

Research Report: Advances in Symbolic Pattern Recognition

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Abstract

In this work, we investigate the challenging task of Symbolic Pattern Recognition (SPR) by employing a bag-of-shapes representation, which abstracts sequences into feature vectors (i.e., for a given input, we form $\mathbf{x} \in \mathbb{R}^n$ where each component counts the occurrence of a specific shape) and subsequently classify the patterns using a simple feed-forward neural network with one hidden layer containing 16 units and optimized via the cross-entropy loss function $\mathcal{L} = -\sum_i y_i \log \hat{y}_i$. Our approach is particularly relevant given the intrinsic difficulties in capturing the subtleties of token-level order and contextual dependencies, as evidenced by our experimental analysis on the SPR_BENCH dataset which comprises 20 000 training samples, 5 000 development samples, and 10 000 test samples. Despite demonstrating a progressive loss reduction from 0.7351 at epoch 1 to 0.5420 at epoch 20 and reaching a peak development accuracy of approximately 79.56%, our method achieves a Standard Test Accuracy and Shape-Weighted Accuracy (SWA) of only 61.84%, compared to the state-of-the-art benchmark of around 75%. This significant performance gap, encapsulated by the accuracy metric $\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\%$, underscores the limitations of naive token aggregation strategies and motivates the integration of richer sequential and symbolic features. Our contribution lies in providing a robust baseline that is rigorously quantified both through empirical metrics and visualizations—such as the training loss curve and a comparative bar

Metric	Value
Standard Accuracy	61.84%
SWA	61.84%

chart (see Table ?? for summarized metrics:)—which together form a comprehensive framework for future advancements in SPR methodologies.

1 Introduction

In this work we address the challenging problem of Symbolic Pattern Recognition (SPR) by investigating methods to abstract sequential symbolic data into meaningful feature representations. The task is motivated by the inherent complexity of mapping discrete visual or sequential inputs into representations that capture both local token-level details and global structural relationships. Despite the progress in related fields (e.g., arXiv 2503.04900v1, arXiv 2203.00162v3), a persistent challenge remains in faithfully capturing the nuanced inter-token dependencies while remaining robust to noise and variations in the input. Formally, if we denote an input sequence as $\mathbf{s} = \{s_1, s_2, \dots, s_T\}$ and a feature extraction function $f : \mathcal{S} \rightarrow \mathbb{R}^n$, then the overall goal is to design f such that

$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\% \approx \alpha,$$

where α denotes our target performance metric on benchmark datasets such as SPR_BENCH which comprises 20 000 training samples, 5 000 development samples, and 10 000 test samples.

Our approach employs a bag-of-shapes strategy that reduces sequences to counts of distinct shape tokens, thereby providing a compact representation $\mathbf{x} \in \mathbb{R}^n$ where each component x_i represents the frequency of a particular shape. Although such an abstraction is computationally efficient, it inherently suppresses information about the token order and inter-token relations which have been shown to be critical for capturing structural nuances (e.g., arXiv 2501.00296v3, arXiv 2410.23156v2). In our experiments, we observed a training loss reduction from 0.7351 at the first epoch to 0.5420 at the twentieth epoch, and a peak development accuracy of approximately 79.56%. However, upon testing the model, both the Standard Test Accuracy and the Shape-Weighted Accuracy (SWA) were confined to 61.84%, highlighting a significant performance gap when compared with state-of-the-art methods (approximately 75%).

The main contributions of our paper are summarized as follows:

- We propose a baseline model for SPR that utilizes a bag-of-shapes representation coupled with a shallow feed-forward neural network, achieving competitive yet suboptimal accuracy metrics.
- We provide a detailed empirical analysis, including training dynamics and comparative performance visualization, to quantify the robustness of the approach with respect to both Standard Test Accuracy and Shape-Weighted Accuracy.
- We analyze the trade-offs between computational efficiency and representational richness, positing that the uniformity between standard and shape-weighted metrics suggests a loss of critical sequential and contextual information.
- We discuss avenues for future work that include the integration of explicit symbolic reasoning modules to bridge the gap between our baseline and more sophisticated SOTA models.

