Solving Problems with Search: 5

Today's Plan

- Project 1 comments and Q&A
- Memory-bound A*
- Solving CSPs with search

Project 1 Questions & Tips

- Use Piazza, read FAQ before posting questions: https://piazza.com/class/ixql4613j9k223?cid=8
- **Questions 1-4**: if you develop a correct solution for DFS, the rest will be easy modifications
- **Run autograder** after *every* question. Until you pass all the test cases, assume your code has bugs.
- Example (incomplete!) implementations: https://github.com/aimacode/aima-python/blob/master/search.py

Tips for Project 1 (cont'd)

- Problems 5-8 depend on code in 1-4. Get that right (and tested) first, before moving on!
- P5/Corners problem: must visit all corners in *single* path
  - Implications for search tree, state info to update
- Heuristics for p6-8: start simple. For extra credit, think back to graph traversal algorithms from cs323.

A* Search: Find bug(s)

```
def astar_search(problem, h=null):
        node = Node(problem.initial)
        frontier = PriorityQueue()
        frontier.append(node, null, null, 0)  //initial cost=0
        explored = set()
        while frontier:
                node = frontier.pop()
                if problem.goal_test(node.state):
                        return node

                explored.add(node.state)
                for (child, action, cost) in problem.getSuccessors(node.state):
                        if child not in explored and child not in frontier:
                                nc = new Node(child, cost+h)
                                frontier.append(nc, cost+h)
        return None
```

```
Node:
    state
    parent
    action_from_parent
    cost
```

Properties of A* w/ consistent heuristics

- Complete?
- Time?
- Space?
- Optimal?

Properties of A* w/ consistent heuristics

- <u>Complete?</u> Yes (unless there are infinitely many nodes with $f \le f(G)$, i.e. step-cost > ε)

- <u>Time/Space?</u> Exponential*: b^d
- <u>Optimal?</u> Yes
- <u>*Optimally Efficient*</u>: Yes (no algorithm with the same heuristic is guaranteed to expand fewer nodes)

* Can be O(n) iff heuristic is exact, or nearly exact (ignoring heuristic computation)

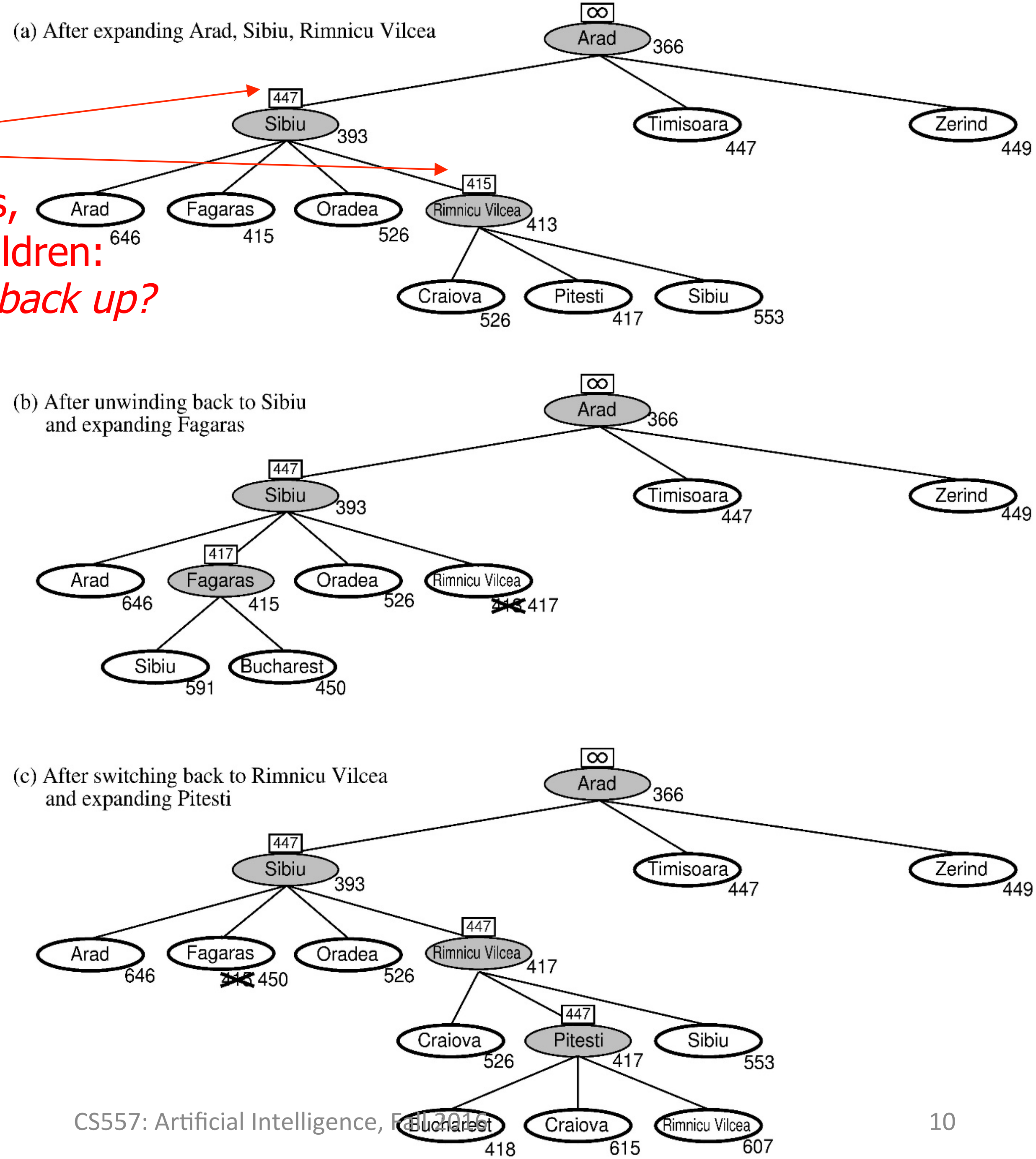
Quiz

- True or False: A* can find a more optimal solution than UCS.
- True or False: A* can expand more nodes than UCS
- True or False: A* with consistent heuristics can expand more nodes than UCS

