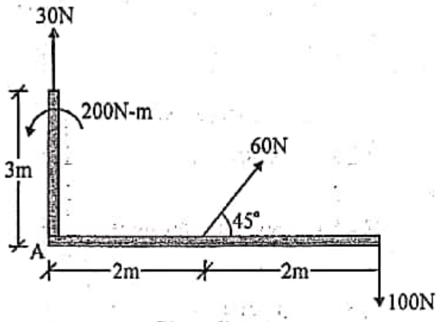
$$\uparrow + \sum F_y = 30 + 60\sin(45) - 100$$
$$= -27.57N$$
$$\therefore \sum F_y = 27.57N(\downarrow)$$



Given diagram

Now,

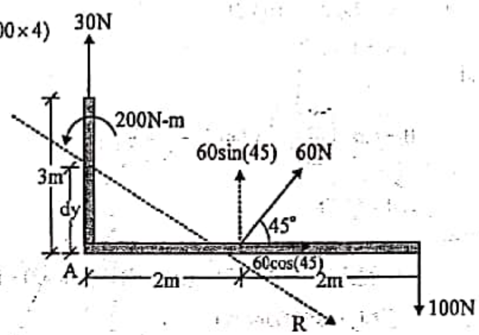
$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$
$$= \sqrt{42.42^2 + 27.57^2}$$
$$\therefore R = 50.59N$$

and

$$\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{27.57}{42.42}\right)$$
$$\therefore \theta = 33.02°$$
$$(\curvearrowright_+) \sum M_A^F = -200 - (60\sin 45 \times 2) + (100 \times 4)$$
$$\therefore \sum M_A^F = 115.14N.m(\curvearrowright)$$

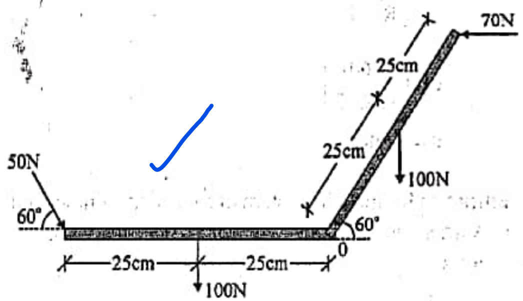
and

$$dy = \left(\frac{\sum M_A^F}{\sum F_x}\right)$$
$$\therefore dy = \left(\frac{115.14}{42.42}\right)$$
$$\therefore dy = 2.714m$$



Example 5 : Determine the resultant of the forces acting on the belt rank shown in figure also locate is it on the belt rank.

Solution :



Given diagram

$$\overset{+}{\overrightarrow{\sum F_x}} = 50\cos(60) - 70$$
$$= -45N$$
$$\therefore \sum F_x = 45N(\leftarrow)$$

and

$$\uparrow + \sum F_y = 50\sin(60) - 100 - 100$$
$$= -24.30N$$
$$\therefore \sum F_y = 24.30N(\downarrow)$$

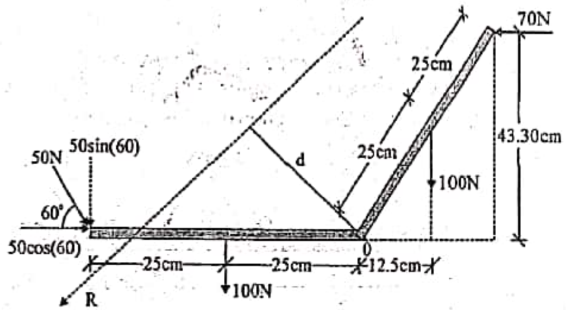
Now,

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$= \sqrt{45^2 + 24.30^2}$$
$$\therefore R = 34.86N$$

and

$$\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$
$$\theta = \tan^{-1}\left(\frac{24.30}{45}\right)$$
$$\therefore \theta = 79.52°$$



Now,

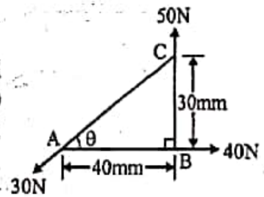
$$(\curvearrowright_+) \sum M_o^F = -(50\sin 60 \times 50) - (100 \times 25) + (100 \times 12.5) - (70 \times 43.30)$$
$$= -6446.06N.cm$$
$$\therefore \sum M_A^F = 6446.06N.cm(\curvearrowleft)$$

and

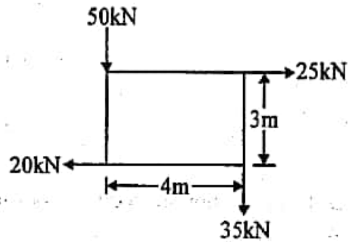
$$d = \left(\frac{\sum M_o^F}{R}\right)$$
$$\therefore d = \left(\frac{6446.06}{34.86}\right)$$
$$\therefore d = 184.91cm$$

- EXERCISE -

1. Define Moment of force.
2. Define Couple and State the characteristics of couple.
3. A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.



4. A system of forces are acting at the corners of a rectangular block as shown in Fig. Determine magnitude of the resultant of such a system of forces.



Example 2 : Find the tension in the spring and the reaction of the wall to the sphere at the point of contact. Self weight of sphere = 100kN

Solution :

Apply Lami's theorem,

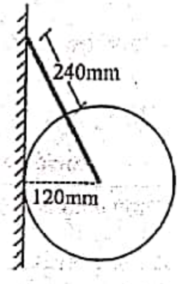
$$\frac{100}{\sin(180-70.52)}=\frac{R}{\sin(90+70.52)}=\frac{T}{\sin(90)}$$

$$\therefore \frac{100}{\sin(180-70.52)}=\frac{R}{\sin(90+70.52)}$$

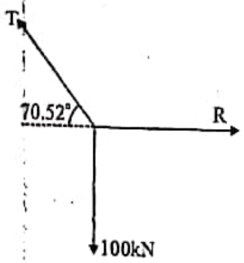
$$\therefore R = 35.35\text{kN}$$

and $$\therefore \frac{100}{\sin(180-70.52)}=\frac{T}{\sin(90)}$$

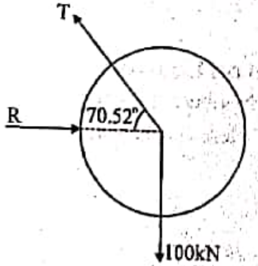
$$\therefore T = 106.06\text{kN}$$



Given diagram



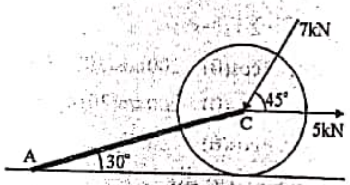
FBD (For Lami's theorem)



FBD of Sphere

Example 3 : A roller of weight 10kN rest on smooth horizontal floor and it is connected to the floor by the AC as shown in figure. Determine the force in bar AC and reaction from the floor if the roller is subjected to the floor as shown in figure.

Solution :



Given diagram

$$\overset{+}{\rightarrow}\Sigma F_x = 0$$

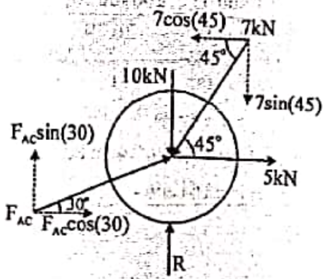
$$\therefore F_{AC}\cos(30)-7\cos(45)+5=0$$

$$\therefore F_{AC}\cos(30)=7\cos(45)-5$$

$$\therefore F_{AC}=\frac{7\cos(45)-5}{\cos(30)}$$

$$\therefore F_{AC} = 0.058\text{kN}$$

and $$\uparrow+\Sigma F_y = 0$$



FBD of Roller

$$\therefore F_{AC}\sin(30)-7\sin(45)-10+R=0$$

$$\therefore R = -0.058\sin(30)+7\sin(45)+10$$

$$\therefore R = 14.92\text{kN}$$

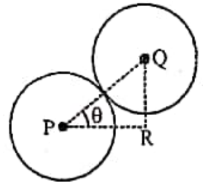
Example 4 : Two sphere, each of weight 1000N and of radius 25cm rest on horizontal channel of width 90cm as shown in figure. Find the reactions on the points of contact A, B and C.

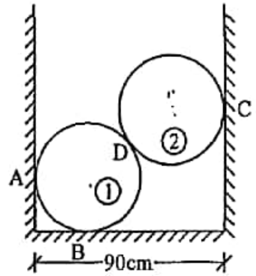
Solution :