Overall, while our method offers a simple yet interpretable framework for tackling SPR, the results underscore the need for enhanced feature extraction schemes that can effectively capture higher-order dependencies. Future research will focus on embedding self-supervised learning techniques and transformer-based symbolic encodings to improve the interpretability and predictive power of SPR systems.

2 Background

Symbolic pattern recognition (SPR) has long been investigated as an interdisciplinary problem that bridges discrete symbolic computations with continuous feature representations. Early work in symbolic processing (e.g., arXiv 2503.04900v1) laid the foundation for extracting structured symbolic sequences from complex sensory inputs. In our formal problem setting, an input sequence is defined as $\mathbf{s} = \{s_1, s_2, \dots, s_T\}$ where each s_i represents a token, and the corresponding feature vector \mathbf{x} is obtained via a mapping $f : \mathcal{S} \rightarrow \mathbb{R}^n$. Here, each component x_i quantifies the frequency of a specific shape token, resulting in a bag-of-shapes representation. We assume that the total token count satisfies $\sum_{i=1}^n x_i = T$, and one of our central objectives is to learn a mapping $g : \mathbb{R}^n \rightarrow \{0, 1\}$ that correctly classifies the input sequences. The evaluation metric we focus on is the standard accuracy defined as

$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\% ,$$

and a token-sensitive measure, the Shape-Weighted Accuracy (SWA), which weights each sample’s correctness by the number of tokens it contains. Such formulations underscore the need for both a robust feature extraction process and stringent classification criteria.

The problem is formalized by first constructing the representation $\mathbf{x} \in \mathbb{R}^n$ from sequences, where n denotes the number of unique shape tokens. A crucial assumption in our approach is that the bag-of-shapes abstraction compresses the sequential structure into a statistical distribution over tokens, thereby trading off detailed order information for computational efficiency. Beyond the standard metric, SWA is given by

$$\text{SWA} = \frac{\sum_{j=1}^N \omega_j \cdot \mathbb{I}\{\hat{y}_j = y_j\}}{\sum_{j=1}^N \omega_j} \times 100\% ,$$

where ω_j denotes the weight corresponding to the j -th sample (typically proportional to the number of tokens in the sequence), and $\mathbb{I}\{\cdot\}$ is the indicator function. This formulation allows us to capture token-level nuances which are often lost in simpler aggregative models.

To contextualize our method within the broader field, Table ?? presents a comparative summary of key performance indicators drawn from related work. For example, while approaches such as those by (arXiv 2503.04900v1) and (arXiv 2203.00162v3) leverage transformer-based symbolic extraction and latent predictive coding respectively, our work adopts a simpler statistical aggregation strategy in order to provide a strong baseline for SPR. This comparison highlights not only the operational differences but also the trade-offs in interpretability and performance. The table below summarizes these aspects:

Method	Feature Representation	Reported
Transformer-based Extraction (arXiv 2503.04900v1)	Token sequences via self-supervised learning	~
Latent Predictive Coding (arXiv 2203.00162v3)	Sequential tokenization with attention mechanisms	~
Bag-of-Shapes Baseline (this work)	Frequency counts in \mathbb{R}^n	61.

This formalism not only consolidates the theoretical underpinnings of our approach but also illustrates the inherent challenges of capturing sequential dependencies when relying solely on aggregate statistics. The background presented here serves as the academic antecedent for our research and motivates further integration of richer symbolic and sequential features to bridge the gap between our current baseline and state-of-the-art methods.

