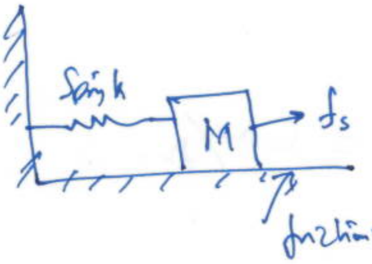
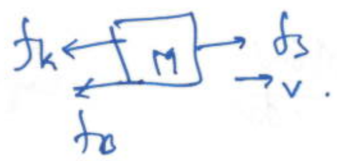
EE201: Mechanical & Electrical Analogy:



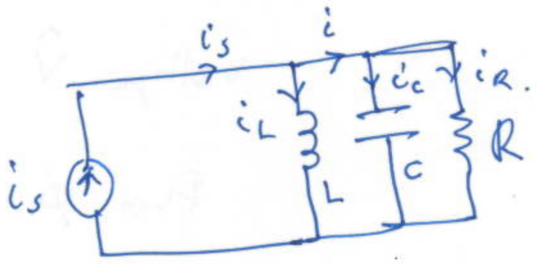


$$f_s - f_k - f_B = \frac{d(Mv)}{dt}.$$

$f_B \to$ friction force is function of velocity

$f_k \to$ spring force is function of displacement.

$$f_s = f_k(x) + f_B(v) + \frac{d(Mv)}{dt}.$$



~~i_s & i_L ...~~

$i_s = i + i_L$

$i = i_c + i_R$

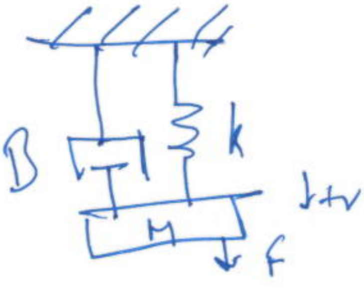
$i_s = i_c + i_L + i_R$

with KCL

$$i_s = i_L + i_c + i_R$$

$$= \frac{1}{L}\int V(t)\,dt + C.\frac{dV}{dt} + V/R.$$

$\equiv$



Compare both Systems:

Mass $f = \frac{M dv}{dt}$ Capacitor $i = C\, dv/dt$.

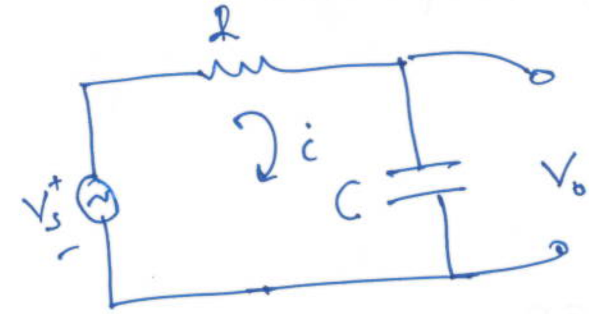
Friction $f = Bv$ Conductor $i = Gv$

Spring $f(t) = f(0) + k\int_0^t v(t')\,dt'$ Inductor $i(t) = i(0) + 1/L \int_0^t v(t')\,dt'$

This sort of relationship is possible for other systems as well.

Mechanical systems	Electric Circuits
Force f_s	Current i_s
velocity v	voltage $\hat{v}$
Displacement x	Flux ϕ
Spring	Inductor
Friction	Resistor
Mass	Capacitor..

