PH101

Vectors
By
Dr. Murtaza
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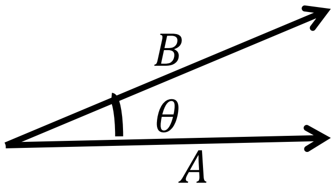
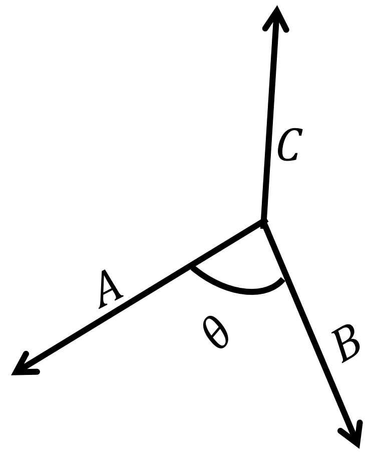
Review

Vector notation $\vec{A} = |A|\,\hat{A}$



Scalar product $\vec{A}.\vec{B} = |A||B|\,Cos\,\theta$



Vector product $\vec{A} \times \vec{B} = \vec{C}$; $|C| = |A||B|\,Sin\,\theta$

Motion under uniform acceleration governed by

$$\begin{pmatrix} v(t) = v_0 + at \\ r(t) = r_0 + v_0 t + \frac{1}{2} a t^2 \\ v(t)^2 = v_0^2 - 2ar(t)^2 \end{pmatrix}$$

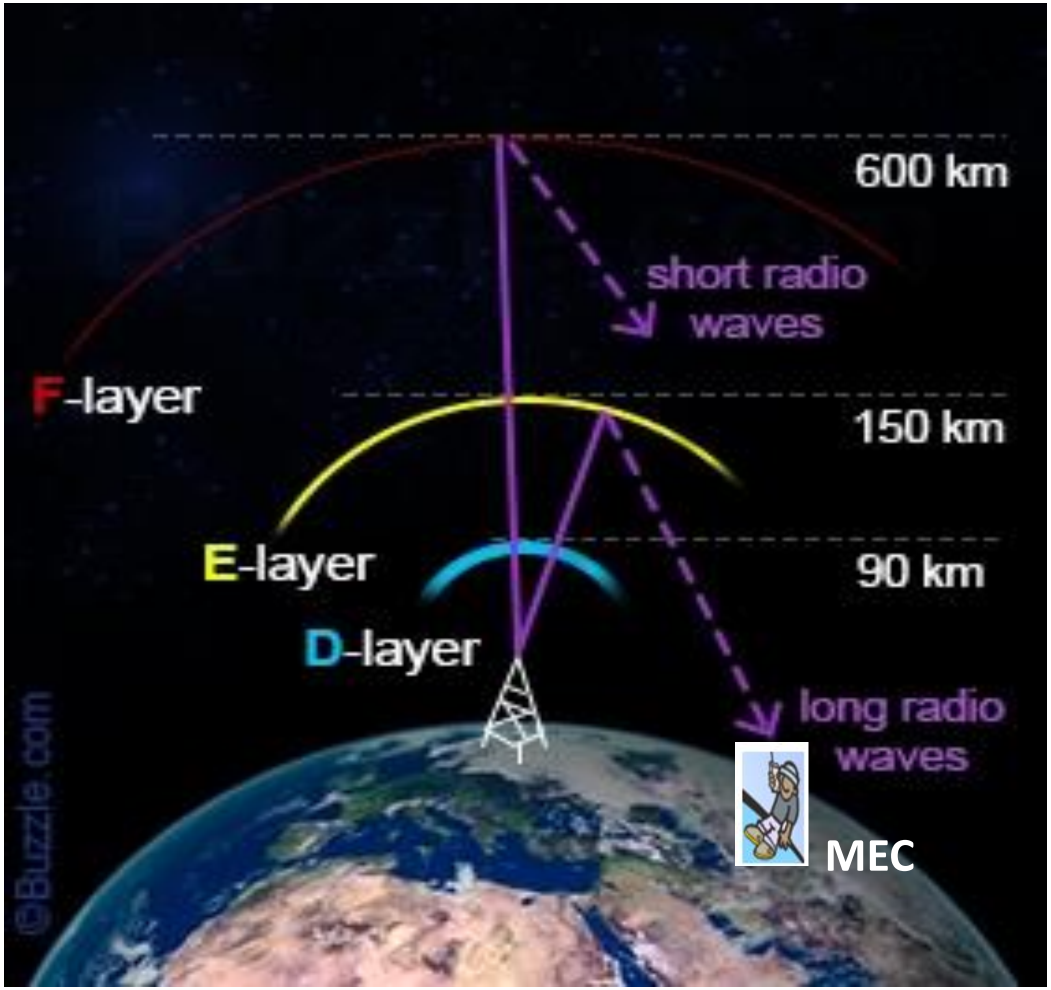
For non uniform acceleration:

