Unit-II : Magnetic Circuit & Electromagnetism

Basic concept of Magnetic flux, Flux density, MMF.

Magnetic Flux

Magnetic Flux is defined as the number of magnetic field lines passing through a given closed surface. It gives the measurement of the total magnetic field that passes through a given surface area. Here, the area under consideration can be of any size and under any orientation with respect to the direction of the magnetic field.

- Magnetic flux is commonly denoted using greek letter Phi .
- Magnetic flux symbol: Φ .

$\phi = B.A = BA \cos\theta$

Where,Φ is the magnetic flux, B is the magnetic field, A is the area,θ the angle at which the field lines pass through the given surface area

Magnetic Flux Unit

Magnetic flux is usually measured with a fluxmeter. The SI and CGS unit of magnetic flux is given below:

- SI unit of magnetic flux is Weber (Wb).
- The fundamental unit is Volt-seconds.
- The CGS unit is Maxwell.
-

Magnetic Flux Density

Magnetic flux density(B) is defined as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field.

- Units of B is Tesla (T) or Kgs−2A−1
- B is a vector quantity

B=F/IL

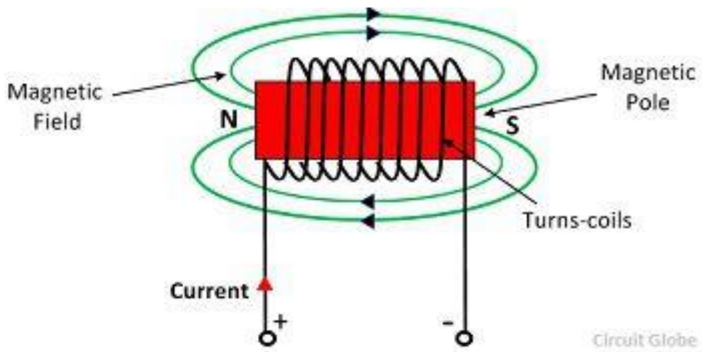
Where, L=length of wire, F=total force acting on the wire, I=current flowing through the wire.

SI unit is Tesla (abbreviated as T) and CGS unit is Gauss (abbreviated as G or Gs).

Magnetomotive Force (MMF)

The current flowing in an electric circuit is due to the existence of electromotive force (EMF) similarly **Magnetomotive force** (MMF) is required to drive the magnetic flux in the magnetic circuit. The magnetic pressure, which sets up the magnetic flux in a magnetic circuit is called Magnetomotive Force.

The SI unit of MMF is **Ampere-turn (AT),** and their CGS unit is G (gilbert). The MMF for the inductive coil



F = N I

Where, N – numbers of turns of inductive coil, I – current

The MMF is also known as the **magnetic potential.** It is the property of material to give rise the magnetic field. The magnetomotive force is the product of the magnetic flux(ϕ) and the magnetic reluctance.

The reluctance(R) is the opposition offers by the magnetic field to set up the magnetic flux on it. The MMF regarding reluctance and magnetic flux is given as

F = ϕ S

Where S – reluctance Φ – magnetic flux
The magnetomotive force can measure regarding magnetic field intensity and the length of the substance.
The magnetic field strength (H) is the force act on the unit pole placed on the magnetic field. MMF regarding field intensity is expressed as

F = H l

Where *H* is the magnetic field strength, and *l*is the length of the substance.

Magnetic flux, Flux density, MMF. Reluctance, Magnetic field intensity and their Relationship

Name	**Symbol**	**Equation**
Magneto-motive Force (MMF)	F (Ampere – Turns) or (A-t)	*F = NI* *F= BIL* *F=* Φ S *F= Hl*
Magnetic Field Intensity	H (A-t/meter)	*H = NI/l* (*l* = path length in meters)
Flux Density	B (Tesla) 1Tesla = 1Wb/meter^2	B = μH = μ *NI/l* = Φ S/*l* = Φ/A
Permiability	μ (H/meter)	μ = μrμo

		Where μr is the relative permeability of the material and **$\mu_o = 4\pi \times 10^{-7}$ H/m** is the permeability of free space.
Flux	Φ (Webers, Wb)	Φ = BA (A is cross-sectional area in square meters)
Reluctance R	S (A-t/Wb)	S = L/μrμoA , S = F/Φ
Flux Linkage	λ (Weber-turns)	λ = NΦ
Inductance	L (Henries)	L = λ/I = N Φ/I