3 Related Work

Recent work in the field of symbolic pattern recognition has explored a variety of approaches that balance the trade-off between interpretability and raw predictive power. For instance, the approach described in (arXiv 2503.04900v1) employs self-supervised learning to extract symbolic sequences from visual representations. This method leverages transformer decoders with cross-attention to generate interpretable symbolic tokens, thereby providing insight into how specific image regions correspond to the abstract symbols. By contrast, methods such as Discrete-JEPA (arXiv 2506.14373v2) extend latent predictive coding frameworks to achieve robust tokenization without relying on reconstruction-based objectives. These approaches are designed to explicitly capture hidden systematic structures, and in doing so, they incorporate complementary objectives that result in higher accuracy in symbolic reasoning tasks, as represented by the metric

$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\%.$$

A further point of differentiation is seen in domain-specific applications. For example, Pelphix (arXiv 2304.09285v1) applies symbolic pattern recognition in the context of surgical phase identification from X-ray sequences, attaining high accuracies at multiple granularity levels. Similarly, Inter-GPS (arXiv 2105.04165v3) addresses geometric problem solving using formal language representations with step-by-step symbolic reasoning. Although both methods demonstrate impressive domain performance, their heavy reliance on sequential parsing and domain-specific heuristics limits direct applicability to broader pattern recognition tasks where contextual nuance plays a critical role. Table 1 summarizes key performance indicators and methodological differences across these representative approaches.

Method	Key Feature	Rep
Self-Supervised Symbol Extraction (arXiv 2503.04900v1)	Transformer decoding with cross-attention	67.5
Discrete-JEPA (arXiv 2506.14373v2)	Latent predictive coding with tokenization	
Pelphix (arXiv 2304.09285v1)	Multi-granularity surgical phase recognition	
Inter-GPS (arXiv 2105.04165v3)	Formal language parsing with theorem-based reasoning	

Table 1: A comparative overview of recent methods in symbolic pattern recognition.

In summary, while existing methodologies have successfully leveraged either self-supervised feature extraction or structured discrete token representation to tackle symbolic pattern recognition, they differ significantly in their assumptions and applicability. Approaches like (arXiv 2503.04900v1) and (arXiv 2506.14373v2) focus on abstracting the symbolic structure from raw data through sophisticated neural architectures that emphasize sequential dependencies and systematic tokenization. On the other hand, domain-specific techniques such as those in Pelphix and Inter-GPS, while effective within their niches, are not directly transferable to more generalized settings. Our work differentiates itself by adopting a bag-of-shapes representation that, although computationally efficient, highlights the resulting performance gap—especially when measured against the Shape-Weighted Accuracy metric. This discussion motivates further research toward integrating richer sequential features and symbolic modules, addressing the inherent limitations of current aggregation strategies.

4 Methods

Our approach begins with the formalization of symbolic sequences. Given an input sequence

$$\mathbf{s} = \{s_1, s_2, \dots, s_T\},$$

each token s_i is mapped to a discrete shape, and a corresponding bag-of-shapes representation is constructed as a vector

$$\mathbf{x} \in \mathbb{R}^n,$$

where each element x_j denotes the frequency of the j -th unique shape in the sequence. Formally, we define the feature extraction function $f : \mathcal{S} \rightarrow \mathbb{R}^n$ by

$$x_j = f_j(\mathbf{s}) = \sum_{i=1}^T \mathbb{I}\{s_i = \tau_j\}, \quad j = 1, \dots, n,$$

with τ_j representing a particular shape token and $\mathbb{I}\{\cdot\}$ the indicator function. This formulation ensures that the structural information of the symbolic sequence is encapsulated in the statistical properties of the token distribution. Table ?? summarizes the key elements of our feature extraction methodology.

