WINTER – 2022

UNIT-1

Q.1 a) Solve $(D^2+3D+2)y = 4\cos^2 x$ **(6)**

b) Solve $(2x+1)^2 \dfrac{d^2y}{dx^2} - 2(2x+1)\dfrac{dy}{dx} - 12y = 6x$ **(7)**

Q.2 a) Solve $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 6y = e^x \cosh 2x$ **(6)**

b) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$ **(7)**

UNIT-2

Q.3 a) Find Laplace transform of $\dfrac{\cos 6t - \cos 4t}{t}$ And evaluate $\displaystyle\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt$ **(4)**

b) Find $L\dfrac{d}{dt}\left(\dfrac{\sin t}{t}\right)$ **(5)**

c) Express the following fu^n in terms of Heaviside unit step function and hence find its Laplace transform if **(5)**

$$f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$$

Q.4 a) If $J_o(t) = \dfrac{1}{\sqrt{1+s^2}}$. Show that $L\{e^{-at}J_o(at)\} = \dfrac{1}{\sqrt{s^2+2as+2a^2}}$ **(4)**

b) Use convolution theorem to obtain inverse Laplace transform of $\dfrac{1}{(s+1)(s^2+1)}$ **(5)**

c) Find the Laplace transform of wave function **(5)**

$$f(t) = \frac{kt}{T}\ 0 < t < T \ \&\ f(t) = f(t+T)$$

UNIT-3

Q.5 a) Solve $(D^2 + 2D + 5)y = e^{-t}\sin t\ y(0), y'(0) = 1$ **(7)**

b) Find Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$ **(6)**

Q.6 a) Solve $\dfrac{d^2x}{dt^2} + x = 6\cos 2t$ with $x = 3,\ \dfrac{dx}{dt} = 1$ at $t = 0$ **(7)**

b) Find Fourier transform of $f(x) = \begin{matrix} 1 \text{ for} |x| < 1 \\ 0 \quad |x| > 1 \end{matrix}$ Hence

find $\displaystyle\int_0^\infty \frac{\sin x}{x}dx$ **(6)**

UNIT-4

Q.7 a) Solve

i) $p^2 + q^2 = z$ **(4)**

ii) $z - px - qy = \sqrt{1 + p^2 + q^2}$ **(4)**

b) Fit a straight line to the data given below. **(5)**

X	15	20	25	30	35	40
y	12	14	18	25	31	44

Q.8 a) Solve

i) $x\dfrac{\partial z}{\partial y} = y\dfrac{\partial y}{\partial x} + xe^{x^2+y^2}$ **(4)**

ii) $p^2 - q^2 = \dfrac{x-y}{z}$ **(4)**

b) Find the line of regression of y on x.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(5)

UNIT-5

Q.9 a) If f(z) = u + iv is analytic function find f(z) if $u - v = (x - y)(x^2 + 4xy + y^2)$ **(7)**

b) Show that the transformation $w = \dfrac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$ **(6)**

Q.10 a) Show that $u = e^x \cos y + x^2 - y^2$ is harmonic function. Find its harmonic conjugate and corresponding analytic function. **(7)**

b) Find the Bilinear transformation which maps the points 0, -1, ∞ from the z – plane into -1, -2, -I, I resp. in to – plane **(6)**

UNIT-6

Q.11 a) Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1, 1, 1) **(7)**

b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ **(6)**

Q.12 a) If $\bar{f} = (x + y + 1)i + j - (x + y)k$ Prove that $\bar{f} \circ \text{curl}\,\bar{f} = 0$ **(4)**

b) If $\bar{f}$ is a solenoidal vector field prove that Curl curl curl curl $\bar{f} = -\nabla^4 \bar{f}$

c) If $\bar{f} = (2xy + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xz)k$ Evaluate $\int_c \bar{f} \circ dr$ where c is a straight line joining points (0, 0, 0) and (1, 1, 1). **(4)**