10: Sine waves and phasors

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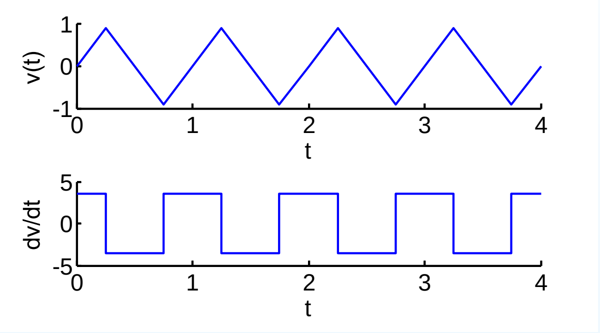

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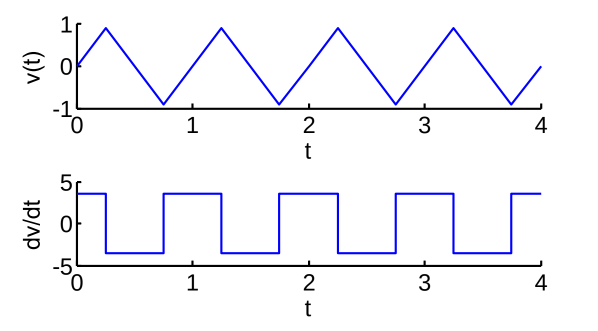

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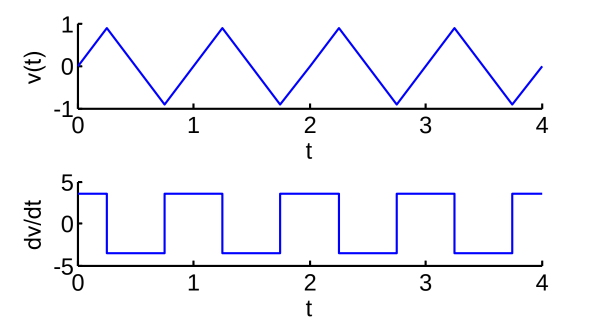

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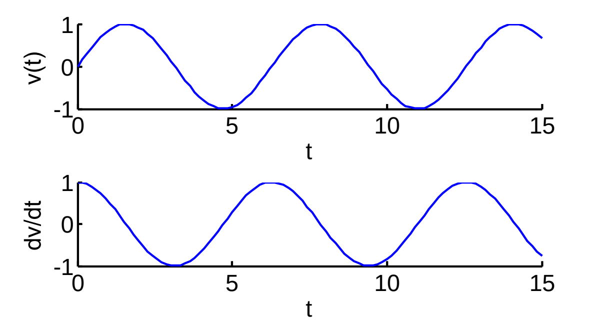

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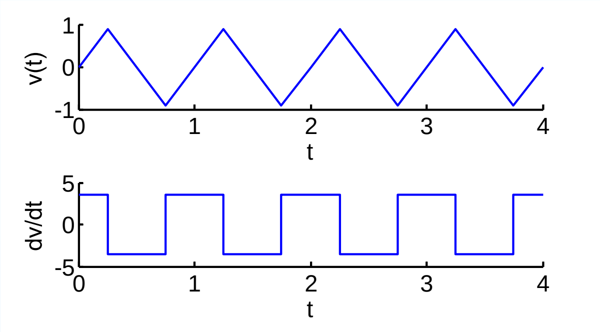

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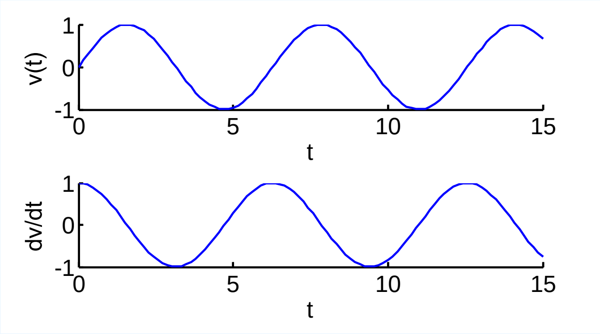

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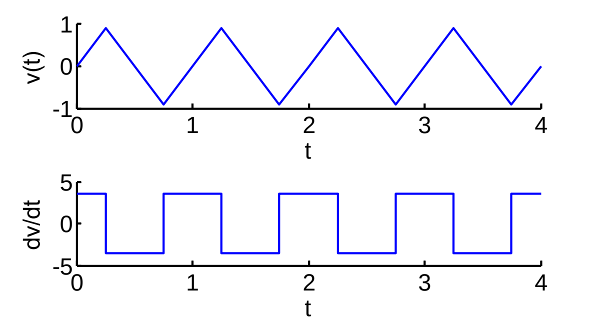

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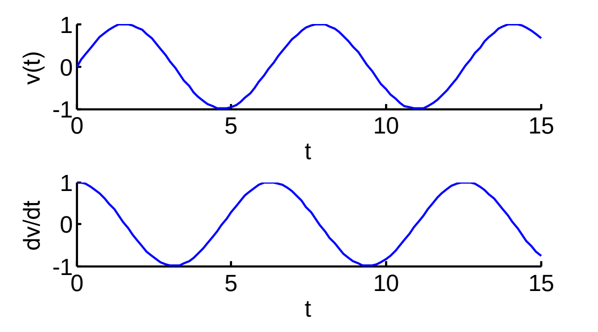

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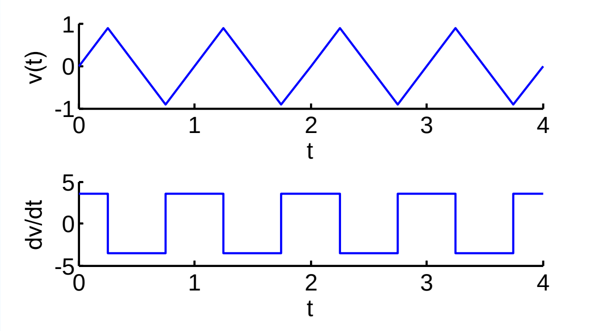

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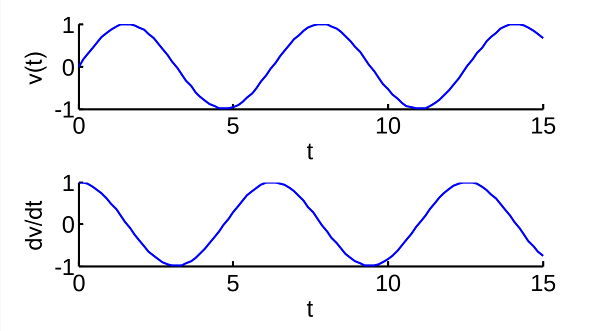

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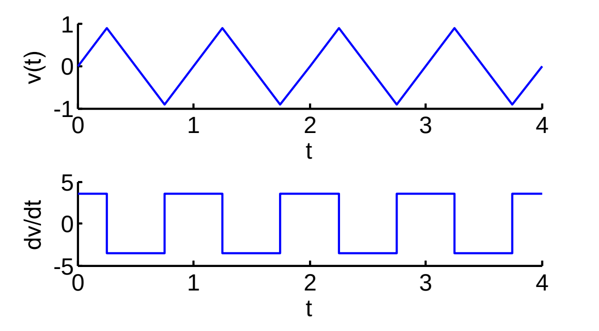

