WINTER-2015

UNIT 1

Q.1 a) Solve: $\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}+6y=e^{-2x}.\sin 2x$. **(6)**

b) Solve: $(3x+2)\dfrac{d^2y}{dx^2}+5(3x+2)\dfrac{dy}{dx}-3y=x^2+x+1$. **(7)**

Q.2 a) Solve by using variation of parameter:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\frac{x^3}{1+x^2}$$ **(6)**

b) Solve: $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+y=\dfrac{\sin(\log x)}{x}$ **(7)**

UNIT 2

Q.3 a) Find Laplace transform of $t^{-3t}.\cos 2t$ **(4)**

b) Find $L^{-1}\left\{\dfrac{e^{-2s}}{s^2+8s+25}\right\}$ **(5)**

c) Solve differential equation by Laplace transform method:

$x''(t) + x'(t) - 2x = 2(1 + t + t^2)$ with

$x = 0,\ x' = 0$ at $t = 0$. **(5)**

Q.4 a) Evaluate $\displaystyle\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt$ **(4)**

b) Use convolution theorem to evaluate:

$$L^{-1}\left\{\frac{1}{(s+1)(s+9)^2}\right\}$$ **(5)**

c) Solve the differential equations by Laplace transform.

$$\frac{dx}{dt}-\frac{dy}{dt}+29=\cos 2t,\quad \frac{dx}{dt}+\frac{dy}{dt}-2x=\sin 2t$$

Subject to $x(0) = 0,\ y(0) = -1$. **(5)**

UNIT 3

Q.5 a) Solve the following difference equations:

i. $y_{n+2} - 4y_n = n^2 + n - 1,$

ii. $y_{n+2} - 2\cos\alpha . y_{n+1} + y_n = \cos n\alpha.$ **(7)**

b) Find inverse z-transform of:

$$\frac{10z}{z^2 - 3z + 2};\ |z| > 2$$ **(4)**

Q.6 a) Solve the difference equations:

i. $y_{n+2} - 5y_{n+1} + 6y_n = 2n^2 + 3$

ii. $y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n = 1$ **(5)**

b) Solve by z-transform:

$y_{k+2} - 2y_{k+1} + y_k = 2^k$ with $y_0 = 2,\ y_1 = 1$ **(4)**

UNIT 4

Q.7 a) Express:

$$f(x) = 1,\quad |x| < 1$$
$$= 0,\quad |x| > 1$$

as a Fourier integral and hence evaluate:

$$\int_0^\infty \frac{\sin\lambda . \cos\lambda x}{\lambda} d\lambda.$$ **(6)**

b) Solve:

i. $x^2(y - z)\, p + y^2(z - x)\, q = z^2(x - y)$

ii. $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \frac{3a^2}{z^2}$ **(4)**

Q.8 a) Solve the integral equation:

$$\int_0^\infty f(t)\cos\lambda t . dt = 1,\quad 0 \le \lambda < 1$$
$$= 2,\quad 1 \le \lambda < 2$$
$$= 0,\quad \lambda > 2$$ **(6)**

b) Solve:

i. $p^2 + q^2 = z^2(x + y)$, **ii.** $yp = 2yx + \log q$. **(4)**

UNIT 5

Q.9 a) If $u = \log_e(x^2 + y^2)$, find analytic function $f(z) = u + iv$. **(4)**

b) Find k such that,

$f(z) = r^2 \cos 2\theta + ir^2 \sin k\theta$ is analytic. **(4)**

c) Expand $f(z) = \dfrac{z}{(z-1)(2-z)}$ in Laurent's series for:

i. $1 < |z| < 2$, **ii.** $|z - 1| > 2$, **iii.** $0 < |z - 2| < 1$. **(5)**

Q.10 a) If f(z) is analytic function with constant amplitude, show that f(z) is constant. **(6)**

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto points $w = 0, 1, \infty$. **(7)**

UNIT 6

Q.11 a) Find the directional derivative of $\phi = 4\, e^{2x-y+z}$ at point (1, 1, –1) in the direction towards the point (–3, 5, 6). **(6)**

b) Prove that $\nabla^2\left(\nabla.\left(\bar{r}/r^2\right)\right) = \dfrac{2}{r^4}$ **(3)**

c) If $\bar{F}_1$ and $\bar{F}_2$ are irrotational; show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. **(4)**

Q.12 a) Find the angle between the surfaces

$x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). **(6)**

b) Evaluate $\iint_S (\nabla \times \bar{F}).d\bar{s}$ for $\bar{F} = (x^2 - y^3)\hat{i} - xyz\hat{j} + y^3\hat{k}$ over the surface $x^2 + 4y^2 + z^2 - 2z = 4$, $z \geq 0$. **(7)**