Unit VI

Part 1.Work Energy Method

Principle:-

Sum of work done by each body is equal to the sum of change in kinetic energy of each body.

Or

Total work done by a system of blocks (bodies) is equal to the total change in Kinetic Energy of the system.

i.e. Total work done = Change in Kinetic Energy

$$TWD = \Delta\ KE$$

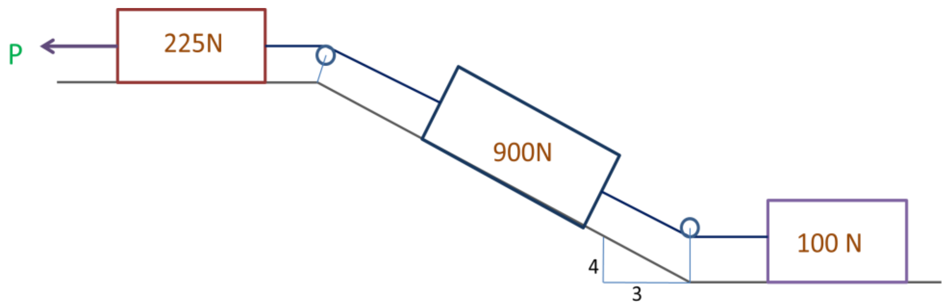
$$FxS = \frac{1}{2}\ x\ m(V^2 - u^2)$$

<u>Steps for solving problems:-</u>

1. Determine direction of motion of each block
2. Determine Kinematic relations.
3. Draw combined FBD of the system. Apply work Energy principle to the system. Do not consider each block separately.

1) Determine the constant force P that will give the system of bodies shown in fig. a velocity of 3 m/s. after moving 4.5m from rest.

(7Marks)

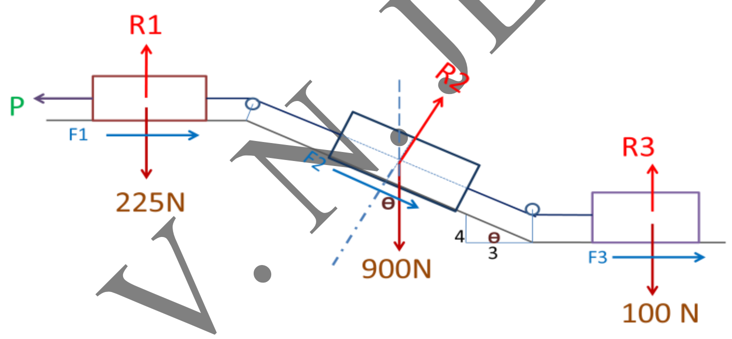


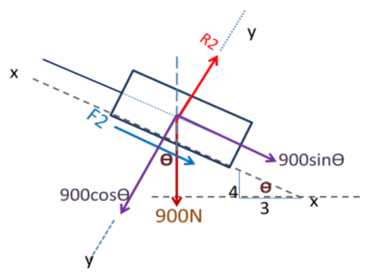
Solution: -

Drawing combined FBD of all blocks

Here velocities of all blocks are same.

Applying work energy principal to the system.