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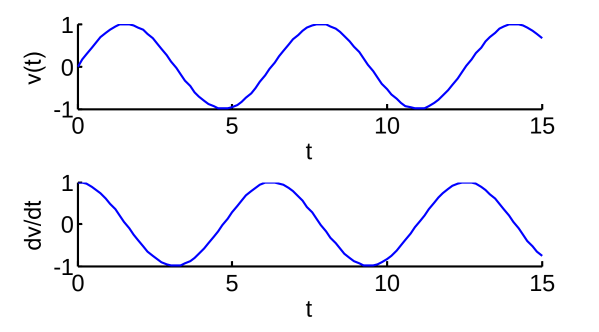
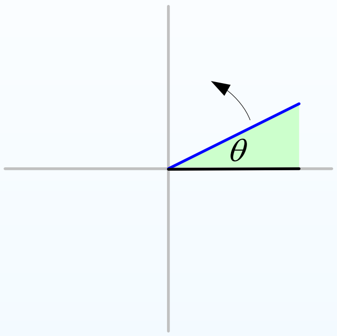


$\sin 2\pi ft$ makes f complete repetitions every time t increases by 1; this gives a *frequency* of f cycles per second, or f Hz.
We often use the *angular frequency*, $\omega = 2\pi f$ instead.
ω is measured in radians per second. E.g. $50\,\mathrm{Hz} \simeq 314\,\frac{\mathrm{rad}}{\mathrm{s}}$.

Rotating Rod

A useful way to think of a cosine wave is as the projection of a rotating rod onto the horizontal axis.



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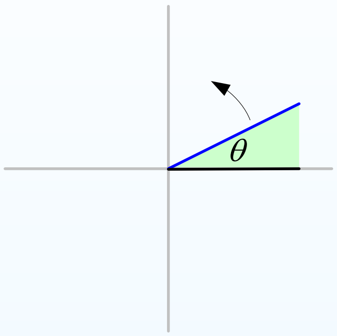

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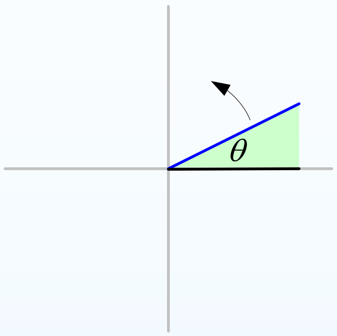

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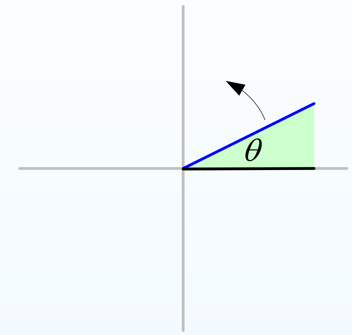

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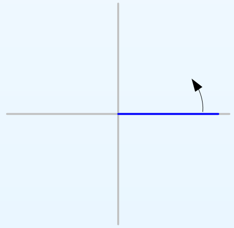

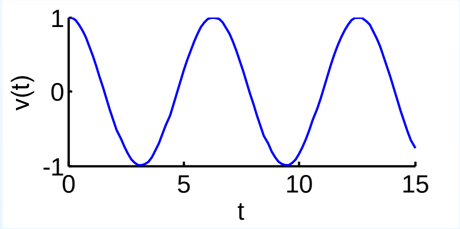

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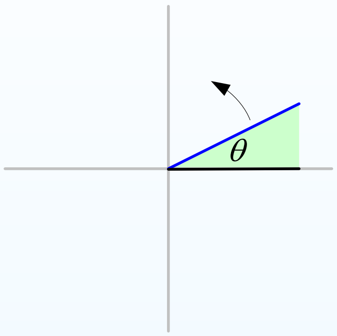

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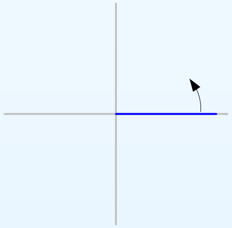

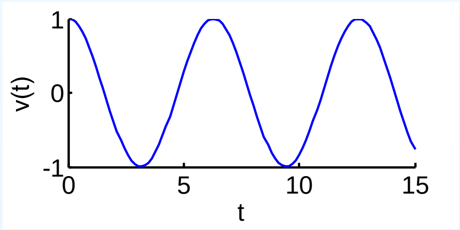

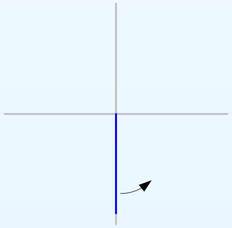

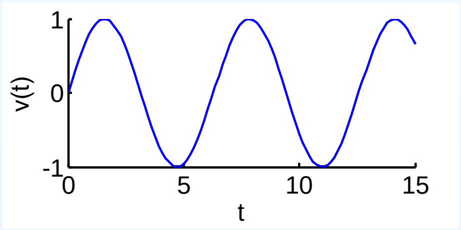

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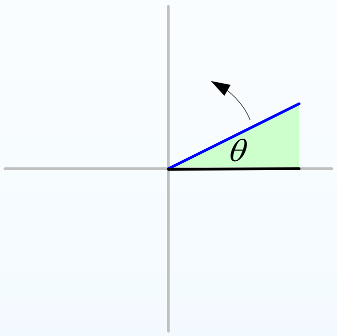

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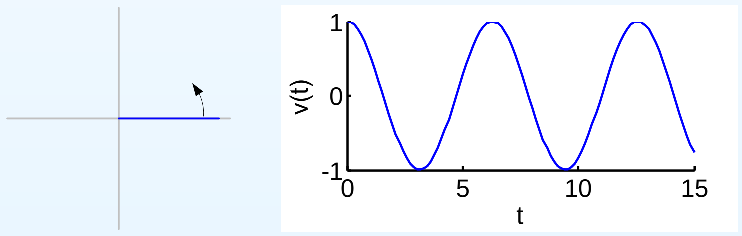

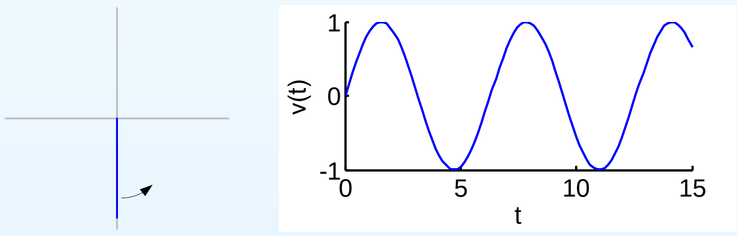

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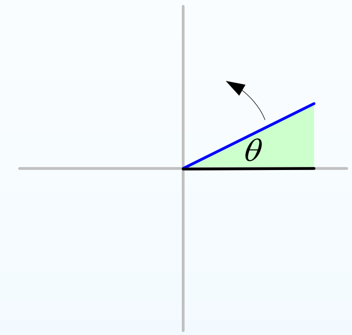

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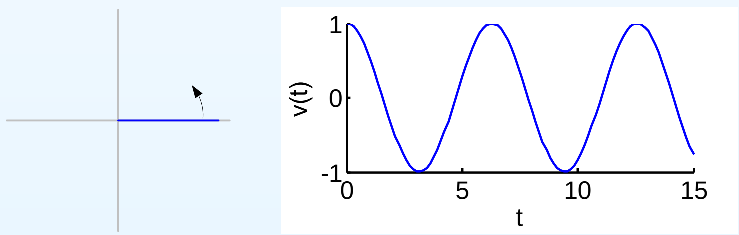
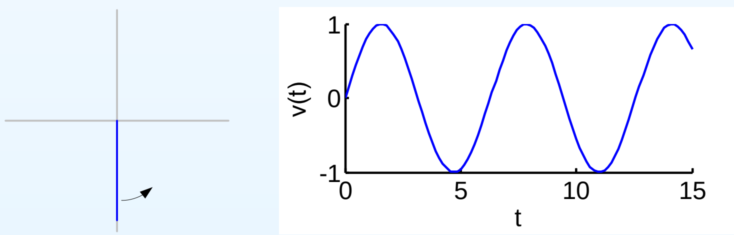