RC System:



output or response.

$$V_o = V_c$$

$$V_s = i.R + V_c$$

$$= C.\frac{dV_c}{dt}.R + V_c.$$

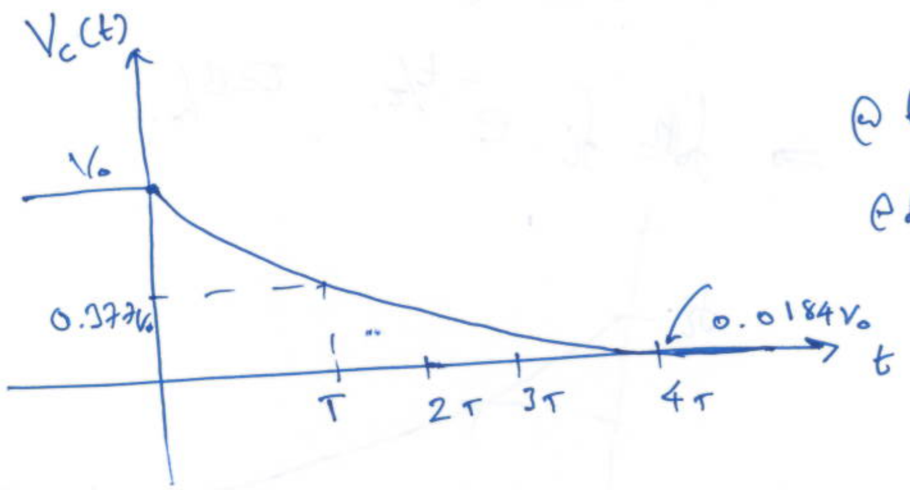
$$= (RC.D + 1)\,V_c. \qquad \tau = RC.$$

$\underline{V_s}$ $\quad V_s = (\tau.D+1)\,V_c.$

$$\text{or} \quad \frac{V_c}{V_s} = \frac{1}{(\tau.D+1)}.$$

if Capacitor as initial voltage of V_o

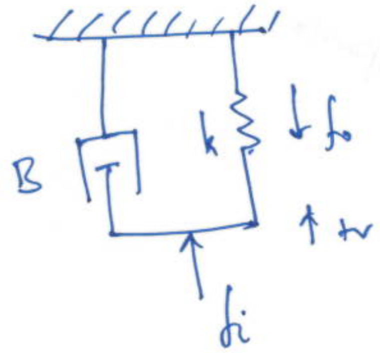
$$V_c(t) = V_o\, e^{(-1/RC)\,t}. \qquad \underline{t \geq 0.}$$



@ $t = \tau$ $\quad$ 38% of V_o

@ $t = 4\tau$ $\quad$ 2% of V_o

Spring - Damper System.



$$f_i - f_B - f_k = 0.$$

$$f_i - B.v - k.x = 0. \qquad f_k = kx = f_o.$$

$$f_i - B.\frac{dx}{dt} - f_o = 0 \qquad f_o = kx$$

$$dx/dt = 1/k \; df_o/dt.$$

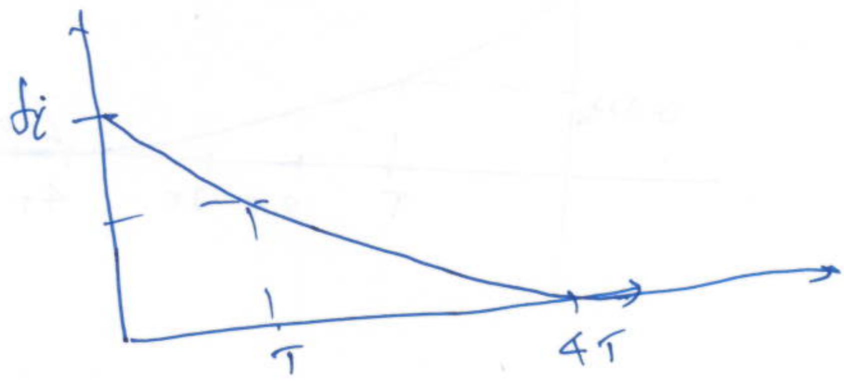
$$f_i - \frac{B}{k}.\frac{df_o}{dt} - f_o = 0.$$

