Lecture -8

PH101

Friction, Viscous and Restoring forces

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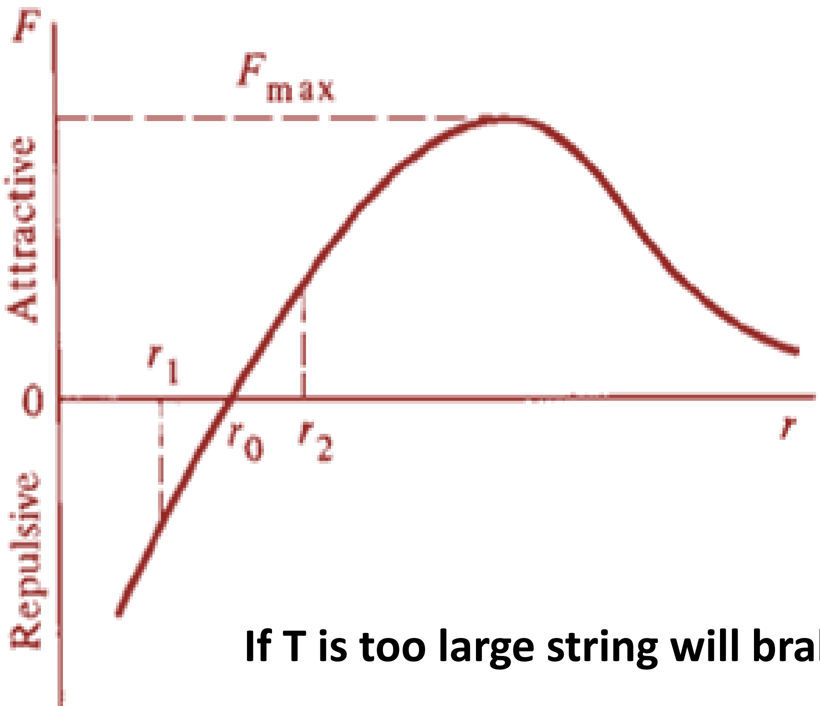
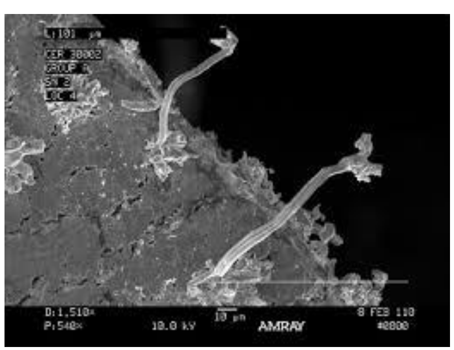
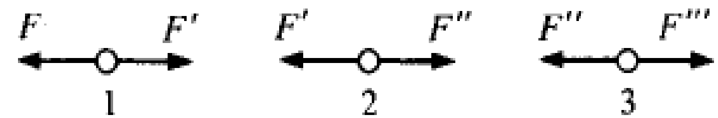
Tension and Atomic Forces



If T is too large string will brake



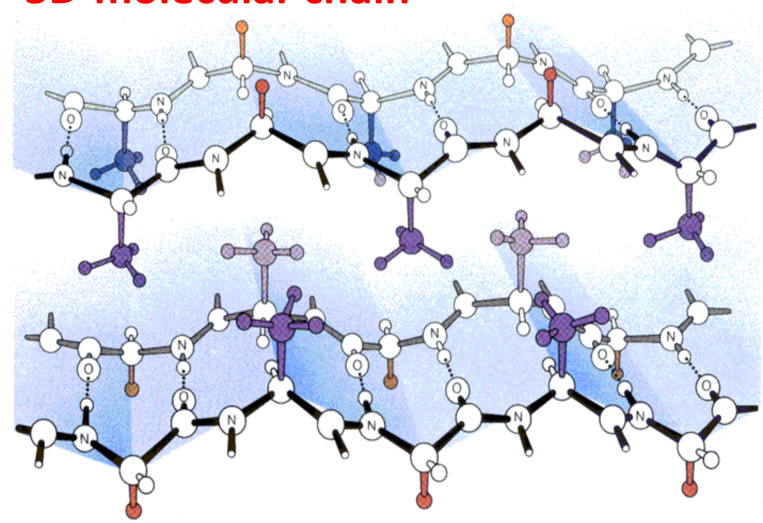
1D-molecular chain



equilibrium, $F = F'$ and $F' = F''$, so that $F'' = F$.

the string “transmits” the force F.