Useful Relationships:

$\Phi = BA = \mu r \mu o HA = \mu r \mu o ANI/l = NI/R$

Comparison between magnetic circuit and electric circuit

Magnetic Circuit	Electric Circuit
Flux $=\frac{MMF}{Reluctance}$	Current $=\frac{EMF}{Resistance}$
MMF (Magnetomotive Force)	EMF (Electromotive Force)
Flux (Wb)	Current (Amp)
Flux Density B (Wb/m^2)	Current Density (A/m^2) or J
Reluctance S = $\frac{l}{\mu r \mu o A}$	Resistance R = $\frac{l}{\rho A}$
Permeance = $\frac{1}{Reluctance}$	Conductance = $\frac{1}{Resistance}$
Reluctivity	Receptivity
Permeability	Conductivity
No perfect magnetic insulator	No. of electric insulator available
No energy consumes in magnetic circuit	Energy consumes by electric circuit
Flux can flow through air also	Electric current cannot flow through air easily.

Series and Parallel Magnetic circuits.

Consider first a simple magnetic circuit, shown in Figure 21.10 with a single core material having uniform cross sectional area A and mean length of flux path l. Reluctance offered to the flow of flux is S. The corresponding electrical representation is rather simple. Due to the fact that NI = φℜ = Hl, the equivalent electrical circuit is also drawn beside the magnetic circuit. Polarity of mmf is decided on the basis of the direction of the flux which is clockwise inside the core in this case. Although in the actual magnetic circuit there is no physical connection of the winding and the core, in the electrical circuit representation mmf and reluctance are shown to be connected. One should not feel disturbed by this as because the relationship between mmf and flux prompted us to draw an electrical equivalent to facilitate easier calculation and neat visualization of the actual problem.

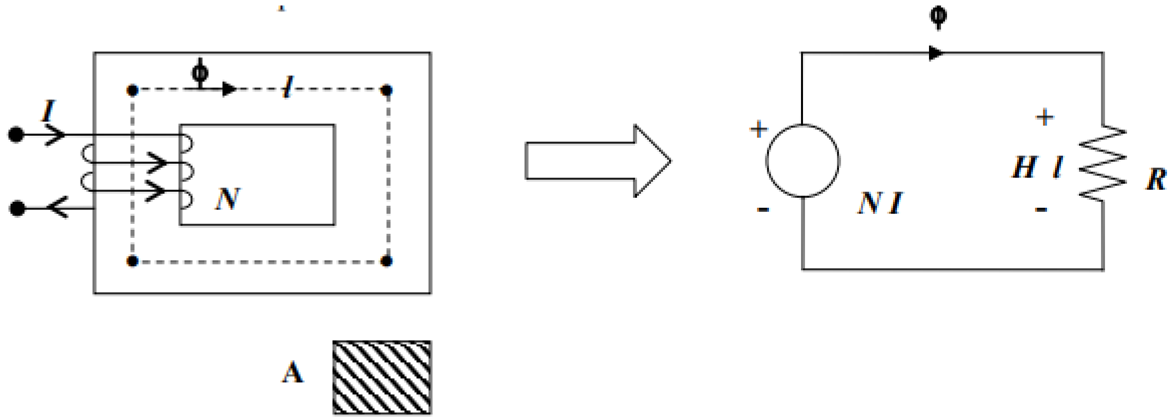


Figure 21.10:

Let us now consider another magnetic circuit which is similar to the earlier one but has a small air gap of length lg as shown in Figure 21.11 and note that it is a series circuit involving two mediums, namely (i) iron and (ii) air. It is a series circuit because same flux (φ) has to flow through the mediums. Hence total reluctance will be the sum of reluctances of iron and air( ) S=Sg +Si air .