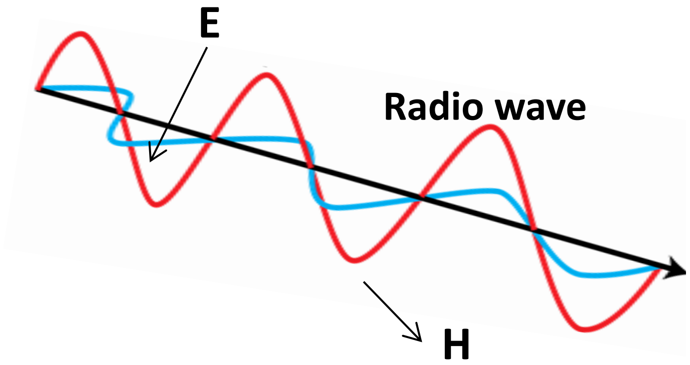
$$v(t_1) = v(t_0) + \int_{t_0}^{t_1} a(t)dt$$

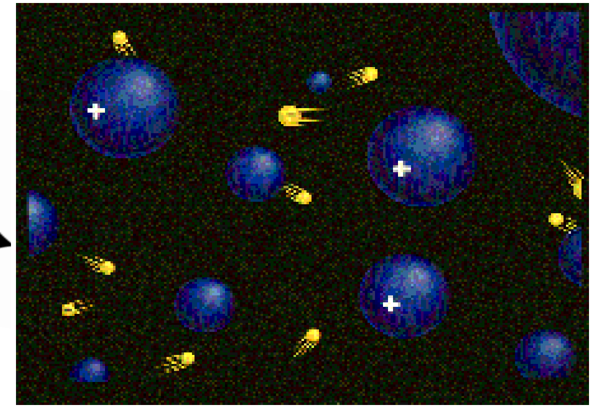
$$r(t_1) = r(t_0) + \int_{t_0}^{t_1} v(t)dt$$

Non uniform acceleration

The effect of a radio wave on an Ionspheric electrons







Electric field exerts force on electrons:

