PH101

Application of Newton's laws of motion

By

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Concept of force and Newton's Laws

Newton's First Law

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

$$\sum \vec{F}_i = \vec{0} \quad \Rightarrow \vec{v} = \text{constant}$$

Newton's Second Law

The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed,

$$\sum_{i=1}^{N} \vec{F}_i = m\,\vec{a}$$

$$\sum_{i=1}^{N} \vec{F}_i = m(\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}\,)\hat{\theta}$$

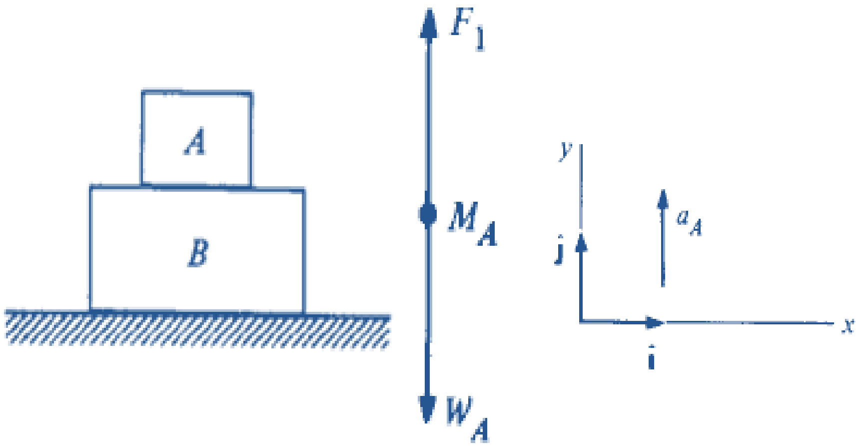
Newton's Third Law

To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

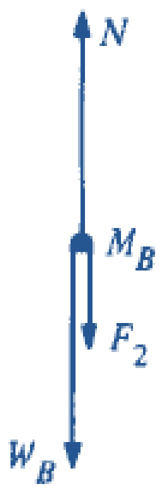
$$\overrightarrow{F_{1,2}} = -\,\overrightarrow{F_{2,1}}$$

Action-reaction pair of forces cannot act on same body; they act on different bodies.

Strategy to solve problems



1. Divide the system into smaller systems of a point mass.
2. Draw a force diagram.
3. Introduce coordinate system.
4. If two bodies in the system interact, the forces between them must be equal and opposite.
5. Introduce constraint equations (if any).



$$\left.\begin{aligned} F_1 - W_A &= m_A a_A \\ N - F_2 - W_B &= m_B a_B \end{aligned}\right\} \text{Equations of motion}$$

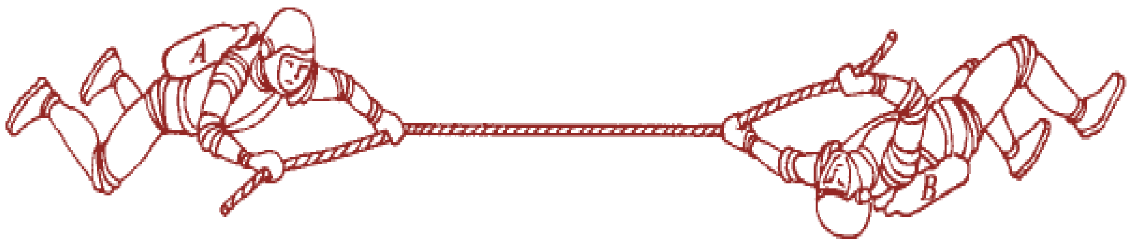
$$F_1 = F_2 \qquad \text{From Newton's third law}$$