$$f_i = \left(B/k . df_o/dt + f_o\right)$$

$$= (\tau.D + 1)\, f_o \qquad \tau = B/k.$$

$$f_o/f_i = \frac{1}{(\tau D + 1)}.$$

$$\Rightarrow f_o(t) = f_i . e^{-t/\tau}. \qquad \tau = B/k.$$

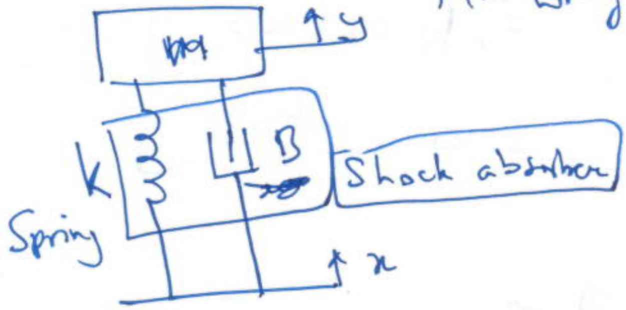


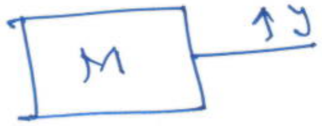
Modelling Suspension:

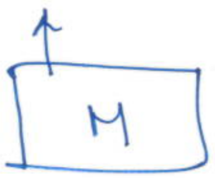
Automotive Suspension

Simple design (in reality there will be a tyre it's own Spring - mass damper system).

M - Wt of Car







$$m\ddot{y} = B(\dot{x}-\dot{y}) + k(x-y)$$

$$m\ddot{y}+B\dot{y}+ky = (B\dot{x}+kx)$$

damper & Spring [Suspension parameters]

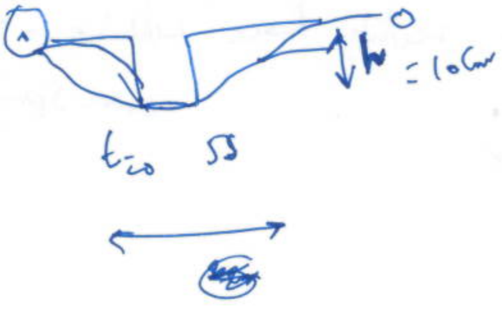
vertical accel of passengers in Car.

$B\dot{x}$ → how fast ht of road changes

kx → function of Road ht.

Based on this model, we get a sense of what our passenger feels when driving over different road Condition.

For Ex:- driving over a sidewalk curb → step function.



$$x(t) = \begin{cases} 0 & t < 0 \\ h & 0 < t < 5 \\ 0 & t > 5 \end{cases}$$ Ramp.

$\dot{x} \to 0.$

$$m\ddot{y} + B\dot{y} + ky = \cancel{B\dot{x}}^{\to 0} + kx$$

$$\ddot{y} + B/m\,\dot{y} + k/m\,y = k/m\,x.$$

$$r_1, r_2 = \frac{-B/m \pm \sqrt{\frac{B^2 - 4km}{m^2}}}{2} \qquad b^2 - 4ac = \frac{B^2}{m^2} - \frac{4.k.m}{m^2}$$

$$= \frac{-B \pm \sqrt{B^2 - 4km}}{2m} \qquad = \left(\frac{B^2 - 4km}{m^2}\right).$$

