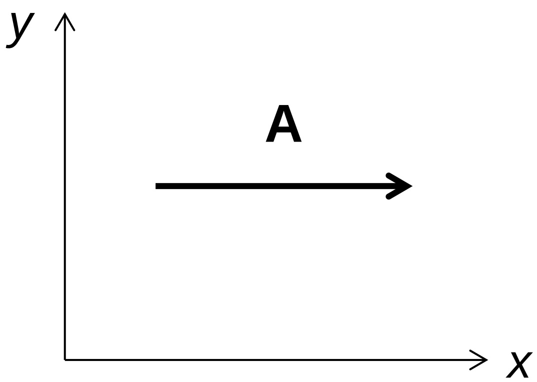
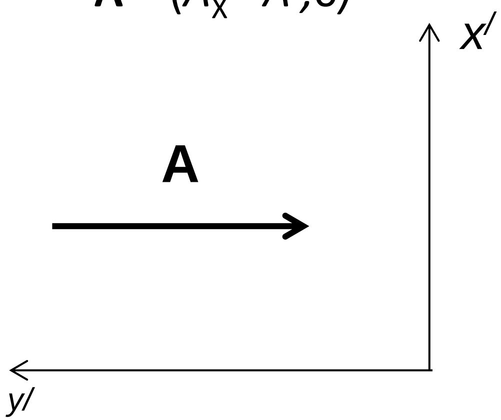
PH101
Lecture 3

07.08.14



A = (A_X =A ,0)



A = (0, A_X/ = - A)

Restricting to one single coordinate system,

If **A** = **B** $\Rightarrow$

This single vector eq. would represent 3 scalar eqns:

$A_x = B_x$

$A_y = B_y$

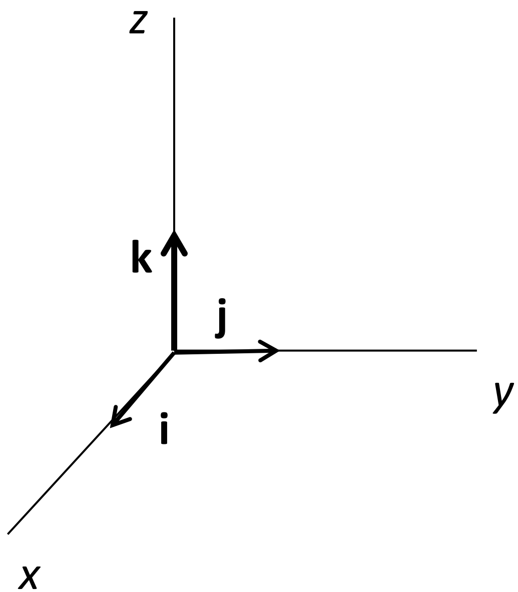
$A_z = B_z$

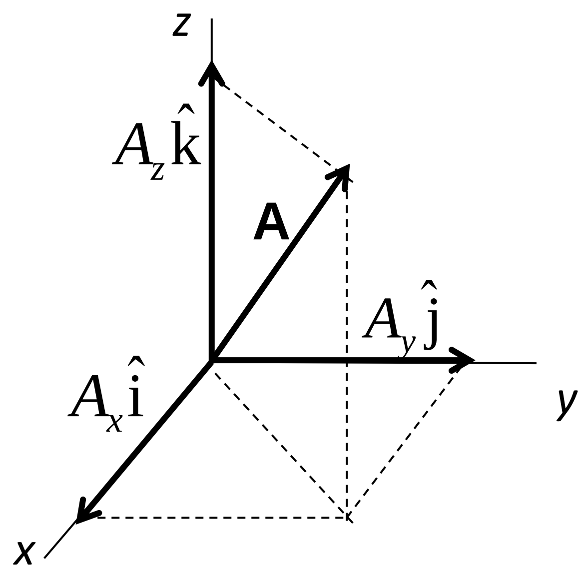
Law of vector addition: $\vec{A} + \vec{B} = \left(A_x + B_x, A_y + B_y, A_z + B_z\right)$

If **A** = (5,6,-8), **B** = (3,5.2), **A ± B** ? $\left|\vec{A}\right|, \left|\vec{B}\right|$?

