PH101
Lecture16

02.09.14

Recap:

1. $\oint \vec{F} . d\vec{s}$ Represents work done by the component of **F** in the direction of displacement

   Or by the component of displacement in the direction of force **F**

   Physical word "Work" means the above integral and not the physiological definition of work that we normally use in ordinary circumstances

e.g. if one holds a 20 kg weight off the ground for a while, physics wise he/she is doing no work though he sweats and breath harder as if he/she were running up a flight of stairs!

But running up the stairs amounts to doing work in physics

Clearly physiological definition differs from physical definition!!

We may ask why do we want to calculate work?

Work done on a particle by a force or the resultant of forces acting on it equals change in its K.E. $\Rightarrow$ if an object is pushed, it picks up speed

$$\frac{1}{2}m\Delta v^2 = \vec{F}.\Delta\vec{s} \Rightarrow \Delta v^2 = \frac{2}{m}\vec{F}.\Delta\vec{s}$$

2. Considering a 1-D system like mass on a spring

difference in P.E. ΔU is given by

$$U_a - U_b = -\int_{s_a}^{s_b} F(x)\,dx$$

As the particle moves from x to $x + \Delta x$, the change ΔU:

$$U(x+\Delta x) - U(x) \equiv \Delta U = -\int_{x}^{x+\Delta x} F(x)\,dx$$

Since, Δx is very small,

$F(x+\Delta x) \approx F(x)$ may be assumed to be constant across

x to $x + \varphi x$

$$\Rightarrow \Delta U \approx -F(x)(x+\Delta x - x) = -F(x)\Delta x$$

$$\Rightarrow F(x) = -\frac{\Delta U}{\Delta x}$$

$$\text{Lt } \Delta x \to 0, \; F(x) = -\frac{dU}{dx}$$

$$U = -\int F(x)\,dx$$

$\Rightarrow$ P.E. is the negative integral of the force

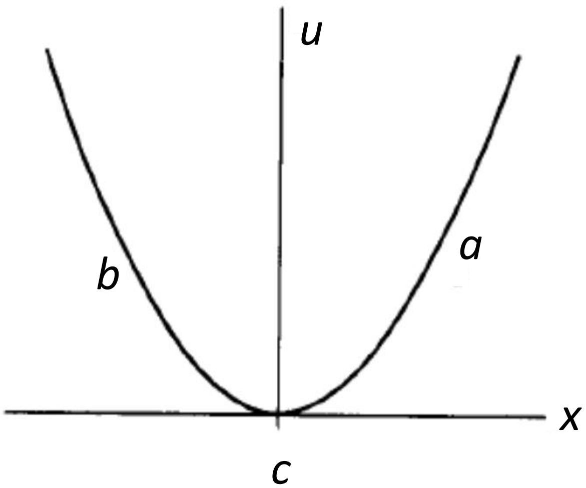
or

$\Rightarrow$ Force is the negative derivative of the P. E.

The result

$$F(x) = -\frac{dU}{dx}$$

is extremely useful in visualizing the stability of a system



Work needed to compress a linear spring a distance x from its equilibrium point is

$$U(x) = k\,x^2/2$$

i.e. P.E. is a parabolic relation

At *a*, $\text{slope}: \dfrac{dU}{dx} > 0$ $\Rightarrow$ Force is negative

At *b*, $\text{slope}: \dfrac{dU}{dx} < 0$ $\Rightarrow$ Force is positive

At *c*, $\text{slope}: \dfrac{dU}{dx} = 0$ $\Rightarrow$ Force vanishes

$\Rightarrow$ Force is directed towards the origin irrespective of which way the particle is displaced

Force vanishes only at the origin

Minimum of the P. E. coincides with the equilibrium position

$\Downarrow$

A stable equilibrium because any displacement of the particle in any direction

$\downarrow$

Leads to a force which pushes the particle towards its equilibrium point

If $\dfrac{dU}{dx} = 0$ occurs at a maximum of *U*, equilibrium is not stable

$\downarrow$ Why ?

