SUMMER-2015

UNIT 1

Q.1 a) Solve: $(D^2 + 3D + 2)y = e^{e^x}$ **(6)**

b) Solve by method of variation of parameters:

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$ **(7)**

Q.2 a) Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$. **(6)**

b) Solve: $\frac{d^2y}{dx^2} + 4y = \sin 3x + e^x + x^2$. **(7)**

UNIT 2

Q.3 a) Find the Laplace Transform of:

$e^{at}[2 \cos bt - 3 \sin bt]$ **(4)**

b) Using Convolution Theorem find inverse Laplace Transform of: $\frac{1}{s^2(s^2+1)}$ **(5)**

c) Find Laplace Transform of:

$$f(t) = \frac{t}{a}, \quad 0 < t < a$$

$$= \frac{1}{a}(2a - t), \quad a < t < 2a$$ **(5)**

Q.4 a) Evaluate: $\int_0^\infty t^3 e^{-t} \sin t\, dt$ **(4)**

b) Solve the differential equation using Laplace Transform:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}, \; y(0) = y'(0) = 1.$$ **(5)**

c) Show that: $L^{-1}\left(\frac{1}{s}\cos\left(\frac{1}{s}\right)\right) = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4!)^2} \ldots\ldots$ **(5)**

UNIT 3

Q.5 a) Solve the difference equation: **(8)**

i. $y_{n+2} - 5y_{n+1} - 6y_n = 4^n$, $y_0 = 0$, $y_1 = 1$.

ii. $y_{n+2} - 2y_{n+1} - 4y_n = 0$.

b) Solve $y_{n+2} - 4y_n = 0$

using z-transform, given $y_0 = 0$, $y_1 = 2$. **(5)**

Q.6 a) Solve the difference equation:

i. $y_{n+2} - 7y_{n+1} + 10y_n = 12e^{3n} + 4^n$

ii. $y_{n+2} + 3y_{n+1} + 2y_n = \sin\left(\frac{n\pi}{2}\right)$ **(5)**

b) Find inverse z-transform of: $\frac{1}{z-2}$.

Using direct division method. **(5)**

UNIT 4

Q.7 a) Solve the following partial differential equation: **(8)**

i. $xp + yq = nz$

ii. $x(y^2 - z^2)\, p + y(z^2 - x^2)\, q + z(y^2 - x^2) = 0$.

b) Find Fourier sine transform of:

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$ **(6)**

Q.8 a) Solve the following partial differential equation:

i. $\left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2 = \frac{3a^2}{z^2}$, **ii.** $pq = x^m y^n z^{2l}$. **(8)**

b) Find Fourier Transform of

$$f(x) = 1 - x^2, \; |x| \le 1$$
$$= 0, \quad |x| > 1$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$ **(6)**

UNIT 5

Q.9 a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic and determine the analytic function $f(z) = u + iv$ as an analytic function of z. **(7)**

b) Find the bilinear transformation which maps the point $z = -1, 0, 1$ from z-plane into $w = 0, i, 3i$ in w-plane. **(6)**

Q.10 a) If f(z) is an analytic function of z, prove that:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ **(7)**

b) Expand: $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$. **(6)**

UNIT 6

Q.11 a) Find the directional derivative at (1, 2, 3) of $V = xy + yz + zx$ in the direction of the vector $3i + 4j + 5k$. **(6)**

b) Find the divergence and curl of the vector:

$$\overline{V} = xyzi + 3x^2yj + (xz^2 - y^2z)k.$$ **(7)**

Q.12 a) Determine the constant a so that the vector:

$$\overline{V} = (x+3y)i + (y-2z)j + (x+az)k$$ is solenoidal. **(3)**

b) If $u\overline{F} = \nabla v$ where u, v are scalar fields show that $\overline{F}.\text{curl}\overline{F} = 0$. **(4)**

c) If $\overline{F} = (2x^2 - 3z)i - 2xyj - 4xk$, then evaluate:

$\iiint_V \nabla.\overline{F}\, dV$ where V is bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$. **(6)**