3D-molecular chain



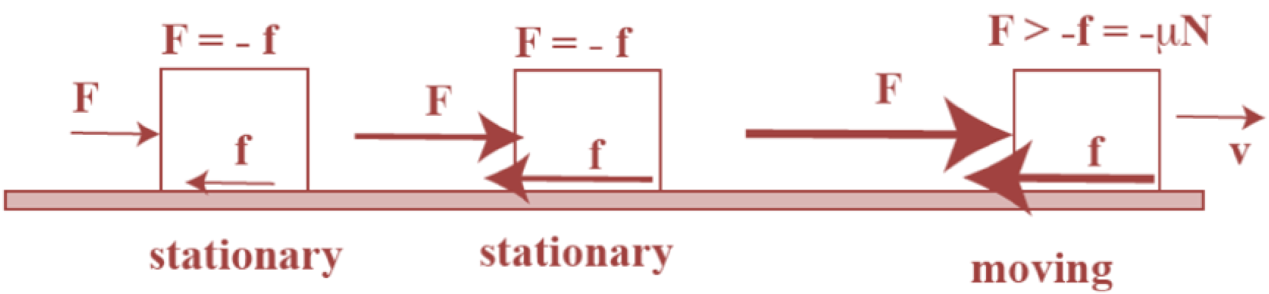
Friction

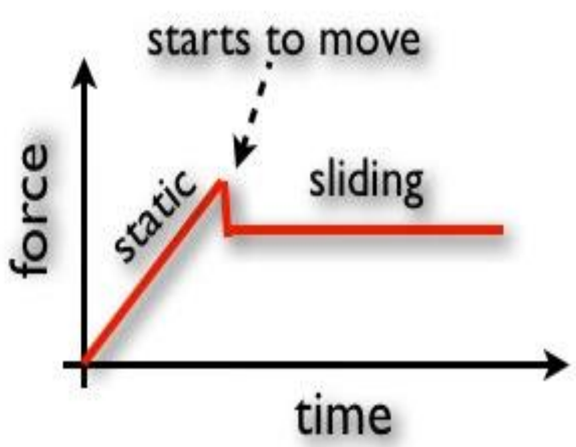
Normal Force

The force exerted by a surface on a body in contact with it can be resolved into two components, one perpendicular to the surface and one tangential to the surface. The perpendicular component is called the *normal* force and the tangential component is called *friction*.

Friction

When a body slides across a surface, the friction force is directed opposite to the instantaneous velocity and has magnitude μN.





In summary, we take the force of friction f to behave as follows:

1. For bodies not in relative motion,

$0 \leq f \leq \mu N.$

f opposes the motion that would occur in its absence.

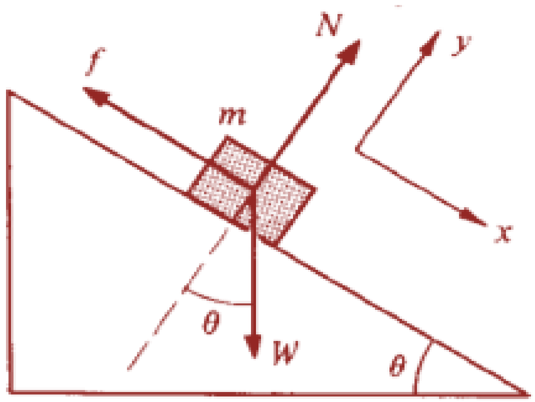
2. For bodies in relative motion,

$f = \mu N.$

f is directed opposite to the relative velocity.

1-Block and wedge with friction

A block of mass m rests on a fixed wedge of angle θ. The coefficient of friction is μ. Find the value of θ at which the block starts to slide.



