WINTER-2013

UNIT 1

Q.1 a) Solve: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{e^x}$ **(6)**

b) Solve: $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$ **(7)**

Q.2 a) Solve: $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+y=\frac{\sin(\log x)}{x}$ **(6)**

b) Solve by the method of variation of parameters

$(D^2 - 1)y = (1 + e^{-x})^2$ **(7)**

UNIT 2

Q.3 a) Evaluate: $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}dt$ **(4)**

b) Find f(t) if $\bar{f}(s)$ is given by $\frac{1}{2}\log\left(\frac{S^2+b^2}{S^2+a^2}\right)$ **(4)**

c) Use convolution theorem to evaluate

$L^{-1}\left\{\frac{1}{(S+1)(S+9)^2}\right\}$ **(6)**

Q.4 a) Evaluate: $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt$ **(4)**

b) Find: $L^{-1}\frac{1}{(S^2+a^2)^2}$ **(4)**

c) Solve the differential equation by using Laplace transform.

$\frac{d^2x}{dt^2}+x=6\cos 2t$ with x = 3, $\frac{dx}{dt}=1$, at t = 0. **(6)**

UNIT 3

Q.5 a) Solve the difference equations

i. $y_{n+2} - 2y_{n+1} + y_n = n^2 2^n$ **(4)**

ii. $y_{n+2} - 5y_{n+1} + 6y_n = 2n^2 + 3$ **(4)**

b) Find inverse Z-transform of $\dfrac{2Z^2 - 10Z + 3}{(Z-3)^2(Z-2)}$ **(5)**

Q.6 a) Solve difference equations

i. $y_{n+2} - 4y_{n+1} + 3y_n = 3^n$ **(4)**

ii. $u_{n+2} - 16u_n = \cos\dfrac{n}{2}$ **(4)**

b) Solve by Z-transform $y_{n+2} + 5y_{n+1} + 6y_n = 6^n + 3n$ **(5)**

UNIT 4

Q.7 a) Solve the following partial differentials equations.

i. $x^2p + y^2q = (x + y) z$, **ii.** $p^2 + q^2 = x + y$. **(8)**

b) Find the Fourier transform of

$$f(x) = 1 - x^2 \quad |x| < 1$$
$$= 0 \quad |x| > 1$$

Hence evaluate $\displaystyle\int_0^\infty \left(\frac{\sin x - \cos x}{x^3}\right)\cos\frac{x}{2}\,dx$ **(6)**

Q.8 a) Solve the following partial differential equations

i. $yzp + xzq + 2xzq + 2xy = 0$ **(4)**

ii. $p \tan x + q \tan y = \tan z$ **(4)**

b) Find the Fourier transform of

$$f(x) = 1, \quad |x| < 1$$
$$= 0 \quad |x| > 1$$

Hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$ **(6)**

UNIT 5

Q.9 a) Show that $u = y^3 - 3x^2y$ is a harmonic function. Find its harmonic conjugate and the corresponding analytic function. **(7)**

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. **(6)**

Q.10 a) Expand the function $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ in the region.

i. $|z| < 2$, **ii.** $2 < |z| < 3$

by Laurent's series. **(7)**

b) If $u = \lambda(1 + \cos\theta)$, find v so that $u + iv$ is analytic function. **(6)**

UNIT 6

Q.11 a) Find the directional derivative of

$\phi = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$. **(7)**

b) Prove that

i. $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi.\nabla\psi + \psi\nabla^2\phi$ **(3)**

ii. $\nabla.\left(r^3 \bar{r}\right) = 6r^3$ **(3)**

Q.12 a) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$ prove that

i. $\nabla.\left(r^n \bar{r}\right) = (n+3)r^n$ **(3)**

ii. $\nabla \times \left(r^n \bar{r}\right) = 0$ **(3)**

b) A vector field is given by $\bar{F} = (\sin y)i + x(1 + \cos y)j$

Evaluate the line integral over a circular path given

$x^2 + y^2 = a^2$, $z = 0$. **(7)**