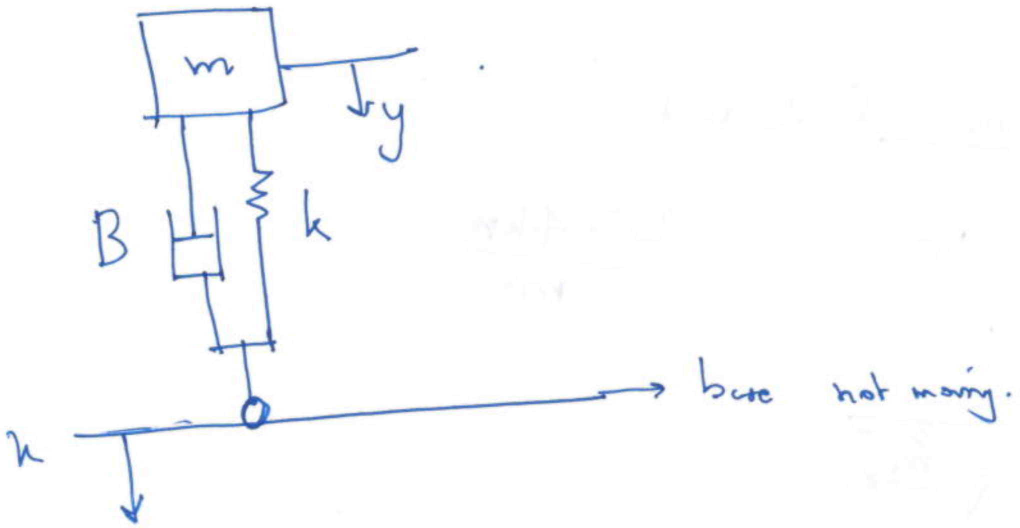
$b^2 - 4ac > 0 \Rightarrow y_h = c_1 e^{r_1 t} + c_2 e^{r_2 t}.$

$b^2 - 4ac = 0 \Rightarrow y_h = c_1 e^{rt} + c_2 t.e^{rt}$

$b^2 - 4ac < 0.$

What if there are multiple speed breaker

$\Rightarrow$ $x = A \cos \omega t.$



$$m \frac{d^2y}{dt} + B.\frac{dy}{dt} + ky = 0.$$

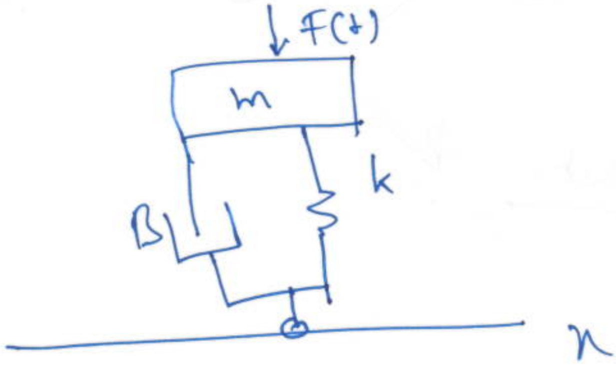
if base also moving

$$m \frac{d^2y}{dt} + B.\frac{dy}{dt} + ky = B.\frac{dx}{dt} + k.x.$$

~~If explained force~~

If external force is acting,

$$\ddot{y} + B/m(\dot{y}-\dot{x}) + k/m(y-x) = F(t).$$



Let's Consider. Smooth road.

$$\Rightarrow b^2 - 4ac = \frac{B^2 - 4km}{m^2}$$

① over damped $B^2 > 4km$.

roots $\dfrac{-\frac{b}{m} \pm \sqrt{\frac{B^2-4km}{m^2}}}{2}$

$= -\frac{b}{2m} \pm \frac{\sqrt{B^2-4km}}{2m}$.

$$y_h = c_1 e^{\frac{-b - \sqrt{B^2-4km}}{2m} t} + c_2 e^{\left(-b/2m + \frac{\sqrt{B^2-4km}}{2m}\right) t}$$

$$= e^{-b/2m}\left[c_1 e^{-\frac{\sqrt{B^2-4km}}{2m} t} + c_2 e^{\frac{\sqrt{B^2-4km}}{2m} t} \right]$$

$r_1 = -b/2m$, $r_2 = -B/2m$

System behavior:

(A) Critical damping: $\Rightarrow B^2 = 4km.$

↳ a critically damped system converges to zero as fast as possible without oscillating. An example → door hinge.

$$y_h = (C_1 + C_2 t)\, e^{(b/2m)t}.$$

$\Rightarrow B/_{2m}$ determines speed of damping

(B) Over-damping $\Rightarrow$ $\dfrac{B}{2\sqrt{mk}} > 1.$

$$B^2 > 4mk.$$

Over damped door takes longer to close.

