

Assumptions:

1. The capacity of a S-T cut is equal to $\sum c(e)$, $\forall e$.
2. The flow is equal to the outgoing flow from vertices A minus the incoming flow of the remaining vertices $V - A$ for any subset of vertices A, or $\text{flow} = \text{o_flow}(A) - \text{in_flow}(V - A)$.
3. The set of vertices reachable from source s in graph G that do not include the max flow path, after the max flow is calculated through the Ford–Fulkerson algorithm, is equal to A and $(V - A)$ vertices is equal to B.

Therefore, for total weight of the min S-T cut to be equal to the max flow,

I. all the outgoing edges of the cut must be fully use all their capacity and

II. the incoming edges of the cut must have no flow,
effectively maximizing $\text{o_flow}(A)$ and minimizing $\text{in_flow}(B)$.

Proof by Contradiction:

In G, there exists an outgoing edge $e(x, y)$ where vertex x is in the cut and vertex y is not, that is **not** using its full capacity. This would imply that there is a path from s to y, meaning A and B are not unique sets. This is a contradiction, proving I.

In G, there exists an incoming edge $e(y, x)$ where vertex x is in the cut and vertex y is not, that is **carrying** flow. This would imply that there is a path from s to y, meaning A and B are not unique sets. This is a contradiction, proving II.