

WINTER-2018

UNIT 1

Q.1 a) Without constructing Truth table, show that

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R \quad (7)$$

b) Obtain principal conjunctive normal form of

$$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P) \quad (7)$$

Q.2 a) For the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$ draw circuit diagram using AND, OR and NOT gates only. (7)

b) Obtain the principle conjunctive normal form of $P \rightarrow (P \wedge (Q \rightarrow P))$. (7)

UNIT 2

Q.3 a) Show that

$(\exists x)M(x)$ follows logically from the premises:

$$(x)(H(x) \rightarrow M(x)) \text{ and } (\exists x)H(x) \quad (7)$$

b) Symbolize the following statement:

i. Rover is a dog, **ii.** All dogs are animal,

iii. Some dogs are black. (6)

Q.4 a) Show without constructing truth tables that the following statements cannot be true simultaneously.

$$R \vee M, R \vee S, \sim M, \sim S. \quad (7)$$

b) Show that $R \vee S$ follows logically from the premises

$$(C \vee D, C \vee D) \rightarrow \sim H, \sim H \rightarrow (A \wedge \sim B)$$

$$\text{and } (A \wedge \sim B) \rightarrow (R \vee S) \quad (6)$$

UNIT 3

Q.5 a) Let $X = \{1, 2, \dots, 7\}$ and $R = \{\langle x, y \rangle \mid x - y \text{ is divisible by } 3\}$ show that R is an equivalence relation. Draw the graph of R . (6)

b) Given the relation matrices M_R and M_S find $\overline{M_R}$, $\overline{M_S}$, $\overline{M_{R \circ S}}$ and Show that $M_{\overline{R \circ S}} = M_{\overline{S} \circ \overline{R}}$. (7)

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Q.6 a) Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagram and formal proof. (7)

b) Explain the following operations on set (6)

i) Union, **ii.** Intersection,

iii. Relative Complement, **iv.** Absolute Complement.

UNIT 4

Q.7 a) Write down the composition table for $(Z_7, +_7)$ and $(Z_7, *_7)$. (7)

b) What is coset? Find the left coset of $\{[0], [4]\}$ in group $\langle Z_7, +_7 \rangle$. (7)

Q.8 a) Let $Q_P = \langle \{E, T, F\}, \{a, +, *, (,), E, \phi\} \rangle$ where ϕ is the set of production as $E \rightarrow E + T$, $E \rightarrow T$, $T \rightarrow T * F$, $T \rightarrow F$, $F \rightarrow (E)$, $F \rightarrow a$. Generate derivation for $a * a + a$. (7)

b) Show that $\langle Z_7, +_7, *_7 \rangle$ is a commutative ring with identity. (7)

UNIT 5

Q.9 a) Draw the diagram of the lattices $\langle S_n, D \rangle$ for $n = 4, 6, 8, 12, 15, 45, 60, 75$ and 210 . (6)

b) For the following function (7)

a. $f = x + y + z$, **b.** $f = \overline{xyz} + \overline{xy}z + xy\overline{z}$.

Given

i. Circuit diagram representation.

ii. Truth table representation.

iii. k-map representation.

Q.10 a) Obtain sum of product and product of sum canonical form of the following expression.

i. $x_1 x_2 + x_3$, **ii.** $[(x_1 + x_2) (x_3 x_4)]$. (6)

b) In any Boolean algebra show

i. $a = 0 \Leftrightarrow ab' + a'b = b$, **ii.** $a = b \Leftrightarrow ab' + a'b = 0$. (7)

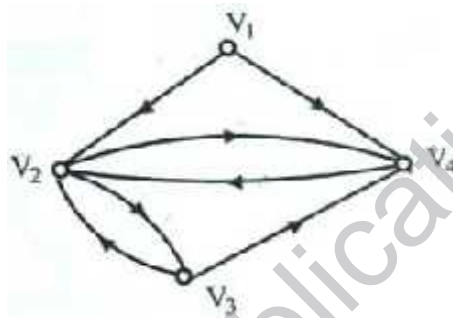
UNIT 6

Q.11 a) Explain with example. (6)

- i.** Graph, **ii.** Indegree, **iii.** Outdegree,
- iv.** Tree, **v.** Path, **vi.** Cycle,
- vii.** Complete binary tree.

b) Obtain adjacency matrix A of the digraph shown below. Find the elementary paths of length 1 and 2 from V_1 to V_4 . Show that there is also a simple path of length 4 from V_1 to V_4 .

(7)



Q.12 a) Give the directed tree representation of the formula $(P \vee (\neg P \wedge Q)) \wedge ((\neg P \wedge Q) \wedge \neg R)$ (6)

b) Explain with example

i. Adjacency matrix, **ii.** Path matrix. (7)