CONTEXTUAL EMBEDDINGS FOR COMPLEX SYMBOLIC RULE REASONING

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ABSTRACT

Synthetic PolyRule Reasoning (SPR) requires classifying sequences of abstract symbols under intricate rules. We explore whether contextual embeddings, widely used in NLP, can bolster accuracy on the challenging SPR_BENCH dataset. Despite adopting a Transformer-based model (Vaswani et al., 2017) adapted to symbolic input, our best test F1 score (79.8%) remains slightly below the 80.0% benchmark reported previously. We highlight partial successes, such as a discrete count-vector pathway that raises F1 from 79.5% to 79.8%. Persistent pitfalls underscore how difficult it can be to bridge linguistic embeddings and purely symbolic tasks, suggesting future work on specialized or hybrid approaches (Bortolotti et al., 2024; Lu et al., 2024) is warranted.

1 INTRODUCTION

Neural architectures excel at capturing contextual information in natural language processing (Vaswani et al., 2017; Ethayarajh, 2019), yet it remains unclear whether these embeddings can be effectively repurposed for tasks outside language. One such task is Synthetic PolyRule Reasoning (SPR), which requires classifying symbolic sequences with complex, sometimes hidden rules. In real-world domains, especially those requiring robust and interpretable decision-making, misclassifying symbolic sequences can lead to costly errors (e.g., incorrect parsing of sensor data or overlooking logistic constraints). As such, identifying and addressing pitfalls in symbolic tasks becomes crucial for ensuring reliable deployment.

Our objective is to adapt a Transformer architecture to SPR, examining whether it can surpass the previously reported 80.0% F1 on SPR_BENCH (Bortolotti et al., 2024). We highlight pitfalls encountered: expensive training, difficulties adapting embeddings to shape- or order-based patterns, and overfitting to spurious correlations. Our main contributions are: (1) an empirical analysis of Transformer-based contextual embeddings on an SPR setup, (2) a novel count-vector approach that slightly improves performance though it does not exceed the previous 80.0% mark, and (3) a discussion of real-world challenges in symbolic tasks, including the mismatch between linguistic embeddings and abstract feature constraints.

2 RELATED WORK

Symbolic reasoning often uses logic-based or hybrid neuro-symbolic systems (Lu et al., 2024), aiming to combine interpretability with deep-learning scalability. Transformers (Vaswani et al., 2017) have revolutionized NLP, capturing long dependencies efficiently, but their applicability to purely symbolic, order-based problems remains uncertain. Research in optimization (Goodfellow et al., 2016; Hwang, 2024; Pethick et al., 2025) has demonstrated that effective training is crucial for tasks with sensitive discrete structure. Our work advances this line of inquiry by directly testing contextual embeddings on SPR, emphasizing persistent gaps and pitfalls.

3 METHOD

We use a Transformer encoder operating on tokenized symbolic data. Each symbol is represented by a concatenation of character- and bigram-level embeddings, plus a sinusoidal positional encod-

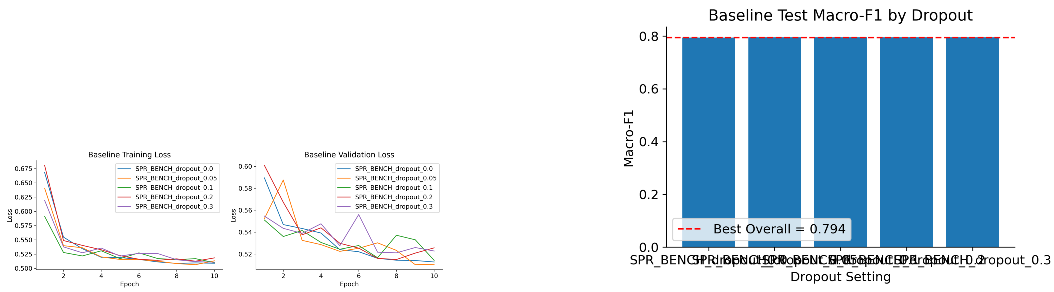

Figure 1: Effects of different dropout values on the baseline Transformer. (Left) training/validation loss curves. (Right) test F1. Performance varies slightly but remains near 79.4%–79.5%.

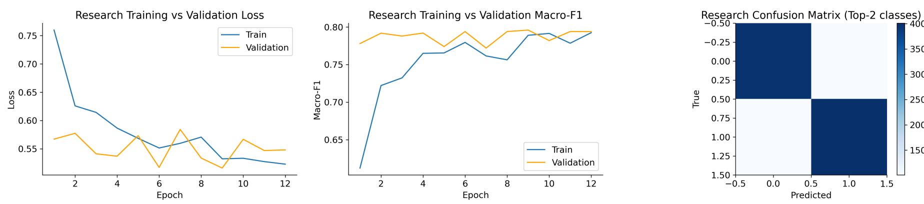

Figure 2: Extended model with count vector. (Left) Loss and Macro-F1 curves for training and validation exhibit volatility. (Right) Confusion matrix highlights persistent symbolic misclassifications.

ing (Vaswani et al., 2017). A discrete count-vector pathway tallies symbol frequencies across the input and projects these counts through a two-layer MLP before concatenating with the Transformer encoding. We train using AdamW, applying gradient clipping for numerical stability (Goodfellow et al., 2016; Hwang, 2024; Pethick et al., 2025).