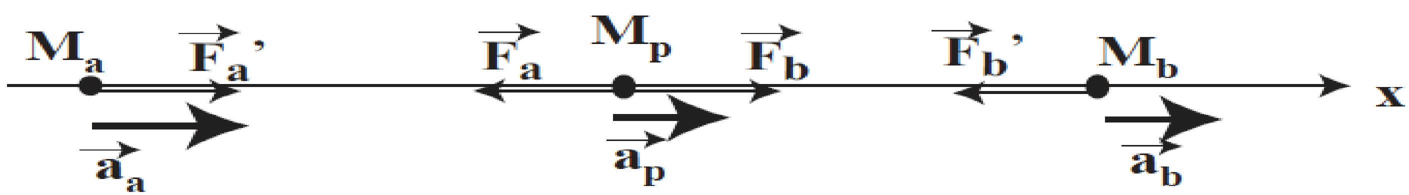
$$\left.\begin{aligned} a_A &= 0 \\ a_B &= 0 \end{aligned}\right\} \text{Constraint equations}$$

Applications of Newton's Laws

1-The astronauts' tug-of-war

Two astronauts, A and B with mass M_a and M_b, initially at rest in free space, pull on either end of a rope that has mass M_p (but negligibly small). Astronaut A is stronger than Astronaut B. Find their motion, if each pulls the rope as hard as they can.

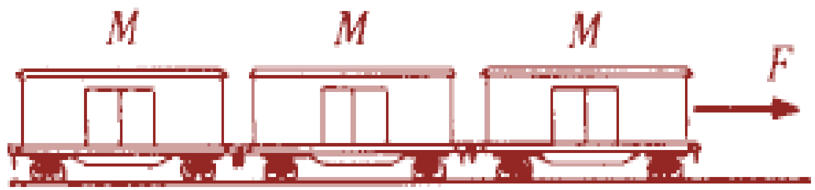


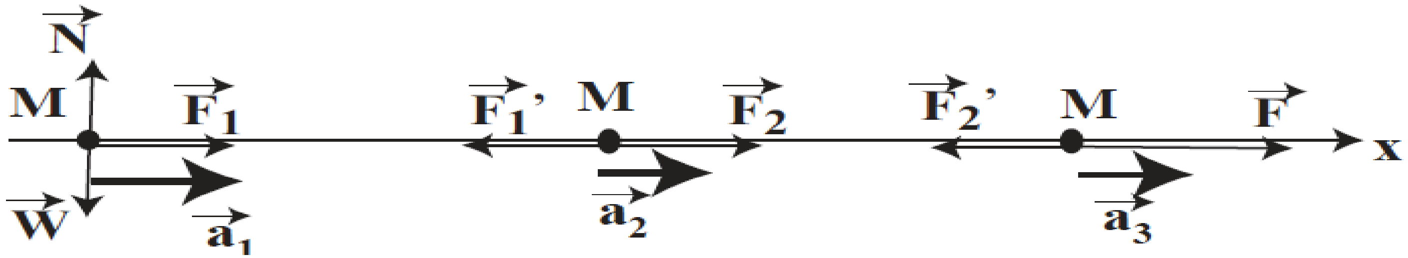


Although astronaut A is stronger than astronaut B, we still have $\overrightarrow{F_{a'}} = -\overrightarrow{F_{b'}}$ from the Newton's third law, i.e., the force exerting on B by A is the same as the force on A by B.

2-Freight Train

Three freight cars, each with mass M, are pulled by a force F. Ignore the friction. Determine the force on each car and their acceleration.





According to the Newton's third law, we have $\overrightarrow{F_1} = -\overrightarrow{F_{1'}}$ and $\overrightarrow{F_2} = -\overrightarrow{F_{2'}}$

The cars move together, therefore $\overrightarrow{a_1} = \overrightarrow{a_2} = \overrightarrow{a_3} = \vec{a}$

$$\overrightarrow{F_1} = M\vec{a}$$
$$\overrightarrow{F_2} - \overrightarrow{F_1} = M\vec{a}$$
$$\vec{F} - \overrightarrow{F_2} = M\vec{a}$$

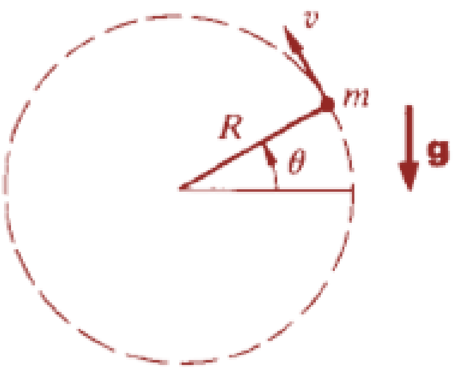
$a = F/(3M)$, $F_1 = F/3$, and $F_2 = 2F/3$

$F_n = nM \times F/NM$: Force is proportional to number of car pulled

Constraints: Since the cars are joined, they are constrained to have same acceleration. However, this is not always true.

