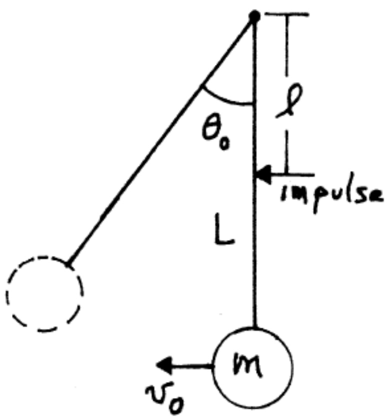
Tutorial Sheet - 6

PHYSICS-101

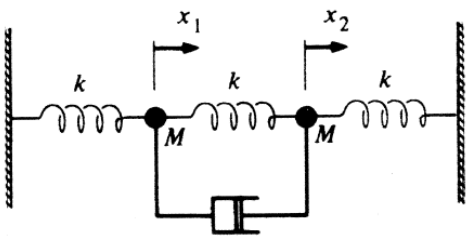
Assignment problems: 2, 4, 6, 10 Date: 07.10.2014

1) A 0.45 kg mass is attached to a spring with a force constant of 26.0 N/m and released from rest a distance of 3.25 cm from the equilibrium position of the spring. (a) What is the period of the mass? (b) Use conservation of energy to find the speed of the mass when it is half way to the equilibrium position. (c) What is the maximum speed of the mass? (d) What is the magnitude of the maximum acceleration of the mass? (T = 0.83 s, V = 0.21 m/s, V_{max} = 0.25 m/s, a_{max} = 1.9 m/s^2)

2) The pendulum of a grandfather's clock activates an escapement mechanism every time is passes through the vertical. The escapement is under tension (provided by a hanging weight) and gives the pendulum a small impulse distance *l* from the pivot. The energy transferred by the impulse compensates for the energy dissipated by friction, so that the pendulum swings with a constant amplitude. Find out, the impulse needed to sustain the motion of a pendulum of length *L* and mass *m*, with an amplitude of swing θ_0 and quality factor Q? {Ans: Impulse $= m\left(\frac{\pi\theta_0}{2Q}\right)\sqrt{gL}$}



3) Two particles, each of mass M, are hung between three identical springs. Each spring is massless and has spring constant *k*. The masses are connected as shown to a dashpot of negligible mass. The dashpot exerts a force of *b*v, where v is the relative velocity of its two ends. The force opposes the motion. Let x_1 and x_2 be the displacement of the two masses from equilibrium.

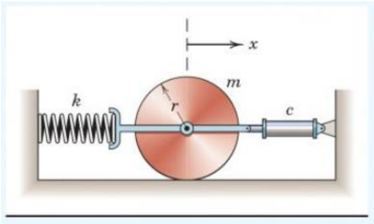


   (a) Find the equation of motion for each mass. (b) Show that the equation of motion can be solved in terms of the new dependent variables $y_1 = x_1 + x_2$ and $y_1 = x_1 - x_2$. (c) Show that if the masses are initially at rest and mass 1 is given initial velocity v_0, the motion of the masses after a sufficiently long time is $x_1 = x_2 = (v_0/2\omega)* \sin(\omega t)$

4) Show that for a lightly damped forced oscillator near resonance,

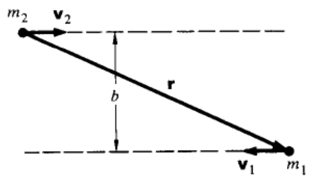
$$\frac{average\ energy\ stored\ in\ the\ oscillator}{average\ energy\ dissipated\ per\ radian} = \frac{\omega_0}{\gamma} = Q$$

5) Derive the equation of motion for the homogenous circular cylinder, which rolls without slipping. If the cylinder mass is 50 kg, the cylinder radius 0.5 m, the spring constant 75 N/m and, the damping co-efficient 10 N.s/m. Determine (a) the undamped natural frequency (b) the damping ratio (c) the damped natural frequency (d) the period of the damped system. In addition, determine x as a function of time if the cylinder is releases from rest at the position x = -0.2 m when t = 0.



6) Let a particle be subject to an attractive central force of the form $f(r)$ where r is the distance between the particle and the centre of the force. (a) Find $f(r)$, if all circular orbits are to have identical areal velocities, $\dot{A}$. (b) a particle in a central field moves in the spiral orbit $r = c\theta^2$ Determine the force function. (c) In part (b), determine how the angle θ varies with time.

7) Two non-interacting particles m_1 and m_2 move towards each other with velocities $\mathbf{v_1}$ and $\mathbf{v_2}$. Their paths are offset by a distance b as shown in the sketch. Investigate the equivalent one body description of this system.



8) A satellite of mass m orbits the earth in a circle of radius r_0. One of its engines is fired briefly toward the center of the earth, changing the energy of the satellite but not its angular momentum. Find the new orbit of the satellite?

9) A satellite of mass m = 2,000 kg is in elliptic orbit about the earth. At perigee (closest approach to the earth) it has an altitude of 1,100 km and at apogee (farthest distance from the earth) its altitude is 4,100 km. What are the satellite's energy E and angular momentum L? How fast is it traveling at perigee and at apogee?

10) A space vehicle is in circular orbit about the earth. The mass of the vehicle is 3,000 kg and the radius of the orbit is $2R_e$. = 12,800 km. It is desired to transfer the vehicle to a circular orbit of radius 4R. (a). What is the minimum energy expenditure required for the transfer? (b) An efficient way to accomplish the transfer is to use a semi-elliptical B orbit (known as a Hohmann transfer orbit), as shown. What velocity changes are required at the points of intersection, A and B? *Useful info: ISRO has used some of these principles of problems 9 and 10 to put MOM in to the Mars orbit which resulted in substantial fuel savings!!!*