PH101
Lecture 6

12.08.14

Acceleration

Linear acceleration in the radial direction

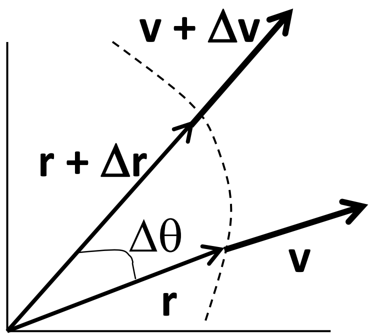
Linear acceleration in the tangential direction

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathrm{r}} + \left(r\ddot{\theta} + 2\dot{r}\,\dot{\theta}\right)\hat{\theta}$$

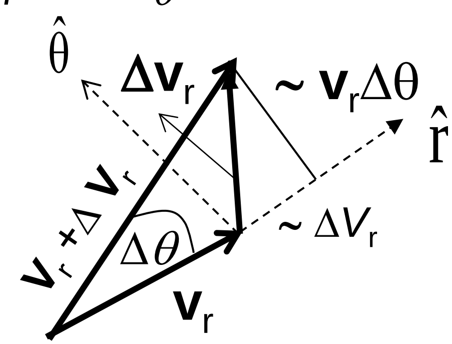
Centripetal acceleration

Coriolis acceleration

Instantaneous velocity:



$$\vec{v} = \dot{r}\,\hat{\mathrm{r}} + r\dot{\theta}\,\hat{\theta} = v_r\hat{\mathrm{r}} + v_\theta\hat{\theta}$$



- Radial comp of $\Delta\mathbf{v}_r$: $\Delta v_r\hat{\mathrm{r}}$

- Tangential comp of $\Delta\mathbf{v}_r$: $v_r\Delta\theta\,\hat{\theta}$

Thus contribution of radial comp to acceleration:

$$\lim_{\Delta t \to 0} \left(\frac{\Delta v_r}{\Delta t} \hat{\mathrm{r}} \right) = \frac{dv_r}{dt} \hat{\mathrm{r}} = \ddot{r}\, \hat{\mathrm{r}}$$

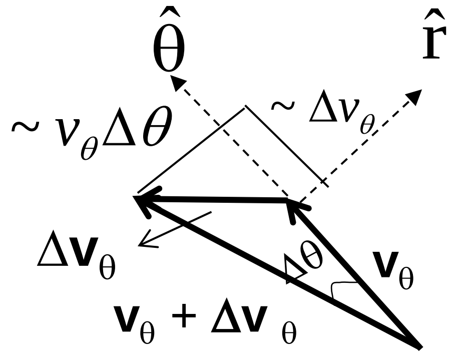
Likewise contribution of tangential comp to acceleration:

$$\lim_{\Delta t \to 0} \left(v_r \frac{\Delta \theta}{\Delta t} \hat{\theta} \right) = v_r \frac{d\theta}{dt} \hat{\theta} = \dot{r}\, \dot{\theta}\, \hat{\theta}$$

One-half of Coriolis acceleration that arises from?

Change in direction of $\mathbf{v}_r$

Tangential vel: $v_\theta \hat{\theta} = r\dot{\theta}\,\hat{\theta}$



change in direction of $\hat{\theta}$

$\Rightarrow \Delta \mathbf{v}_\theta$ gives an inward radial comp. $\rightarrow -v_\theta \Delta\theta\,\hat{r}$

whose contribution to acceleration:

$$\lim_{\Delta t \to 0}\left(-v_\theta \frac{\Delta\theta}{\Delta t}\hat{r}\right) = -v_\theta \frac{d\theta}{dt}\hat{r} = -v_\theta\,\dot{\theta}\,\hat{r} = -r\,\dot{\theta}^2\hat{r}$$

?

Centripetal acceleration

Tangential comp. of $\Delta \mathbf{v}_\theta$: $\Delta v_\theta \hat{\theta}$

Since $v_\theta = r\dot{\theta}$

$$\Rightarrow \Delta v_\theta = \underbrace{r\Delta\dot{\theta} + \Delta r\dot{\theta}}_{\Rightarrow \text{ Two terms}}$$

Thus its contribution to acceleration:

$$\lim_{\Delta t \to 0} \left(\frac{\Delta v_\theta}{\Delta t} \hat{\theta} \right) =$$

$$\lim_{\Delta t \to 0} \left(r \frac{\Delta\dot{\theta}}{\Delta t} + \frac{\Delta r}{\Delta t} \dot{\theta} \right) \hat{\theta} = \left(r\ddot{\theta} + \dot{r}\dot{\theta} \right) \hat{\theta}$$

2nd half of Coriolis acceleration due to change in radial distance, which induces change in tangential speed

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2 \right) \hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} \right) \hat{\theta}$$

Foundation of Newtonian Mechanics

Newton's laws combine

- Certain definitions
- Observations from nature
- Partly intuitive concepts
- Certain unexplained assumptions on properties of space and time

Ernst Mach after about two hundred years later examined assumptions of Newtonian Mechanics and provided a more critical analysis

His analysis played an imp role in special theory of relativity

Prior to Newton, no theory could satisfactorily explain planetary motion

After Newton's laws, even slight deviations from Kepler's laws could be explained

i.e. perturbations on the planets like Uranus due to Jupiter and Saturn

Other examples:

- Motions of pendulum
- Oscillators with springs and weights in them

Galileo made a significant contribution through his discovery of his *principle of inertia*:

If an object is left alone without any external disturbance, it will continue to move at a const. velocity in a straight line if it was originally moving or

Continues to stand still if it was just standing still

But what we observe in nature is on the contrary!

If we slide a block across a table it stops.

It was <u>not</u> left to itself ; it is rubbing against the table!

It reqd. certain imagination to find the right rule

Next thing was to find how an object changes its speed if something is affecting it

That was the contribution of Newtonian Mechanics

Newton wrote down three laws of motion

1st law is a mere restatement of Galilean principle of inertia

2nd law gave a specific way to determine how the velocity changes under influence of a force/forces

Newton’s laws provided a direct introduction to classical mechanics

However there are other alternate approaches

e.g. formulations of Lagrange and Hamilton

Energy rather than force is taken as the fundamental concept

These different formulations of classical mechanics are physically equivalent though