WINTER-2017

UNIT 1

Q.1 a) $(D^2 + 5D + 6)y = e^{-2x} \sin 2x + 4x^2e^x$ **(6)**

b) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = 4\sec^2(2x)$$ **(7)**

Q.2 a) Solve:

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x-2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$ **(7)**

b) Evaluate: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ **(6)**

UNIT 2

Q.3 a) Evaluate using Laplace transform: $\int_0^\infty \frac{[e^{-t} - e^{-3t}]}{t}dt$ **(4)**

b) Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$ **(4)**

c) Solve the differential equation by Laplace transform: **(6)**

$(D^2 + 2D + 5)\, y = e^{-t} \sin t.$ $\quad Y(0) = 0,\ y'(0) = 1$

Q.4 a) Using convolution theorem, find inverse Laplace transform of $\frac{1}{(S+1)(S^2+1)}$ **(4)**

b) Find Laplace transform of the function **(4)**

$f(t) = 1/a,\ 0 < t < a$

$= \frac{1}{a}(2a - t).\ a < t < 2\,a$

c) Use Laplace transform method to solve the following differential equation: **(6)**

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t} \text{ if } y(0) = 1,\ y'(0) = -2$$

UNIT 3

Q.5 a) Solve: $u_{n+2} - 2u_{n+1} + u_n = n^2 2^n$ **(4)**

b) Solve: $u_{n+2} - 2u_{n+1}\ u_n = 3n + 4$. **(4)**

c) Find inverse z-transform of: $\dfrac{z}{z^2 + 11z + 30}$ **(5)**

Q.6 a) Solve: $u_{n+2} - 16u_n = \cos n/2$. **(4)**

b) Solve: $u_{n+2} - 7u_{n+1} + 10u_n = 12e^{3n} + 4^n$. **(4)**

c) Solve by using z-transform $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$ with $y(0) = 0$, $y(1) = 1$. **(5)**

UNIT 4

Q.7 a) Solve the following partial differential equations:

i) $(mz - ny)\, p + (nx - lz)\, q = ly - mx$. **(4)**

ii) $x^2p^2 + y^2q^2 = z^2$ **(4)**

b) Express the function: $f(x) = \begin{cases} 1 & \text{for} |x| \le 1 \\ 0 & \text{for} |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate: $\displaystyle\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$ **(5)**

Q.8 a) Find the Fourier sine transform of following function:

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(5)**

b) Solve the following partial differential equations:

i) $4xyz = pq + 2px^2y + 2qxy^2$. **(4)**

ii) $p^2 - pq = 1 - z^2$ **(4)**

UNIT 5

Q.9 a) Use Cauchy's Integral formula to evaluate:

$$\oint_c \frac{e^{2z}}{(z+1)^4}\, dz \quad c: |z| = 2$$ **(5)**

b) Find the Taylors series of the function $f(z) = \frac{(z+1)}{(z-3)(z-4)}$ about z = 2. **(5)**

c) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$ **(4)**

Q.10 a) Evaluate using residue theorem

$$\int_c \frac{(1-2z)}{z(z-1)(z-2)}\, dz \quad c: |z| = 1.5$$ **(5)**

b) Find the analytic function of, $f(z) = u(r, \theta) + iv(r, \theta)$ such that $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$ **(4)**

c) Expand $\frac{1}{z^2 - 3z + 2}$ in the region **i)** $1 < |z| < 2$ **ii)** $|z| > 2$. **(4)**

UNIT 6

Q.11 a) Find div $\vec{F}$ and Curl $\vec{F}$

where $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$ **(6)**

b) Find whether following vector field. $\vec{F}$ is conservative and find ϕ such that $\vec{F} = \nabla\phi$ and also evaluate $\int_c \vec{F}.d\bar{r}$, along the curve joining the points (1, –2, 1) to (3, 1, 4). $\vec{F} = (2xy + z^3)i + x^2 j + 3z^2 x\, k$. **(7)**

Q.12 a) Find the directional derivative of $\phi = e^{2x} \cos yz$ at the origin in the direction of the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$. **(6)**

b) Evaluate the surface integral: $\iint_S (yzi + zxj + xyk)\, ds$

where s is the surface of sphere $x^2 + y^2 + z^2 = 1$ in the first octant. **(7)**