Block on string (Radial and tangential acceleration)

Mass m is whirled on the end of a string length R. The motion is in a vertical plane in the gravitational field of the earth. The forces on m



Radial equation

$$-(T + W\sin\theta) = ma_r$$
$$= m(\ddot{r} - r\dot{\theta}^2).$$

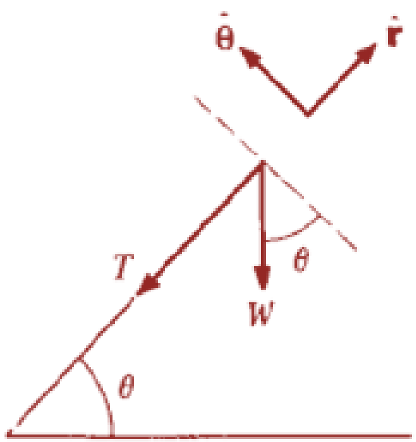
Since $r = R =$ constant, $a_r = -R(\dot{\theta}^2) = -v^2/R$

$$T = \frac{mv^2}{R} - W\sin\theta$$

The string can pull but not push, so that T cannot be negative.

hence $mv^2/R \geq W\sin\theta$

The maximum value of W sinθ *occurs when the mass is vertical up, i. e mv^2/2 > W. If this condition is failed, mass starts to fall and $\ddot{r}$ is no longer zero.*



Tangential equation

$$-W\cos\theta = ma_\theta$$
$$= m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

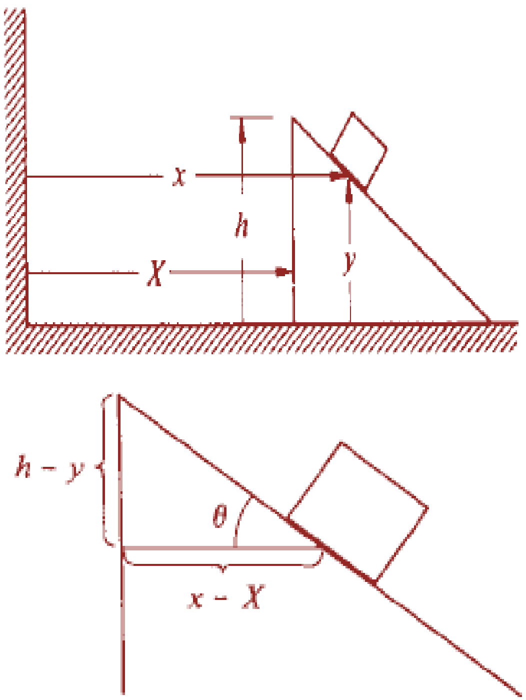
Since $\dot{r} = 0$; $a_\theta = R\ddot{\theta}$

$$= -\frac{W\cos\theta}{m}$$

The mass does not move with constant speed; it accelerates tangentially. On the downswing the tangential speed increases, on the upswing it decreases.

