479.72 \text{ N}, a = 13.72 \text{ m/s}^2$$

- 1) Two bodies in fig. are separated by a spring. Their motion down the incline is resisted by a force $P = 200 \text{ N}$. $\mu = 0.3$ under A & 0.1 under B. Determine the force under spring.



for body A



$$\tan \theta = 3/4, \theta = 36.86^\circ$$

$$\sum F_y = 0$$

$$N - 400 \cos 36.86 = 0$$

$$N = 320.04$$

$$f = \mu N = 0.3 \times 320.04$$

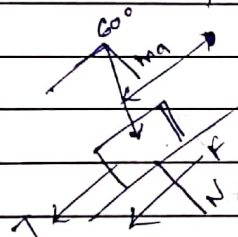
$$f = 96.01 \text{ N}$$

$$\sum F_x = 0$$

$$T + 200 - 96.01 - 400 \sin 36.86 - \frac{400 a_A}{9.81} = 0$$

$$T - 40.77 a_A = 135.95 \quad (1)$$

for body B



$$\sum F_y = 0$$

$$N - 600 \cos 36.86 = 0$$

$$N = 480.06$$

$$f = 0.1 \times 480.06$$

$$f = 48.00 \text{ N}$$

$$\sum F_x = 0$$

$$T - 48 - 600 \sin 36.86 - \frac{600 a_B}{9.81} = 0$$

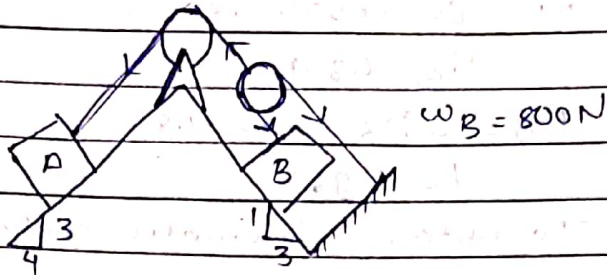
$$T - 61.16 a_B = 407.91$$

$$T - 61.16 a_B = -407.91$$

$$T = -81.58 \text{ N}$$

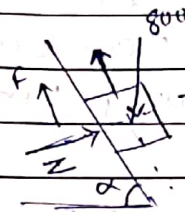
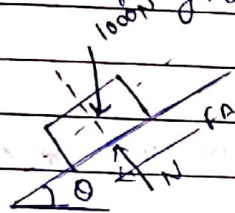
$$a_A = 5.33 \text{ m/s}^2$$

- ① Determine the velocity of block B after block A has moved 6m from rest. Use D'Alembert principle



Soln :

FBD of given assembly is



$$\theta = \tan^{-1}(3/4) = 36.86^\circ$$

$$\alpha = \tan^{-1}(4/3) = 53.13^\circ$$

Consider block B at rest

$$\sum F_x = 0$$

$$T - 800 \sin 53.13^\circ = 0$$

$$T = 640 \text{ N}$$

With the value of T resultant force acting on block A.

$$2T - 1000 \sin 36.86^\circ = 0$$

$$2(640) - 1000 \sin 36.86^\circ = 0$$

$$T = 680.13 \text{ N}$$

Block A will move up the plane and block B will move down the plane. To determine friction force.

Form FBD of A.

