An Introduction to Dark Energy

Nihar Dalal

April 2025

1 Introduction

In 1998, two independent teams of researchers (the Supernova Cosmology Project and the High-Z Supernova Search Team) made the same exciting discovery: the universe is expanding at an accelerating rate. We've known that the universe has been expanding for a while now, thanks to the work of several astronomers (including Edwin Hubble), who found that light from distant galaxies was redshifted.

Definition

Redshift, often denoted by the letter z, is the phenomenon of light seeming redder if the source is moving away from us with some high velocity v. You may have heard of Doppler shifts, which are the basis for redshifts. Just like sirens sound longer as an ambulance drives away from you, the wavelength of light gets stretched as the source moves away from you, causing the light to look redder.

The key idea behind the work of both of these research teams (who shared the Nobel prize in 2011) was to use a specific type of supernova, called a **Type Ia** supernova, to measure distances and redshifts to these objects further out in the universe. More on that later!

The exciting thing is that we still use the same method today, although with a lot more precise data, and a better control on the systematics in making these measurements. That means the error bars get smaller, and our conclusions get stronger! Supernovae also play a very important part in the DESI results that came out last month, which is why this project is super exciting and relevant right now, and we might get a lot of questions at our poster. But have no fear, I'll try my best to prepare us as much as possible with this document :-)

2 Cosmology 101

Although cosmology is a very developed and complicated field today, it can mostly be described by a few equations. The first of these equations is thanks to Albert Einstein, and called the "Einstein Field Equations". Here it is:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1}$$

This is beyond the scope of our project, and rather quite a complicated equation to truly understand and solve (despite how simple it looks!). What it basically means is that the curvature/shape of the universe, which is given by this $G_{\mu\nu}$ thing is related to the stuff/energy inside the universe itself, represented by this $T_{\mu\nu}$ thing. There are some deep conclusions that can be further explored in a general relativity course, but we don't have to worry too much about that.

The other equation that really matters, and is somewhat derivative of this Einstein stuff, is the Friedmann equation. Before we jump to it, here's another definition.

Definition

The **scale factor** a represents the scale of the expanding universe. You can think of it as a variable that describes the age of the universe, or even the size of the universe. Today, at present day, we set the scale factor to $a = 1$. When the universe was just born, we set $a = 0$. It is very convenient to think of distances between objects in terms of a, so we have a constant scale for the actual distance between objects at different times in the universe without having to worry about the expansion that is always happening. For example, if two galaxies are 10 Mpc (big unit of distance) apart today, in the past, when $a = 0.5$, they would have really physically been $10a = 5\,\text{Mpc}$ apart. But it's more convenient for us to think of them as 10 comoving Mpc across at all points in time, because we don't have to keep switching between numbers. See Figure 1 for an example. Another useful fact that basically comes from wavelengths is that redshift and scale factor are inherently related by $a = \frac{1}{1+z}$, so the two quantities are often used interchangeably to describe the universe when it was older (and smaller).

Here's the Friedmann equation as promised:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{2}$$

$\dot{a}$ refers to the time derivative of the scale factor a. ρ is the density of the universe, k is the curvature, and Λ is a mysterious cosmological constant. G is just Newton's gravitational constant, and $H = \frac{\dot{a}}{a}$ is something we refer to as the Hubble parameter. The value of H at $a = 1$ (i.e. today) is the Hubble constant H_0, which is the current expansion rate of the universe, i.e. $\dot{a}$, which is the time derivative of the scale of the universe - that makes sense!

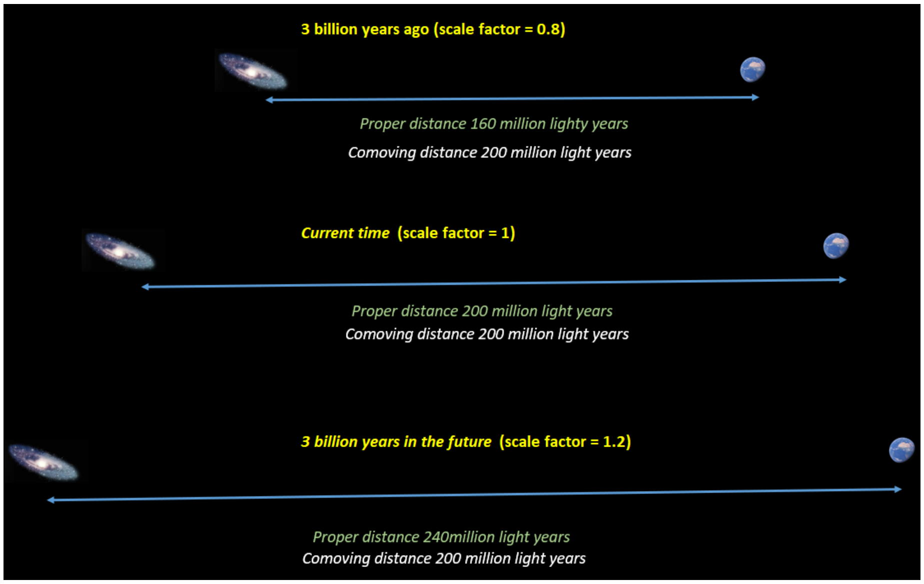

Figure 1: The use of comoving coordinates and scale factors to describe distances

2.1 Curvature

As we've discussed, the universe may have some curvature. This is a bit hard to visualize, but imagine how we live on a curved surface, but it looks flat all the time on earth. The same idea can hold in 3D, where the inherent geometry of the universe could also potentially be curved. We parametrize the curvature with this k number in the Friedmann equation, and it can take on one of three values.

