Lecture -

PH101

The Everyday Forces of Physics

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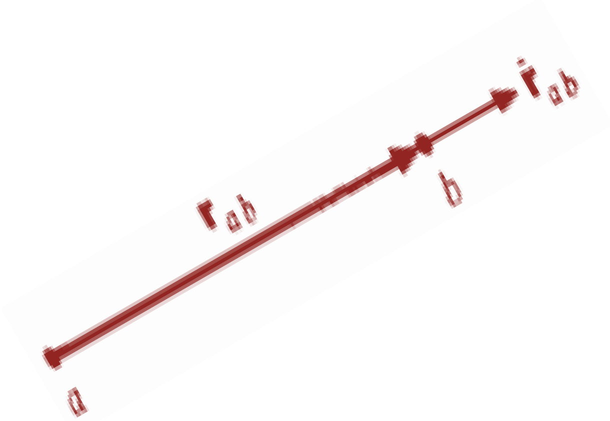
Four fundamental interaction exist in the universe:

1-Gravity

2-Electromagnetic

3-Strong and 4-Weak interactions

1- Gravitational force



$$\boxed{\mathbf{F}_b = -\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{ab}} \; ; \; |\mathbf{r}_{ab}| = r, \text{ unit vector } \hat{\mathbf{r}}_{ab} = \mathbf{r}_{ab}/r$$

The negative sign indicates that the force is attractive.

$$\mathbf{F}_a = -\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{ba} = +\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{ab} = -\mathbf{F}_b,$$

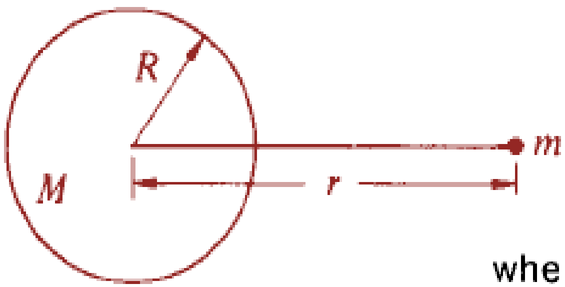
since $\hat{\mathbf{r}}_{ba} = -\hat{\mathbf{r}}_{ab}$. The forces are equal and opposite, and Newton's third law is automatically satisfied.

At the surface of the earth, the gravitational force is $\mathbf{F} = -\frac{GM_em}{R_e^2}\hat{\mathbf{r}}$, the acceleration due to gravity is

Gravitational field: $\boxed{\mathbf{g} = -\frac{GM_e}{R_e^2}\hat{\mathbf{r}}}$

Weight: $\mathbf{W} = -G\frac{M_em}{R_e^2}\hat{\mathbf{r}} \boxed{= m\mathbf{g}}$

To calculate the force between a particle of mass m and a uniform thin spherical shell of mass M and Radius R as follows:



$$\mathbf{F} = -G\frac{Mm}{r^2}\hat{\mathbf{r}} \qquad r > R$$

$$\mathbf{F} = 0 \qquad r < R,$$

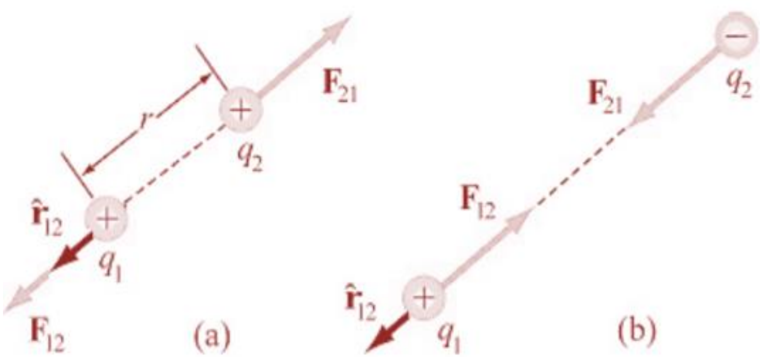
Exercise ???

where r is the distance from the center of the shell to the particle.

