

## SUMMER-2015

### UNIT 1

**Q1. a)** Explain with example,

- i. Tautology    ii. Contradiction    iii. Equivalent formulas  
(8)

**b)** Obtain the principle disjunctive normal form of

$$(P \wedge Q) \vee (\neg P \wedge Q) \vee (Q \wedge R). \quad (6)$$

**Q2. a)** Show the following equivalences,

$$\text{i. } A \rightarrow (P \vee C) \Leftrightarrow (A \wedge \neg P) \rightarrow C \quad \text{ii. } (P \rightarrow C) \wedge (Q \rightarrow C) \Leftrightarrow (P \vee Q) \rightarrow C \quad (8)$$

**b)** Obtain the principal conjunctive normal form of  $\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$ .  
(6)

### UNIT 2

**Q3. a)** Determine whether the conclusion C follows logically from the premises  $H_1$  and  $H_2$ :

$$\begin{aligned} \text{i. } H_1: P \rightarrow Q \quad H_2: \neg P \quad C: Q \\ \text{ii. } H_1: \neg P \quad H_2: P \Leftrightarrow Q \quad C: \neg(P \wedge Q) \end{aligned} \quad (8)$$

**b)** Show that  $(\exists x)M(x)$  follows logically from the premises  $(x)(H(x) \rightarrow M(x))$  and  $(\exists x)H(x)$ .  
(7)

**Q4. a)** Show that  $S \vee R$  is tautological implied by  $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ .  
(7)

**b)** Symbolize the following statements

$$\text{i. Some cats are black. ii. All Indians are brave} \quad (6)$$

### UNIT 3

**Q5. a)** Explain with example:

$$\begin{aligned} \text{i. Set} \quad \text{ii. Subset} \quad \text{iii. Equality of set} \\ \text{iv. Empty set} \end{aligned} \quad (6)$$

**b)** Show that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$   
(7)

By i. Formal proof.                      ii. Venn diagram

**Q6. a)** Let  $X = \{1, 2, 3, 4, 5, 6, 7\}$  and  $R = \{ \langle x, y \rangle \mid x - y \text{ is divisible by } 3 \}$ . Show that  $R$  is an equivalent relation. Draw the graph of  $R$ . (7)

**b)** Let  $X = \{1, 2, 3\}$  and  $f, g, h$  and  $s$  be the functions from  $X$  to  $X$  given by,

$$f = \{ \langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle \} \quad g = \{ \langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 3, 3 \rangle \}$$

$$h = \{ \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle \} \quad s = \{ \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle \}$$

find  $f, g, g.f, f.h.g, g.s, f.s$  and  $s.s$ . (7)

#### UNIT 4

**Q7. a)** Explain: i. Group ii. Subgroup (7)

**b)** Convert the following infix expressions to prefix and postfix,

$$\text{i. } (A + B)/(C - D) \quad \text{ii. } (A * B) + (C * (D/F)) \quad (6)$$

**Q8. a)** What is coset? Find the coset of  $\{[0], [3]\}$  in the group  $\langle \mathbb{Z}_6, +_6 \rangle$  (7)

**b)** Write down the composition table for

$$\text{i. } \langle \mathbb{Z}_7, +_7 \rangle \quad \text{ii. } \langle \mathbb{Z}_7, *_7 \rangle \quad (6)$$

#### UNIT 5

**Q9. a)** Expand the following functions into their sum-of-product form:

$$\text{i. } f(x, y, z) = xy' + y'z' \quad \text{ii. } f(w, x, y, z) = xy + w'yz \quad (8)$$

**b)** In any Boolean algebra, show that

$$\text{i. } a = b \Leftrightarrow ab' + a'b = 0 \quad \text{ii. } a = 0 \Leftrightarrow ab' + a'b = b \quad (6)$$

**Q10. a)** Draw the diagram of the lattice  $\langle S_n, D \rangle$  for  $n = 4, 12, 15, 60$ . (5)

**b)** Use the K-map representation to find a minimal sum-of-products expression for the following functions:

$$\text{i) } f(a, b, c) = \Sigma(0, 1, 4, 6) \quad \text{ii) } f(a, b, c, d) = \Sigma(0, 5, 7, 8, 12, 14) \quad (8)$$

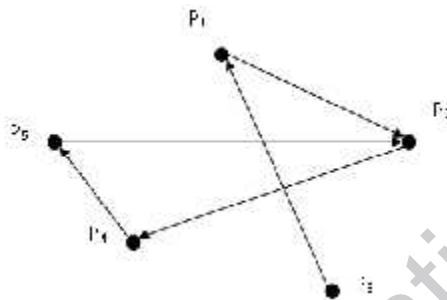
## UNIT 6

**Q11. a)** Explain with example,

i. Graph ii. Indegree and outdegree iii. Tree. **(6)**

**b)** Show that in a complete binary tree the total number of edges is given by  $2(n_t - 1)$ , where  $n_t$  is the number of terminal nodes. **(7)**

**Q12. a)** Obtain adjacency matrix and path matrix of the digraph given below, **(7)**



**b)** Give the directed tree representation of the following formula  $(P \vee (\neg P \wedge Q)) \wedge (\neg P \wedge Q) \wedge \neg R$  **(6)**