EE101 Assignment-3 Solutions.

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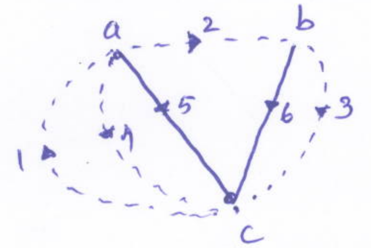


Proper tree, oriented graph.

Node Analysis

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
A= \begin{array}{c} a \\ b \end{array} & \begin{bmatrix} -1 & 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

Step 1:

$$Ai_b = 0$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix} = 0$$

But $i_{b_1} = I_{s_1}$. So taking it separate

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{s_1} \quad -(1)$$

Step 2:

$$\begin{bmatrix} i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_2}D^{-1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_4} & 0 & 0 \\ 0 & 0 & 0 & C_5 D & 0 \\ 0 & 0 & 0 & 0 & C_6 D \end{bmatrix} \begin{bmatrix} V_{b_2} \\ V_{b_3} \\ V_{b_4} \\ V_{b_5} \\ V_{b_6} \end{bmatrix}$$

where $D \Rightarrow \frac{d}{dt}$

$D^{-1} \Rightarrow \int dt$.

Step : 3

$$V_b = A^T \cdot V_n$$

$$\begin{bmatrix} V_{b_1} \\ V_{b_2} \\ V_{b_3} \\ V_{b_4} \\ V_{b_5} \\ V_{b_6} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}$$

V_{b_1} is the voltage across current source. Hence we need to express it in terms of other node voltages. From the matrix

$$\boxed{V_{b_1} = -V_a}$$

$$\begin{bmatrix} V_{b_2} \\ V_{b_3} \\ V_{b_4} \\ V_{b_5} \\ V_{b_6} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}$$

Step-4

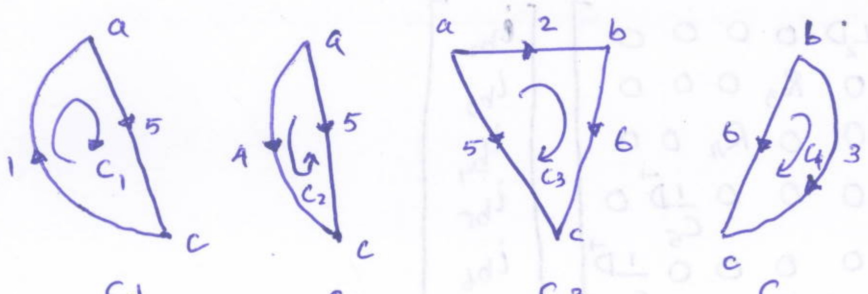
Substituting in (1)

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{L_2}D^{-1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_4} & 0 & 0 \\ 0 & 0 & 0 & C_5D & 0 \\ 0 & 0 & 0 & 0 & C_6D \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{s_1}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{L_2}D^{-1} & 0 & \frac{1}{R_4} & C_5D & 0 \\ -\frac{1}{L_2}D^{-1} & \frac{1}{R_3} & 0 & 0 & C_6D \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{s_1}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{L_2}D^{-1} + \frac{1}{R_4} + C_5D & -\frac{1}{L_2}D^{-1} \\ -\frac{1}{L_2}D^{-1} & \frac{1D^{-1}}{L_2} + C_6D + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{s_1}$$

Mesh Analysis



C1 C2 C3 C_4.

Step 1.

$$B = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline C1 & 1 & 0 & 0 & 0 & 1 & 0 \\ C2 & 0 & 0 & 0 & 1 & -1 & 0 \\ C3 & 0 & 1 & 0 & 0 & -1 & 1 \\ C4 & 0 & 0 & 1 & 0 & 0 & -1 \end{array}$$

$$BV_b = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_{b_1} \\ V_{b_2} \\ V_{b_3} \\ V_{b_4} \\ V_{b_5} \\ V_{b_6} \end{bmatrix} = 0.$$

Now V_{b_1} is a current source. Since we don't have the V-I relation for it, we need to eliminate it. From the matrix

$$V_{b_1} + V_{b_5} = 0$$

or $\boxed{V_{b_1} = -V_{b_5}}$

Now we can remove first row or B matrix. Now the first colum is all zeros, which enables us to remove the first column along with V_{b_1}.

