

## 5. Combinational Circuits.

A Digital system consist of two types of circuits namely i) combinational logic circuit ii) Sequential logic circuit.

In Any logic circuit if output depends only on present input Such type of logic circuit is called as combinational logic circuit.

e.g code Converter

Mux (Multiplexer) | Demux

Encoder, Decoder etc.

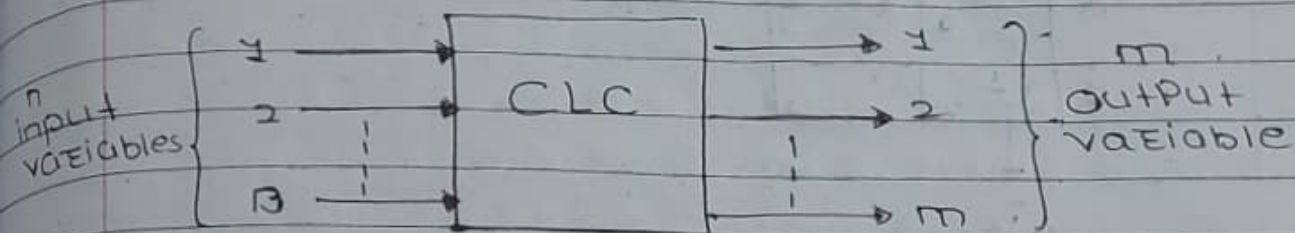
On the other hand if output of logic circuit depend not only on the present input but also on the past outputs. Such type of logic circuit is called Sequential logic circuit.

e.g

A combinational logic circuit consist of input variable, logic gates and output variable. The logic gates accepts input from \_\_\_\_\_ and produces output signals.

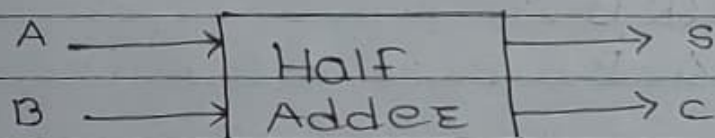
$$A \oplus B = \overline{A}B + A\overline{B}$$

$$A \odot B = A \odot B = \overline{A}B + A\overline{B}$$



### Half Adder

A logic circuit that performs the Arithmetic addition of two bits is called Half Adder. The Half Adder needs two inputs and two binary output i.e. Sum and carry. Fig shows the block diagram of Half adder.



I/P		O/P	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

for Sum

	B	0	1	
A				
0		0	(1)	$\bar{A}B$
1	(1)	2	3	$A\bar{B}$

$$\therefore S = A\bar{B} + \bar{A}B$$

$$S = A \oplus B$$

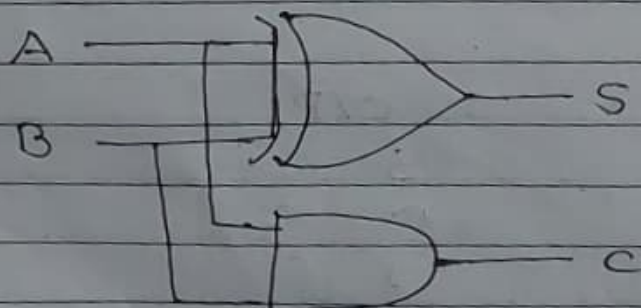
for Carry

	B	0	1	
A				
0		0	1	
1		2	(1)	$AB$

$$\therefore C = AB$$

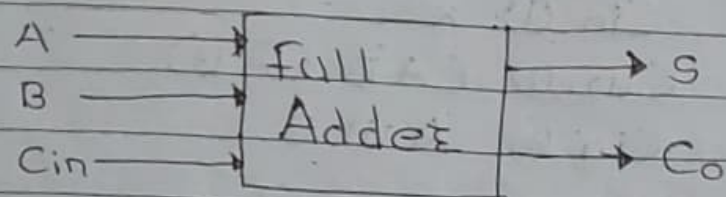
$$S = A \oplus B$$

$$C = A \cdot B$$



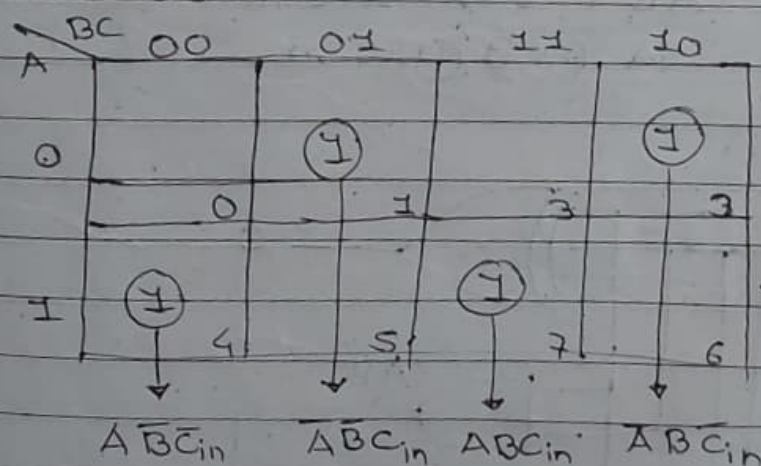


# Full Adder



I/P			O/P	
A	B	C	S	Co
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

for Sum



$$\begin{aligned}
 S &= \bar{A}\bar{B}\bar{C}_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in} \\
 &= \bar{C}_{in}(\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB) + C_{in}(\bar{A}\bar{B} + AB) \\
 &= \text{Assume } \bar{A}\bar{B} + \bar{A}B = X
 \end{aligned}$$

$$\begin{aligned}
 &= \bar{C}_{in} x + C_{in} \bar{x} \quad (A \oplus B = \bar{A}B + A\bar{B}) \\
 &= C_{in} \oplus x \\
 &= C_{in} \oplus (A\bar{B} + \bar{A}B) \\
 &= C_{in} \oplus A \oplus B
 \end{aligned}$$

$$\therefore S = A \oplus B \oplus C_{in}$$

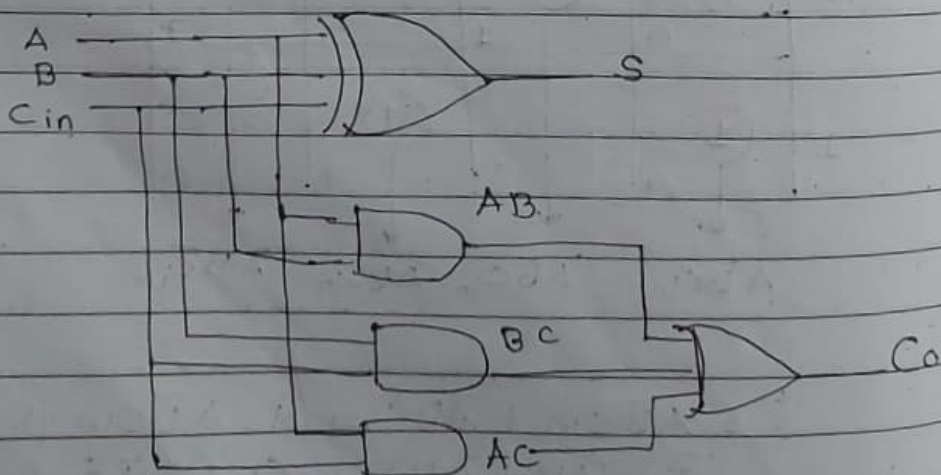
for carry :-

A \ B C <sub>in</sub>	00		01		11		10	
	0	1	0	1	0	1	0	1
0	0	1	0	1	1	1	0	0
1	1	0	1	0	0	0	1	1

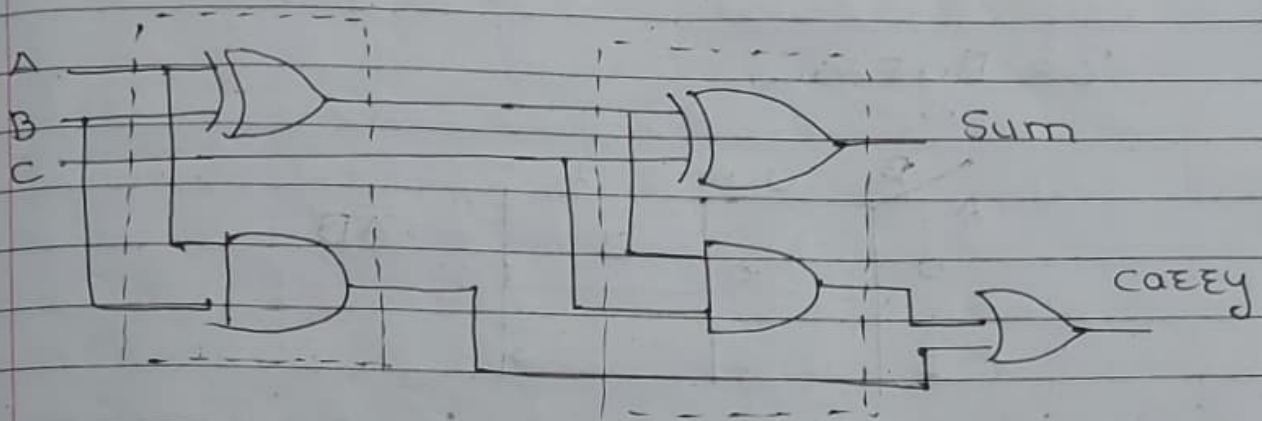
The truth table is represented as a 4x4 grid. The columns are labeled with B C<sub>in</sub> pairs: 00, 01, 11, 10. The rows are labeled with A: 0, 1. The cells contain the sum S. Circled cells indicate the carry C: (A=0, B C<sub>in</sub>=11) is 1; (A=1, B C<sub>in</sub>=01) is 1; (A=1, B C<sub>in</sub>=11) is 0; (A=1, B C<sub>in</sub>=10) is 1. Arrows point from these circled cells to the expressions AC, BC, and AB respectively.

$$C = AB + BC_{in} + AC_{in}$$

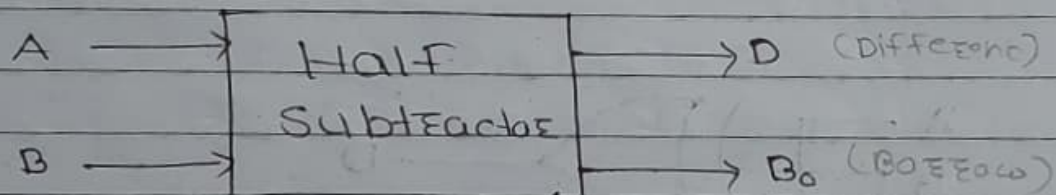
$$S = A \oplus B \oplus C_{in}$$



- Realisation of Full Adder using two half Adder.