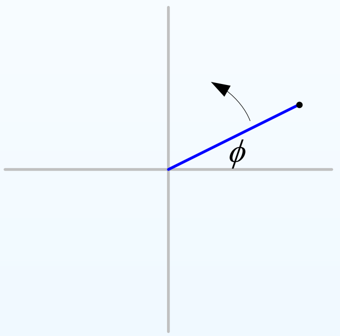
$$v = \cos 2\pi f t \qquad\qquad v = \sin 2\pi f t = \cos\left(2\pi f t - \frac{\pi}{2}\right)$$

$\sin 2\pi f t$ *lags* $\cos 2\pi f t$ by $90°$ (or $\frac{\pi}{2}$ radians) because its peaks occurs $\frac{1}{4}$ of a cycle later (equivalently $\cos$ *leads* $\sin$) .

Phasors

If the rod has length A and starts at an angle ϕ then the projection onto the horizontal axis is

$A\cos(2\pi f t + \phi)$



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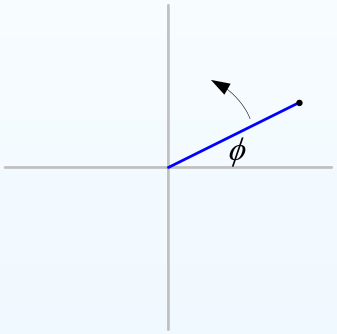

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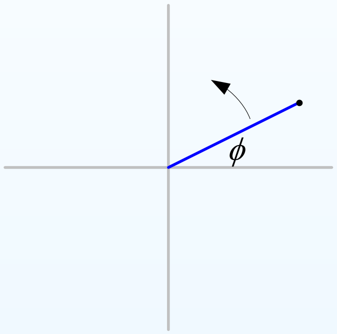

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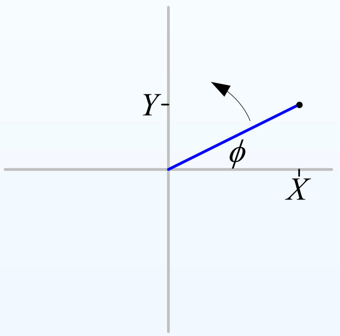

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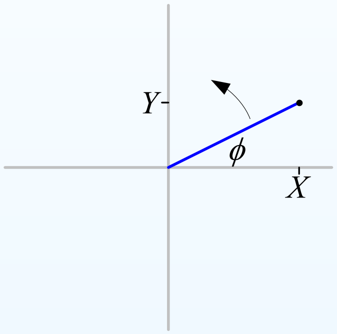

At time $t = 0$, the tip of the rod has coordinates (X, Y).

If we think of the plane as an Argand Diagram (or complex plane), then the complex number $X + jY$ corresponding to the tip of the rod at $t = 0$ is called a *phasor*.

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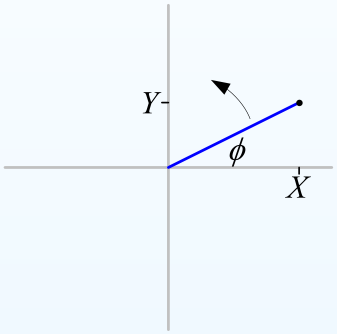

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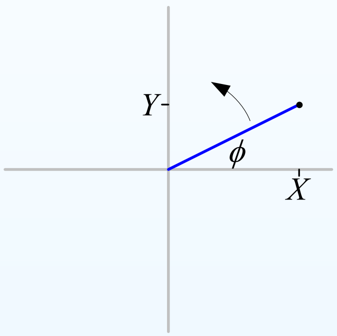

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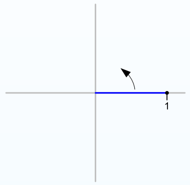
The *argument* of the phasor, $\phi=\arctan\frac{Y}{X}$, gives the phase shift relative to $\cos 2\pi ft$.
If $\phi>0$, it is *leading* and if $\phi<0$, it is *lagging* relative to $\cos 2\pi ft$.

Phasor Examples

$V = 1, f = 50\,\mathrm{Hz}$



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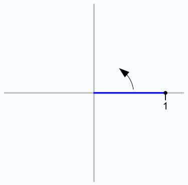

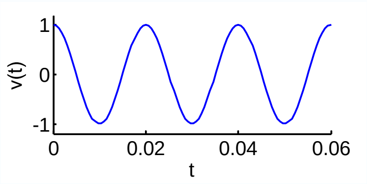

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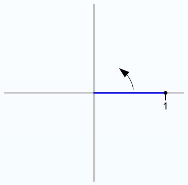

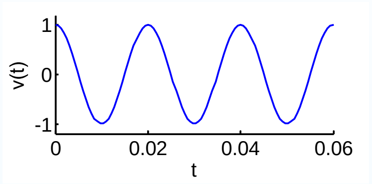

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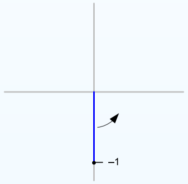

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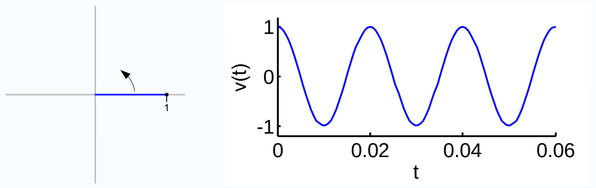

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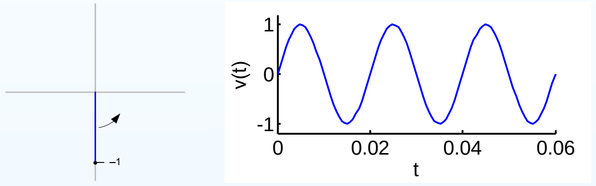

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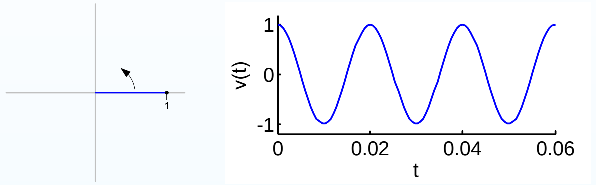