In Δ PQR,

$$\cos(\theta)=\frac{PR}{PQ}=\frac{(90-25-25)}{25+25}$$

$$\therefore \theta = 36.86°$$





Given diagram

Now,

Consider FBD of sphere-2,

Apply condition of equilibrium,

$$\uparrow+\Sigma F_y = 0$$

$$\therefore R_D\sin(36.86)-1000=0$$

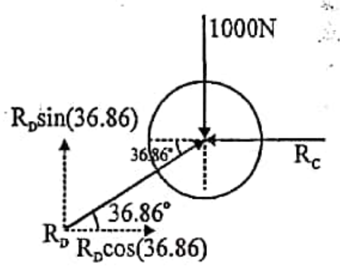
$$\therefore R_D = 1666.66\text{N}$$

and $$\overset{+}{\rightarrow}\Sigma F_x = 0$$

$$\therefore R_D\cos(36.86)-R_C=0$$

$$\therefore R_C = 1666.66\cos(36.86)$$

$$\therefore R_C = 1333.33\text{N}$$



FBD of Sphere - 2

Now,

Consider FBD of sphere-1,

Apply condition of equilibrium,

$$\overset{+}{\rightarrow}\Sigma F_x = 0$$

$$\therefore R_A - R_D\cos(36.86) = 0$$

$$\therefore R_A = 1666.66\cos(36.86)$$

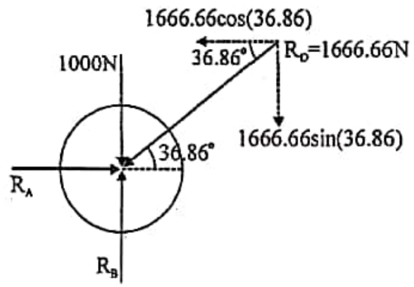
$$\therefore R_A = 1333.32\text{N}$$

and $$\uparrow+\Sigma F_y = 0$$

$$\therefore R_B - R_D\sin(36.86)-1000=0$$

$$\therefore R_B = 1666.66\sin(36.86)+1000$$

$$\therefore R_B = 2000\text{N}$$



FBD of Sphere - 1

ii) Web,

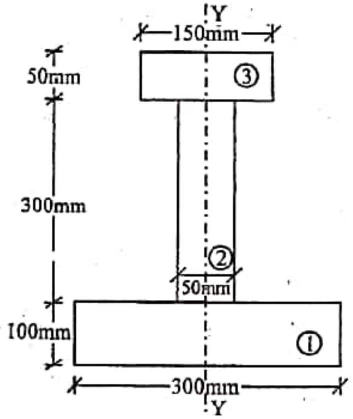
$$a_2 = 300 \times 50 = 15000\text{mm}^2$$

and $$y_2 = 100 + (300/2) = 250\text{mm}$$

iii) Top Flange,

$$a_3 = 150 \times 50 = 7500\text{mm}^2$$

and $$y_3 = 100 + 300 + (50/2) = 425\text{mm}$$



We know that distance between centre of gravity of the section and bottom of the flange.

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(30000 \times 50) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500}$$

$$\bar{y} = 160.7\text{mm}$$

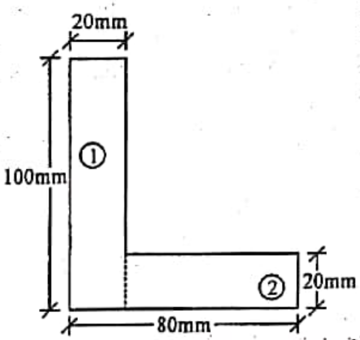
5.5 Problems on Centre of Gravity of Unsymmetrical Sections

Example 1 : Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.

Solution :

As the section is not symmetrical about any axis, therefore we have to find out thevalues of x and y for the angle section. Split up the section into two rectangles as shown in figure.

Let, left face of the vertical section and bottom face of the horizontal section be axes ofreference.



i) Rectangle-1,

$$a_1 = 100 \times 20 = 2000\text{mm}^2$$

$$x_1 = (20/2) = 10\text{mm}$$

and $$y_1 = (100/2) = 50\text{mm}$$

ii) Rectangle-2,

$$a_2 = 60 \times 20 = 1200\text{mm}^2$$

$$x_2 = 20 + (60/2) = 50\text{mm}$$

and $$y_2 = (20/2) = 10\text{mm}$$

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} \qquad \therefore \bar{x} = 25\text{mm}$$

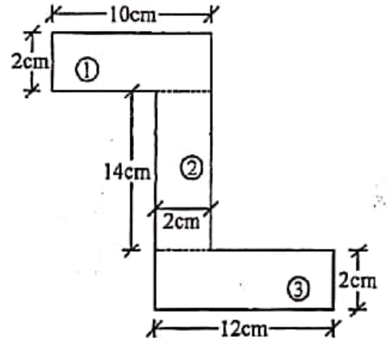
$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} \qquad \therefore \bar{y} = 35\text{mm}$$

Example 2 : Find the centroid of lamina as shown in figure.

Solution :

As the section is not symmetrical about any axis, therefore we have to find out thevalues of x and y for the angle section. Split up the section into three rectangles as shown in figure.

Let, left face of the vertical section and bottom face of the horizontal section be axes of reference.



i) Rectangle-1,

$$a_1 = 10 \times 2 = 20\text{cm}^2$$

$$x_1 = (10/2) = 5\text{cm}$$

and $$y_1 = 2 + 14 + (2/2) = 17\text{cm}$$

ii) Rectangle-2,

$$a_2 = 14 \times 2 = 28\text{cm}^2$$

$$x_2 = 8 + (2/2) = 9\text{cm}$$

and $$y_2 = 2 + (14/2) = 9\text{cm}$$

iii) Rectangle-3,

$$a_3 = 12 \times 2 = 24\text{cm}^2$$

$$x_3 = 8 + (12/2) = 14\text{cm}$$

and $$y_3 = (2/2) = 1\text{cm}$$

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(20 \times 5) + (28 \times 9) + (24 \times 14)}{20 + 28 + 24} \qquad \therefore \bar{x} = 9.56\text{cm}$$

and

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(20 \times 17) + (28 \times 9) + (24 \times 1)}{20 + 28 + 24} \qquad \therefore \bar{y} = 8.56\text{cm}$$

$$I_{YY1}=\left(\frac{hb^3}{12}\right)=\left(\frac{50\times150^3}{12}\right)=14.0625\times10^6\,mm^3$$

and M.I. of rectangle (2) about Y-Y axis,

$$I_{YY2}=\left(\frac{hb^3}{12}\right)=\left(\frac{150\times50^3}{12}\right)=1.5625\times10^6\,mm^4$$

Now, Moment of Inertia of the whole section about Y-Y axis,

$$I_{YY}=I_{YY1}+I_{YY2}$$

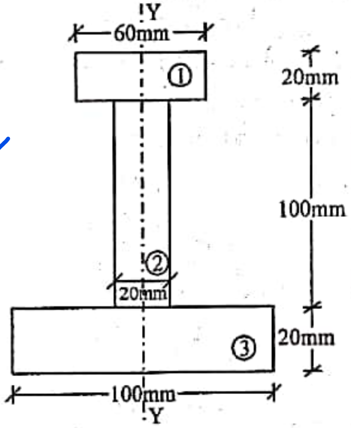
$$I_{YY}=(14.0625\times10^6)+(1.5625\times10^6)$$

$$\therefore I_{YY}=15.625\times10^6\,mm^4$$

Example 2 : An I-section is made up of three rectangles as shown in figure. Find themoment of inertia of the section about the horizontal axis passing through the centre of gravity ofthe section.

Solution : The given I-section is shown in figure. First, let us find out CG of the section. As the section is symmetrical about Y-Y axis, therefore its centreof gravity will lie on this axis. Split up the whole section into three rectangles viz., 1, 2 and 3 as shown in figure.

Let bottom of the bottom flange be the axis of reference.



i) Rectangle-1,

$$a_1=60\times20=1200mm^2$$

and $\quad y_1=20+100+(20/2)=130mm$

ii) Rectangle-2,

$$a_2=20\times100=2000mm^2$$

and $\quad y_2=20+(100/2)=70mm$

iii) Rectangle-3,

$$a_3=100\times20=200mm^2$$

and $\quad y_3=(20/2)=10mm$

We know that distance between centre of gravity of the section and bottom of the Rectangle - 3

$$\bar{y}=\frac{a_1y_1+a_2y_2+a_3y_3}{a_1+a_2+a_3}=\frac{(1200\times130)+(200\times70)+(2000\times10)}{1200+2000+2000}\qquad\therefore\bar{y}=60.8mm$$

Now,

Moment of inertia about X-X axis,

M.I. of rectangle (1) about an axis through its centre of gravity and parallelto X-X axis.

$$I_{G1}=\left(\frac{bh^3}{12}\right)=\left(\frac{60\times20^3}{12}\right)=40\times10^3\,mm^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1=(y_1-\bar{y})=130-60.8=69.2mm$$

$\therefore$ Moment of inertia of rectangle (1) about X-X axis

$$I_{XX1}=I_{G1}+a_1h_1^2=(40\times10^3)+[1200\times(69.2)^2]=5786\times10^3\,mm^4$$

Similarly, M.I. of rectangle (2) about an axis through its centre of gravity andparallel to X-X axis,

$$I_{G2}=\left(\frac{bh^3}{12}\right)=\left(\frac{20\times100^3}{12}\right)=1666.7\times10^3\,mm^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2=(y_2-\bar{y})=70-60.8=9.2mm$$

$\therefore$ Moment of inertia of rectangle (2) about X-X axis

$$I_{XX2}=I_{G2}+a_2h_2^2=(1666.7\times10^3)+[2000\times(9.2)^2]=1836\times10^3\,mm^4$$

Similarly, M.I. of rectangle (3) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G3}=\left(\frac{bh^3}{12}\right)=\left(\frac{100\times20^3}{12}\right)=66.7\times10^3\,mm^4$$

and distance between centre of gravity of rectangle (3) and X-X axis,

$$h_2=(\bar{y}-y_3)=60.8-10=50.8mm$$

$\therefore$ Moment of inertia of rectangle (3) about X-X axis

$$I_{XX3}=I_{G3}+a_3h_3^2=(66.7\times10^3)+[2000\times(50.8)^2]=5228\times10^3\,mm^4$$

Now, Moment of Inertia of the whole section about X-X axis,

$$I_{XX}=I_{XX1}+I_{XX2}+I_{XX3}$$

$$I_{XX}=(1666.7\times10^3)+(1836\times10^3)+(5228\times10^3)$$

$$\therefore I_{XX}=12850\times10^3\,mm^4$$

Example 3 : Find the moment of inertia about the centroidal X-X and Y-Y axes of the angle section shown in figure.

Solution : First of all, let us find the centre of gravity of the section. As the section is notsymmetrical about any section, therefore we have to find out the values of and for the anglesection. Split up the section into two rectangles 1 and 2 as shown in figure.