### \* Half Subtractor



I/P		O/P	
A	B	D	B <sub>0</sub>
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

⇒ for Difference

A \ B	0	1
0	0	1
1	1	0

$\bar{A}B$  (for the 1 in the first row, second column)  
 $A\bar{B}$  (for the 1 in the second row, first column)

$$\therefore D = \overline{A}B + A\overline{B}$$

$$D = A \oplus B$$

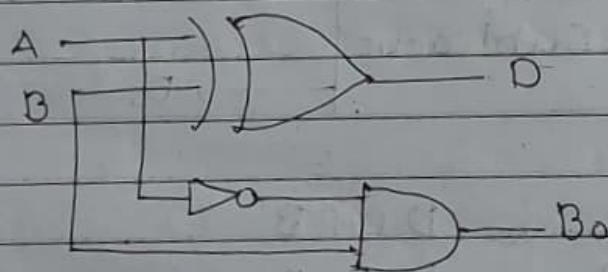
for Boolean

A \ B	0	1
0	0	1
1	1	0

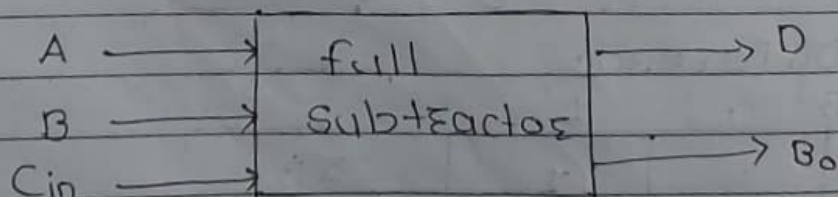
$$B_0 = \overline{A}B$$

$$D = A \oplus B$$

$$B_0 = \overline{A}B$$



\* Full Subtractor





I/P				O/P	
A	B	C	Cin	D	Bo
0	0	1	0	0	0
0	0	0	1	1	1
0	1	1	0	1	1
0	1	0	1	0	1
1	0	1	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0
1	1	0	1	1	1

→ for Difference

A	BCin			
	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$\downarrow$   $\downarrow$   $\downarrow$   $\downarrow$   
 $\bar{A}\bar{B}\bar{C}_{in}$   $\bar{A}\bar{B}C_{in}$   $A\bar{B}C_{in}$   $\bar{A}B\bar{C}_{in}$

$$\begin{aligned}
 S = D &= \bar{A}\bar{B}\bar{C}_{in} + \bar{A}\bar{B}C_{in} + A\bar{B}C_{in} + \bar{A}B\bar{C}_{in} \\
 &= \bar{C}_{in}(\bar{A}\bar{B} + \bar{A}B) + C_{in}(\bar{A}\bar{B} + AB) \\
 &\quad \text{Assume } \bar{A}\bar{B} + \bar{A}B = x \\
 &= \bar{C}_{in}x + C_{in}\bar{x} \\
 &= C_{in} \oplus x \\
 &= C_{in} \oplus (\bar{A}\bar{B} + \bar{A}B) \\
 &= C_{in} \oplus A \oplus B
 \end{aligned}$$



$$\therefore D = A \oplus B \oplus \text{Cin}$$

FOR BOEEOC

	BCin	00	01	11	10
A					
0		0	1	1	1
1		1	1	1	0

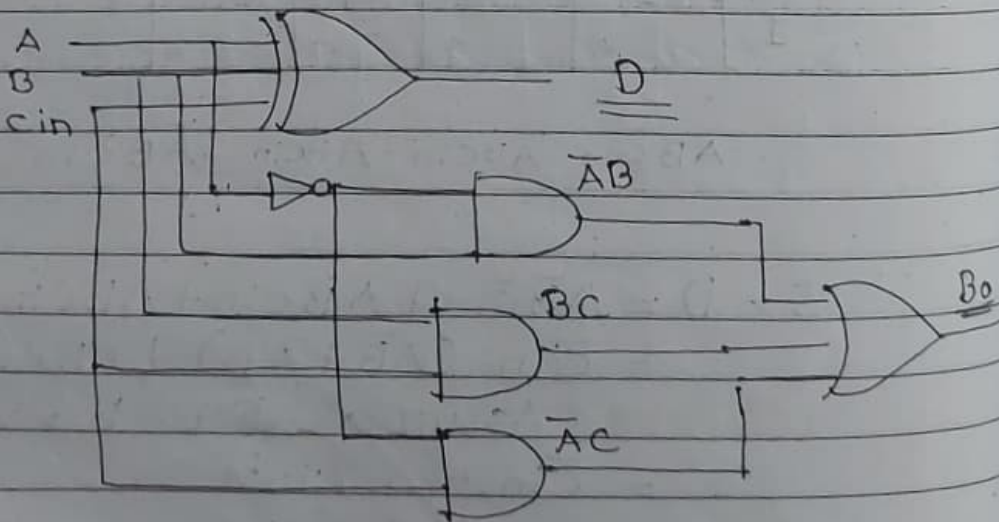
Arrows from the truth table indicate the following expressions:
 

- From the first row (A=0, BCin=01, 11, 10):  $\bar{A}C$
- From the second row (A=1, BCin=11):  $BC$
- From the third row (A=1, BCin=10):  $\bar{A}B$

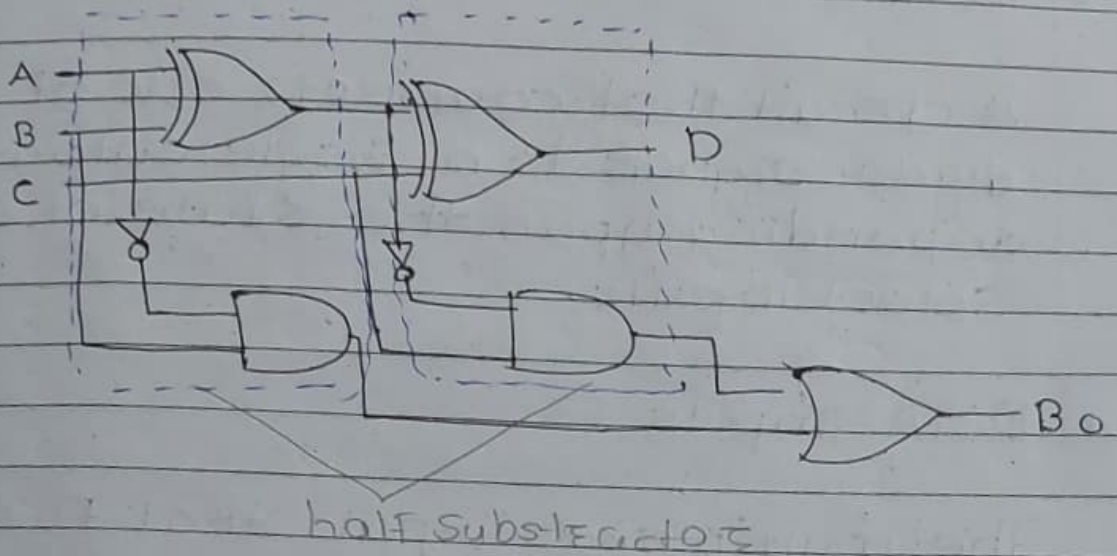
$$B_0 = \bar{A}C + BC + \bar{A}B$$

$$\therefore D = A \oplus B \oplus \text{Cin}$$

$$B_0 = \bar{A}C + BC + \bar{A}B$$



- Realisation of full subtractor using two half subtractor



imp

## \* Multiplexer (MUX)

### • Definition of Multiplexer :-

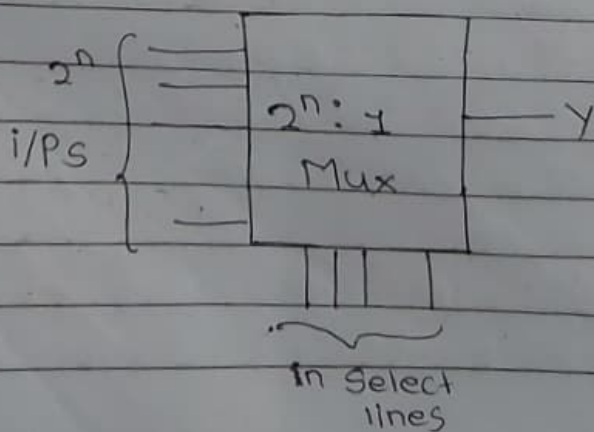
A circuit that connects one of several digital signals to a single output depending upon the state of several select inputs.

### • DATA Inputs :-

The multiplexer inputs that feed a digital signal to the output when selected. (maximum of inputs is  $2^n$ )

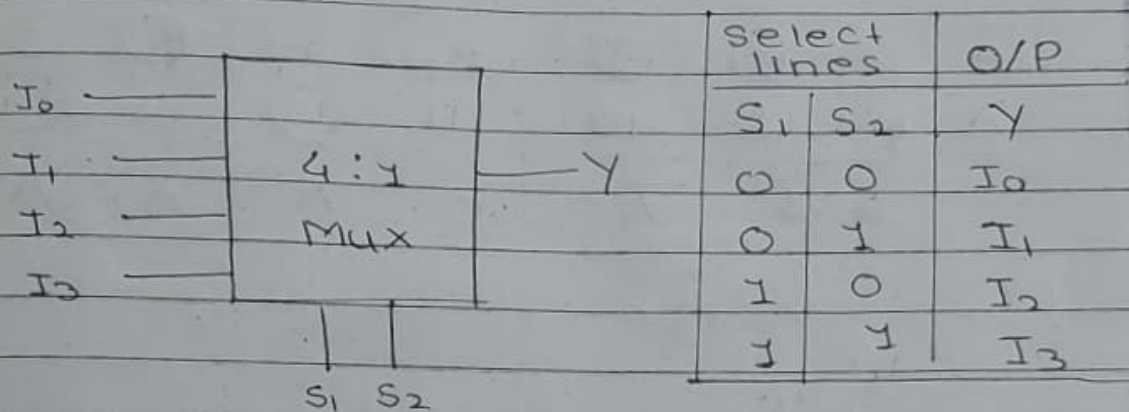
### • Select Inputs :- (Select lines)

