This document is a list of all the common mistakes we saw too many times in the first Mid Term Exam, September 2014 at MEC.

1 Common mistakes in the whole exam.

- In English, a new sentence starts with a capital letter (even in a Mathematics exam!).
- English again, we prefer to write "there is no element in E" and **not** "there are no elements".
- Avoid all abbreviations in an exam (especially the symbol &).
- Try to keep your answer script *clean* and *readable*.
- Try to *highlight* or *underline* every result.
- Always specify which question you are solving ($5.a$), $1.b$) etc).
- Try to "talk more", by explaining just a little bit more what you are doing and how you proceed. An answer to an entire question cannot be just maths equations, and the symbols $\therefore$ or $\because$ are not enough. However, this does not mean that one has to give long and unnnecessary explanation. Just suffices to make all equations etc as part of sentences.
- Whenever new variables (not used in the question) are used please introduce them carefully (by using statements like where $x \in \mathbb{R}$ or let x be such that...).
- Do not be confused about the difference between proving : an equality ("stuff $= \cdots =$ one other thing"), an implication ("If we have P then $\dots$ then we have the result", or $P \implies \dots \implies$ result), and an equivalence ("P is true if and only if $\dots$ iff Q", or $P \Leftrightarrow \cdots \Leftrightarrow Q$).
- English again : in written English, one do not use abbreviation : "We can't" will be written "We cannot".
- When we use binary operators ($=, \in, \subset, <, >, \leqslant, \geqslant$ etc), we try to not mix them with text : we do not write "So the minimum is > 0", but "So the minimum is greater than 0" or "So $\min(E) > 0$".
- When you introduce a variable, no need to quote it : "Let $'b'$ be a lower bound" can really be written "Let b be a lower bound", without ambiguity usually.
- Try to practice and get used to write properly the common symbols : $\forall$ is not V, $\mathbb{R}$ is not just R, $\mathbb{Z}$ is not Z, etc. It can be really confusing in some cases!

- Usually, it is useless and time-consuming to *write again what was the question before you start answering it*. If you start (5.a), do not loose time by writing "We have to show that $(f(x))^2 + (g(x))^2 = 1, \forall x \in \mathbb{R}$". Just start. We know what is the question, and you also know because you have the question paper just in front in you.
- English again : try to not forget to add a "n" at the end of the pronoun "a" before another word starting with a vowel. We write "Let a be <u>an</u> element of E", "<u>an</u> upper bound", "<u>an</u> even function" etc.
- Try to avoid words or sentence structures as "... is nothing else but ... ", "obviously", "surely", or "clearly". It gives bad impression of overconfidence (especially if you are wrong).

2 Common mistakes done in problem 1 (on sets in $\mathbb{R}$).

- The empty set is unique (so do not say "an empty set"). It is written $\emptyset$, not $\{\emptyset\}$ (this is a set with one elemen,t $\emptyset$, ie $\{\emptyset\}$ is a *subset* and not an element of $\mathcal{P}(\mathbb{R})$, set of all subsets of $\mathbb{R}$),
- The empty set has lower and upper bounds, in fact, any number $k \in \mathbb{R}$ is both! Indeed the statement "$\forall s \in E, s \leqslant k$" is always true because the first part is impossible (no such element $s \in E$ if $E = \emptyset$). It is a very classical case of "False implies Anything", that you have to keep in mind !
- To prove that a statement is *true* for any subset $E \subset \mathbb{R}$, a proof *in the general case* is **required**. Considering an example **cannot be enough**!
- But to disprove a statement (like (b) or (c)), giving one counter-example is enough : the statement is not true for every set.
- Writing "Let $E \overset{\text{def}}{=} \{x : x \in E\}$" does not define anything! Every set is "the set of its elements", but that does not say anything.
- Similarly, writing "Let $S \overset{\text{def}}{=} \{x : x \in \mathbb{R}, 0 < x < 3\}$" can be done way quicker by saying "Let $S = (0, 3)$". Shortcut notations for intervals are practical!
- A set **is not a function** ! This problem was focusing *on sets only*, so finding a vague counter-example with a function is not enough, unless you precise "With the set $E \overset{\text{def}}{=} \{f(n) : n \in \mathbb{N}\}$", or "And with $E \overset{\text{def}}{=} \{g(x) : x \in \mathbb{R}\}$" for example.
- The minimum $m = \min(E)$ (when it exists) is *already a lower bound*, no need to write $m - \delta$ (for one positive δ) to have a lower bound!
- An *inequality* is **not** a set. Writing "E has both maximum and minimum, $0 \leqslant x \leqslant 1$" does not define E as $\{x : x \in \mathbb{R}, 0 \leqslant x \leqslant 1\} = (0, 1)$.