2- Electrostatic force

The force exerted on q_1 due to the interaction between q_1 and q_2 is given by Coulomb's Law,

$$\vec{\mathbf{F}}_{1,2} = k_e \frac{q_1 q_2}{r_{1,2}^{\,2}} \hat{\mathbf{r}}_{1,2}$$



The electric field at $\mathbf{r}$ due to a charge q at the origin is

$$\mathbf{E} = k\frac{q}{r^2}\hat{\mathbf{r}}.$$

Coulomb's Law and the Universal Law of Gravitation

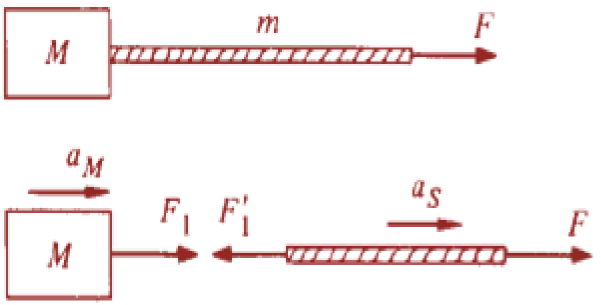
Contact forces

By contact forces we mean the forces which are transmitted between bodies by short-range atomic or molecular interactions. Examples include the pull of a string, the surface force of sliding friction, and the force of viscosity between a moving body and a fluid.

Tension—The Force of a String

Block on string

Consider a block of mass M in free space pulled by a string of mass m. A force F is applied to the string, as shown. What is the force that the string "transmits" to the block?



$$F_1 = Ma_M$$

$$F - F_1' = ma_S.$$

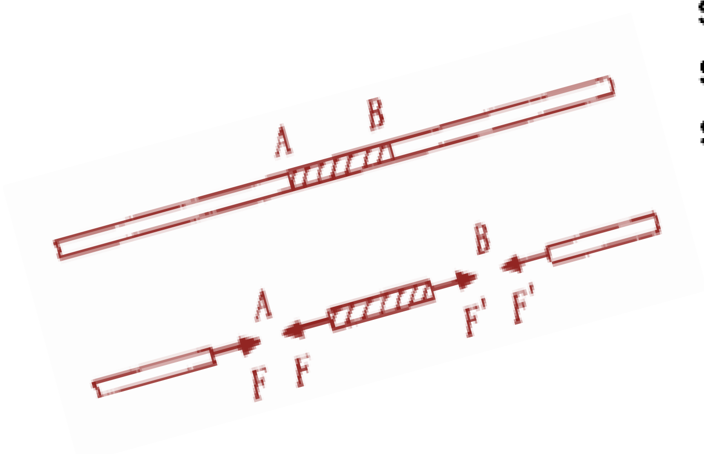
Assuming that the string is inextensible, it accelerates at the same rate as the block, giving the constraint equation $a_S = a_M$. Furthermore, $F_1 = F_1'$ by Newton's third law. Solving for the acceleration, we find that

$$a = \frac{F}{M + m}$$

$$F_1 = F_1'$$
$$= \frac{M}{M+m} F$$

The force on the block is less than F; the string does not transmit the full applied force. However, if the mass of the string is negligible compared with the block, $F_1 = F$ to good approximation.

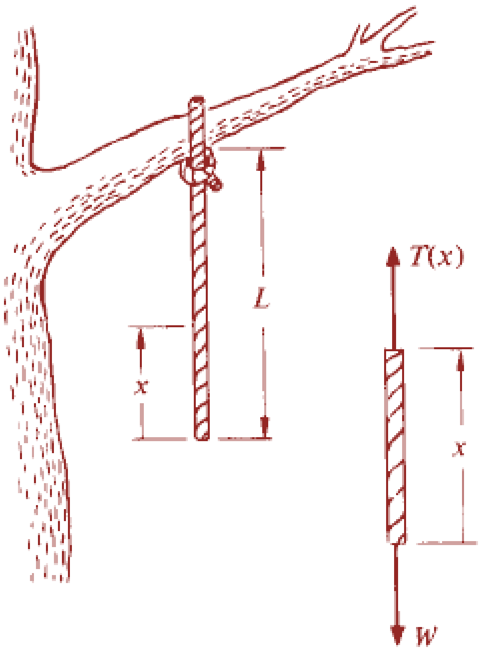
We can think of a string as composed of short sections interacting by contact forces. Each section pulls the sections to either side of it, and by Newton's third law, it is pulled by the adjacent sections. The magnitude of the force acting between adjacent sections is called *tension*.



If there are external forces on the section, or if the string is accelerating, the tension generally varies along the string.

Dangling rope

A uniform rope of mass M and length L hangs from the limb of a tree. Find the tension a distance x from the bottom.