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_{b_2} \\ V_{b_3} \\ V_{b_4} \\ V_{b_5} \\ V_{b_6} \end{bmatrix} = 0. \quad \text{— (1)}$$

Step-2.

$$\begin{bmatrix} v_{b_2} \\ v_{b_3} \\ v_{b_4} \\ v_{b_5} \\ v_{b_6} \end{bmatrix} = \begin{bmatrix} L_2D & 0 & 0 & 0 & 0 \\ 0 & R_3 & 0 & 0 & 0 \\ 0 & 0 & R_4 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_5}D^{-1} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{C_6}D^{-1} \end{bmatrix} \begin{bmatrix} i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix}$$

Step-3.

$$I_b = B^T I_m$$

$$\begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_{m_1} \\ I_{m_2} \\ I_{m_3} \\ I_{m_4} \end{bmatrix}$$

But $I_{m_1} = I_{s_1}$

$$\Rightarrow \boxed{I_{b_1} = I_{s_1}}$$

$$\begin{bmatrix} i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} I_{s_1}$$

Step-4

Now substituting in (1).

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} L_2D & 0 & 0 & 0 & 0 \\ 0 & R_3 & 0 & 0 & 0 \\ 0 & 0 & R_4 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_5}D^{-1} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{C_6}D^{-1} \end{bmatrix} \left[\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} I \right.$$

$$\begin{bmatrix} 0 & 0 & R_4 & -\frac{1}{C_5}D^{-1} & 0 \\ L_2D & 0 & 0 & -\frac{1}{C_5}D^{-1} & \frac{1}{C_6}D^{-1} \\ 0 & R_3 & 0 & 0 & -\frac{1}{C_6}D^{-1} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & 0 & R_4 & -\frac{1}{C_5}D^{-1} & 0 \\ L_2D & 0 & 0 & -\frac{1}{C_5}D^{-1} & \frac{1}{C_6}D^{-1} \\ 0 & R_3 & 0 & 0 & -\frac{1}{C_6}D^{-1} \end{bmatrix}}^{=0} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$=D$

$$\begin{bmatrix} R_4+\frac{1}{C_5}D^{-1} & \frac{1}{C_5}D^{-1} & 0 \\ \frac{1}{C_5}D^{-1} & L_2D+\frac{1}{C_5}D^{-1}+\frac{1}{C_6}D^{-1} & -\frac{1}{C_6}D^{-1} \\ 0 & -\frac{1}{C_6}D^{-1} & R_3+\frac{1}{C_6}D^{-1} \end{bmatrix} \begin{bmatrix} i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{C_5}D^{-1} \\ 0 \end{bmatrix} I_s = 0 \therefore$$

$$\begin{bmatrix} R_4+\frac{1}{C_5}D^{-1} & \frac{1}{C_5}D^{-1} & 0 \\ \frac{1}{C_5}D^{-1} & L_2D+\frac{1}{C_5}D^{-1}+\frac{1}{C_6}D^{-1} & -\frac{1}{C_6}D^{-1} \\ 0 & -\frac{1}{C_6}D^{-1} & R_3+\frac{1}{C_6}D^{-1} \end{bmatrix} \begin{bmatrix} i_{m_2} \\ i_{m_3} \\ i_{m_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{C_5}D^{-1} \\ 0 \end{bmatrix} I_s.$$

(2). This question you can solve in the same way. Let's try applying KCL and KVL, whe-ther we can get the same answer.

1. Node analysis.

Applying KCL at node 'a'.

$$-I_{s_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} + \frac{d(V_a - V_c)}{5\,dt} = 0$$

But V_c is given as KV_b.

$$-I_{s_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} + C_5 D(V_a - KV_b) = 0. \quad —(1)$$

$D \Rightarrow \frac{d}{dt}$

Applying KCL at node 'b'

$$-\frac{(V_a - V_b)}{R_3} + CDV_b = 0 \quad —(2)$$

At node c, we alredy know $V_c = KV_b$. $—(3)$.