In the absence of friction, the block would slide down the plane; hence the friction force f points up the plane.

$$m\ddot{x} = W \sin\theta - f \qquad m\ddot{y} = N - W\cos\theta = 0.$$

When sliding starts, f has its maximum value μN, and $\ddot{x} = 0$. The equations then give

$$W \sin\theta_{\max} = \mu N$$
$$W \cos\theta_{\max} = N.$$

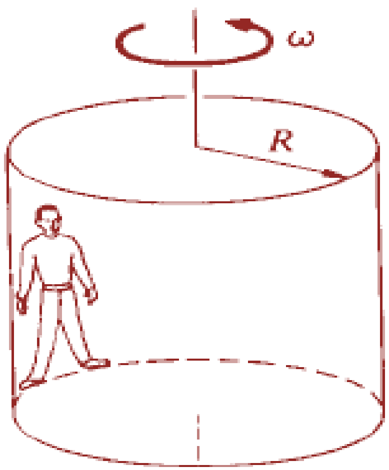
$$\Rightarrow \quad \tan\theta_{\max} = \mu$$

Notice that as the wedge angle is gradually increased from zero, the friction force grows in magnitude from zero toward its maximum value μN, since before the block begins to slide we have

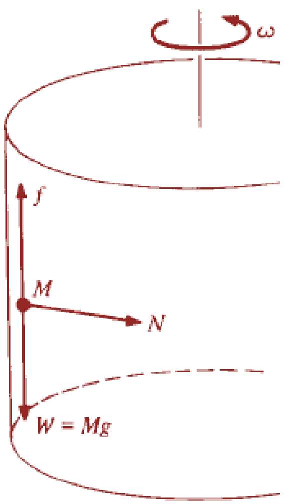
$$f = W\sin\theta \qquad \theta \le \theta_{\max}.$$

2-Spinning Terror

The Spinning Terror is an amusement park ride—a large vertical drum which spins so fast that everyone inside stays pinned against the wall when the floor drops away. What is the minimum steady angular velocity ω which allows the floor to be dropped away safely?



Suppose that the radius of the drum is R and the mass of the body is M. Let μ be the coefficient of friction between the drum and M. The forces on M are the weight W, the friction force f, and the normal force exerted by the wall, N, as shown below.



The radial acceleration is $R\omega^2$ toward the axis, and the radial equation of motion is

$$N = MR\omega^2 \text{ static friction, } f \leq \mu N = \mu MR\omega^2$$

M to be in vertical equilibrium,

$$f = Mg, \quad Mg \leq \mu MR\omega^2$$

$$\omega^2 \geq \frac{g}{\mu R} \Rightarrow \boxed{\omega_{\min} = \sqrt{\frac{g}{\mu R}}}$$

The larger the radius R, or the larger the coefficient of friction, the slower the cylinder can rotate to hold the object onto the wall.

KK problem

2.6 In a concrete mixer, cement, gravel, and water are mixed by tumbling action in a slowly rotating drum. If the drum spins too fast the ingredients stick to the drum wall instead of mixing.

Assume that the drum of a mixer has radius R and that it is mounted with its axle horizontal. What is the fastest the drum can rotate without the ingredients sticking to the wall all the time? Assume $g = 32\ \mathrm{ft/s^2}$.

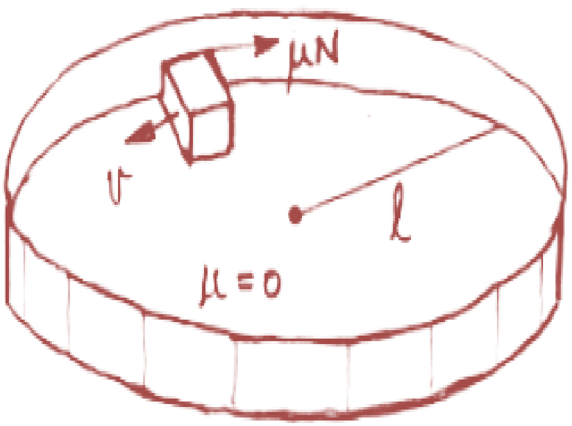
2.35 KK problem

A block of mass m slides on a frictionless table. It is constrained to move inside a ring of radius l which is fixed to the table. At $t = 0$, the block is moving along the inside of the ring (i.e., in the tangential direction) with velocity v_0. The coefficient of friction between the block and the ring is μ.

a. Find the velocity of the block at later times.