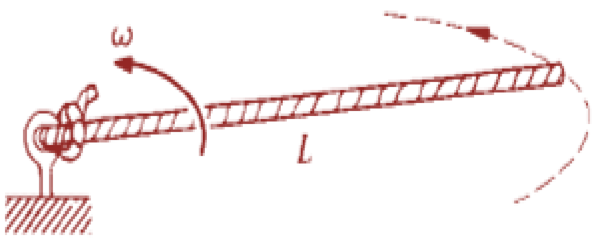
The total force on the section is zero since it is at rest. $W = Mg(x/L)$

$$T(x) = \frac{Mg}{L}x.$$

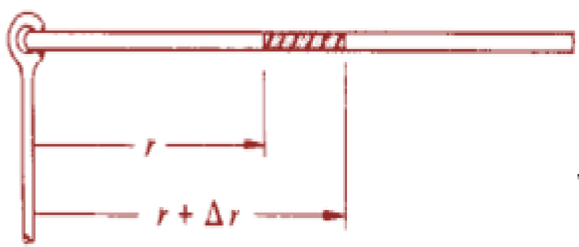
At the bottom of the rope the tension is zero, while at the top the tension equals the total weight of the rope Mg.

Whirling rope

A uniform rope of mass M and length L is pivoted at one end and whirls with uniform angular velocity ω. What is the tension in the rope at distance r from the pivot? Neglect gravity.



Consider the small section of rope between r and $r + \Delta r$. The length of the section is Δr and its mass is $\Delta m = M\,\Delta r/L$. Because of its circular motion, the section has a radial acceleration. Therefore, the forces pulling either end of the section cannot be equal, and we conclude that the tension must vary with r.



The inward force on the section is $T(r)$, the tension at r, and the outward force is $T(r + \Delta r)$. Treating the section as a particle, its inward radial acceleration is $r\omega^2$.

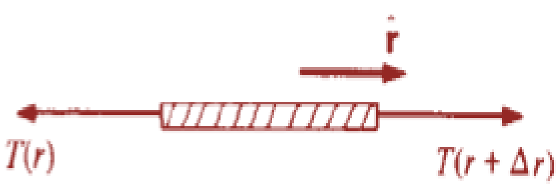
The equation of motion for the section is

$$T(r + \Delta r) - T(r) = -(\Delta m) r\omega^2$$

$$= -\frac{M r\omega^2\,\Delta r}{L}.$$

$$\frac{dT}{dr} = \lim_{\Delta r \to 0} \frac{T(r + \Delta r) - T(r)}{\Delta r}$$

$$= -\frac{M r\omega^2}{L}$$



$$\int_{T_0}^{T(r)} dT = -\int_0^r \frac{M\omega^2}{L}\, r\, dr,$$

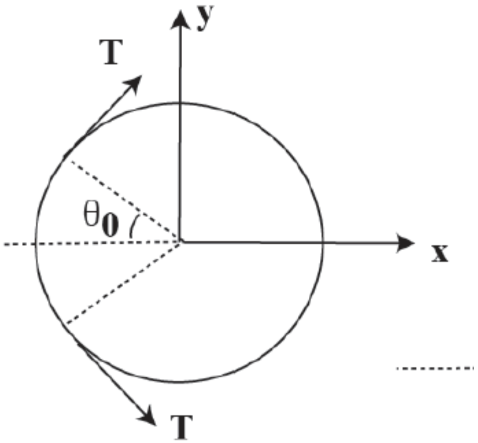
where T_0 is the tension at $r = 0$.

Sincethe end of the rope at $r = L$ is free, the tension there must be zero.

$$\boxed{T(r) = \frac{M\omega^2}{2L}\,(L^2 - r^2).}$$

Force on a pulley from a string

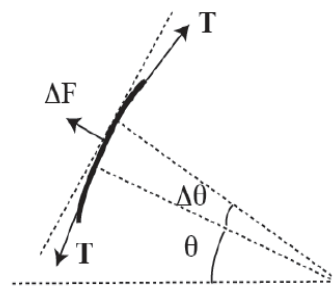
A string with constant tension T is deflected through angle $2\theta_0$ by a smooth fixed pulley. What is the force on the pulley?