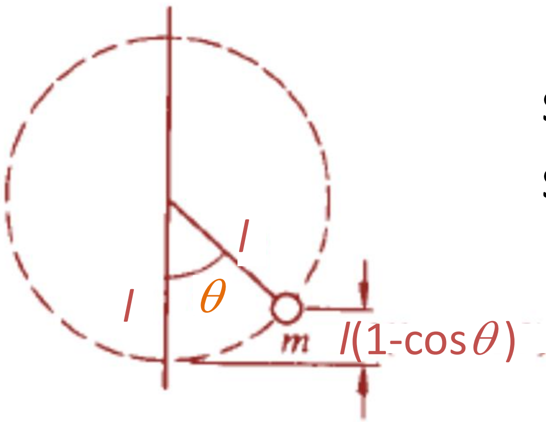
Positive displacement produces a positive force

$\Downarrow$

Which tends to increase the displacement

Likewise a negative displacement produces a negative force

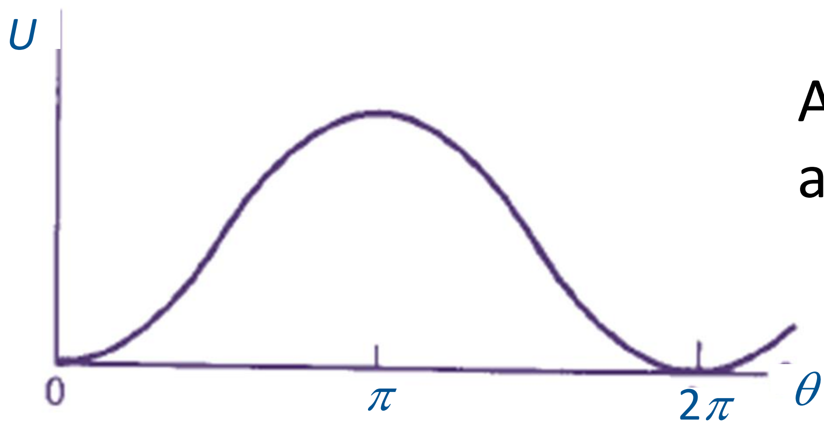
Both positive and negative forces $\Rightarrow$ Larger displacement



If we take the P.E. of a pendulum of length *l* supporting a mass *m* to be 0 at the bottom of its swing we have

$$U = mgl(1-\cos\theta)$$

It is in equilibrium for θ = 0 or = π



At θ = 0, it will hang downward as long as we want

But at $\theta = \pi$, it will not hang vertically up for long . Why?

Because though $\dfrac{dU}{dx} = 0$

U has a max there $\Rightarrow$ equilibrium is *not stable*

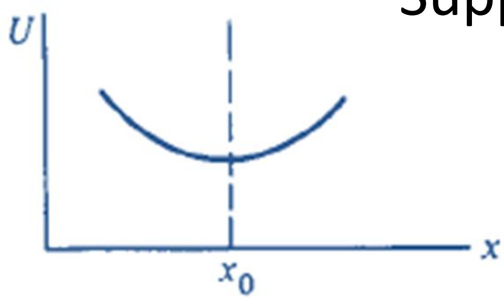
$\Rightarrow$ Sketch of a P. E. curve is extremely useful because it yields information about stability almost <u>intuitively obvious</u>

$\Rightarrow$ At the minimum, it is a point of stable equilibrium and at the maximum it is a point of unstable equilibrium

