NB : this document sums up some important techniques you should already be familiar with!

Trigonometry

Solve a trigonometric equation To solve a simple trigonometric equation, you can use one of these equivalences:

$$\begin{aligned}
\cos x = \cos a &\Leftrightarrow x \equiv a[2\pi] \text{ or } x \equiv -a[2\pi], \\
\sin x = \sin a &\Leftrightarrow x \equiv a[2\pi] \text{ or } x \equiv \pi - a[2\pi], \\
\tan x = \tan a &\Leftrightarrow x \equiv a[2\pi].
\end{aligned}$$

To solve a trigonometric equation of the form $a\cos x + b\sin x = c$, with $a, b, c \in \mathbb{R}$, such that $(a, b) \neq (0, 0)$, simplify it with this trick: there exists $\theta \in \mathbb{R}$ such that $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ and $\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$. Therefore, $a\cos x + b\sin x = c \Leftrightarrow \cos(x-\theta) = \frac{c}{\sqrt{a^2+b^2}}$.

Compute a trigonometric sum To compute a sum of trigonometric functions (circular or hyperbolic), we usually express them as exponentials (real, or pure imaginary), then we apply the binomial theorem or geometric identity, to finally come back to trigonometric notation by taking the real (or pair, or impair) part.

Compute with the reciprocal hyperbolic or trigonometric functions

To compute with the reciprocal hyperbolic or trigonometric functions, one can use the logarithmic expressions.

However, you should be cautions about the fact that the reciprocal trigonometric functions, $\arccos(x)$, $\arcsin(x)$ and $\arctan(x)$, are *peculiar* antecedents of x for the functions cos, sin and tan. Therefore, remember to specify its unique value among all the possibles antecedents of x, by localize them:

To show that $t = \arccos(x)$:

(i) verify that $\cos(t) = x$,

(ii) localize $t \in [-\pi/2, \pi/2]$.

To show that $t = \arcsin(x)$:

(i) verify that $\sin(t) = x$,

(ii) localize $t \in [0, \pi]$.

To show that $t = \arctan(x)$:

(i) verify that $\tan(t) = x$,

(ii) localize $t \in (-\pi/2, \pi/2)$.

Handle the “usual” functions

How to simplify an expression or prove an equality? Three basic approaches:

- use an appropriate variable transformation,
- differentiate and integrate (after **checking the differentiability**),
- use a logarithmic expression (if there is reciprocal hyperbolic functions).

How to resolve an equation? Three basic approaches: (after rewriting the equation as $f(x) = 0$):

- study the function f, to prove the uniqueness of the solution, and try to compute it,
- reason by *analysis and synthesis*. Assuming that x is a solution, you can try to transform the equation into a polynomial equation on x.