NB : this document sums up some important techniques you should already be familiar with!

Some redaction advices

For now on, you be asked to write proofs, step by step, so you will have to start writing your solutions, in the right order, even before being able to conclude. When reading the question, first focus on the hypotheses, then on the conclusion, to highlight what is being asked to you? And then ask yourself how to solve it (using definitions and methods)?

You will also be really thoughtful about all the variables used in your proofs, remember that every variable should be introduced *before* being used! But you will avoid the abusive use of existential and universal quantifiers ($\exists, \forall$). They are logical symbols, to be used only in formulas, but not in English sentences.

General approaches to prove an assertion

If P is an assertion to be proved, usually we use one of these methods:

- proof by deduction (show that for a certain A, we have $A \Rightarrow P$, and then prove A),
- proof by cases consideration,
- proof by the absurd (show that if P is false, then we arrive to an absurdity).

General approaches to prove a consequence

If $P \Rightarrow Q$ is a consequence to be proved, usually we use one of these methods:

- the direct proof: "Assume P true, use it to show Q.",
- the contrapositive proof, *i.e.* show $\text{Non}(Q) \Rightarrow \text{Non}(P)$,
- proof by the absurd: "Assume P true and Q false, use them to conclude to an absurdity.",

General approaches to prove an equivalence

If $P \Leftrightarrow Q$ is an equivalence to be proved, usually we use one of these methods:

- proceed by double implication: $P \Rightarrow Q$ and $Q \Rightarrow P$,
- reason by equivalences: $P \Leftrightarrow P_2 \Leftrightarrow \cdots \Leftrightarrow P_n \Leftrightarrow P$ (with a sufficient number of intermediate steps),
- proof by two cases consideration (P is either true or false): $P \Rightarrow Q$ and $\mathrm{Non}(P) \Rightarrow \mathrm{Non}(Q)$.

General approaches to prove an universal or existential property

Universal property To prove $\forall x \in E, P(x)$ (the formula $P(x)$ depends on the variable x):

- your proof should start by: "Let $x \in E$", **fixed and arbitrary**,
- then you justify than $P(x)$ is true.

Remark 1. *When $E = \mathbb{N}$, you can also reason **by induction** (*i.e. *, proof $P(0)$ and $\forall n \in \mathbb{N}, P(n) \Rightarrow P(n+1)$).*

Existential property To prove $\exists x \in E, P(x)$, one can try to *find* a peculiar x that satisfy P, for instance by solving an equation, but it is not always easy! Otherwise, we usually prove an existential property thanks to an other existential property, seen in the lectures.

A property of existence and uniqueness To prove $\exists! x \in E, P(x)$, you can proceed with one of these approaches:

- regardless of the order, prove:
  - **existence**: with the previous methods,
  - **uniqueness**: "Let x and $x' \in E$ such that $P(x)$ and $P(x')$. Show $x = x'$."
- or by *analysis and synthesis*:
  - **Analysis**: "Assume there is $x \in E$ such that $P(x)$." Then you prove that x has to be equal to a certain x_0,
  - **Synthesis**: you verify that x_0 satisfies P (*i.e.* $P(x_0)$ is true).