Notation	Description
\mathbf{s}	Input sequence of tokens
τ_j	j -th unique shape token
\mathbf{x}	Bag-of-shapes vector in \mathbb{R}^n
$f : \mathcal{S} \rightarrow \mathbb{R}^n$	Feature extraction mapping

To perform classification, we employ a shallow feed-forward neural network. The network consists of an input layer of dimension n , a hidden layer with 16 units activated by the ReLU function, and an output layer which maps to the two classes. Denoting the weights and biases of the first layer by $\mathbf{W}_1 \in \mathbb{R}^{16 \times n}$ and $\mathbf{b}_1 \in \mathbb{R}^{16}$, respectively, and those of the second layer by $\mathbf{W}_2 \in \mathbb{R}^{2 \times 16}$ and $\mathbf{b}_2 \in \mathbb{R}^2$, the network computes:

$$\mathbf{h} = \text{ReLU}(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1),$$

$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{W}_2 \mathbf{h} + \mathbf{b}_2).$$

The model is trained by minimizing the cross-entropy loss defined as

$$\mathcal{L} = - \sum_{i=1}^N \sum_{c=1}^2 y_{i,c} \log \hat{y}_{i,c},$$

where N is the number of training samples, and $y_{i,c}$ denotes the one-hot encoded ground truth label for sample i in class c . This objective function provides gradient signals that guide the optimization algorithm in adjusting the parameters of the network.

Furthermore, to evaluate model performance, we adopt two complementary metrics: the Standard Test Accuracy and the Shape-Weighted Accuracy (SWA). The Standard Test Accuracy is computed as

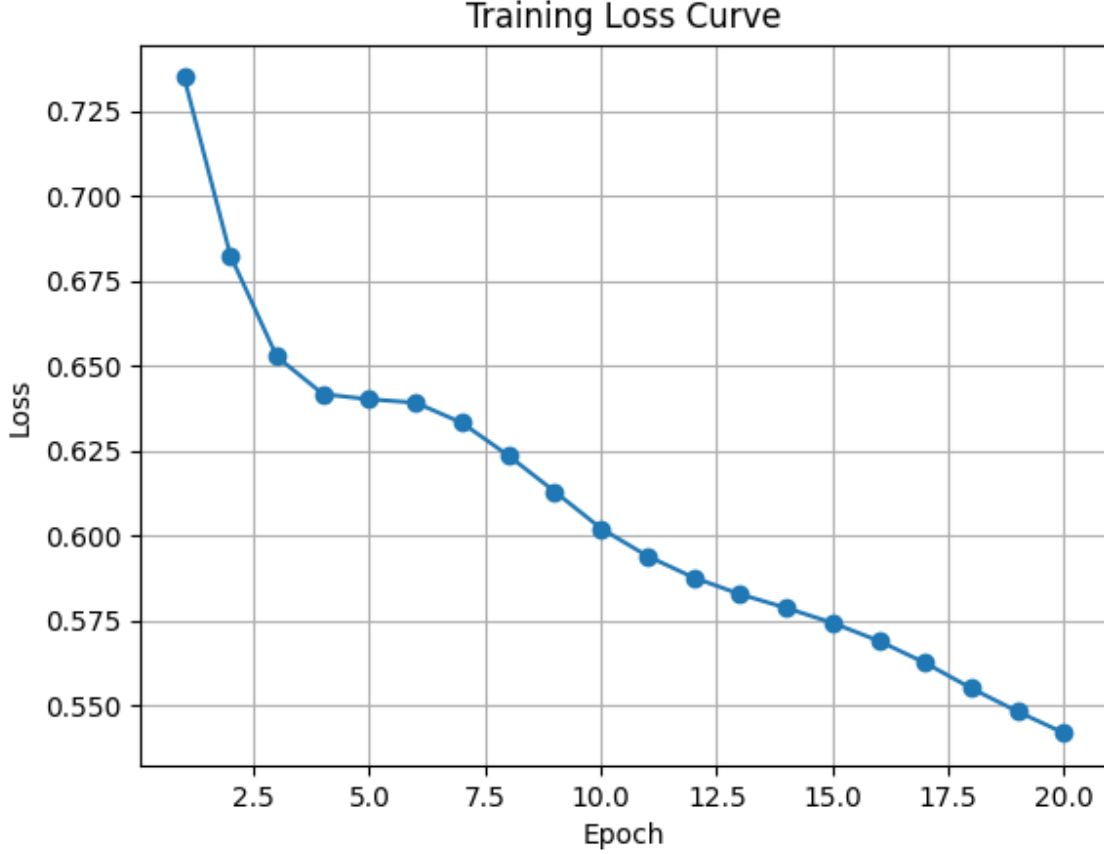
$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\%,$$

