SUMMER-2014

UNIT 1

Q.1 a) Solve: $(D^2 - 3D + 2)\, y = xe^{3x} + \sin 2x$. **(6)**

b) Solve: $x^3 \dfrac{d^3y}{dx^3} + 2x^2 . \dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$. **(7)**

Q.2 a) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x.$$ **(7)**

b) Solve: $(D^2 + 3D + 2)\, y = \sin (e^x)$. **(6)**

UNIT 2

Q.3 a) Find the Laplace transform of: $\int_0^t e^t \dfrac{\sin t}{t} dt$. **(4)**

b) Using convolution theorem, find inverse of:

$$\overline{f(s)} = \frac{1}{(s-2)(s+2)^2}.$$ **(4)**

c) Solve the differential equation using Laplace transform:

$(D^2 + 4D + 8)\, y = 1,\ y(0) = 0,\ y'(0) = 1.$ **(5)**

Q.4 a) Find the inverse Laplace transform of:

$$\frac{s^2 + 2s - 4}{(s^2 + 2s + 5)(s^2 + 2s + 2)}.$$ **(4)**

b) Evaluate: $\int_0^\infty e^{-2t} \dfrac{\sinh t \sin t}{t} dt$. **(4)**

c) Express:

$$\begin{aligned} f(t) &= \cos t, && 0 < 1 < \pi \\ &= \cos 2t, && \pi < 1 < 2\pi \\ &= \cos 3t, && t > 3\pi \end{aligned}$$

in terms of unit step function and hence find its Laplace transform. **(5)**

UNIT 3

Q.5 a) Solve the difference equations:

i. $(\Delta^2 + \Delta + 1)\,y = x^2$, **ii.** $u_{n+2} + u_n = \cos n/2$. **(8)**

b) Solve $y_{n+2} + 2y_{n+1} + y_n = 0$ using z-transform, given $y_0 = y_1 = 0$. **(6)**

Q.6 a) i. Show that inverse z-transform of:

$$\frac{1}{(z-a)^3} = \frac{(k-1)(k-2)a^{k-3}}{2}.$$ **(5)**

ii. Prove that:

$$Z\{(k+1)a^{k+1}\} = \frac{az^2}{(z-a)^2}.$$ **(5)**

b) $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$. **(4)**

UNIT 4

Q.7 a) Solve the following partial differential equations:

i. $pq = x^m.y^n.z^{2l}$, **(4)**

ii. $x(z^2 - y^2)\frac{dz}{dx} + y(x^2 - z^2)\frac{dz}{dy} = z(y^2 - x^2)$. **(4)**

b) Find Fourier transform of:

$$f(x) = x, \quad 0 < x < 1/2$$
$$= 1 - x, \quad \frac{1}{2} < x < 1$$
$$= 0, \quad x > 1.$$ **(6)**

Q.8 a) Solve the following partial differential equations:

i. $(y + z)\,p + (z + x)\,q = x + y$ **(4)**

ii. $x^2p^2 + y^2q^2 = z^2$. **(4)**

b) Express the function:

$$f(x) = 1; \ |x| \le 1$$
$$= 0; \ |x| > 0, \text{ as}$$

Fourier integral and hence evaluate: $\int_0^\infty \frac{\sin\lambda\cos\lambda}{\lambda}$. **(6)**

UNIT 5

Q.9 a) If f(z) is analytic show that:

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)|f(z)|^4 = 16.|f(z)|^2.|f'(z)|^2 .$$ **(7)**

b) Find the bilinear transformation which maps the point $z = -1, 0, 1$ form z-plane into $w = 0, i, 3i$ in w-plane. **(6)**

Q.10 a) Expand $f(z) = \dfrac{z}{(z+1)(z+2)}$, about $z = -2$. **(6)**

b) Show that the transformation $w = \dfrac{2z+3}{z-4}$, maps the circle $x^2 + y^2 - 4x = 0$ into straight line $4u + 3 = 0$. **(7)**

UNIT 6

Q.11 a) Prove that:

i. $\nabla^2(\phi\psi) = \phi\nabla^2\psi \neq 2\nabla\phi.\nabla\psi \neq \psi\nabla^2\phi$

ii. $\nabla(\phi\nabla\psi + \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$. **(6)**

b) If the directional derivative of: $\phi = ax^2y + by^2z + cz^2x$ at a point (1, 1, 1) has maximum magnitude is in the direction parallel to the line $\dfrac{x-1}{2} = \dfrac{y-3}{-2} = \dfrac{z}{1}$, find the value of a, b, c.

(7)

Q.12 a) A vector field given by: $\overline{F} \sin yi + x(1+\cos y)j$.

Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2$, $z = 0$. **(6)**

b) Evaluate: $\iint\limits_S \overline{F} \circ \hat{n}\, dS$, where $\overline{F} = zi + xj - 3y^2zk$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. **(7)**