Let, left face of the vertical section and bottom face of the horizontal section be axes ofreference.

i) *Rectangle-1,*

$$a_1 = 20 \times 100 = 2000\text{mm}^2$$

$$x_1 = (b/2) = (20/2) = 10\text{mm}$$

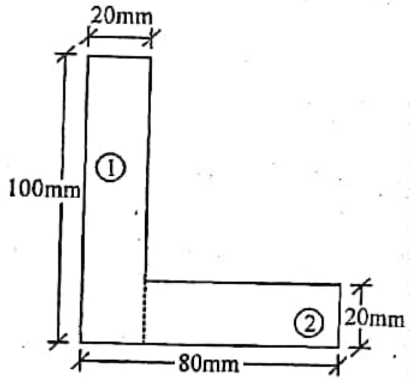
and $$y_1 = (h/2) = (100/2) = 50\text{mm}$$

ii) *Rectangle-2,*

$$a_2 = 60 \times 20 = 1200\text{mm}^2$$

$$x_2 = 20 + (b/2) = 20 + (60/2) = 50\text{mm}$$

and $$y_2 = (h/2) = (20/2) = 10\text{mm}$$



We know that distance between centre of gravity of the section, bottom and left face,

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} \qquad \therefore \bar{x} = 25\text{mm}$$

and $$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} \qquad \therefore \bar{y} = 35\text{mm}$$

Now,

Moment of inertia about X-X axis,

We also know that, M.I. of rectangle (1) about an axis through its centre of gravity and parallelto X-X axis.

$$I_{G1} = \left(\frac{bh^3}{12}\right) = \left(\frac{20 \times 100^3}{12}\right) = 1.667 \times 10^6\text{mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = (y_1 - \bar{y}) = 50 - 35 = 15\text{mm}$$

$\therefore$ Moment of inertia of rectangle (1) about X-X axis

$$I_{XX1} = I_{G1} + a_1h_1^2 = (1.667 \times 10^6) + [2000 \times (15)^2] = 2.117 \times 10^6\text{mm}^4$$

Similarly, M.I. of rectangle (2) about an axis through its centre of gravity andparallel to X-X axis,

$$I_{G2} = \left(\frac{bh^3}{12}\right) = \left(\frac{60 \times 20^3}{12}\right) = 0.04 \times 10^6\text{mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = (\bar{y} - y_2) = 35 - 10 = 25\text{mm}$$

$\therefore$ Moment of inertia of rectangle (2) about X-X axis

$$I_{XX2} = I_{G2} + a_2h_2^2 = (0.04 \times 10^6) + [1200 \times (25)^2] = 0.79 \times 10^6\text{mm}^4$$

Now, Moment of Inertia of the whole section about X-X axis,

$$I_{XX} = I_{XX1} + I_{XX2}$$

$$I_{XX} = (2.117 \times 10^6) + (0.79 \times 10^6)$$

$$\therefore I_{XX} = 2.907 \times 10^6\text{mm}^4$$

Now, Moment of inertia about Y-Y axis,

We also know that, M.I. of rectangle (1) about an axis through its centre of gravity and parallelto Y-Y axis.

$$I_{G1} = \left(\frac{hb^3}{12}\right) = \left(\frac{100 \times 20^3}{12}\right) = 0.067 \times 10^6\text{mm}^4$$

and distance between centre of gravity of rectangle (1) and Y-Y axis,

$$h_1 = (\bar{x} - x_1) = 25 - 10 = 15\text{mm}$$

$\therefore$ Moment of inertia of rectangle (1) about Y-Y axis

$$I_{YY1} = I_{G1} + a_1h_1^2 = (0.067 \times 10^6) + [2000 \times (15)^2] = 0.517 \times 10^6\text{mm}^4$$

Similarly, M.I. of rectangle (2) about an axis through its centre of gravity andparallel to Y-Y axis,

$$I_{G2} = \left(\frac{hb^3}{12}\right) = \left(\frac{20 \times 60^3}{12}\right) = 0.36 \times 10^6\text{mm}^4$$

and distance between centre of gravity of rectangle (2) and Y-Y axis,

$$h_2 = (x_2 - \bar{x}) = 50 - 25 = 25\text{mm}$$

$\therefore$ Moment of inertia of rectangle (2) about Y-Y axis

$$I_{YY2} = I_{G2} + a_2h_2^2 = (0.36 \times 10^6) + [1200 \times (25)^2] = 1.11 \times 10^6\text{mm}^4$$

Now,Moment of Inertia of the whole section about Y-Y axis,

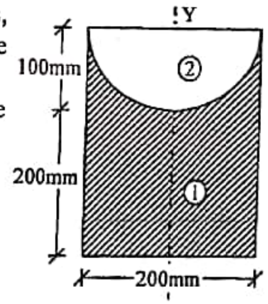
$$I_{YY} = I_{YY1} + I_{YY2}$$

$$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6)$$

$$\therefore I_{YY} = 1.627 \times 10^6\text{mm}^4$$

Example 4 : Determine the Moment of Inertia of the shaded area about the centroidal axis as shown in figure.

Solution : As the section is symmetrical about Y-Y axis, therefore its centreof gravity will lie on this axis. Split up the whole section into two figures viz., 1 and 2 as shown in figure. Let, left face of the vertical section and bottom face of the horizontal section be axes ofreference.



i) *Rectangle,*

$$a_1 = 200 \times 300 = 60000\text{mm}^2$$

and $$y_1 = (h/2) = (300/2) = 150\text{mm}$$

ii) Semicircle,

$$a_2 = \frac{\pi}{2} \times r^2 \frac{\pi}{2} \times 100^2 = 15707.96\text{mm}^2$$

and $$y_2 = 300 - \frac{4r}{3\pi} = 300 - \frac{4 \times 100}{3\pi} = 257.55\text{mm}$$

We know that distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(60000 \times 150) - (15707.96 \times 257.55)}{60000 - 15707.96} \qquad \therefore \bar{y} = 111.85\text{mm}$$

Now,

Moment of inertia about X-X axis,

We also know that, M.I. of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G1} = \left(\frac{bh^3}{12}\right) = \left(\frac{200 \times 300^3}{12}\right) = 450 \times 10^6 \text{mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = (y_1 - \bar{y}) = 150 - 111.85 = 38.15\text{mm}$$

$\therefore$ Moment of inertia of rectangle (1) about X-X axis

$$I_{XX1} = I_{G1} + a_1 h_1^2 = (450 \times 10^6) + [60000 \times (38.15)^2] = 537.32 \times 10^6 \text{mm}^4$$

Similarly, M.I. of semicircle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = 0.11r^4 = 0.11 \times 100^4 = 11 \times 10^6 \text{mm}^4$$

and distance between centre of gravity of semicircle (2) and X-X axis,

$$h_2 = (y_2 - \bar{y}) = 257.55 - 111.85 = 145.7\text{mm}$$

$\therefore$ Moment of inertia of semicircle (2) about X-X axis

$$I_{XX2} = I_{G2} + a_2 h_2^2 = (11 \times 10^6) + [15707.96 \times (145.7)^2] = 344.45 \times 10^6 \text{mm}^4$$

Now, Moment of Inertia of the whole section about X-X axis,

$$I_{XX} = I_{XX1} - I_{XX2}$$

$$I_{XX} = (537.32 \times 10^6) - (344.45 \times 10^6)$$

$$\therefore I_{XX} = 192.87 \times 10^6 \text{mm}^4$$

Now,

Moment of inertia about Y-Y axis,

We also know that, M.I. of rectangle (1) about an axis through its centre of gravity and parallel to Y-Y axis.

$$I_{YY1} = \left(\frac{hb^3}{12}\right) = \left(\frac{300 \times 200^3}{12}\right) = 200 \times 10^6 \text{mm}^4$$

Similarly, M.I. of semicircle (2) about an axis through its centre of gravity and parallel to Y-Y axis,

$$I_{YY2} = \frac{\pi}{128} D^4 = \frac{\pi}{128} \times 200^4 = 39.269 \times 10^6 \text{mm}^4$$

Now, Moment of Inertia of the whole section about Y-Y axis,

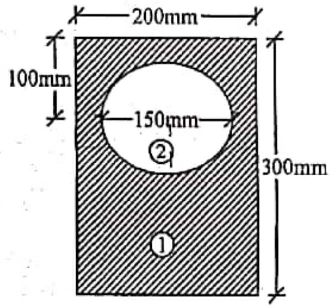
$$I_{YY} = I_{YY1} - I_{YY2}$$

$$I_{YY} = (200 \times 10^6) - (39.269 \times 10^6)$$

$$\therefore I_{YY} = 160.73 \times 10^6 \text{mm}^4$$

Example 5 : Find the moment of inertia of a hollow section shown in fig. about an axis passing through its centre of gravity or parallel X-X axis.

Solution : As the section is symmetrical about Y-Y axis, therefore centre of a gravity of the section will lie on this axis.



Let $\bar{y}$ be the distance between centre of gravity of the sectionfrom the bottom face.

i) Rectangle,

$$a_1 = 200 \times 300 = 60000\text{mm}^2$$

and $$y_1 = (h/2) = (300/2) = 150\text{mm}$$

ii) Circle,

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} \times 150^2 = 17670\text{mm}^2$$

and $$y_2 = 300 - 100 = 200\text{mm}$$

We know that distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(60000 \times 150) - (17670 \times 200)}{60000 - 17670}$$

$$\therefore \bar{y} = 129.1\text{mm}$$

Now,

Moment of inertia about X-X axis,

We also know that, M.I. of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G1} = \left(\frac{bh^3}{12}\right) = \left(\frac{200 \times 300^3}{12}\right) = 450 \times 10^6 \text{mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = (y_1 - \bar{y}) = 150 - 129.1 = 20.9\text{mm}$$

$\therefore$ Moment of inertia of rectangle (1) about X-X axis

Now,

We know that,

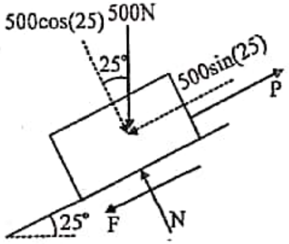
$$F = \mu.N = 0.3 \times 453.15 = 135.94N$$

and

$$\overline{\Sigma F_x^{+}} = 0$$

$$\therefore P - 500\sin(25) - 135.94 = 0$$

$$\therefore P = 347.24N$$



FBD of Body

Example 2 : Block A of mass 300kg is resting on an inclined plane as shown in figure. Take $\mu = 0.3$, Find the angle α of the inclination of the plane with the horizontal to maintain equilibrium of the system.

