PH101
Lecture13

26.08.14

$$\vec{F} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0$$

$\Rightarrow$ Total momentum is constant

This is law of conservation of momentum

Derivation of law of conservation of momentum relied on Newton's 3rd law and appears to be natural consequence of Newtonian mechanics

Law of conservation of momentum is ubiquitous as it is found to hold in areas where Newtonian mechanics is inadequate e.g. QM and relativity

Law of conservation of momentum can be generalized to apply to even EM fields, which possess momentum but no mass!

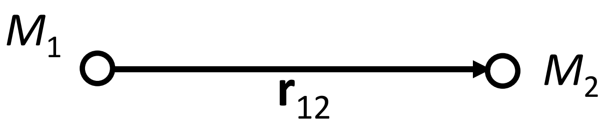
From this perspective Law of conservation of momentum is considered more fundamental than Newtonian mechanics

From this point of view, Newton's 3rd law can be stated to be a simple consequence of conservation of momentum for interacting particles

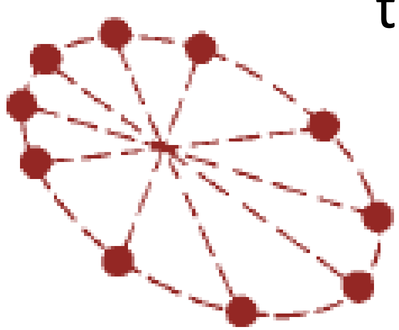
Center of mass coordinate system for a two-body problem

<u>A three body system</u>

Earth, Moon and Sun

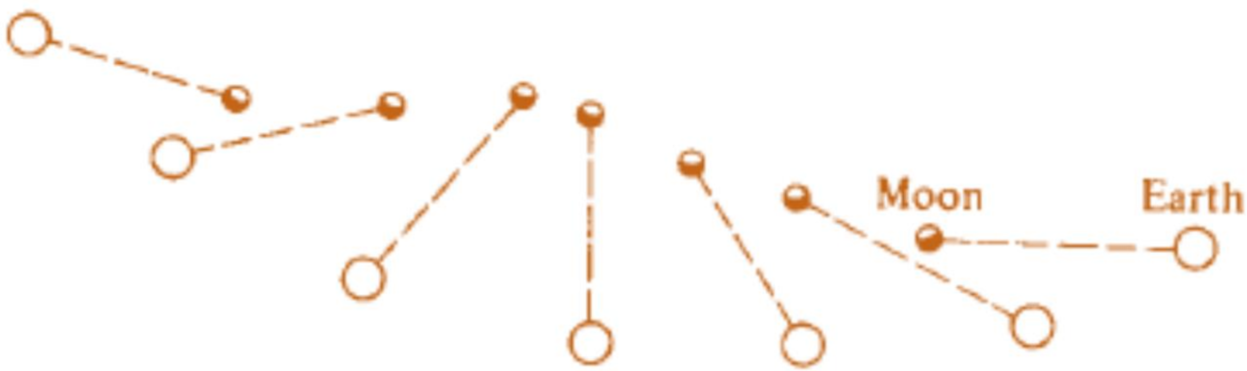


M_1 and M_2 bound under the influence of gravity such that $\mathbf{r}_{12}$ traces out an ellipse

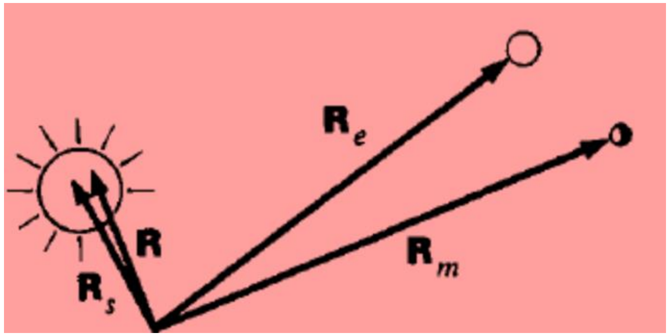


In the absence of Sun

Earth and Moon will move in elliptic orbit about their center of mass



To a good approximation center of mass of Earth and Moon will move in elliptic orbit about the Sun as a system like a single planet $\Rightarrow$ total motion will be result of two simultaneous elliptical orbits