For this circuit basic equations can be developed as follows: Let,

Number of turns = N

Exciting current i = I in A

Mean length of the flux path through iron = li in m

Length of the flux path through air = lg in m

Cross sectional area = A in m

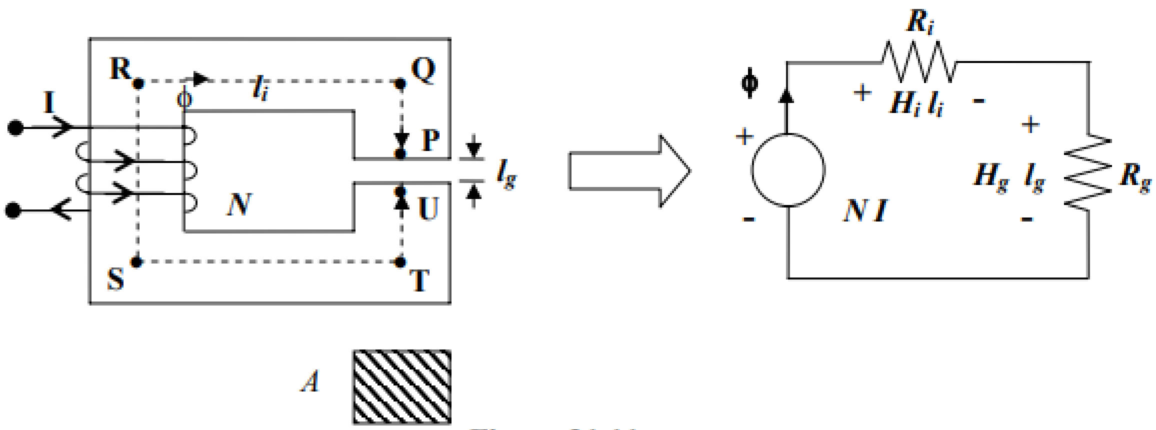


Figure 21.11

If the cross sectional area A is constant throughout, flux density B = φ/A will be also constant both in iron and air path. Since value of permeabilities are different for iron and air, the corresponding values of H too will be different.

H required for iron $Hi = \frac{B}{\mu 0\ \mu r}$

H required for air, $Hg = \frac{B}{\mu 0}$

Ampere circuital law gives $NI = H_g\, l_g + H_i\, l_i$

$$= \frac{B\ lg}{\mu 0} + \frac{B\ li}{\mu 0\ \ \mu r}$$

$$= \frac{(\varphi/A) lg}{\mu 0} + \frac{(\varphi/A)\ li}{\mu 0\ \ \mu r}$$

$$= \frac{\varphi lg}{\mu 0\ \ A} + \frac{\varphi li}{\mu 0\ \ \mu r\ A}$$

$$= \varphi S_g + \varphi Si$$

$$NI = \varphi\ (S_g + Si)$$

$$\varphi = \frac{NI}{(Sg + Si)}$$

For parallel circuit

Important equations

The various equations which will hold good are written below:

$\varphi = \varphi 1 + \varphi 2$

$NI = Hl + H_1 l_1 + Hglg = S\varphi + (S1+Sg)\varphi 1$ balance in loop1

$(S1+Sg)\varphi 1 = S2\varphi 2$ mmf balance in loop2

$H_1 l_1 + Hglg = H_2 l_2$ mmf balance in loop2

$NI = H_l\, l_1 + H_2 l_2$mmf balance in the outer loop.

Leakage and fringing of flux.

Leakage flux & Fringing effect Strictly speaking all the flux produced by the mmf will not be confined to the core. There will be some flux lines which will complete their paths largely through the air as depicted in figure 21.6. Since the reluctance (discussed in the following section) or air is much higher compared to the reluctance offered by the core, the leakage flux produced is rather small. In our discussion here, we shall neglect leakage flux and assume all the flux produced will be confined to the core only.

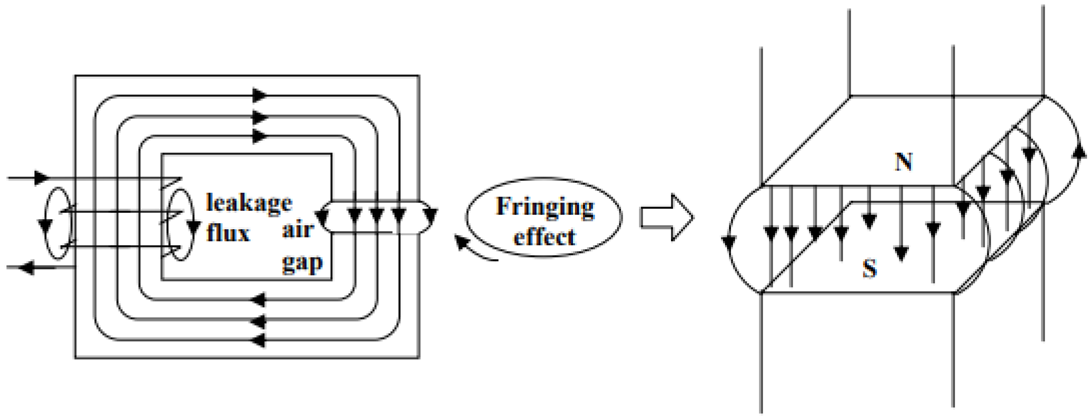


Figure 21.6:

In the magnetic circuit of figure 21.6 an air gap is present. For an exciting current, the flux lines produced are shown. These flux lines cross the air gap from the top surface of the core to the bottom surface of the core. So the upper surface behaves like a north pole and the bottom surface like a south pole. Thus all the flux lines will not be vertical and confined to the core face area alone. Some lines of force in fact will reach the bottom surface via bulged out curved paths outside the face area of the core. These flux which follow these curved paths are called fringing flux and the phenomenon is called fringing effect. Obviously, the effect of fringing will be smaller if the air gap is quite small. Effect of fringing will be appreciable if the air gap length is more. In short, the effect of fringing is to make flux density in the air gap a bit less than in the core as in the air same amount of flux is spread over an area which is greater than the core sectional area. Unless otherwise specified, we shall neglect the fringing effect in our following discussion. Effect of fringing sometimes taken into account by considering the effective area in air to be about 10 to 12% higher than the core area.

2. In the practical magnetic circuit (as in figure 21.5), the thickness (over which the lines of forces are spread = b-a) are much smaller compared to the overall dimensions (a or b) of the core. Under this condition we shall not make great mistake if we calculate H at 2 (b-a) mr = and take this to be H everywhere within the core. The length of the flux path corresponding to the mean radius i.e., 2 ml = πrm is called the mean length. This assumption allows us to calculate the total flux φ produced within the core rather easily as enumerated below:

• Calculate the mean length lm of the flux path from the given geometry of the magnetic circuit. • Apply Ampere's circuital law to calculate H = NI/lm

• Note, this H may be assumed to be same everywhere in the core.

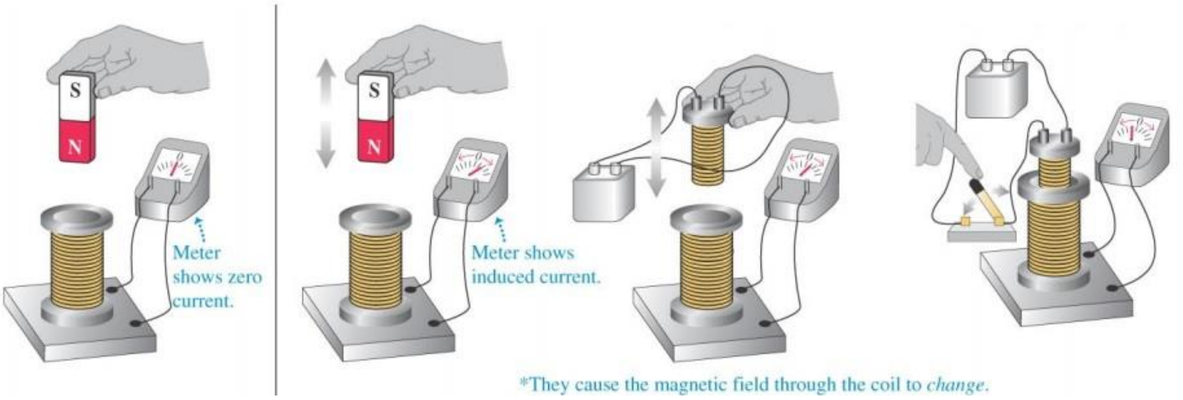
• Calculate the magnitude of the flux density B from the relation B = μoμrH.

• Total flux within the core is φ = BA, where A is the cross-sectional area of the core.

Principles of Electromagnetic induction

<u>Induction Experiments (Faraday / Henry)</u>

An induced current (and emf) is generated when: (a) we move a magnet around a coil, (b) move a second coil toward/away another coil, (c) change the current in the second coil by opening/closing a switch.



