WINTER-2016

UNIT 1

Q.1 a) Solve: $(D^3+1)y = \sin 3x - \cos^2\left(\frac{x}{2}\right)$. **(6)**

b) Solve by the method of variation of parameter:

$$(D^2+9)y = \frac{1}{1+\sin 3x}. \quad \textbf{(7)}$$

Q.2 a) Solve: $(x^2D^2 - 3xD + 1)y = \frac{\sin(\log x)}{x}$. **(7)**

b) Solve: $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = e^{2x}(1+x)$. **(6)**

UNIT 2

Q.3 a) If $L\left\{\frac{1}{\sqrt{\pi x}}\right\} = \frac{1}{\sqrt{S}}$ then find Laplace transform of $\frac{2\sqrt{x}}{\sqrt{x}}$.

Hence evaluate $\int_0^\infty e^{-4x} x^{\frac{1}{2}} dx$. **(4)**

b) Use convolution theorem to evaluate: $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$. **(5)**

c) Using Laplace transform solve the equation,

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 20\sin 2t \text{ with } x = -1,\ \frac{dx}{dt} = 2 \text{ at } t = 0. \quad \textbf{(5)}$$

Q.4 a) Show that: $L^{-1}\left\{\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right\} = \int_0^t \frac{2}{u}(1-\cos u)du$. **(4)**

b) Express f(t) in terms of unit step function and hence find its Laplace transform if,

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases} \quad \textbf{(5)}$$

c) Find the Laplace transform of f(t) if,

$$f(t) = \frac{1}{a}, \qquad 0 < t < a$$

$$= \frac{1}{a}(2a - t), \qquad a < t < 2a$$

and f(t) = f(t + 2a). **(5)**

UNIT 3

Q.5 a) Solve: $u_{n+2} - 5u_{n+1} + 6u_n = 4^n.n^2$. **(4)**

b) Find the inverse Z-transform of: $\dfrac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. **(4)**

c) Use z-transform to solve the difference equation.

$u_{n+1} + 2u_{n+1} + u_n = n$, given that u(0) = u(1) = 0. **(5)**

Q.6 a) Solve: $u_{n+2} - 2u_{n+1} + 2u_n = \cos\left(\frac{n\pi}{2}\right)$ **(4)**

b) Show that: $z\{(k+1)a^{k+1}\} = \dfrac{az^2}{(z-a)^2}$. **(4)**

c) Use z-transform to solve the difference equation,

$u_{n+2} + 6u_{n+1} + 9u_n = 2^n$

given that u(0) = u(1) = 0. **(5)**

UNIT 4

Q.7 a) Solve: $(x^2 - yz)\, p + (y^2 - xz)\, q - z^2 - xy$. **(4)**

b) Solve: $z^2(p^2 + q^2) = x^2 + y^2$. **(4)**

c) Using Fourier sine integral formula, prove that:

$$\int_0^\infty \frac{\sin xs . \sin sx}{1 - s^2} ds = \begin{cases} \frac{\pi}{2} \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$ **(6)**

Q.8 a) Solve: $q^2 = z^2p^2(1 - p^2)$. **(4)**

b) Solve: $(1 - y^2)\, xq^2 + y^2p = 0$. **(4)**

c) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. **(6)**

UNIT 5

Q.9 a) Construct an analytic function

$F(z) = u + iv$, if $u - v = (x - y)\,(x^2 + 4xy + y^2)$. **(7)**

b) Find the Bilinear transformation which maps the points $z = \infty, i, 0$ into $w = 0, i, \infty$. **(6)**

Q.10 a) If $f(z)$ is an analytic function with constant modulus, then show that $f(z)$ is constant. **(6)**

b) Expand $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the region. **(7)**

i. $0 < |z| < 3$,

ii. $0 < |z - 3| < 3$,

iii. $3 < |z - 3| < 6$.

UNIT 6

Q.11 a) Find the directional derivative of $\phi = e^{2x} \cos yz$ at origin in the direction of tangent to the curve $x = b \sin t$, $y = b \cos t$, $z = bt$ at $t = \frac{\pi}{4}$. **(6)**

b) Prove that:

i. $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, **ii.** $\nabla^2\left(\nabla . \frac{\bar{r}}{r^2}\right) = \frac{2}{r^4}$. **(7)**

Q.12 a) Show that the vector field,

$\overline{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$ is irrotational and hence find a scalar function ϕ such that $\overline{F} = \nabla\phi$. Also evaluate $\int_C \overline{F}.d\overline{r}$ along any curve joining point (1, –2, 1) to (3, 1, 4). **(7)**

b) Find the values of constants m and n so that the surfaces $mx^2 - nyz = (m + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at the point (1, –1, 2). **(6)**