The multiplexer inputs that selects the digital inputs. (maximum n select lines)





e.g. Put  $n=2$   
 $i/p = 2^n = 2^2 = 4$   
 $n$  Select lines  $= 2$

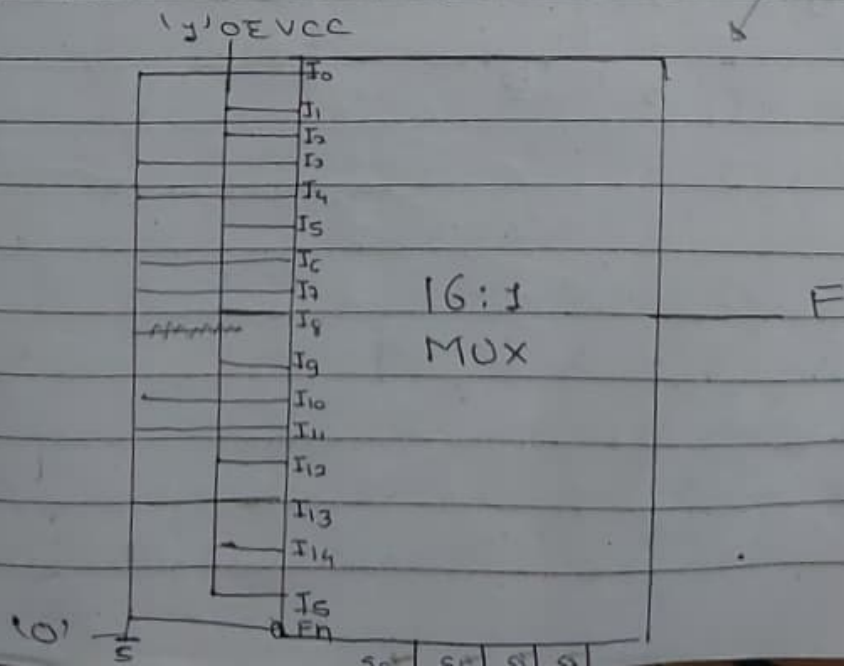


- Implement the following Boolean expression using i) 16:1 Mux  
 2) 8:1 Mux.

$$F = \sum m(1, 2, 5, 8, 9, 12, 14, 15)$$

Ans:-

i) for 16:1 Mux.

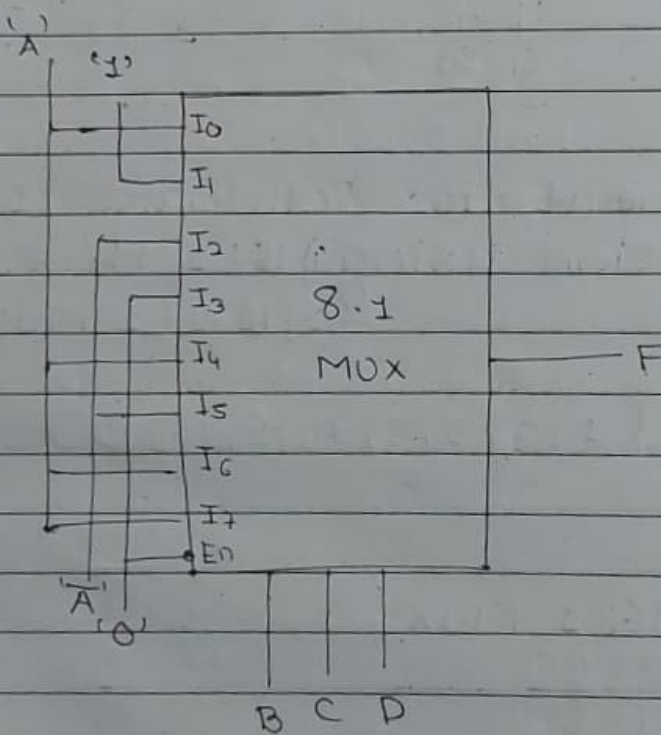


active low mux

ii) for 8:1 Mux

$I_0$   $I_1$   $I_2$   $I_3$   $I_4$   $I_5$   $I_6$   $I_7$

$\bar{A}$	0	(1)	(2)	3	4	(5)	6	7
A	(8)	(9)	10	11	(12)	13	(14)	(15)
	A	1	$\bar{A}$	0	A	$\bar{A}$	A	A

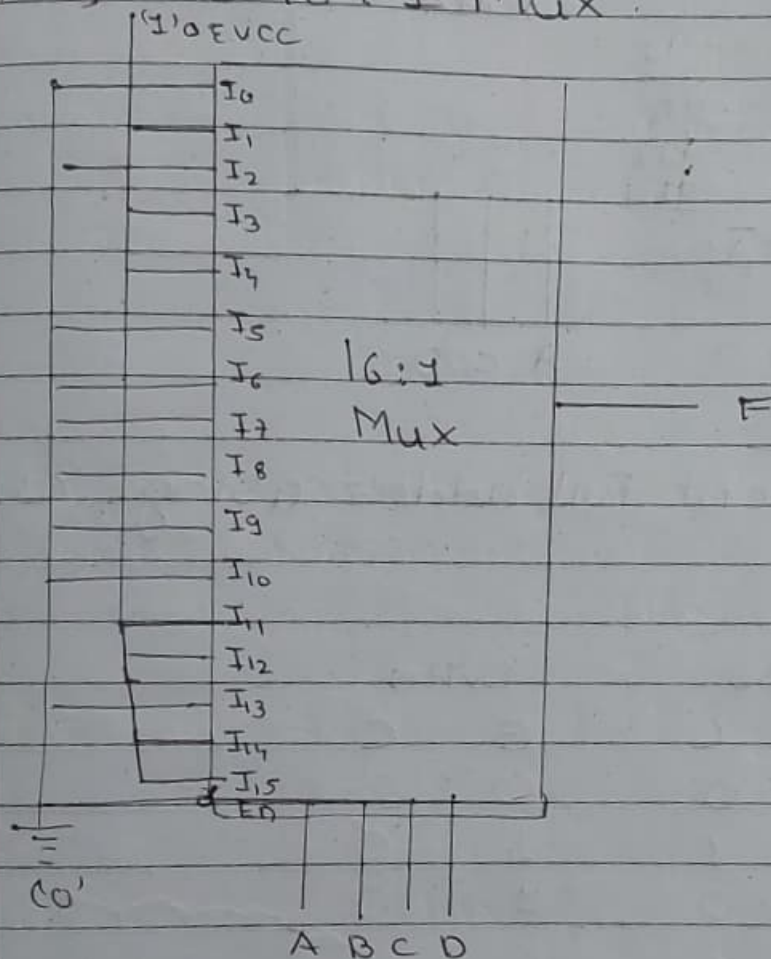


Q. 1) 16:1 Mux.

2) 8:1 Mux.

$$F(A, B, C, D) = \sum m(1, 3, 4, 11, 12, 13, 14, 15)$$

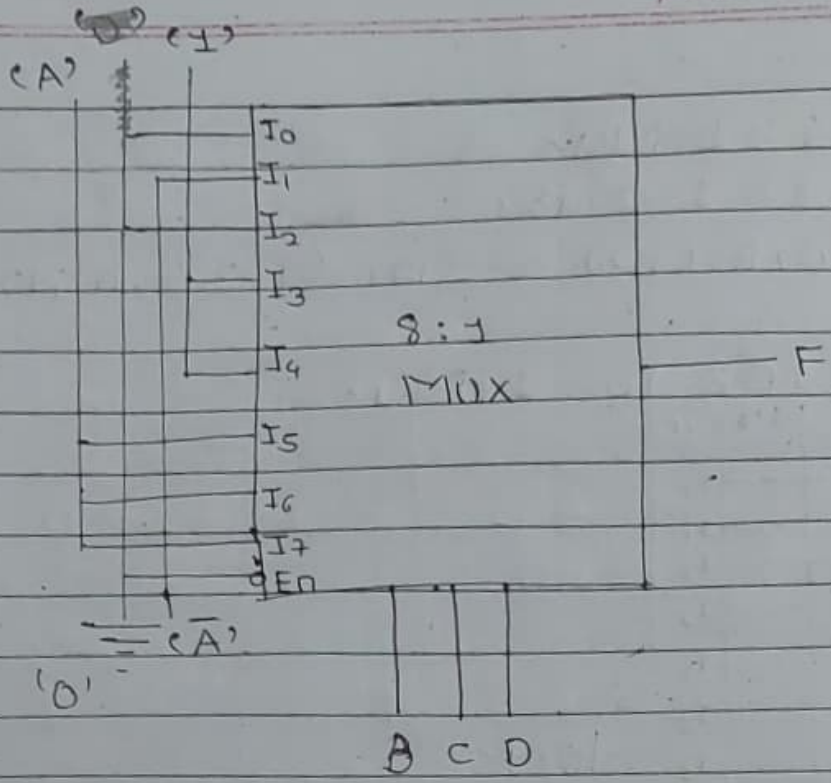
soln: i) for 16:1 Mux.



ii) for 8:1 Mux.

	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
$\bar{A}$	0	(1)	2	(3)	(4)	5	6	7
A	8	9	10	(11)	(12)	(13)	(14)	(15)
$\bar{O}$	$\bar{A}$	0	1	1	A	A	A	A



