WINTER-2012

UNIT 1

Q.1 a) Solve: $(D^2 + 5D + 4)\, y = x^2 + 7x + 9$ **(7)**

b) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$ **(6)**

Q.2 a) Solve: $\frac{d^2y}{dx^2} - 4y = \sinh x$ **(6)**

b) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$ **(7)**

UNIT 2

Q.3 a) Find $L^{-1}\left\{\frac{2S^2 - 4}{(S+1)(S-2)(S-3)}\right\}$ **(4)**

b) If $L\{f(t)\} = \bar{f}(s)$, when $f(t) = e^{-5t}.\sin t$ **(5)**

c) Solve the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,\ x(0) = 0,\ x'(0) = 1.$$ **(5)**

Q.4 a) Find Laplace transform of

i. $t\, e^{-t} \cosh t$, **ii.** $(t - 1)^2\, U(t - 1)$. **(6)**

b) Evaluate $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\, dt$ **(4)**

c) Find the Laplace transform of the rectified semi-wave function defined by

$$f(t) = \sin \omega t, \quad 0 < t < \frac{\pi}{\omega}$$

$$= 0, \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$$ **(4)**

UNIT 3

Q.5 a) Solve the following difference equation:

i. $y_{n+2} - 4y_n = 9n^2$, **ii.** $y_{n+2} - 3y_{n+1} - 4y_n + 6 = 0$. **(8)**

b) i. Find the inverse z-transform of $\dfrac{z}{z^2 + 7z + 10}$ **(3)**

ii. Find z-transform of $\dfrac{1}{n!}$ **(2)**

Q.6 a) If z{f(k) = F(z) then prove that $z\{kf(k)\} = -z\dfrac{d}{dz}F(z)$. **(4)**

b) Solve by z-transform

$x(k + 2) - 3x(k + 1) + 2x(k) = 4^k$.

where x(0) = 0, x(!) = 1 **(5)**

c) Solve the difference equation $(\Delta^2 + \Delta + 1)\, y = x^2$ **(4)**

UNIT 4

Q.7 a) Find the Fourier sine transform of $\dfrac{e^{-ax}}{x}$. **(7)**

b) Solve the following partial differential equations.

i. $x^2(y - z)\, p + y^2(z - x)\, q = z^2(x - y)$, **ii.** $p^2 - q^2 = x - y$. **(8)**

Q.8 a) Find the Fourier transform of

$f(x) = 1, \; |x| < 1$

$= 0 \; |x| > 1$

Hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\, dx$ **(7)**

b) Solve the following partial differential equation

i. $pq = P + q$, **ii.** $p^2 - q^2 = \dfrac{x - y}{z}$ **(7)**

UNIT 5

Q.9 a) Determine the analytic function

$f(z) = u + iv$ whose real part is $e^{-x}(x \sin y - y \cos y)$. **(6)**

b) Find Bilinear transformation which maps the points -1, i, 1 of the Z-plane onto l, i, $-l$ of the W plane respectively. **(7)**

Q.10 a) Show that the function

$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1$ is harmonic and find the corresponding analytic function $f(z) = u + iv$. **(7)**

b) Find the Taylor's series of $f(z) = \dfrac{1}{z^2 + 4}$ about $z = -i$ **(6)**

UNIT 6

Q.11 a) Find the directional derivative $f(x, y, z) = x^2y^2z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$. **(6)**

b) i. If $\bar{r} = xi + yj + zk$ show that $\nabla r^n = nr^{n-2}\bar{r}$ **(3)**

ii. Prove that $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$. **(4)**

Q.12 a) A vector field is given by

$\bar{F} = (x^2 - y^2 + x)i - (2xy + y)j$ show that the field is irrotational and find its scalar potential. Hence evaluate the line integral TF-.dr- from (1, 2) to (2, 1). **(7)**

b) If $u = x + y + z$, $v = x^2 + y^2 + z^2$,

$w = xy + yz + zx$ prove that

$\nabla u.(\nabla v \times \nabla w) = 0$ **(6)**