$$\sum F_y = 0$$

$$N_A - 1000 \cos 36.86^\circ = 0$$

$$N_A = 800 \text{ N}$$

$$f_A = \mu N_A = 0.2 \times 800$$

$$f_A = 160 \text{ N}$$

Form FBD of B

$$\sum F_y = 0$$

$$N_B - 800 \cos 53.13^\circ = 0$$

$$N_B = 480 \text{ N}$$

$$F_B = \mu N_B = 0.2 \times 480$$

$$F_B = 96 \text{ N}$$

To obtain kinematic relation

$$\omega_{DA} + \omega_{DB} = 0$$

$$2TSA - TSB = 0$$

$$2SA = SB$$

$$2VA = VB$$

$$2a_A = a_B \quad \text{--- (1)}$$

To apply kinematic equation for block A

$$\sum F_x - ma = 0$$

$$2T - 1000 \sin 36.86 - 160 = 1000/9.81 a_A$$

$$2T - 101.93 a_A = 759.86$$

$$\text{from eqn (1)} \quad a_A = a_B/2$$

$$2T - 50.96 a_B = 759.86 \quad \text{--- (2)}$$

For block B

$$\sum F_x = ma_B$$

$$800 \sin 53.13^\circ - 96 - T = 800/9.81 a_B$$

$$T + 81.54 a_B = 544 \quad \text{--- (3)}$$

Solving eqn (2) & (3), we get

$$T = 419 \text{ N}$$

$$a_B = 1.53 \text{ m/s}^2$$

Determining velocity of block B after block A has moved 6m

$$SA = 6 \text{ cm}$$

$$SB = 2 \times 6 = 12 \text{ cm}$$

&

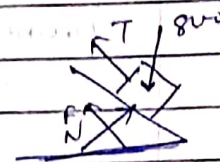
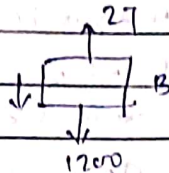
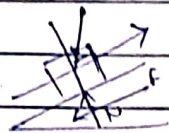
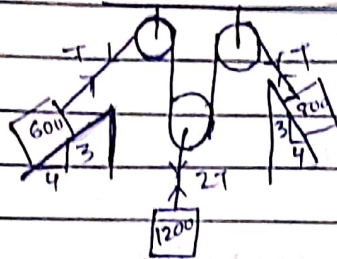
$$VB = 0 \text{ (rest)} \quad a_B = 1.53 \text{ m/s}^2$$

Using, $V_B^2 = u_B^2 + 2a_B \cdot S_B$

$= 0 + 2 \times 1.53 \times 12$

$V_B = 6.05 \text{ m/s}$

(2) Determine the accelⁿ of block A & B from given fig.



$\theta = \alpha = \tan^{-1} 3/4$

$\alpha = 0.25$

$\theta = \alpha = 36.86^\circ$

$\sum F_x = 0$

$2T = 1200$

$T = 600 \text{ N}$

with this value of T resultant force acting on block A.

$R = \sum F_x = 0$

$T - 600 \sin 36.86^\circ = 0$

$600 - 600 \sin 36.86^\circ = 0$

240.08 N

\therefore Block A will move upwards.

from FBD of block A

$\sum F_y = 0$

$N_A - 600 \cos 36.86^\circ = 0$ $N_A = 480.06 \text{ N}$

$F_A = \mu \cdot N_A = 0.25 \times 480.06$

$F_A = 120.015 \text{ N}$

As resultant force is greater than frictional force motion of block will take resultant.

$$R = \sum F_x = 0$$

$$T - 800 \sin 36.86 = 0$$

$$600 - 800 \sin 36.86 = 0 \\ = 120.11 \text{ N}$$

Hence block C will move upward.

From FBD of C

$$N_c - 800 \cos 36.86 = 0$$

$$N_c = 640.08$$

$$F_c = \mu N_c = 160.02$$

To obtain kinematic relation,

$$\omega_A + \omega_B = 0$$

$$T_{CA} - (2T_{AB}) = 0$$

$$a_A = 2a_B$$

By condition of equilibrium

$$\sum F_x - \omega/g \cdot a_A$$

$$T - 600 \sin 36.86 - 120.15 = 600 a_A \\ 9.81$$

$$T - 61.16 a_A = 479.93$$

$$T - 122.32 a_A = 479.93 \quad \text{--- (2)}$$

Now,

$$\sum F_x - \omega/g \cdot a_B = 0$$

$$-2T + 1200 = 1200/9.81 \times a_B$$

$$2T + 122.32 = 1200 a_B$$

$$2T - 1200 a_B = -122.32$$

$$T - 600 a_B = -61.16 \quad \text{--- (3)}$$

$$\text{eqn (2)} \times 2$$

$$\text{(2)} \Rightarrow T - 244.64 a_B = 479.93 \quad \text{--- (4)}$$

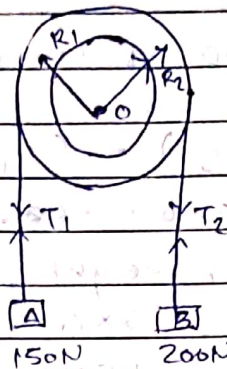
Solving (3) & (4)

$$T = 538.84 \text{ N}$$

$$a_B = 0.9996 \text{ m/s}^2 \approx 1 \text{ m/s}^2$$

$$a_A = 1.999 \text{ m/s}^2 \approx 2 \text{ m/s}^2$$

- ③ The pulley assembly shown in fig, weight 150N & has centroidal radius of gyration of 2m . The block are attached to the assembly by cord wrapped around the pulleys. Determine the acc'n of each body & tension in each cord.



$$W = 150\text{N}$$

$$K = 2\text{m}$$

$$R_1 = 3\text{m}$$

$$R_2 = 1.5\text{m}$$

Soln -

$$W = 150\text{N}$$

$$R_1 = 3\text{m}$$

$$I = mK^2 = \frac{150}{9} \times 2^2 = 61.22 \text{ kgm}^2$$

$$I = 612.2 \text{ gm}^2$$

Block B moves downward with displacement S_B , while the block A moves upward with displacement S_A .

