PH101
Lecture 2

05.08.14

What does Mechanics deal with?

Deals with concepts like velocity, momentum, angular momentum, energy, etc.

Is important in everyday physical world as well as in the understanding of phenomena on the atomic and cosmic scales.

e.g. *Heat* is governed by motion and hence K.E. of atoms – higher the temperature of a body, greater is the K.E. of its atoms

Mathematics helps in expressing complex ideas in a concise manner

Vectors is a good example of how useful is Mathematics in the study of Physics

discovered by a physicist Willard Gibbs of Yale University in USA

In the 19th century notation, Newton's second law :

$$F_i = ma_i; i = x, y, z$$

In vector notation,

$$\vec{F} = m\vec{a}$$

Vectors simplify the form of equations

Study of motion involves quantities like displacement

Speed	Mass	Power
Velocity	Momentum	Work
Acceleration	Energy	.
		.
		.

Scalar or vector

Examples of vectors:

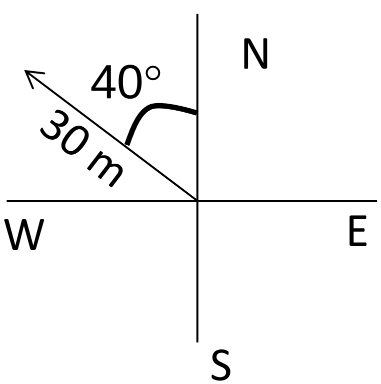
Displacement

Velocity

Acceleration

A Christmas gift is placed outside the class room. To find it displace yourself by 30 m

30 m at 40° west of north w.r.t the center of the main exit door



1cm ≡ 5m

- Likewise, velocity of a vehicle of say 60 km/h is incomplete unless stated with the direction along which it is moving e.g. south because velocity is a vector quantity.

- A vector is specified by both its magnitude and direction

- Vectors **B** & **C** are equal if their length & direction are same

- Magnitude of a vector is indicated as

$$\left|\vec{A}\right|$$

Unit vector: $$\hat{A} = \frac{\vec{A}}{\left|\vec{A}\right|} \Rightarrow \vec{A} = \left|\vec{A}\right|\hat{A}$$

Multiplication of a vector by a scalar:

$$b\vec{A} = \vec{C}$$

Direction & length of ***C*** ?

Parallel to ***A*** & *b* times greater

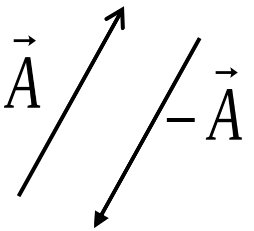
$$\left|\vec{C}\right| = b\left|\vec{A}\right|$$

$$\Rightarrow$$

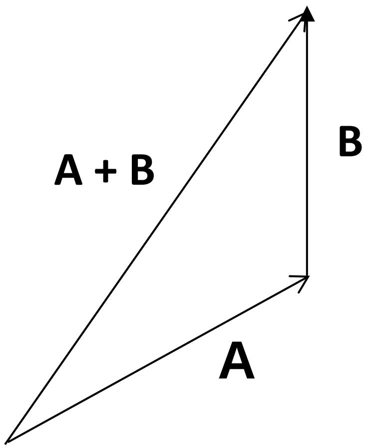
Unit vector $\hat{C}$?

Multiply $\vec{A}$ by -1 ?

A new vector antiparallel to original $\vec{A}$

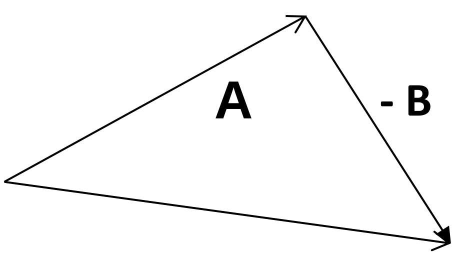


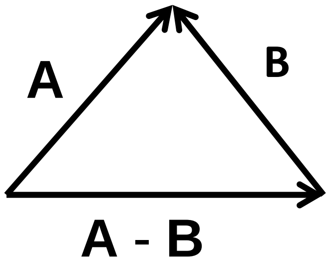
A + **B** ?