Base vectors

$$\vec{A} = A_x\hat{\mathrm{i}} + A_y\hat{\mathrm{j}} + A_k\hat{\mathrm{k}}$$





Dot and cross products between these base vectors:

$$\hat{\mathrm{i}}.\hat{\mathrm{i}} = \hat{\mathrm{j}}.\hat{\mathrm{j}} = \hat{\mathrm{k}}.\hat{\mathrm{k}} = 1$$

$$\hat{\mathrm{i}}.\hat{\mathrm{j}} = \hat{\mathrm{j}}.\hat{\mathrm{k}} = \hat{\mathrm{k}}.\hat{\mathrm{i}} = 0$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}},\ \hat{\mathrm{j}}\times\hat{\mathrm{k}} = \hat{\mathrm{i}},\ \hat{\mathrm{k}}\times\hat{\mathrm{i}} = \hat{\mathrm{j}}$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{i}} = \hat{\mathrm{j}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}}\times\hat{\mathrm{k}} = 0$$

A X B ?

$$A_x\vec{\mathrm{i}}\times\vec{B} = A_x\vec{\mathrm{i}}\times\left(B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}} + B_k\hat{\mathrm{k}}\right)$$

$$= A_x\left(B_y\hat{\mathrm{k}} - B_z\hat{\mathrm{j}}\right)$$

A X B in the form of a determinant:

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z\end{vmatrix}$$

$$=\hat{i}\left(A_y B_z - A_z B_y\right)+\hat{j}\left(A_z B_x - A_x B_z\right)+\hat{k}\left(A_x B_y - A_y B_y\right)$$

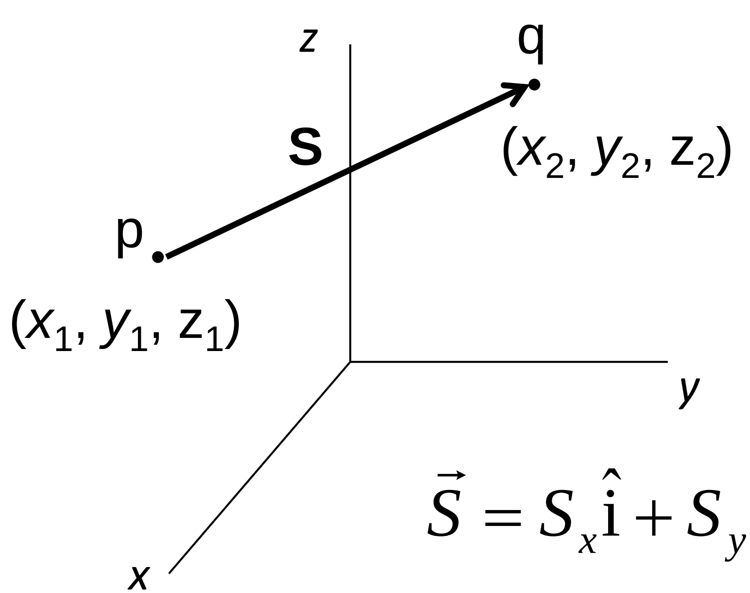
$$\vec{A}=\hat{\mathrm{i}}+3\hat{\mathrm{j}}-\hat{\mathrm{k}}$$

