PH101

Differential equations of physical system and assignment

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Differential equation

If number of student in MEC increased (growth rate) $y'=dy/dx$ (x=time, 1yr) equal to the present population $y(x)$.

The population model is differential equation

$$y'=y$$

General solution for this, $y = k\,e^x$

Free fall stone $y''= d^2y/dx^2 = g$ **(x=time)**

$$y'= gx + v_0 \Rightarrow y=y_0+1/2\,gx^2 + xv_0$$

First-order-differential equation

$F(x, y, y') = 0$ Or $y' = f(x,y)$

$y = x^2$ is a solution of 1^{st} order eq^n $xy' = 2y$

Separable differential equation

$$g(y)y' = f(x)$$

$$g(y)dy = f(x)dx$$

$$\int g(y)dy = \int f(x)dx + c$$

Example $9yy' + 4x = 0$

Sol^n of ellipses; $x^2/9 + y^2/4 = c$

Reduction of Separable : $y = ux$ for $y' = g(y/x)$ 1D eq^n

Linear differential equation

$$y' + p(x)\, y = r(x)$$

Its linear ín the unknown function of y and y',
whereas p as well as r on the right might function of x.

If r(x) = 0, then homogeneous otherwise nonhomogeneous

If p and r continuous in interval then formula

$$y' + p(x)\, y = 0$$

$$dy/y = -\, p(x)\, dx \quad \Rightarrow \quad \text{Log}_e\, y = -\int p(x)dx + c\, *$$

$$y(x) = c\, e^{-\int p(x)dx}$$

We may have trivial solution at c=0, y(x)=0

Nonhomogeneous 1D equation

$$y' + p(x)\, y = r(x)$$

Integrating factor (I.F.) = $e^{\int p\, dx}$

Solution: $\quad y(x) = \dfrac{\int I.F. \times r(x) dx + c}{I.F.}$

If r(x) is zero, $y(x) = c\, e^{-\int p(x) dx}$ *And if p(x) is zero then* $y(x) = \int r(x) dx + c$

Example $\quad y' - y = e^{2x}$

Sol^n : $p(x) = -1$, $r(x) = e^{2x}$ *and* I.F. $= e^{-x}$

$$y(x) = e^{2x} + c\, e^{x}$$

Second-order-differential equation

Linear *2^{nd} order differential eqn^n*

$$y'' + p(x)\, y' + q(x)\, y = r(x)$$

If $r(x) = 0$, then homogeneous otherwise nonhomogeneous

Second-order- homogeneous equation with constant coefficients

$$ay'' + by' + cy = 0$$ a, b and c are constants

If $a=0$, then reduced 1^{st} order eq^n $by' + cy = 0$ has solution $y = c\, e^{-kx}$

This gives us idea to assume solution $y=e^{\lambda x}$

After substitution $(a\lambda^2 + b\lambda + c)\, e^{\lambda x} = 0$

$$a\lambda^2 + b\lambda + c = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

General solution is $\quad y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 X}$

C_1 and C_2 are constants that determined by value of y(x) at two different points.

Case1 if b^2-4ac > 0, two real roots
Case2 if b^2-4ac=0 a real and equal roots
Case3 if b^2-4ac$<$0 complex conjugate roots

Example

$$d^2 x/dt^2 + w^2 x = 0$$

$$x'' + w^2 x = 0$$

Substitute $x = e^{\lambda t}$

Characteristic eq^n $\lambda^2 + w^2 = 0$

$$\lambda = \pm\, i\, w$$

$$x(t) = C_1\, e^{iwt} + C_2\, e^{-iwt}$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$x(t) = A \cos wt + B \sin wt$$

Surprize test

Q.1 A mass m is constrained to the surface of a smooth disk radius *R* by a spring with spring constant k and unstretched length *R/2*, as illustrated below. Write the equation of motion of the mass if the disk is in a horizontal plane, including the affect of friction (coefficient of kinetic friction μ).