First Mid Term Exam @MEC

When: 08 of September 2014

What: MA 101 : Calculus and Introduction to analysis

Problem 1 E is a subset of $\mathbb{R}$ here.

1.a) Answer is: False. ($E = \emptyset$)

Proof: If $m \in \mathbb{R}$ is maximum of E, thanks to definition Def. 1.1, we know that $m \in E$. Or E is empty ($E = \emptyset$), so $m \in E$ is a contradiction.

Conclusion: E cannot have a maximum.

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1.b) Answer is: False.

Proof: With a counter example, like $E = (-\infty, 0]$, E has a maximum, 0, but no lower bound. Therefore, the statement "if E has a maximum then it is lower bounded" is wrong. (in the general case).

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1.c) Answer is: False.

Proof: Again with a simple counter example.

If $E = [0, 1) = \{x : x \in \mathbb{R}, 0 \leq x < 1\}$

E has a supremum equal to 1,

but no maximum (because $1 \notin E$).

Hence, the statement "if E has a supremum, then it has a maximum" is wrong (in the general case). $\circ$

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1.d) Answer is: True.

Proof: (Here, an example IS NOT ENOUGH)

Assume (as given) that E has a minimum.

Let $m \in \mathbb{R}$ be this minimum.

With Def. 1.1 we know that

- $m \in E$ (useless here)

- and $\forall x \in E, m \leq x$.

Therefore m is already a good lower-bound for E: it is smaller than or equal to any element x of E.

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Conclusion. E is lower bounded

+1/1 Bonus for cleanliness and presentation.