SUMMER-2013

UNIT 1

Q.1 a) Show the following equivalence: **(7)**

i. $P \rightarrow (Q \vee R)\ (P \rightarrow Q) \vee (P \rightarrow R)$

ii. $(\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Rightarrow (\neg P \wedge Q)$

b) Define the following terms:- **(6)**

i. Statement
ii. Tautological Implication
iii. Well formed formula
iv. Normal Forms.

Q.2 a) Show the following implications without constructing truth tables: **(6)**

i. $P \rightarrow (P \rightarrow R) \Rightarrow (P\ (P \wedge Q))$

ii. $(((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R)) \Rightarrow (Q \rightarrow R)$

b) Obtain PCNF **(6)**

i. $(\neg P \rightarrow R) \vee (Q \leftrightarrow P)$
ii. $(P \wedge Q) \vee (\neg P \wedge R)$.

UNIT 2

Q.3 a) Show that the conclusion C follows from premises $H_1, H_2, \ldots$ in following cases using truth table: **(7)**

i. $H_1 : \neg Q, H_2 : P \rightarrow Q,\ C : \neg P$

ii. $H_1 : (P \vee Q),\ H_2 : P \rightarrow R,\ H_3 : Q \rightarrow R,\ C : R$

b) Show that $R \wedge (P \vee Q)$ is valid conclusion of premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$ using rules of inference. **(6)**

Q.4 a) Show that $R \rightarrow S$ can be derived from premises

$P \rightarrow (Q \rightarrow S,)\ \neg R \vee P$ and Q. **(13)**

i. Show that $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$, using indirect proof method.

ii. Show that following premises are inconsistent:

$E \rightarrow S,\ S \rightarrow H,\ A \rightarrow \neg H,\ E \wedge A$.

UNIT 3

Q.5 a) Draw the Benn diagram showing: **(7)**

i. $A \cap B = A \cap C$ but $B \neq C$

ii. $A \cup B = A \cup C$ but $B \neq C$

iii. $A \cap B = A \cap C$ but B is not subset of C

b) Let R and S be given relations as: **(6)**

$R = \{\langle 1,2\rangle,\langle 3,4\rangle\langle 2,32\rangle\}$

$S = \{\langle 4,2\rangle,\langle 2,5\rangle,\langle 3,1\rangle\langle 1,3\rangle\}$

Find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$,

$R \circ S$, $S \circ S$.

Q.6 a) Given the relation matrices M_R and M_S, find

$M_{\overline{(R.S)}}.M_{\overline{R}}.M_{\overline{S}}.M_{\overline{S.R}}.M_{\overline{R.S}}$. **(7)**

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$ **(7)**

b) Define the following terms: **(6)**

i. Equal Sets **ii.** Power Set

iii. Relative Complement **iv.** Absolute Complement.

UNIT 4

Q.7 a) Define the algebraic system:

i. $\langle Z_4, +_4, *_4\rangle$

ii. $\langle Z_6^* +_6\rangle$ where $Z_6^* = Z_6 - \{[0]\}$. **(7)**

b) Explain the following terms with example:

i. Group **ii.** Semigroup

iii. Monoid **iv.** Direct Product of Groups.

Q.8 a) Show that with every element in a group is its own inverse then the group must be abelian. **(7)**

b) Let $\langle G,*\rangle$ is group where, $G = \langle \alpha, \beta, \gamma, \delta\rangle$ and $*$ is given by composition table:

*	α	β	γ	δ
α	α	β	γ	δ
β	β	α	δ	γ
γ	γ	δ	β	α
δ	δ	γ	α	β

Find out the identity element and inverse of each element of group with justification. **(7)**

UNIT 5

Q.9 a) Let is integer and S_R contains all divisors of n which are less than or equal to n Let D denote relation of division such that a, b ε S_n and $a \leq b$ if a divides b. Draw the lattice $< S_n\ D >$ for various values of n = 5, 15, 45, 75. **(7)**

b) Prove the following Boolean identities: **(7)**

i. $a \oplus (a'^{*}b) = a \oplus b$ **ii.** $a * (a' \oplus b) = a * b$

iii. $(a * b) \oplus (a * b') = a$ **iv.** $(a * b * c) \oplus (a * b) = a * b$

Q.10 a) Simplify following Boolean Expression: **(7)**

i. $(a * b)' \oplus (a \oplus b)'$ **ii.** $(a'^{*}b'^{*}c) \oplus (a * b'^{*}c) \oplus (a * b'^{*} c')$

iii. $(a \oplus b') * (b \oplus c') * (c \oplus a')$

b) Obtain the sum of product canonical form of following using three variables x_1, x_2, and x_3: **(7)**

i. $x_1 + x_2$ **ii.** $(x_1 + x_2)' + (x'_1 * x_3)$

UNIT 6

Q.11 a) Define the terms: **(6)**

i. Isomorphic Graphs **ii.** Strongly Connected Graph

iii. Simple Graph **iv.** Parallel Edges.

b) Give the Warshall’s algorithm for path matrix with example. **(7)**

Q.12 a) Show that in a complete binary tree the total number of edges is given by $2(n_t - 1)$ where n_t is the total number of terminal nodes. **(6)**

b) Traverse the following tree with Preorder Postorder and Inorder: **(7)**