Solution :

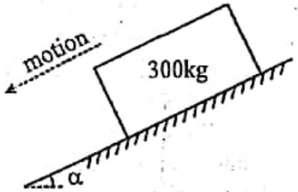
Consider FBD of body,

Apply condition of equilibrium,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N - 2942\cos(\alpha) = 0$$

$$\therefore N = 2942\cos(\alpha)$$



Given Diagram

Now,

We know that,

$$F = \mu N = 0.3 \times 2942\cos(\alpha) = 882.6\cos(\alpha)$$

and

$$\overline{\Sigma F_x^{+}} = 0$$

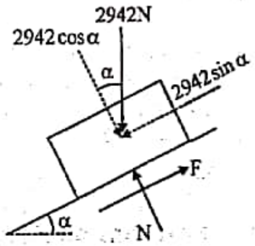
$$\therefore -2942\sin(\alpha) + 882.6\cos(\alpha) = 0$$

$$\therefore -2942\sin(\alpha) = -882.6\cos(\alpha)$$

$$\therefore \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{882.6}{2942}$$

$$\therefore \theta = \tan^{-1}(0.3)$$

$$\therefore \theta = 16.69°$$



FBD of Body

Example 3 : Find the least value of P that will start the system of block as shown in figure moving in right, Take $\mu = 0.3$ for all the contact surfaces and assume the pulley is frictionless.

Solution :

Consider FBD of Block-A,

Apply condition of equilibrium,

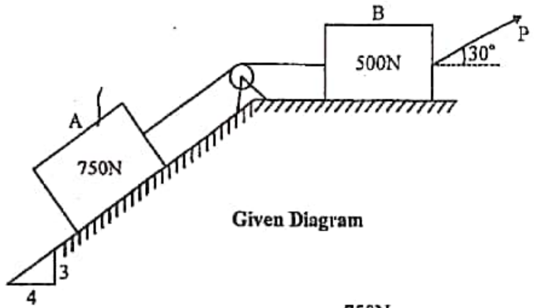
$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_A - 750\cos(36.86) = 0$$

$$\therefore N_A = 600N$$

$$F_A = \mu_A.N_A = 0.3 \times 600$$

$$\therefore F_A = 180N$$



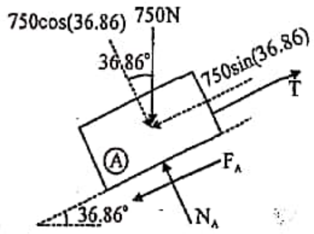
Given Diagram

and

$$\overline{\Sigma F_x^{+}} = 0$$

$$\therefore T - 180 - 750\sin(36.86) = 0$$

$$\therefore T = 629.89N$$



FBD of Block - A

Now,

Consider FBD of Block-B,

Apply condition of equilibrium,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_B - 500 + P\sin(30) = 0$$

$$\therefore N_B = 500 - P\sin(30)$$

We know that,

$$F_B = \mu_B.N_B = 0.3 \times [500 - P\sin(30)]$$

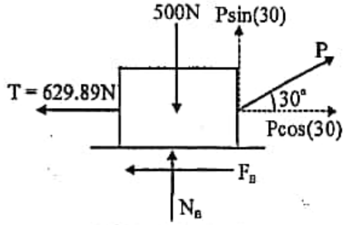
$$\therefore F_B = [150 - 0.15P]$$

and

$$\overline{\Sigma F_x^{+}} = 0$$

$$\therefore P\cos(30) - 629.89 - (150 - 0.15P) = 0$$

$$\therefore P = 767.58N$$



FBD of Block - B

Example 4 : Find the least value of P that will start the system of block as shown in figure moving in right, Take coefficient of friction under each block is 0.30.

Solution :

Consider FBD of Block-A,

Apply condition of equilibrium,

$$\uparrow + \Sigma F_y = 0$$

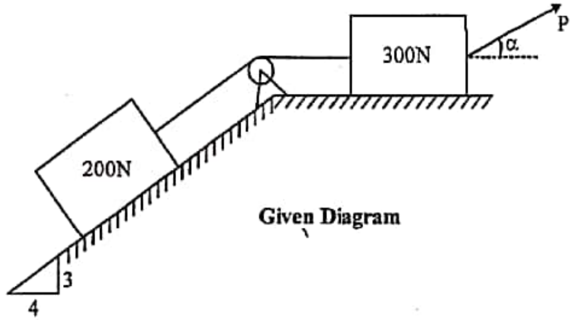
$$\therefore N_A - 200\cos(36.86) = 0$$

$$\therefore N_A = 160N$$

$$F_A = \mu_A.N_A$$

$$= 0.3 \times 160$$

$$\therefore F_A = 148N$$



Given Diagram

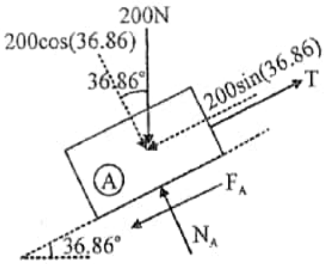
$\therefore$

and $\therefore$

$$\overrightarrow{\Sigma F_x} = 0$$

$$\therefore T - 148 - 200\sin(36.86) = 0$$

$$\therefore T = 168N$$



FBD of Block - A

Now,

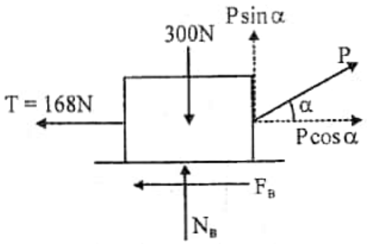
Consider FBD of Block-B,

Apply condition of equilibrium,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_B - 300 + P\sin(\alpha) = 0$$

$$\therefore N_B = 300 - P\sin(\alpha)$$



FBD of Block - B

We know that,

$$F_B = \mu_B . N_B = 0.3 \times [300 - P\sin(\alpha)]$$

$$\therefore F_B = [90 - 0.3P\sin(\alpha)]$$

and

$$\overrightarrow{\Sigma F_x} = 0$$

$$\therefore P\cos(\alpha) - 168 - [90 - 0.3P\sin(\alpha)] = 0$$

$$\therefore P[\cos(\alpha) + 0.3\sin(\alpha)] = 258$$

$$\therefore P = \frac{258}{[\cos(\alpha) + 0.3\sin(\alpha)]} \quad \text{............(1)}$$

For 'P' to be minimum, denominator should be maximum.

$$\therefore \frac{d}{d\alpha}(\cos\alpha + 0.3\sin\alpha) = 0$$

$$\therefore -\sin\alpha + 0.3\cos\alpha = 0$$

$$\therefore 0.3\cos\alpha = \sin\alpha$$

$$\therefore \tan\alpha = 0.3$$

$$\therefore \alpha = 16.7°$$

~~μ = tan φ~~

~~0.3 = tan φ~~

~~φ = tan 0.3~~

~~φ = 16.69~~

Putting in eq. (i), we get,

$$\therefore P = \frac{258}{[\cos(16.7) + 0.3\sin(16.7)]}$$

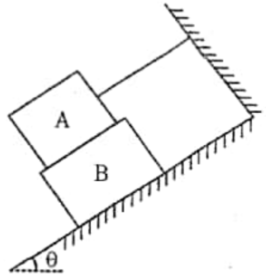
$$\therefore P = 247N$$

Example 5 : Block 'A' weighs 300N and Block 'B' weighs 900N and the block is parallel to the inclined. Coefficient of friction for all the contact surfaces is 0.25. Determine the angle 'θ' of the inclined for which motion of block 'B' impends.

Solution :

Consider FBD of Block-A,

Apply condition of equilibrium,



Given Diagram

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_A - 300\cos\theta = 0$$

$$\therefore N_A = 300\cos\theta$$

$$F_A = \mu_A . N_A = 0.25 \times 300\cos\theta$$

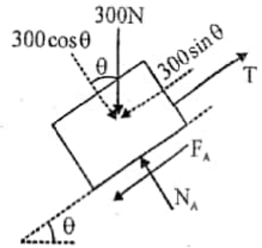
$$\therefore F_A = 75\cos\theta$$

and

$$\overrightarrow{\Sigma F_x} = 0$$

$$\therefore T - 300\sin\theta - 75\cos\theta = 0$$

$$\therefore T = (300\sin\theta + 75\cos\theta) \quad \text{............(i)}$$



FBD of Block - A

Now,

Consider FBD of Block-B,

Apply condition of equilibrium,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_B - 900\cos\theta - 300\cos\theta = 0$$

$$\therefore N_B = 1200\cos\theta$$

We know that,

$$F_B = \mu_B . N_B = 0.25 \times 1200\cos\theta$$

$$\therefore F_B = 300\cos\theta$$

and

$$\overrightarrow{\Sigma F_x} = 0$$

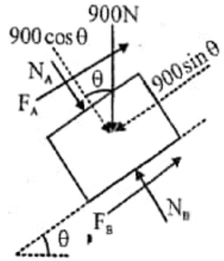
$$\therefore 300\cos\theta + 75\cos\theta - 900\sin\theta = 0$$

$$\therefore 375\cos\theta = 900\sin\theta$$

$$\therefore \frac{\sin\theta}{\cos\theta} = \frac{375}{900}$$

$$\therefore \theta = \tan^{-1}\left(\frac{375}{900}\right)$$

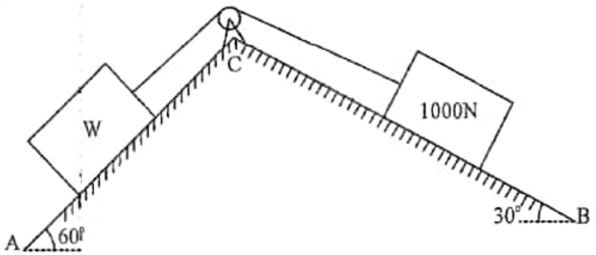
$$\therefore \theta = 22.61°$$



FBD of Block - B

Example 6 : Two plane AC and BC inclined at 60^0 and 30^0 to the horizontal meet at pt. 'C' as shown in fig. Block weighs 1000N rest on the inclined plane BC and its tight by the row passing over the pulley to a block weighing 'W' and resting on plane AC. If $\mu_{BC} = 0.28$ and $\mu_{AC} = 0.20$. Find the least value of 'W' for the equilibrium of the system.

