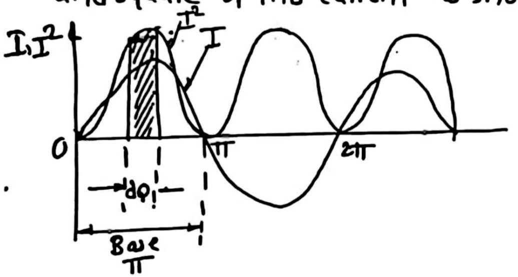
2) Analytical Method. (W=11,15)

- The above graphical method is very good way of finding rms voltage of a alternating waveform that is non-sinusoidal in nature.
- when dealing with pure sinusoidal waveform we can make life a little bit easier by using an analytical or mathematical way of finding the rms value.

Consider sinusoidal varying alternating current and square of this current as shown in fig.



The current $I = I_m \sin\theta$ while

$$I^2 = I_m^2 \sin^2\theta$$

Average value of square of current over half cycle is

$$= \frac{\text{Area of curve over half cycle}}{\text{length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} I^2 d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I^2 d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi}\frac{(1-\cos 2\theta)}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\int_0^{\pi} 1 \, d\theta - \int_0^{\pi}\cos 2\theta \, d\theta\right]$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}[\pi]$$

$$= \frac{I_m^2}{2}$$

Hence Root mean Square value i.e. rms value can be calculated as

$$I_{rms} = \sqrt{\text{Mean or Average of square of current}}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

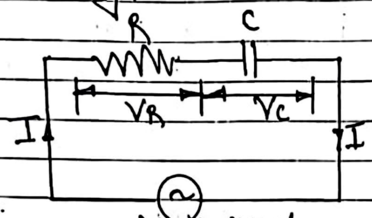
$$I_{rms} = 0.707 I_m$$

This relation shows that rms value of AC is about 70.7 % of its peak value.

Similarly,

$$V_{rms} = 0.707 V_m$$

* A.C. Through Series R-C Circuit



$V = V_m \sin \omega t$

The series combination is connected across a.c. supply given by

$$V = V_m \sin \omega t$$

Circuit draws a current I, then there are two voltage drops

1) $V_R = I \cdot R$.
2) $V_C = I X_C$

$$= \frac{I}{2\pi f C}$$

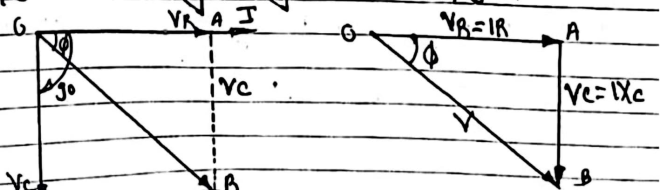
According Kirchoff Voltage law

$$\bar{V} = \bar{V_R} + \bar{V_C}$$
$$\bar{V} = \bar{IR} + \bar{IX_C}$$

9 SUN

In case of resistance, voltage and current are in phase

In case of capacitance, current leads the voltage by 90° ie voltage lags current by 90°.



From the tringle.

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_C)^2}$$

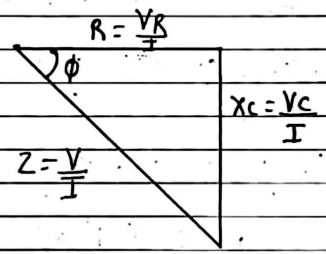
$$V = IZ$$

where

$Z = \sqrt{R^2 + (X_C)^2}$ is the impedance of the circuit

* Impedance:

Similar, in R-C. ckt in this case also, the impedance is nothing but opposition to the flow of alternating current.



X-component of impedance is R and is given by

$$R = Z \cos \phi$$

and Y component of impedance is X_C and given by

$$X_C = Z \sin \phi.$$

But, as direction of the X_C is the negative Y direction the reactangular form of the impedance is denoted as

$$Z = R - jX_C$$

while in Polar form

$$Z = |Z| \angle -\phi \ \Omega$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$\phi = \tan^{-1}\left[\frac{-X_c}{R}\right].$$

Power and power triangle

The current leads the voltage by an angle ϕ

$I = I_m \sin(\omega t + \phi)$ As current leads the V.

Power is given by

$$P = V \times I$$

$$P = V_m \sin\omega t * I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin\omega t \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$P = \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

As $\cos(-\phi) = \cos\phi$.

Now second term is cosine term whose average value over a cycle is zero. Hence average power consumed by the ckt is

$$P_{av} = \frac{V_m I_m}{2}\cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi.$$

$$P = VI\cos\phi.$$

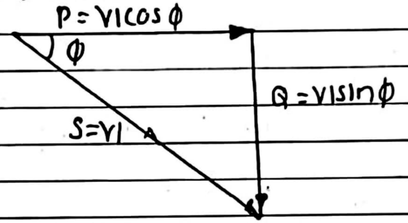
The total voltage V

$$V = V_R + V_C$$

Multiply by I.

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\overline{VI} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$



Thus various powers are

* Apparent power (S) = VI VA or KVA
* True Power (P) = $VI\cos\phi$ W or KW
* Reactive Power (Q) = $VI\sin\phi$. VAR or KVAR

Ex: A resistance of 75 Ω and 60 μF are connected in series across 230V, 50Hz supply. Find 1) X_c 2) Current 3) Power factor 4) Active and reactive components of power

Given: $R = 75\,\Omega$; $C = 60\,\mu F$

$V = 230V$, $f = 50Hz$

1) $X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}$

$X_c = 53.051\,\Omega$

2) $I = \frac{V}{Z}$ $Z = R - jX_c = 75 - j\,53.05$

$Z = 91.867 \angle -35.27°\ \Omega$

$|Z| = 91.867\,\Omega$ $\phi = -35.27°$

* Similarities:

Electric Circuit	Magnetic Circuit.
Path traced by the current is called as electric circuit	Path traced by magnetic flux is called as magnetic circuit.
E.M.F is the driving force in electric circuit, which is measured in Volt (V).	M.M.F is the driving force in magnetic circuit which is measured in ampertums (AT).
Current (I) in electric ckt measured in ampers	3. Flux (ϕ) in the magnetic ckt measured in Webers.
Resistance oppose the flow of current. unit is ohm(Ω)	4. Reluctance is opposed by magnetic path to the flux. Unit is Amper turn/weber
$R = \rho \frac{l}{a}$	$S = \frac{l}{\mu_0 \mu_r a}$
$I = \frac{emf}{Resistance}$	$\phi = \frac{m.m.f}{reluctance}$
Current density (δ) $= \frac{I}{a}$ A/m^2	The flux density (B) $= \frac{\phi}{a}$ Wb/m^2
Conductane $= \frac{1}{R}$	Permeance $= \frac{1}{S}$
Kircholl current and voltage law is applicable to electric ckt.	Kircholf m.m.f law and flux law is applicable to the magnetic ckt

No	Electric Circuit.	Magnetic Circuit.
1)	In electric circuit the current actually flow ie. there is movement of electrons.	Actually magnetic flux do not flow it exists.
2)	Number of good electrical insulators are available.	2) No perfect magnetic insulator exists.
3)	Energy must be supplied to the electric circuit to maintain the flow of current.	3) Energy is required to create the magnetic flux, but not required to maintain it.
4)	At normal conditions no current can pass through the air gap.	4) Flux can exists in air gap even at normal condition.
5)	Resistance and conductivity are independent of current density.	5) The reluctance, permeance and permeability are dependent on flux density.
6)	In electric circuit it is possible to point out where emf is acting.	6) In magnetic circuit it is difficult to find out exactly where the mmf is acting.

29 SUN