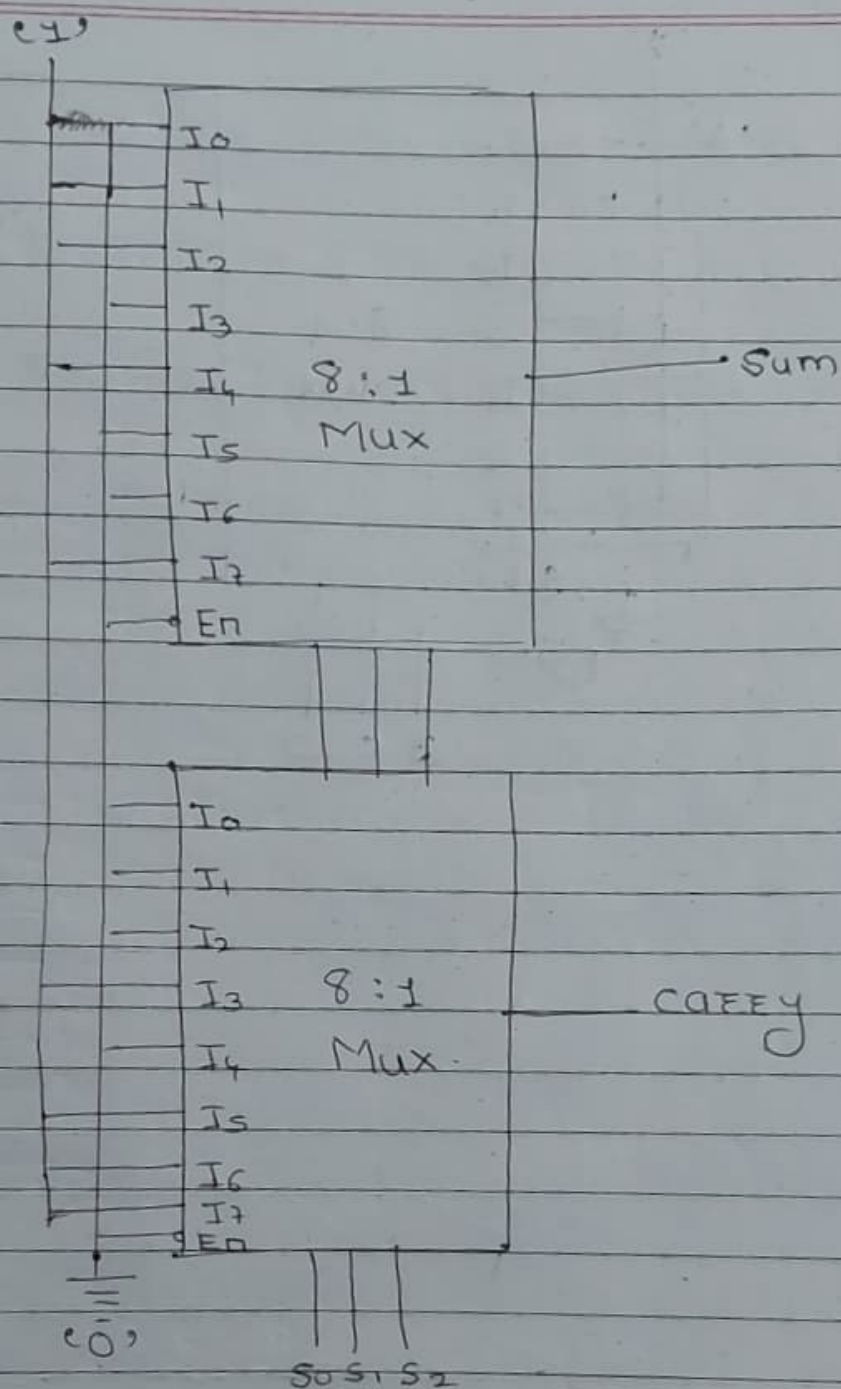


Q. Implement full adder using 8:1 Mux.

I/Ps			O/Ps	
A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \sum m(1, 2, 4, 7)$$

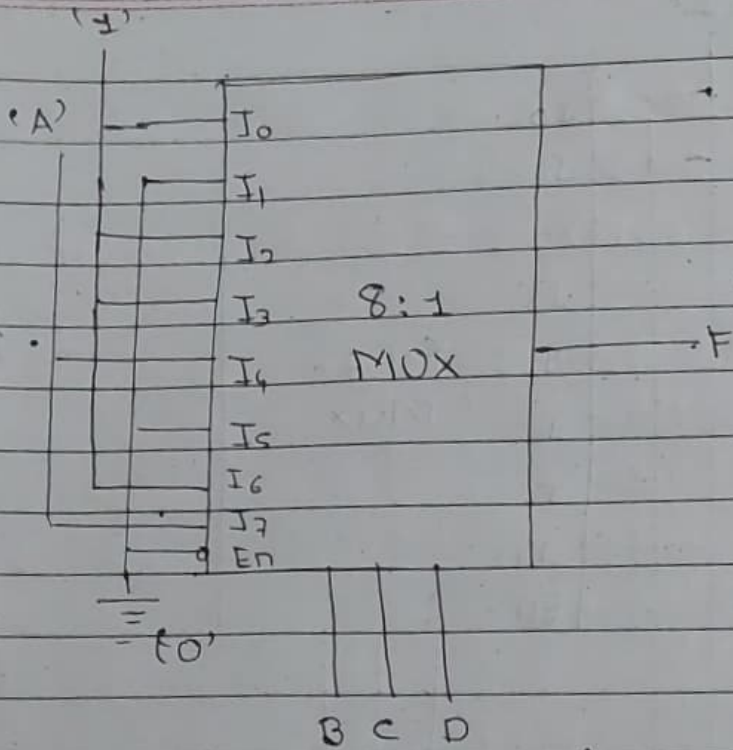
$$C = \sum m(3, 5, 6, 7)$$



Q.  $F(A, B, C, D) = \sum m(0, 2, 6, 10, 11, 12, 15) + d(3, 8, 14)$

i) 8:1 MUX.

	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
$\bar{A}$	(0)	1	(2)	(3)	4	5	(6)	7
A	(8)	9	(10)	(11)	(12)	13	(14)	(15)
	1	0	1	1	A	0	1	A





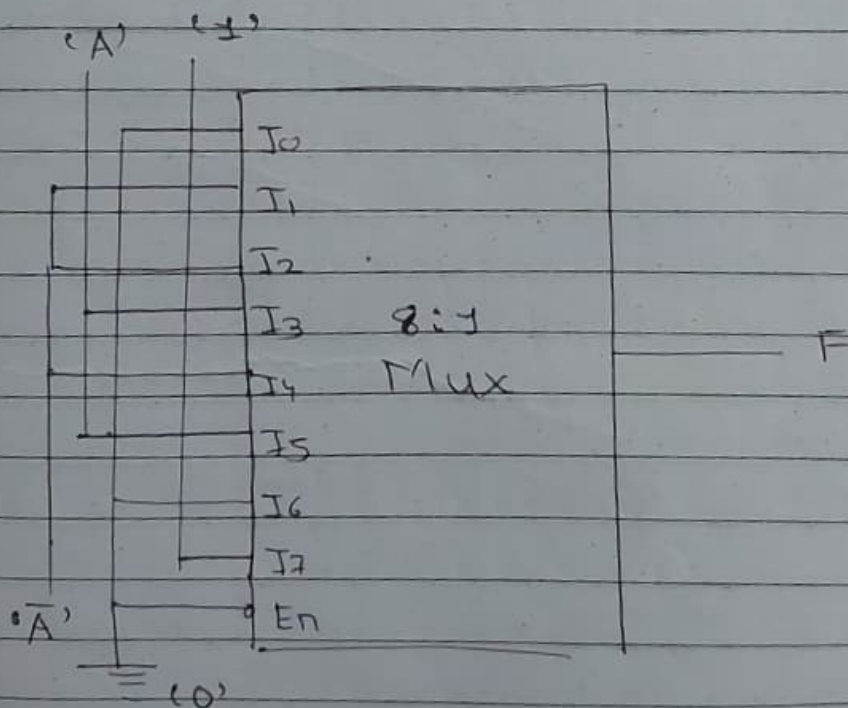
Q.  $F(A, B, C, D) = \pi M(0, 3, 5, 6, 8, 9, 10, 12, 14)$

i) 8:1 Mux.

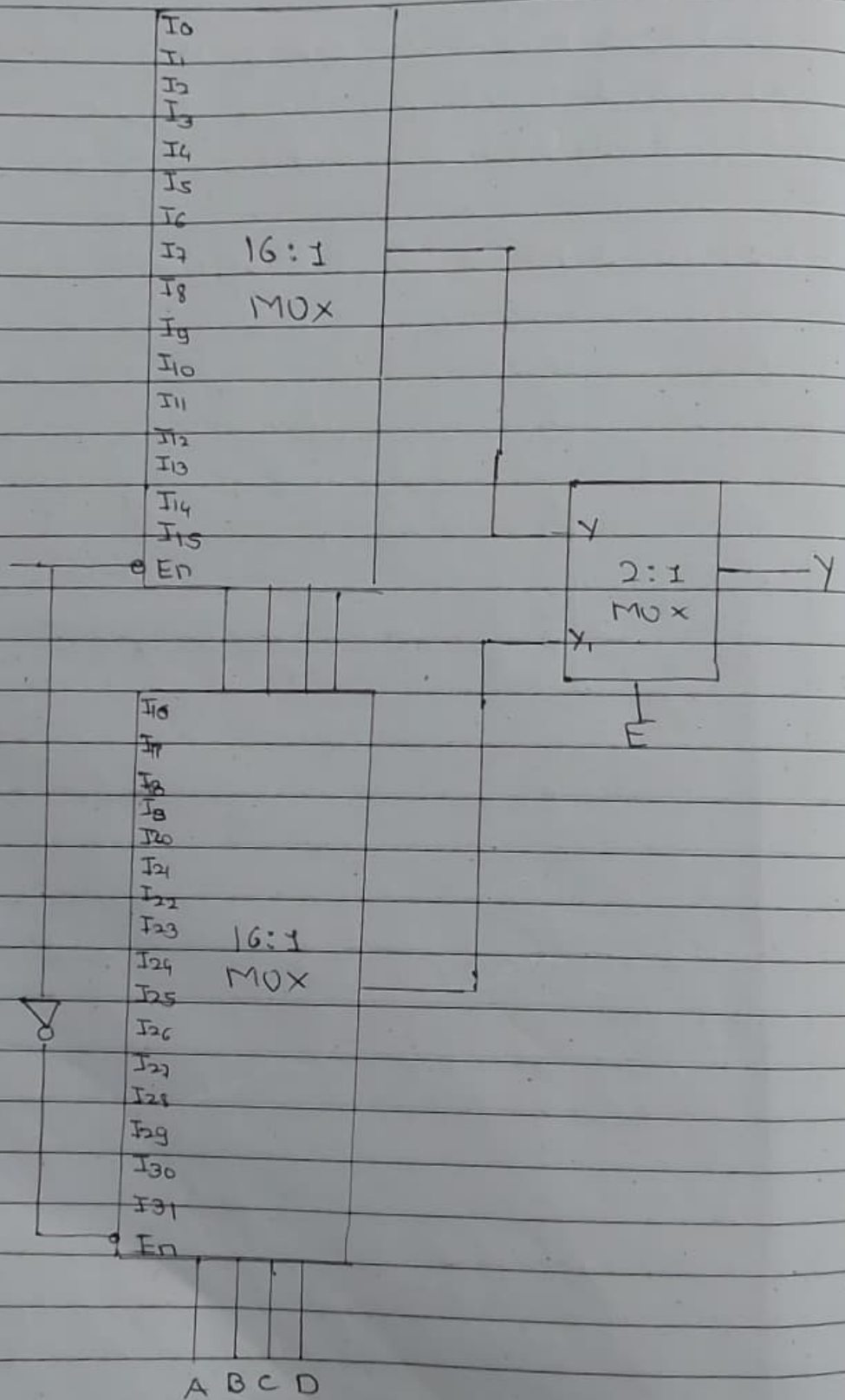
$\Rightarrow F(A, B, C, D) = \Sigma M(1, 2, 4, 7, 11, 13, 15)$

	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
A	0	1	2	3	4	5	6	7
$\bar{A}$	8	9	10	11	12	13	14	15

