WINTER-2011

UNIT 1

Q.1 a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). **(7)**

b) If $\overline{F} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$; Computer the circulation of $\overline{F}$ about a circle C, in the xy-plane with centre at origin and radius 2; circle C is transversed in the positive direction. **(7)**

Q.2 a) i. $\overline{F_1}$ and $\overline{F_2}$ are irrotational; show that $\overline{F_1} \times \overline{F_2}$ is solenoidal. **(3)**

ii. Prove that $\nabla.\left(r^3 \overline{r}\right) = 6r^3$ **(4)**

b) Find the directional derivative of $\phi = 4.e^{2x-y+z}$ at a point (1, 1, –1) in the direction towards the points (–3, 5, 6). **(7)**

UNIT 2

Q.3 a) Solve $(D^2 - 2D + 1)\, y = x.e^x.\sin x$. **(6)**

b) Solve $(3x+2)^2 \dfrac{d^2y}{dx^2} + (3x+2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 1$ **(7)**

Q.4 a) Solve: $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 4\cos^2 x$ **(6)**

b) Solve by method of variation of parameter

$\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x.\log x$. **(7)**

UNIT 3

Q.5 a) Find the Laplace transform of $\dfrac{1-\cos t}{t^2}$ **(4)**

b) Evaluate $\displaystyle\int_0^\infty \frac{e^{-t}.\sin^2 t}{t}.dt$ **(4)**

c) Solve by Laplace transform

$(D^2 - 1)\, x = a.\cos ht$

$x(0) = 0,\ x^1(0) = 0.$ **(6)**

Q.6 a) Find inverse Laplace transform of $\frac{2s-3}{s^2+4s+13}$ **(4)**

b) Find the Laplace transform of the square wave function of period 'a' defined as

$f(t) = 1$, when $0 < t < a/2$

$= -1$, when $a/2 < t < a$. **(4)**

c) Use convolution theorem to evaluate

$$L^{-1}\left\{\frac{1}{(s+1)(s+9)^2}\right\}$$ **(6)**

UNIT 4

Q.7 a) Solve the following difference equations:

i. $u_{n+1} - 2u_{n+2} - 5u_{n+1} + 6u_n = 0$

ii. $y_{n+2} - 2\cos\alpha . y_{n+1} + y_n = \cos\alpha n$ **(5)**

b) Solve by Z-transform method

$y_{k+2} - 2y_{k+1} + y_k = 2^k$ with $y_n = 2, y_1 = 1$ **(5)**

Q.8 a) Solve the different equation

$u_{n+2} + 9u_n = n.2^n + 3^n + 7$ **(4)**

b) Find Z-transform of $\frac{1}{n(n+1)}$ **(4)**

c) Find the inverse Z-transform of $\frac{3z^2+2}{(5z-1)(5z+2)}$ **(5)**

UNIT 5

Q.9 a) Express $f(x) = 1, \ |x| < 1$

$= 0, \ |x| > 1$

as a Fourier-integral and hence evaluate $\int_0^\infty \frac{\sin - \cos\lambda x}{\lambda}.d\lambda$ **(6)**

b) Solve:

i. $pq + p + q = 0$, **ii.** $z^2(p^2x^2 + q^2) = 1$ **(7)**

Q.10 a) Using the Fourier sine transform of e^{-ax} $(a > 0)$,

show that $\int_0^\infty \frac{x.\sin kx}{a^2 + x^2} dx = \frac{\pi}{2}.e^{-ak}(k > 0)$ **(7)**

b) Solve: $x^2(y - z)\, p + y^2\,(z - x)\, q = z^2(x - y)$ **(6)**

UNIT 6

Q.11 a) Show that the polar from of Cauchy-Riemann equations are $\frac{\delta u}{\delta r} = \frac{1}{r}.\frac{\delta v}{\delta \theta};\ \frac{\delta v}{\delta r} = -\frac{1}{r}\frac{\delta v}{\delta \theta}$

Deduce that $\frac{\delta^2 u}{\delta r^2} + \frac{1}{r}.\frac{\delta u}{\delta r} + \frac{1}{r^2}.\frac{\delta^2 u}{\delta \theta^2} = 0$ **(7)**

b) Find the Laurent's series of $\frac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$. **(6)**

Q.12 a) If f(z) is an analytic function of z, prove that

$$\left(\frac{\delta^2}{x^2} + \frac{\delta^2}{\delta y^2}\right).\log|f(z)| = 0$$ **(6)**

b) Find the bilinear transformation which maps 1, i, –1 to the points 2, i, –2 respectively. **(7)**