FBD for Block 2

Total work done = Change in Kinetic Energy

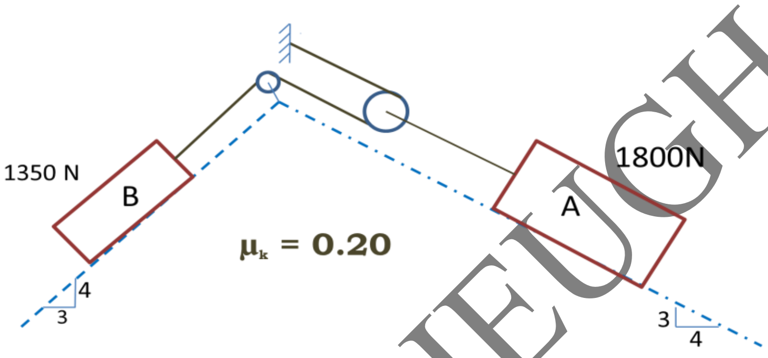
TWD = Δ KE

(WD) 1 + (WD) 2+ (WD) 3 = Δ KE1+Δ KE2+Δ KE3

$$(P - FA)\,xS + (-FB - 900\sin 53.13)\,xS + (-FC)\,x\,S$$
$$= \frac{1}{2}\,x\frac{225}{9.81}\,x\,(3^2 - 0) + \frac{1}{2}\,x\frac{900}{9.81}\,x\,(3^2 - 0) + \frac{1}{2}\,x\frac{100}{9.81}\,x\,(3^2 - 0)$$

$$(P - 0.2\,x225 - 0.2\,x\,900\cos 53.13 - 900\sin 53.13 - 0.2\,x100)\,x\,4.5$$
$$= \frac{1}{2}\,x\frac{(225 + 900 + 100)}{9.81}\,x\,(3^2 - 0)$$
$$P = 1076.86\text{ N}$$

2) In what distance will body A as shown attain a velocity of 3 m/s starting from rest.



Solution:-

Given S_A =? VA= 3 m/s, u_A=0, u_B= 0 $\Theta_1 = \tan^{-1}(4/3) = 53.13$, $\Theta_1 = \tan^{-1}(3/4) = 36.86$,

1. To decide direction of motion.

Assuming the system at rest. Considering FBD of block B.

Making algebraic sum of all the forces which are responsible for restrict the motion of block

$\sum Fx = 0;$

T – 1350 sin53.13 = 0

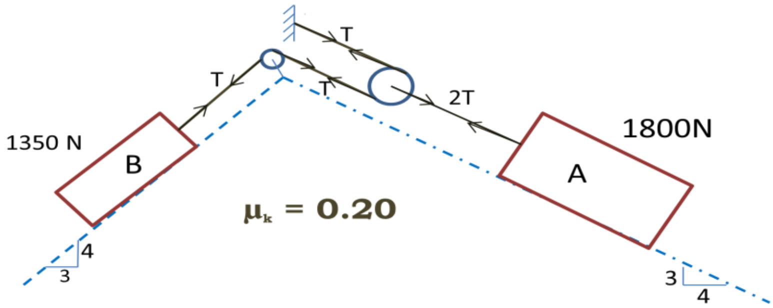
T = 1079.9 N: 2T = 2159.9N

Considering block A,

2T-1800sin 36.86

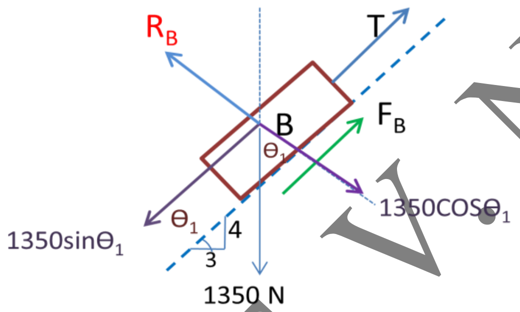
= 1079.9 N < 2T

Block A will move in upward direction and Block B will move in downward direction.

