Project #2 : Kinematic Decoupling

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Inverse Kinematics problems can be de-coupled into two simpler problems:

- Inverse Position Kinematics
- Inverse Orientation Kinematics

Assumption: The first three joints are entirely responsible for *Positioning* the end-effector and any additional joints are responsible for *Orienting* the end-effector

For six-DOF manipulator spherical wrist, the inverse kinematic problem can be separated into two simpler problem. This can be done using two sets of equations representing rotational and positional equations:

- $R_0(q_1, ..., q_6) = R$
- $d_0^6(q_1, ..., q_6) = d$

We can solve the first 3 joint variables to get O_c to get the orientation of end effector relative to $\mathrm{O}_3X_3Y_3Z_3$
$\mathrm{R} = \mathrm{R}_3R_6$

Examples of Kinematic Decoupling are:

- Three-link Robot

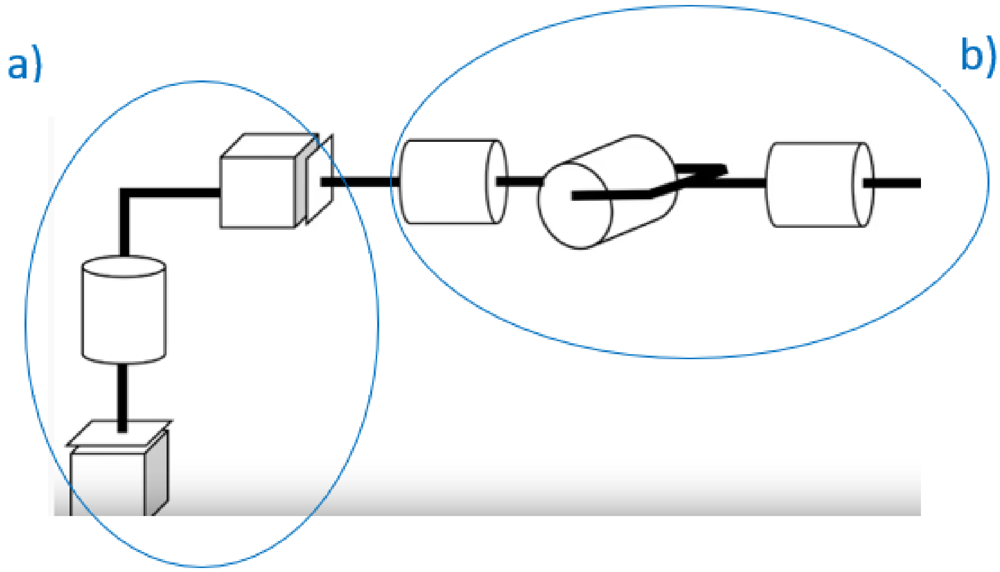

Figure 1: 6-DOF Manipulator

- SCARA Arm
- Spherical Wrist

Example: To be solved for the below 6-DOF manipulator.

As can be seen, the problem can be separated into two parts:

- a) Cylindrical Manipulator
- b) Spherical Wrist

a) First, Solve for the first three joints

$\theta_2 = tan^{-1}(y/x)$ $\quad$ $\mathrm{d}_1 = Z - a_2 - a_1$

$\mathrm{d}_3 = \sqrt{X^2 + Y^2} - a_3 - a_4$

Forward Kinematics on first three joints:

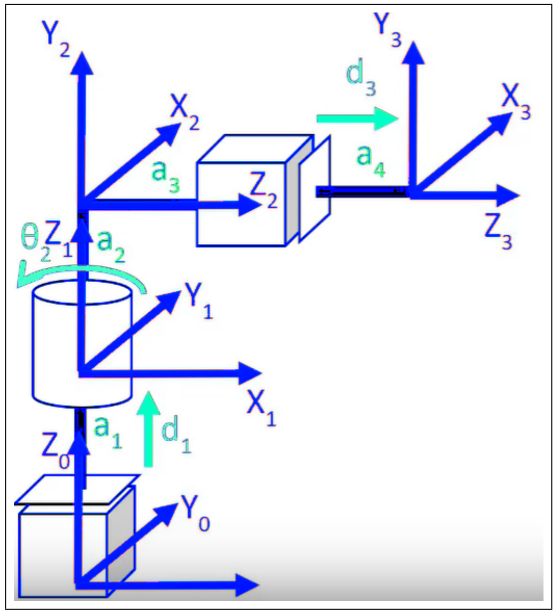

Figure 2: Solving for the first three joints

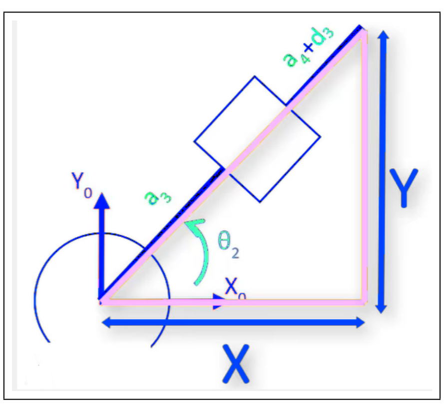

Figure 3: Top View

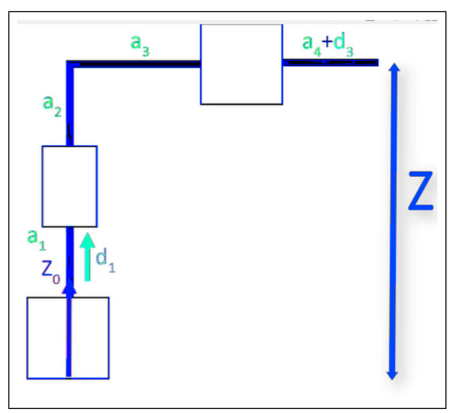

Figure 4: Side View

We can use the rotation matrix method to find the rotation from frame 3 to frame 0. This is because there is only rotation at joint number 2. Hence, can be shown as $R_0^3 = \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 \\ C\theta_2 & 0 & S\theta_2 \\ 0 & 1 & 0 \end{bmatrix}$

R_0^6 is what we want the rotation of the end-effector to be.