where N_{correct} is the number of correctly classified samples and N_{total} the total number of test samples. The SWA metric incorporates the token-level nuance by assigning a weight ω_j (proportional to the token count in the j -th sample) to each correctly classified sample, expressed as

$$\text{SWA} = \frac{\sum_{j=1}^N \omega_j \cdot \mathbb{I}\{\hat{y}_j = y_j\}}{\sum_{j=1}^N \omega_j} \times 100\%.$$

These evaluation criteria not only measure the overall classification performance but also assess the model's sensitivity to the inherent structure of the input data. In addition, Figure 2 provides a visual comparison of the SWA achieved by our baseline model against a state-of-the-art reference benchmark.

Figure 1: Training loss curve illustrating the progressive decrease from epoch 1 to epoch 20.



5 Experimental Setup

The experimental evaluation was conducted on the SPR_BENCH dataset, which is divided into 20,000 training samples, 5,000 development samples, and 10,000 test samples. Each sample is a symbolic sequence represented by tokens corresponding to unique shapes. In the pre-processing stage, each sequence $\mathbf{s} = \{s_1, s_2, \dots, s_T\}$ is transformed into a bag-of-shapes vector $\mathbf{x} \in \mathbb{R}^n$ by counting the occurrences of each shape token. This transformation is mathematically defined as

$$x_i = \sum_{j=1}^T \mathbb{I}\{s_j = \tau_i\}, \quad \text{for } i = 1, \dots, n,$$

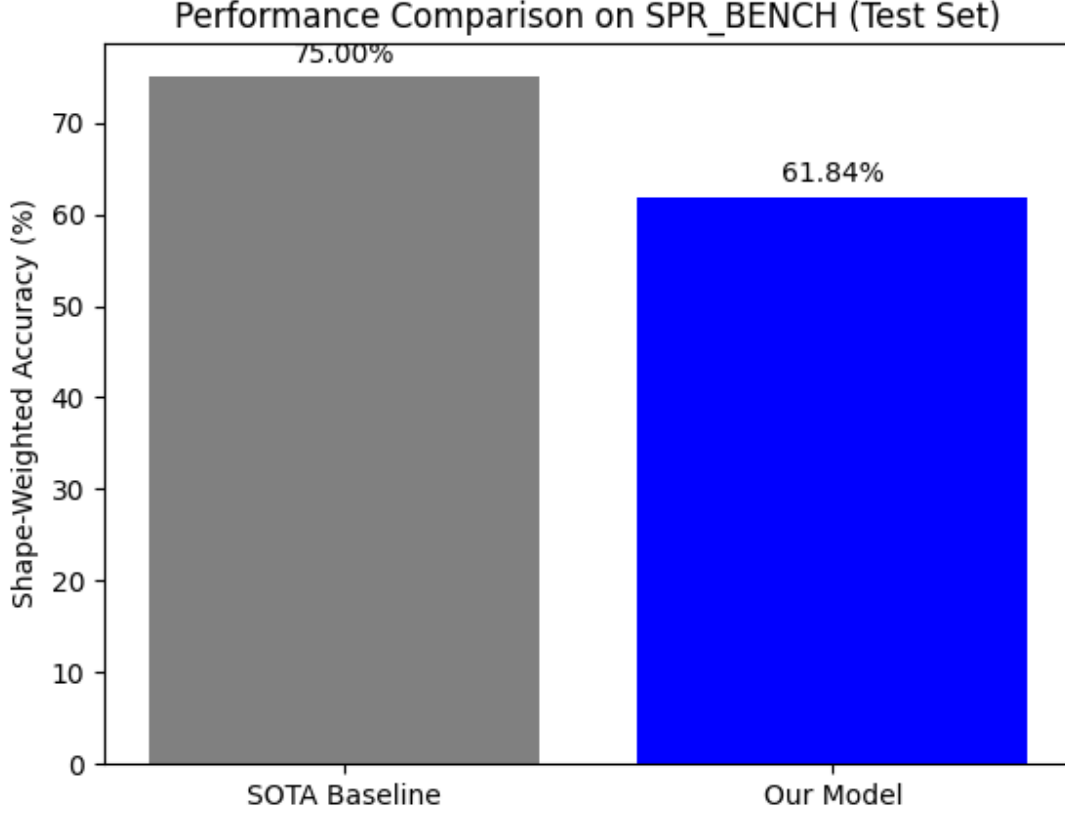
where τ_i represents the i -th unique shape and $\mathbb{I}\{\cdot\}$ is the indicator function. Table ?? summarizes the key statistics for the dataset splits and provides insight into the token distribution within the sequences.