$$\boldsymbol{F = ma = -eE}$$

where electric field in x direction given as $\boldsymbol{E = E_0 \ sin\, wt}$

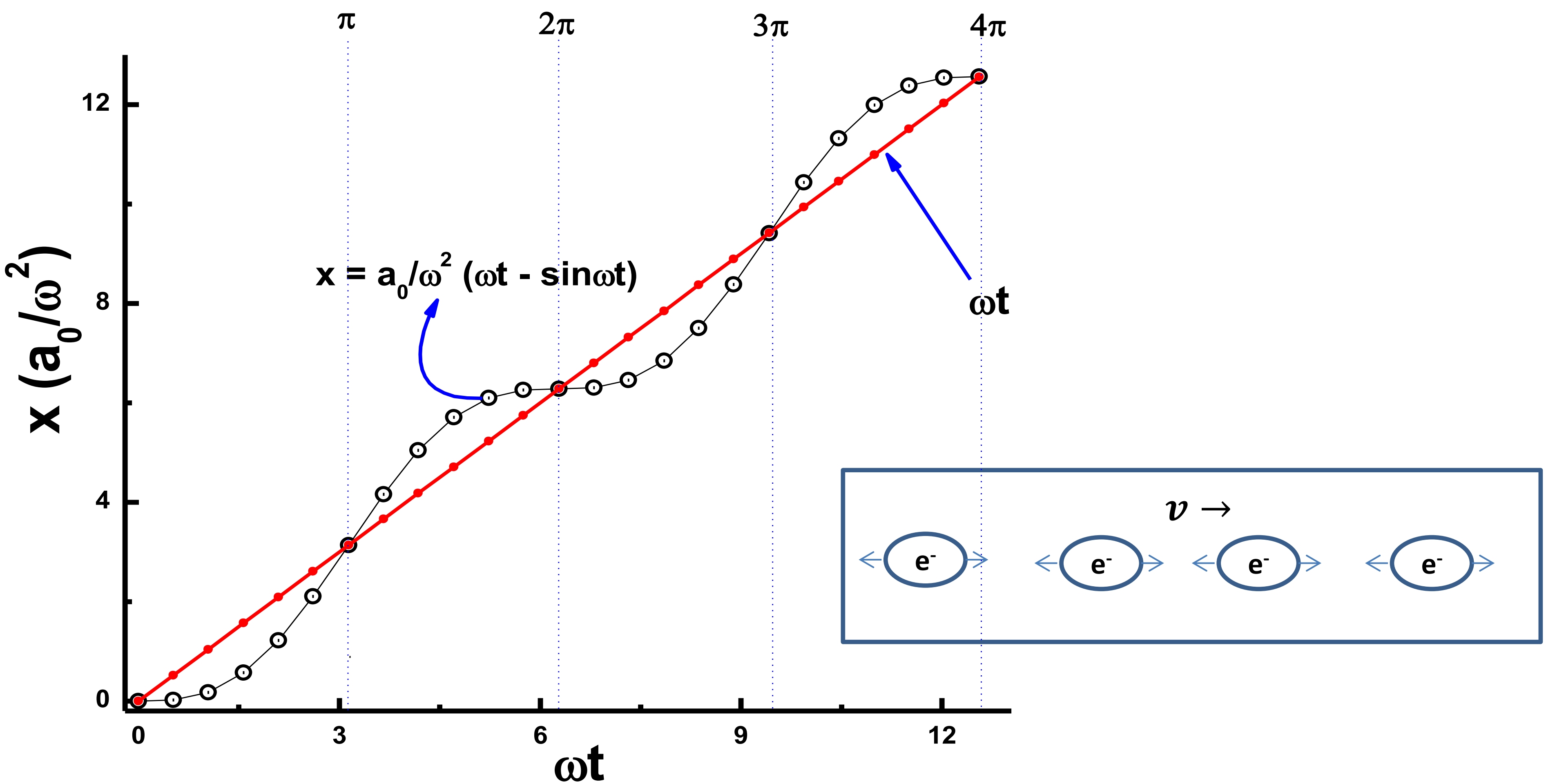
$$\boldsymbol{a = \frac{-eE_0}{m} \ \mathbf{sin\ wt}}$$

$$\boldsymbol{a(t) = a_0 \ \mathbf{sin\ wt}} \qquad \textbf{where } \boldsymbol{a_0 = \frac{-eE_0}{m}}$$

$$\mathbf{v}(\boldsymbol{t}) = \int_0^t \boldsymbol{a(t)dt} = \boldsymbol{a_0} \int_0^t \mathbf{sin\ wt}\ \boldsymbol{dt} = \frac{\boldsymbol{a_0}}{\boldsymbol{w}} \ (\mathbf{1 - cos\ wt})$$