$$\vec{B}=4\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}$$

$$\vec{A}\times\vec{B}\,?$$

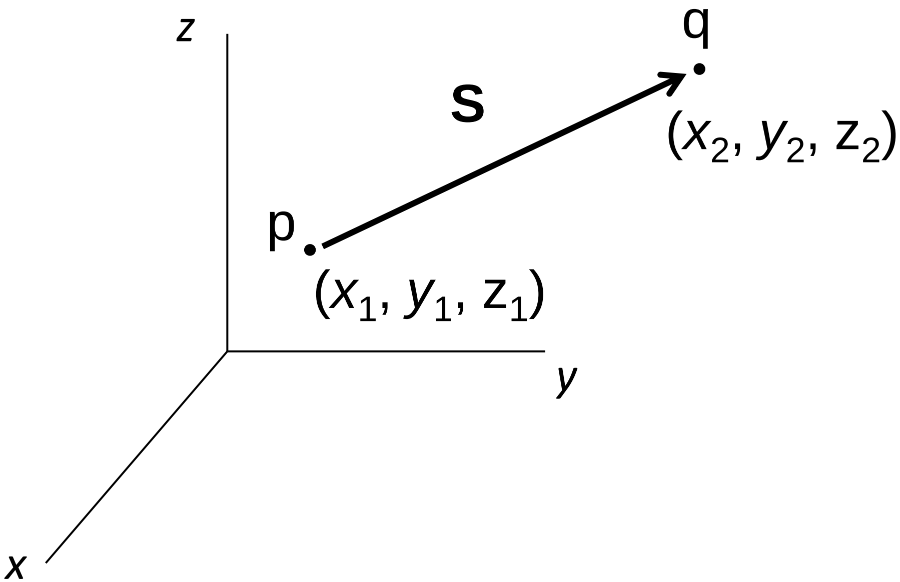
Application of vectors in Kinematics

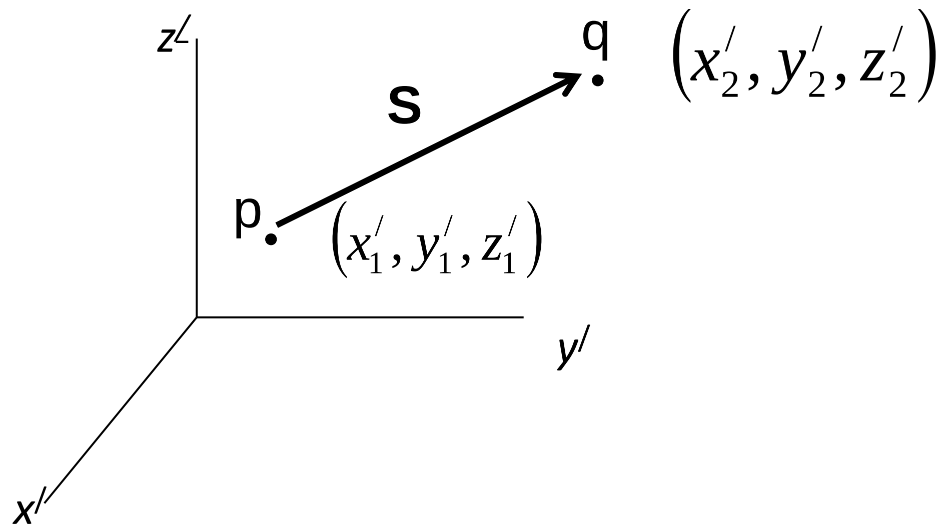
Position and Displacement vector:



$$\vec{S} = S_x\hat{\mathrm{i}} + S_y\hat{\mathrm{j}} + S_z\hat{\mathrm{k}}$$: Displacement vector

$$S_x = x_2 - x_1$$



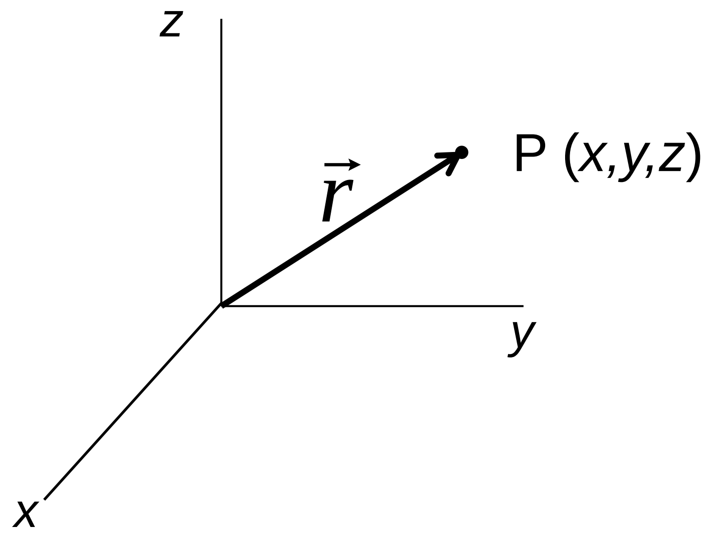


S = (8m, 0, 0)