Memory Bounded Heuristic Search: Recursive BFS

- How can we solve the memory problem for A* search?
- Idea: Try something like iterative deepening DFS, but let's not forget everything about the branches we have partially explored.
- *We remember the best f-value we have found so far in the branch we are deleting.*

best alternative
over fringe nodes,
which are not children:
i.e. do I want to back up?



RBFS changes its mind
very often in practice.

This is because the
f=g+h become more
accurate (less optimistic)
as we approach the goal.
Hence, higher level nodes
have smaller f-values and
will be explored first.

Problem: We should keep
in memory whatever we can.

Simple-Memory Bounded A*

- This is like A*, but when memory is full we delete the worst node (largest f-value).
- Like RBFS, we remember the best descendent in the branch we delete.
- If there is a tie (equal f-values) we delete the oldest nodes first.
- simple-MBA* finds the optimal *reachable* solution given the memory constraint.
- Time can still be exponential.

A Solution is not reachable
if a single path from root to goal
does not fit into memory

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms keep a single "current" state, try to improve it

What is Search For?

- Assumptions about the world: a single agent, deterministic actions, fully observed state, discrete state space

- Planning: sequences of actions
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance

- **Identification: assignments to variables**
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are specialized for identification problems

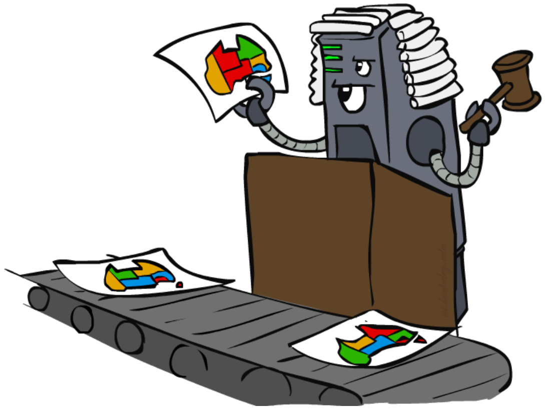
Constraint Satisfaction Problems

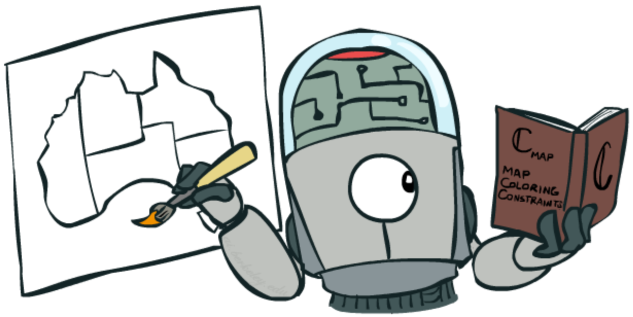
- Standard search problems:
  - State is a "black box": arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything

- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables

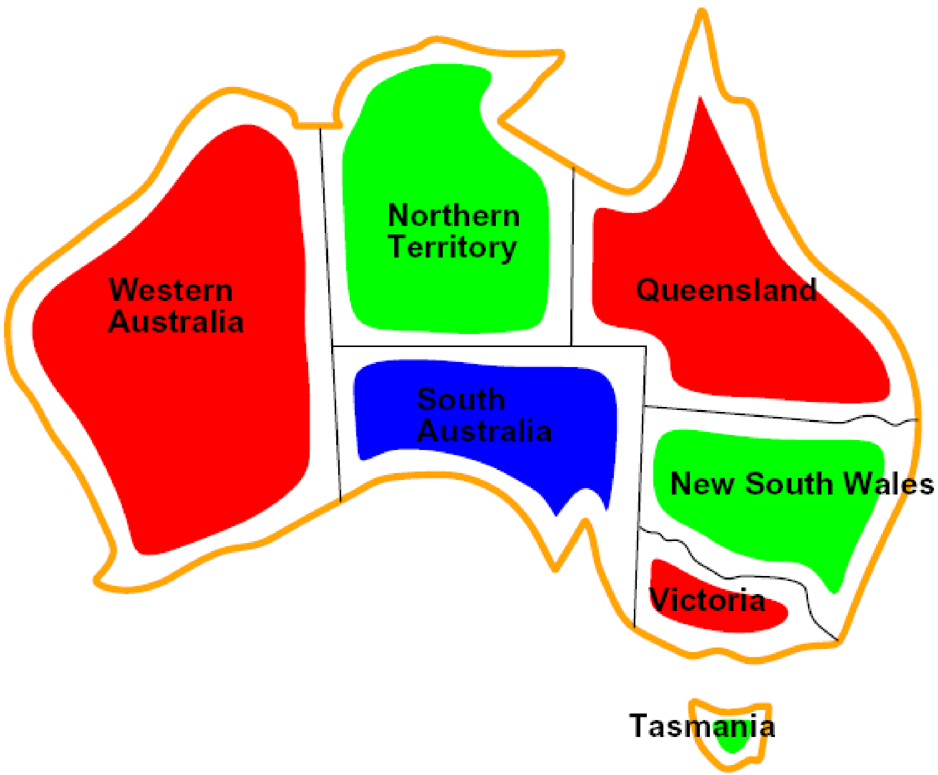
- Simple example of a *formal representation language*

