SUMMER – 2023

UNIT-1

Q.1. a) Solve $(D^2 + 5D + 6)\,y = e^{-2x} \sin 2x + 4x^2 e^x$ **[6]**

b) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + 4y = 4\sec^2(2x)$ **[7]**

Q.2. a) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ **[6]**

b) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ **[7]**

UNIT-2

Q.3. a) Find Laplace transform of $t\, e^{3t} \sin 2t$ **[4]**

b) Find $f(t)$ if $\bar{f}(s) = \text{lag}\frac{(s+b)}{(s+a)}$ **[5]**

c) If $L\{J_0(t)\} = \frac{1}{\sqrt{1+s^2}}$ show that $\int_0^\infty J_0(t)dt = 1$ **[5]**

Q.4. a) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$ **[4]**

b) Use convolution theorem to find Inverse. Laplace transform of $\frac{1}{(s+1)(s^2+1)}$ **[5]**

c) Find Laplace transform of the functions **[5]**

$$f(t) = \frac{t}{a} \quad 0 < t < a = \frac{1}{a}(2a - t) \quad a < t < 2a$$

UNIT-3

Q.5. a) Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} + 5y = e^{-t}\sin t$ **[7]**

$Y(0) = 0,\ y'(0) = 1$

b) Express the function **[6]**

$$f(x) = 1 \quad \text{for} |x| \le 1$$

$$0 \quad \text{for} |x| > 1$$

as a Fourier integral. Hence evaluate

$$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$$

Q.6. a) Solve $\frac{d^2y}{dt^2} + 9y = 18t$ if $y(0) = 0$, & $y\left(\frac{\pi}{2}\right) = 0$ **[7]**

b) Find the Fourier transform of **[6]**

$$f(x) = 1 \quad |x| \le 1$$

$$= 0 \quad |x| > 1$$

Hence evaluate

$$\int_0^\infty \frac{\sin x}{x} dx$$

UNIT-4

Q.7. a) Solve

i) $x\,p \tan y = q - 1$ **[4]**

ii) $(mz - ny)p + (nx - lz)\,q = ly - mx$ **[4]**

b) Fit a second degree parabola to the following data by least square method. **[5]**

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

Q.8. a) Solve i) $p^2 - pq = 1 - z^2$ **[4]**

ii) $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \frac{x-y}{xy}$ **[4]**

b) Find the equations of the lines of regression for the data **[5]**

X	62	64	65	69	70	71	72	74
y	126	125	139	145	165	152	180	208

UNIT-5

Q.9. a) Find the analytic function **[7]**

$$F(z) = u + iv \quad \text{if } \frac{u}{v} = \cot y$$

b) Expand $f(z) = \dfrac{1}{z^2 - 3z + 2}$ in the region **[6]**

i) $|z| < 1$

ii) $1 < |z| < 2$

Q.10. a) If $f(z) = u + iv$ is an analytic function and $(u - v) = (x - y)(x^2 + 4xy + y^2)$ find $f(z)$ in terms of z. **[7]**

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$. **[6]**

UNIT-6

Q.11. a) Find div of $\bar{f}$ and curl $\bar{f}$ where $\bar{f} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$ **[7]**

b) Evaluate the line integral $\int_C \bar{f}.d\bar{r}$ where c is the circle $x^2 + y^2 = 1$ in xy-plane and $\bar{f} = (2x^2 - y^2)i + (x^2 + y^2)j$ **[7]**

Q.12. a) Show that $\bar{f} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational. Hence find a scalar function ϕ such that $\bar{f} = \nabla\phi$ **[7]**

b) Find the directional derivative of $\phi = e^{2x} \cos yz$ at the origin in the direction of the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$. **[7]**