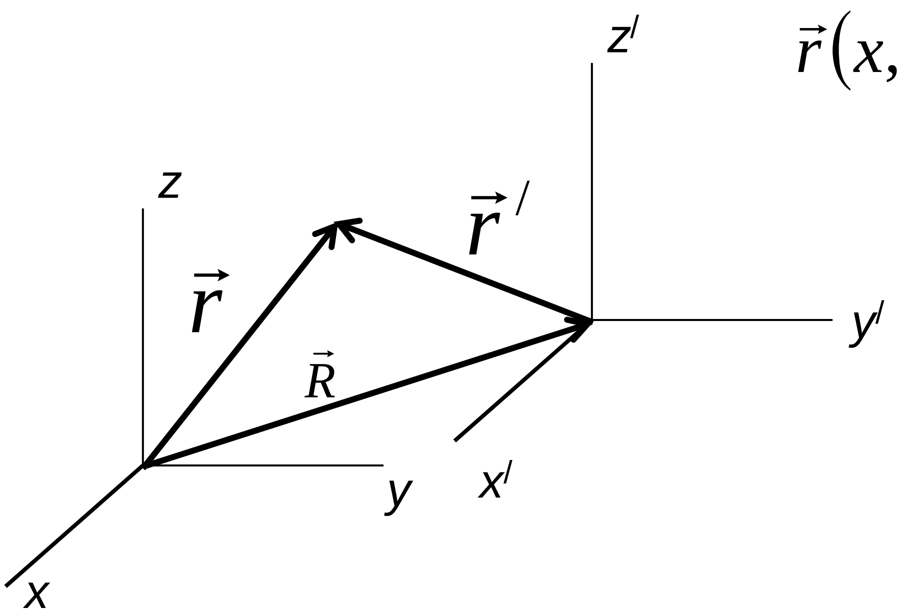
$$\frac{\vec{S}}{\left|\vec{S}\right|} = ?$$

$$\hat{S}$$

Position vector

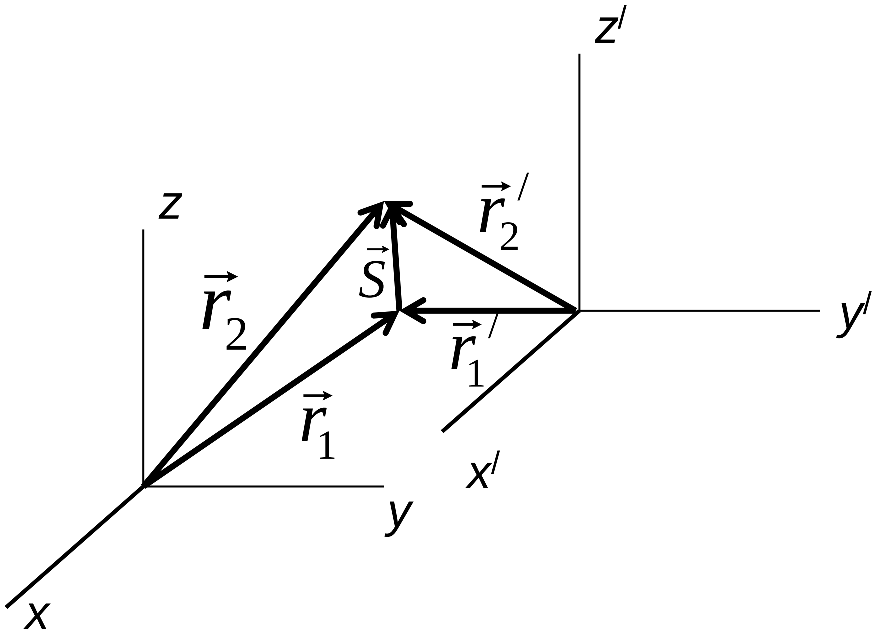


Position of an arbitrary point:



$$\vec{r}(x, y, z) = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$

$$\vec{r}^{\,/} = \vec{r} - \vec{R}$$



$$\vec{r}_1 = \vec{r}_2 - \vec{S} \qquad \Rightarrow \vec{S} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_2 = \vec{r}_2^{\,/} + \vec{R} \qquad \Rightarrow \vec{S} = \left(\vec{r}_2^{\,/} + \vec{R}\right) - \left(\vec{r}_1^{\,/} + \vec{R}\right)$$

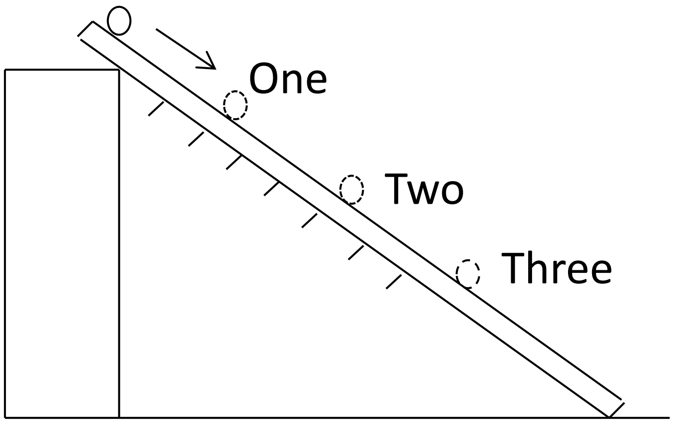
$$\vec{r}_1 = \vec{r}_1^{\,/} + \vec{R} \qquad = \vec{r}_2^{\,/} - \vec{r}_1^{\,/}$$

Conclusion: $\vec{r}$ and $\vec{r}^{\,/}$ depend on coordinate system but **S** does not!

Motion:

Concerned with where/ When?

Galileo's experiment on motion with a ball rolling down an inclined plane:



How far the ball went in *how long* time?

Location of the ball was marked at equal intervals of time: 1, 2, 3, …..rel to its instant of release

Those distances turned out to be proportional to 1, 4, 9,16, …..

Conclusion:

$$\text{Distance}\,(s) \propto (\text{time})^2 \Rightarrow s \propto t^2$$

Let *x* be the coordinate in 1D of a moving object along a line

Average velocity:

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

Instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \Rightarrow v = \frac{dx}{dt}$$

This idea was invented independently by Newton and Leibnitz

– the subject of *differential calculus* was formed!

Similarly instantaneous acceleration:

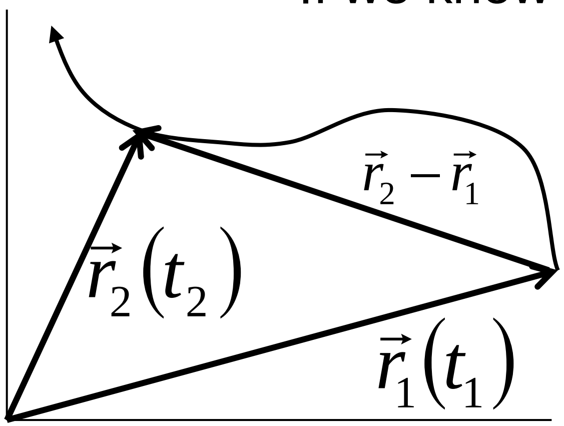
$$a = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t} \Rightarrow a = \frac{dv}{dt}$$

What is speed ?