Ans. $v_0/[1 + (\mu v_0 t/l)]$

b. Find the position of the block at later times.



There are two forces acting on the block in the plane of the table. They are the normal force exerted by the ring and the frictional force.

normal force merely makes the block move in the circular path that we assumed. $N = \frac{mv^2}{l}$

We can now write the equation of motion for the particle by Newton's second law

$$m\frac{dv}{dt} = -\mu N = -\frac{\mu m v^2}{l} \Rightarrow \frac{dv}{dt} = -\frac{\mu}{l}v^2$$

$$\int_{v_0}^{v(t)} \frac{dv}{v^2} = -\int_0^t \frac{\mu\, dt}{l} \quad \Rightarrow \quad \frac{1}{v_0} - \frac{1}{v(t)} = -\frac{\mu t}{l}$$

$$v(t) = v_0\left(1 + \frac{\mu v_0 t}{l}\right)^{-1}$$

$$\theta(t) = \frac{1}{\mu}\ln\left(1 + \frac{\mu v_0}{l}t\right)$$

Viscosity

A body moving through a liquid or gas is retarded by the force of viscosity exerted on it by the fluid. Unlike the friction force between dry surfaces, the viscous force has a simple velocity dependence; it is proportional to the velocity.

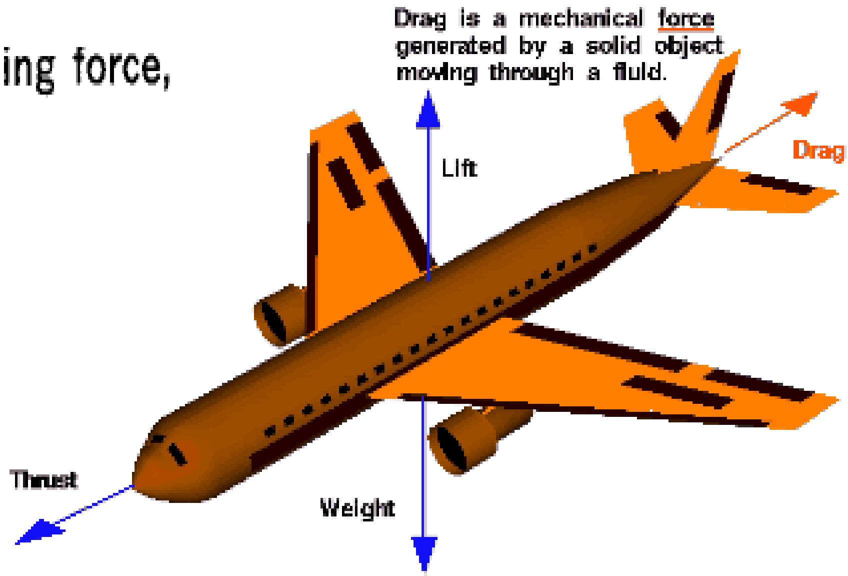
Viscosity arises because a body moving through a medium exerts forces which set the nearby fluid into motion. By Newton's third law the fluid exerts a reaction force on the body.

We can write the viscous retarding force in the form

$\mathbf{F}_v = -C\mathbf{v}$, where C is a constant which depends on the fluid

When the only force on a body is the viscous retarding force, the equation of motion is

$$\boxed{-C\mathbf{v} = m\frac{d\mathbf{v}}{dt}}$$

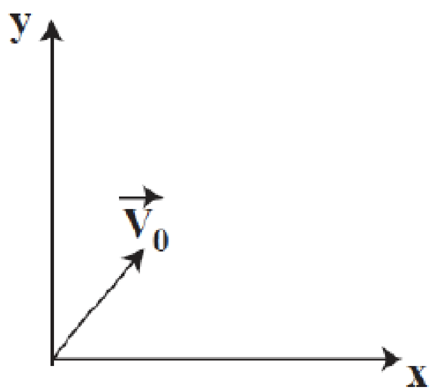


Air drag

For a cannon ball firing at an initial velocity $\vec{v}_0$, suppose that there is an air drag force that is given as $\vec{f} = -mk\vec{v}$, where $\vec{v}$ is velocity vector, m is the mass of the cannon ball, and k is a constant. What's velocity as a function of time during the flight?