$$\Downarrow$$

At the bottom of the PE valley the system is stable

At the top of the PE hill, the system is unstable

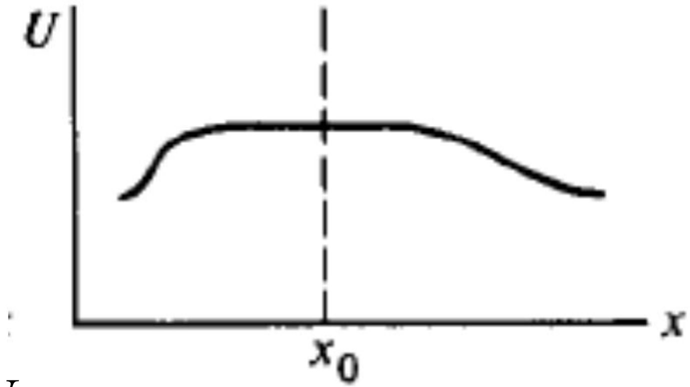


Suppose point x_0 corresponds to equilibrium where

$$\frac{dU}{dx} = 0$$

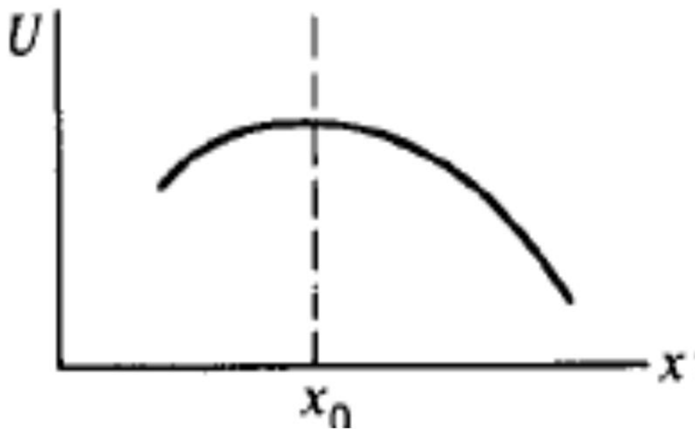
To test for stability, we need to find out if x_0 corresponds to a max or a min for which one needs to find if

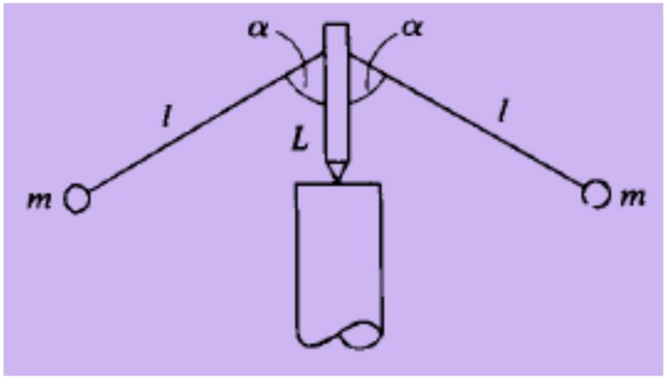
$$\frac{d^2U}{dx^2} > 0 \Rightarrow \quad \text{the equilibrium is stable}$$



$$\frac{d^2U}{dx^2} = 0 \text{ and if higher order derivatives are also } 0 \Rightarrow$$

U is const. around x_0 $\Rightarrow$ the system is in a cond. of neutral stablility





Teeter toy

$$\frac{d^2U}{dx^2} < 0 \Rightarrow \text{the equilibrium is unstable}$$

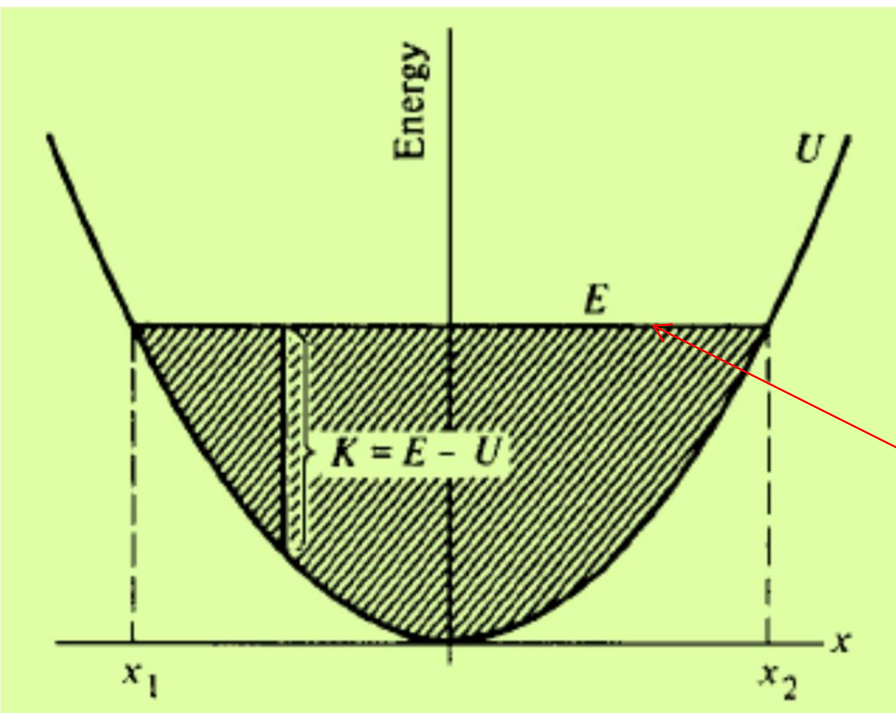
Utility of energy diagram for motion in 1D:

E and *U* are plotted as a function of *x*

$\Rightarrow$ K.E., $K = E - U$ is obtainable by inspection

Since K.E. can not be negative, the motion is constrained to regions where

$$U \le E$$



Energy diagram for a harmonic oscillator whose P.E. is given by

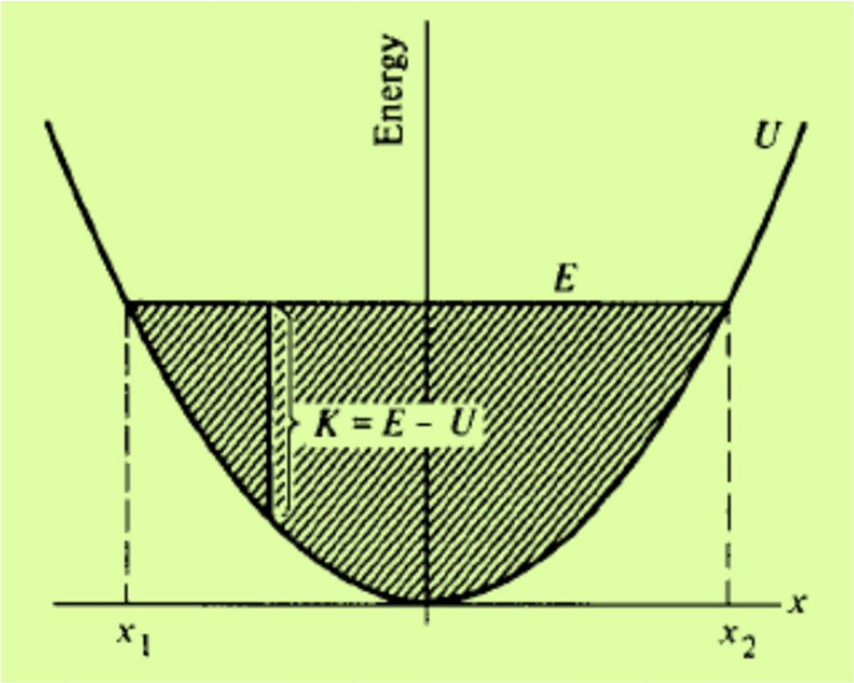
$$U = \frac{1}{2}kx^2$$

is a parabola centered at the origin

E is represented by a horizontal line

The motion is constrained within the shaded region where

$$E \ge U$$



The limits of the motion at $x_{1,2}$ are called turning points

K of the system is greatest at the origin and as it moves away in either direction the speed is slowed down by the spring and comes to rest at either of $x_{1,2}$

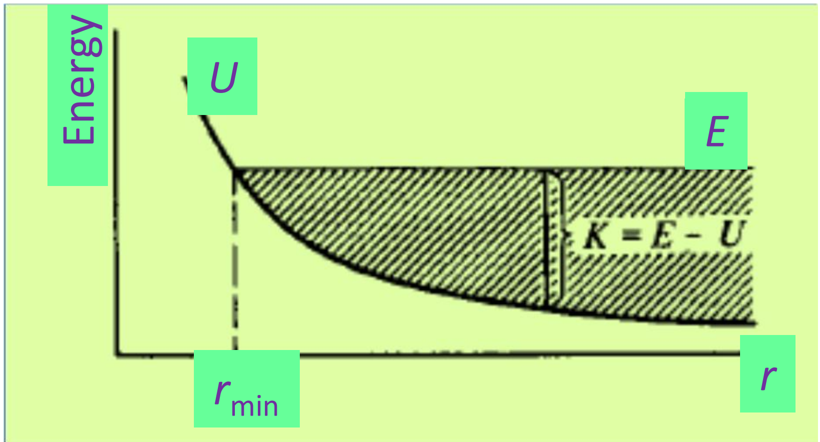
The reverse process then takes place i.e. it moves with an increasing K.E. towards the origin and the cycle gets repeated