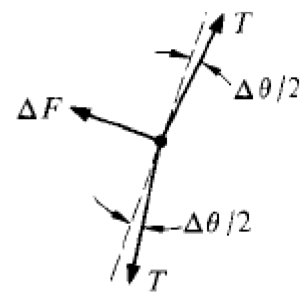


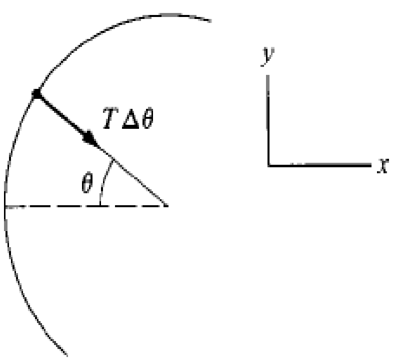
Consider the section of string between θ and $\theta + \Delta\theta$. The force diagram is drawn below, center. ΔF is the outward force due to the pulley

$$\Delta F - 2T \sin\frac{\Delta\theta}{2} = 0 \Longrightarrow \text{For small } \Delta\theta, \quad \Delta F = 2T\frac{\Delta\theta}{2} = T\Delta\theta$$

Thus the element exerts an inward radial force of magnitude $T\,\Delta\theta$ on the pulley.







The element at angle θ exerts a force in the x direction of $(T\,\Delta\theta)\cos\theta$. The total force in the x direction is $\Sigma T\cos\theta\,\Delta\theta$, where the sum is over all elements of the string which are touching the pulley. In the limit $\Delta\theta \to 0$, the sum becomes an integral. The total force in the x direction is therefore

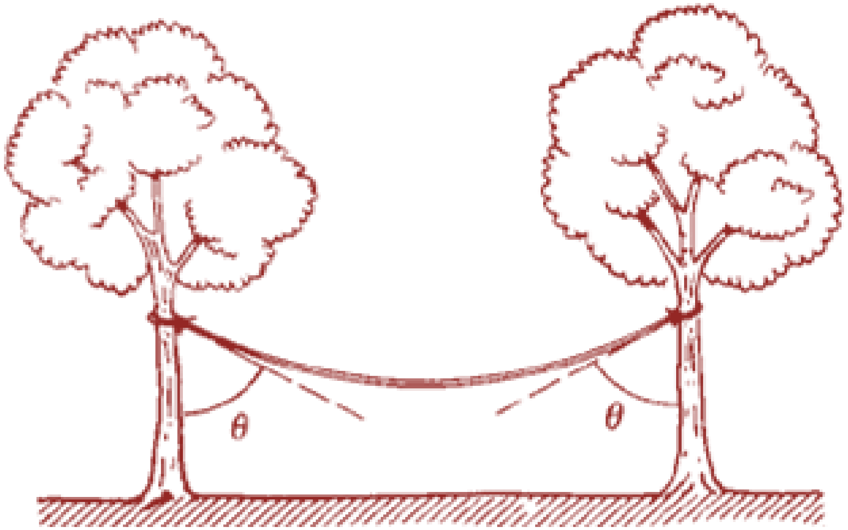
$$\int_{-\theta_0}^{\theta_0} T\cos\theta\,d\theta = 2T\sin\theta_0.$$

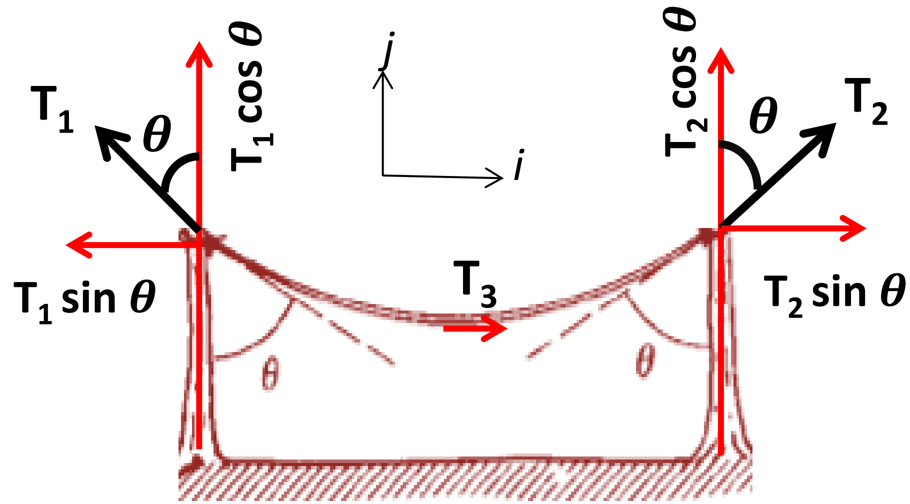
KK prob.2.21

A uniform rope of weight W hangs between two trees. The ends of the rope are the same height, and they each make angle θ with the trees. Find