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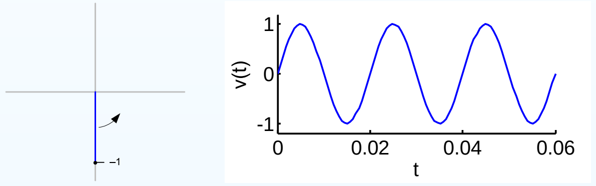
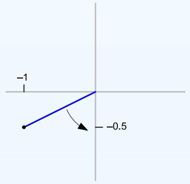


$$V = -1 - 0.5j$$



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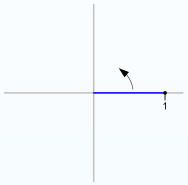
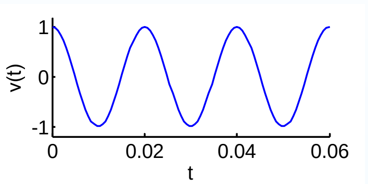
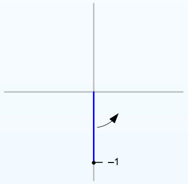
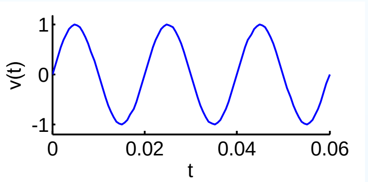




$$V = -j$$
$$v(t) = \sin 2\pi f t$$





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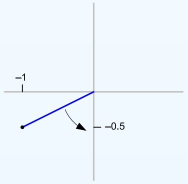
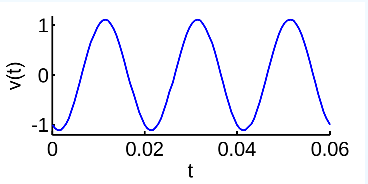
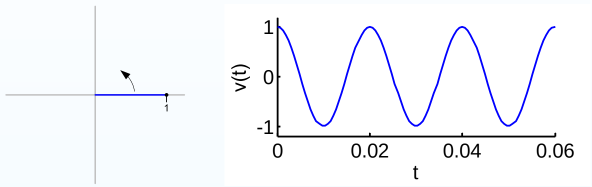




Phasor Examples

$$V = 1,\ f = 50\,\mathrm{Hz}$$
$$v(t) = \cos 2\pi f t$$



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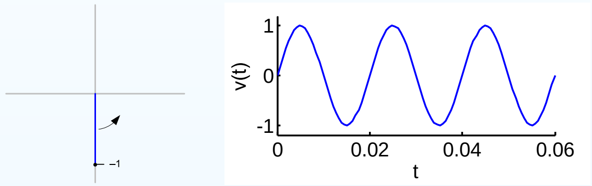

$$V = -1 - 0.5j = 1.12\angle -153°$$
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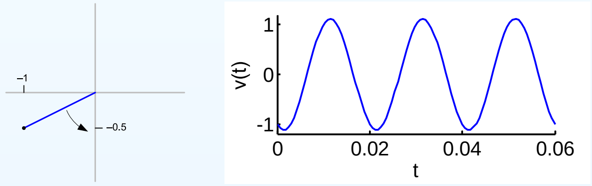

Phasor Examples

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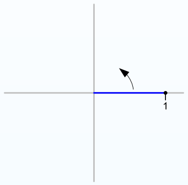

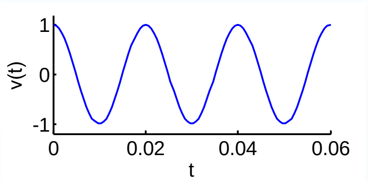

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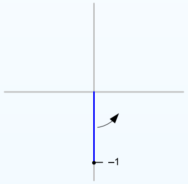

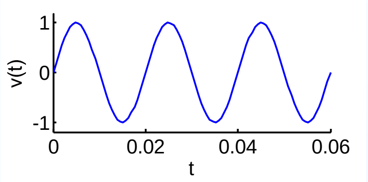

$$V = -1 - 0.5j = 1.12\angle -153^\circ$$
$$v(t) = -\cos 2\pi f t + 0.5 \sin 2\pi f t$$
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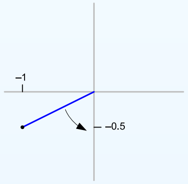

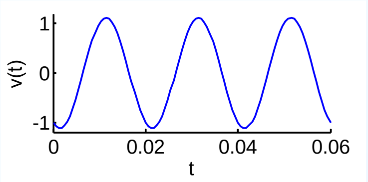

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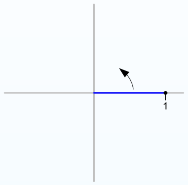
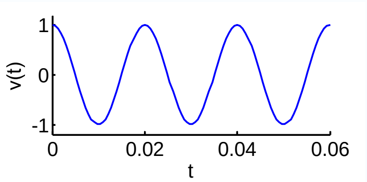

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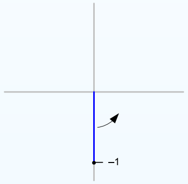
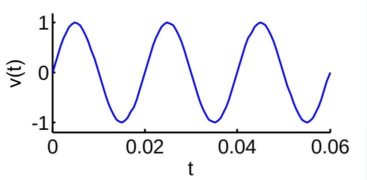

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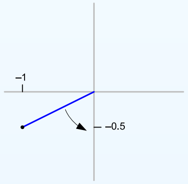
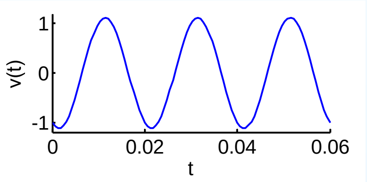
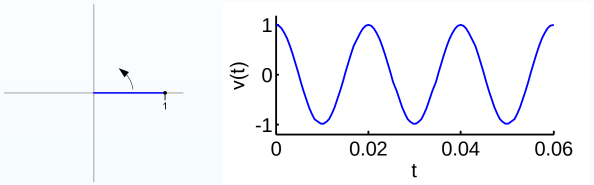



$$V = X + jY$$

Phasor Examples

$$V = 1,\ f = 50\,\mathrm{Hz}$$
$$v(t) = \cos 2\pi f t$$



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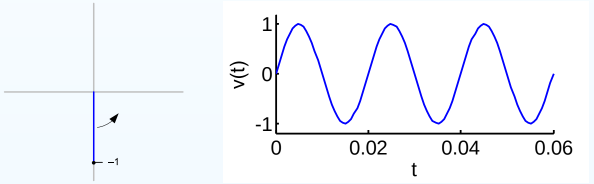

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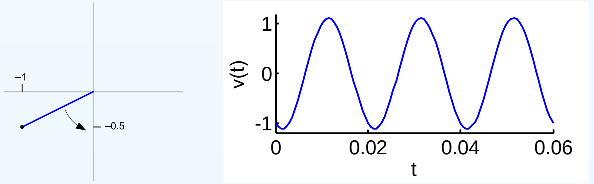

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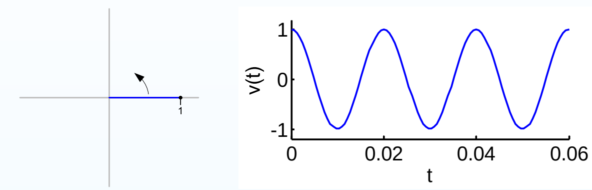

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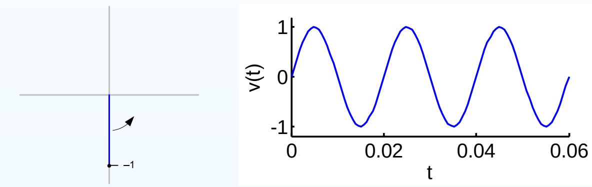

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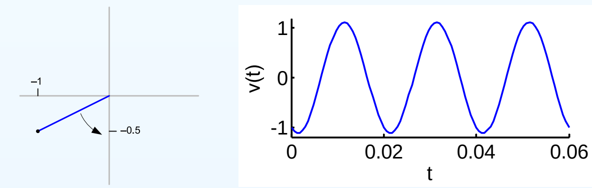

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Beware minus sign.