$$\text{speed} = |\vec{v}|$$

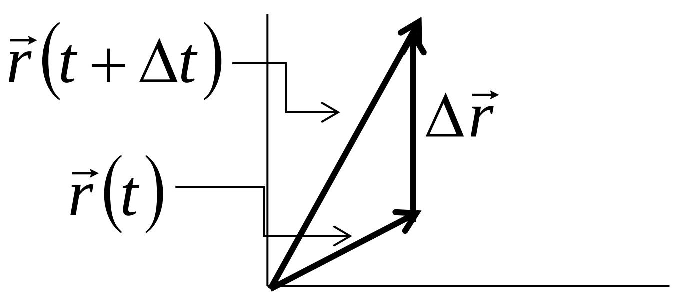
Consider motion of a particle in a plane such that over a certain interval
It traces out a path

If we know the instantaneous positions at times $t + \Delta t$ and t



$$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

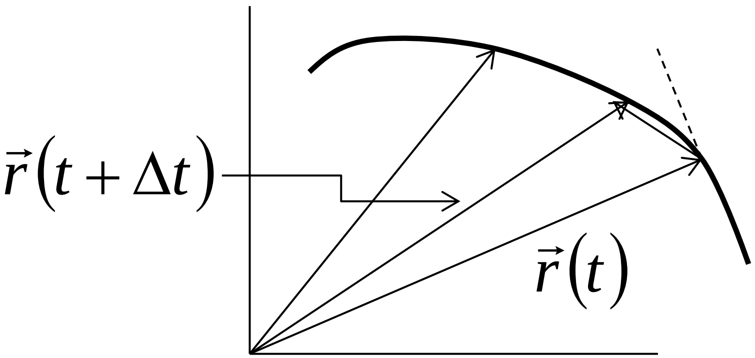
$$\Rightarrow \quad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



Alternatively, $r = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{k}$

$$\Rightarrow$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}$$



$$\Delta\vec{r} \approx \frac{d\vec{r}}{dt}\Delta t = \vec{v}\,\Delta t$$

Exact

Limit $\Delta t \rightarrow 0$

$\vec{v}$ is parallel to $\Delta\vec{r}$

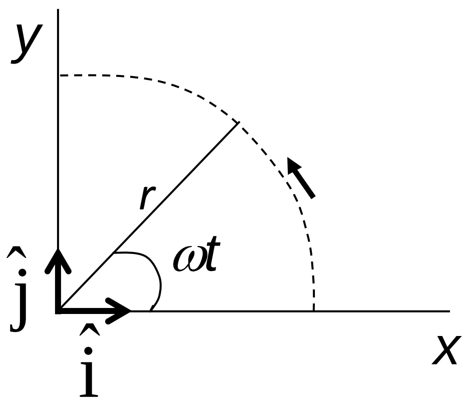
Instantaneous $\vec{v}$ is tangent to the trajectory

Uniform circular motion:

A particle is moving in the *xy* plane described through

$$\vec{r} = r\left(\cos\omega t\,\hat{\mathrm{i}} + \sin\omega t\,\hat{\mathrm{j}}\right);\ r \text{ and } \omega \text{ are constants}$$

Trajectory, velocity and acceleration:



$$\left|\vec{r}\right| = \left(r^2\cos^2\omega t + r^2\sin^2\omega t\right)^{1/2} = r$$

$\Rightarrow$ Circle; starting at (r, 0) it moves ccw and completes one revolution in a time period T such that $\omega T = 2\pi$

$$x = r\cos\omega t$$
$$y = r\sin\omega t$$

$$\vec{v} = \frac{d\,\vec{r}}{dt} = r\omega\left(-\sin\omega t\,\hat{\mathrm{i}} + \cos\omega\, t\,\hat{\mathrm{j}}\right)$$

$$\left|\vec{v}\right| = r\omega$$

$$\vec{v}.\vec{r} = \omega r^2\left(-\cos\omega t\sin\omega t + \cos\omega t\sin\omega t\right) = 0$$