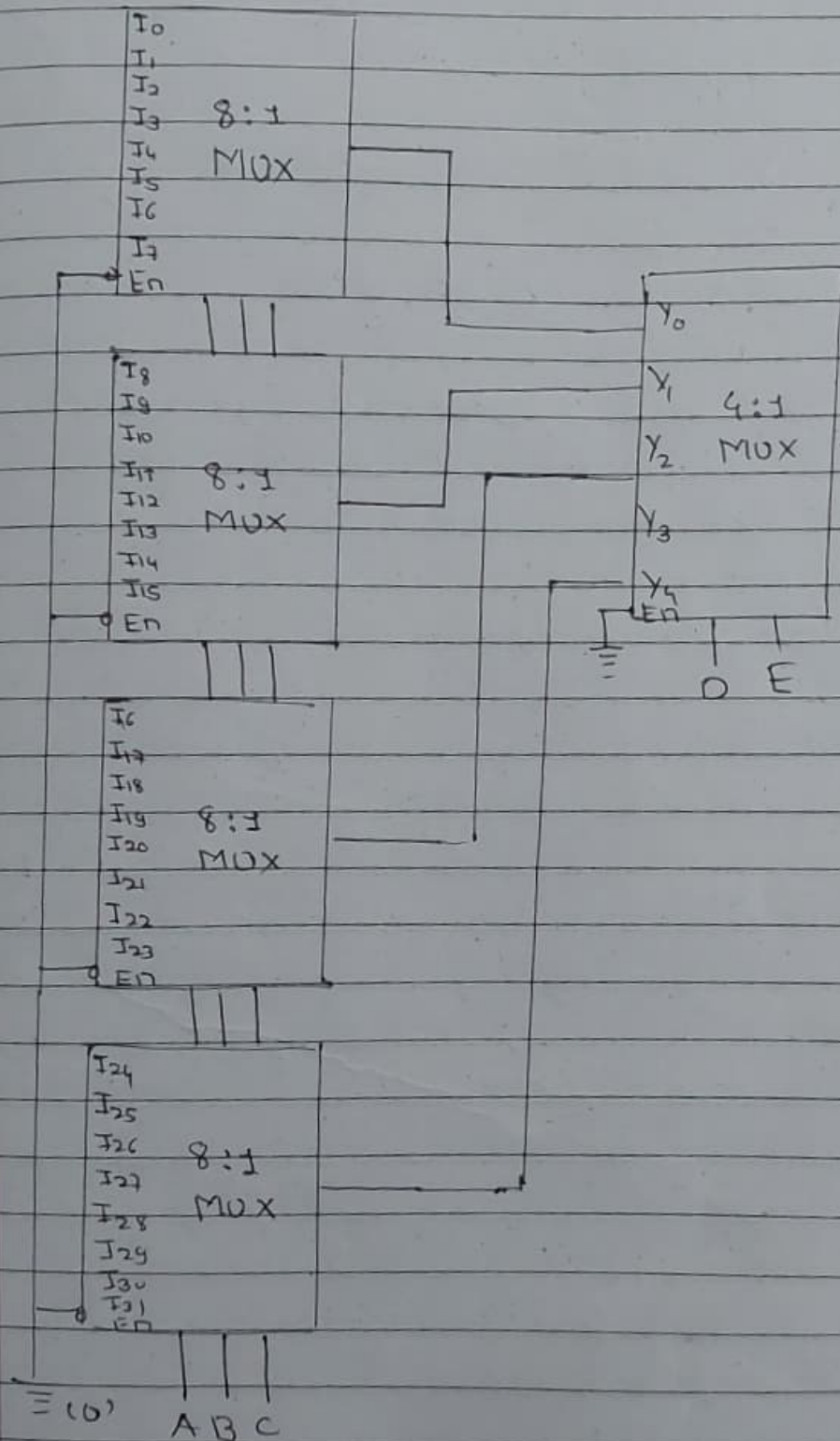
	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
$\bar{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	0	A	A	$\bar{A}$	A	$\bar{A}$	0	1



Ans:- 2) 16:1 MUX & 2:1 MUX



Ans: 3) 8:1 MUX and 4:1 MUX

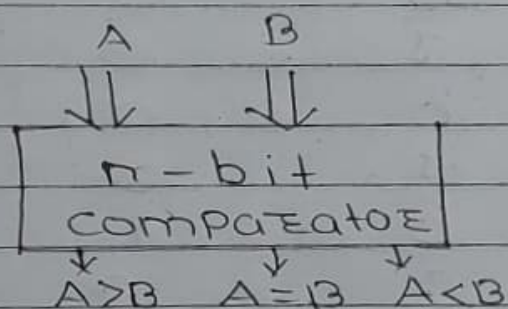




## • Comparator

A comparator is the special combinational circuit design to compare the relative magnitude of two binary numbers.

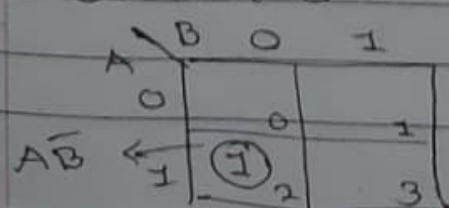
The fig shows the block diagram of  $n$  bit comparator that receives  $n$  bits numbers  $A$  and  $B$  as inputs and outputs are  $A > B$ ,  $A = B$  and  $A < B$ .



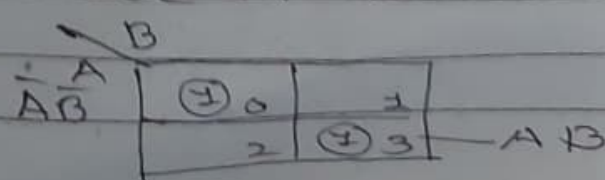
Q. Design one bit Comparator

I/PS		O/PS		
A	B	$A > B$	$A = B$	$A < B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

For  $A > B$



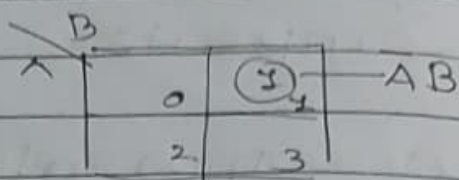
For  $A = B$



$$= \bar{A}\bar{B} + AB$$

$$= A \odot B$$

for  $A < B$



Q. Design two bit comparator

$A_1$	$A_0$	$B_1$	$B_0$	$A > B$	$A = B$	$A < B$
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

for  $A > B$

$A, A_0$	$B_1 B_0$ 00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	8	9	11	10
10	12	13	15	14

$A_1 B_1$

for  $A > B$

for  $A < B$

$A, A_0$	$B_1 B_0$ 00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	8	9	11	10
10	12	13	15	14

$A_1 A_0 B_0$

$A_1 B_1$

$A_0 B_1 B_0$

$$A < B = \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0 + A_1 \bar{A}_0 B_0$$

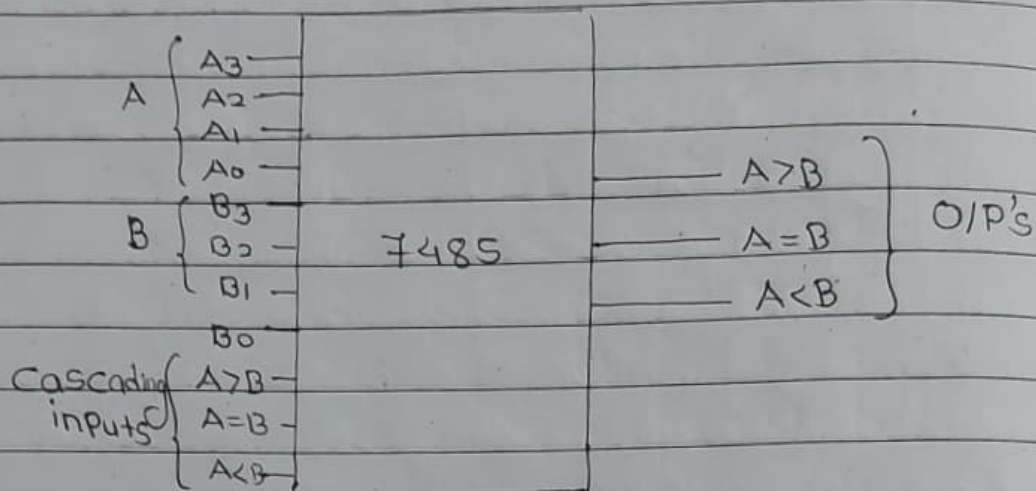
For  $A = B$

$A_1 A_0$	$B_1 B_0$ 00	01	11	10	$\bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0$
00	(1) 0	1	3	2	$\bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0$
01	4	(1) 5	7	6	$A_1 \bar{A}_0 \bar{B}_1 \bar{B}_0$
11	11	12	(1) 14	13	$A_1 \bar{A}_0 B_1 \bar{B}_0$
10	7	8	10	(1) 9	$A_1 \bar{A}_0 B_1 B_0$

$$\begin{aligned}
 Y &= \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 \bar{A}_0 \bar{B}_1 B_0 + A_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 \\
 &\quad + A_1 \bar{A}_0 B_1 \bar{B}_0 \\
 &= \bar{A}_1 B_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (A_0 \bar{B}_0 + \bar{A}_0 B_0)
 \end{aligned}$$



Q. Design four Bit comparator



I/P's A, B	Cas I/P's			O/P's		
	A > B	A = B	A < B	A > B	A = B	A < B
A > B	x	x	x	1	0	0
A = B	1	0	0	1	0	0
	0	1	0	0	1	0
	0	0	1	0	0	1
A < B	x	x	x	0	0	1

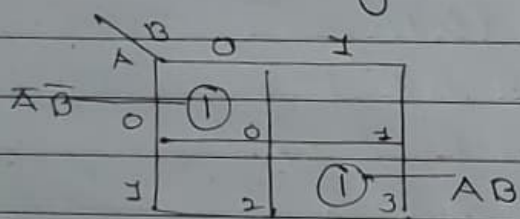
# • Parity checker CKT :-

- Design a circuit to check the parity of two incoming two bits words using logic gates.

