Tutorial Sheet - 5

FIRST SEMISTER 2014
PHYSICS-101

Date: 19.09.2014 Tut. Sheet **05**

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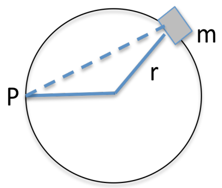
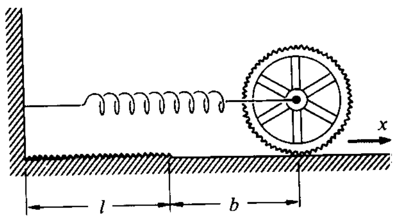
1. A particle of mass *m* is moving in a circle of radius *r* with constant speed v_0 as shown in the figure below. Calculate the angular momentum $\boldsymbol{L}$ and torque $\boldsymbol{\tau}$ about the point P and show that $\frac{dL}{dt} = \tau$.
   (Result: $\tau = 2mv^2 \cos\theta \sin\theta$)



2. A book about 20 cm in length slips out of your hand with its shorter side parallel to the horizontal. Assume that the book does not open during the fall. (1) Draw schematically the position and orientation of the book at four different instances after it starts falling. (2) Calculate approximately the angle through which the book would rotate before hitting the ground. Take the height through which the book falls to be 1m and the coefficient of friction between the hand and the book to be 0.5. (Result: approximately 250^0)

3. A wheel with fine teeth is attached to the end of a spring with constant *k* and unstretched length *l*. For *x*>*l*, the wheel slips freely on the surface, but for *x*<*l* the teeth mesh with the teeth on the ground so that it can not slip. Assume that all the mass of the wheel is in its rim.



   a. The wheel is pulled to *x*=*l*+*b* and released. How close will it come to the wall on its first trip?
   b. How far out will it go as it leaves the wall?
   c. What happens when the wheel next hits the gear track?

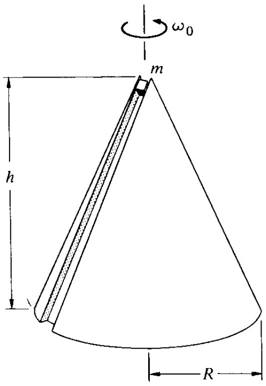
   Result: $b/\sqrt{2}$, b/2

4. A cone of height h and base radius R is free to rotate about a fixed vertical axes. It has a thin groove cut in the surface. The cone is set rotating freely with angular speed ω_0 and a small block of mass m is released in the top of the frictionless groove and allowed to slide

under gravity. Assume that the block stays in the groove. Take the moment of inertia of the cone about the vertical axis to be I_0.

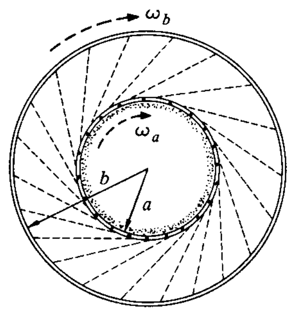
a. What is the angular velocity of the cone when the block reaches the bottom?

b. Find the speed of the block in inertial space when it reaches the bottom.



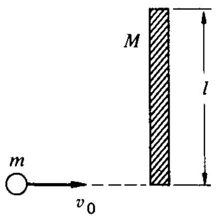
Result: $\omega_f = \left[\frac{I_0}{I_0 + MR^2}\right]\omega_0$, $V_f = \left[\frac{I_0}{m}\left(\omega_0^2 - \frac{I_0}{I_0 + mR^2}\right)\right]$

5. A drum of mass M_A and radius a rotates freely with initial angular velocity $\omega_A(0)$. A second drum with mass M_B and radius b>a is mounted on the same axis and is at rest, although it is free to rotate. A thin layer of sand with mass M_S is distributed on the inner surface of the smaller drum. At t=0, small perforations in the inner drum are opened. The sand starts to fly out at a constant rate λ and sticks to the outer drum. Find the subsequent angular velocities of the two drums ω_A and ω_B. Ignore the transit time of the sand. (Result: $\omega_B = \frac{\lambda t a^2 \omega_A(0)}{(\lambda t + m_B)b^2}$)

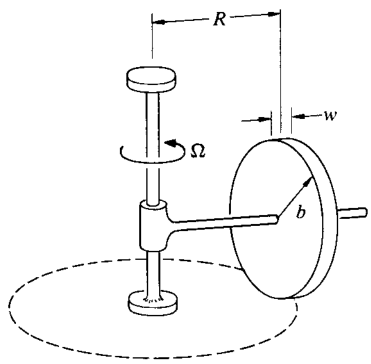


6. A boy of mass m runs on ice with velocity v_0 and steps on the end of a plank of length l and mass M which is perpendicular to his path.
   a. Describe quantitatively the motion of the system after the boy is on the plank. Neglect friction with the ice.
   b. One point on the plank is at rest immeadiately after the collision. Where is it?

      Result: $2l/3$ from the boy

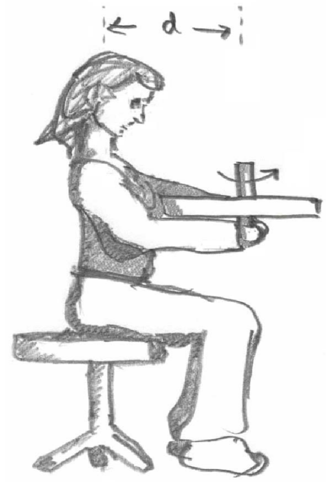


7. In an old-fashioned rolling mill, grain is ground by a disk-shaped millstone which rolls in a circle on a flat surface driven by a vertical shaft. Because of the stone's angular momentum, the contact force with the surface can be considerably greater than the weight of the wheel.
   Assume that the millstone is a uniform disk of mass M, radius b, and width w, and that it rolls without slipping in a circle of radius R with angular velocity Ω. Find the contact force. Assume that the millstone is closely fitted to the axle so that it can not tip, and that w << R. Neglect friction.



Result: $N = M\left(g + \frac{1}{2}b\Omega^2\right)$

8. A person is sitting on a stool that is initially not rotating and is holding a spinning wheel. The moment of inertia of the person and the stool about a vertical axis passing through the center of the stool is $I_{S,P}$. The moment of inertia of the wheel about an axis, perpendicular to the plane of the wheel, passing through the center of mass of the wheel is $I_w = (1/4)I_{S,P}$. The mass of wheel is m_w. Suppose that the person holds the wheel as shown in the sketch such that the distance of an axis passing through the center of mass of the wheel to the axis of rotation of the stool is *d* and that $m_w d^2 = (1/3)I_w$. Suppose the wheel is spinning initially at an angular speed ω. The person then turns the spinning wheel upside down. You may ignore any frictional torque in the bearing of the stool. What is the angular speed of the person and stool after the spinning wheel is turned upside down?



(Result: $\omega_f = \frac{3}{8}\omega_s$)

9. An ice-skater starts his pirouette with arms outstretched, rotating at 1.5 rev/s. Estimate his rotational speed (in revolutions per second) when he brings his arms flat against his body. (*I assume that his body is cylindrical in shape with 40 cm in diameter. His arms are rod shaped and stretched to 1 m length. The mass of the skater and his arms are 50 kg and 4 kg respectively. Make further assumptions if necessary!!).*

10. A solid cylinder and a hollow cylinder of the *same mass* are rolled down an incline. Which gets to the bottom first? Do they arrive at the same time? What would happen if you dropped the two from the same height ignoring the resistance of air? (*Hint: Use the energy conservation and make assumptions if necessary!!*)