Constraints

a. WEDGE AND BLOCK



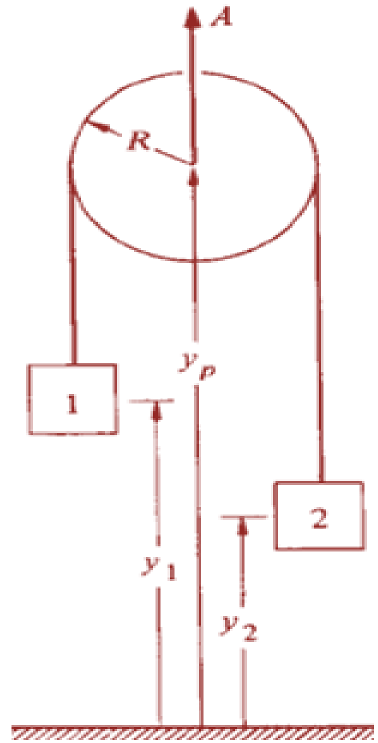
From the geometry

$$(x - X) = (h - y)\cot\theta$$

Differentiating twice

$$\ddot{x} - \ddot{X} = -\ddot{y}\cot\theta$$

b. MASSES AND PULLEY



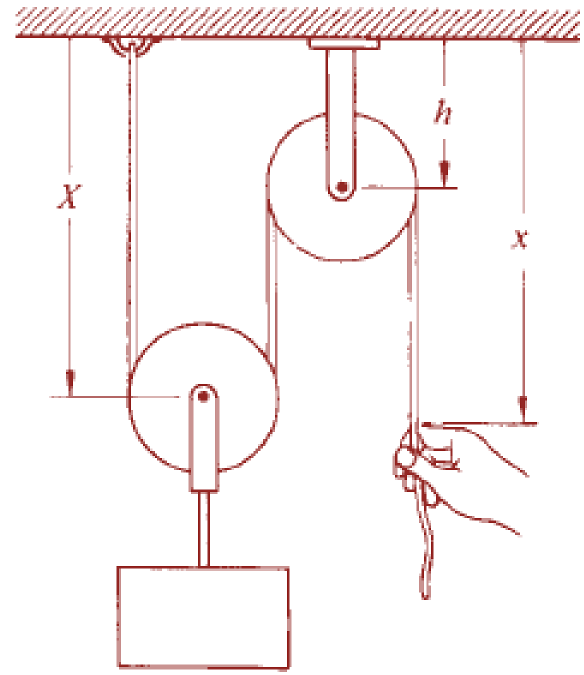
$$l = \pi R + (y_p - y_1) + (y_p - y_2)$$

Differentiating twice

$$0 = 2\ddot{y}_p - \ddot{y}_1 - \ddot{y}_2$$

$$A = \tfrac{1}{2}(\ddot{y}_1 + \ddot{y}_2) \;:\; A = \ddot{y}_p$$

c. PULLEY SYSTEM



$$l = X + \pi R + (X - h) + \pi R + (x - h)$$

Differentiating twice

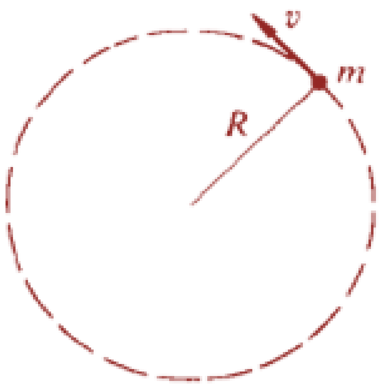
where R is the radius of the pulleys

$$\ddot{X} = -\tfrac{1}{2}\ddot{x}.$$

Dynamics of rotational motion

Block on string (Radial acceleration)

Mass m whirls with constant speed v at the end of a string of length R. Find the force on m in the absence of gravity or friction.

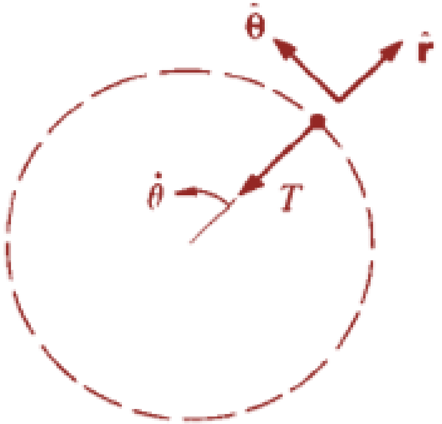


The only force on m is the string force T, which acts towards the centre.