Now writing equations 1 and 2 in matrix format

$$\begin{bmatrix} \frac{1}{R_2}+\frac{1}{R_3}+C_5D & \frac{-1}{R_3}-KC_5D \\ \frac{-1}{R_3} & \frac{1}{R_3}+CD \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{s_1}$$

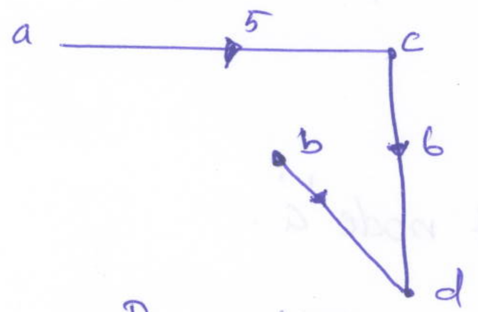
$$V_c = KV_b$$

This is the required solution.

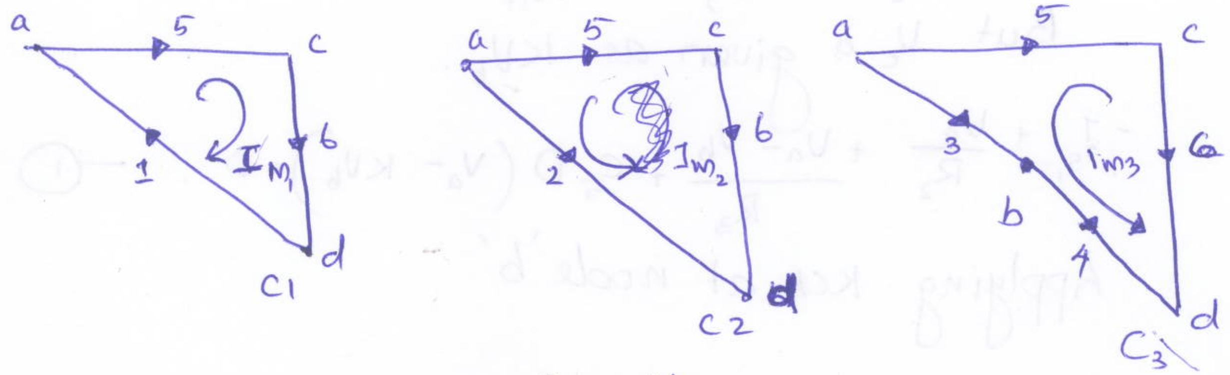
See the Matrix is asymmetric. Check Q1 answer. It is symmetric.

Mesh Analysis

Again we try with KVL to see whether we get the same answer.



Proper tree.



Circuits.

Now writing KVL for each circuit,

From C1, $I_{m_1} = I_{s_1}$ — (1)

From C_2,

$$\cancel{-\frac{1}{C_5}D^{-1}I_{s_1} + \frac{1}{C_5}D^{-1}I_{m_3} + I_{m_2}R_2 - KV_b - \frac{1}{C_5}D^{-1}I_{m_2} = 0}$$

(6)

$$-\frac{1}{C_5}D^{-1}\left[I_{s_1} - I_{m_2} - I_{m_3}\right] + I_{m_2}R_2 - e_{s_6} = 0$$

$$e_{s_6} = KV_b.$$

But $V_b = \frac{1}{C_4}D^{-1}I_{m_3}$ (from C_3)

$$-\frac{1}{C_5}D^{-1}I_{s_1} + \frac{1}{C_5}D^{-1}I_{m_2} + \frac{1}{C_5}D^{-1}I_{m_3} + I_{m_2}R_2 - \frac{1}{C_4}D^{-1}I_{m_3} = 0 \quad -(2)$$

From C_3

$$I_{m_3}R_3 - \frac{1}{C_4}D^{-1}I_{m_3} - K\frac{1}{C_4}D^{-1}I_{m_3} - \frac{1}{C_5}D^{-1}\left(I_{s_1} - I_{m_2} - I_{m_3}\right) = 0 \quad -(3)$$

Writing equations (2) and (3) in matrix format

$$\begin{bmatrix} \frac{1}{C_5}D^{-1} + R_2 & \frac{1}{C_5}D^{-1} - \frac{1}{C_4}D^{-1} \\ \frac{1}{C_5}D^{-1} & R_3 - \frac{1}{C_4}D^{-1} - \frac{KD^{-1}}{C_4} + \frac{1}{C_5}D^{-1} \end{bmatrix} \begin{bmatrix} I_{m_2} \\ I_{m_3} \end{bmatrix} = \begin{bmatrix} +\frac{1}{C_5}D^{-1} \\ \frac{1}{C_5}D^{-1} \end{bmatrix} I_{s_1}$$

$$I_{m_1} = I_{s_1}$$