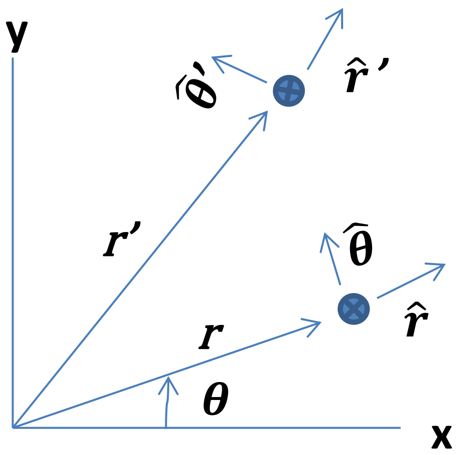
$$x(t) = \int_0^t v(t)dt = \frac{a_0}{w}\int_0^t (1 - \cos wt)dt = \frac{a_0}{w}t - \frac{a_0}{w^2}\sin wt$$

$$x = \frac{a_0}{w^2}(wt - \sin wt)$$



Motion in plane polar coordinates

-Direction of unit vectors $(\hat{\mathrm{r}}, \widehat{\theta})$ of polar coordinates change with in plane motion



$$\boldsymbol{r} = r\,\hat{\boldsymbol{r}}(\theta)$$

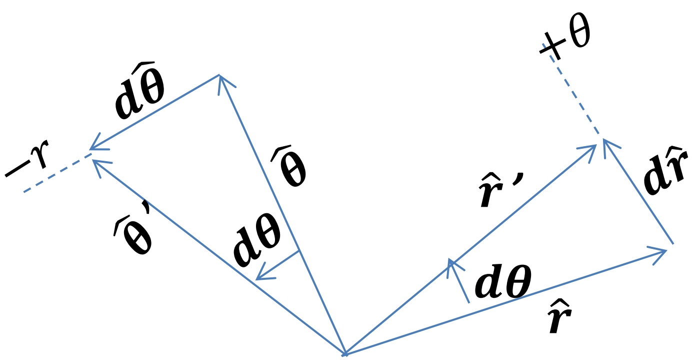
-Unit vectors are function of $\boldsymbol{\theta}$ but not $\boldsymbol{r}$

$$d\hat{r} = 1\,d\theta\hat{\theta} \quad \& \quad d\hat{\theta} = -1\,d\theta\hat{r}$$

Time derivatives

$$\dot{\hat{r}} = \dot{\theta}\,\hat{\theta} \qquad \dot{\hat{\theta}} = -\dot{\theta}\,\hat{r}$$

Position, velocity and accelerations



$$\boldsymbol{r} = r\,\hat{\boldsymbol{r}}(\theta)$$

$$\boldsymbol{v} = \underbrace{\dot{\boldsymbol{r}}\,\hat{\boldsymbol{r}}}_{v_r} + \underbrace{r\dot{\boldsymbol{\theta}}\,\widehat{\boldsymbol{\theta}}}_{v_\theta}$$

$$v = \sqrt{(v_r^2 + v_\theta^2)}$$

$$\boldsymbol{a} = (\ddot{\boldsymbol{r}} - r\dot{\boldsymbol{\theta}}^2)\hat{\boldsymbol{r}} + (r\ddot{\boldsymbol{\theta}} + 2\,\dot{\boldsymbol{r}}\,\dot{\boldsymbol{\theta}}\,)\widehat{\boldsymbol{\theta}}$$