Phasor Examples

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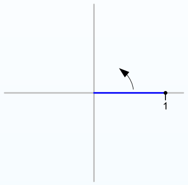

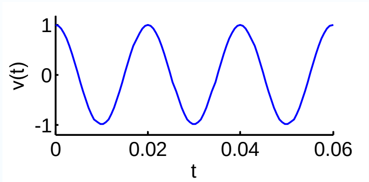

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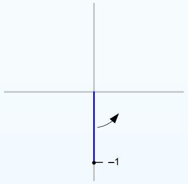

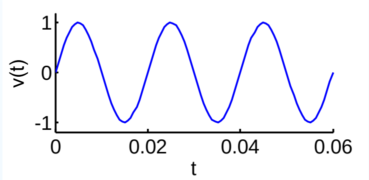

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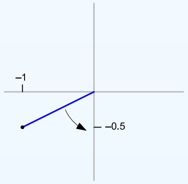

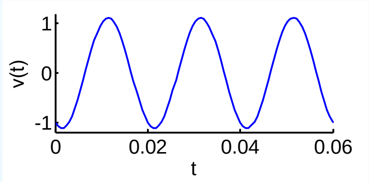

$$V = X + jY$$
$$v(t) = X \cos 2\pi f t - Y \sin 2\pi f t$$

Beware minus sign.

$$V = A\angle\phi$$

Phasor Examples

$$V = 1,\ f = 50\,\mathrm{Hz}$$
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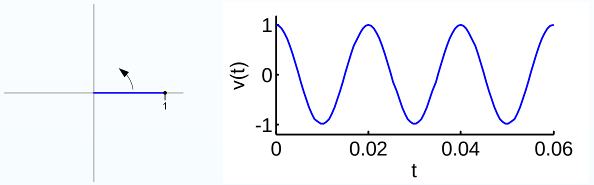

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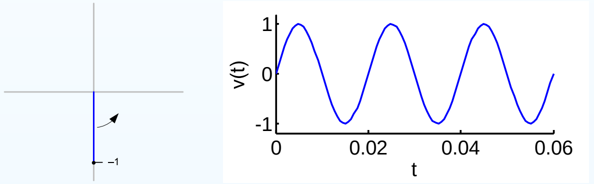

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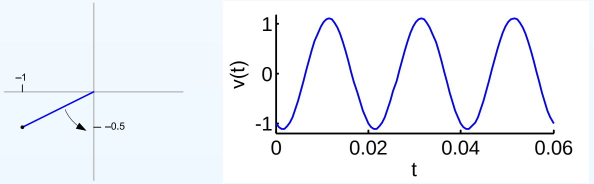

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Phasor Examples

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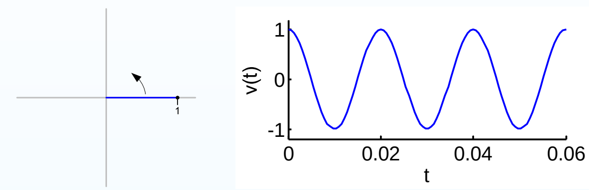

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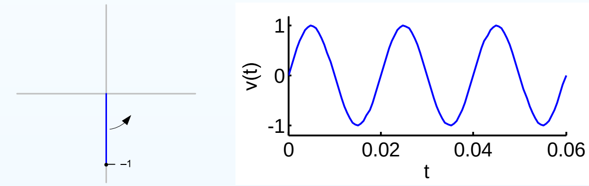

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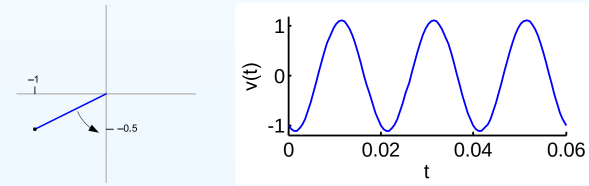

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Phasor Examples

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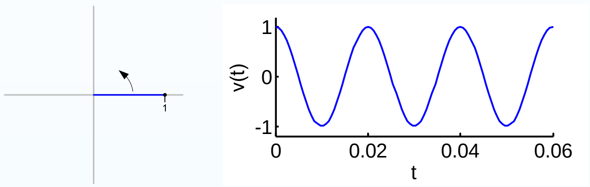

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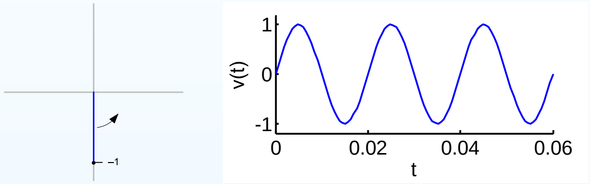

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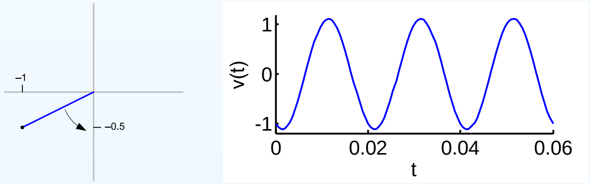

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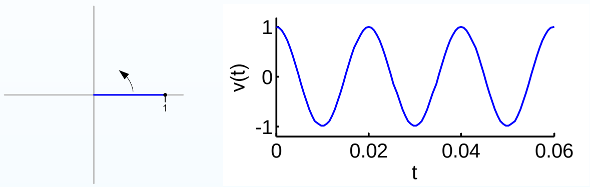
A phasor represents an entire waveform (encompassing all time) as a single complex number. We assume the frequency, f, is known.

Phasor Examples

$V = 1,\ f = 50\,\mathrm{Hz}$
$v(t) = \cos 2\pi f t$



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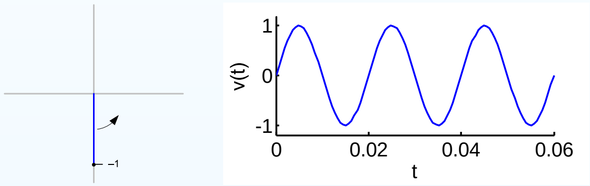

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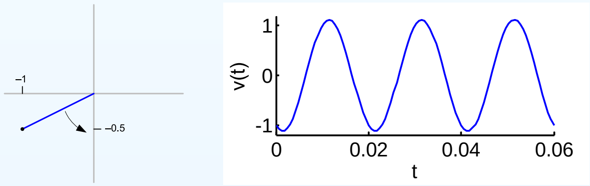

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A phasor is not time-varying, so we use a capital letter: V.
A waveform is time-varying, so we use a small letter: $v(t)$.

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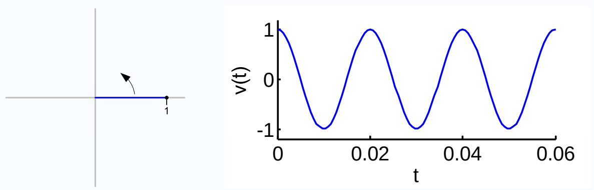

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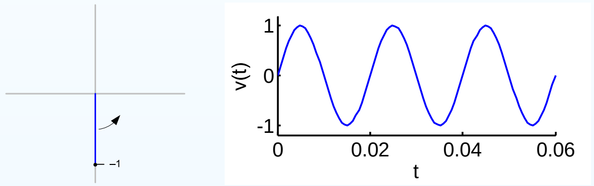

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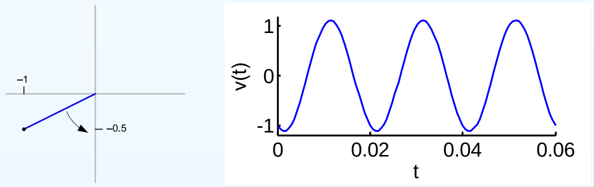

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Casio: $\mathrm{Pol}(X, Y) \to A, \phi$, $\mathrm{Rec}(A, \phi) \to X, Y$. Saved $\to X$ & Y mems.