Center of mass of the overall system will lie at

$$\vec{R} = \frac{M_{\mathrm{e}}\vec{R}_{\mathrm{e}} + M_{\mathrm{m}}\vec{R}_{\mathrm{m}} + M_{\mathrm{s}}\vec{R}_{\mathrm{s}}}{M_{\mathrm{e}} + M_{\mathrm{m}} + M_{\mathrm{s}}}$$

Mass of Sun is so much greater than that of Moon or Earth

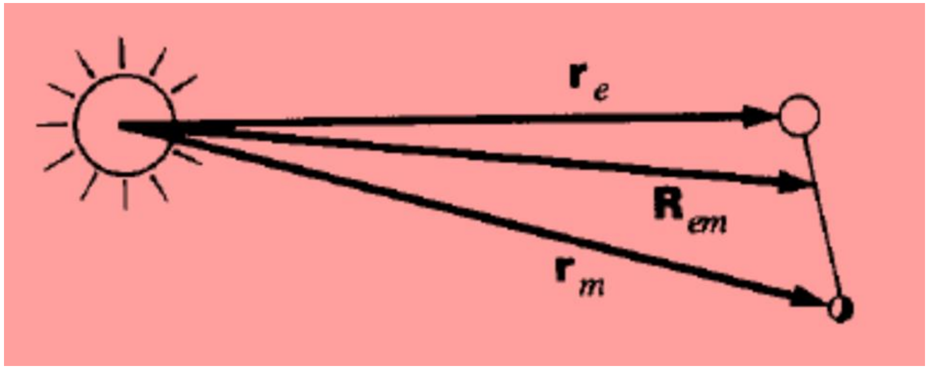
that one may assume $\vec{R} \approx \vec{R}_s$

$\Rightarrow$ Center of mass of this 3-body system could be taken to lie at the center of the Sun

Further due to negligible external forces, Sun is effectively at rest in an inertial frame of reference

Thus one can set up a coordinate system with center of Sun as the origin

$\Rightarrow$ **R** = 0



At a particular instant of time, let position vectors of earth & moon w.r.t sun are

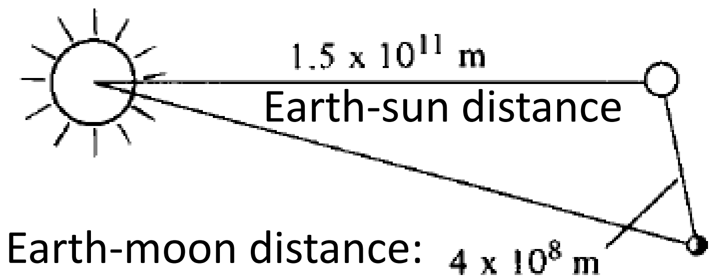
$\mathbf{r}_e$ and $\mathbf{r}_m$

$\Rightarrow$ Center of mass lies at $$\vec{R}_{\mathrm{em}} = \frac{M_{\mathrm{e}}\vec{r}_{\mathrm{e}} + M_{\mathrm{m}}\vec{r}_{\mathrm{m}}}{M_{\mathrm{e}} + M_{\mathrm{m}}}$$

External force on the earth-moon system:

$$\vec{F} = -GM_{\mathrm{s}} \overbrace{\left(\frac{M_{\mathrm{e}}}{r_{\mathrm{e}}^2} \hat{r}_e + \frac{M_{\mathrm{m}}}{r_{\mathrm{m}}^2} \hat{r}_m \right)}^{\text{Gravitational pull of the Sun}}$$

Thus the eq of motion of their center of mass will be



$$\left(M_{\mathrm{e}} + M_{\mathrm{m}} \right) \ddot{\vec{R}}_{\mathrm{em}} = \vec{F}$$