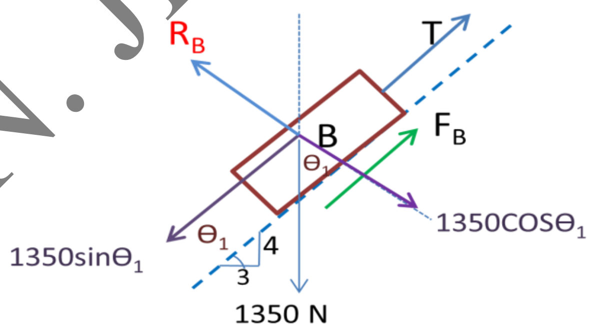


2. Kinematic relations.

$$WD_B = WD_A$$
$$T\,S_B = 2T\,S_A$$
$$S_B = 2\,S_A$$
$$V_B = 2V_A$$
$$a_B = 2\,a_A$$



FBD for block B



FBD for block B

3. Applying Work Energy Principle

Total work done = Change in Kinetic Energy

$$TWD = \Delta\ KE$$

$$(WD)_A + (WD)_B = \Delta\ KE_A + \Delta\ KE_B$$

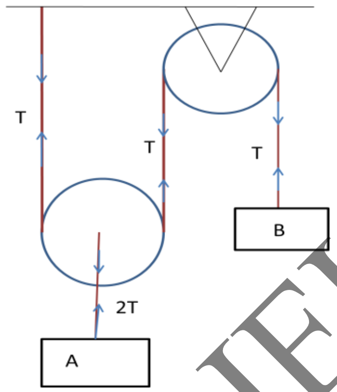
$$(-FA - 1800\sin 36.86) \times S_A + (-FB + 1350\sin 53.13)\,S_B =$$
$$\frac{1}{2} \times \frac{1800}{9.81} \times (v_A{}^2 - u_A{}^2) + \frac{1}{2} \times \frac{1350}{9.81} \times (v_B{}^2 - u_B{}^2)$$

$$(-0.2 \times 1800\cos 36.86 - 1800\sin 36.86) \times S_A + (-0.2 \times 1350\cos 53.13 +$$

$$1350\sin 53.13)\text{x } 2\, S_A \;=\; \frac{1}{2}\,\text{x}\,\frac{(1800)}{9.81}\,\text{x}\,(3^2-0)+\frac{1}{2}\text{x}\frac{(1350)}{9.81}\,\text{x}\,((2\text{x}3)^2-0)$$

$$S_A = 7.05 \text{ m}$$

3) To what distance will body A as shown move in changing its velocity from 1.8 m/s to 3.6 m/s. Take W_A= 900N & W_B = 1350N



Solution:-

1. To decide direction of motion.

Assuming the system at rest. Considering block A.

$\sum F_y = 0$; 2T-900 = 0

T = 450N; T = 1350 N

W_B = 1350N < 2T

As TB > TA block B will move in down direction and Block A will move in upward direction.

2. Kinematic relations.

WDA = WDB

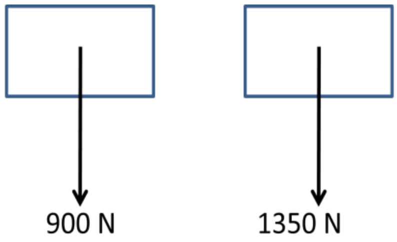
T SA = 2 T SB

SA = 2 SB

VA = 2 V B

aA= 2 aB

Applying Work Energy Principle



FBD for block A FBD for block B

Total work done = Change in Kinetic Energy

$$TWD = \Delta\ KE$$

$$(WD)\ A + (WD)\ B = \Delta\ KEA + \Delta\ KEB$$

$$(-900)\ x\ SA\ +\ (1350)\ SB = \frac{1}{2}\ x \frac{900}{9.81} x\ (3.6^2 - 1.8^2)\ +\ \frac{1}{2}\ x \frac{1350}{9.81} x\ (V_B^2 - u_B^2)$$

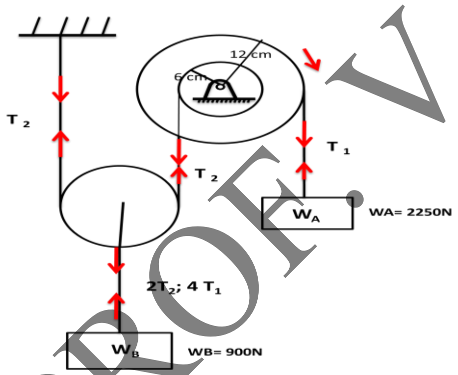
$$(-900) x\ SA\ +\ (1350)\ {}^{SA}/_{2}$$

$$= \frac{1}{2}\ x \frac{900}{9.81} x\ (3.6^2 - 1.8^2) + \frac{1}{2}\ x \frac{1350}{9.81} x\ [({}^{3.6}/_{2})^2 - ({}^{1.8}/_{2})^2]$$

SA= 0.1617 m

4) Determine the velocity of body A after it has moved 3.6 m starting from rest. Assume the pulleys as frictionless and having negligible weight.