\therefore for kinematic relation is using,

$$S_A = r\theta \Rightarrow S_A = 3\theta$$

$$V_A = 3\omega \Rightarrow a_A = 3\alpha$$

$$S_B = r\theta \Rightarrow S_B = 1.5\theta$$

$$V_B = 1.5\omega \Rightarrow a_B = 1.5\alpha$$

From FBD of block A, we get,

$$\sum F_x = ma_A$$

$$T_1 - 100 - \frac{W}{g} a_A = 0$$

$$T_1 - 100 = \frac{100}{9.8} \times 3\alpha$$

$$T_1 = 30.61\alpha + 100 \quad \text{--- (1)}$$

from FBD, of block B, we get

$$\sum F_x = ma_B$$

$$-T_2 + 200 = w/g a_B = 0$$

$$-T_2 + 200 = 200/9.8 \times 1.5 \alpha$$

$$-T_2 + 200 = 200/9.81 \times 1.5 \alpha$$

$$T_2 = 200 - 30.61 \alpha \quad \text{--- (2)}$$

from FBD of compound pulley, we get

$$\sum M_O = I \alpha$$

$$(T_2 \times 1.5) - (T_1 \times 3) = I \alpha$$

$$1.5 \times (200 - 30.61 \alpha) - 3 \times (30.61 + 100) = 6$$

$\alpha = 0$, we get,

$$a_A = 3\alpha = 3 \times 0 = 0 \text{ m/s}^2$$

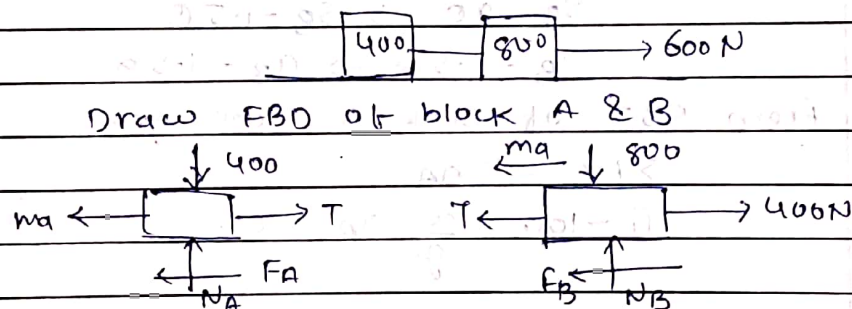
$$a_B = 1.5\alpha = 1.5 \times 0 = 0 \text{ m/s}^2$$

putting this value in eqⁿ (1) & (2) we get,

$$T_1 = 100 + 30.61 \alpha = 100 \text{ N}$$

$$T_2 = 200 - 30.61 \alpha = 200 \text{ N}$$

- (4) Two weights 800N & 400N are connected by a thread and move along a rough horizontal plane under the action of 400N applied to 800N weight as shown in fig. The coefficient of friction betⁿ sliding surface of weights and plane is 0.3 using D'Alembert principle accⁿ of block & tension in string.



Draw FBD of block A & B

From FBD of block A

$$\sum F_y = 0$$

$$-400 + N_A = 0$$

$$N_A = 400 \text{ N}$$

$$F_A = \mu N_A = 0.3 \times 400 = 120 \text{ N}$$

$$\Sigma F_x = 0 \quad \text{--- (By law of friction)}$$

$$T - F_A - m a_A = 0$$

$$T - F_A - 400/9.81 a_A = 0$$

$$T - F_A - 40.77 a_A = 0$$

$$T - 40.77 a_A = 120 \quad \text{--- (1)}$$

Now,

$$T = 400$$

From FBD of block B

$$\Sigma F_y = 0$$

$$-800 + N_B = 0$$

$$N_B = 800 \text{ N}$$

$$F_B = \mu N_B = 0.3 \times 800 = 240$$

Now,

$$\Sigma F_x = 0$$

$$-T - 800/9.81 a_B - F_B + 600 = 0$$

$$-T - 81.54 a_B - 240 + 600 = 0$$

$$-T - 81.54 a_B = -360 \quad \text{--- (2)}$$

$$T + 81.54 a = 360 \quad \text{--- (2)}$$

subtracting eqn (1) from (2)

$$(81.54 + 40.77) a = 360 - 240$$

$$122.31 a = 120$$

$$a = 0.98 \text{ m/s}^2$$