Solution :



Given Diagram

For the least value of 'W', In this case, Motion of 1000N block moving rightward.

Consider FBD of 1000N Block,

Apply condition of equilibrium,

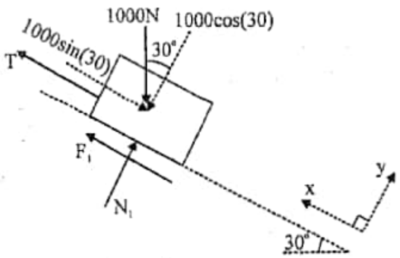
$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_1 - 1000\cos(30) = 0$$

$$\therefore N_1 = 866.02N$$

$$F_1 = \mu_1 . N_1 = 0.28 \times 866.02$$

$$\therefore F_1 = 242.48N$$



FBD of 1000N Block

and

$$\Sigma \overset{+}{F_x} = 0$$

$$\therefore -T - 242.48 - 1000\sin(30) = 0$$

$$\therefore T = 257.52N$$

Now,

Consider FBD of Block-W,

Apply condition of equilibrium,

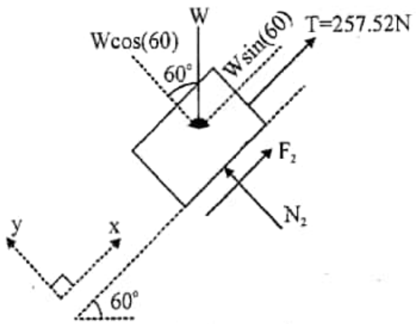
$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_2 - W\cos(60) = 0$$

$$\therefore N_2 = W\cos(60)$$

We know that,

$$F_2 = \mu_2 . N_2$$



FBD of Block - W

$$= 0.20 \times W\cos(60)$$

$$\therefore F_2 = 0.1W$$

and

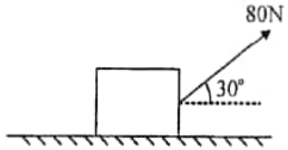
$$\Sigma \overset{+}{F_x} = 0$$

$$\therefore 257.52 - 0.1W - W\sin(60) = 0$$

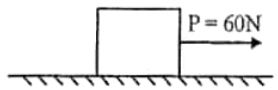
$$\therefore W = 266.57N$$

- EXERCISE -

1. State Coulomb's law of dry friction.
2. Explain the terms:
   a) Angle of friction. b) Cone of friction. c) Limiting friction. d) Angle of repose.
3. A pull of 80N at 30^0 up the horizontal is required to just move a block of weight (W). Calculate the weight of block if $\mu = 0.25$



4. A body weighing 240N is resting on a horizontal surface. It is just on the point of moving when a horizontal force of 60N is applied on it. Find coefficient of friction.



5. For the block shown in figure. Calculate the force 'P' required to just move it up the plane. Take $\mu = 0.25$

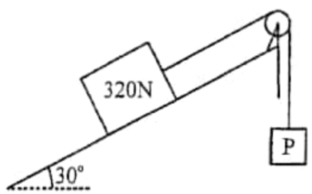


Fig. 9.1

In the case of a bridge, such as shown in figure 9.2 (a), the load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. Like the roof truss, the bridge truss loading is also coplanar, figure 9.2 (b).

When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, for example, joint A in figure 9.1 (a) and 9.2 (a). This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.

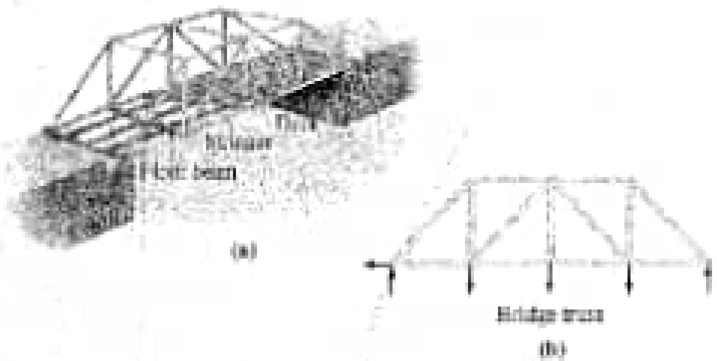


Fig. 9.2

9.2 Classification of truss

Though there are many types of frames, yet from the analysis point of view, the frames may be classified into the following two groups:

1. Perfect frame
2. Imperfect frame

1. **Perfect frame :** A perfect frame is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape. The simplest perfect frame is a triangle, which contains three members and three joints as shown in Fig. 9.3 It will be interesting to know that if such a structure is loaded, its shape will not be distorted. Thus, for three jointed frame, there should be three members to prevent any distortion.

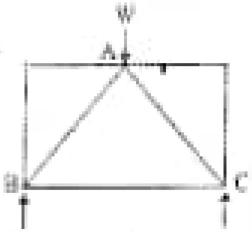


Fig. 9.3

It will be further noticed that if we want to increase a joint, to a triangular frame, we require two members as shown by dotted lines in Fig. 9.3 Thus we see that for every additional joint, to a triangular frame, two members are required.

The no. of members, in a perfect frame, may also be expressed by the relation:

$m = (2j - 3)$

m = No. of members, and j = No. of joints

2. **Imperfect frame :** An imperfect frame is that which does not satisfy the equation: $m = (2j - 3)$ or in other words, it is a frame in which the no. of members are more or less than $(2j - 3)$. The imperfect frames may be further classified into the following two types:
   a) Deficient frame
   b) Redundant frame

a) **Deficient frame :** A deficient frame is an imperfect frame, in which the no. of members are less than $(2j-3)$.

b) **Redundant frame :** A redundant frame is an imperfect frame, in which the no. of members are more than $(2j-3)$.

9.3 Assumptions made in the analysis of truss

To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions :

All loadings are applied at the joints :

In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.

The members are joined together by smooth pins :

The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, as shown in figure 9.4 (a), or by simply passing a large bolt or pin through each of the members, figure 9.4 (b). We can assume these connections act as pins provided the center lines of the joining members are concurrent, as in figure 9.4.

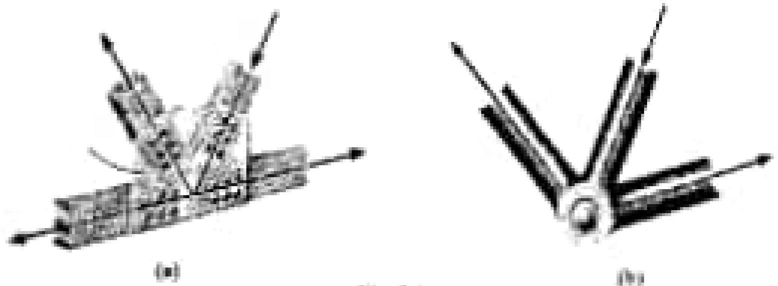


Fig. 9.4

$\therefore 1500\sin(53.13) + F_{BC} +$

$948.66\sin(18.43) - 1200 = 0$

$\therefore F_{BC} = 299.91N(T)$

Consider FBD of joint (C),

$\uparrow + \Sigma F_y = 0$

$\therefore -400 + 299.91 - F_{CD}\sin(45) = 0$

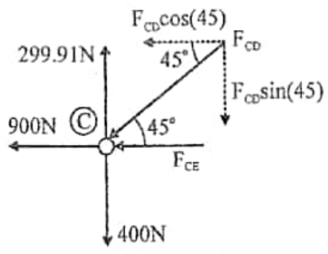
$\therefore F_{CD} = -141.54N$

$\therefore F_{CD} = 141.54N(T)$

$\overrightarrow{\Sigma F_X^+} = 0$

$\therefore -900 - F_{CE} - [-141.54\cos(45)] = 0$

$\therefore F_{CE} = 799.91N(T)$



FBD of Joint (C)

Consider FBD of joint (E),

$\overrightarrow{\Sigma F_X^+} = 0$

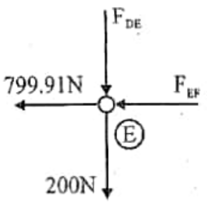
$\therefore -799.91 - F_{EF} = 0$

$\therefore F_{EF} = 799.91N(T)$

$\uparrow + \Sigma F_y = 0$

$\therefore -200 - F_{DE} = 0$

$\therefore F_{DE} = 200N(T)$



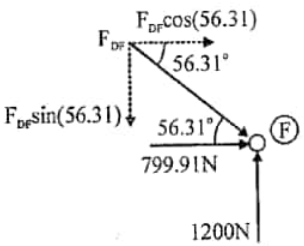
FBD of Joint (E)

Consider FBD of joint (F),

$\uparrow + \Sigma F_y = 0$

$\therefore 1200 - F_{DF}\sin(56.31) = 0$

$\therefore F_{DF} = 1442.22N(C)$

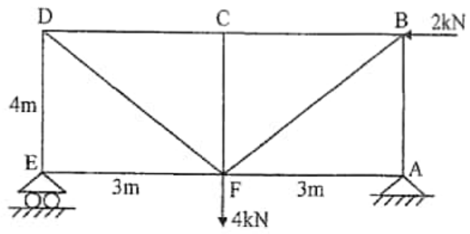


FBD of Joint (F)

Member	Magnitude	Nature
F_{AB}	1500N	C
F_{AC}	900N	T
F_{BD}	948.66N	C
F_{BC}	299.91N	T
F_{CD}	141.54N	T
F_{CE}	799.91N	T
F_{EF}	799.91N	T
F_{DE}	200N	T
F_{DF}	1442.22N	C

Example 5 : A truss shown in figure is subjected to two point loads at point P and F. Find the forces in all the members of the truss and tabulate the forces in members.