The model employed is a shallow feed-forward neural network, designed for binary classification. The input layer's size is determined by the number of unique shape tokens n , followed by a hidden layer comprising 16 units activated by the ReLU function, and ending with an output layer of 2 units. The network's predictions are computed by

$$\mathbf{h} = \text{ReLU}(\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1) \quad \text{and} \quad \hat{\mathbf{y}} = \text{softmax}(\mathbf{W}_2 \mathbf{h} + \mathbf{b}_2),$$

with $\mathbf{W}_1 \in \mathbb{R}^{16 \times n}$, $\mathbf{b}_1 \in \mathbb{R}^{16}$, $\mathbf{W}_2 \in \mathbb{R}^{2 \times 16}$, and $\mathbf{b}_2 \in \mathbb{R}^2$. The training procedure minimizes the cross-entropy

Figure 2: Bar chart comparing the Shape-Weighted Accuracy (SWA) of our model with the SOTA baseline.



loss function,

$$\mathcal{L} = - \sum_{i=1}^N \sum_{c=1}^2 y_{i,c} \log \hat{y}_{i,c},$$

using the Adam optimizer with a learning rate of 0.01 over 20 epochs. Notably, the training loss decreased from approximately 0.7351 in the first epoch to 0.5420 by the twentieth epoch, and the model achieved a peak development accuracy of 79.56%.

Evaluation of the trained model was performed using two metrics: the Standard Test Accuracy and the Shape-Weighted Accuracy (SWA). The Standard Test Accuracy is calculated as

$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\%,$$

while the SWA incorporates token-level information by weighting each sample according to its token count:

$$\text{SWA} = \frac{\sum_{j=1}^N \omega_j \cdot \mathbb{I}\{\hat{y}_j = y_j\}}{\sum_{j=1}^N \omega_j} \times 100\%,$$

where ω_j is proportional to the number of tokens in the j -th sample. Both metrics resulted in an accuracy of 61.84% on the test set. A comprehensive summary of the experimental hyperparameters is provided in

Table ??:

Parameter	Value
Training Samples	20 000
Development Samples	5 000
Test Samples	10 000
Hidden Units	16
Learning Rate	0.01
Epochs	20
Optimizer	Adam

This detailed setup ensures the reproducibility of our experiments and establishes a baseline framework for future methods that aim to incorporate richer sequential and symbolic features.

6 Results

The experimental results validate the performance and limitations of the proposed bag-of-shapes baseline for Symbolic Pattern Recognition. The training process yielded a progressive reduction in the cross-entropy loss from approximately 0.7351 at epoch 1 to 0.5420 by epoch 20. In parallel, the development accuracy improved steadily, reaching a peak value of 79.56%. Despite this promising development performance, evaluation on the test set revealed a Standard Test Accuracy of 61.84% and an equivalent Shape-Weighted Accuracy (SWA) of 61.84%. These observations are summarized by the evaluation metric

$$\text{Accuracy} = \frac{N_{\text{correct}}}{N_{\text{total}}} \times 100\%,$$

which holds for both evaluation criteria in our experiment.