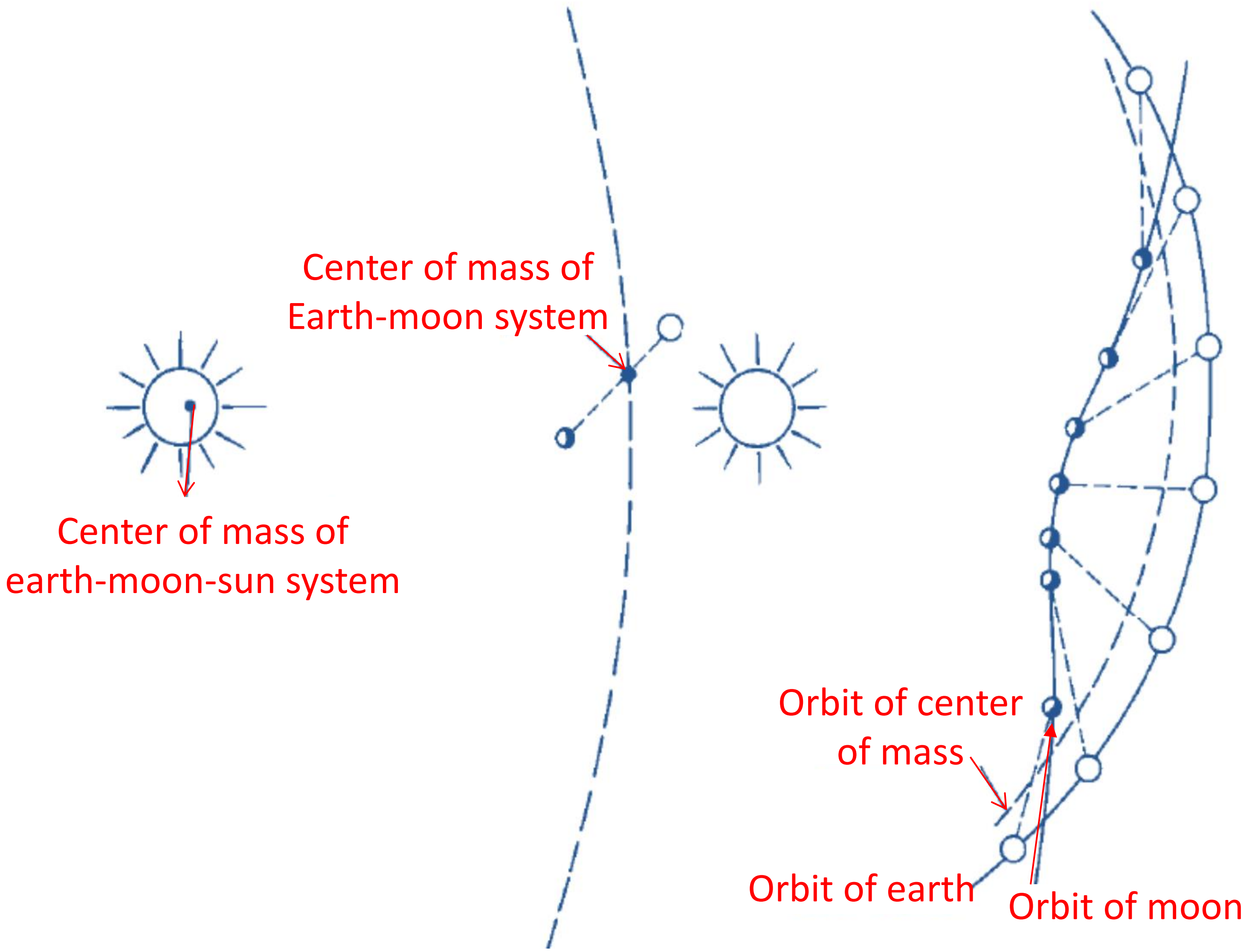
It is ok to assume

$$r_{\mathrm{e}} \approx r_{\mathrm{m}} \approx R_{\mathrm{em}}$$

$$\Rightarrow \left(M_{\mathrm{e}} + M_{\mathrm{m}} \right) \ddot{\vec{R}}_{\mathrm{em}} \approx -\frac{GM_{\mathrm{s}}}{R_{\mathrm{em}}^2} \left(M_{\mathrm{e}} \hat{r}_e + M_{\mathrm{m}} \hat{r}_{\mathrm{m}} \right) \approx -\frac{GM_{\mathrm{s}}}{R_{\mathrm{em}}^2} \left(M_{\mathrm{e}} + M_{\mathrm{m}} \right) \hat{R}_{\mathrm{em}}$$

