SUMMER-2016

UNIT 1

Q.1 a) Solve: $(D^2 + 2D + 2)y = \sin h\, x + x^2$. **(6)**

b) Solve by method of variation of parameter:

$(D^2 + D)\, y = (1 + e^x)^{-1}$. **(7)**

Q.2 a) Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \log x$. **(6)**

b) Solve $\left(D^2 - D - 2\right)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$. **(7)**

UNIT 2

Q.3 a) Find the Inverse Laplace Transform of: $\frac{s+1}{s^2+s+1}$. **(4)**

b) Evaluate $\int_0^\infty t^3 e^{-t} \sin t\, dt$. **(5)**

c) Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$,

$y(0) = 0$, $y'(0) = 1$, $\frac{d}{dt} = D$ using Laplace Transform. **(5)**

Q.4 a) Find Laplace Transform of $\frac{d}{dt}\left(\frac{\sin t}{t}\right)$. **(4)**

b) Find Inverse Laplace Transform of $\frac{S^2+1}{\left(S^3+3S^2+2S\right)}$. **(7)**

c) Find the Laplace Transform of f(t) if:

$f(t) = a \sin pt, \quad 0 < t < \pi/p$

$\quad\; = 0, \quad \pi/p < t < 2\pi/p$

and $f(t) = f(t + 2\pi/p)$ **(5)**

UNIT 3

Q.5 a) Solve $(\Delta^2 + \Delta + 1)\, y = x^2$. **(5)**

b) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, using Z-Transform, with $y(0) = y(1) = 0$. **(5)**

c) Find Inverse Z-Transform of $\dfrac{3z^2 + 2z + 1}{z^2 - 3z + 2}$. **(4)**

Q.6 a) Solve $(E^2 - 5E + 6)\, y = x + 2^x$. **(5)**

b) Show that inverse Z-Transform of:

$$\frac{1}{(z-a)^3} = \frac{(k-1)(k-2)}{2} a^{k-3}.$$ **(5)**

c) Find Z-Transform of: Ka^{K-1}, $K \geq 1$. **(4)**

UNIT 4

Q.7 a) Solve: $pq = x^m y^n z^{2l}$. **(4)**

b) Solve $yp + xq + pq = 0$. **(4)**

c) Express the function:

$$f(x) = 1, \; |x| \leq 1$$
$$= 0, \; |x| > 1$$

as Fourier Integral, hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$. **(4)**

Q.8 a) Solve $(3y - 2z)\, p + (z - 3x)\, 1 = 2x - y$. **(5)**

b) Solve $yp = 2xy + \log q$. **(4)**

c) Find the Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$. **(5)**

UNIT 5

Q.9 a) Show that $4 = e^x(x \cos y - y \sin y)$ is harmonic and find the analytic function $f(z) = u + iv$. **(7)**

b) Find Bilinear transformation which maps the points $z = 0, -1, \infty$ from z-plane into $w = -1, -2 - i, i$ respectively in the w-plane. **(6)**

Q.10 a) If f(z) is an analytic function of z and f(z) = u + iv then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(7)**

b) Expand $\frac{z^2-1}{z^2+5z+6}$ about z = 0 in the region 2 < |z| < 3.

(6)

UNIT 6

Q.11 a) Find the directional derivatives of $f(x, y, z) = x^2y^2z^2$ at the point (1, 1, –1) in the direction of the tangent to the curve $x = e^t$, y = 2 sin t, z = t – cos t, at t = 0. **(6)**

b) A vector field is given by:

$\bar{F} = (x^2 - y^2 + x)i - (2xy + y)j$, show that the field is irrotational and find its scalar potential. **(7)**

Q.12 a) If $\bar{F} = (2x^2 - 3z)i - 2xyj - 4xk$ then evaluate:

$\iiint_V \nabla \circ \bar{F}\, dv$, where v is bounded by x = 0, y = 0, z = 0 and 2x + 2y + z = 4. **(6)**

b) If $\bar{r} = xi + yj + zk$, show that:

i. grad $r = \frac{\bar{r}}{r}$. **ii.** grad $(l/r) = \frac{\bar{r}}{r^3}$. **iii.** $\nabla r^n = nr^{n-2}\bar{r}$. **(7)**