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$



Solution :

To determine the support reaction,

Apply condition of equilibrium,

$(\curvearrowright_+)\Sigma M_E^F = 0$

$\therefore (4\times 3) - (2\times 4) - (R_{AV}\times 6) = 0$

$\therefore R_{AV} = 0.67kN(C)$

$\uparrow + \Sigma F_y = 0$

$\therefore R_F + 0.67 - 4 = 0$

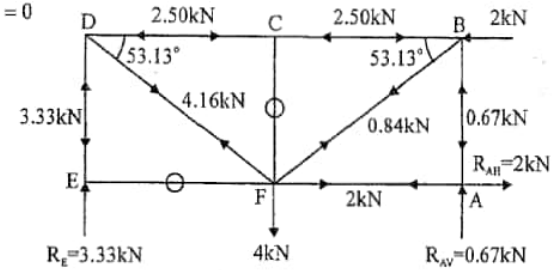
$\therefore R_F = 3.33kN(C)$

$\overrightarrow{\Sigma F_X^+} = 0$

$\therefore -R_{AH} + 2 = 0$

$\therefore R_{AH} = -2kN$

$\therefore R_{AH} = 2kN(T)$



FBD of Truss

Apply method of joint

Consider FBD of joint (E),

$\uparrow + \Sigma F_y = 0$

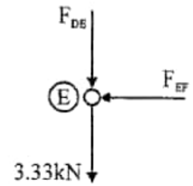
$\therefore 3.33 - F_{DE} = 0$

$\therefore F_{DE} = 3.33kN(C)$

$\overrightarrow{\Sigma F_X^+} = 0$

$\therefore -F_{EF} = 0$

$\therefore F_{EF} = 0$



FBD of Joint (E)

Consider FBD of joint (D),

$\uparrow + \sum F_y = 0$

$\therefore 3.33 + F_{DF}\sin(53.13) = 0$

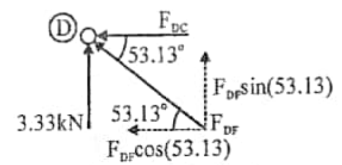
$\therefore F_{DF} = -4.16kN$

$\therefore F_{DF} = 4.16kN(T)$

$\sum \overrightarrow{F_X} = 0$

$\therefore -F_{DC} - [-4.16\cos(53.13)] = 0$

$\therefore F_{DC} = 2.50kN(C)$



FBD of Joint (D)

Consider FBD of joint (C),

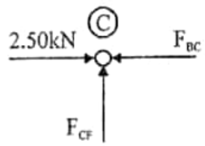
$\sum \overrightarrow{F_X} = 0$

$\therefore 2.50 - F_{BC} = 0$

$\therefore F_{BC} = 2.50kN(C)$

$\uparrow + \sum F_y = 0$

$\therefore F_{CF} = 0$



FBD of Joint (C)

Consider FBD of joint (D),

$\uparrow + \sum F_y = 0$

$\therefore -4 + 4.16\sin(53.13)$

$-F_{BF}\sin(53.13) = 0$

$\therefore F_{BF} = -0.84kN$

$\therefore F_{BF} = 0.84kN(T)$

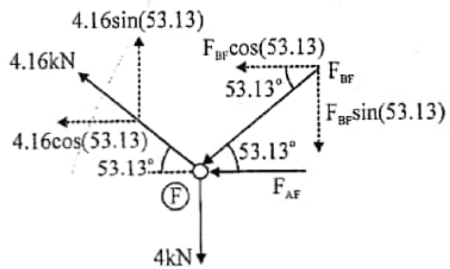
$\sum \overrightarrow{F_X} = 0$

$\therefore -F_{AF} - 4.16\cos(53.13)$

$-[-0.84\cos(53.13)] = 0$

$\therefore F_{AF} = -1.99kN$

$\therefore F_{AF} = 1.99kN(T)$



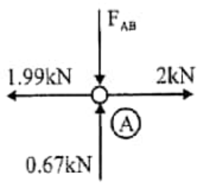
FBD of Joint (D)

Consider FBD of joint (A),

$\uparrow + \sum F_y = 0$

$\therefore 0.67 - F_{AB} = 0$

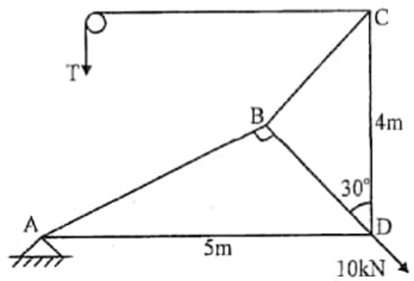
$\therefore F_{AB} = 0.67kN(C)$



FBD of Joint (A)

Member	Magnitude	Nature
F_{DE}	3.33kN	C
F_{EF}	0	--
F_{DF}	4.16kN	T
F_{DC}	2.50kN	C
F_{BC}	2.50kN	C
F_{CF}	0	--
F_{BF}	0.84kN	T
F_{AF}	1.99kN	T
F_{AB}	0.67kN	C

Example 6 : Determine the tension in the wire and axial forces in all the members of truss as shown in figure.



Solution :
To determine the support reaction,
Apply condition of equilibrium,

$(\curvearrowright +)\sum M_A^F = 0$

$\therefore (10\cos(30)\times 5) - (T\times 4) = 0$

$\therefore T = 10.82kN$

$\uparrow + \sum F_y = 0$

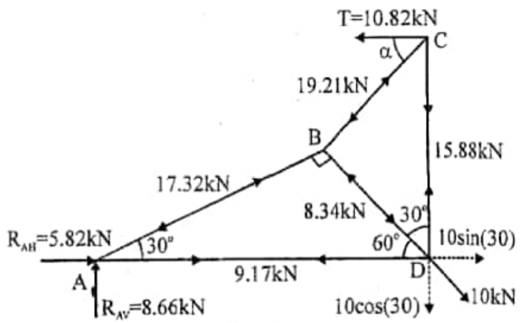
$\therefore R_{AV} - 10\cos(30) = 0$

$\therefore R_{AV} = 8.66kN(C)$

$\sum \overrightarrow{F_X} = 0$

$\therefore R_{AH} + 10\sin(30) - 10.82 = 0$

$\therefore R_{AH} = 5.82kN(C)$



FBD of Truss

Apply method of joint,
Consider FBD of joint (A),

$$\therefore [-231.82 - 1200\sin(15) + T] = \frac{1200}{9.81} \times a$$

$$\therefore T - 122.32a = 542.40 \quad \text{............(i)}$$

Now,

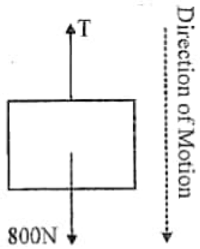
Consider FBD of 800N Block,

Apply Equation of Kinetic Translation,

$$\Sigma F = \frac{W_2}{g} \times a$$

$$\therefore (-T + 800) = \frac{800}{9.81} \times a$$

$$\therefore T + 81.54a = 800 \quad \text{............(ii)}$$



FBD of 800N Block

Equating equation (i) and (ii), we get,

$$T = 696.96N \quad \text{and} \quad a = 1.26 m/sec^2$$

Now,

For the Distance move by the Block after 3sec from rest,

We know that,

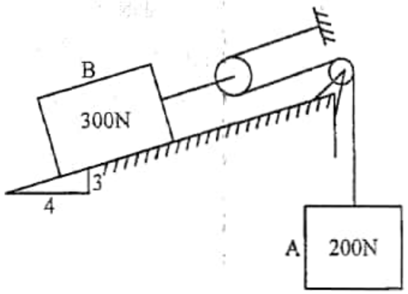
$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 1.26 \times 3^2$$

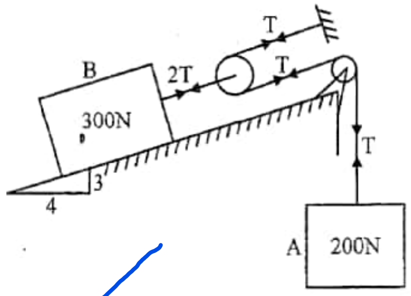
$$\therefore s = 5.67m$$

14.7 Problems on rectilinear motion of several interconnected particles / bodies

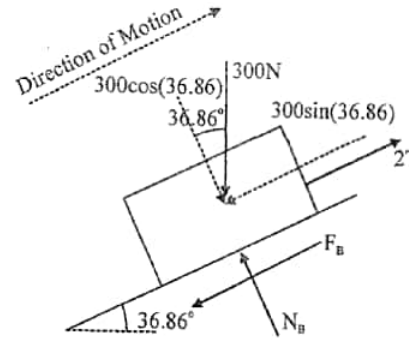
Example 1 : Determine the acceleration of block B and tension in cord supported that block. Take $\mu = 0.2$.