Thus c.m. of earth and moon moves around the sun like a planet of mass $\equiv \left(M_{\mathrm{e}} + M_{\mathrm{m}} \right)$

Total motion is the combination of this elliptical orbit and the orbit traced out by the earth and moon about their center of mass



- Differential relation between force and momentum is

$$\vec{F} = \frac{d\vec{P}}{dt}$$

- In integral form , it can be written as

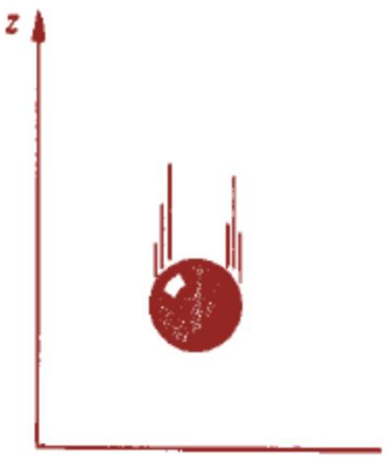
$$\vec{P}(t) - \vec{P}(0) = \int_0^t \vec{F}\,dt$$

- Change in momentum equals time integral of the force and does not depend on detailed time dependence of the force
- To produce the same amount of change in momentum, one could either use a small force over a longer duration

or

- use a large force over only a shorter time interval
- Integral: $\int_0^t \vec{F}\,dt$ is called *impulse*

#1. A rubber ball of mass 0.2 kg falls to the floor. It hits the floor with a speed of 8 m/s and rebounds with approxly. the same speed. If a high-speed camera records the duration of contact of the ball with the floor as 10^{-3}sec , what can be said about the force exerted on the ball by the floor?



Just before hitting the floor, momentum of the ball

$$\vec{P}_a = -0.2\text{x } 8\hat{\text{k}} \text{ kg.m/s}$$

Just after hitting the floor, momentum of the ball

$$\vec{P}_a = 0.2\text{x } 8\hat{\text{k}} \text{ kg.m/s}$$

The impulse

$$\int_{t_a}^{t_b} \vec{F}\, dt = \vec{P}_b - \vec{P}_a = 1.6\,\hat{\text{k}} \text{ - } (\text{-}1.6\,\hat{\text{k}}) = 3.2\,\hat{\text{k}} \text{ kg.m/s}$$

Since precise variation of **F** with *t* is not known, we can find the avg. force exerted by the floor on the ball.

Let collision time be Δt (= $t_b - t_a$)

Avg force acting during the collision will be

$$\vec{F}_{avg}\Delta t = \int_{t_a}^{t_a+\Delta t} \vec{F}\,dt = 3.2\,\hat{k}\ \text{kg.m/s}$$

$$\Rightarrow \vec{F}_{av} = \frac{3.2\,\hat{k}\ \text{kg.m/s}}{10^{-3}\,s} = 3200\,\hat{k}\ \text{N}$$

Directed upward

Pretty large force!

More correctly one shld include gravitational force also; however corresponding correction is negligible $\vec{F} = \vec{F}_{floor} + \vec{F}_{gravity}$

$$-\int_0^{10^{-3}} M\,g\,\hat{k}\,dt = -0.2\times 9.8\times 10^{-3}\,\hat{k} = -\underbrace{1.96x10^{-3}}_{\text{Insignificant}}\,\hat{k}\ \text{Kg.m/s}$$

