NB : This document sums up as much symbols, notations and conventions used for the Maths 101 course as possible. If something is missing, please tell us.

Shortcuts

i.e. : *id est*[1], "that is",

e.g. : *exempli gratia*[1], "for example" (does **not** come from "example given").

Basic notations for functions

$f(x)$: the *value* of the function f at the point x,

$f(A)$: the *image set* of A by f, *i.e.* $\{f(x) : x \in A\}$,

$f(B)$: the *pre-image set* of B by f, *i.e.* $\{y \in B : \exists x \in A / f(x) = y\}$,

$\to$: to specify the domain for a function, *e.g.* $\sin : \mathbb{R} \to \mathbb{R}$,

$f_{|A}$: the restriction of the function $f : E \to F$ to the subset $A \subset E$.

Shortcuts for sums and products

$\sum_{k=1}^{n} f(k)$: sum $f(1) + f(2) + \cdots + f(n-1) + f(n)$ for $n \geqslant 1$,

$\prod_{k=1}^{n} g(k)$: product $g(1) \times g(2) \times \cdots \times g(n-1) \times g(n)$ for $n \geqslant 1$,

$\bigtimes_{k=1}^{n} E_k$: Cartesian product for spaces : $E_1 \times E_2 \times \cdots \times E_{n-1} \times E_n$ for $n \geqslant 1$,

$\bigoplus_{k=1}^{n} E_k$: direct sum for spaces : $E_1 \oplus E_2 \oplus \cdots \oplus E_{n-1} \oplus E_n$ for $n \geqslant 1$.

Logical binary predicates

$\Rightarrow$: implies, *e.g.* $(x > 0) \Rightarrow (x \geqslant 0)$,

$\Leftrightarrow$: is equivalent to, *i.e.* implies and is implied by,

$\neg$: logical negation, *e.g.* $\neg(\omega > 0) \equiv \omega \leqslant 0$.

Binary predicates for set

$\in$: belongs to, *e.g.* $n \in \mathbb{N}$,

$\notin$: do not belong to, *e.g.* $-1 \notin \mathbb{R}_+$,

$\subset$: is included in, *e.g.* $\mathbb{N} \subset \mathbb{R}$,

$\supset$: contains, *e.g.* $\mathbb{Z} \supset \{-1, 0, 1\}$,

$\subseteq$: to precise that inclusion can be an equality, at the opposite of $\subsetneq$ which exclude the equality,

$\setminus$: difference of two sets, *e.g.* $\mathbb{R} \setminus \mathbb{R}_- = \mathbb{R}_+^*$,

$\cap$: intersection of two sets (also $\bigcap$), *e.g.* $\mathbb{C} \cap \mathbb{Z} = \mathbb{Z}$,

$\cup$: union of two sets (also $\bigcup$), *e.g.* $\{0\} \cup \mathbb{N}^* = \mathbb{N}$,

[1] It is latin.

$\uplus$: disjunct union of two sets (also $\biguplus$), *e.g.* $\mathbb{R}_-^* \uplus \mathbb{R}_+^*$ (also $\sqcup$ or $\bigsqcup$),

$E_1 \times E_2$: Cartesian product of two sets E_1 and E_2, *i.e.* $\{(x, y) : x \in E_1, y \in E_2\}$,

$E_1 \oplus E_n$: direct sum of two sets $E_1, E_2 \subset E$, *i.e.* $\{x + y : x \in E_1, y \in E_2\}$, where $E_1 \cap E_2 = \{0_E\}$.

Usual constants and functions

π : (pi), usually defined by the quotient of any circle perimeter by its diameter,

e : base of the exponential function, *i.e.* $\mathrm{e} \overset{\text{def}}{=} \mathrm{e}^1 = \exp(1)$,

$+\infty$: positive infinite, $-\infty$ negative infinite, ∞ infinite,

sign : sign function, *i.e.* $\mathrm{sign} : \mathbb{R} \to \mathbb{R}, 0 \mapsto 0, x \mapsto 1$ if $x > 0, -1$ if $x < 0$,

exp : exponential function, *i.e.* $\exp : \mathbb{R} \to \mathbb{R}, x \mapsto \exp(x) = \mathrm{e}^x$,

ln : Neperian logarithm function, *i.e.* $\ln : \mathbb{R}_+^* \to \mathbb{R}, x \mapsto \ln(x)$,