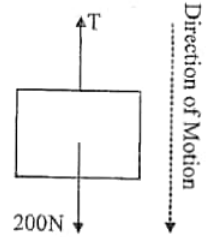
Given Diagram



Solution :



FBD of Block (B)



FBD of Block (A)

Note : *In this FBD, Direction of motion and frictional force is shown after determination of the dire ction of motion. (i.e. After first step) Same will follow for further problems.*

i) Determine the Direction of Motion,

Consider Block-A at rest,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore T - 200 = 0$$

$$\therefore T = 200N$$

With this value of T, determine the resultant force acting on other block (i.e Block-B)

$$\therefore R = \overrightarrow{\Sigma F_x}^{+}$$

$$= 2T - 300\sin(36.86)$$

$$= (2 \times 200) - 300\sin(36.86)$$

$$\therefore R = 220.04N(\rightarrow)$$

As the Resultant is positive, Block-B will move towards right and thus, Block-A will move downward.

ii) Determine the Frictional force,

Consider FBD of Block-B,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_B - 300\cos(36.86) = 0$$

$$\therefore N_B = 240.03N$$

and

$$F_B = \mu . N_B = 0.2 \times 240.03 = 48N$$

iii) Obtain Kinematic relation,

Work-done by tensile forces are equated to zero,

$$T_A s_A + T_B s_B = 0$$

$$-T \cdot S_A + 2T \cdot S_B = 0$$

$$-Ts_A = -2Ts_B$$

$$s_A = 2s_B$$

lly, $v_A = 2v_B$

lly, $a_A = 2a_B$

iv) Apply Kinetic equation,

Consider FBD of Block-A,

Apply Equation of Kinetic Translation,

$$\Sigma F = \frac{W_A}{g} \times a_A$$

$$\therefore (-T + 200) = \frac{200}{9.81} \times 2a_B \qquad (\because a_A = 2a_B)$$

$$\therefore T + 40.77a_B = 200 \qquad \text{...........(i)}$$

Now,

Consider FBD of Block-B,

Apply Equation of Kinetic Translation,

$$\Sigma F = \frac{W_B}{g} \times a_B$$

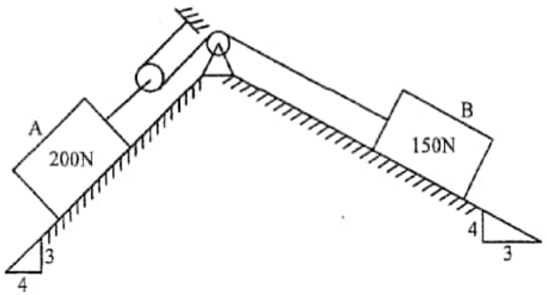
$$\therefore (-48 - 300\sin(36.86) + 2T) = \frac{300}{9.81} \times a_B$$

$$\therefore 2T - 30.58a_B = 227.95 \qquad \text{...........(ii)}$$

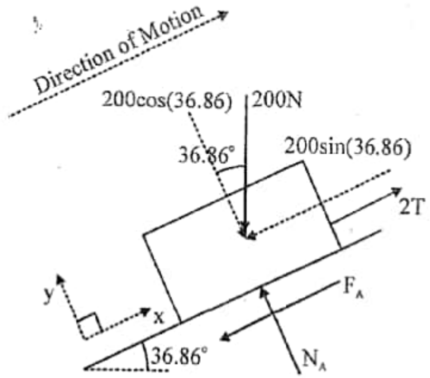
Equating equation (i) and (ii), we get,

$T = 137.43N$ and $a = 1.53 m/sec^2$

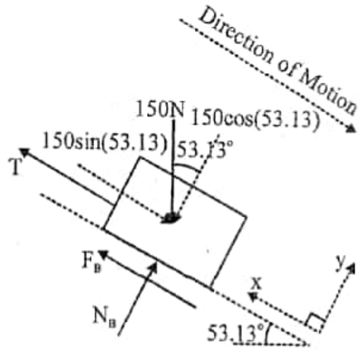
Example 2 : Determine the velocity of Block-B after Block-A has move 6m starting from rest. Use D'Alemberts principle. Take $\mu = 0.2$.



Solution :



FBD of Block (A)



FBD of Block (B)

i) Determine the Direction of Motion,

Consider Block-B at rest,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore 150\sin(53.13) - T = 0$$

$$\therefore T = 120N$$

With this value of T, determine the resultant force acting on other block (i.e Block-A)

$$\therefore R = \overset{+}{\overline{\Sigma F_x}}$$

$$= 2T - 200\sin(36.86)$$

$$= (2 \times 200) - 200\sin(36.86)$$

$$\therefore R = 120.02N(\rightarrow)$$

As the Resultant is positive, Block-A will move towards right & thus, Block-B will also move rightwards.

ii) Determine the Frictional force,

Consider FBD of Block-A,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_A - 200\cos(36.86) = 0$$

$$\therefore N_A = 160.02N$$

and

$$F_A = \mu . N_A = 0.2 \times 160.02 = 32N$$

Now,

Consider FBD of Block-B,

$\therefore T = 211.76N$

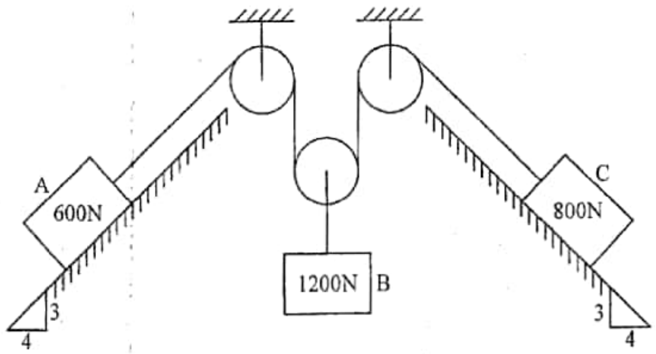
Putting the value of $T = 211.76N$ in equation (i), (ii) and (iii), we get,

$\therefore a_A = 4.03\,m/sec^2$

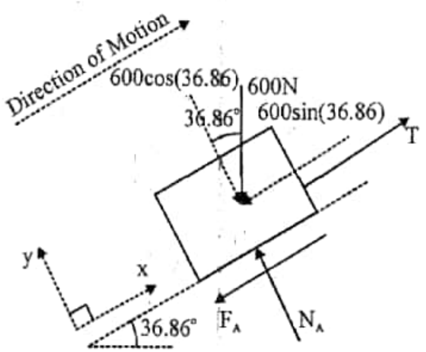
$\therefore a_B = 1.15\,m/sec^2$

$\therefore a_C = 2.88\,m/sec^2$

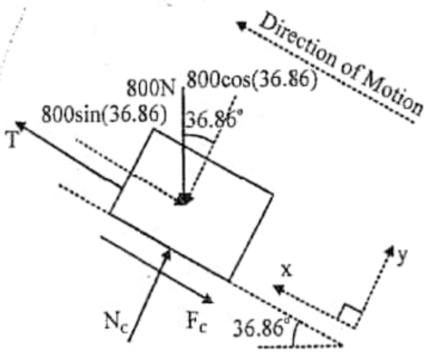
Example 8 : If the coefficient of kinetic friction is 0.25 under each block in the system shown in figure. What will be the acceleration of block-B?



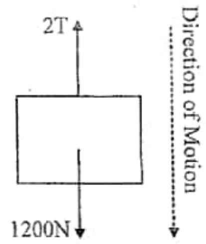
Solution :



FBD of Block (A)



FBD of Block (C)



FBD of Block (B)

i) ***Determine the Direction of Motion,***

Consider Block-B at rest,

$$\uparrow + \sum F_y = 0$$

$$\therefore 2T - 1200 = 0$$

$$\therefore T = 600N$$

With this value of T, determine the resultant force acting on other block (i.e Block-A & C)

Consider Block-A,

$$\therefore R_A = \sum \overset{+}{F_x}$$

$$= T - 600\sin(36.86) = 600 - 600\sin(36.86)$$

$$\therefore R_A = 240.08N(\rightarrow)$$

and Consider Block-C,

$$\therefore R_C = \sum \overset{+}{F_x}$$

$$= -T + 800\sin(36.86) = -600 + 800\sin(36.86)$$

$$\therefore R_C = 120.11N(\leftarrow)$$

As the Resultant of Block-A is positive, thus Block-A will move rightwards & the Resultant of Block-C will negative, thus Block-C will move leftward & thus Block-B will move downward.

ii) ***Determine the Frictional force,***

Consider FBD of Block-A,

$$\uparrow + \sum F_y = 0$$

$$\therefore N_A - 600\cos(36.86) = 0$$

$$\therefore N_A = 480.06N$$

and

$$F_A = \mu . N_A = 0.25 \times 480.06 = 120.01N$$

Now,

Consider FBD of Block-C,

$$\uparrow + \Sigma F_y = 0$$

$$\therefore N_C - 800\cos(36.86) = 0$$

$$\therefore N_C = 640.08N$$

Here,

and $F_C = \mu . N_C = 0.25 \times 640.08 = 160.02N$

As the Resultant of Block-C is less than that of Frictional force, Block-C will not move.

iii) Obtain Kinematic relation,

Work-done by tensile forces are equated to zero,

$$T_A s_A + T_B s_B = 0$$

$$Ts_A - 2Ts_B = 0$$

$$Ts_A = 2Ts_B$$

$$s_A = 2s_B$$

lly, $v_A = 2v_B$

lly, $a_A = 2a_B$

iv) Apply Kinetic equation,

Consider FBD of Block-A,

Apply Equation of Kinetic Translation,

$$\Sigma F = \frac{W_A}{g} \times a_A$$

$$\therefore [T - 120 - 600\sin(36.86)] = \frac{600}{9.81} \times 2a_B \qquad (\because a_A = 2a_B)$$

$$\therefore T - 122.32a_B = 480 \qquad \text{............(i)}$$

Now,

Consider FBD of Block-B,

Apply Equation of Kinetic Translation,

$$\Sigma F = \frac{W_B}{g} \times a_B$$

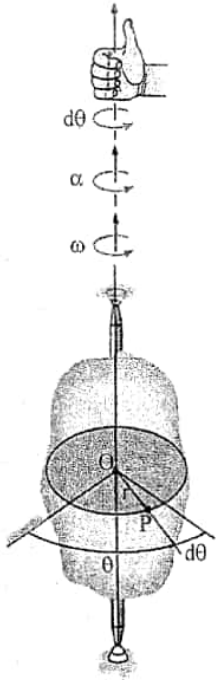
$$\therefore (-2T + 1200) = \frac{1200}{9.81} \times a_B$$

$$\therefore 2T + 122.32a_A = 1200$$

Equating equation (i) and (ii), we get,

$$T = 560N \text{ and } a_A = 0.625 \text{ m/sec}^2$$

14.8 Rotation of rigid body about a fixed axis :

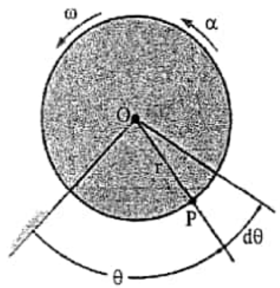


(a)

When a body rotates about a fixed axis, any point P located in the body travels along a circular path. To study this motion it is first necessary to discuss the angular motion of the body about the axis.