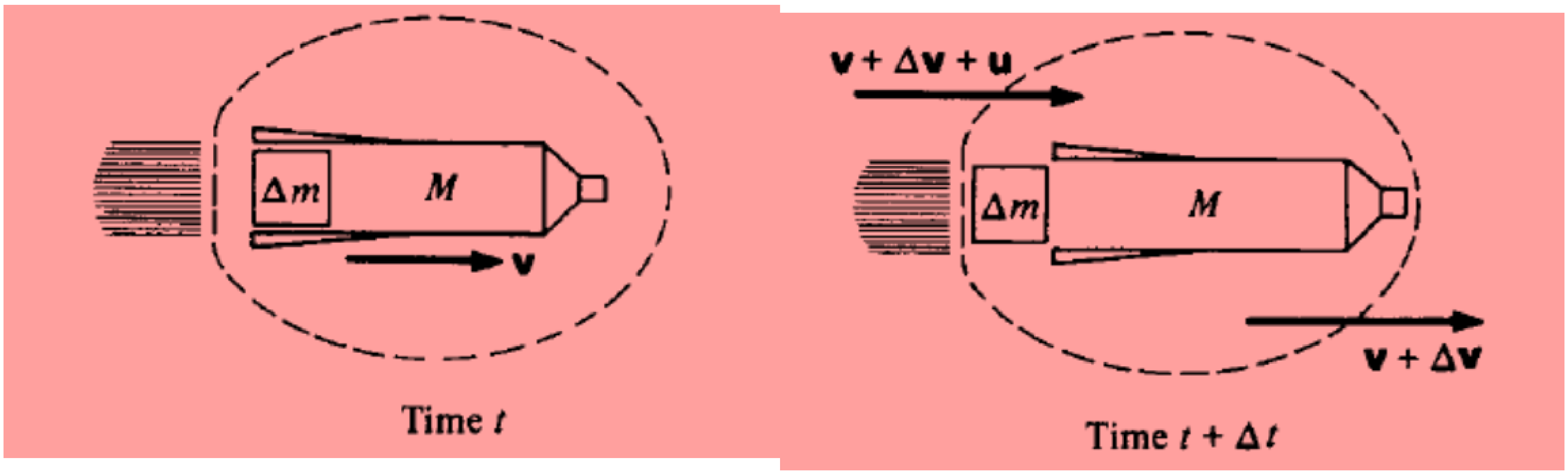
However over a longer period of time gravity can produce larger change in momentum

- Contact force during short duration collision are huge as compared to impulses due to other forces of moderate strength e.g. gravity, friction
- Even if initial and final velocities are equal, a quick collision is more violent than a slow collision!
- e.g. a hammer can produce a force much greater than what the carpenter can produce on his own
- The hammerhead rebounds in a very short time compared to the time of its swing $\Rightarrow$ force driving the hammer gets amplified
- This concept is utilized (in reverse i.e. prolong the duration of collision) to prevent bodily injury in an accident e.g. car seat belt, helmet of a bike rider/a cricket player
- to cushion the impact with ground while jumping from a height one instinctively flexes rather than jumping rigidly

Newton's law when applied to a system when there is a flow of mass

Examples:

- Reaction force of the gas in a rocket as it accelerates



- Reaction force on a fire hose

- To calculate acceleration of a snow ball which grows in size as it rolls downhill

Recall

$\vec{F} = \dfrac{d\vec{P}}{dt}$ was obtained for a system composed of a set of particles

$\downarrow$

When applied in integral form

$$\int_{t_a}^{t_b} \vec{F}\, dt = \vec{P}(t_b) - \vec{P}(t_a)$$

It is essential to keep track of all the particles throughout the time interval t_a to t_a that were originally in the system $\Rightarrow$ mass of the system can not change during that time interval