Phasor arithmetic

Phasors

$$V = X + jY$$

Waveforms

$$v(t) = X\cos\omega t - Y\sin\omega t$$

where $\omega = 2\pi f$.

Phasor arithmetic

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$$a \times v(t)$$

Phasor arithmetic

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where $\omega = 2\pi f$.

$$a \times v(t) \;= aX\cos\omega t - aY\sin\omega t$$

Phasor arithmetic

Phasors	Waveforms
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aV	$a \times v(t) \; = aX\cos\omega t - aY\sin\omega t$

Phasor arithmetic

Phasors

$$V = X + jY$$

$$aV$$

Waveforms

$$v(t) = X\cos\omega t - Y\sin\omega t$$

where $\omega = 2\pi f$.

$$a \times v(t) \; = aX\cos\omega t - aY\sin\omega t$$

$$v_1(t) + v_2(t)$$

Phasor arithmetic

Phasors	Waveforms
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$V_1 + V_2$	$v_1(t) + v_2(t)$

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Adding or scaling is the same for waveforms and phasors.

Phasor arithmetic

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Adding or scaling is the same for waveforms and phasors.

$$\frac{dv}{dt} = -\omega X\sin\omega t - \omega Y\cos\omega t$$

Phasor arithmetic

Phasors	Waveforms
$V = X + jY$	$v(t) = X\cos\omega t - Y\sin\omega t$ where $\omega = 2\pi f$.
aV	$a \times v(t) = aX\cos\omega t - aY\sin\omega t$
$V_1 + V_2$	$v_1(t) + v_2(t)$

Adding or scaling is the same for waveforms and phasors.

$$\begin{aligned}\frac{dv}{dt} &= -\omega X\sin\omega t - \omega Y\cos\omega t \\ &= (-\omega Y)\cos\omega t - (\omega X)\sin\omega t\end{aligned}$$

Phasor arithmetic

Phasors	Waveforms
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Adding or scaling is the same for waveforms and phasors.

$\dot{V} = (-\omega Y) + j(\omega X)$	$\frac{dv}{dt} = -\omega X\sin\omega t - \omega Y\cos\omega t$ $= (-\omega Y)\cos\omega t - (\omega X)\sin\omega t$

Phasor arithmetic

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$$\begin{aligned}\dot{V} &= (-\omega Y) + j(\omega X) \\ &= j\omega(X + jY)\end{aligned}$$

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$$\begin{aligned}\dot{V} &= (-\omega Y) + j\,(\omega X)\\ &= j\omega\,(X + jY)\\ &= j\omega V\end{aligned}$$

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Differentiating waveforms corresponds to multiplying phasors by $j\omega$.

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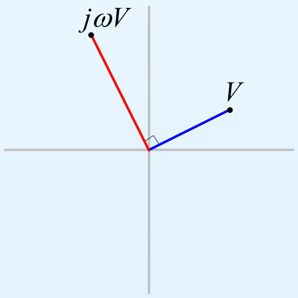

Phasor arithmetic

Phasors	Waveforms
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Adding or scaling is the same for waveforms and phasors.

$$\dot{V} = (-\omega Y) + j(\omega X) = j\omega(X + jY) = j\omega V$$

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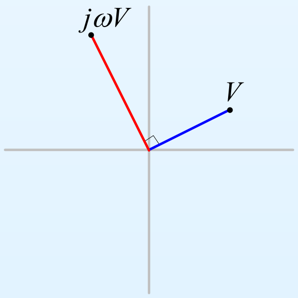
Differentiating waveforms corresponds to multiplying phasors by $j\omega$.

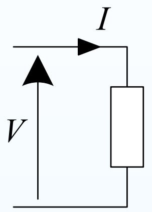


Rotate anti-clockwise 90° and scale by $\omega = 2\pi f$.

Complex Impedances

Resistor:

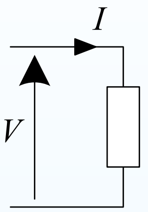
$$v(t) = Ri(t)$$



Complex Impedances

Resistor:

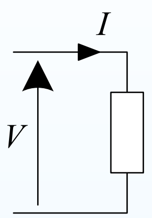
$$v(t) = Ri(t) \Rightarrow V = RI$$



Complex Impedances

Resistor:

$$v(t) = Ri(t) \Rightarrow V = RI \Rightarrow \frac{V}{I} = R$$



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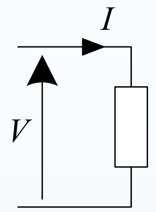
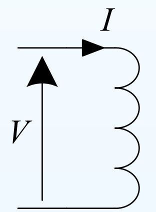


Inductor:

$$v(t) = L\frac{di}{dt}$$



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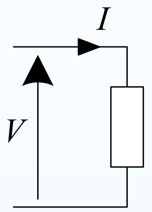
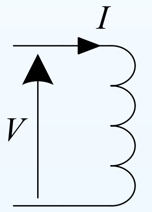


Inductor:

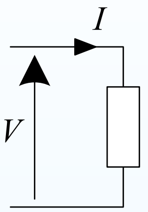
$$v(t) = L\frac{di}{dt} \Rightarrow V = j\omega LI$$



Complex Impedances

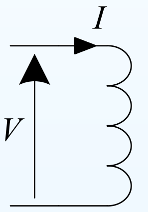
Resistor:

$$v(t) = Ri(t) \Rightarrow V = RI \Rightarrow \frac{V}{I} = R$$



Inductor:

$$v(t) = L\frac{di}{dt} \Rightarrow V = j\omega LI \Rightarrow \frac{V}{I} = j\omega L$$



Complex Impedances

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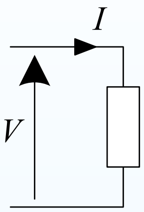

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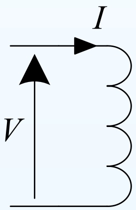
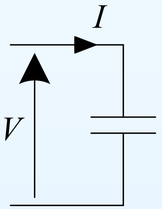


Capacitor:

$$i(t) = C\frac{dv}{dt}$$



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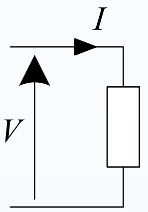

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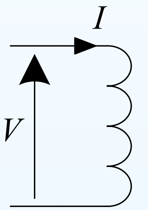
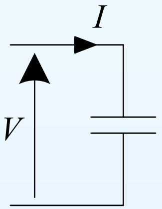


Capacitor:

$$i(t) = C\frac{dv}{dt} \Rightarrow I = j\omega CV$$



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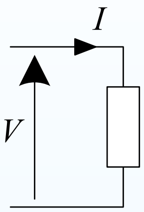
$$v(t) = Ri(t) \Rightarrow V = RI \Rightarrow \frac{V}{I} = R$$



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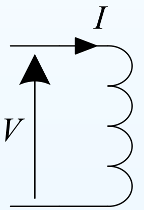
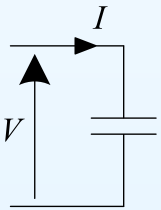


Capacitor:

$$i(t) = C\frac{dv}{dt} \Rightarrow I = j\omega CV \Rightarrow \frac{V}{I} = \frac{1}{j\omega C}$$



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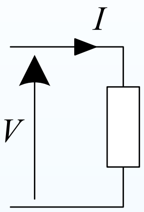

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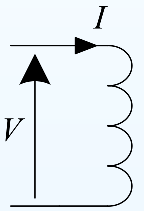

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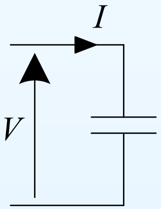

For all three components, phasors obey Ohm’s law if we use the *complex impedances* $j\omega L$ and $\frac{1}{j\omega C}$ as the “resistance” of an inductor or capacitor.