cos : cosine, sin : sinus, tan : tangent ($\mathbb{R} \to \mathbb{R}$),

arccos : arc cosine, arcsin : arc sinus, arctan : arc tangent (reciprocal),

cosh : hyperbolic cosine, sinh : hyperbolic sinus, tanh : hyperbolic tangent ($\mathbb{R} \to \mathbb{R}$),

arccosh : arc hyperbolic cosine, arcsinh : arc hyperbolic sinus, arctanh : arc hyperbolic tangent (reciprocal),

det : determinant, com : comatrix,

E : integer part (lower), *i.e.* $\mathrm{E}(x) \overset{\text{def}}{=} \max\{n \in \mathbb{Z}, n \leqslant x\}$,

floor : lower integer round, *i.e.* $\mathrm{floor}(x) \overset{\text{def}}{=} \lfloor x \rfloor = \mathrm{E}(x)$,

ceil : upper integer round, *i.e.* $\mathrm{ceil}(x) \overset{\text{def}}{=} \lceil x \rceil = \mathrm{E}(x) + 1$,

$\delta_{i,j}$: KRONECKER's symbol, $\delta_{i,j} : (k, l) \mapsto 1$ if $k = i$ and$l = j, \mapsto 0$ otherwise,

γ : usually, EULER's constant,

$\Gamma(x)$: usually, EULER's Gamma function,

I_n : identity matrix of size $n \geqslant 0$.

Usual sets

$\varnothing$: *the* empty set (it is unique, even if included in any set),

$\{x : P(x)\}$: set of all elements x satisfying verifying a certain property[2] $P(x)$, *e.g.* $\{x \in \mathbb{R} : x \geqslant 0\} = \mathbb{R}_+$,

$\mathbb{N}$: non-negative natural numbers, $\mathbb{N} = \{0, 1, \dots, n-1, n, n+1, \dots\}$,

$\mathbb{Z}$: (ring of the) integer numbers, $\mathbb{Z} = \{\dots, -n-1, -n, -n+1, \dots, -1, 0, 1, \dots, n-1, n, n+1, \dots\}$,

$\mathbb{Q}$: (field of the) rational numbers, $\mathbb{Q} \overset{\text{def}}{=} \left\{\frac{p}{q}, (p, q) \in \mathbb{Z} \times \mathbb{N}^*\right\}$,

$\mathbb{R}$: (field of the) real numbers, $\mathbb{R} \overset{\text{def}}{=} \left\{\lim_{n\to\infty} r_n, (r_n)_{n\in\mathbb{N}} \in \mathbb{Q}^{\mathbb{N}} \text{ being convergeant}\right\}$,

- $[a, b]$: segment (close interval), *i.e.* $[a, b] \overset{\text{def}}{=} \{x \in \mathbb{R} : a \leqslant x \leqslant b\}$,
- (a, b) : open interval, *i.e.* $(a, b) \overset{\text{def}}{=} \{x \in \mathbb{R} : a < x < b\}$,
- $[a, b)$: half-open on the right interval, *i.e.* $[a, b) \overset{\text{def}}{=} \{x \in \mathbb{R} : a \leqslant x < b\}$,
- $(a, b]$: half-open on the left interval, *i.e.* $(a, b] \overset{\text{def}}{=} \{x \in \mathbb{R} : a < x \leqslant b\}$,

[2] If you are curious, this page on Wikipédia explains *how* and *why* we have to limit the expressiveness of this way to define set.

$\mathbb{C}$: (field of the) complex numbers, $\mathbb{C} \stackrel{\text{def}}{=} \{a + ib : a, b \in \mathbb{R}\}$,

$\mathbb{K}$: any (commutative) field, usually understood as "$\mathbb{R}$ or $\mathbb{C}$",

E^n : product set, set of the n-tuples of E, *i.e.* $\{(x_1, \dots, x_n) : \forall 1 \leqslant i \leqslant n, x_i \in E\}$, *e.g.* $\mathbb{R}^2 \simeq \mathbb{C}$,

$\mathbb{M}_{n,m}(A)$: set of $n \times m$-sized matrices ($n, m \in \mathbb{N}$), also written as $M_{n,m}(A)$,

Operators for sets

Card : cardinal of a set, *i.e.* number of its elements, also written $\mathrm{Card}(E) \stackrel{\text{def}}{=} |E| \stackrel{\text{def}}{=} \#E$,

$\mathcal{P}(E)$: set of subsets of E (also written 2^E),

E^* : the set E without its zero[3] 0, *i.e.* $E^* \stackrel{\text{def}}{=} E \backslash \{0\}$,