⇒

I/P's		O/P's	
A	B	$\Sigma E$	$\Sigma O$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

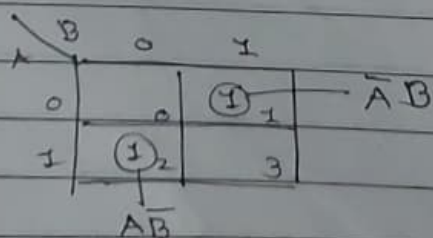
for even Parity :-



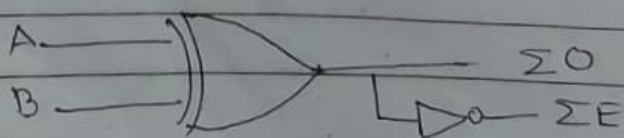
$$Y = \overline{A}\overline{B} + AB$$

$$= A \odot B = \overline{A \oplus B}$$

for odd parity :-



$$Y = \overline{A}B + A\overline{B} = A \oplus B$$



Q. Design a logic ck+ to check the odd even parity for three input bits.

I/P's			O/P's	
A	B	C	$\Sigma E$	$\Sigma O$
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

for even parity.

A \ BC	BC				
	00	01	11	10	
$\bar{A}\bar{B}\bar{C}$	1		1		$\bar{A}BC$
0	0	1	3	2	
1		1		1	$ABC$
	4	5	7	6	

$\downarrow$   
 $\bar{A}BC$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + ABC$$

$$= \bar{B}(\bar{A}\bar{C} + AC) + B(\bar{A}C + AC)$$

$$= \bar{B}(A \odot C) + B(A \oplus B)$$

$$= \bar{B}(A \oplus C) + B(A \oplus C)$$

$$= \bar{B}x + Bx$$

$$= B \odot x = \overline{B \oplus x}$$

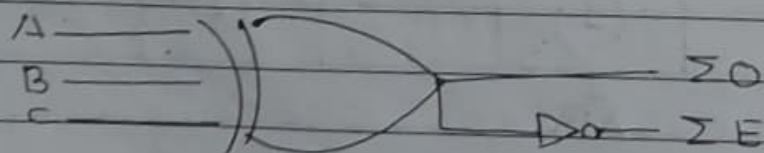
$$= \overline{B \oplus A \oplus C} = A \odot B \odot C$$

for odd parity :-

		BC			
		00	01	11	10
A	$\bar{A}\bar{B}\bar{C}$	0	1	3	2
	$\bar{A}\bar{B}C$	1	4	7	6
		0	1	3	2
		4	5	7	6

↓  
ABC

$$\begin{aligned}
 Y &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C} \\
 &= \bar{C}(\bar{A}\bar{B} + \bar{A}B) + C(\bar{A}\bar{B} + A\bar{B}) \\
 &= \bar{C}(A \oplus B) + C(\bar{A} \oplus \bar{B}) \\
 &= \bar{C}(A \oplus B) + C(\overline{A \oplus B}) \\
 &\quad \text{Let } A \oplus B = x \\
 &= \bar{C}x + C\bar{x} \\
 &= C \oplus x \\
 &= C \oplus A \oplus B \\
 &= A \oplus B \oplus C
 \end{aligned}$$



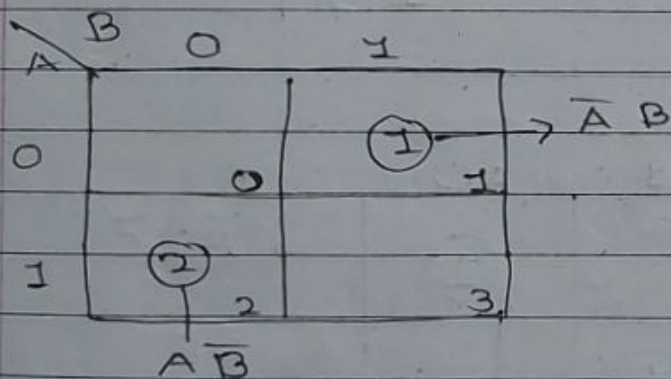


# Parity generator

Q. Design two bit even parity generator using logic gates.

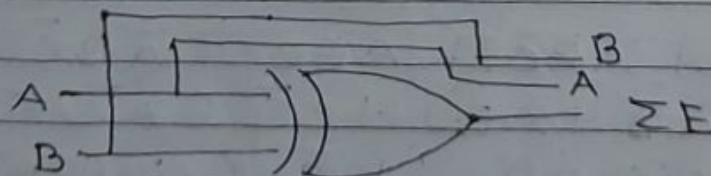
Soln:-

I/P's		O/P
A	B	$\Sigma E$
0	0	0
0	1	1
1	0	1
1	1	0



$$= \bar{A}B + A\bar{B}$$

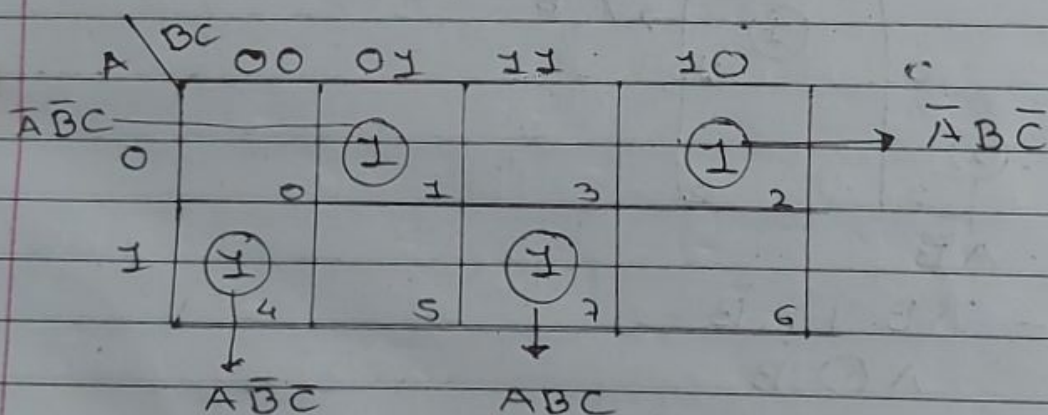
$$= A \oplus B$$



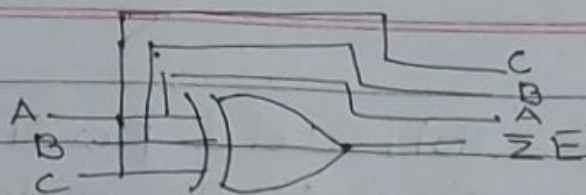
Q.5 Design three bit even parity generator using logic gates

Soln:-

I/p's			O/P's
A	B	C	$\Sigma E$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



$$\begin{aligned}
 &= \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}B\bar{C} \\
 &= C(\bar{A}\bar{B} + AB) + \bar{C}(A\bar{B} + \bar{A}B) \\
 &= C(A \odot B) + \bar{C}(A \oplus B) \\
 &= C\bar{X} + \bar{C}X \\
 &= C \oplus X \\
 &= C \oplus A \oplus B \\
 &= A \oplus B \oplus C
 \end{aligned}$$



Q. Design two bit odd parity generator using logic gates.

Soln:-

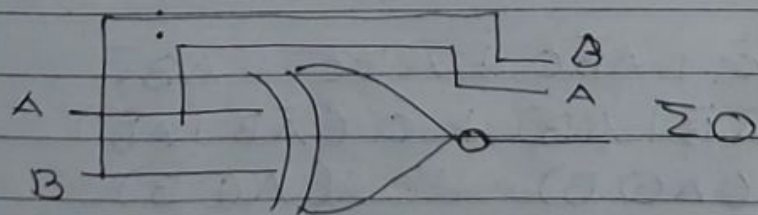
I/P's		O/P's
A	B	$\Sigma O$
0	0	1
0	1	0
1	0	0
1	1	1

	B	0	1
A	0	1	0
1	1	0	1

AB

$$= \bar{A}\bar{B} + AB$$

$$= A \oplus B$$

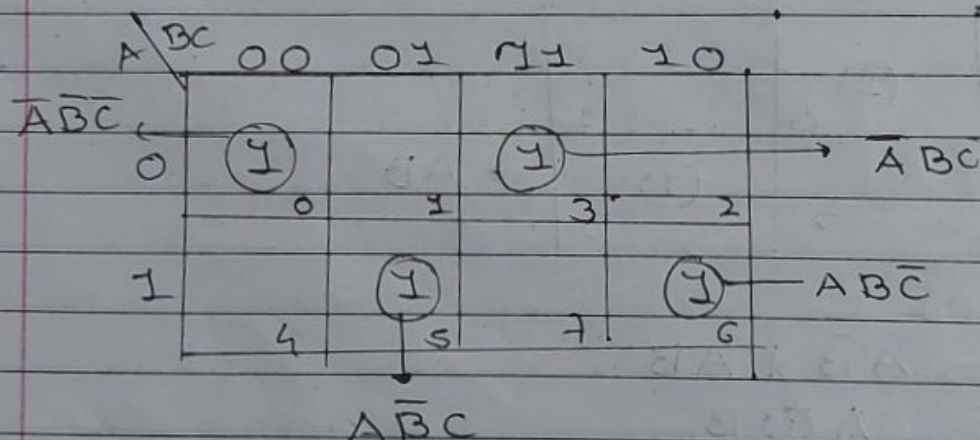




Q. Design three bit odd parity generator using logic gates

Soln:-

I/P's			O/P's
A	B	C	$\Sigma O$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



$$\begin{aligned}
 &= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} \\
 &= \bar{C}(\bar{A}\bar{B} + A\bar{B}) + C(\bar{A}B + A\bar{B}) \\
 &= \bar{C}(A \odot B) + C(A \oplus B) \\
 &= \text{Let } x = A \oplus B \\
 &= \bar{C}\bar{x} + Cx \\
 &= C \odot x \\
 &= C \odot (A \oplus B)
 \end{aligned}$$



