WINTER-2014

UNIT 1

Q.1 a) Solve: $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$. **(6)**

b) Solve: $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. **(7)**

Q.2 a) Solve: $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3.\log x$. **(6)**

b) Apply the method of variation of parameters to solve:

$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. **(7)**

UNIT 2

Q.3 a) Find the Laplace transform of:

$\int_0^\infty te^{-2t} \sin t\, dt$. **(4)**

b) Find the inverse Laplace transform of $\log \frac{s+1}{s-1}$. **(4)**

c) Use convolution theorem to find: $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$. **(4)**

Q.4 a) Find the inverse Laplace transform of:

$\frac{21s-33}{(s+1)(s-2)^3}$. **(7)**

b) Use Laplace transform method to solve the equation:

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} 5x = e^{-t} \sin t$, when x(0) = 0 and x'(0) = 1. **(7)**

UNIT 3

Q.5 a) Solve:

i. $y_{n+2} - 3y_{n+1} + 2y_n = 5^n$, **ii.** $u_{n+2} - 2u_{n+1} + u_n = n^2.2^n$. **(8)**

b) Find the z-transform of sin (5k + 3). **(5)**

Q.6 a) Solve: $y_{x+3} - 3y_{x+2} - 4y_{x+1} + 12y_x = 4 - x$. **(4)**

b) Find the inverse z-transform of: $\dfrac{4z}{z-2}$ if $|z| > |2|$. **(4)**

c) Solve the difference equation:

$6y_{k+2} - y_{k+1} + y_k = 0$, $y(0) = 0$, $y(1) = 1$ by z-transform. **(5)**

UNIT 4

Q.7 a) Solve the following equations:

i. $\dfrac{y^2 z}{x} p + xzq = y^2$. **ii.** $z^2(p^2 + q^2 + 1) = a^2$. **(8)**

b) Find the Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$. **(5)**

Q.8 a) Solve the following equations:

i. $z = px + qy + \sin(p + q)$, **ii.** $z^2(p^2 + q^2) = x^2 + y^2$. **(8)**

b) Find the Fourier transform of:

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| \le | \\ 0 & \text{if } |x| > | \end{cases}.$$ **(5)**

UNIT 5

Q.9 a) Show that the function $u(x, y) = 4xy - 3x + 2$ is harmonic. Construct the corresponding analytic function $f(z) = u(x, y) + i\,v(x, y)$. Express f(z) in terms of complex variable z. **(6)**

b) Expand the following function in Laurent's series:

$f(z) = \dfrac{1}{z^2 - 4z + 3}$, for $1 < |z| < 3$. **(7)**

Q.10 a) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. **(6)**

b) Determine the analytic function:

$$w = u + iv, \text{ if } v = \log(x^2 + y^2) + x - 2y. \quad \textbf{(7)}$$

UNIT 6

Q.11 a) Find the divergence and curl of:

$$\bar{v} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}. \quad \textbf{(4)}$$

b) Show that the vector:

$\bar{A} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal. **(3)**

c) Evaluate $\iint \bar{F} \circ \hat{n}\, ds$, where s is the surface of the plane $2x + y + 2z = 6$ in the first octant and $F = (x + y)^2 \hat{i} - 2x\hat{j} + 2yz\hat{k}$.

(7)

Q.12 a) Evaluate the line integral:

$\int_C \left[(x^2 + xy)dx + (x^2 + y^2)dy\right]$, where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$. **(7)**

b) Find the directional derivative of $\Phi = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in the direction towards the point $(-3, 5, 6)$. **(7)**