a. The tension at either end of the rope

b. The tension in the middle of the rope





At the end points

$$T_1 \sin\theta = T_2 \sin\theta;\ \ T_1 = T_2$$
$$2T_1 \cos\theta = W$$
$$T_1 = W/2\cos\theta \ \ \text{and for}\ \theta = 45^o$$
$$T_1 = W/\sqrt{2}$$

At the middle

$$T_1 \sin\theta = T_3 \quad \Longrightarrow \quad W/2 * \tan\theta = T_3$$

$$T_1 \cos\theta = W/2$$

$$and\ for\ \theta = 45^o \quad T_3 = W/2$$

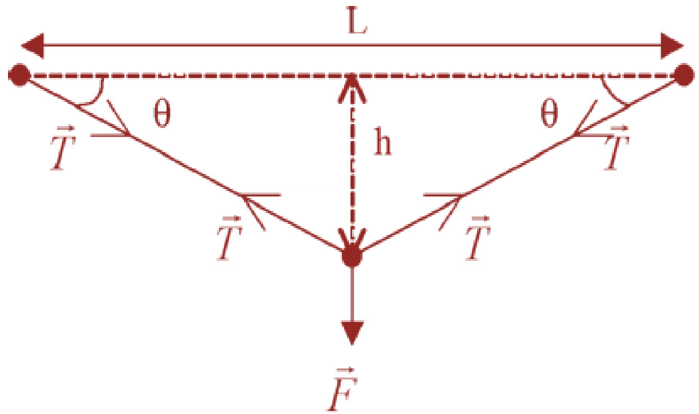
Example-2

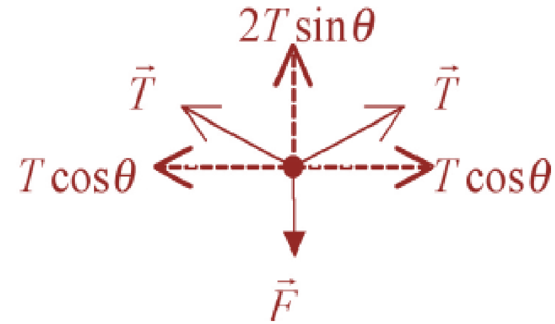
Suppose a rope is tied rather tightly between two trees that are 30 m apart. You grab the middle of the rope and pull on it perpendicular to the line between the trees with as much force as you can. Assume this force is 1000 N (about 225 lb), and the point where you are pulling on the rope is $h = 1\text{m}$ from the line joining the trees.

a) What is the magnitude of the force tending to pull the trees together?

b) Give an example of a situation where you think this may be of practical use.

(a)





$$\hat{\mathbf{i}}: T\cos\theta - T\cos\theta = 0$$
$$\hat{\mathbf{j}}: \quad 2T\sin\theta - F = 0.$$

$\Rightarrow$ $T = F/(2\sin\theta)$. With, $L = 30\,\mathrm{m}$ and $h = 1\,\mathrm{m}$

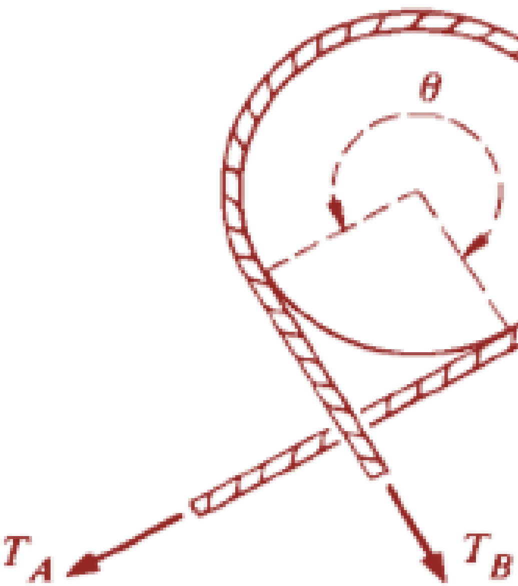
$$\tan\theta = h/(L/2) = 1/15 \Rightarrow \theta = \tan^{-1}(1/15) = 3.81° \Rightarrow \sin\theta = 0.067$$