4.2 Faraday's Experiment

Let us study first the experiment conducted by **Faraday** to get understanding of electromagnetic induction.

Consider a coil having 'N' turns connected to a galvanometer as shown in the Fig. 4.1. Galvanometer indicates flow of current in the circuit, if any. A permanent magnet is moved relative to coil, such that magnetic lines of force associated with coil get changed. Whenever, there is motion of permanent magnet, galvanometer deflects indicating flow of current through the circuit.

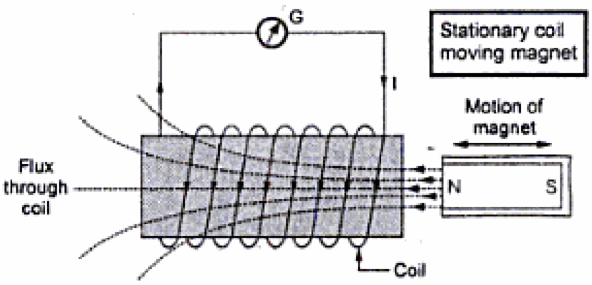


Fig. 4.1 Faraday's experiment

(4 - 1)

Similarly greater is the deflection if conductor is moved quickly in magnetic field.

In both cases, basically there is **change of flux lines with respect to conductor** i.e there is cutting of the flux lines by the conductor in which e.m.f. induced.

With this experiment Faraday stated laws called **Faraday's Laws of Electromagnetic Induction.**

This phenomenon of cutting of flux lines by the conductor to get the induced e.m.f. in the conductor or coil is called ***electromagnetic induction.***

Thus, to have induced e.m.f. there must exist,

1) A coil or conductor.
2) A magnetic field (permanent magnet or electromagnet).
3) Relative motion between conductor and magnetic flux (achieved by moving conductor with respect to flux or moving with respect to conductor.)

4.3.1 First Law

Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f. gets induced in that coil or circuit.

4.3.2 Second Law

The magnitude of the induced e.m.f. is directly proportional to the rate of change of flux linkages (flux × turns of coil).

$$\boxed{\text{Flux linkages} = \text{Flux} \times \text{Number of turns of coil}}$$

The law can be explained as below.

Consider a coil having N turns. The initial flux linking with a coil is ϕ_1

$$\therefore \qquad \text{Initial flux linkages} = N\phi_1$$

In time interval t, the flux linking with the coil changes from ϕ_1 to ϕ_2.

$$\therefore \qquad \text{Final flux linkages} = N\phi_2$$

$$\therefore \qquad \text{Rate of change of flux linkages} = \frac{N\phi_2 - N\phi_1}{t}$$

Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$\therefore \qquad e \propto \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \qquad e = K \times \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \qquad e = N\frac{d\phi}{dt}$$

4.4 Nature of the Induced E.M.F.

E.M.F. gets induced in a conductor, whenever there exists change in flux with that conductor, according to Faraday's Law. Such change in flux can be brought about by different methods.

Depending upon the nature of methods, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f. and
2) Statically induced e.m.f.

With K as unity to get units of e as volts, $d\phi$ is change in flux, dt is change in time hence ($d\phi$ / dt) is rate of change of flux.

Now as per Lenz's law (discussed later), the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.

Thus such an induced e.m.f. is mathematically expressed alongwith its sign as,

$$\therefore \qquad \boxed{e = -N\frac{d\phi}{dt} \quad \text{volts}}$$

4.7.1 Self Inductance

According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence the self induced e.m.f. will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep it to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. **So any change in current through coil is opposed by the coil.**

This property of the coil which opposes any change in the current passing through it is called **Self Inductance or Only Inductance.**

It is analogous to electrical inertia or electromagnetic inertia.

4.7.2 Magnitude of Self Induced E.M.F.

From the Faraday's law of electromagnetic induction, self induced e.m.f. can be expressed as

$$e = -N\frac{d\phi}{dt}$$

Negative sign indicates that direction of this e.m.f. is opposing change in current due to which it exists.