- Allows useful general-purpose algorithms with more power than standard search algorithms





CSP Examples



Example: Map Coloring

- Variables: $\mathrm{WA},\ \mathrm{NT},\ \mathrm{Q},\ \mathrm{NSW},\ \mathrm{V},\ \mathrm{SA},\ \mathrm{T}$

- Domains: $D = \{\mathrm{red}, \mathrm{green}, \mathrm{blue}\}$

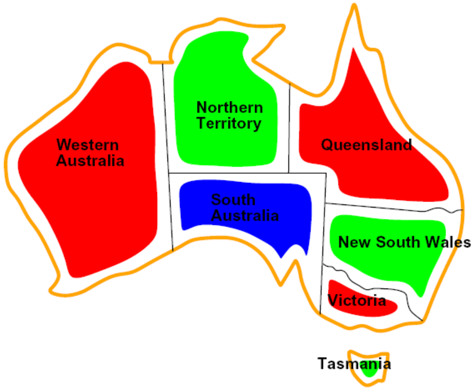
- Constraints: adjacent regions must have different colors

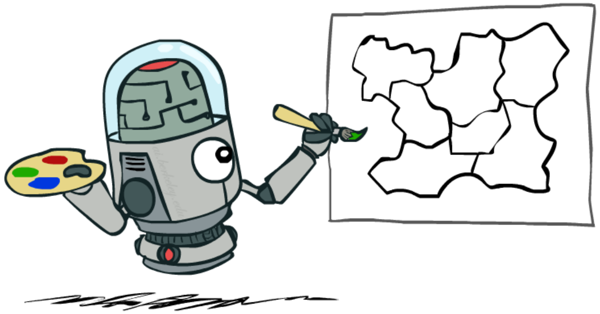
  Implicit: $\mathrm{WA} \neq \mathrm{NT}$

  Explicit: $(\mathrm{WA}, \mathrm{NT}) \in \{(\mathrm{red}, \mathrm{green}), (\mathrm{red}, \mathrm{blue}), \ldots\}$

- Solutions are assignments satisfying all constraints, e.g.:

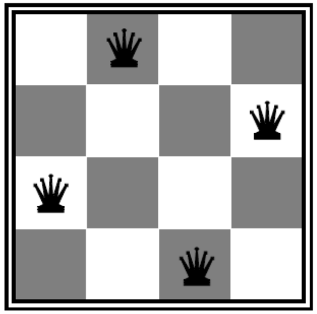
  $\{\mathrm{WA}{=}\mathrm{red},\ \mathrm{NT}{=}\mathrm{green},\ \mathrm{Q}{=}\mathrm{red},\ \mathrm{NSW}{=}\mathrm{green},$
  $\mathrm{V}{=}\mathrm{red},\ \mathrm{SA}{=}\mathrm{blue},\ \mathrm{T}{=}\mathrm{green}\}$

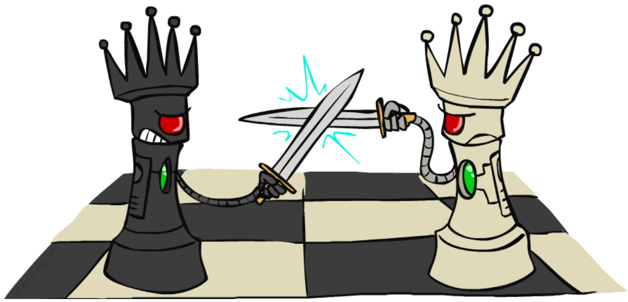




Example: N-Queens

- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$
  - Constraints





$$\forall i, j, k \;\; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

Example: N-Queens

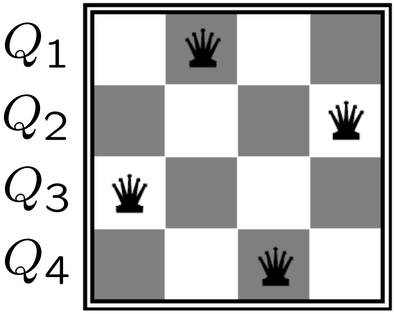
- Formulation 2:
  - Variables: Q_k

    $\{1, 2, 3, \ldots N\}$
  - Domains:

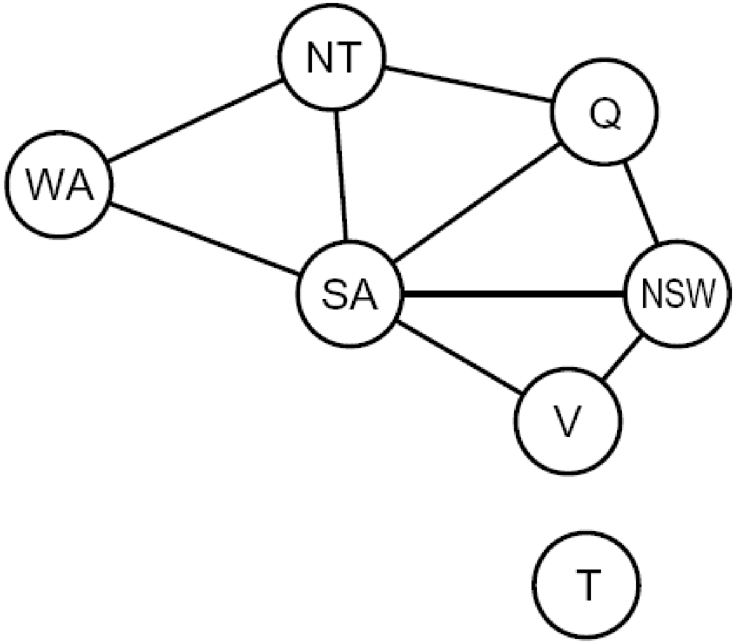
    Implicit: $\forall i, j \ \text{non-threatening}(Q_i, Q_j)$
  - Constraints:

    Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

    $\ldots$

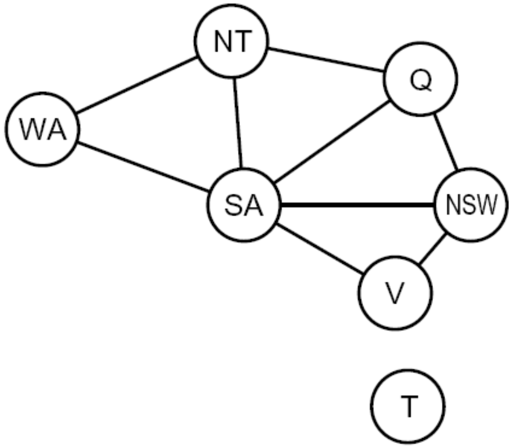


Constraint Graphs



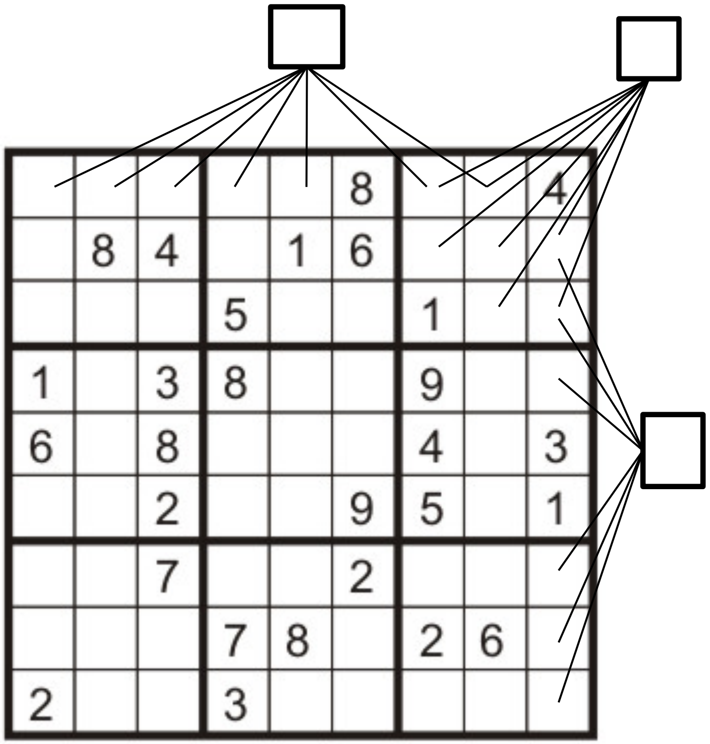
Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



[Demo: http://aispace.org/constraint/](http://aispace.org/constraint/)

Example: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

  9-way alldiff for each column

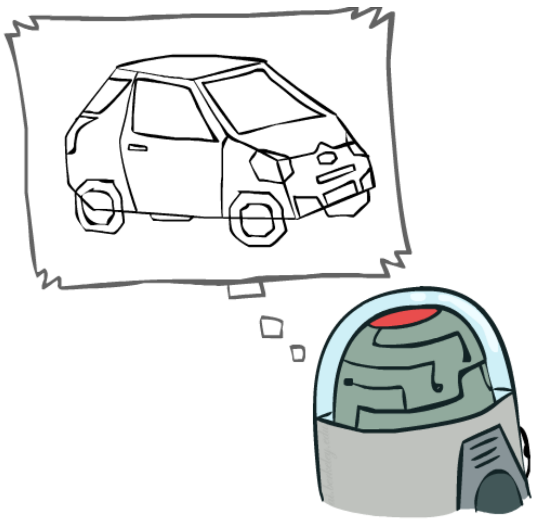
  9-way alldiff for each row

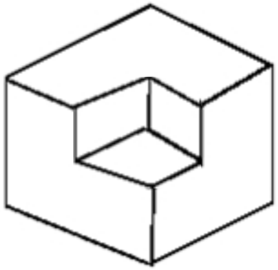
  9-way alldiff for each region

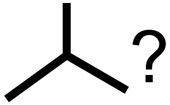
  (or can have a bunch of pairwise inequality constraints)

Example: The Waltz Algorithm

- The Waltz algorithm is for interpreting line drawings of solid polyhedra as 3D objects
- An early example of an AI computation posed as a CSP

