

# Imp Question Bank.

Date : \_\_\_\_\_

Page : \_\_\_\_\_

## UNIT - I.

① Solve :

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin \log(1+x)$$

$$② (D^3 + 1)y = \sin 3x - \cos^2 \frac{x}{2}$$

$$③ (D^2 + 5D + 6)y = e^{-2x} \sin 2x + 4x^2 e^x$$

④ Solve by vop.

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = \cos e^x$$

$$⑤ (2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 6x$$

$$⑥ (D^4 - m^4)y = \sin mx$$

$$⑦ x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$$

$$⑧ \frac{d^2y}{dx^2} + y = \operatorname{cosec} x \text{ by vop.}$$

$$⑨ \text{Solve } (D^2 + 2D + 2)y = \sinh x + x^2$$

$$⑩ \text{Solve } x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \log x$$

## UNIT - II

① using convolution thm, find  $\mathcal{L}^{-1} \left[ \frac{(s+2)^2}{(s^2+4s+8)^2} \right]$

② find  $f(t)$  if  $\bar{F}(s) = \log \left( \frac{s+b}{s+a} \right)$

③ Find Laplace transform of the periodic fun,

$$f(t) = \frac{t}{a}, \quad 0 < t < a$$

$$= \frac{1}{a} (2a - t), \quad a < t < 2a$$

$$\text{and } f(t) = f(t + 2a)$$

④ find  $\mathcal{L}^{-1} \left[ \frac{s}{(s+1)(s+9)^2} \right]$

⑤ find  $\mathcal{L} \left[ \frac{d}{dt} \left( \frac{\sin t}{t} \right) \right]$

⑥ evaluate  $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$

⑦ find  $f(t)$  if  $\bar{F}(s) = \frac{1}{2} \log \frac{(s-1)}{(s+1)}$

⑧ using Convolution thm. find  $\mathcal{L}^{-1} \left[ \frac{1}{(s+1)(s^2+1)} \right]$

⑨ find L.T of  $f(t)$  if,  $f(t) = a \sin pt, \quad 0 < t < \frac{\pi}{p}$   
 $= 0, \quad \frac{\pi}{p} < t < \frac{2\pi}{p}$

$$\text{and } f(t) = f(t + \frac{2\pi}{p})$$



### UNIT - III

① Solve by using Laplace transform  
 $\frac{d^2x}{dt^2} + x = 6 \cos 2t$  with  $x=3$ ,  
 $\frac{dx}{dt} = 1$  at  $t=0$ .

② using Fourier cosine integral s.t.  
 $\int_0^\infty \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi e^{-kx}}{2k}, \quad x > 0, k > 0$

③ solve by using Laplace transform  
 $(D^2 + 2D + 5)y = e^{-t} \cdot \sin t$ , where  
 $y(0) = 0$  and  $y'(0) = 1$ .

④ Find Fourier sine transform of  
 $\frac{-|x|}{e}$  and evaluate  $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$

⑤ using Fourier integral representation, s.t.  
 $\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, \quad (x > 0)$

⑥ Express the function:  $f(x) = 1, |x| \leq 1$   
 $= 0, |x| > 1$   
as Fourier integral. Hence evaluate  
 $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$

⑦ Find Fourier sine transform of  $f(x) = \frac{e^{-ax}}{x}$

## UNIT - VI

- (1) Find the D.O. of surface  $\phi = x^2y + y^2z + z^2x$  at point  $(3, 1, 2)$  is the direction of Normal to the surface  $xyz = 6$  at the point  $(1, 2, 3)$ .

- (2) A fluid motion is given by  $\vec{v} = (y \sin z - \sin x)\vec{i} + (x \sin z + 2yz)\vec{j} + (xy \cos z + y^2)\vec{k}$ . Is the motion is irrotational? If so, find scalar potential function.

- (3) Evaluate  $\iint_S \vec{F} \cdot \vec{n} \, ds$ , whr.

$\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$  and  $S$  is the surface of the plane  $x^2 + y^2 + z^2 = 1$  in the 1st octant.

- (4) Find the angle betn the normal to the surface  $x \log z - y^2 = -4$  and  $x^2y + z = 3$  at pt  $(-1, 2, 1)$ .

- (5) Find the directional derivative of  $f(x, y, z) = x^2y^2z^2$  at pt  $(1, 1, -1)$  in the direction of tangent to the curve  $x = e^t$ ,  $y = 2 \sin t$ ,  $z = t - \cos t$  at  $t = 0$ .



(6) A vector field is given by  $\vec{F} = (x^2 - y^2 + x)\mathbf{i} - (2xy + y)\mathbf{j}$  s.t the field is irrotational and find scalar potential.

(7) If  $\vec{F} = (2x^2 - 3z)\mathbf{i} - 2xy\mathbf{j} - 4xz\mathbf{k}$  then evaluate  $\iiint_V \nabla \cdot \vec{F} \, dv$ , where

$V$  is bounded by  $x=0$ ,  $y=0$ ,  $z=0$  and  $2x + 2y + z = 4$ .

## UNIT - V

① Find the analytic function whose real part is  $u = e^{2x} (x \cos 2y - y \sin 2y)$ .

② Find 'p' such that function  $f(z) = \frac{1}{2} \log (x^2 + y^2) + i \tan^{-1} \left( \frac{px}{y} \right)$  is analytic.

③ S.T. the transformation  $w = z^2$  maps the circle  $|z-1| = 1$  into the cardioid  $\rho = 2(1 + \cos \phi)$  where  $w = \rho e^{i\phi}$ .

④ If  $f(z)$  is regular analytic function of  $z$ . p.T

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

⑤ Expand  $f(z) = \sin z$  in Taylor series at  $z = \frac{\pi}{6}$ .

⑥ S.T. under transformation  $w = \frac{z+3}{z-4}$

the circle  $|z-2| = 2$  in  $z$  plane is transform into straight line  $4u + 3 = 0$  in  $w$ -plane.



7) expand the function  $\frac{1}{(z+1)(z+2)}$

in the region

i)  $0 < |z| < 1$ , ii)  $1 < |z| < 2$

iii)  $|z| > 2$

8) expand  $\frac{1}{z^2 - 4z + 3}$  in the region.

i)  $|z| < 1$ ,

ii)  $1 < |z| < 3$

iii)  $|z| > 3$

iv)  $0 < |z-1| < 2$ .



## UNIT - IV

① solve: p.d.e.

①  $p \tan x + q \tan y = \tan z$

②  $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \frac{3a^2}{z^2}$

③  $(mz - ny)p + (nx - lz)q = ly - mx$

④  $\sqrt{p} + \sqrt{q} = 2x$

⑤  $yp + xq + pq = 0$

⑤ If a cap R related with v by relation  $R = a + bv + cv^2$ , find, a, b, c for corresponding data.

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.9

⑥ obtain eqn of line of regression y on x.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

⑦ find the coefficient of correlation.

x	10	14	18	22	26	30
y	18	12	24	6	30	36