Radial acceleration

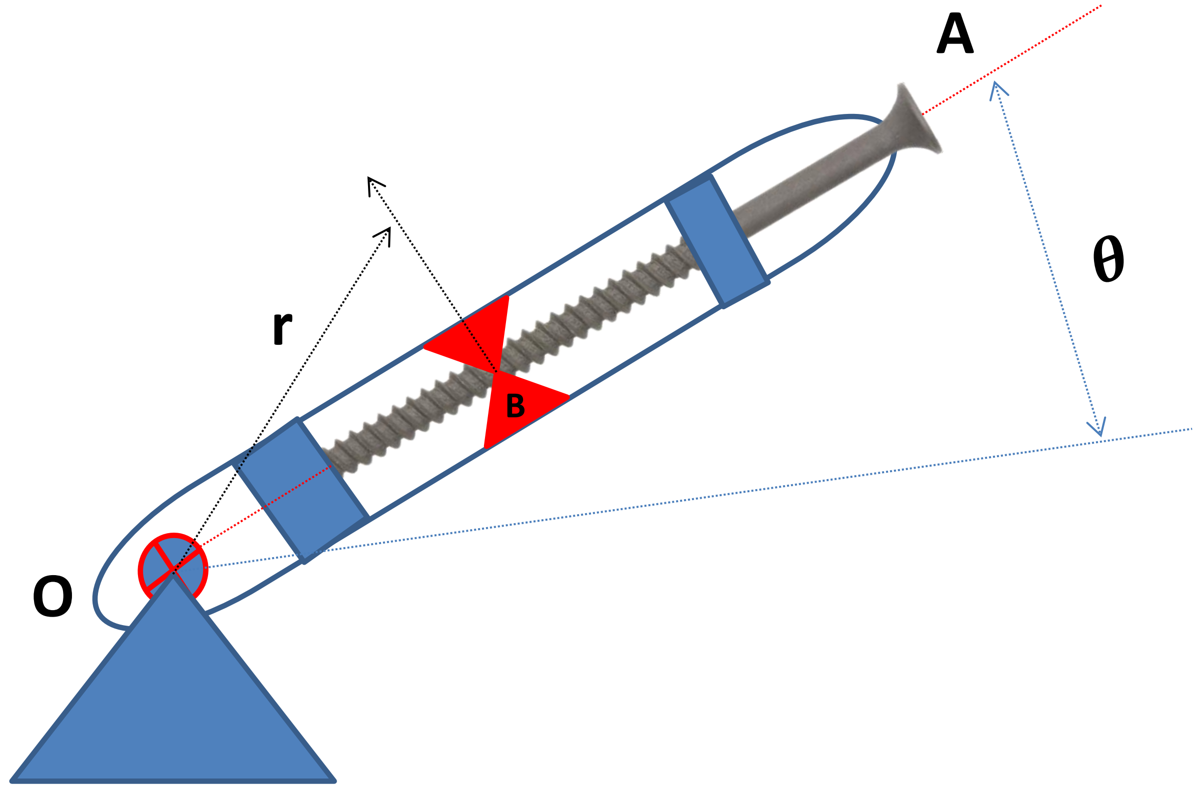
Centripetal acceleration

Tangential acceleration

Coriolis acceleration

a= $\sqrt{(a_r^2 + a_\theta^2)}$

Example



Rotation of the radially slotted arm is governed by $\theta=0.2\,t + 0.02\,t^3$, where θ is in radians and t is in seconds. Simultaneously, the power screw in the arm engages the slider B and controls its distance from O according to $r = 0.2 + 0.04\,t^2$, where r is in meters and t is in seconds. Calculate the magnitudes of the velocity and acceleration of the slider for the instant when t = 3 s.

Hints:

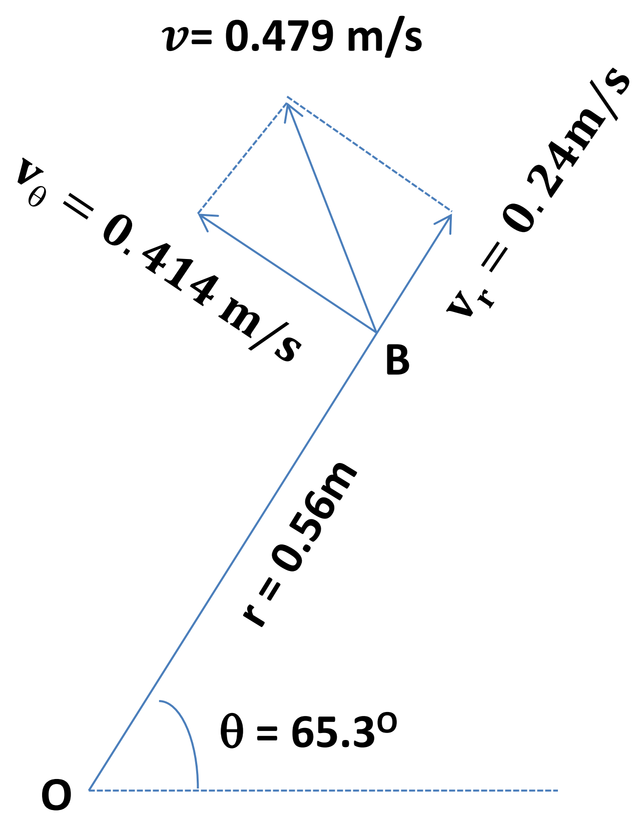
The coordinates (r, θ) and their time derivatives used in velocity and acceleration formulas

