SUMMER-2019

UNIT-1

Q.1 a) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. **(6)**

b) Solve $(D^2 + a^2)y = \sec ax$ by method of VOP. **(7)**

Q.2 a) Solve $(D^2 + 5D + 6)y = e^{-2x}\sin 2x + 4x^2e^x$. **(6)**

b) Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$. **(7)**

UNIT-2

Q.3 a) Show that $L^{-1}\left\{\frac{1}{s}\cos\frac{1}{s}\right\} = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4i)^2} - \frac{t^6}{(6!)^2}$. **(4)**

b) Find the L.T. of the following periodic functions

$f(t) = \frac{kt}{T}; 0 < t < T$ and given that $f(t) = f(t + T)$ **(5)**

c) If $f(t) = e^t \cos t;\ 0 < t < \pi$

$\quad = e^t \sin t;\ t > \pi$

Then find $L\{f(t)\}$ **(5)**

Q.4 a) Solve the D.E. by using Laplace transformation.

$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}; y(0) = 1 \text{ and } y'(0) = -1$ **(4)**

b) Using convolution theorem $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$. **(5)**

c) Find the L.T. of $\frac{d}{dt}\left(\frac{\sin t}{t}\right)$. **(5)**

UNIT-3

Q.5 a) Solve $y_{x+2} - 7y_{x+1} + 12y_x = \cos x$. **(5)**

b) Find the inverse Z-transform of $\dfrac{z}{z^2 + 7z + 10}$. **(4)**

c) Find the Z-transform of k^2a^{k-1}; $k \geq 1$. **(4)**

Q.6 a) i. Solve $y_{n+2} - 2\cos\alpha\, y_{y+1} + y_n = \cos \alpha n$ **(4)**

ii. Solve $u_{n+2} - 16u_n = \cos\dfrac{n}{2}$ **(4)**

b) Solve $y(n + 3) - 3y(n + 1) + 2y(n) = 0$ given that $y(0) = 4$, $y(1) = 0$, $y(2) = 8$ using z-transformation. **(5)**

UNIT-4

Q.7 a) Using Fourier integral prove that

$$\int_0^\infty \left(\frac{1-\cos\lambda\pi}{\lambda}\right)\sin\lambda x\,d\lambda = \frac{\pi}{2};\quad 0 \leq x \leq \pi$$

$$= 0;\ x > \pi$$ **(5)**

b) Solve the partial differential equations

i. $(y + z)p + (z + x)q = (x + y)$ **(4)**

ii. $yp = 2xy + \log q$ **(4)**

Q.8 a) Using Fourier integral prove that

$$\int_0^\infty \left(\frac{\sin\pi\lambda\sin x\lambda}{1-\lambda^2}\right)d\lambda = \frac{\pi}{2}\sin x;\quad 0 \leq x \leq \pi$$

$$= 0;\ x > \pi$$

b) Solve partial differential equation:

i. $(3y - 2z)p + (z - 3x)q = 2x - y$ **(4)**

ii. $p^2 - pq = 1 - z^2$ **(4)**

UNIT-5

Q.9 a) If f(z) = u + iv is an analytic function, the complex variable z and (u - v) = $e^x(\cos y - \sin y)$ then find f(z). **(5)**

b) Find the Bilinear transformation which maps the points -1, i, 1 of w-plane respectively. **(4)**

c) If f(z) is an analytic function of z then

prove $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f'(z)|=0$ **(4)**

Q.10 a) If f(z) = u + iv is analytic and

$u - v = e^x[(x - y)\cos y - (x + y)\sin y]$ then find f(z) if f(0) = 1. **(1)**

b) Expand $\frac{1}{z^2-3z+2}$ in the region

i. $|z| > 2$, **ii.** $0 < |z - 1| < 1$ **(1)**

UNIT-6

Q.11 a) Find the rate of change $\phi = xyz$ in the direction of the normal to the surface $x^2y + y^2x + yz^2$ at the point (-1, 2, 1). **(7)**

b) Prove that

i. $d\,iv\left(\frac{\bar{r}}{r^3}\right)=0$

ii. $d\,iv\,(\text{grad}\,r^n) = n(n + 1)r^{n-2}$

Where r = xi + yi + zk **(7)**

Q12 a) If $\bar{F}=(2x^2-3z)i-2xyj-4xk$ then evaluate $\iiint\limits_{v}\nabla\circ\bar{F}dv$ where v is bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4 **(7)**

b) If $\bar{A}=(3x^2+6y)i-14yzj+20xz^2k$ then evaluate the line integral. $\oint\bar{A}\circ d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the curve c,x = t, y = t^2 and z = t^3. **(7)**