(8-9 marks)



Solution:

1. To decide direction of motion.

Assuming the system at rest. Considering block A.

T1= 2250N

$\sum$ M@o = 0; 12T1 = 6T2

2T1 = T2

Consider block B

∴ 2T2 = 4T1 = 4 x 2250 N = 9000 N

WB < 4T1

∴ Block B will move in upward direction and Block A will move in downward direction.

2. Kinematic relations.

4T1SA = T1SB

4 SA = SB

4VA = V B

4aA= aB

Applying Work Energy Principle

Total work done = Change in Kinetic Energy

TWD = Δ KE

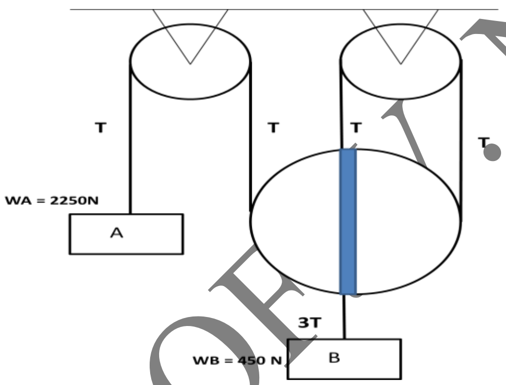
(WD) A + (WD) B = Δ KEA+Δ KEB

(2250) x SA + (-900) SB= $\frac{1}{2}$ x $\frac{2250}{9.81}$x (VA2-uA2) + $\frac{1}{2}$ x $\frac{900}{9.81}$x (VB2-uB2)

(2250) x3.6+ (-900) x 14.4 = $\frac{1}{2}$ x $\frac{2250}{9.81}$x (VA2-0) + $\frac{1}{2}$ x $\frac{900}{9.81}$x (16 VA2-0)

VA= 2.393 m/s

5) Determine the velocity of body B after it has moved 3.0 m starting from rest. Assume the pulleys as frictionless and having negligible weight.



1. To decide direction of motion.

Assuming the system at rest. Considering block A.

T= 2250N

3T = 3 x 2250 = 6750 N

WB < 3T

∴ Block B will move in upward direction and Block A will move in downward direction.

2. Kinematic relations.

TSA = 3TSB

SA = 3SB

VA =3 VB

aA=3 aB

Applying Work Energy Principle

Total work done = Change in Kinetic Energy

TWD = Δ KE

(WD) A + (WD) B = Δ KEA+Δ KEB

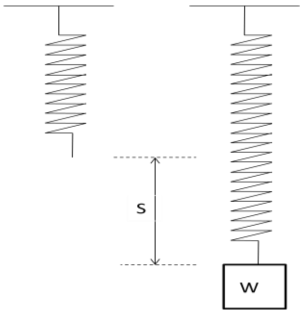
$$(2250)\text{ x SA } + (-450)\text{ SB} = \frac{1}{2}\text{ x}\frac{2250}{9.81}\text{x }(\text{VA}^2 - \text{uA}^2) + \frac{1}{2}\text{ x}\frac{450}{9.81}\text{x }(\text{VB}^2 - \text{uB}^2)$$

$$(2250)\text{x}9 + (-450)\text{x } 3 = \frac{1}{2}\text{ x}\frac{2250}{9.81}\text{x }(3\text{VB}^2 - 0) + \frac{1}{2}\text{ x}\frac{450}{9.81}\text{x }(\text{VB}^2 - 0)$$