E_+ : the subset of non-negative[3] elements of E, *i.e.* $E_+ \stackrel{\text{def}}{=} E \cap [0, +\infty)$,

E_- : the subset of non-positive[3] elements of E, *i.e.* $E_- \stackrel{\text{def}}{=} E \cap (-\infty, 0]$,

E^c : complementary set of E (**when this is not ambiguous**),

δE : frontier of the set E, *e.g.* $\delta \mathcal{B}_2(0,1) = \mathcal{C}_2(0,1)$ (ball and sphere being centered at 0 and of radii 1, in 2D),

E° : interior of the set E, *i.e.* $E^\circ \stackrel{\text{def}}{=} E \backslash \delta E$ (also written $\mathrm{int}(E)$),

$\overline{E}$: cloture of E, *i.e.* $\overline{E} \stackrel{\text{def}}{=} \left\{ \lim\limits_{n \to +\infty} u_n, (u_n)_{n \in \mathbb{N}} \in E^{\mathbb{N}} \text{ being convergent} \right\}$,

$d(x, y)$: distance between two points x and y, usually $d(x,y) = |x - y|$,

$d(x, A)$: distance from the point x to the subset A ($\stackrel{\text{def}}{=} \inf\limits_{a \in A} |x - a|$),

$d(A, B)$: distance from two subsets A and B ($\stackrel{\text{def}}{=} \inf\limits_{(a,b) \in A \times B} |a - b|$),

$A + B$: sum of two subsets A and B ($= \{a + b : (a, b) \in A \times B\}$),

$A - B$: subtraction of two subsets A and B ($= \{a - b : (a, b) \in A \times B\}$),

$\mathrm{diam}(A)$: diameter of a subset A, *i.e.* $\mathrm{diam}(A) \stackrel{\text{def}}{=} \sup \left\{ |x - y|, (x, y) \in A^2 \right\}$,

χ_A : characteristic function for $A \subset E$, *i.e.* $E \to \{0, 1\}, x \mapsto 1$ if $x \in A, 0$ otherwise,

$\dim(E)$: dimension of a space E, $\dim_{\mathbb{K}}(A)]$: dimension of a subset $A \subset E$, has a $\mathbb{K}$-vectorial-space.

Orders and relationships

$<$: (strictly) lower than, *e.g.* $1 < \sqrt{2}$,

$\leqslant$: lower or equal than, *e.g.* $1 \leqslant 1$,

$>$: (strictly) greater than, *e.g.* $1 > \mathrm{e}^{-1}$,

$\geqslant$: greater or equal than, *e.g.* $2 \geqslant 2$,

$\equiv$: "same meaning", or semantical identity for formulas : $\neg(\neg P) \equiv P$,

$\simeq$: similar, *e.g.* for numerical approximations : $\pi \simeq 3.14$,

$\sim$: numerically equivalent, for a limit, *e.g.* $\dfrac{\sin(7x)}{x} \underset{x \to 0, x \neq 0}{\sim} 7$.

[3] When this as a meaning.

Other binary operators

$\circ$: composition of functions, if $f : E \to F, g : F \to G$, then $f \circ g : E \to G, x \mapsto f(g(x))$,

$\mathbf{x}.\mathbf{y}$: inner product, also written $\langle \mathbf{x}, \mathbf{y} \rangle$, *e.g.* $[1;2;3].[4;5;6] = 1*4+2*5+3*6 = 32$,

$\mathbf{x} \times \mathbf{y}$: vectorial product, *e.g.* $[1;2;3] \times [4;5;6] = [-3;6;-6]$,

$\mid$: divisibility (for integers, or polynomials), *e.g.* $1007 \mid 2014$,

$\nmid$: non-divisibility (for integers, or polynomials), *e.g.* $2013 \nmid 2014$,

$. \mod .$: used to write equality *modulo* another number, *e.g.* $17 = 2 \mod 5$.

Usual unary operators

$\mathcal{R}e$: real part for a complex number, *i.e.* $\mathcal{R}e(a + ib) = a$,

$\mathcal{I}m$: imaginary part for a complex number, *i.e.* $\mathcal{I}m(a + ib) = b$,

$\sup$: best upper-bound of a set (smaller upper-bound),

$\inf$: best lower-bound of a set (bigger lower-bound),

$\lim$: limit, usually by precising $\lim\limits_{x\to 0}$ to say "when x tends to 0",

$\limsup$: upper limit (also written $\overline{\lim}$, or sometimes $\lim \uparrow$),

$\liminf$: lower limit (also written $\underline{\lim}$, or sometimes $\lim \downarrow$),

$\arg$: **one** argument (for a complex number), *e.g.* $\arg(\mathrm{e}^{i\pi/4}) = \pi/4$ but $\pi/4 + 2\pi$ is also a valid argument.