$$= \bar{C} (A \oplus B) + C (A \oplus B)$$

$$\Rightarrow \text{Let } A \oplus B = x$$

$$\therefore = \bar{C} x + C x$$

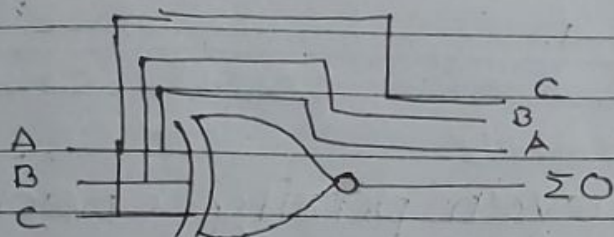
$$= C \odot x$$

$$= \overline{C \oplus x}$$

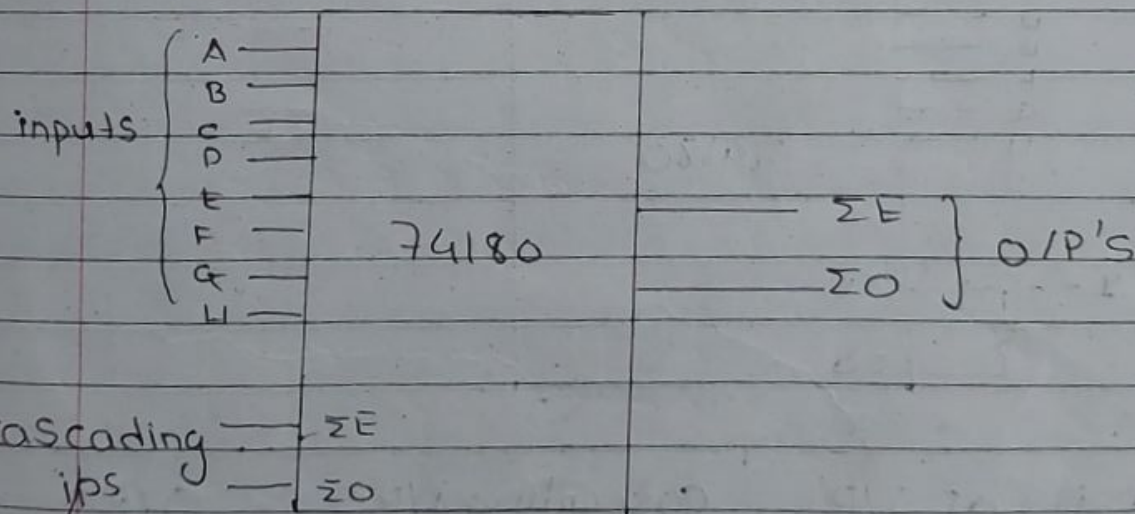
$$= \overline{C \oplus A \oplus B}$$

$$= C \odot A \odot B$$

$$= A \odot B \odot C$$



- Design Using 74180 (generalized)



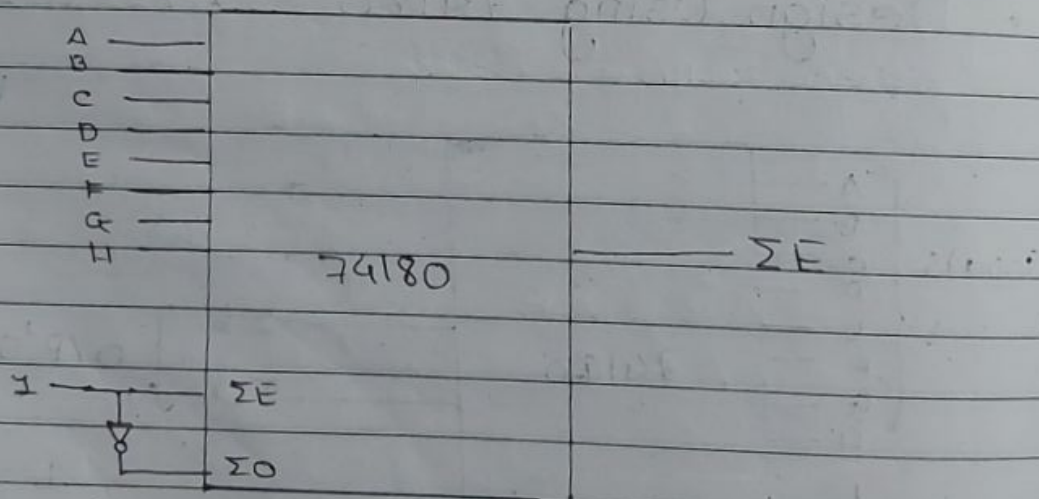
Even + even = even  
 Odd + odd = even  
 even + odd = odd  
 odd + even = odd

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Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

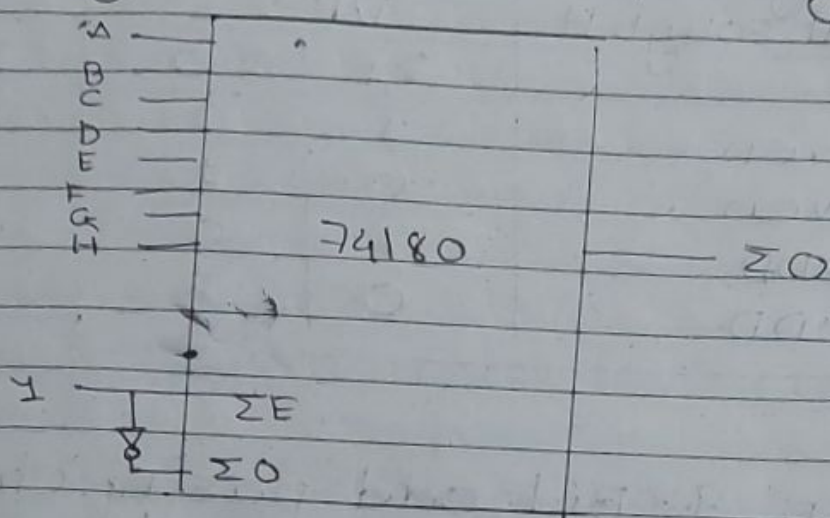
parity of i/p's A through H	cascading i/p's		O/p's	
	$\Sigma E$	$\Sigma O$	$\Sigma E$	$\Sigma O$
Even	1	0	1	0
Even	0	1	0	1
ODD	1	0	0	1
ODD	0	1	1	0

- Design 8 bit even parity checker using IC 74180



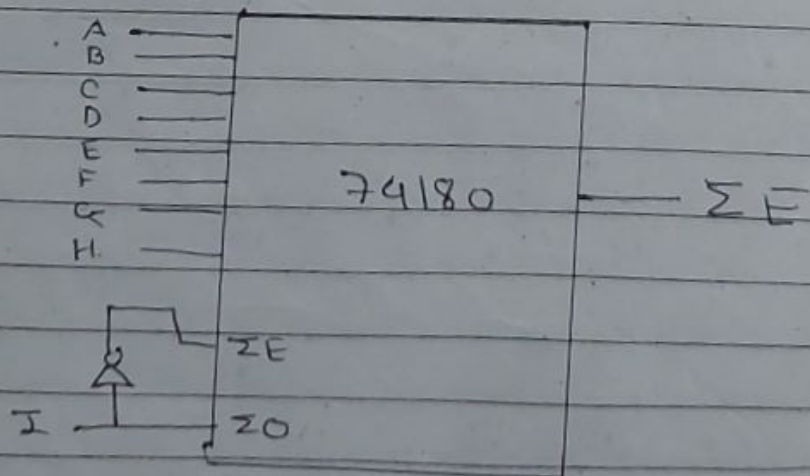
Parity of i/p's A through H	cascading i/p's		O/p's	
	$\Sigma E$	$\Sigma O$	$\Sigma E$	$\Sigma O$
Even	1	0	1	0
odd	1	0	0	1

Q. Design 8 bit odd parity checker



Parity of i/p's A through H	cascading i/p's		O/P's	
	$\Sigma E$	$\Sigma O$	$\Sigma E$	$\Sigma O$
Even	1	0	1	0
Odd	1	0	0	1

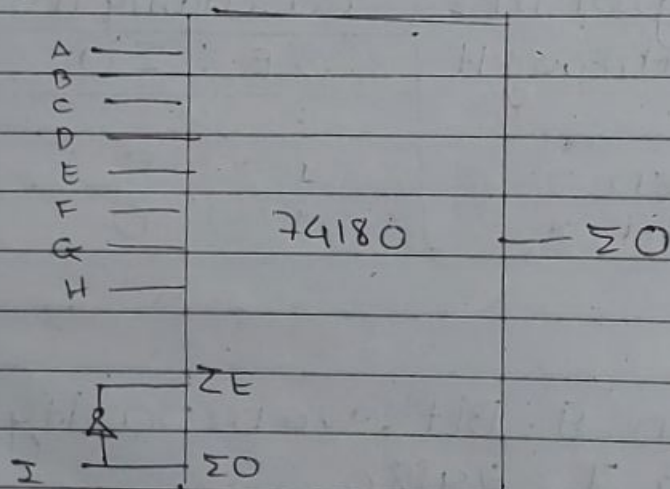
Q. Design 9 - Bit even parity checker Using IC 74180





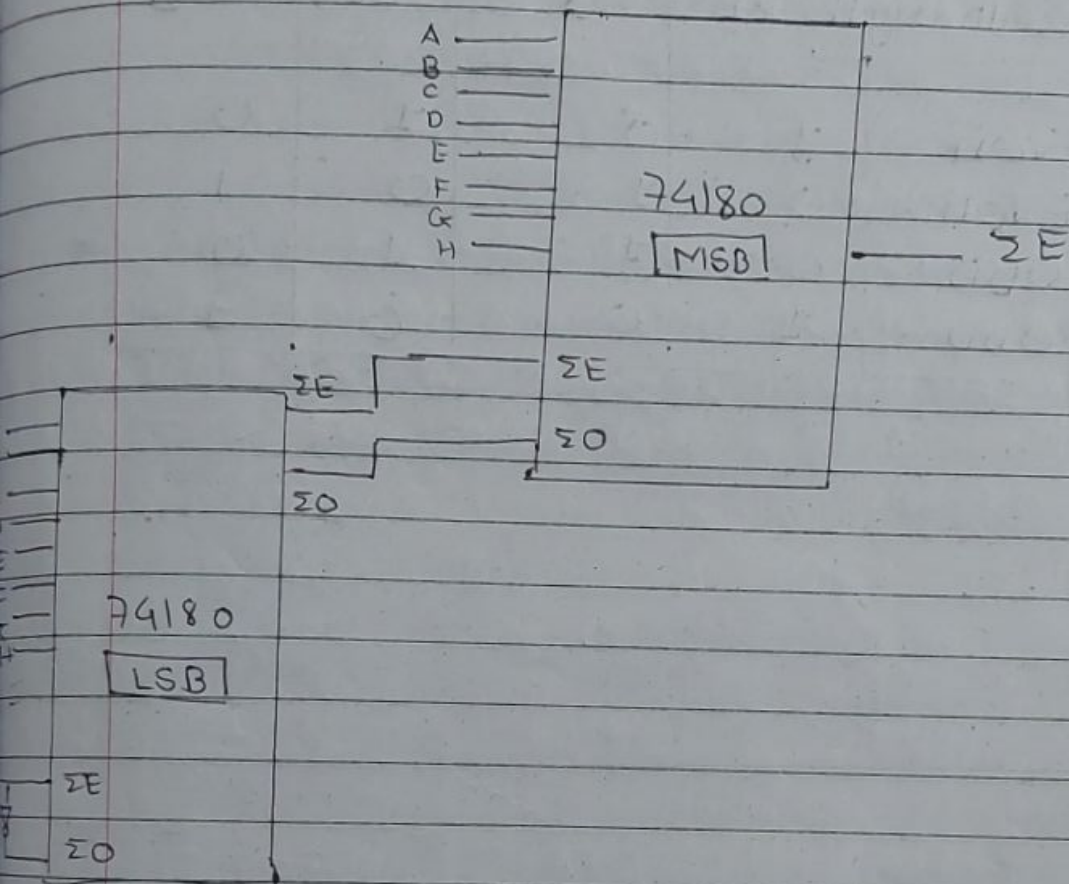
Parity of i/p's A through H	cascading i/p's		O/P's	
	$\Sigma E$	$\Sigma O$	$\Sigma E$	$\Sigma O$
Even	1	0	1	0
Even	0	1	0	1
ODD	1	0	0	1
ODD	0	1	1	0

Q. Design 8-Bit odd parity checker  
Using IC 74180





a. Design 16-Bit even parity checker.