$r = 0.2t + 0.04\,t^2\ ;\ \dot{r} = 0.08t\ ;\ \ddot{r} = 0.08$

At t = 3s, r = 0.56m; $\dot{r} = 0.24m/s$; $\ddot{r} = 0.08\frac{m}{s^2}$

$\theta = 0.2\,t + 0.02\,t^3$; $\dot{\theta} = 0.2 + 0.06t^2\ ;\ \ddot{\theta} = 0.12t$

At t = 3s, θ = 65.3°; $\dot{\theta} = 0.74\,rad/s$; $\ddot{\theta} = 0.36\frac{rad}{s^2}$



The velocity components

$$v_r = \dot{r} = 0.24m/s$$

$$v_\theta = r\dot{\theta} = 0.414\,m/s$$

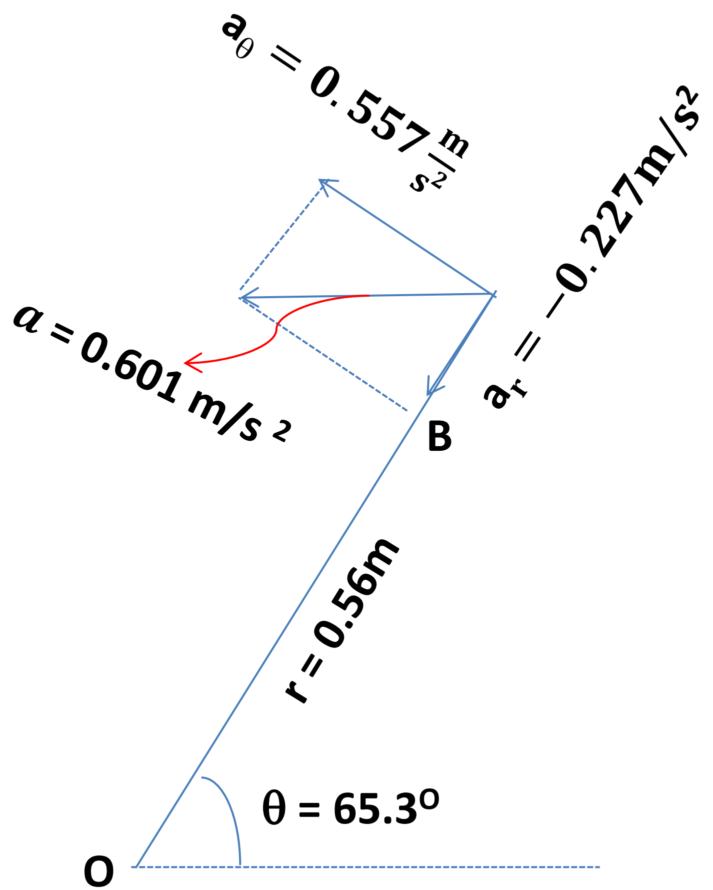
$$v = \sqrt{(v_r^2 + v_\theta^2)} = 0.479\text{ m/s}$$

