Project #3 : Controls

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1 Objective

A simplified version of 2DOF of an end-effector is attached to the robot. Where the Force Fa represents a force created by robot's actuators. Force Fe represents a force exerted by the environment at the point in time when a robot interacts with its surrounds.
Following are the tasks:

- Find a Mathematical Model of the system as shown above. Here, we have choose to control the distance between the two masses.
- We have assume the Spring Coefficient (K) = 200 N/m ; Damping Coefficient (B) = 4 Ns/m ; Masses for both body(M_r and M) = 5 Kg.
- We have designed PID controller using SIMULINK to achieve an periodic robot move when a step environment force is applied on the system.
- We have tuned PID controller on MATLAB and recorded step response of the control system.

2 Mathematical Model

As shown in the figure below, the end effector consists of two masses connected by Spring and Damper between them and external force F_e is applied on to the system. The actuator force F_a works an reaction for to the system.

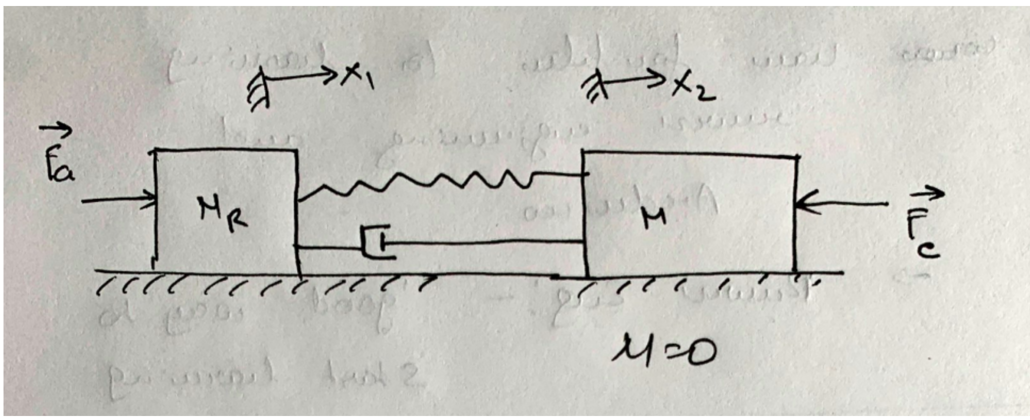

Figure 1: 2 Degree of Freedom System

Here, in order for the admittance control we have decided to control the displacement between the two masses when an external force acts upon the system. Thus transfer function will be:

$$G(S) = \frac{X_1(S) - X_2(S)}{F_e(S)} \tag{1}$$

where,
$X_1(S)$: Displacement of body Mr.
$X_2(S)$: Displacement of body M.
$F_e(S)$: External Force.
$G(S)$: Transfer function of the system.

Solving the mathematical model for the body 1 using free body diagram:

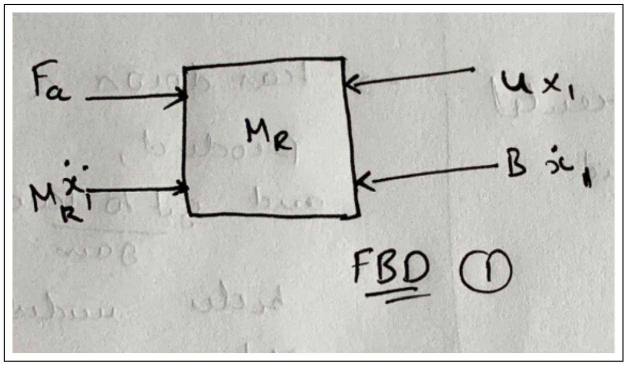

Figure 2: Body 1 - Free Body Diagram

$$F_a + M_r\ddot{x}_1 = K(x_1 - x_2) + B(\ddot{x}_2 - \ddot{x}_1) \tag{2}$$

Using the zero initial conditions and converting mathematical model into Laplace transformation, we get:

$$F_a + M_r S^2 X_1(S) = KX_1(S) - KX_2(S) + BSX_1(S) - BSX_2(S) \tag{3}$$
$$F_a + X_1(S)(M_r S^2 - K - BS) + X_2(S)(K + BS) = 0 \tag{4}$$

Solving the mathematical model for the body 2 using free body diagram:

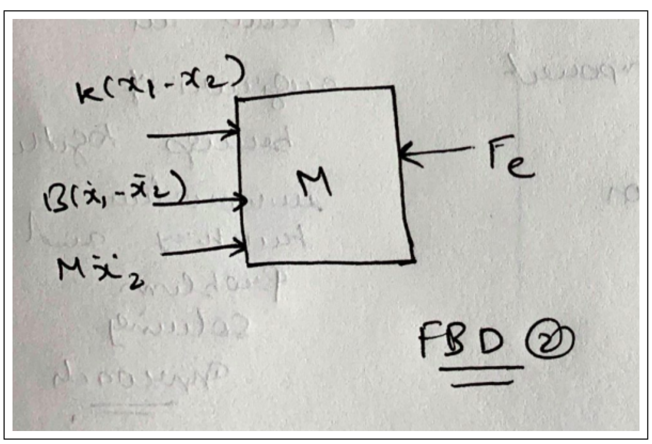

Figure 3: Body 2 - Free Body Diagram

$$F_e = M\ddot{x}_2 + K(x_1 - x_2) + B(\ddot{x}_2 - \ddot{x}_1) \tag{5}$$

Again,using the zero initial conditions and the mathematical model into Laplace transformation, we get:

$$F_e = KX_1(S) - KX_2(S) + BSX_1(S) - BSX_2(S) + MS^2X_2(S) \tag{6}$$

From equation(4) & equation(6), we get:

$$\begin{bmatrix} -M_rS^2 + K + BS & -K - BS \\ K + BS & -K - BS - MS^2 \end{bmatrix} \begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} = \begin{bmatrix} F_a \\ F_e \end{bmatrix}$$

$$\begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -K - BS + MS^2 & K + BS \\ -K - BS & -M_rS^2 + K + BS \end{bmatrix} \begin{bmatrix} F_a \\ F_e \end{bmatrix} \tag{7}$$

Thus,

$$X_1(S) - X_2(S) = \frac{1}{\Delta}(F_a(MS^2) + F_e(M_rS^2)) \tag{8}$$

where, $\Delta = (-M_rS^2 + BS + K)(-K - BS + MS^2) - (K + BS)(-K - BS)$.

When we want to consider environmental force F_e, we set the actuation force $F_a = 0$ & vice-versa using law of super-position.
Therefore,

$$G(S) = \frac{X_1(S) - X_2(S)}{F_e(S)} = \frac{M_rS^2}{\Delta} \tag{9}$$

Thus, assuming the Spring Coefficient (K) = 200 N/m; Damping Coefficient (B) = 4Ns/m; and corresponding masses M_r and M

$$G(S) = \frac{10S^2}{(-5S^2 + 200 + 4S)(-200 - 4S + 5S^2) - (200 + 4S)(-200 - 4S)} \tag{10}$$

On simplifying further,

$$G(S) = \frac{5S^2}{-25S^4 + 40S^3 + 2000S^2} \tag{11}$$

Equation (11) is the final transfer function equation.

3 PID Controller on Simulink

Once we obtained the transform function from mathematical model, we designed a control system using a PID controller in Simulink. As per the objective, that the system should aperiodic robotic motion(i.e no oscillations/overshoot), on tuning the PID controller we were able to achieve the system with the required characteristics.

Below image shows the design of the control system:

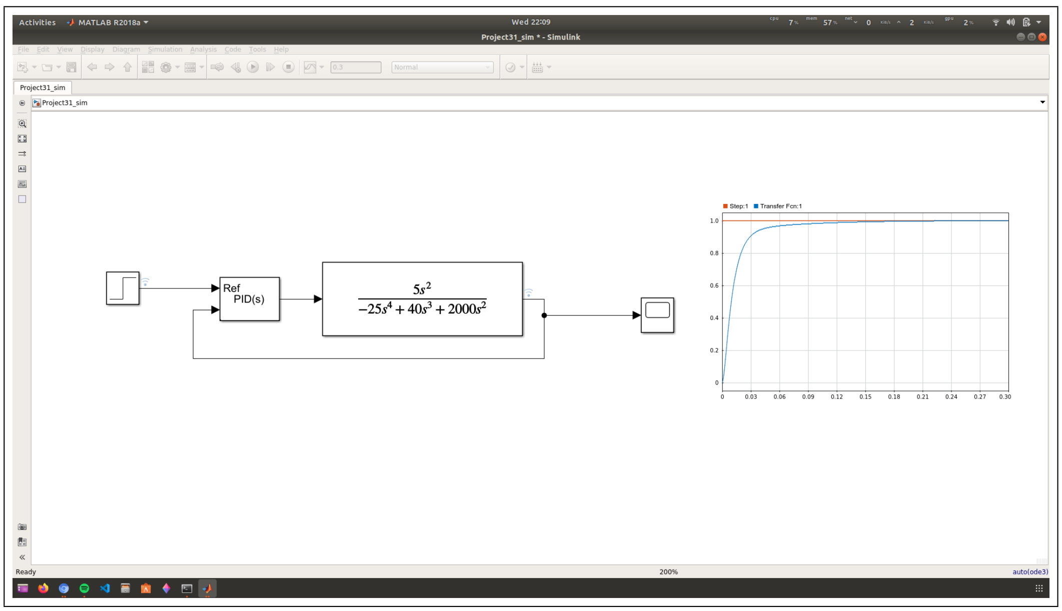

Figure 4: Control Design and Output - Simulink

On tuning the rise time(t_r) and damping ratio (ζ) of the controller we were able to achieve step response of the system.
Below image shows the gains required for the system:

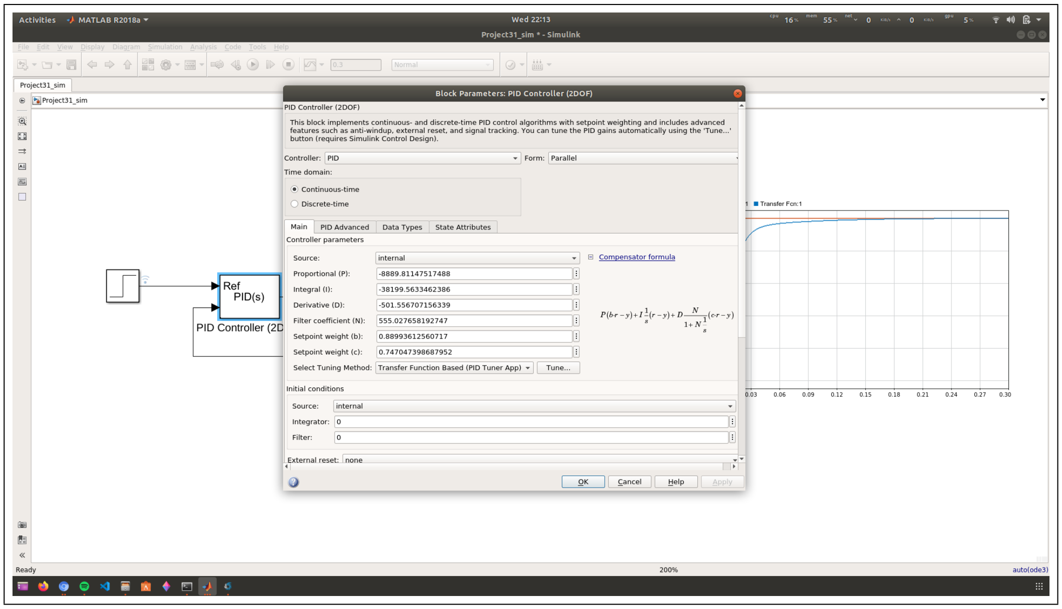

Figure 5: Gains of the Controller - Simulink

The corresponding gains were:

- Proportional (P) = -8889.811
- Integral (I) = -38199.56
- Derivative (D) = -501.55

The response:

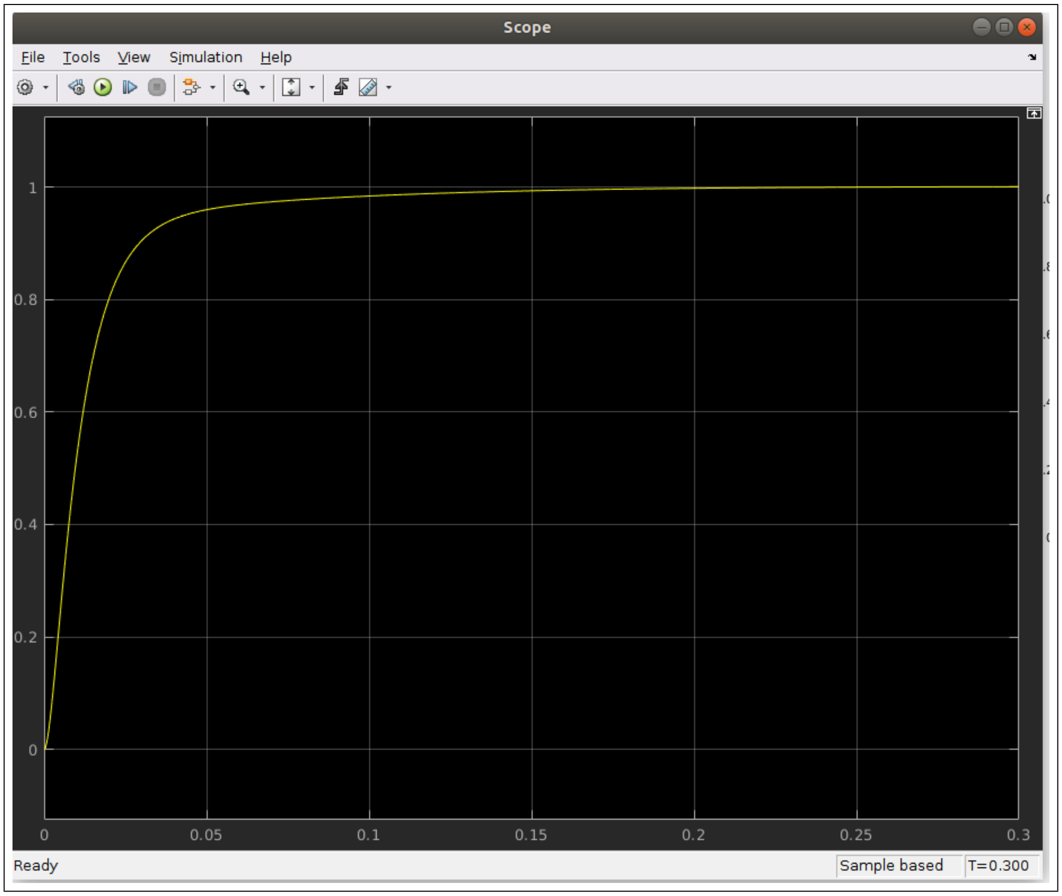

Figure 6: Output of the system - Simulink

4 Matlab - Step Response

We implemented the system using Matlab, and achieved the same results that we arrived from Simulation.

```
G_CL  = tf([5 0 0],[-25 40 2000 0 0]);
C2tf = tf(C);
Cr = C2tf(1);
Cy = C2tf(2);
T  = Cr*feedback(G_CL,Cy,1);
step(T);
grid on;
```

C

```
C =

                          1                 s
  u = Kp (b*r-y) + Ki --- (r-y) + Kd -------- (c*r-y)
                          s              Tf*s+1

  with Kp = -8.89e+03, Ki = -3.82e+04, Kd = -502, Tf = 0.0018, b = 0.89, c = 0.747

Continuous-time 2-DOF PIDF controller in parallel form.
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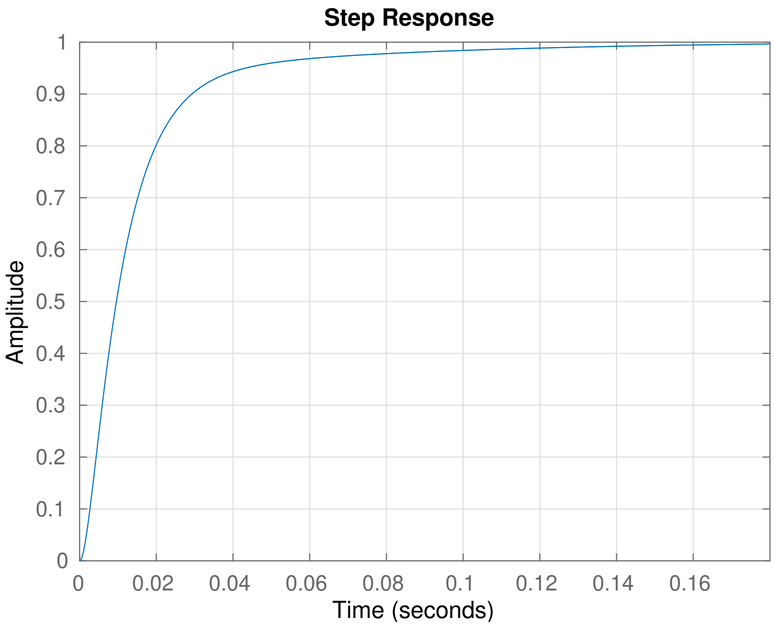

5 Conclusion

Thus using 2DOF PID Controller we were able to achieve the step response of the system with aperiodic motion of the robot end-effector.