- $k = 0$: A flat universe, which is nice and easy to work with, like a sheet of paper. The universe is infinite.
- $k = +1$: A spherical universe, where the universe is closed. Some paths may lead you to the same point again, seemingly parallel lines can intersect, and triangle angles can add to $> 180°$. The eventual fate of the universe is to collapse as it closes in on itself with expansion.
- $k = -1$: A hyperbolic universe, where the universe is open. All paths diverge, triangles can add to $< 180°$. The universe does still have infinite space to expand

2.2 Cosmological Constant

I mentioned that this Λ term was constant, but mysterious. It actually goes back to the Einstein equations which I told us not to worry about, where he realized (if the universe wasn't expanding, which was the belief at the time) there needed to be some constant term (on either side of the equation) so that the universe wouldn't just collapse on itself under the force of gravity, and can stay 'static'. However, people found out quickly that the universe is indeed expanding. It turns out that if you keep this constant term with an expanding universe, then the expansion gets faster over time, which could be a nice simple explanation for an accelerating expansion that we measure today.

Here's a nice way to think about it. Let's say that empty space (vacuum) has some energy associated with it. If you expand space (i.e. add more vacuum), you add more vacuum energy, and this leads to even more energy to cause the expansion, so the expansion gets faster and faster as we add more and more space! But why do we call it a constant? The name comes from the role Λ plays in the Einstein Field Equations. But for practical purposes it's constant in terms of energy density. If you look at the units of the Friedmann equation (we can ignore the units of G), Λ has essentially the same units as energy density, so the **density** is constant.

2.3 Components of the Universe

We can do some reframing of the Friedmann equation by defining a critical density ρ_c

$$\rho_c = \frac{3H^2}{8\pi G} \tag{3}$$

If the universe is more dense than ρ_c, and we assume $\Lambda = 0$, then the universe would be too dense, and collapse in on itself, like a closed universe. Likewise, if $\rho < \rho_c$, it would expand forever. Another way of thinking of critical density is solving for exactly when $k = 0$ in the Friedmann equation.

We can define different density parameters Ω for objects by dividing their densities by the critical density.

$$\Omega_i = \frac{\rho_i}{\rho_c} = \frac{8\pi G\rho_i}{3H^2} \tag{4}$$

With some work, we can rewrite the Friedmann equation as

$$\frac{H^2}{H_0^2} = \Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda \tag{5}$$

where Ω_R is the radiation energy density parameter, Ω_M is the matter density parameter, Ω_k is the curvature density parameter, and Ω_Λ is the dark energy density parameter today. All quantities are their values today. In practice, we can let Ω_R and Ω_k go to 0 because they are just super tiny.

The matter density should make sense, as matter density goes like mass/volume, and volume is exactly like a^{-3}. As space expands, you expect the matter density to drop like a^3. We once again see that Ω_Λ is constant; it quite literally stays the same density throughout all time, and across all scales, if it is a constant. But is it??

2.4 Evolving Dark Energy?

Let's suppose that Λ isn't actually a constant, and maybe it changes how it behaves over time. What would that look like?

Definition

The **equation of state** parameter w describes simply the ratio of an objects density to its pressure. That is $w = \frac{\rho}{P}$. A cosmological constant, by definition has $w = -1$, or a negative pressure associated with density. If you think about space expanding as you add more of it, that kinda makes sense. Matter, on the other hand has a $w = 0$, because it doesn't really apply much pressure in comparison to its mass. Radiation has a $w = \frac{1}{3}$, which you will learn in future physics/astronomy classes. It turns out that these exponents of the scale factors are all related to w; specifically component Ω_i goes as $a^{-3(1+w)}$. Double check this for matter and radiation, and the cosmological constant!

The idea is to change the equation of state for dark energy. If it was evolving, then we would expect its equation of state to also evolve with time. We can Taylor expand w in terms of the scale factor a. But rather than expanding around $a = 0$, which would have been at the birth of the universe, we expand around today, $a = 1$. Specifically we let

$$w = w_0 + w_a(1 - a) \tag{6}$$

so the equation of state today is given by w_0, and it changes over time with slope w_a. A cosmological constant corresponds to $w_0 = -1$, and $w_a = 0$.

3 Type Ia Supernovae

4 DESI Results