$$-T = ma_r$$
$$= m(\ddot{r} - r\dot{\theta}^2).$$

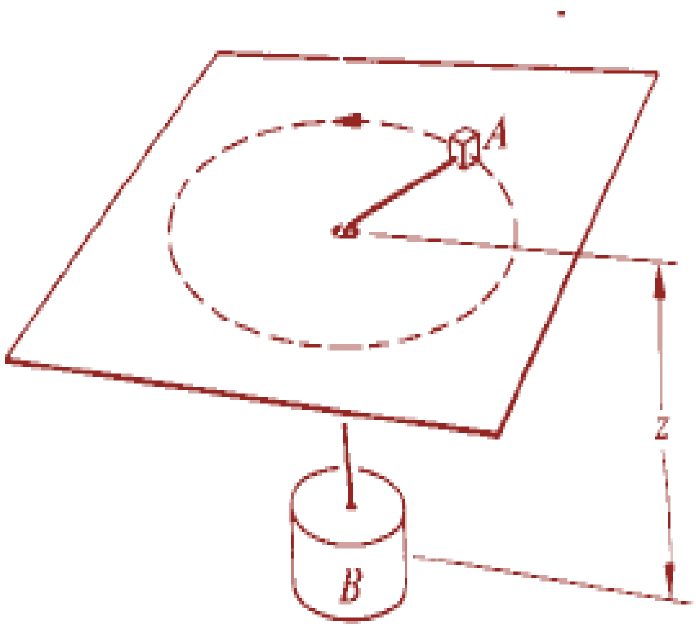
$\ddot{r} = \ddot{R} = 0$ and $\dot{\theta} = v/R$. Hence $a_r = -R(v/R)^2 = -v^2/R$ and

$$T = \frac{mv^2}{R}.$$



No outward force on m, but a person who does whirling feels this force. Its magnitude is equal to the pulling force but has opposite direction.

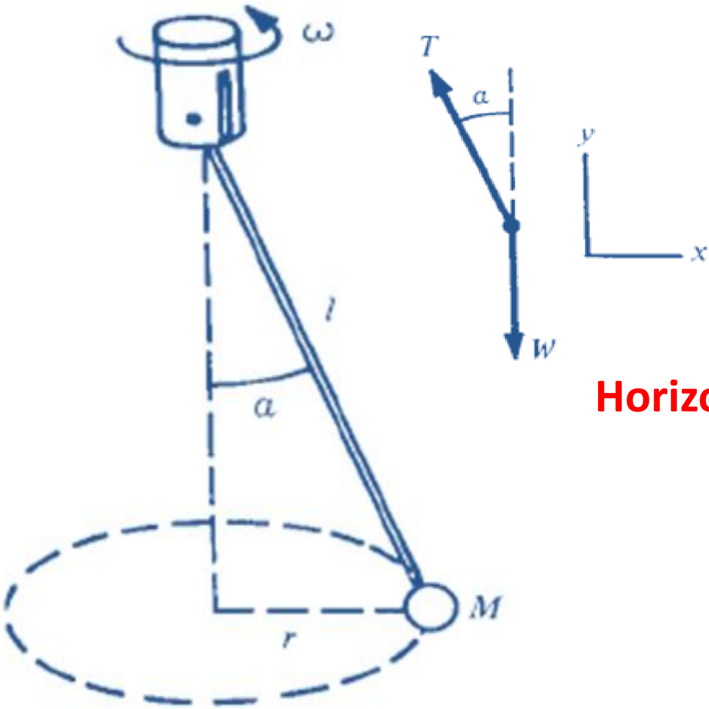
Problem



A horizontal frictionless table has a small hole in its center. Block A on the table is connected to block B hanging beneath by a string of negligible mass which passes through the hole.

Initially, B is held stationary and A rotates at constant radius r_0 with steady angular velocity ω_0. If B is released at $t = 0$, what is its acceleration immediately afterward?

Conical pendulum



Only two forces acting on mass M that are string force (T) and force of gravity (W).

Vertical eq^n $T\cos\alpha - W = 0$

because y is constant and $\ddot{y}$ is therefore zero.