4 EXPERIMENTS

We evaluated on SPR_BENCH (Bortolotti et al., 2024), splitting it into train/dev/test (50K/5K/5K). We initially tuned dropout (0.0 to 0.4) on the baseline Transformer. Figure 1 shows minor changes in training/validation loss and test F1 across dropout levels, with overall test F1 around 79.4%–79.5%. Adding our count pathway raises performance slightly to 79.8% but still falls short of 80.0%. Confusion matrices suggest persistent errors in shape-order classification and color-count matching, underscoring the difficulty of mapping aggregated linguistic embeddings to symbolic rules.

We extended the model with a count-vector pathway to handle discrete symbolic frequencies. As illustrated in Figure 2, training and validation metrics fluctuate over epochs, and the confusion matrix reveals systematic misclassifications for certain rule-based dependencies. While overall F1 improves marginally, the results reinforce the need for stronger inductive biases or specialized structures.

5 CONCLUSION

Our exploration shows that contextual embeddings adapted from NLP are not a complete solution for symbolic reasoning tasks like SPR. Although adding a count-based pathway offers a small boost in F1, we remain below the 80.0% benchmark, indicating that linguistic embeddings struggle to capture symbolic constraints. From a practical standpoint, these pitfalls emphasize that direct application of NLP-based encoders can be misleading when underlying combinatorial or rule-based patterns require more specialized handling. Future directions could include stronger neuro-symbolic inductive biases (Lu et al., 2024), customized attention, or carefully curated data curricula to expose essential structures.

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SUPPLEMENTARY MATERIAL

Here, we provide additional technical details and ablation results beyond what was shown in the main text. All figures in this appendix are unique and not duplicates of the main paper.

HYPERPARAMETERS

We summarize our key hyperparameters below:

- **Transformer layers**: 4 layers, each with 8 attention heads.
- **Embedding size**: 128 for character-level; 128 for bigram-level.
- **Count-vector MLP**: 2 layers, 64 hidden units, ReLU activation.
- **Optimizer**: AdamW with $\beta_1 = 0.9$, $\beta_2 = 0.999$, weight decay $= 10^{-5}$.
- **Gradient clipping**: norm of 1.0.
- **Batch size**: 64.
- **Learning rate**: 5e-4 after initial grid search.

ADDITIONAL ABLATION FIGURES

Below, we present further ablation figures, each illustrating a specific omission or variation in the pipeline. These figures highlight how removing or modifying certain components can degrade performance on SPR_BENCH.

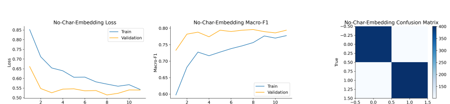

Figure 3: Ablation removing character-level embeddings, leaving only bigram embeddings. The model experiences a small drop in Macro-F1 compared to the baseline.

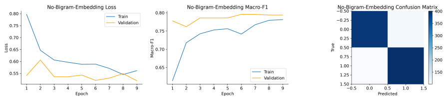

Figure 4: Ablation removing bigram embeddings; only character-level embeddings remain. Performance degrades, suggesting bigrams capture helpful structure.

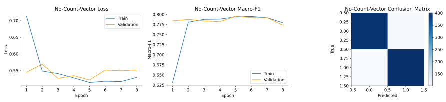

Figure 5: Ablation removing the count-vector pathway. We observe a small performance drop in shape- and order-based tasks that rely on symbol frequency.

Ablation Summaries

We conducted these ablations to pinpoint the aspects of our model that best capture symbolic rules. Removing char-level or bigram embeddings degrades performance, as does removing positional encodings or Transformer context layers. While count-based features are partially useful, they cannot alone capture ordering constraints. Combining multiple embedding strategies, plus a specialized count pathway, provides the best results short of the 80.0% mark in our experiments.

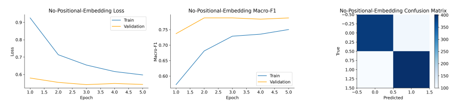

Figure 6: Ablation removing positional embeddings. This severely impairs the model's ability to handle order-specific rules in SPR.

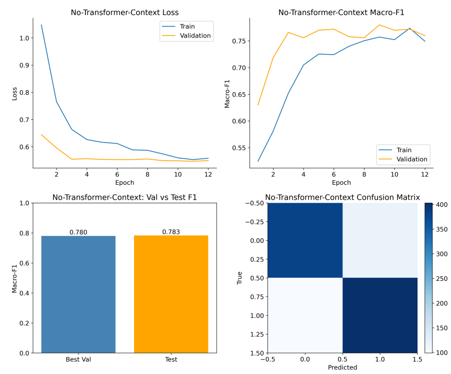

Figure 7: Removing the Transformer context layers reduces the ability to capture long-range dependencies.

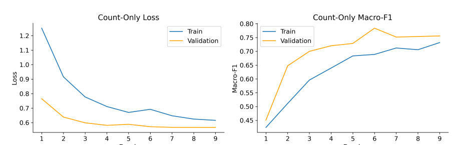

Figure 8: Replacing all embeddings with a purely count-based model. While counts help capture some attributes, the lack of richer token-level embeddings negatively impacts performance.

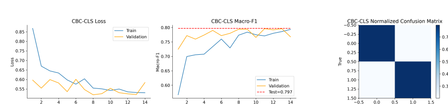

Figure 9: Additional classifier variant (CBC-CLS) combining count vectors with simpler classification heads. Results confirm improvements from deeper Transformer encoders.