- Similarly, an *expression* (like $\frac{1}{n}$ or $\frac{n}{n+1}$) does not define a set! You have to precise range of values for n (or x, or ...) : $E \stackrel{\text{def}}{=} \left\{\frac{1}{n} : n > 0\right\}$ or $E \stackrel{\text{def}}{=} \left\{\frac{n}{n+1} : n > -1\right\}$.

3 Common mistakes done in problem 2 (on complex numbers).

- Writing $\frac{1}{0}$ is **never allowed**. Similarly ∞ cannot be treated as a regular number. For example, $\tan^{-1}(\infty)$ is wrong but $\lim_{x\to\infty} \tan^{-1}(x)$ may be valid.
- The formula for the principal argument $\theta = \arctan(\frac{y}{x})$ (or $\tan^{-1}$!) is **not true for every complex number** $z = x+iy$! First of all, x has to be nonzero. There are 5 different cases for the formula. Read that Wikipédia page `https://en.wikipedia.org/wiki/atan2` for more details.
- You **cannot** write the square root of a complex number (or a negative real number) with the symbol $\sqrt{}$! This symbol has meaning only for non-negative real number (ie for $u \in \mathbb{R}_+$).
- When we write $xy = 1$ implies $x = \frac{1}{y}$ one has to justify why $y \neq 0$.
- Similarly, $\log_{\mathrm{e}}$ (or any logarithm $\log_a$) is defined **only for positive real number**. So for the question (2.c), writing $\mathrm{e}^z = u \Leftrightarrow z = \log_{\mathrm{e}}(u)$ *has no meaning if* $u \notin \mathbb{R}_+^*$.
- There appears to be a confusion between principal argument and argument of complex numbers.
- In Problem (2.c), one cannot conclude that e^z is unimodular. Think why!
- Similarly, it is not clear why something like $e^z = e^{i\theta}$ implies $z = i\theta$. This needs to be proven.
- The argument for the complex number 0 *is not defined*!

4 Common mistakes done in problem 3 (on sequences and sequences).

- Even if n is implicitely taken as greater than 1 in all the problem, when you write $a_n = \frac{1}{n(n+3)}$, **you have to precise** (quickly) that $n \neq 0$, so a_n *is well defined* (and similarly for any fraction).

- Do not manipulate limits of sequences *before being sure that they exist* ! Do not start directly by saying $\lim\limits_{n\to+\infty} \dfrac{n/2+\sin(n)}{3n+4\sqrt{n}+1} = \lim\limits_{n\to+\infty} \dfrac{1/2+\sin(n)/n}{3+4\sqrt{n}/n+1} = \cdots = \frac{\frac{1}{2}}{\frac{1}{3}} = \dfrac{1}{6}$: at first **you cannot be sure that the limit will exist**!

- The same is also true for limits of functions.

- You cannot do what ever you want with series. Main results on series are only true under certain hypotheses ! For instance, for $(3.b)$, $\sum\limits_{n=1}^{+\infty} \dfrac{1}{n(n+3)}$ is converging (of sum $\dfrac{11}{18}$), but even if $\dfrac{1}{n(n+3)} = \dfrac{1}{3}\left(\dfrac{1}{n} - \dfrac{1}{n+3}\right)$, **you CANNOT write** $\dfrac{1}{3}\left(\sum\limits_{n=1}^{+\infty} \dfrac{1}{n} - \sum\limits_{n=1}^{+\infty} \dfrac{1}{n+3}\right)$: the two series $\sum\limits_{n=1}^{+\infty} \dfrac{1}{n}$ and $\sum\limits_{n=1}^{+\infty} \dfrac{1}{n+3}$ **diverge** and their sum is $+\infty$ (and you know that you cannot write $(+\infty) - +(\infty)$: this *has no meaning AT ALL*).

- Do not get confused with convergence of sequence and series.

