Number of Method Calls per Hanoi solve() Implementation

k (# of rings)	Dynamic •	Recursive	-(/					
3	12	15						
4	15	31	4500000					
5	18	63	4000000				7	
6	21	127	400000					
7	24	255	3500000					
8	27	511	3000000					
9	30	1023	2500000					
10	33	2047	2500000					
11	36	4095	2000000				7	
12	39	8191	1500000					
13	42	16383	1500000					
14	45	32767	1000000				7	
15	48	65535	500000					
16	51	131071						
17	54	262143	0 0	5	10	15	20	25
18	57	524287	0	5	10	15	20	23
19	60	1048575	Dynamic Recursive					
20	63	2097151		,				
21	66	4194303						

Pattern Between K and the Number of the Method Calls

Assuming k, the number of rings, is at least 3:

Recursive Implementation

Exponential increase of the number of method calls by factor of 2 (plus an additional 1) for every additional ring k OR $f(k) = 2^{k+1}$.

This pattern is due to the fact that for every movement of a ring to its destination tower, the method has to call itself to determine the submovements that are needed to accomplish that movement. These submovements may have sub-movements of their own, resulting in an exponential increase in method calls to execute the movement.

Dynamic Implementation

Linear increase of 3 method calls for every additional ring k OR f(k) = 3*k + 3.

This pattern is due to the fact that the implementation saves each movement of a ring from a starting tower to a destination tower, storing them in a hash map so that fetching the movements requires O(1) complexity. Therefore, only a linear increase in method calls is needed to calculate the movements of an additional ring or, more simply, only the additional number of movements that only relate to the additional ring need to be calculated.