Now, We Know that:

$$R_0^6 = R_0^3 R_3^6$$

We can solve for R_3^6 as follows:

$$(R_0^3)^{-1} R_0^6 = (R_0^3)^{-1} R_0^3 R_3^6$$

$$R_3^6 = (R_0^3)^{-1} R_0^6$$

b) Solving the remaining half:

Second, to find R_3^6, we can go from frame 3 to frame 6, can be done using DH parameters or rotation matrix method. This can be done by first finding R_3^4 , R_4^5 and R_5^6 and then multiplying them to get R_3^6.

$$R_3^6 = \begin{bmatrix} -S\theta_4 C\theta_5 C\theta_6 - C\theta_4 S\theta_6 & S\theta_4 C\theta_5 S\theta_6 - C\theta_4 C\theta_6 & -S\theta_4 S\theta_5 \\ C\theta_4 C\theta_5 S\theta_6 - S\theta_4 S\theta_6 & -C\theta_4 C\theta_5 C\theta_6 - S\theta_4 C\theta_6 & C\theta_4 S\theta_5 \\ -S\theta_5 C\theta_6 & S\theta_5 S\theta_6 + C\theta_5 & C\theta_5 \end{bmatrix}$$

Now, suppose we have the final R_0^6 specified as some matrix, then we can specify the R_3^6 and then solve for the angles

Now, to solve for the angles defined in R_3^6, we can compare the matrix we found using $R_3^6 = (R_0^3)^{-1} R_0^6$

And the above formula

$$R_3^6 = \begin{bmatrix} -S\theta_4 C\theta_5 C\theta_6 - C\theta_4 S\theta_6 & S\theta_4 C\theta_5 S\theta_6 - C\theta_4 C\theta_6 & -S\theta_4 S\theta_5 \\ C\theta_4 C\theta_5 S\theta_6 - S\theta_4 S\theta_6 & -C\theta_4 C\theta_5 C\theta_6 - S\theta_4 C\theta_6 & C\theta_4 S\theta_5 \\ -S\theta_5 C\theta_6 & S\theta_5 S\theta_6 + C\theta_5 & C\theta_5 \end{bmatrix}$$

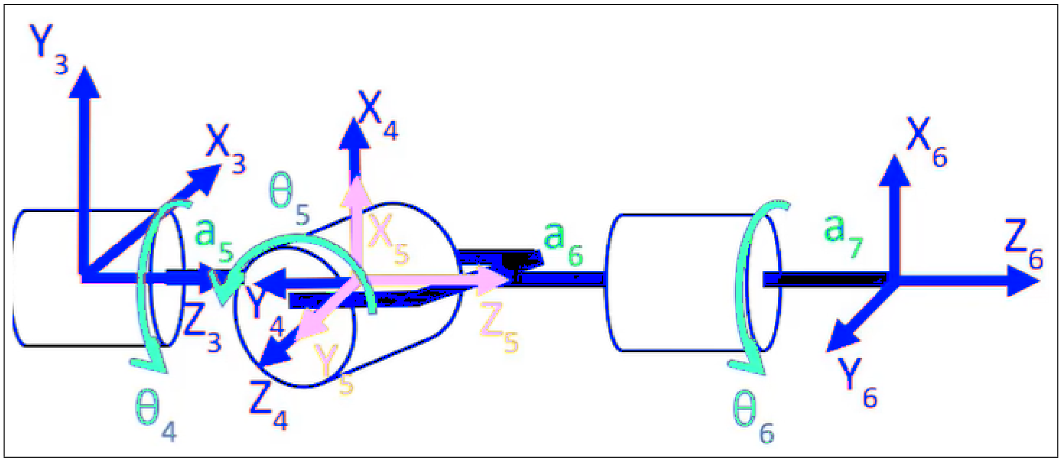

Figure 5: Last Three Joints

Solving for an example value of R_0^6 using Matlab:

Suppose, we have length X=10, Y=0 and $R_0^6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

MATLAB CODE TO FIND ANGLES:

```
clear all; clc; close all;
X=10;
Y=0;
Theta_2=atan(Y/X);
R0_6=[1 0 0; 0 1 0; 0 0 1];
R0_3=[-sin(Theta_2) 0 cos(Theta_2);
    cos(Theta_2) 0 sin(Theta_2);
    0 1 0];
R3_6 = inv(R0_3)*R0_6
Theta_5 = acos(R3_6(3,3))
Theta_6 = acos(R3_6(3,1)/sin(Theta_5))
Theta_4 = acos(R3_6(2,3)/sin(Theta_5))


R3_6 =
```

```
     0     1     0
     0     0     1
     1     0     0


Theta_5 =

    1.5708


Theta_6 =

     0


Theta_4 =

     0
```

Thus we decoupled and solved for the angles in this given examples.