VB= 1.6 m/s

Problems on spring

Work done by spring



We can say that F α s

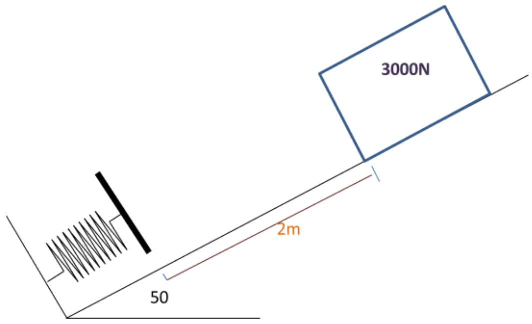
F = ks

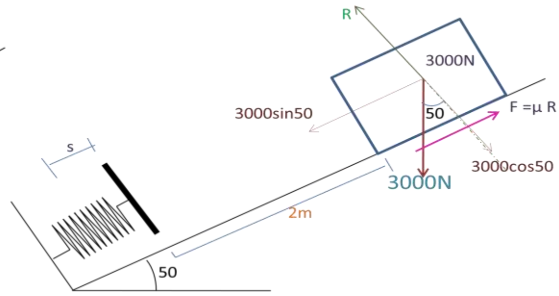
∴K = F/s

Where k is known as stiffness of spring or modulus of spring or spring constant. Its SI unit is N/m

$$\text{Work done by spring } = \frac{1}{2}\text{k}s^2$$

6) A 3000 N block starting from rest as shown, slides down the 50 degree inclined plane. After travelling 2 m. it strikes a spring whose modulus is 20 N/mm. If coefficient of friction is 0.2, determine the maximum deformation of the spring and maximum velocity of the block.





Applying Work Energy Principle

Total work done = Change in Kinetic Energy

$$TWD = \Delta\ KE$$

$$(WD)_{block} + (WD)_{spring} = \Delta\ KE_{block} + \Delta\ KE_{spring}$$

$$(3000\sin 50 - F) \text{ x } (2 + S) + \frac{1}{2} ks^2 = 0 + 0$$

$$(3000\sin 50 - 0.2 \text{ x} 3000\cos 50) \text{ x } (2 + S) + \frac{1}{2} \text{x} 20 \text{x} 10^3 \text{x} s^2 = 0 + 0$$

s = 0.26 m

∴ **Maximum deformation of the spring (s) = 0.26 m**

To find maximum velocity of the block

Velocity is maximum when, $\frac{dv}{dt} = 0$ or acceleration is zero.

And acceleration is zero when force is zero.

∴ Net force exerted by spring = Net force exerted by block

$$ks' = 3000\sin 50 - F$$

$$20 \text{ x} 10^3 s' = 3000\sin 50 - 0.2 \text{ x} 3000\cos 50$$

$$s' = 0.095 \text{ m}$$

Applying Work Energy Principle

Total work done = Change in Kinetic Energy

$$TWD = \Delta\ KE$$

$$(WD)_{block} + (WD)_{spring} = \Delta\ KE_{block} + \Delta\ KE_{spring}$$

$$(3000\sin 50 - F) \text{ x } (2 + S') + \frac{1}{2} ks'^2 = 0 + 0$$

$$(3000\sin 50 - 0.2 \text{ x } 3000\cos 50) \text{ x } (2 + 0.095) + \frac{1}{2} \text{x} 20 \text{x} 10^3 \text{x} 0.095^2 = \frac{1}{2} \text{ x} \frac{3000}{9.81} \text{x } (Vmax^2 - 0) + 0$$

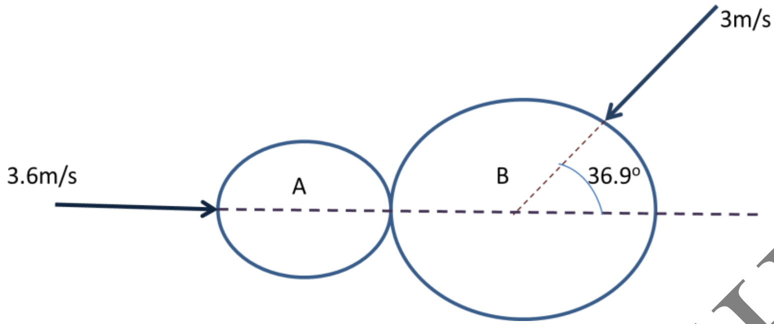
V_{max} = 5.16 m/s

∴Maximum velocity of the block **(V_{max}) = 5.16 m/s**

UNIT VI

PART 2.IMPULSE MOMENTUM METHOD

7) Just before collision, two discs on a horizontal surface have the velocities as shown. Knowing that 90 N Disc A rebounds to the left with a velocity of 1.8 m/s, determine the rebound velocity of the 135 N disc B. Assume the colliding surfaces as smooth.