The acceleration components at t=3 sec

$$a_r = (\ddot{r} - r\dot{\theta}^2) = -0.227\frac{m}{s^2}$$

$$a = (r\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}\,) = 0.557\frac{m}{s^2}$$

$$a = \sqrt{(a_r^2 + a_\theta^2)} = 0.601\frac{m}{s^2}$$

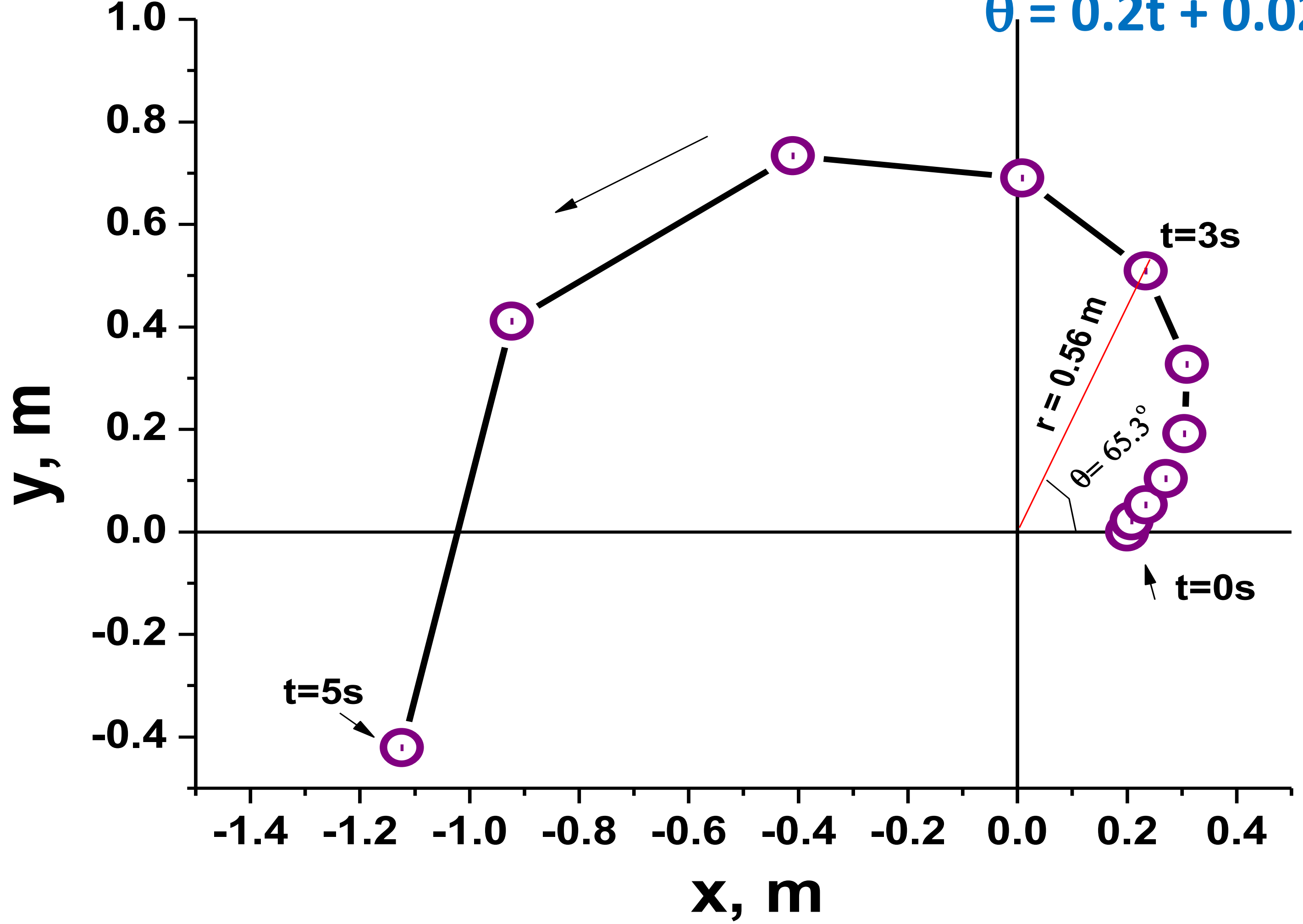


Sketch the motion of slider B over time interval $0 \le t \le 5s$

Using $x = r\cos\theta$, $y = r\sin\theta$ and

$r = 0.2 + 0.04\,t^2$

$\theta = 0.2t + 0.02\,t^3$



Thank you