It is a good example of bounded motion

As E increases , $x_{1,2}$ move farther and farther off but the particle can never move away freely

With decrease in E, amplitude decreases until for $E = 0$, the particle lies at rest at $x = 0$

Case when *U* does not increase indefinitely



Distance of closest approach

Consider the case of a particle constrained to a radial line

It is acted upon by a repulsive inverse square law force

$$\vec{F} = A\frac{\hat{r}}{r^2}$$

$$\Rightarrow U = \frac{A}{r}$$

But the motion is not bounded for large *r*

Since *U* decreases with *r*

If the particle is shot towards the origin, KE would gradually reduce since *U* correspondingly increases until it comes to rest and then moves out towards ∞

Interaction between two atoms

At large separations, they weakly attract each other with van der Waals force

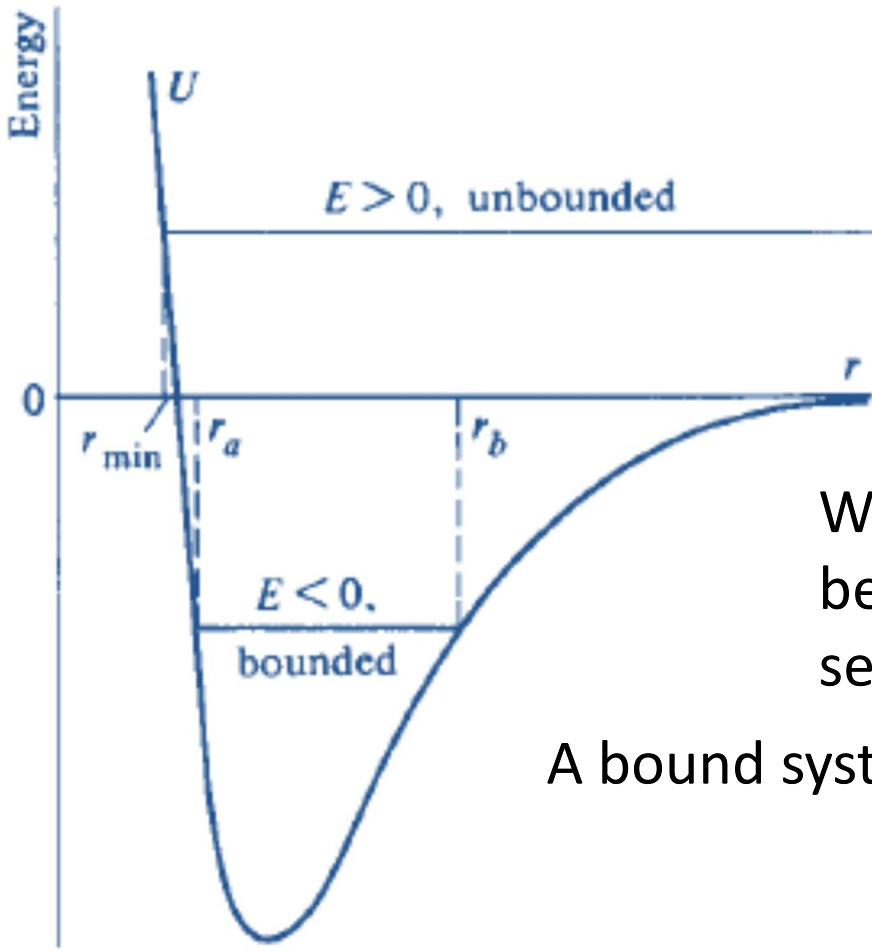
$$\propto \frac{1}{r^7}$$

As they approach, electron cloud start overlapping producing strong forces – either attractive or repulsive depending on details of electron configuration

U decreases with decrease in *r*

At very short distances they repel each other $\Rightarrow$ *U increases* rapidly with decrease in *r*

Energy diagram for a typical attractive two atoms



For $E > 0$ unbounded

Due to the very steep P.E. curve

r_{min} does not change appreciably as E is decreases

When E is negative, motion is bounded between r_a & r_b for both small & large separations

A bound system of two atoms is a molecule!