Solution: By applying law of conservation of momentum equation

In X direction

$\xrightarrow{+}$ $$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$\frac{90}{9.81} x 3.6 + \frac{135}{9.81} \text{ x } (-3\cos 36.9) = \frac{90}{9.81} \text{ x } (-1.8) + \frac{135}{9.81} \text{ x } v_{2x}$$

$v_{2x} = 1.2$ m/s

In Y direction

$+\uparrow$

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

$$\frac{90}{9.81} \text{ x } 0 + \frac{135}{9.81} \text{ x } (-3\sin 36.9) = \frac{90}{9.81} \text{ x } (0) + \frac{135}{9.81} \text{ x } v_{2y}$$

$v_{2y} = -1.80$ m/s

$$v_2 = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$

$$v_2 = \sqrt{(1.2)^2 + (-1.80)^2}$$

$V_2 = 2.16$ m/s

$$\theta = tan^{-1}\frac{v_{2y}}{v_{2x}};$$

$$\theta = tan^{-1}\frac{-1.8}{1.2}$$

$\theta = 56.30^0$

8) Direct central impact occurs between two discs as shown. If coefficient of restitution is 0.6, determine average impact force for the time of impact lasting 0.02 second.

Solution:

By applying law of conservation of momentum equation

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\frac{270}{9.81} \text{x} 3.0 + \frac{135}{9.81} \text{ x } (-6) = \frac{270}{9.81} \text{ x } (v_1) + \frac{135}{9.81} \text{ x} (v_2)$$

$$2v_1 + v_2 = 0 - - - - - - - - - - - - - - (1).$$

We have

$$e = \frac{v_2 - v_1}{u_1 - u_2} ;$$

$$0.6 = \frac{v_2 - v_1 1}{3-(-6)}$$

$$v_2 - v_1 = -5.4$$

$$v_2 - v_1 = -5.4 \ldots \ldots \ldots (2)$$

Solving equation 1 & 2

v_1 = -1.8 m/s

∴V_2 = 3.6 m/s

Now by applying impulse momentum equation

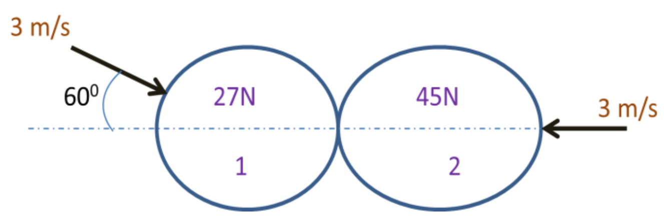
$$F \text{ x } t = m_1 (v_1 - u_1)$$

$$F \text{ x } 0.02 = \frac{270}{9.81}(-1.8 - 3)$$

F = -6605 N

$$\therefore \text{ Impact force F } = -6605 \text{ N}$$

9) A 27 N ball moving at 3 m/s acting at 60^0 to the horizontal plane collides with a 45 N ball moving horizontally leftward at 3m/s. At the moment of impact, line joining the centre of balls is horizontal. If e = 0.6, determine magnitude and direction of velocity of each block after impact.



Solution:

By applying law of conservation of momentum equation

In X direction

$\xrightarrow{}$ +

$$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$$

$$\frac{27}{9.81}\text{x}3\cos 60 + \frac{45}{9.81}\text{ x }(-3) = \frac{27}{9.81}\text{ x }(v_1\text{x}) + \frac{45}{9.81}\text{ x }(v_2)$$

$$2.75v_{1x} + 4.58\ v_2 = 8.24\ldots\ldots\ldots\ldots(1)$$

We have

$$e = \frac{v2-v1}{u1-u2};\qquad 0.6 = \frac{v2-v1x}{3cos60-(-3)};$$

$$v_{1x} + v_2 = 2.7\ \ldots\ldots\ldots\ldots\ (2)$$

Solving equation 1 & 2

$$v_{1x} = -9.6\text{ m/s}$$

$$\therefore \mathbf{V_2 = 6.9\ m/s}$$

In Y direction

+↑

$$m_1u_{1y} + m_2u_{2y} = m_1v_{1y} + m_2v_{2y}$$

$$\frac{27}{9.81}\text{x }(-3\sin 60) + \frac{45}{9.81}\text{ x }(0) = \frac{27}{9.81}\text{ x }(v_1y) + \frac{45}{9.81}\text{ x }0$$

$$V_{1y} = -2.5\text{ m/s}$$

$$v_1 = \sqrt{(v_{1x})^2 + (v_{1y})^2}$$

$$V_1 = 3.9\text{ m/s}$$

$$\theta = tan^{-1}\frac{v2y}{v2x};\qquad \boldsymbol{\theta = 50.19^0}$$

10) A bullet weighing 0.3 N and moving at 660m/s penetrates a 45 N block and emerges out with a velocity of 180 m/s as shown. How far and how does the block will then move? Take μ =0.4.

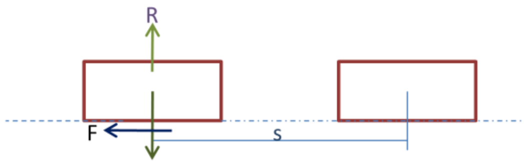
Solution: By applying law of conservation of momentum equation

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\frac{0.3}{9.81}\text{x}660 + \frac{45}{9.81}\text{ x }(0) = \frac{0.3}{9.81}\text{ x }(180) + \frac{45}{9.81}\text{ x }v_2$$

$$v_2 = 3.2\text{ m/s}$$

v_2 is the velocity with which the block 2 will start moving.



Applying work energy principle on the block only

$$F x s = \frac{1}{2} x\ m\ x\ (v_2 - u_2)$$

$$-0.4 \text{ x } 45 x s = \frac{1}{2} x \frac{45}{9.81} \text{ x } (0^2 - 3.2^2)$$

s = 1.3 m.

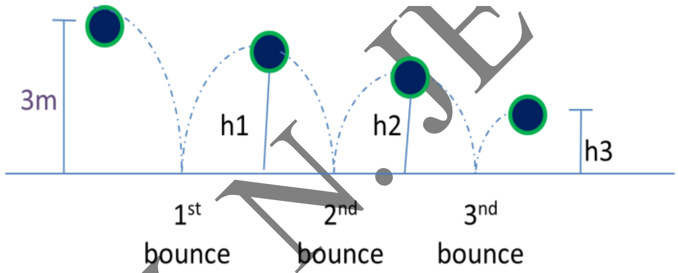
Now by applying impulse momentum equation

$$F \text{ x } t = m_1 (v_1 - u_1)$$

$$-0.4 \text{ x } 45 \text{ x } t = \frac{45}{9.81}(0 - 3.2)$$

t = 0.81 sec.

11) A golf ball is dropped on a floor from a height of 3m.If e = 0.984, determine the velocity of ball after 3rd bounce.



Solution: - velocity before 1st bounce (v) = $\sqrt{2gh} = \sqrt{(2x9.81x3)}$ = 7.67 m/s

Velocity after 1st bounce (v_1) = v x e = 7.67 x 0.98 = 7.54 m/s

We know

$$v_1 = \sqrt{(2gh_1)}$$

$$7.54 = \sqrt{(2x9.81xh_1)}$$

h_1 = 2.89m

Velocity after 2nd bounce (v_2) = v_1 x e = 7.54 x 0.98 = 7. 41 m /s

We know

$$v_2 = \sqrt{(2gh_2)}$$

$$7.41 = \sqrt{(2x9.81xh_2)}$$

h_2 = 2.79m

Velocity after 3rd bounce (v_3) = v_2 x e = 7.41 x 0.98 = **7. 29 m /s**