Notations for the limits

$\to$: for a limit : $n \to +\infty$ means that "n tends to $+\infty$",

$x \to a^+$: means that x tends to a **by being lower than** a,

$x \to a^-$: means that x tends to a **by being greater than** a,

$f(x^+)$: limit of f at x **for smaller values**, *i.e.* $f(x^+) \overset{\text{def}}{=} \lim\limits_{y\to x^+} f(y)$,

$f(x^-)$: limit of f at x **for bigger values**, *i.e.* $f(x^-) \overset{\text{def}}{=} \lim\limits_{y\to x^-} f(y)$.

Autres notations

g^+ : non-negative part of g, *i.e.* $g^+ \overset{\text{def}}{=} \max\{g(x), 0\}$,

g^- : non-positive part of g, *i.e.* $g^- \overset{\text{def}}{=} \min\{g(x), 0\}$,

$\overline{z}$: conjugate of a complex number, *i.e.* $\overline{a + ib} \overset{\text{def}}{=} a - ib$,

$|z|$: absolute value of a number (real or complex), *e.g.* $|a + ib| \overset{\text{def}}{=} \sqrt{a^2 + b^2}$ or $|-2.73| = 2.73$,

$\mathbf{x}, \mathbf{y}$: vector x, y. Sometimes also written as $\overrightarrow{u}, \overrightarrow{v}$ etc,

$|\mathbf{x}|$: norm of the vector $\mathbf{x}$, *e.g.* $|[1;2;3]| = \sqrt{1^2 + 2^2 + 3^2} \simeq 3.741$,

$\|.\|$: unified notation for a norm, $\|.\|_p$: norme p, $\|.\|_{+\infty}]$: infinite norm,

$(u_n)_{n\in\mathbb{N}}$: the sequence of $u_n \in E$, *i.e.* the function $u : \mathbb{N} \to E, n \mapsto u_n$.

Derivatives and differential calculus

f' : derivative[4] of f, *e.g.* $\ln'(x) = \dfrac{1}{x}, \forall x > 0$. May be written $\dot{x}$ (in physics),

f'' : second derivative of f (or $\ddot{x}$ in physics)), f''' third derivative, $\dots$, $f^{(n)}$ n^{th} derivative for $n \geqslant 0$,

f^n : n^{th} power of f for $n \geqslant 0$ (**be careful, it is not** a derivative $f^{(n)}$!),

$D_j f$: partial derivative of f for the j^{th} coordinate,

∇f : gradient of f (also written $\mathbf{Grad} f$),

$\mathrm{Div} f$: divergence of f.

Integrals

$\displaystyle\int_a^b f(x)\mathrm{d}x$: integral of the function $f : x \mapsto f(x)$ on (a, b),

$\displaystyle\int f$: integral of the function f on its domain (**when it is not ambiguous**),

$\displaystyle\oint f(x)\mathrm{d}x$: curvilinear integral of f,

$\displaystyle\iint_{(a,b)\times(c,d)} f(x, y)\mathrm{d}x\mathrm{d}y$: double integral of the 2-variables function $f : (a, b) \times (c, d), (x, y) \mapsto f(x, y)$.

Usual functions spaces

$\mathcal{F}(E, F)$: functions from E to F, *i.e.* F^E,

$\mathcal{C}(E, F)$: continuous functions from E to F,

$\mathcal{C}(E)$: continuous functions from E to E,

$\mathcal{C}_b(E, F)$: bounded and continuous functions from E to F,

$\mathcal{C}^k(E, F)$: k-time differentiable functions, of k^{th} derivatives being continuous, from E to F ($k \in \mathbb{N}$),

$\mathcal{C}^\infty(E, F)$: infinitely differentiable functions, from E to F,

$\mathcal{L}(E, F)$: linear applications from E to F,

$\mathcal{L}(E)$: linear applications from E to E.

$\mathcal{L}_c(E, F)$: continuous and linear applications from E to F,

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Want more? A list of mathematical symbols : `en.wikipedia.org/wiki/Table_of_mathematical_symbols`, on Wikipédia, and some others lists on mathematical notations, on Wikipédia.

[4] If it exists!