Typical diatomic molecule energy diagram

If two atoms collide they can not form a molecule unless energy is lost by some means to make E negative.

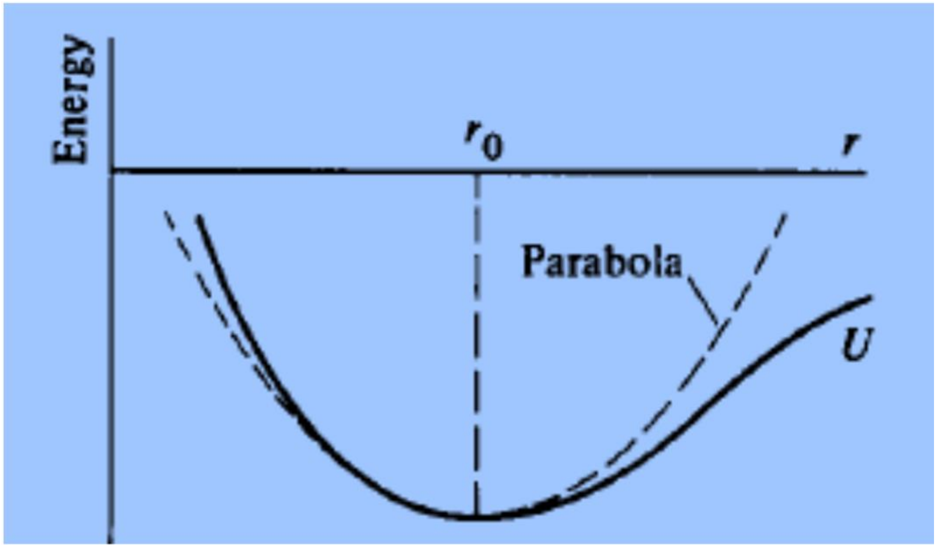
In general a third body is necessary for carrying off the excess energy

Example, atomic hydrogen is quite stable in gas phase, and if a piece of platinum is inserted in there, the atoms join to form molecules

Hydrogen atoms tightly adhere to the surface of Pt and if a collision occurs between two atoms on the surface the excess energy is released to the surface and the molecule which is not strongly attracted to the surface leaves

Energy delivered to the surface is so large that Pt glows brightly

Small oscillations in a bound system



Every bound system oscillates like a harmonic oscillator if it is slightly perturbed from its equilibrium position

Near the min, the shape of the curve is parabolic like that of a harmonic oscillator potential

If the total energy is low enough that motion is restricted to the Parabolic region, system will indeed behave like a harmonic oscillator

This can be verified mathematically

Any well behaved function *f* (*x*)

can be expanded in a Taylor's series about a point $x_0 \Rightarrow$

$$f(x) = f(x_0) + (x - x_0) f^{/}(x)\Big|_{x_0} + \frac{1}{2}(x - x_0)^2 f^{//}(x)\Big|_{x_0} + \ldots\ldots$$

Likewise expanding *U* (*r*) about the point r_0

$$f(r) = U(r_0) + (r - r_0)\underbrace{\frac{dU}{dr}\bigg|_{r_0}}_{\downarrow} + \frac{1}{2}(r - r_0)^2 \frac{d^2U}{dr^2}\bigg|_{r_0} + \ldots\ldots$$

= 0 because *U* is minimum at $r = r_0$

For sufficiently small displacements, terms beyond the 3rd term can be ignored $\Rightarrow$

$$U(r) \cong U(r_0) + \frac{1}{2}(r - r_0)^2 \frac{d^2U}{dr^2}\bigg|_{r_0} \equiv \text{const} + \frac{1}{2}k(r - r_0)^2$$