1. **If matching M is a maximum, then M has no augmenting path.**

Proof by Contradiction
If M has an augmenting path, then it has an alternating path between two free vertices. Switching each alternate edge from out of the matching into the matching will therefore give a matching with one more edge, proving that M cannot be a maximum.

2. **If matching M has no augmenting path, then M is a maximum.**

Proof by Contradiction
If M is not a maximum matching, then there exists an M’ that is a maximum matching. All vertices in either graph but not in both graphs must make a subgraph S with a highest degree of 2. The edges in S alternate between M and M’. Since M’ must contain more edges than M, S must contain at least one path P that contains more edges of M’ than of M. Therefore, the beginning and end edges of P are in M’ not M, so P is an augmenting path for M.