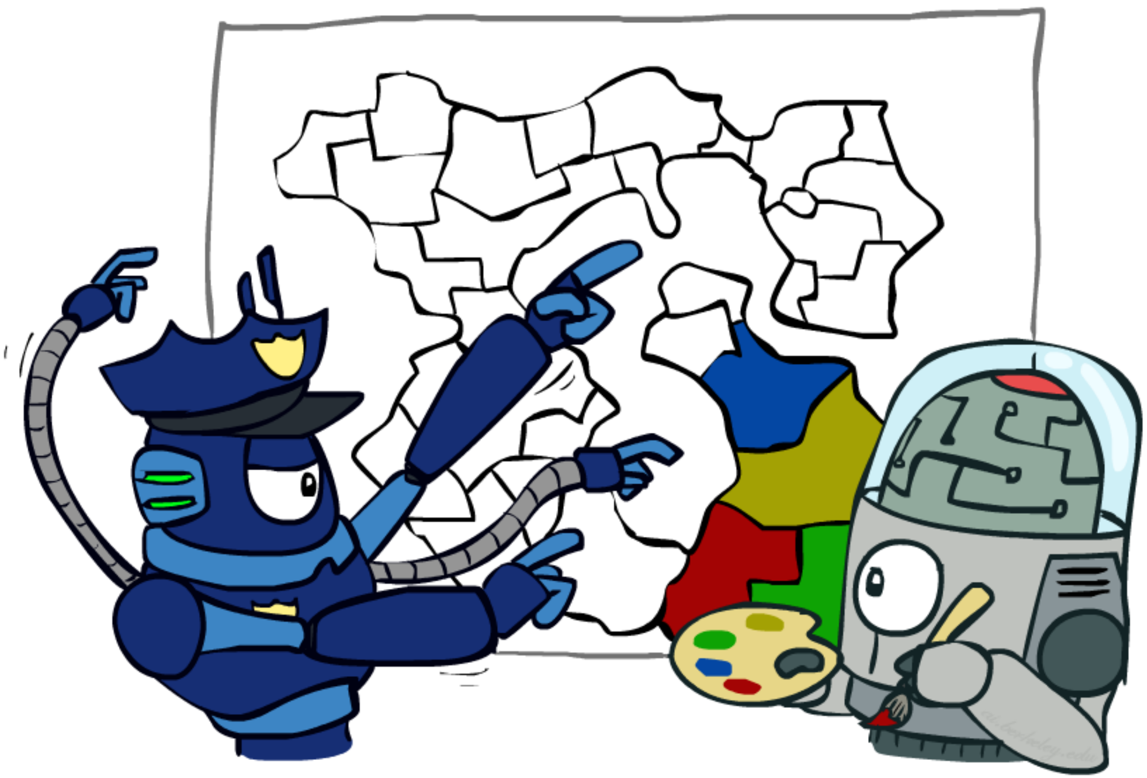






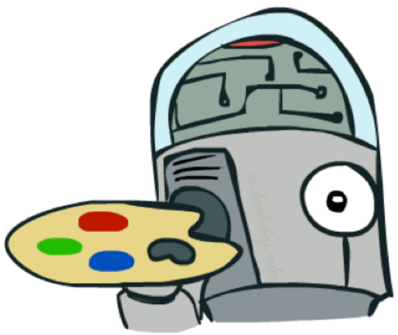
- Approach:
  - Each intersection is a variable
  - Adjacent intersections impose constraints on each other
  - Solutions are physically realizable 3D interpretations

Varieties of CSPs and Constraints



Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by LP methods (see cs170 for a bit of this theory)

Varieties of Constraints

- Varieties of Constraints
  - Unary constraints involve a single variable (equivalent t reducing domains), e.g.:

    $$SA \neq green$$

  - Binary constraints involve pairs of variables, e.g.:

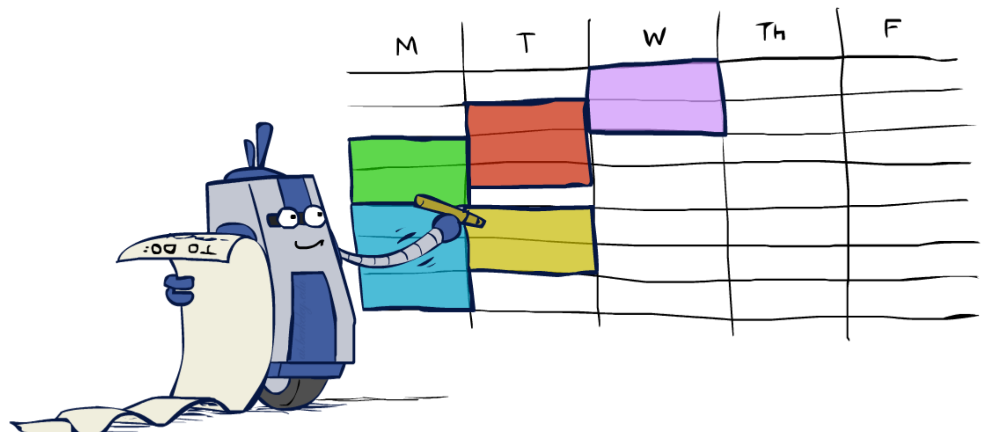
    $$SA \neq WA$$

  - Higher-order constraints involve 3 or more variables:
    e.g., cryptarithmetic column constraints

- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

Real-World CSPs

- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Circuit layout
- Fault diagnosis
- … lots more!



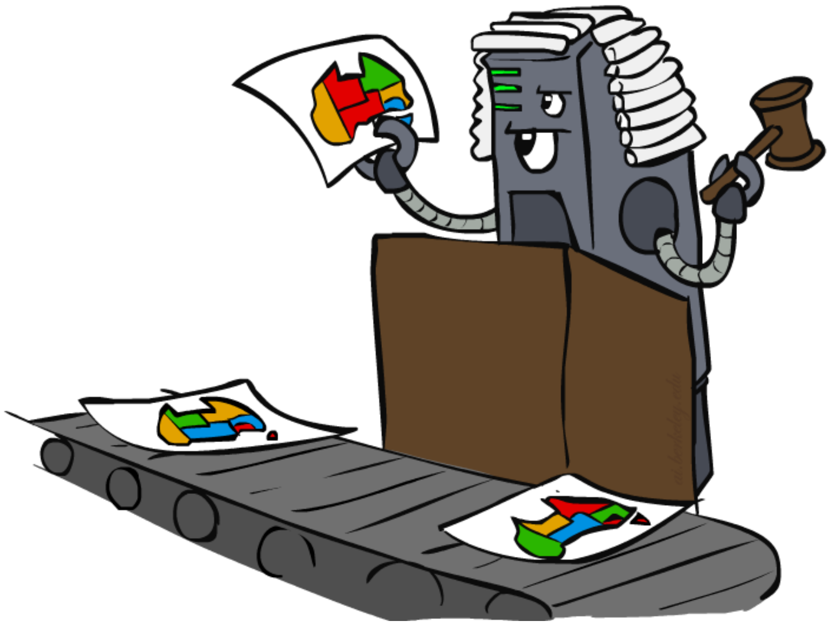
- Many real-world problems involve real-valued variables…