- Convergence of a series is related to the convergence of sequence of n^{th} partial sum (the series $\sum\limits_{n=1}^{+\infty} a_n$ converges iff the sequence $S_n \overset{\text{def}}{=} \sum\limits_{k=1}^{n} a_k$ converges, for $n \to +\infty$).

- For a sequence, you *cannot* define left and right hand limit.

- All the convergence tests covered in lectures are applied for $a_n > 0$, no convergence test for series can be applied to test convergence of a sequence.

5 Common mistakes done in problem 4 (on continuity).

- Again, **do not forget to introduce yourself the variable you use**. Writing $h(x)$ without having previously say what or where is x is *an imprecision.*

- Most of you proved the following, using the "ε, δ" definition that $\lim\limits_{x\to c} x = c$ (defining $\delta = \varepsilon$ is enough in this easy case). Then use $(\lim\limits_{x\to c} x) * (\lim\limits_{x\to c} x) = c * c$ to conclude about $\lim\limits_{x\to c} x^2$. But the same δ does not work to prove $\lim\limits_{x\to c} x^2 = c^2$ with the "ε, δ" definition.

- Some of you wrote, $|x-c||x+c| < \varepsilon$ then[1] $\delta = \dfrac{\varepsilon}{|x+c|}$. But the δ has to be obtained in terms of ε and c, not x !
  You have to fix one $\varepsilon > 0$, then decide what δ to take such that the wanted inequality

[1] And a lot of you forget to specify that $|x+c| \neq 0$ when you divide.

is true for any x close enough of c (ie $|x-c| < \delta$). In this case, one has to find a bound for the term $|x+c|$ ($2|c| + \delta$ if $|x-c| < \delta$), then **choose a good** δ.

- If you use the theorem about limit of product being product of limits (ie $\lim_{x\to c} f(x)g(x) = (\lim_{x\to c} f(x))(\lim_{x\to c} g(x))$ when both limit exist), in this case, you will have $f(x) = g(x) = x$, and after you write $|f(x)g(x) - f(c)g(c)| \leqslant (|x| + |c|)(|x+c|)$, so the proper value of δ has to be obtained in terms of ε (Refer the lecture notes of Prof. Vijay).
- While finding $h(x) = f(g(x))$, most of you wrote, $h(x) = 1$ for $|x| \leqslant 1$, and $h(x) = 0$ for $|x \rhd 1$. But, it has to be noted that, while finding the domain of h you have to look at the domain and the definition of g. $f(g(x))$ is defined only when g can be applied to x **and** f can be applied to $g(x)$.
- Most of you wrote, $|2 - x^2| \leqslant 1$ implies $1 \leqslant x^2 \leqslant 3$ implies $1 \leqslant x \leqslant \sqrt{3}$. But $-1 \leqslant x \leqslant -\sqrt{3}$ also satisfies the condition $|2 - x^2| \leqslant 1$.
- You have to apply $\varepsilon - \delta$ definition of left and right limits, for checking the continuity of the functions f, g and h at the reference points, or at least do one of them carefully, and do the other points more quickly.

6 Common mistakes done in problem 5 (on functions).

- *Obviously*, proving a property like $(f(x))^2 + (g(x))^2 = 1$ for *only one value* $x = 100$ *or* $x = 0$ cannot be enough to conclude that the property is true **for every real number** x (ie $\forall x \in \mathbb{R}$)!
- $g(x)$ and $f(x)$ **are numbers, values : not functions !**. The function is g or f. So we do not say "$g(x)$ is an even function", but "g is even".
- Almost none of you think of *properly* introducing the variables you work with. Saying $(f(x))^2 + (g(x))^2 = 1$ without specifying **what is** x or **in which set** x **can be taken** is *meaningless* !
- Similarly, for question (c) and (d), many of you say : "let $x = 0$ and $y = x$ in $g(x-y) = g(x)g(y) + f(x)f(y)$". Here, x will be 0, OK, but what will be y? x ? So $y = x = 0$, so $y = 0$, and you conclude $g(0) = (g(0))^2 + (f(0))^2$, which is not what we want. You can proceed *more precisely and more quickly*, saying "Let $x \in \mathbb{R}$, then $g(100 - x) = g(100)g(x) + f(100)f(x)$ thanks to the given result". That is enough.
- The name of a variable is not important. If the question ask you to prove $g(x) = g(-x), \forall x \in \mathbb{R}$, and you conclude your computation with $g(y) = g(-y)$, with y being any real number, *that is already enough.*