

SECTION - APART - A

1a) REGRESSION: Regression is a statistical technique that models and analyzes the relationship between a dependent (target) variable and one or more independent (predictor) variables. The goal is to determine the strength and character of the relationship, predict future values, and identify trends.

CORRELATION: Correlation quantifies the degree to which two variables move in relation to each other. It ranges from -1 to 1, where 1 indicates a perfect positive relationship, -1 indicates a perfect negative relationship, 0 indicates no relationship.

METHODS OF ESTIMATION OF REGRESSIONS

1. Ordinary least squares (OLS): minimizes the sum of squared residuals.
2. Maximum Likelihood Estimation (MLE): Finds parameter values that maximize the likelihood of making the observations given the parameters.
3. Generalized Least squares (GLS): Generalizes OLS to allow for heteroscedasticity or autocorrelation.
4. Ridge Regression: Adds a penalty equal to the square of the magnitude of coefficients to the OLS objective function to prevent overfitting.
5. Lasso Regression: Similar to ridge regression but uses an L1 penalty, which can shrink some coefficients to zero, effectively performing variable selection.

1.b) The assumptions of OLS are:

1. Linearity: The relationship between the dependent variable and the independent variables is linear.
2. Independence: Observations of the dependent variable are independent of each other.
3. Homoscedasticity: The variance of the error terms is constant across all the levels of the independent variables.
4. No Multicollinearity: Independent variables are not perfectly linearly related.
5. Normality of Errors: The error terms are normally distributed.

1.c) Detection:

1. Linearity: Use scatter plots of observed v.s predicted values or residuals vs predicted values.
2. Independence: Durbin - Watson test for autocorrelation of residuals.
3. Homoscedasticity: Plot residual vs fitted values or conduct the Breusch - Pagan Test.
4. Multicollinearity: Check variance Inflation Factor (VIF); VIF values above 10 indicate significant multicollinearity.
5. Normality: Use Q-Q plots or conduct the Shapiro-Wilk test.

Correction:

1. Linearity: Apply ~~transitions~~ transformations (e.g. logarithmic, polynomial) to the values.
2. Independence: Use time series models or include lagged variables if data is time dependent.



2. Homoscedasticity: Use robust standard errors, transform the dependent variable or apply weighted least squares.

4. Multicollinearity: Remove or combine correlated variables, or use regularization techniques like ridge regression.

5. Normality: Transform the dependent variable (eg log transformation) or use non-parametric methods if normality cannot be achieved.

1.d)  $R^2$  (Coefficient of Determination):

• Definition:  $R^2$  is a statistical measure that represents the proportion of the variance <sup>of</sup> the dependent variable that's explained by the independent variables in the model.

• Formula  $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$ , where  $SS_{res}$  is the sum of squares of the residuals  
 $SS_{tot}$  is the total sum of squares.

INTERPRETATION:

• 0 to 1 Range: Ranges from 0 to 1

• An  $R^2$  of 0 indicates that the model does not explain any of the variance in the dependent variable.

• An  $R^2$  of 1 indicates that the model explains the variance.

USAGE:

• Goodness-of-fit: Helps assess how well the model fits the data

• Comparisons: Used to compare the explanatory power of different models.

• Limitations: Does not indicate whether a regression model is adequate; higher  $R^2$  does not imply causation

### 1.e) Parametric Tests:

- Assumptions: assume that data follows a certain distribution (typically normal distribution)
- Examples:
  1. t-test: compares the means of two groups.
  2. ANOVA (analysis of variance): compares the means of three or more groups.
  3. Pearson correlation: Measures the linear relationship between two continuous variables.

### Non Parametric Tests:

- Assumptions: Do not assume a specific distribution for the data.
- Examples:
  1. Mann-Whitney U Test: compares differences between two independent groups when the dependent variable is ordinal or continuous but not normally distributed.
  2. Kruskal-Wallis Test: compares differences between three or more independent groups on a non-normally distributed dependent variable.
  3. Spearman correlation: Measures the strength and direction of the association between two variables.



SECTION - BPART - A

- 2) 1. Probability Distributions: A probability distribution is a function that describes the likelihood of obtaining the possible values that a random variable can take.

TYPES OF PROBABILITY DISTRIBUTIONS:

1. Discrete Probability Distributions: where the random variable can take on a finite or countable number of values.

## • Examples:

1. Binomial Distribution

2. Poisson Distribution

3. Geometric Distribution

2. Continuous Probability Distributions: Distributions where the random variable can take on an infinite number of values within a given range.

## • Examples

1. Normal Distribution

2. Exponential Distribution

3. Uniform Distribution

2.2) Parameters of a Probability Distribution:

1. Mean ( $\mu$ ): The average or expected value of the random variable.

• Impact: Shifts the distribution left or right along the x-axis without altering its shape.

## 2. Variance ( $\sigma^2$ ) and standard deviation ( $\sigma$ )

measures of the spread or dispersion of the distribution

• Impact: Variance & standard deviation determine the width of the distribution. A larger variance / standard deviation result in a wider, flatter dist, while a smaller variance / standard deviation results in narrower, taller dist.

## 3. Skewness: measures the asymmetry of the distribution.

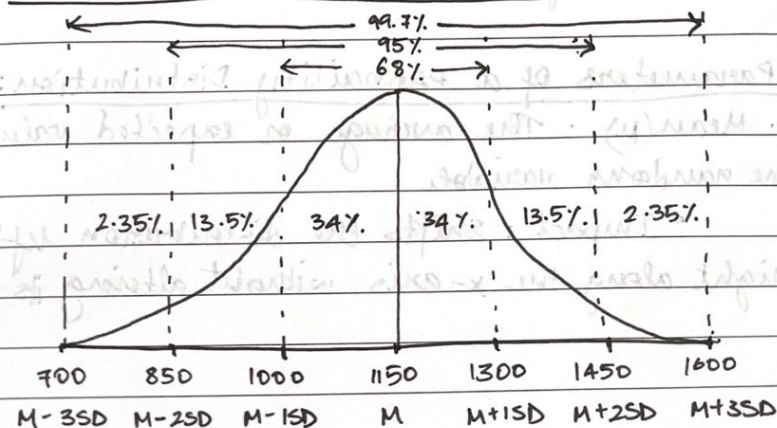
• Impact: positive skewness indicates a longer right tail, while negative skewness indicates a longer left tail.

## 4. Kurtosis: measures the "tailedness" of the distribution.

• Impact: High kurtosis indicates heavy tails and a sharper peak, while low kurtosis indicates light tails and a flatter peak.

## 2) 5. Total Area Under the Normal Distribution Curve: The total area under the normal distribution curve is 1

### Normal Distribution curve.





- 2.4) • Mean  $\pm 1$  standard deviation: Approx 68.27% of the data falls within this range.
- Mean  $\pm 2$  standard deviation: Approx 95.45% of the data falls in this range.
- Mean  $\pm 3$  standard deviation: Approx 99.73% of the data falls within this range.