Dijkstra's Algorithm
Proof by Induction

Let *S* be the set of all vertices on the shortest path found by Dijkstra's algorithm.
Prove that the distances[*u*] is the minimum length between the vertex *u* and the target vertex *t*.

Base Case:
When $|S| = 1$, it is true.

Induction Case:
Assume for $|S| = k$ and prove for $|S| = k + 1$.
Suppose that node *i* with distance[*i*], was added to S, assuming that it is the shortest path to i, but to show contradiction, distance[*i*] is not the shortest path.
Then, there must be at least one node in *S* along the shortest path *u* to *i*.
Let *j* be the first node in *S* on the shortest path from *u* to *i*.
Then the length of shortest path, through *j*, from *u* to *i* = distance[*j*] + length[*j, i*].
Since distance[*i*] is not the shortest, distance[*j*] + length[*j, i*] < distance[*i*], and since length[*j, i*] must be positive, distance[*j*] < distance[*i*].
This contradicts that distance[*i*] is the minimum path to *I*, proving Dijkstra's algorithm.