small angle approximation $\sin\theta \sim \tan\theta = h/(L/2)$

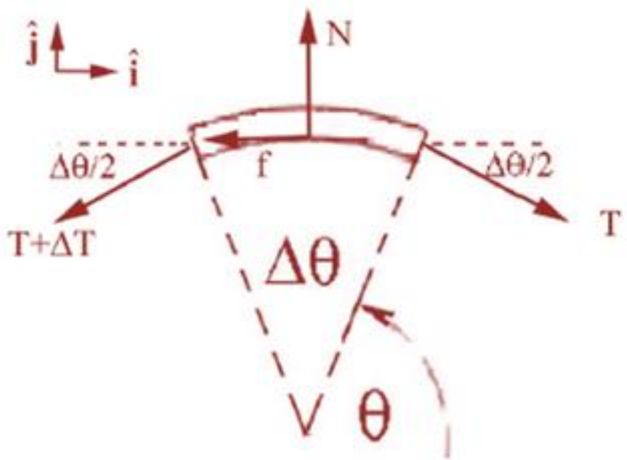
$$T = \frac{F}{2\sin\theta} = (1000\,\mathrm{N})\times 15/2 = 7500\,\mathrm{N}$$

(b) If the applied force is vertically downward, you're walking a tightrope, and the trees should be big enough to hold up to roughly 7-8 times your weight. If the applied force is horizontal, maybe one of the trees will come down, and you might have firewood without having to resort to a chainsaw. If one end of the rope is attached to a large enough tree and the other to a car in a ditch, you can apply several times the pulling force to try to move the car.

KK prob.2.24



A device called a capstan is used aboard ships in order to control a rope which is under great tension. The rope is wrapped around a fixed drum, usually for several turns (the drawing shows about three-fourths turn). The load on the rope pulls it with a force T_A, and the sailor holds it with a much smaller force T_B. Can you show that $T_B = T_A e^{-\mu\theta}$, where μ is the coefficient of friction and θ is the total angle subtended by the rope on the drum?



Tangential component

$$\hat{\mathbf{i}}: T\cos(\Delta\theta/2) - f_s - (T+\Delta T)\cos(\Delta\theta/2)$$

Radial component

$$\hat{\mathbf{j}}: -T\sin(\Delta\theta/2) + N - (T+\Delta T)\sin(\Delta\theta/2)$$

For small angles $\Delta\theta$, $\cos(\Delta\theta/2) \cong 1$ and $\sin(\Delta\theta/2) \cong \Delta\theta/2$.

By the static equilibrium condition the sum of the x-components of the forces is zero,

$$\hat{\mathbf{i}}: -f_s - \Delta T = 0 \qquad .....(1)$$

Similarly

$$\hat{\mathbf{j}}: -T\Delta\theta + N = 0 \qquad .....(2)$$

The just slipping condition is that the magnitude of the static friction attains its maximum value

$$f_s = (f_s)_{\max} = \mu_s N$$

From eq^n 1 and 2

$$\frac{\Delta T}{\Delta\theta} = -\mu_s T$$

The derivative of tension with respect to the angle θ is defined to be the limit

$$\frac{dT}{d\theta} \equiv \lim_{\Delta\theta \to 0} \frac{\Delta T}{\Delta\theta},$$

Integrate both sides, noting that when $\theta = 0$, the tension is equal to force of the load T_A, and when angle $\theta = \theta_{A,B}$ the tension is equal to the force T_B the sailor applies to the rope,

$$\int_{T=T_A}^{T=T_B} \frac{dT}{T} = -\int_{\theta=0}^{\theta=\theta_{A,B}} \mu_s \, d\theta \;\longrightarrow\; \ln\left(\frac{T_B}{T_A}\right) = -\mu_s \theta_{A,B}$$

the tension decreases exponentially, $\boxed{T_B = T_A e^{-\mu_s \theta_{A,B}},}$

Since the tension decreases exponentially, the sailor need only apply a small force to prevent the rope from slipping.

Q.1 A number of trucks are to be hauled by a rope passed round a hydraulic capstan; assuming a coefficient of friction of 0.25 between the rope and the capstan and a constant manual pull of 30 lb., derive an expression for the maximum pull on the trucks in terms of complete 2.5 turns of rope round the capstan?