The flux can be expressed as,

$$\phi = (\text{Flux / Ampere}) \times \text{Ampere} = \frac{\phi}{I} \times I$$

Now for a circuit, as long as permeability 'μ' is constant, ratio of flux to current (i.e. B/H) remains constant.

$$\therefore \quad \text{Rate of change of flux} = \frac{\phi}{I} \times \text{rate of change of current}$$

$$\therefore \qquad \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -N.\frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

4.7.3 Expressions for Coefficient of Self Inductance (L)

$$L = \frac{N\phi}{I} \quad \text{... (1)}$$

But $\phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S}$

$\therefore$ $L = \frac{N.NI}{I.S}$

$\therefore$ $$\boxed{L = \frac{N^2}{S} \quad \text{henries}} \quad \text{... (2)}$$

Now $S = \frac{l}{\mu a}$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$\therefore$ $$\boxed{L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \quad \text{henries}} \quad \text{... (3)}$$

Where l = length of magnetic circuit

a = area of cross-section of magnetic circuit through which flux is passing.

The constant $\frac{N\phi}{I}$ in this expression is nothing but the quantitative measure of the property due to which coil opposes any change in current.

So this constant $\frac{N\phi}{I}$ is called **coefficient of self inductance and denoted by 'L'.**

$\therefore$ $$\boxed{L = \frac{N\phi}{I}}$$

It can be defined as flux linkages per ampere current in it. Its unit is **henry (H).**

A circuit possesses a **self inductance of 1 H** when a current of 1 A through it produces flux linkages of 1 Wb-turn in it.

$\therefore$ $$\boxed{e = -L\frac{dI}{dt} \quad \text{volts}}$$

4.8.2 Definitions of Mutual Inductance and its Unit

1) The coefficient of mutual inductance is defined as the flux linkages of the coil per ampere current in other coil.
2) It can also be defined as equal to e.m.f. induced in volts in one coil when current in other coil changes uniformly at a rate of one ampere per second.

Similarly its unit can be defined as follows :

1. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current of one ampere flowing through one coil produces a flux linkage of one weber turn in the other coil.
2. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f. of one volt in the other coil.

4.8.3 Expressions of the Mutual Inductance (M)

1) $$\boxed{M = \frac{N_2 \phi_2}{I_1}}$$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linking with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$\therefore \quad \boxed{M = \frac{N_2 K_1 \phi_1}{I_1}}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

$$\therefore \quad M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$\boxed{M = \frac{K_1 N_1 N_2}{S}}$$

If all the flux produced by coil A links with coil B then $K_1 = 1$.

$$M = \frac{N_1 N_2}{S}$$

4) Now $\quad S = \frac{l}{\mu a}$ and $K_1 = 1$

Then $\quad M = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l}$

$$\therefore \quad \boxed{M = \frac{N_1 N_2 a \mu_0 \mu_r}{l}}$$

5) If second coil carries current I_2, producing flux ϕ_2, the part of which links with coil A i.e. ϕ_1 then,

$$\phi_1 = K_2 \phi_2 \quad \text{and} \quad M = \frac{N_1 \phi_1}{I_2}$$

$$M = \frac{N_1 K_2 \phi_2}{I_2}$$

Now $\quad \phi_2 = \frac{N_2 I_2}{S}$

$$\therefore \quad M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$\therefore \quad \boxed{M = \frac{K_2 N_1 N_2}{S}}$$

If entire flux produced by coil B_2 links with coil 1, $K_2 = 1$ hence,

$$M = \frac{N_1 N_2}{S}$$

4.8.4 Coefficient of Coupling or Magnetic Coupling Coefficient

We know that, $M = \frac{N_2 K_1 \phi_1}{I_1}$ and $M = \frac{N_1 K_2 \phi_2}{I_2}$

Multiplying the two expressions of M,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

$$\therefore \quad M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

But $\frac{N_1 \phi_1}{I_1}$ = Self inductance of coil 1 = L_1

$\frac{N_2 \phi_2}{I_2}$ = Self inductance of coil 2 = L_2

$$\therefore \quad M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 L_2} = K\sqrt{L_1 L_2}$$

where $\boxed{K = \sqrt{K_1 K_2}}$

The K is called **coefficient of coupling.**

If entire flux produced by one coil links with other then $K = K_1 = K_2 = 1$ and maximum mutual inductance existing between the coil is $M = K\sqrt{L_1 L_2}$.

This gives an idea about magnetic coupling between the two coils. When entire flux produced by one coil links with other, this coefficient is maximum i.e. Unity.

It can be defined as the ratio of the actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance.

The expression for K is,

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$