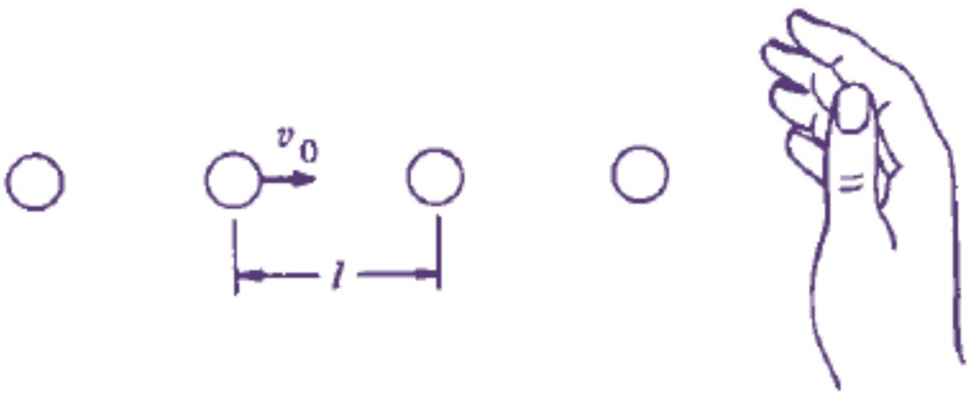
Momentum transport

Intense stream of water from a hose pipe connected to e.g. a fire Hydrant can be used to break through the wall of a burning building

How is it that it exerts so much force as real as the force transmitted by a rigid steel rod?

A person would feel strong push if stands in front of water jet

The push results from the transfer of momentum to you



Consider the stream directed to your hand is made of a series of uniform droplets which collide with you without rebound and run down your hand

Let each droplet has

Mass = m Travels with velocity = v_0

As each droplet hits there is an instantaneous large force for a short time

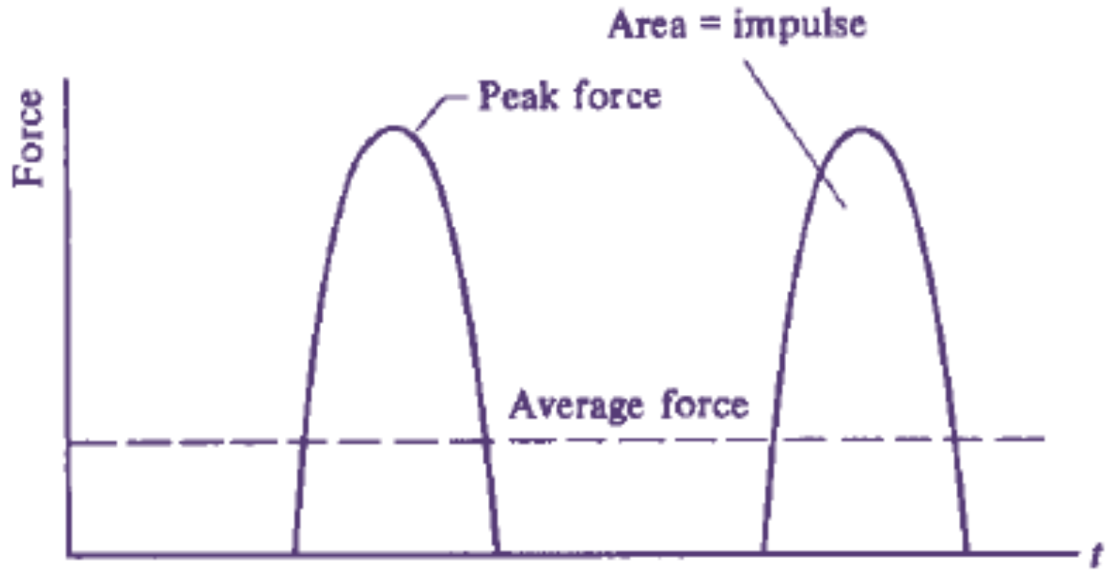
Not known

Impulse on each droplet due to your hand :

$$I_{\text{droplet}} = \int_{\text{1collision}} F\,dt = \Delta p$$
$$= m\left(v_{\text{f}} - v_0\right)$$
$$= -mv_0$$

Impulse on your hand is equal and opposite: $I_{\text{hand}} = mv_0$

You do not feel shock due to each droplet rather an avg force



For many droplets hitting your hand, you feel an avg force F_{avg}

Area under F_{avg} during collision period T (betn two collisions) is identical to the impulse due to one drop

$$F_{av}T = \int_{1\,\mathrm{collision}} F\,dt = \frac{mv_0}{l / v_0} = \frac{m}{l} v_0^2$$

$$\int F\,dt = mv_0; T = \frac{l}{v}$$