Horizontal eq^n The bob is accelerating in the $\hat{r}$ direction with $a_r = -\omega^2 r$.

$$-T\sin\alpha = -Mr\omega^2.$$

Since $r = l\sin\alpha$ we have

$$T\sin\alpha = Ml\omega^2\sin\alpha$$

or

$$T = Ml\omega^2.$$

Hence $Ml\omega^2\cos\alpha = W \Rightarrow \boxed{\cos\alpha = \frac{g}{l\omega^2}}$

Time period of revolution $T' = 2\pi\sqrt{\frac{l\cos\alpha}{g}}$

$$\boxed{\cos\alpha = \frac{g}{l\omega^2}}$$

For $\omega \to \infty$, $\cos\alpha \to 0$ and $\alpha \to \pi/2$

At high speeds the bob flies out until it is almost horizontal. However, at low speeds the solution does not make sense. As $\omega \to 0$,

$\cos\alpha \to \infty$

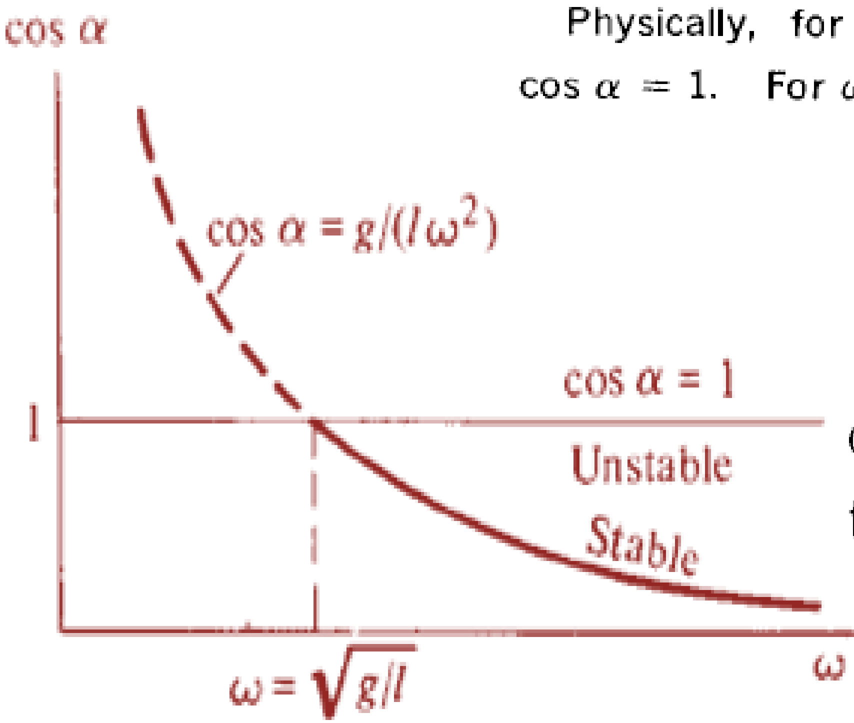
Which does not make any sense because $\cos\alpha \le 1$

Our solution predicts $\cos\alpha > 1$ for $\omega < \sqrt{g/l}$. When $\omega = \sqrt{g/l}$, $\cos\alpha = 1$ and $\sin\alpha = 0$; the bob simply hangs vertically.

The only possible solution is cos α =1, where T = W, for all ω values. The solution corresponds to the pendulum hanging straight down.

Physically, for $\omega \le \sqrt{g/l}$ the only acceptable solution is $\alpha = 0$, $\cos\alpha = 1$. For $\omega > \sqrt{g/l}$ there are two acceptable solutions:

1. $\cos\alpha = 1$
2. $\cos\alpha = \dfrac{g}{l\omega^2}$.



Solution 1 corresponds to the bob rotating rapidly but hanging vertically. Solution 2 corresponds to the bob flying around at an angle with the vertical. For $\omega > \sqrt{g/l}$, solution 1 is unstable—if the system is in that state and is slightly perturbed, it will jump outward.


Example of conical pendulum
Light house
Thank you