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

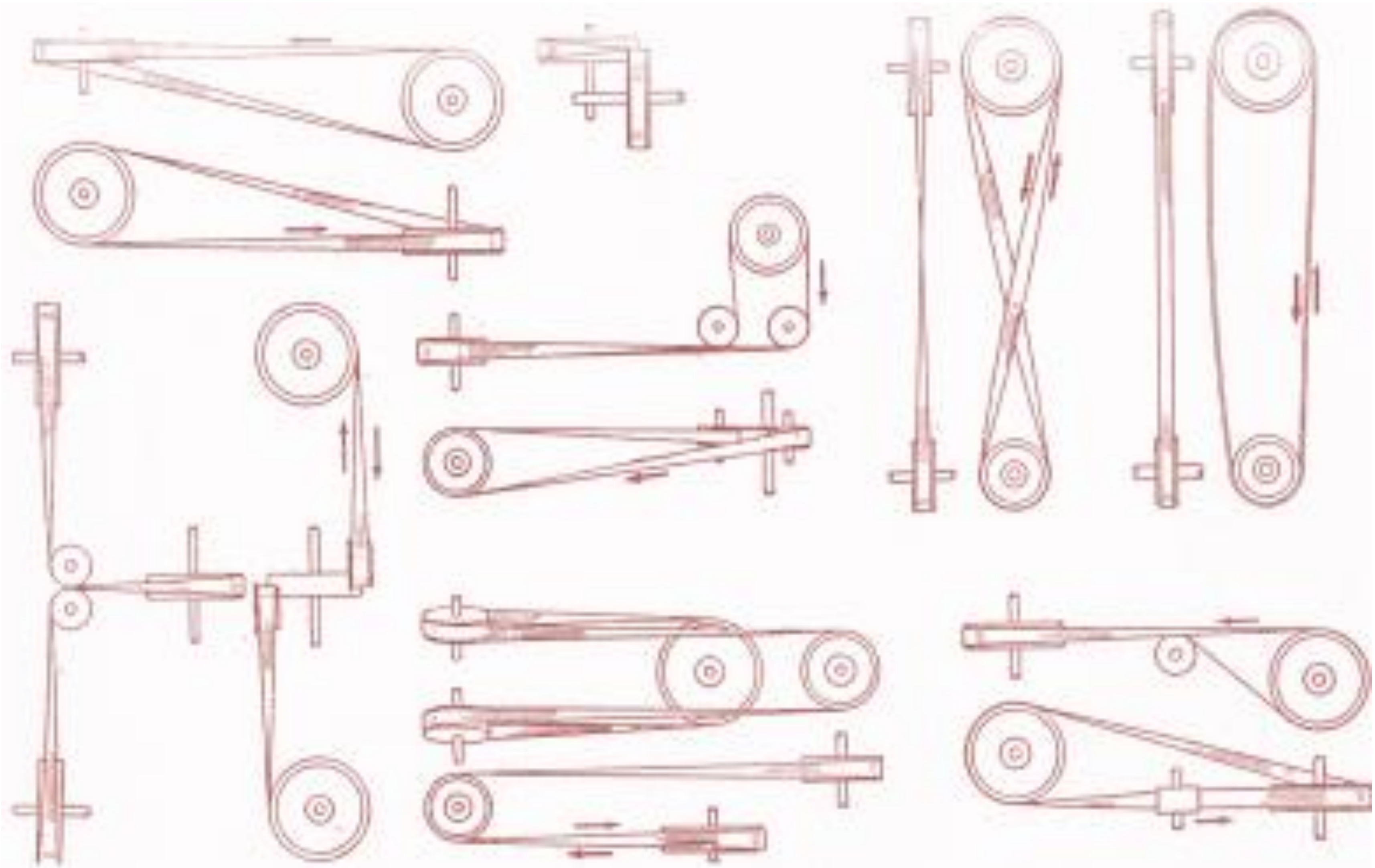
From the question:

$T_2 = 30\ lb.$, $\mu = 0.25$ and $\theta = 2\,\pi\, n\ radians$

From which:

$$T_1 = 30\ e^{0.25 \times 2\,\pi\, n} = 30(4.82)^n$$

When $n = 2.5$, $T_1 = 1520\ lb.$



THANK YOU