• For LSB

Parity of i/p's A through H	cascading i/p's		O/P's	
	ΣE	ΣO	ΣE	ΣO
Even	1	0	1	0
Even Even	0	1	0	1
ODD	1	0	0	1
ODD ODD	0	1	1	0

FOE MSB

Parity of i/P's at the output	cascading i/P's		O/P's	
	$\Sigma E$	$\Sigma O$	$\Sigma E$	$\Sigma O$
Even	1	0	1	0
Even	0	1	0	1
Odd	0	1	1	0
Odd	1	0	0	1

## • 4 bit Parallel Adder (Binary)

### Comparison of Serial

Sl. no	Parameter of comparison	Parallel	Serial
1.	Clock pulse	Not Essential	Essential
2.	Principle of operation	All the bits are added simultaneously	One pair of bits is added at a time
3.	Result is in	parallel form	Serial form
4.	Time taken to complete addition	Very short	long
5.	No. of components	less	more
6.	cost	less	more
7.	circuit complexity	less	more

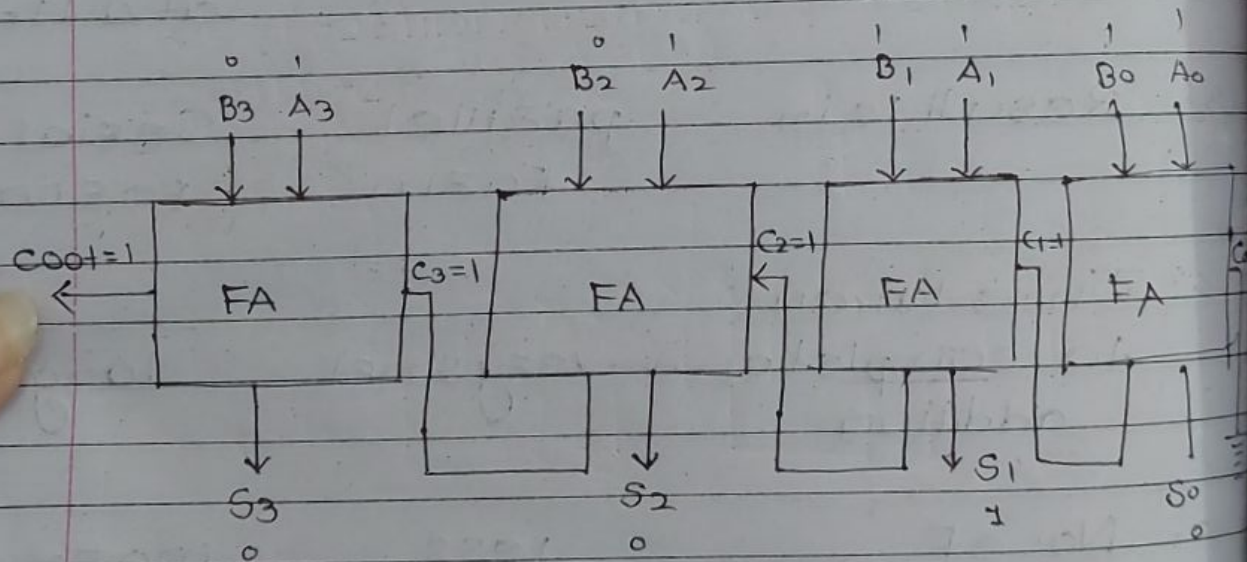


## Parallel Binary Adder

A circuit consisting of 'n' full adders that will add to 'n' bit nos. the output consist of 'n' sum bit & a carry bit.

## Cascade

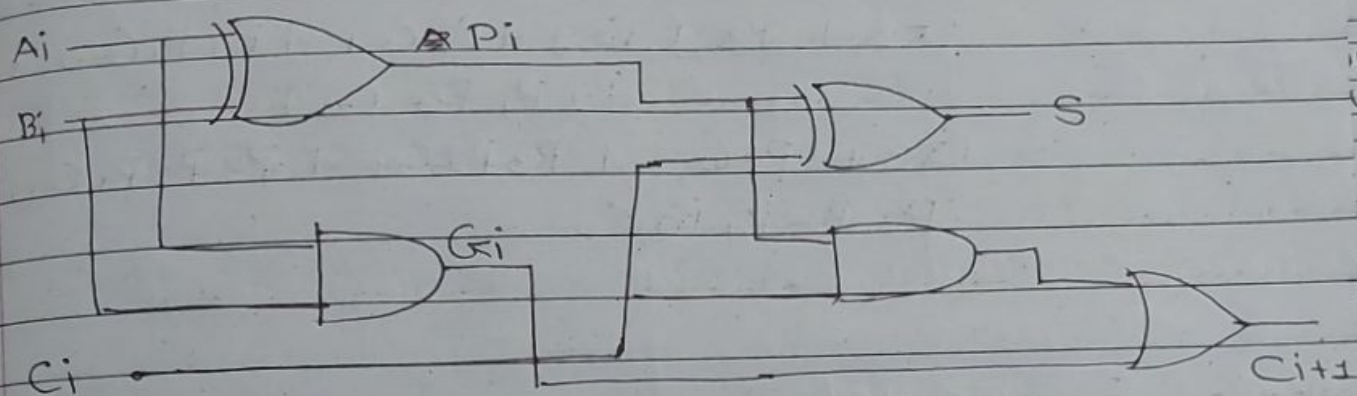
To connect an output of device to input of another device often for the purpose of expanding the no. of bits available for particular function.





# CARRY LOOK AHEAD ADDER (4 bit)

0, 1, 2, 3



$$C_{i+1} = A_i \oplus B_i + C$$

$$C_{i+1} = A_i \cdot B_i + (A_i \oplus B_i) C_i$$

$$P_i = A_i \oplus B_i \quad \& \quad G_i = A_i \cdot B_i$$

$$\therefore \boxed{C_{i+1} = G_i + (P_i) C_i}$$

$$\text{if } i = 0$$

$$\therefore C_1 = G_0 + P_0 C_0$$

$$\text{if } i = 1$$

$$C_2 = G_1 + P_1 C_1$$

$$= G_1 + P_1 (G_0 + P_0 C_0)$$

$$\text{if } i = 2$$

$$C_3 = G_2 + P_2 C_2$$

$$= G_2 + P_2 (G_1 + P_1 (G_0 + P_0 C_0))$$

$$= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$$

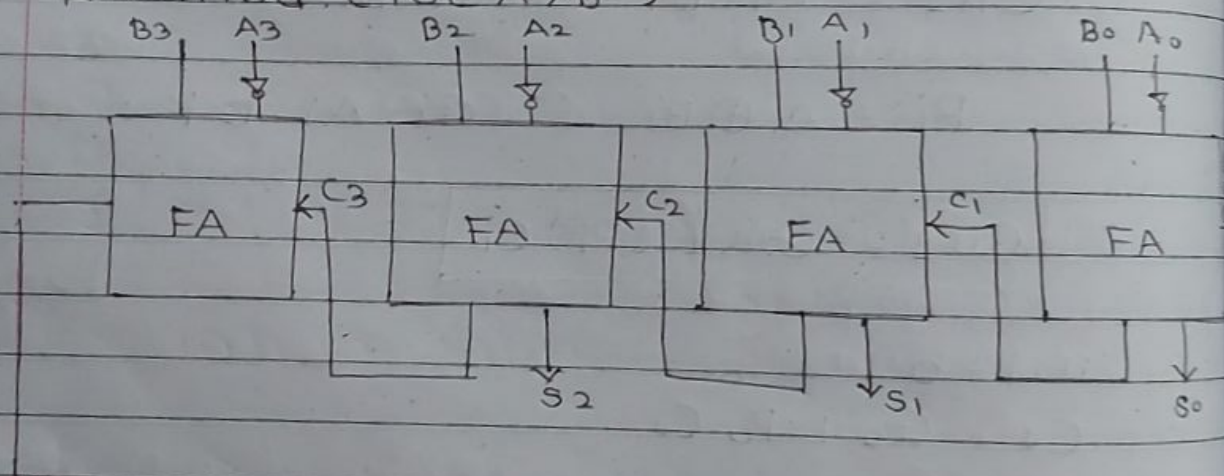
if  $i = 3$

$$\begin{aligned}
 C_4 &= G_3 + P_3 C_3 \\
 &= G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0) \\
 &= G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0
 \end{aligned}$$

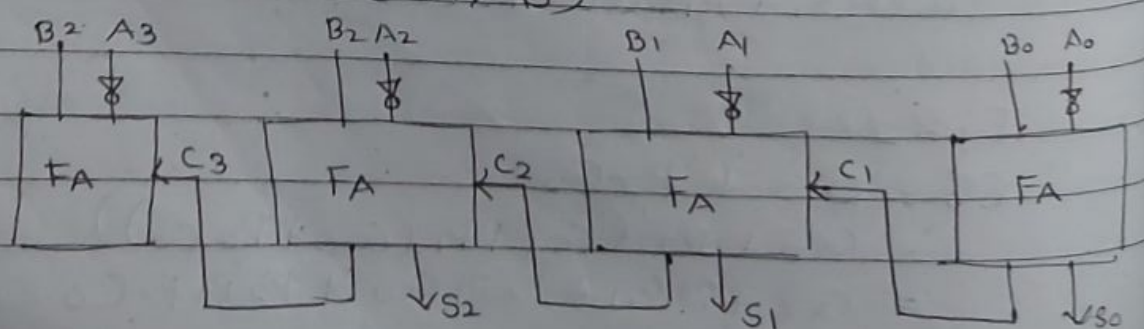
Parallel

#### • 4-bit Subtractor ( $A > B$ )

Subtraction using 1's complement method (for  $A > B$ )



Subtraction using 2's complement method (for  $A > B$ )



# 4 bit parallel Adder / Subtractor