A - **B** ?

A - B = A + (- B)





A + (- B) = A - B

Laws in vector algebra

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

Commutative law

$$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$$

$$c\left(d\vec{A}\right)=\left(cd\right)\vec{A}$$

Associative law

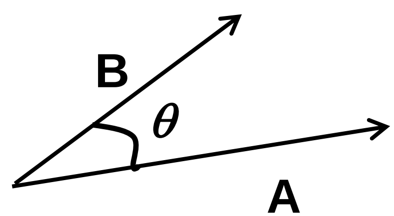
$$\left(c+d\right)\vec{A}=c\vec{A}+d\vec{A}$$

$$c\left(\vec{A}+\vec{B}\right)=c\vec{A}+\vec{c}B$$

Distributive law

Scalar or dot product

$$\vec{A}.\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$



Since $\left|\vec{B}\right|\cos\theta$: projection of **B** along **A** $\Rightarrow \vec{A}.\vec{B}=\left|\vec{A}\right|\times$ Projection **B** on **A**

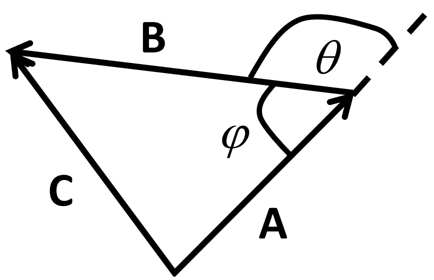
If
$$\vec{A}.\vec{B} = 0$$

- Either $\left|\vec{\mathrm{A}}\right| = 0$ or $\left|\vec{B}\right| = 0$

  - **A** is perpendicular to **B** $\Rightarrow \theta = \pi/2$
  - Dot product of two non-zero vectors could be 0!
  - **A.A** = A^2

<u>Cosine law</u>

For $\vec{C} = \vec{A} + \vec{B}$

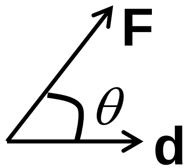


$$\vec{C}.\vec{C} = \left|\vec{C}\right|^2 = \left(\vec{A} + \vec{B}\right).\left(\vec{A} + \vec{B}\right)$$

$$\left|\vec{C}\right|^2 = \left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 + 2\left|A\right|\left|\vec{B}\right|\cos\theta \qquad C^2 = A^2 + B^2 - 2AB\cos\phi$$

Example:

Work done in the form of a dot product

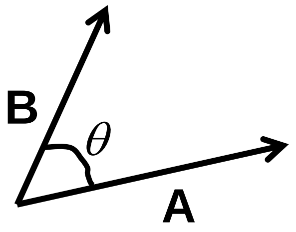


$$W = \left(F\cos\theta\right)d$$

$$W = \vec{F}.\vec{d}$$

Cross product/Vector product

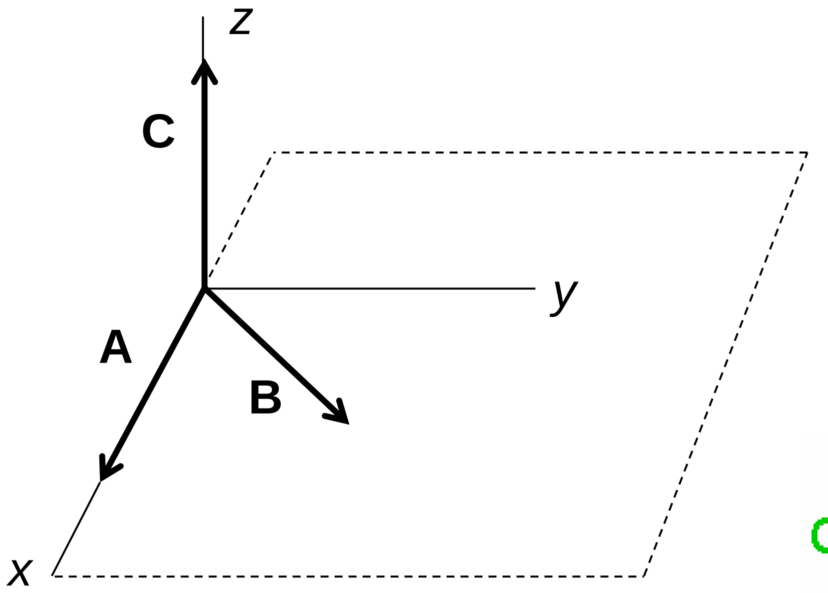
$$\vec{C} = \vec{A}\times\vec{B}$$

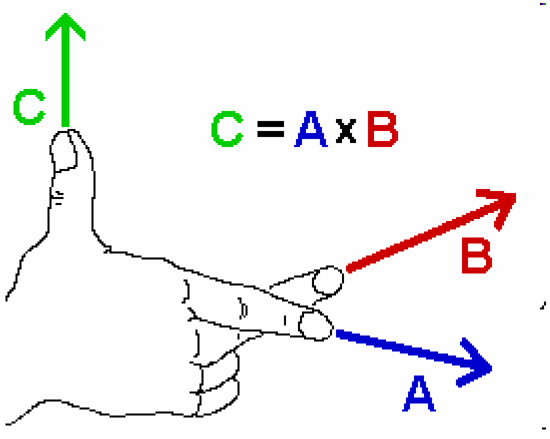


$$\left|\vec{C}\right| = \left|\left|\vec{A}\right|\left|\vec{B}\right|\right|\sin\theta$$

For θ = 0, $\left|\vec{C}\right| = 0$ even if $\left|\vec{A}\right|, \left|\vec{B}\right| \neq 0$

C is normal to the plane containing **A** and **B**





A, **B**, **C** form a triple

Right handed system:

e.g. right hand screw with the axis perpendicular to **A** & **B**

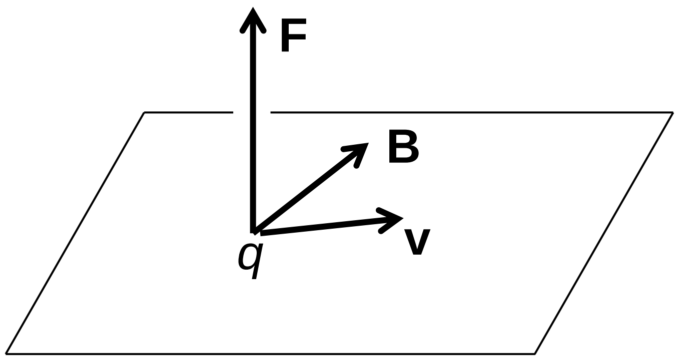
Vector product is anti-commutative:

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$$

Role of vector products in real world Physic:

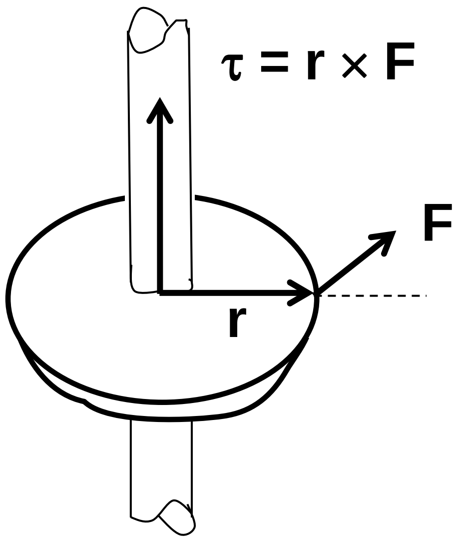
Lorentz force:

A moving (v) charged particle of charge q gets deflected in a magnetic field B due to the force:



$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Torque:



$\tau = \mathbf{r} \times \mathbf{F}$ Along the axis of rotation

r is a vector from the axis about which $\boldsymbol{\tau}$ is applied

Component of **F** perpendicular to **r** produces torque

Torque is a measure of an applied force to produce a twist

Vector area:

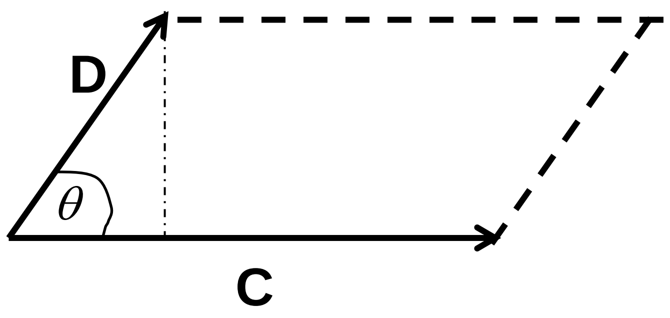
X product can be used to describe area

Example

Rate at which water in a stream flows through a wire loop of given area

Is the loop parallel or perpendicular to the flow?

Consider area of a parallelogram formed by **C** & **D**:



Area = base x height: $$A = C\,D\sin\theta$$

$$= \left|\vec{C}\times\vec{D}\right|$$

$$\vec{A} = \vec{C}\times\vec{D}$$

Direction is somewhat arbitrary

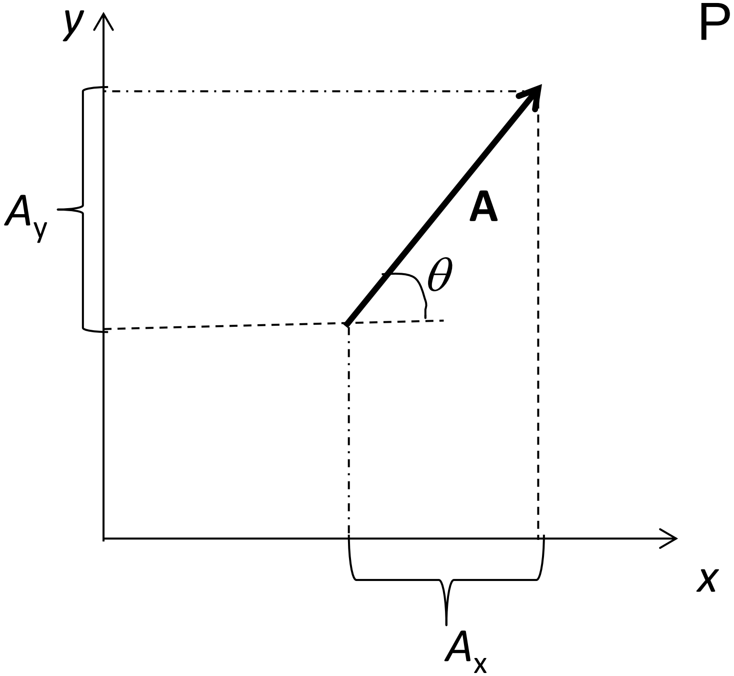
This vector product assigns the direction to the area!

Components of a vector:

A has a meaning independent of a coordinate system

Due to which vectors are so useful

But components of **A** depends on the coordinate system!



Projections of **A** along the two coordinates:

A_x & A_y $\Rightarrow$ $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

$$\left|\vec{A}\right| = \sqrt{\left(A_x^2 + A_y^2\right)}$$

In 3-D

$$\left|\vec{A}\right| = \sqrt{\left(A_x^2 + A_y^2 + A_z^2\right)}$$