(C) Under damped. $0 \leq \dfrac{B}{2\sqrt{mk}} < 1$

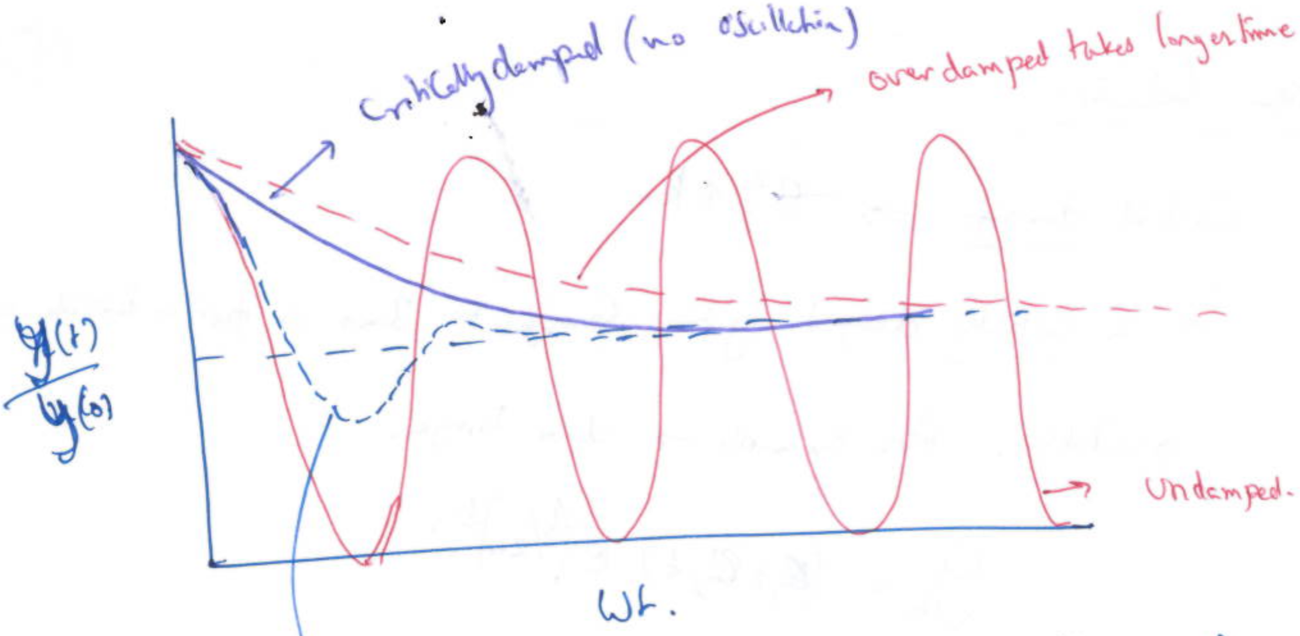
$$\Rightarrow 0 \leq B^2 < 4mk.$$

Complex roots.

The system will oscillate @ natural damped frequency ω_d, which is a function of natural frequency and damping ratio.

An under damped would close quickly & hit the door!

or oscillate if swinging door.



Under damped will oscillate @ ω_d (damping frequency)

$\boxed{\text{Undamped}} \rightarrow \underline{B = 0.}$

$$\ddot{y} + k/m\, y = 0.$$

Solution is $\frac{d^2y}{dt^2} = -k/m\, y$

$$\omega_n^2 = k/m.$$

$$d^2y/dt^2 + \omega_n^2 y = 0.$$

roots $= \pm i\omega_n$

$$\underline{y(t) = \cos\omega_n t + \frac{1}{\omega_n}\sin\omega_n t.}$$

amplitude doesn't decrease with time.

$$\underline{\omega_n = k/m \uparrow \text{ as } k\uparrow\ m\downarrow}.$$

frequency of swinging