Standard Search Formulation

- Standard search formulation of CSPs

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints

- We'll start with the straightforward, naïve approach, then improve it

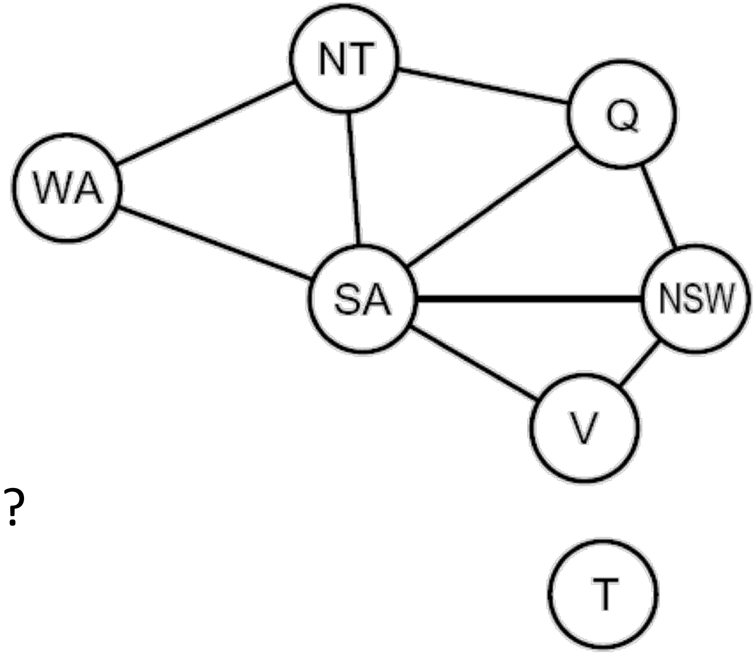


Search Methods

- What would BFS do?

- What would DFS do?

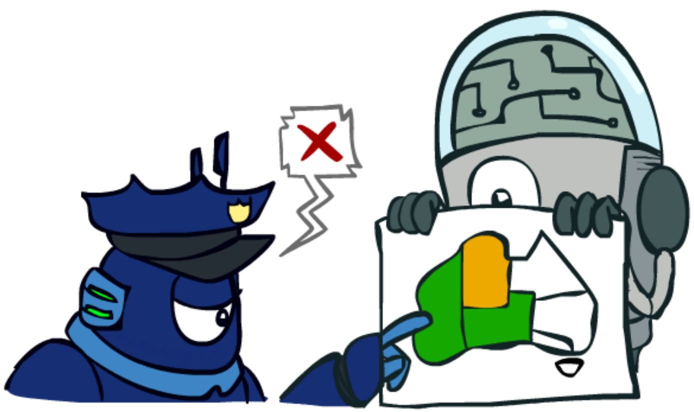
- What problems does naïve search have?



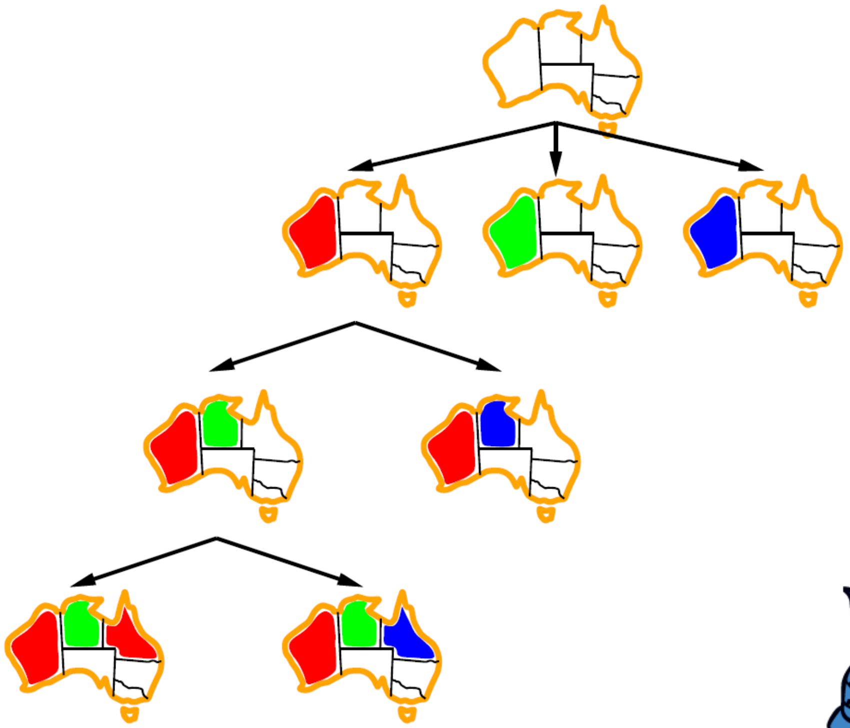
[Demo: coloring -- dfs]

