NB : this document sums up some important techniques you should already be familiar with!

Compute a finite sum or a finite product

We know how to compute finite sums and products:

- thanks to an inductive reasoning,
- thanks to a changing of index, or a telescoping,
- by using a geometric identity or the binomial theorem (for a sum).

Changing of index

We can compute $\sum_{i=0}^{q-p} x_{p+i}$ by changing from i to the new index $k := p + i$:

(1) we write $k = p + i$,

(2) we replace i by $k - p$ in the sum,

(3) we precise the new bound for k: when i varies from 0 to $q - p$, k varies from p to q.

Exercise 1: Let $0 \leqslant p \leqslant n$, show that $\prod_{k=0}^{p-1} \frac{n-k}{p-k} = \binom{p}{n}$.

Deal with inequalities

Bound a sum To upper-bound (resp. lower-bound, bound) the sum $\sum_{k=0}^{n} x_k$,

(1) for $k \in \{0, \dots, n\}$, find an upper-bound (resp. lower-bound, bound) M_k for the term x_k,

(2) then add each term in the right order: $\sum_{k=0}^{n} x_k \leqslant \sum_{k=0}^{n} M_k$.

Exercise 2: Prove: $\forall n \in \mathbb{N}^*, \frac{1}{n} \leqslant \sum_{k=0}^{n} \frac{1}{n^2+k^2} \leqslant \frac{n+1}{n^2}$.

Compute an integer part To compute an integer part, we usually come back to the definition, by bounding the real number x by two consecutive integers. To conclude $\lceil x \rceil = n$, prove $n \leqslant x < n + 1$.

Bound the absolute value of a sum We usually use on of the two triangular inequalities.

Solve a polynomial equation

For an equation of degree lower (or equals) than 2, classical formulas can be used. For an equation of degree lower than 3, we try to decrease the degree by a factorization (obvious root, remarkable identity $(a+b)^2 = a^2+2ab+b^2$ etc) or by a changing of unknown.

Exercise 3: In $\mathbb{R}$, solve the equation: $x^3 - 9x + \frac{20}{x}$.

Solve a linear equations system

We usually use the GAUSS elimination method.