Complex Impedances

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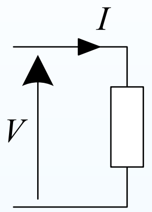

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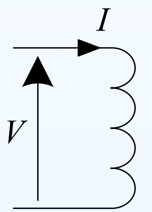

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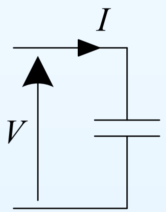
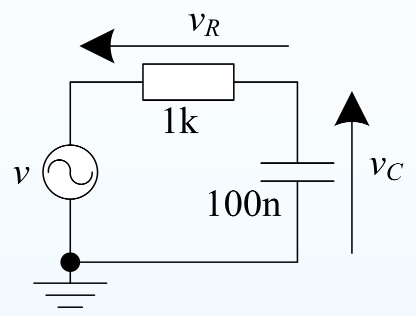


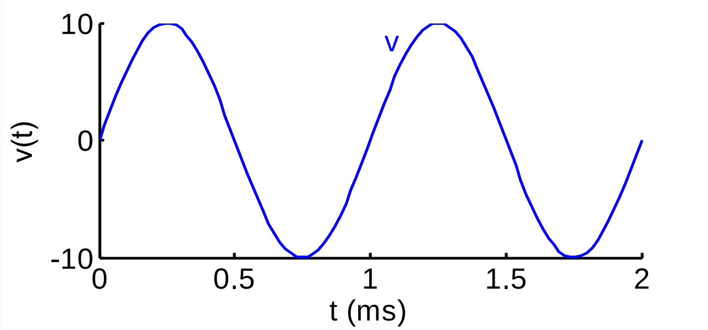
For all three components, phasors obey Ohm's law if we use the *complex impedances* $j\omega L$ and $\frac{1}{j\omega C}$ as the "resistance" of an inductor or capacitor.

If all sources in a circuit are sine waves having the same frequency, we can do circuit analysis exactly as before by using complex impedances.

Phasor Analysis

Given $v = 10 \sin \omega t$ where $\omega = 2\pi \times 1000$, find $v_C(t)$.





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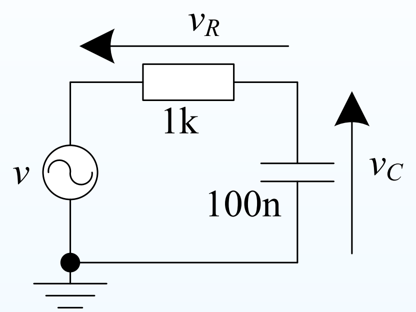

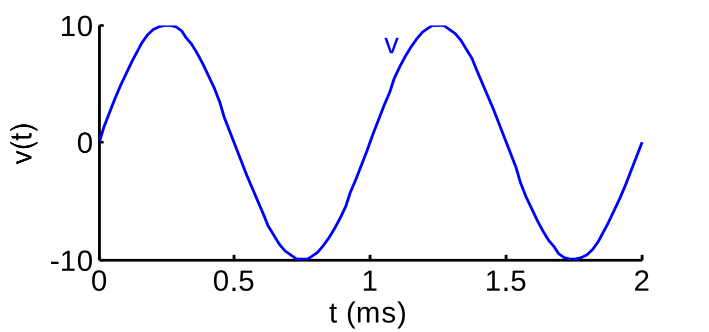

(1) Find capacitor complex impedance

$$Z = \frac{1}{j\omega C} = \frac{1}{6.28j \times 10^{-4}} = -1592j$$

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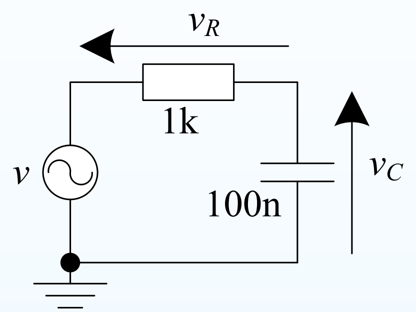

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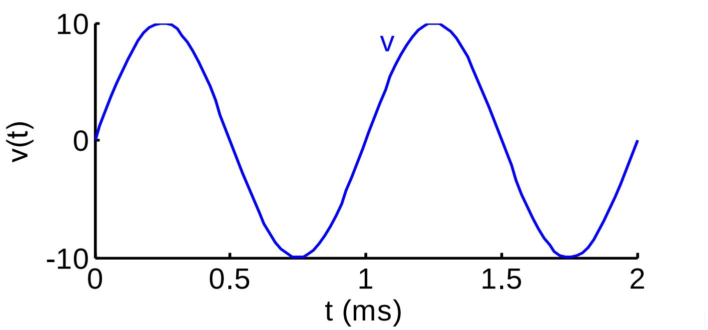
(2) Solve circuit with phasors

$$V_C = V \times \frac{Z}{R+Z}$$



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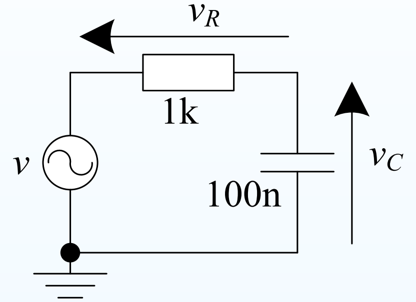

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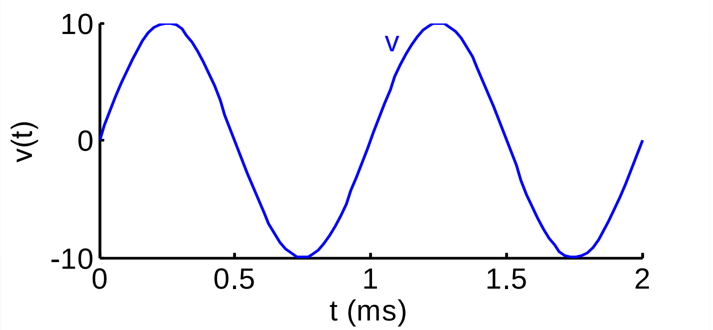

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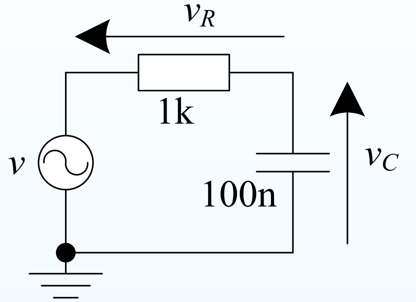

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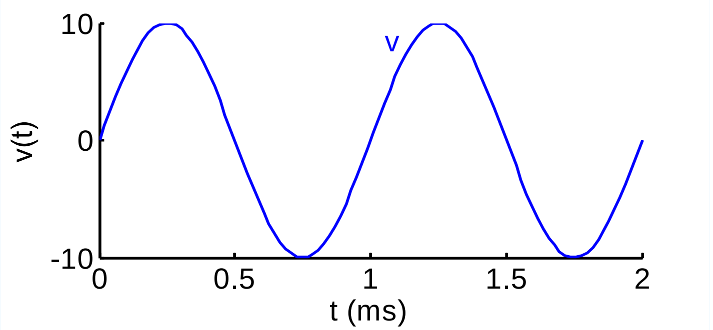
$$V_C = V \times \frac{Z}{R+Z}$$
$$= -10j \times \frac{-1592j}{1000-1592j}$$
$$= -4.5 - 7.2j = 8.47\angle - 122°$$