$\Rightarrow$ **v** i.e. d**r**/dt is perpendicular to **r**

Acceleration:

$$\vec{a} = \frac{d\,\vec{v}}{dt} = r\omega.\omega\left(-\cos\omega t\,\hat{\mathrm{i}} - \sin\omega t\,\hat{\mathrm{j}}\right)$$

$$= -\omega^2\,\vec{r}$$

For a known **a** (*t*), corresponding **v** is obtained through integration

$$\frac{d\,\vec{v}}{dt} = \vec{a}\left(t\right) \Rightarrow \int_{t_1}^{t_2} d\,\vec{v} = \int_{t_1}^{t_2} \vec{a}\left(t\right)dt$$

$$\vec{v}\left(t_2\right) - \vec{v}\left(t_1\right) = \int_{t_1}^{t_2} \vec{a}\left(t\right)dt$$

$$\vec{v}\left(t'\right) = \vec{v}\left(t_0\right) + \int_{t_1}^{t_2} \vec{a}\left(t'\right)dt'$$

Likewise, position can be found through $\frac{d\,\vec{r}\left(t\right)}{dt} = \vec{v}\left(t\right) \Rightarrow \vec{r}\left(t\right) = \vec{r}_0 + \int_0^t \vec{v}\left(t'\right)dt'$

Problem:

A ping-pong ball is released near the surface from the moon with a velocity **V_0** = (0, 5, -3) m/s. It accelerates downward with acceleration **a** =(0,0, -2) m/s^2. What will be its velocity after 5 sec?

$$\vec{v}\left(t'\right) = \vec{v}\left(t_0\right) + \int_{t_1}^{t_2} \vec{a}\left(t'\right) dt' \equiv \vec{v}_0 + \int_0^t \vec{a}\left(t'\right) dt'$$

$$\left.\begin{aligned} v_x(t) &= v_{0x} + \int_0^5 0.dt' = 0 \\ v_y(t) &= v_{0y} + \int_0^5 0.dt' = 5 \\ v_z(t) &= v_{0z} + \int_0^5 (-2).dt' = -13 \end{aligned}\right\} \Rightarrow \text{(0, 5, -13) m/sec}$$

Case of uniform acceleration:

$\Rightarrow$ **a** is constant

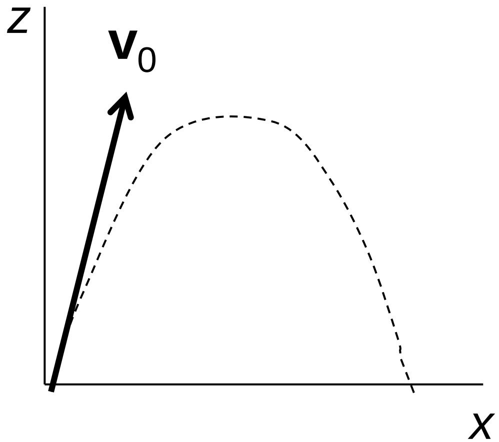
$$\vec{v}(t) = \vec{v}_0 + \vec{a}t \Rightarrow \vec{r}(t) = \vec{r}_0 + \int_0^t \left(\vec{v}_0 + \vec{a}\,t'\right) dt'$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0\,t + \frac{1}{2}\vec{a}\,t^2$$

True only for uniform acceleration!

Example on motion in a uniform gravitational field:

A free falling ball under the influence of earth's gravity experiences a constant downward acceleration of *g*. Assuming that it is confined to *xz* plane, obtain its trajectory by assuming *z*-axis to be vertically upwards.



$$\vec{a} = -g\,\hat{\mathrm{k}}$$

Assuming it to be released at $t = 0$ with an initial velocity $\mathbf{v}_0$ and assuming $\mathbf{r}_0 = 0$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\vec{a}\, t^2$$

$$x = v_{0x} t$$

$$z = v_{0z} t - \frac{1}{2} g t^2$$

$$\Rightarrow \text{trajectory}: z = v_{0z}\frac{x}{v_{0x}} - \frac{1}{2} g \frac{x^2}{v_{0x}^2} = \frac{v_{0z}}{v_{ox}} x - \frac{g}{2v_{0x}^2} x^2$$

Example on motion in a non-uniform acceleration: Effect of a radio wave on an ionospheric electron

To be discussed in a review class