$$\vec{F}_{net} = -mg\,\hat{\mathbf{j}} - mk\vec{v} = -(mg + mkv_y)\,\hat{\mathbf{j}} - mkv_x\,\hat{\mathbf{i}},$$

$$-kv_x\,\hat{\mathbf{i}} - (mg + mkv_y)\,\hat{\mathbf{j}} = m\left(\frac{dv_x}{dt}\,\hat{\mathbf{i}} + \frac{dv_y}{dt}\,\hat{\mathbf{j}}\right),$$



Treat each component separately and we have

$$-kmv_x = m\frac{dv_x}{dt}, \qquad (1)$$

$$-mg - mkv_y = m\frac{dv_y}{dt} \qquad (2)$$

The solutions are given as follows.

$$v_x = v_{x0}e^{-kt}, \; v_y = (v_{y0} + \frac{g}{k})e^{-kt} - \frac{g}{k}.$$

The air drag causes the horizontal velocity of the ball to decrease with time, and also a terminal velocity for vertical component, $-g/k$. **Exercise????**

2.36 KK Problem

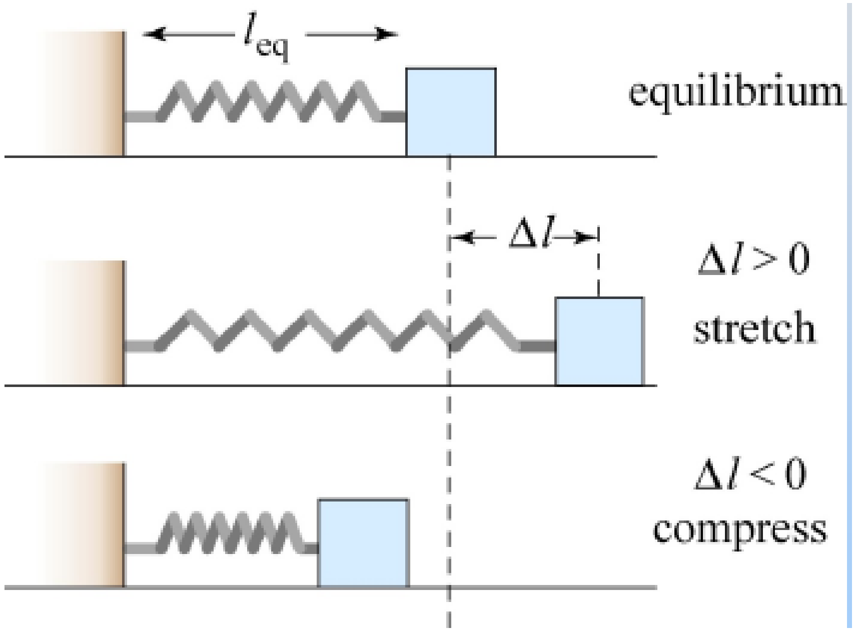
A particle of mass m moving along a straight line is acted on by a retarding force (one always directed against the motion) $F = be^{\alpha v}$, where b and α are constants and v is the velocity. At $t = 0$ it is moving with velocity v_0. Find the velocity at later times.

Ans. $v(t) = (1/\alpha) \ln [1/(\alpha bt/m + e^{-\alpha v_0})]$

Empirical Force Law: Hooke's Law

Consider a mass m attached to a spring

Stretch or compress spring by different amounts produces different accelerations



Hooke's law holds within some reasonable range of extension or compression F=|k| *l*

Direction: restoring spring to equilibrium

Hooke's law holds within some reasonable range of extension or compression