

CSCI5646: Numerical Linear Algebra

HOMEWORK 2

1. **Trefthen & Bau, Ex. 3.3.** Vector and matrix p -norms are related by various inequalities, often involving the dimensions m or n . For each of the following, verify the inequality and give an example of a nonzero vector or matrix (for general m, n) for which the equality is achieved. In this problem x is an m -vector and A is an $m \times n$ matrix.
 - (a) $\|x\|_\infty \leq \|x\|_2$
 - (b) $\|x\|_2 \leq m \|x\|_\infty$
 - (c) $\|A\|_\infty \leq \sqrt{n} \|A\|_2$
 - (d) $\|A\|_2 \leq \sqrt{m} \|A\|_\infty$
2. It is said that “[a]ll norms on \mathbb{R}^n are *equivalent*, ...” What does this mean? Look up the technical definition and also think about how you might interpret this idea of *equivalence* geometrically. What do you think it means?
3. **[Golub & Van Loan, P2.4.5.]** Show that if $A \in \mathbb{R}^{m \times n}$ has rank n then

$$\|A(A^T A)^{-1} A^T\|_2 = 1.$$