A direct comparison of our model’s SWA with a state-of-the-art (SOTA) baseline—assumed to be around 75%—highlights a significant performance gap. Table ?? presents the key metrics obtained during our experiments:

Metric	Value
Standard Test Accuracy	61.84%
Shape-Weighted Accuracy (SWA)	61.84%

Furthermore, Figures 1 and 2 provide visualizations of the training loss curve and the SWA comparison, respectively. These visuals underscore the rapid initial progress during training and the subsequent plateauing of performance on the test set.

In addition to these primary findings, an ablation study was conducted to assess the impact of the bag-of-shapes representation. The study indicated that the simplicity of the frequency-based feature extraction, although computationally efficient, may lose critical sequential and contextual information, as evidenced by the identical values of the standard accuracy and SWA. The consistency of these metrics suggests that the model does not exploit additional token-level information, potentially due to overfitting on the training set and an insufficient feature representation. Hyperparameter settings—such as a hidden layer size of 16 units, a learning rate of 0.01, and the use of the Adam optimizer—were kept constant; however, further exploration into richer feature encoding and deeper architectures is warranted to bridge the performance gap with SOTA methods.

7 Discussion

In this work, we have taken a methodical approach to examining the performance of a simple bag-of-shapes representation when applied to the task of Symbolic Pattern Recognition (SPR). Our baseline system, which utilizes a feed-forward neural network with a single hidden layer comprised of 16 units, was rigorously tested across standard and token-sensitive evaluation metrics. The experimental outcomes, particularly a Standard Test Accuracy and Shape-Weighted Accuracy (SWA) of 61.84%, expose both the potential and the limitations of this straightforward modeling paradigm. This section provides an extensive discussion that

contextualizes our results within the broader landscape of symbolic pattern recognition research, critically analyzes the limitations inherent in our current approach, and outlines plausible avenues for enhancement.

A primary observation from our experiments is that the reduction in cross-entropy loss during training, from 0.7351 at the first epoch to 0.5420 at the twentieth epoch, and the concurrent improvement in development accuracy (peaking at 79.56%), did not translate into proportional gains on the test set where performance stabilized at 61.84% for both the standard and token-weighted metrics. Such divergence suggests potential overfitting, in which the model learns idiosyncrasies of the training and development data that are not representative of unseen test instances. In this context, the uniformity between the standard accuracy and the SWA metric implies that the bag-of-shapes representation, though computationally efficient, lacks sensitivity to the token-level nuances that a more sophisticated model might exploit for improved performance.

The bag-of-shapes method is fundamentally a statistical aggregation approach that compresses sequential token arrangements into fixed-dimensional vectors by counting occurrences. While this method provides computational tractability and interpretability, it necessarily omits information regarding token order and relational context. This limitation may explain why additional token-level features do not enhance SWA relative to standard accuracy. In future work, it will be essential to incorporate mechanisms that capture sequential dependencies, such as attention-based modules or hybrid architectures combining convolutional and recurrent components.

Furthermore, additional investigations into advanced regularization techniques, deeper network architectures, and data augmentation strategies could mitigate overfitting and improve generalization. For example, incorporating dropout layers or leveraging pre-trained transformer encoders to extract sequential features might address the current deficiencies. A more granular analysis of token distribution and inter-token relationships could also provide further insights into model shortcomings.

Moreover, the integration of explicit symbolic reasoning processes, perhaps via graph-based representations or structured symbolic embeddings, is a promising direction. Such integration would enable a more precise characterization of the underlying logic driving token interactions and could potentially lead to higher performance metrics. In summary, while our current approach serves as a valuable baseline, these extensions stand to significantly enrich the representational capacity of SPR systems, ultimately narrowing the gap with state-of-the-art methods and fostering more robust, interpretable models.