SUMMER-2013

UNIT 1

Q.1 a) Solve: $(D^2 + 3D + 2) = x.\sin 3x$. **(6)**

b) Solve by the method of variation of parameters

$(D^2 - 1)y = (1 + e^{-x})^{-2}$. **(7)**

Q.2 a) Solve: $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 2\log x + \dfrac{1}{x} + \dfrac{1}{x^2}$. **(6)**

b) Solve: $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - 3y = x^2$. **(7)**

UNIT 2

Q.3 a) Find the Laplace transform of $\int_0^t e^{-2t}.t.\sin^3 t.dt$. **(4)**

b) Use convolution theorem to find inverse Laplace transform of $\dfrac{1}{(s^2+1)^3}$. **(4)**

c) Solve by Laplace transform

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}.\sin x, \text{ where } y(0) = 0 \text{ and } y'(0) = 1.$$

(5)

Q.4 a) Evaluate $\int_0^\infty \dfrac{\cos at - \cos bt}{t} dt$. **(4)**

b) Find the inverse Laplace transform of $\dfrac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$. **(4)**

c) Using Laplace transform to solve

$$\frac{dx}{dt} - y = e^t, \qquad \frac{dy}{dt} + x = \sin t.$$ **(5)**

UNIT 3

Q.5 a) Solve the difference equations:

i. $u_{n+2} - 2u_{n+1} + 4u_n = 6.$ **(4)**

ii. $y_{n+2} - 2y_{n+1} + y_n = 3n + 4.$ **(4)**

b) Find the inverse z-transform of $F(z) = \frac{4z}{z-a}$.

i. $|z| < |a|$, **ii.** $|z| > |a|$. **(6)**

Q.6 a) Solve the difference equation

$y_{n+2} - 2 \cos \alpha . y_{n+1} + y_n = \cos n\alpha.$ **(6)**

b) Find the z-transform of $\frac{1}{n(n+1)}$. **(4)**

c) Solve the difference equation by z-transform

$u_{k+2} - 5u_{k+1} + 6u_k = 6k$ if $u(0) = u(1) = 0.$ **(4)**

UNIT 4

Q.7 a) Solve the following partial differential equations

i. $\left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2 = \frac{3a^2}{z^2}$. **ii.** $yzp + xzq + 2xy = 0.$ **(8)**

b) Find the Fourier sine and cosine transforms of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(6)**

Q.8 a) Solve the following partial differential equations:

i. $(mz - ny)\, p + (nx - lz)\, q = ly - mx.$ **(4)**

ii. $z^2(p^2 + q^2 + 1) = 1.$ **(4)**

b) Using Fourier integrals, show that

$$\int_0^\infty \frac{\sin \pi\lambda . \sin x\lambda}{1 - \lambda^2} d\lambda = \begin{cases} \frac{1}{2}\pi . \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}.$$ **(6)**

UNIT 5

Q.9 a) If f(z) is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f'(z)^2\right|.$$ **(7)**

b) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscate $\rho^2 = \cos 2\phi$; where $w = \rho e^{i\phi}$. **(6)**

Q.10 a) Find the bilinear transformation which maps the points z = 0, –i, –1 into the points w = i, 1, 0. **(7)**

b) Expand the function in Laurent's series:

$\frac{e^z}{(z-1)^2}$, about the point z = 1. **(6)**

UNIT 6

Q.11 a) A particle moves along the curve

$x = e^{-t}$; $y = 2\cos 3t$; $z = 2\sin 3t$;

where t is the time. Determine its velocity and acceleration vectors and also the magnitudes of velocity and acceleration at t = 0. **(7)**

b) If $\overline{F}\left(2xy+3z^2\right)\hat{i}+\left(x^2+4yz\right)\hat{j}+\left(2y^2+6xz\right)\hat{k}$

evaluate $\int_C \overline{F}.\overline{dr}$, where C is the

i. cirve $x = t$, $y = t^2$, $z = t^3$. **ii.** straight line x = y = z;

in each case joining the points (0, 0, 0) and (1, 1, 1). **(6)**

Q.12 a) Find the directional derivative of

$\phi = x^2y^2 + y^2z^2 + z^2x^2$ at (1, 1, –2) in the direction of tangent to the curve $x = e^{-t}$, y 2 sin t + 1, z = t – cos at at t = 0. **(7)**

b) Prove that:

i. $\nabla.(\phi\nabla\psi - \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$. **ii.** $\nabla^2 f[r] = \frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$. **(3)**