- **Angular Motion :** Since a point is without dimension, it cannot have angular motion. Only lines or bodies undergo angular motion. For example, consider the body shown in Fig. (a) and the angular motion of a radial line 'r' located within the shaded plane.
- **Angular Position :** At the instant shown, the angular position of r is defined by the angle 'u', measured from a fixed reference line to 'r'.
- **Angular Displacement :** The change in the angular position, which can be measured as a differential dU, is called the angular displacement.

  This vector has a magnitude of dU, measured in degrees, radians, or revolutions, where $1\text{rev} = 2\pi\text{rad}$. Since motion is about a fixed axis, the direction of dU is always along this axis. Specifically, the direction is determined by the right-hand rule; that is, the fingers of the right hand are curled with the sense of rotation, so that in this case the thumb, or dU, points upward, Fig. (a). In two dimensions, as shown by the top view of the shaded plane, figure (b), both u and 'du' are counterclockwise, and so the thumb points outward from the page.



(b)

- **Angular Velocity**. The time rate of change in the angular position is called the angular velocity ω (omega). Since dU occurs during an instant of time dt, then,

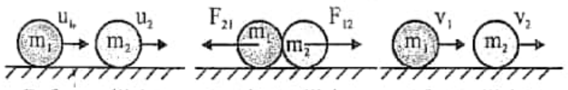
$$\boxed{\omega = \frac{d\theta}{dt}}$$

$$\therefore \frac{-1070.33}{(-P+250)} + \frac{2140.67}{(P-550)} = 0$$

$$\therefore P = 814.18N$$

15.8 Law of Conservation of Momentum

Law of conservation of linear momentum states that total momentum of the system is always conserved if no external force acts on an object or system of objects. Consider a collision between two balls wherein there occurs no energy losses during the collision.



Momentum of the two balls before collision,

$$p_{1i} = m_1u_1$$

$$p_{2i} = m_2u_2$$

Total momentum of the system of two balls before collision, $p_i = p_{1i} + p_{2i} = m_1u_1 + m_2u_2$

During the collision exerts an action force F_{12} on m_2.

In response, from Newton's third law, m_2 exerts a reaction on m_1, that is, F_{21}, such that

$$F_{12} = -F_{21}$$

Negative sign implies that the two forces are directed in opposite directions. After the collision they undergo change in velocity and the corresponding change in momentum.

Momentum of the two balls after collision $p_{1f} = m_1v_1$ and $p_{2f} = m_2v_2$

Total momentum of the system of two balls after collision, $p_f = p_{1f} + p_{2f} = m_1v_1 + m_2v_2$

Also from Newton's Second Law :

$$\text{Force} = \frac{\text{change in momentum}}{\text{time interval}}$$

$$F_{12} = \frac{\text{change in momentum produce in mas } m_2}{\text{collision time}}$$

$$F_{12} = \frac{m_2v_2 - m_2u_2}{t}$$

Similarly,

$$F_{21} = \frac{m_1v_1 - m_1u_1}{t}$$

From Newton's 3rd law

$$F_{12} = -F_{21}$$

$$\frac{m_2v_2 - m_2u_2}{t} = -\left(\frac{m_1v_1 - m_1u_1}{t}\right)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

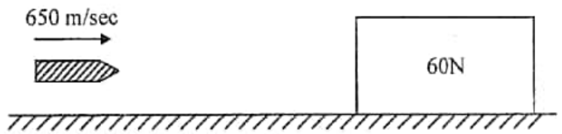
Total momentum before collision = Total momentum after collision

Hence this equation implies that if no external force acts on the system of two colliding balls, the total momentum is conserved.

15.9 Problems on Law of Conservation of Momentum

Example 1 : A bullet weighing 0.5N and moving at 650 m/sec penetrates into 60N body as shown in figure and emerges with a velocity of 180 m/sec. How far and how long does the body then move? Take $\mu = 0.2$.

Solution :



For the velocity of block,

Apply Law of conservation of Momentum.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\left(\frac{W_1}{g} \times u_1\right) + \left(\frac{W_2}{g} \times u_2\right) = \left(\frac{W_1}{g} \times v_1\right) + \left(\frac{W_2}{g} \times v_2\right)$$

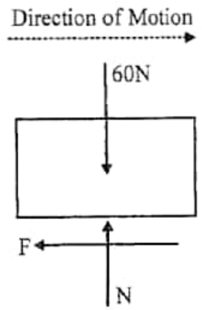
$$\therefore \left(\frac{0.5}{9.81} \times 650\right) + \left(\frac{60}{9.81} \times 0\right) = \left(\frac{0.5}{9.81} \times 180\right) + \left(\frac{60}{9.81} \times v_2\right)$$

$$\therefore v_2 = 3.91 \text{ m/sec}$$

Now,

For Distance after collision,

Consider FBD of Block,



FBD of Block

$$\uparrow + \sum F_y = 0$$

$$\therefore N - 60 = 0$$

$$\therefore N = 60N$$

We know that,

$$F = \mu.N = 0.2 \times 60 = 12N$$

Here,

Apply Work-Energy Equation,

$$\Sigma F.s = \frac{W}{2g} \times (v^2 - u^2)$$

$$\therefore [-12] \times s = \frac{60}{2 \times 9.81} \times (0^2 - 3.91^2)$$

$$\therefore s = 3.89m$$

and

For the time of Travel,

Apply Impluse-Momentum Equation,

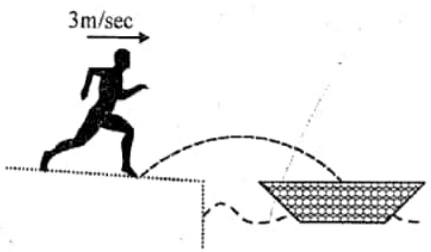
$$\Sigma F.t = \frac{W}{g} \times (v - u)$$

$$\therefore [-12] \times t = \frac{60}{9.81} \times (0 - 3.91)$$

$$\therefore t = 2sec$$

Example 2 : A 800N man moving with a velocity of 3 m/sec jumps into the boat of 3200N. Determine the velocity of boat after jump if its initial velocity is zero.

Solution :



Apply Law of conservation of Momentum.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\left(\frac{W_1}{g} \times u_1\right) + \left(\frac{W_2}{g} \times u_2\right) = \left(\frac{W_1 + W_2}{g}\right)v$$

$$\therefore \left(\frac{800}{9.81} \times 3\right) + \left(\frac{3200}{9.81} \times 0\right) = \left(\frac{800 + 3200}{9.81}\right)v$$

$$\therefore v_2 = 0.6 \text{ m/sec}$$

Example 3 : A 750N hammer fall from a height of 1.2m on a top of pile as shown in fig. The pile is driven 100mm into the ground. Determine the average resistance of the ground to penetration. Assume the weight of pile is 2250N.

Given data :

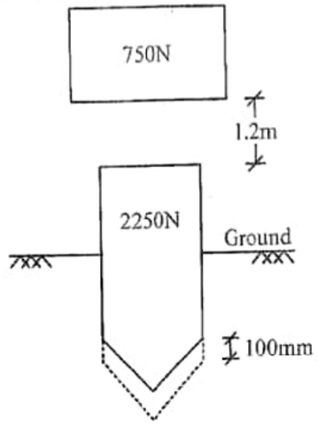
Weight of Hammer, $W_1 = 750N$

Velocity of Hammer, $u_1 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.2}$

$$\therefore u_1 = 4.85 \text{m/sec}$$

Weight of Pile, $W_2 = 2250N$

Velocity of Pile, $u_2 = 0$



Given Diagram

Solution :

Velocity of Pile and Hammer after Impact,

Apply Law of conservation of Momentum.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\left(\frac{W_1}{g} \times u_1\right) + \left(\frac{W_2}{g} \times u_2\right) = \left(\frac{W_1 + W_2}{g}\right)v$$

$$\therefore \left(\frac{750}{9.81} \times 4.85\right) + \left(\frac{2250}{9.81} \times 0\right) = \left(\frac{750 + 2250}{9.81}\right)v$$

$$\therefore v = 1.21 \text{ m/sec}$$

Now,

For Average resistance from the ground,

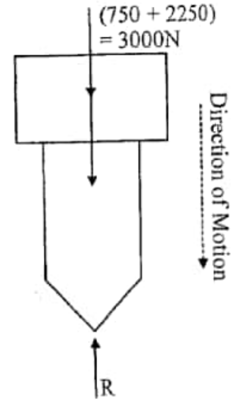
Consider FBD of Block,

Apply Work-Energy Equation,

$$\Sigma F.s = \frac{W}{2g} \times (v^2 - u^2)$$

$$\therefore [3000 - R] \times 0.1 = \frac{3000}{2 \times 9.81} \times (0^2 - 1.21^2)$$

$$\therefore R = 5238.68m$$



FBD of Block

15.10 Impact

Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.