WINTER-2018

UNIT 1

Q.1 a) Solve: $(D^2 + 5D + 6)\, y = e^{-2x} \sin 2x + 4x^2 e^x$. **(6)**

b) Solve by the Method of variation parameters

$(D^2 - 1)\, y = e^{-x} \sin(e^{-x}) + \cos(e^{-x})$ **(7)**

Q.2 a) Solve

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 3x - 7$$ **(7)**

b) Solve

$$\left(D^2 - D - 2\right)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$$ **(7)**

UNIT 2

Q.3 a) Find the Laplace transform of $\int_0^t e^t \frac{\sin t}{t}\, dt$ **(5)**

b) Find f(t) if $\bar{f}\left(s\right) = \frac{1}{2}\log\frac{\left(s^2 - a^2\right)}{s^2}$ **(4)**

c) Solve $\frac{dx}{dt} = 2x - 3y,\ \frac{dy}{dt} = y - 2x$

using L. T being given x(0) = 8, y(0) = 3 **(5)**

Q.4 a) Find the inverse Laplace Transform of

$$\bar{f}\left(s\right) = \frac{s^2 + 2s + 3}{\left(s^2 + 2s + 2\right)\left(s^2 + 2s + 5\right)}$$ **(5)**

b) Solve $\int_0^\infty t^3 e^{-t} \sin t\, dt$ **(4)**

c) Find the Laplace Transform of f(t) if f(t) = a sin pt, $0 < t < \pi/P = 0,\ \pi/P < t < 2\pi/P$ and $f(t) = f\left(t + \frac{2\pi}{P}\right)$ **(5)**

UNIT 3

Q.5 a) Solve the difference equations

$$U_{n+2} - 16u_n = \cos\frac{n}{2}$$ **(5)**

b) Find the Z-Transform of ka^{k-1}, $k \geq 1$ **(4)**

c) Solve by using Z-Transform

$y_{n+2} + 3y_{n+1} + 2y_n = 4^n$ given $y(0) = 0$, $y(1) = 1$ **(4)**

Q.6 a) Solve the difference equation

$(E^2 - 5E + 6)\, y = x + 2^x$ **(5)**

b) Show that inverse Z-Transform of

$$\frac{1}{(z-a)^3} = \frac{(k-1)(k-2)a^{k-3}}{2}$$ **(4)**

c) Find Z-Transform of $\sin(3n + 5)$ **(4)**

UNIT 4

Q.7 a) Solve $pq = x^m y^n z^{2P}$. **(4)**

b) Solve $(mz - ny)p + (nx - Pz)q = Py - mx$ **(4)**

c) Express the function, **(5)**

$$f(t) = t, \quad |x| \leq 1$$
$$= 0, \quad |x| > 1$$

as Fourier integral, hence

evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$

Q.8 a) Solve $4xyz = pq + 2px^2y + 2qxy^2$ **(4)**

b) Solve $yp = 2xy + \log q$ **(4)**

c) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$. **(5)**

UNIT 5

Q.9 a) Construct an analytic function

$f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$ **(7)**

b) Find Bilinear transformation which maps the points $Z = 0, -1, \infty$ and from Z-planes into $W = -1, -2 - i, i$ respectively in the W-plane. **(6)**

Q.10 a) If f(z) is analytic, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^n f = n^2 |f(z)|^{n-2} |f'(z)|^2$$ **(7)**

b) Expand $f(z) = \dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region.

i. $1 < |z| < 4$, **ii.** $|z| > 4$. **(6)**

UNIT 6

Q.11 a) Find the directional derivative $f(x, y, z) = x^2y^2z^2$ at the point (1, 1, -1) in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$. **(7)**

b) A vector field is given by $\overline{F} = (x^2 - y^2 + x)i - (2xy + y)j$

Show that the field is irrotational and find its scalar potential **(7)**

Q.12 a) Find whether the vector field $\overline{F} = (2xy + z^3)i + x^2 j + 3z^2 xk$ is conservative and find ϕ such that $\overline{F} = \nabla\phi$ and also evaluate $\int \overline{F} \circ d\overline{r}$ along any curve joining the points (1, -2, 1) to (3, 1, 4). **(7)**

b) If $\overline{r} = xi + yj + zk$, show that. **(9)**

i. $\text{grad } r = \dfrac{\overline{r}}{r}$

ii. $\text{grad}\left(\dfrac{1}{r}\right) = \dfrac{-\overline{r}}{r^3}$

iii. $\nabla_r n = nr^{n-2}\overline{r}$.