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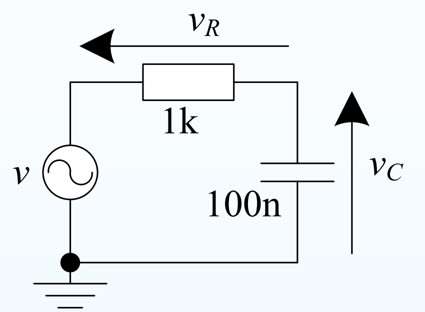

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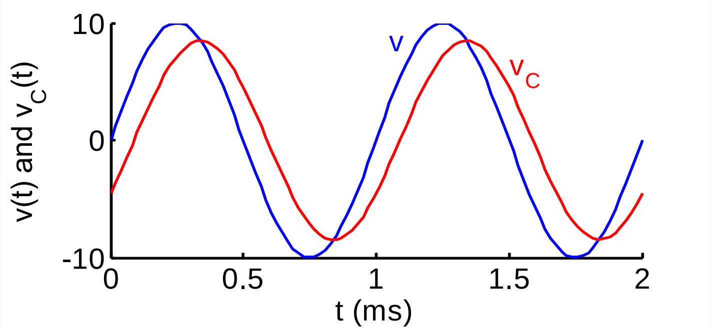

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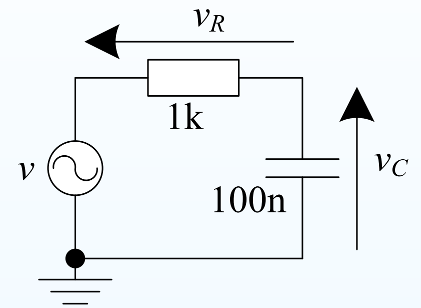

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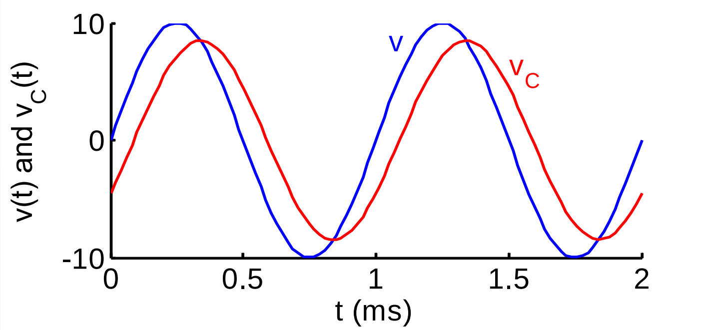
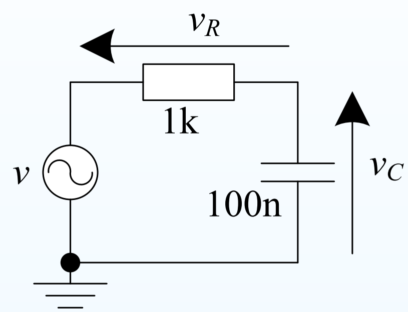


(3) Draw a *phasor diagram*:

$$V = -10j$$
$$V_C = -4.5 - 7.2j$$
$$V_R = V - V_C = 4.5 - 2.8j = 5.3\angle - 32°$$

Phasor Analysis

Given $v = 10\sin\omega t$ where $\omega = 2\pi \times 1000$, find $v_C(t)$.



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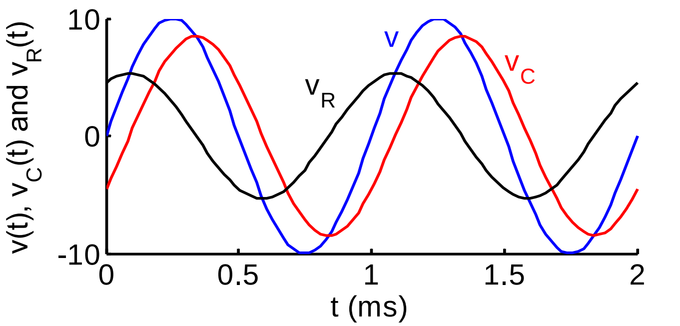

(3) Draw a *phasor diagram*:

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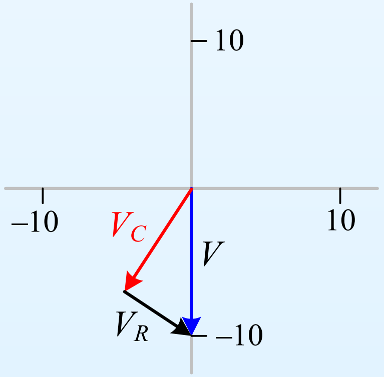
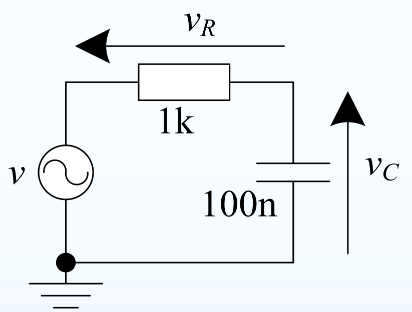


Phasor Analysis

Given $v = 10 \sin \omega t$ where $\omega = 2\pi \times 1000$, find $v_C(t)$.



(1) Find capacitor complex impedance

$$Z = \frac{1}{j\omega C} = \frac{1}{6.28j \times 10^{-4}} = -1592j$$

(2) Solve circuit with phasors

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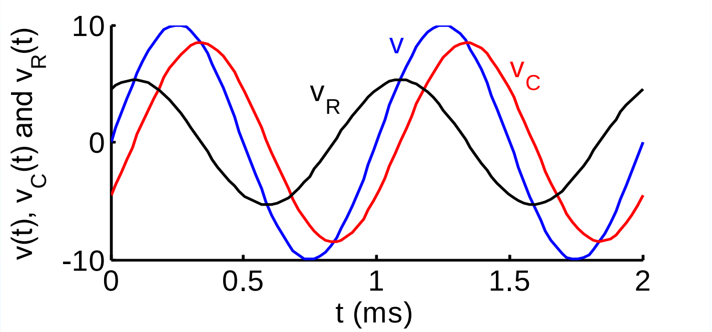

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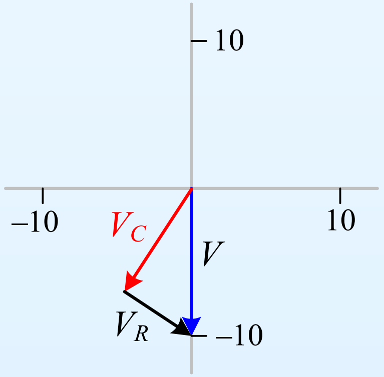
Phasors add like vectors



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Casio fx-991 (available in all exams except Maths) will do complex arithmetic ($+, -, \times, \div, x^2, \frac{1}{x}, |x|, x^*$) in CMPLX mode.

Learn how to use this: it will save lots of time and errors.

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Beware: $G \neq \frac{1}{R}$ unless $X = 0$.

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