WINTER-2019

UNIT-1

Q.1 a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ **(6)**

b) Solve by method of variation of parameter:

$(D^3 + D)y = \text{cosec } x.$ **(7)**

Q.2 a) solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x + x^2e^x$ **(6)**

b) Solve $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = \left(3x^2 + 4x + 1\right)$ **(7)**

UNIT-2

Q.3 a) Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$ **(4)**

b) Using convolution theorem find inverse Laplace Transform of: $\frac{1}{S^2(S^2+1)}$ **(5)**

c) Solve the differential equation using Laplace transform: **(5)**

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-1}.y(0) = y'(0) = 1.$$

Q.4) Find Laplace transform of: $e^{-3t}\int_0^t t\sin 3t\, dt$ **(4)**

b) Find f(t) if f(s) is given by $\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$ **(5)**

c) Find Laplace transfer of half wave rectified since wave **(5)**

defined as f(t) = f(t + 2π/a) and $f(t) = \left(t + \frac{2\pi}{a}\right)$

UNIT-3

Q.5 a) solve the difference equation

$y_{n+2} - 7y_{n+1} + 10y_n = 12\, e^{3n} + 4^n$ **(4)**

b) Find the inverse Z – transform of $\dfrac{(z-4)}{(z-1)(z-2)^2}$ **(4)**

c) Solve $y(k+2) - 3y(k+1) + 2y(k) = 4^k$ given $y(0) = 0$, $y(1) = 1$ using Z-transform. **(5)**

Q.6 a) Solve $y_{n+2} - 2y_{n+1} = n^2 2^n$. **(4)**

b) Find the Z-transform of $k^2 a^{k-1}$; $k \geq 1$. **(4)**

c) Solve $y_{n+2} - 3y_{n+1} + 2y_n = 0$ where $y(0) = 0$ $y(1) = 1$ using z-transform

UNIT-4

Q.7 a) Find Fourier Transform of **(6)**

$$f(x) = 1 - x^2 \ , \ |x| \leq 1$$
$$= 0 \qquad , \ |x| > 1$$

Hence evaluate $\int_0^\infty \dfrac{x\cos x - \sin x}{x^3} \cos\dfrac{x}{2}\, dx$

b) Solve the following partial differential equation:

1. $py = 2xy + \log q$ **(4)**

2. $pq = x^m y^n z^{2l}$ **(4)**

Q.8 a) Using Fourier integral prove that **(6)**

$$\int_0^\infty \left(\frac{\sin \pi\lambda \cdot \sin x\lambda}{1-\lambda^2}\right) d\lambda = \frac{\pi}{2}\sin x ; \quad 0 \leq x \leq \pi$$
$$= 0 \qquad\qquad x > \pi$$

b) Solve the following partial differential equation.

1. $\left(\dfrac{\partial z}{\partial y}\right)^2 + \left(\dfrac{\partial z}{\partial x}\right)^2 = \dfrac{3a^2}{z^2}$ **(4)**

2. $x(y^2 - z^2)p + y(z^2 - x^2)q + z(y^2 - x^2) = 0$ **(4)**

UNIT-5

Q.9 a) if $(z) = u + iv$ is an analysis function of the complex variable z and $u - v = e^x (\cos y - \sin y)$ then find f(z). **(4)**

b) Expand: $f(z) = \dfrac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$ **(4)**

c) Evaluate $\int_c \dfrac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = 3/2$ using Residue theorem. **(5)**

Q.10 a) Evaluate $\int_c \dfrac{z+4}{(z^2+2z+5)} dz$ if C is the circle $|z+1| = 1$. **(4)**

b) Expand the function $f(z) = \dfrac{z^2-1}{(z+2)(z+3)}$ in using region $2 < |z| < 3$ by Laurent's series. **(4)**

c) Find the bilinear transformation which maps the point z =1, i, -1 onto the points. w = i, 0, -i. **(5)**

UNIT-6

Q.11 a) Find the directional derivation of $\phi = e^{2x}\cos yz$ at (0, 0, 0) in the directional of tangent to the curve x = a sint, y = a cost, z = at at $t = \pi/4$. **(6)**

b) A vector field is given by $\overline{F} = (\sin y)i + x(1+\cos y)j$ evaluate the line integral over a circular path given $x^2 + y^2 = a^2$, z = 0. **(7)**

Q.12 a) Show that $\vec{F}$ is irrotational and find the scalars ϕ. Such that $\vec{F} = \nabla\phi$. $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$. **(6)**

b) Prove that:

1. $\operatorname{div}\left(\dfrac{\vec{r}}{r^3}\right) = 0$ **(3)**

2. $\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$ where $\vec{r} = xi + yj + zk$ **(4)**