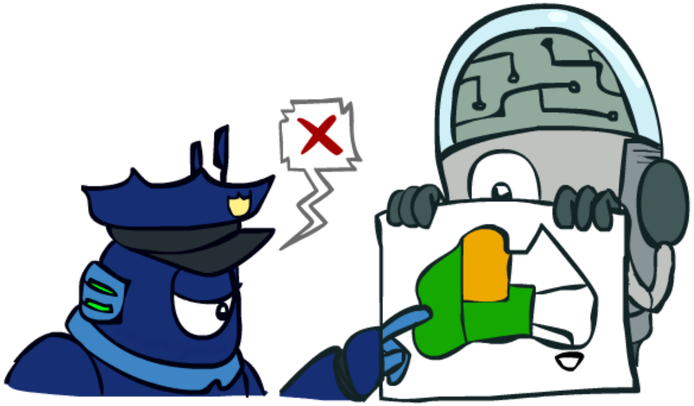
Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constra
  - "Incremental goal test"
- Depth-first search with these two improvements is called *backtracking search* (not the best name)
- Can solve n-queens for n $\approx$ 25



Backtracking Example





Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation
- What are the choice points?

[Demo: coloring -- backtracking]

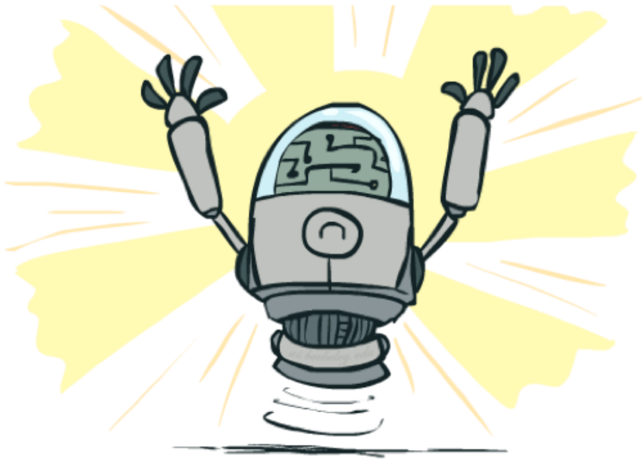
Improving Backtracking

- General-purpose ideas give huge gains in speed

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?

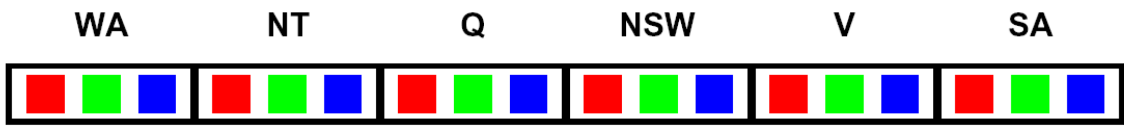
- Filtering: Can we detect inevitable failure early?

- Structure: Can we exploit the problem structure?



Filtering: Forward Checking

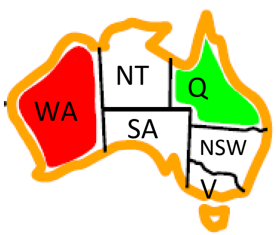
- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment

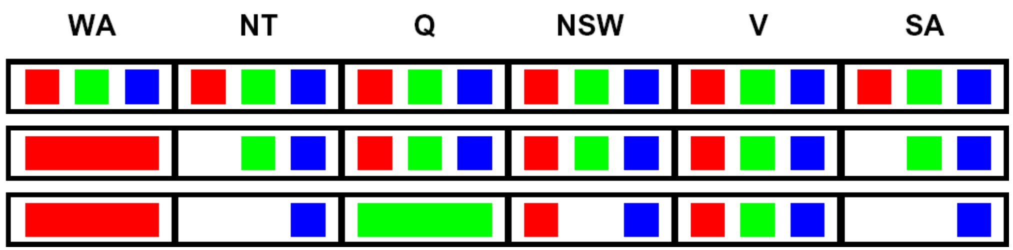


[Demo: coloring -- forward checking]

Filtering: Constraint Propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

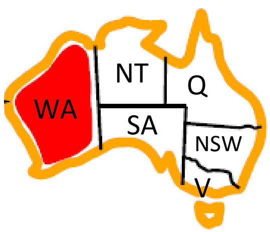


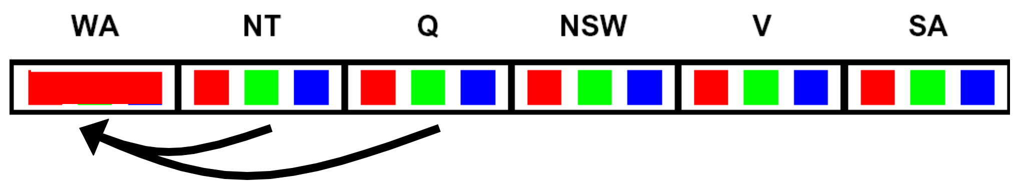


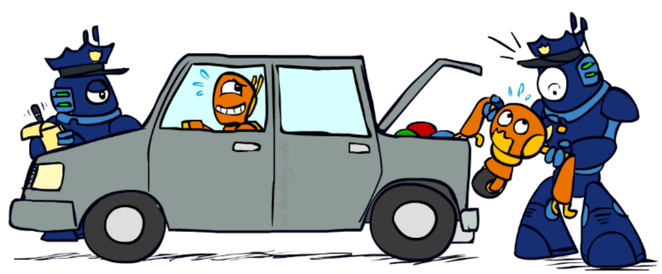
- NT and SA cannot both be blue!
- Why didn’t we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Single Arc

- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint





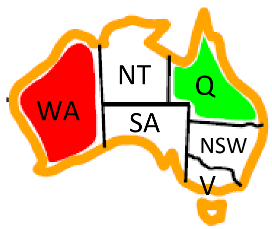


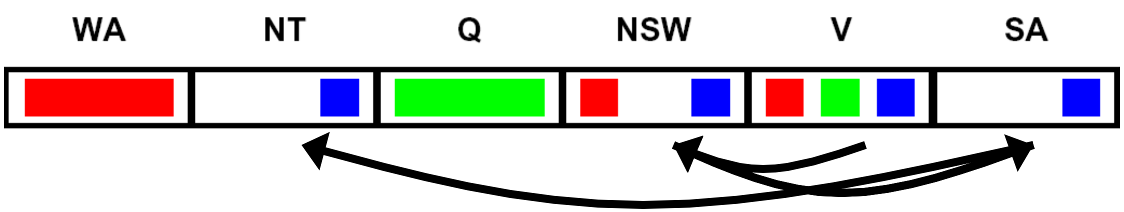
Delete from the tail!

- Forward checking: Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES$(X_i,\ X_j)$ **then**
      for each X_k **in** NEIGHBORS$[X_i]$ **do**
        add $(X_k,\ X_i)$ to *queue*

---

function REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **returns** true iff succeeds
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN$[X_i]$ **do**
    if no value y in DOMAIN$[X_j]$ allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN$[X_i]$; *removed* $\leftarrow$ *true*
  return *removed*

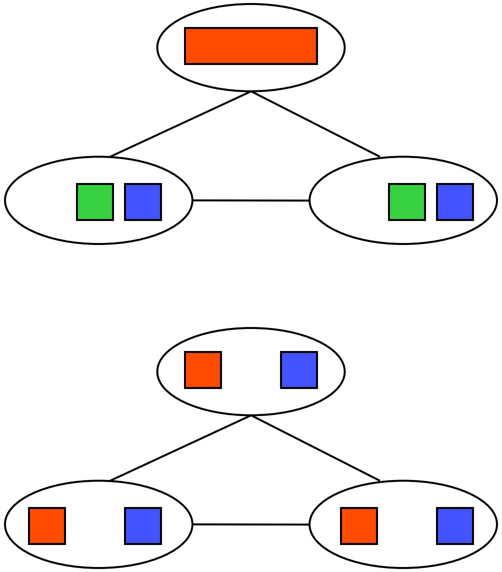
- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

[Demo: CSP applet (made available by aispace.org) -- n-queens]

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

- Arc consistency still runs inside a backtracking search!

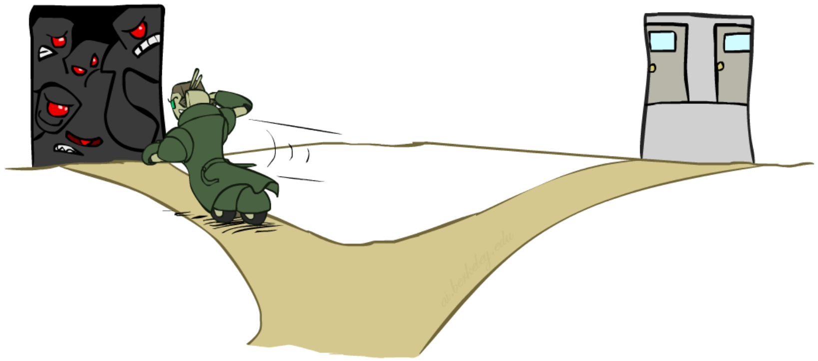


What went wrong here?

Ordering: Minimum Remaining Values

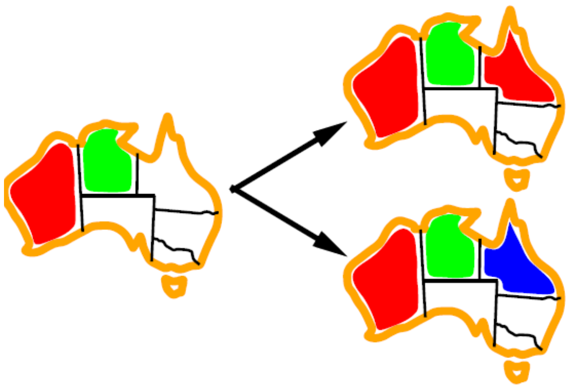
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

- Why min rather than max?
- Also called “most constrained variable”
- “Fail-fast” ordering



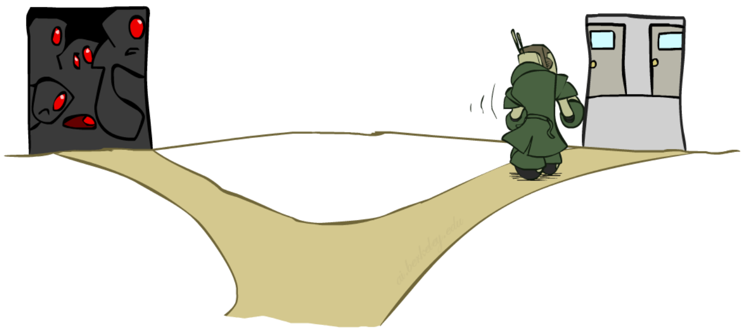
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible



ToDo

- Finish rest of project 1 (Due Wednesday Feb 1)
- Will study approximate and local search on Tue
- Begin adversarial search on Thu