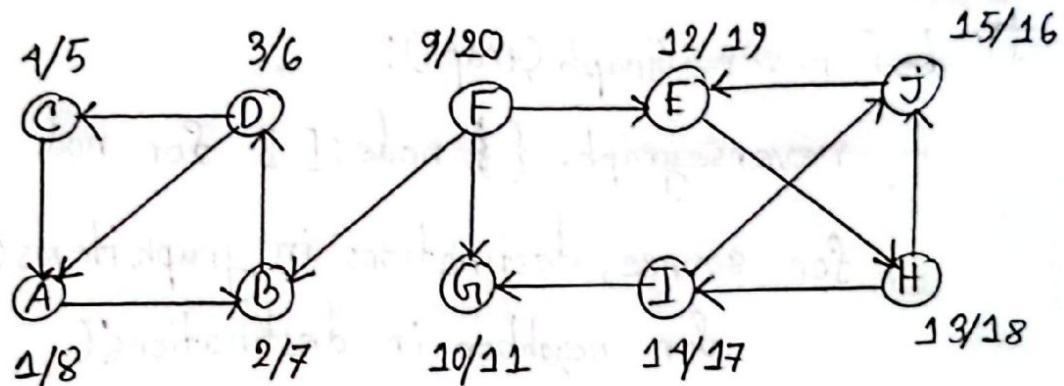


Summer - 23

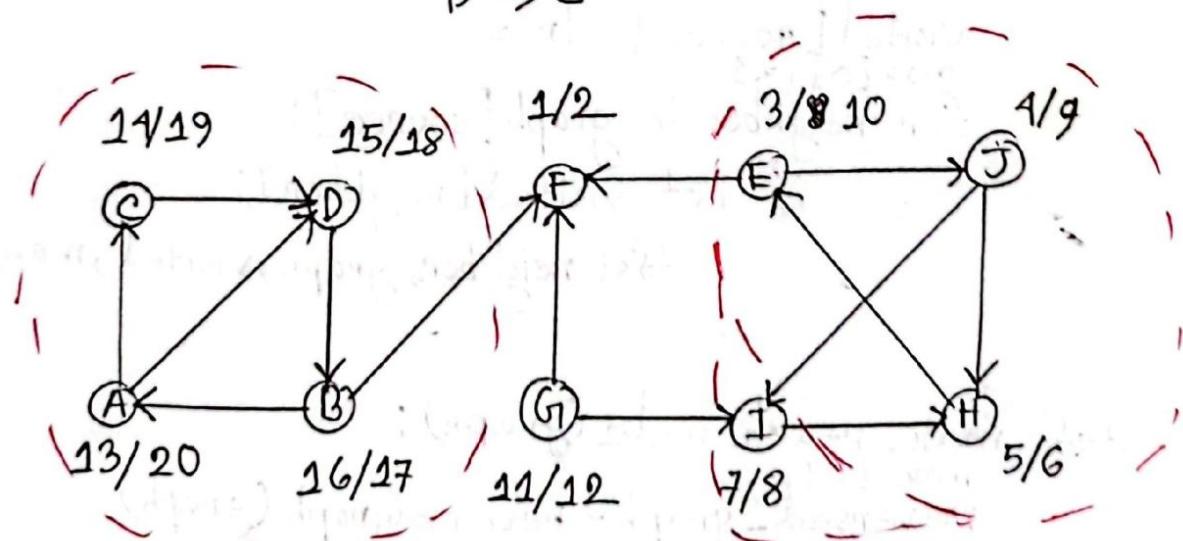
(A)

1) a) Kosaraju's Strongly Connected Components Algorithm

b)



Decreasing order: $F \rightarrow E \rightarrow H \rightarrow I \rightarrow J \rightarrow G \rightarrow A \rightarrow B \rightarrow D \rightarrow C$



Two Groups:

1
Carol
Dan
Bob
Alice

2
Ethan
Jack
Isaac

farhan
farhan

c) Fahim
Glen

d) def reverseGraph(graph):

reversegraph = {
 node: [] for node in graph}

for source, destinations in graph.items():

for neighbor in destinations:

reversegraph[neighbor].append(source)

return reversegraph

def dfs(source, graph, visited, max):

visited[source] = True

max[0] += 1

for neighbor in graph[source]:

if not visited[neighbor]:

dfs(neighbor, graph, visited, max)

def max_reach_node(graph):

max = [0]

reversed_graph = reverseGraph(graph)

max_visit = float("-inf")

max_node = None

for node in graph:

if not visited

\downarrow
 visited = {node: False for node in graph}
 for node in graph:
 dfs(node, graph, visited.copy(), max)
 if max_visit < max[0]:
 max_visit = max[0]
 max_node = node
 max[0] = 0
 return (max_node, max_visit)

2)

a) 0000 | 0100 | 0100 | 1000 | 0100 | 1000
 a | e | e | i | e | i

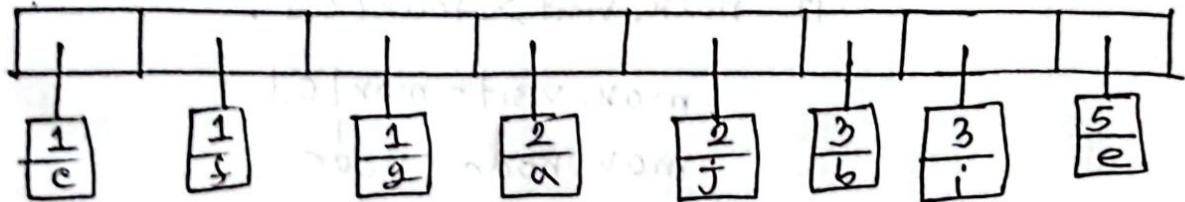
0100 | 1000 | 0001 | 0101 | 0001 | 0001
 e | i | b | f | b | b.

0110 | 0000 | 0100 | 1001 | 0010 | 1001
 g | a | e | j | c | j

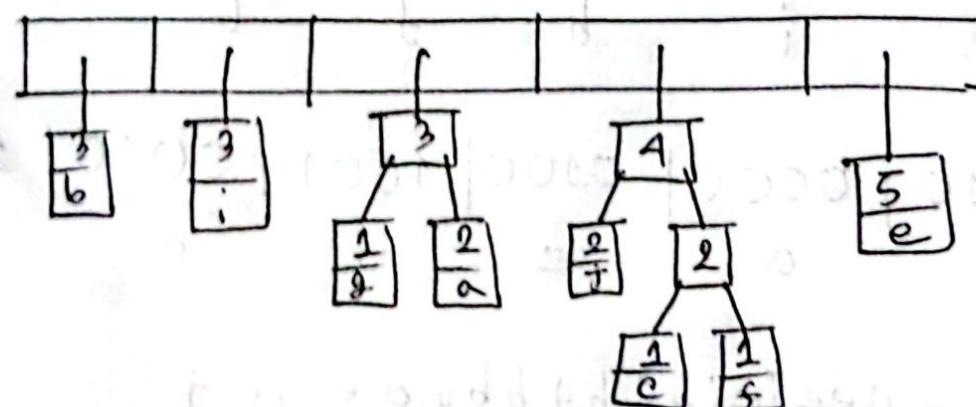
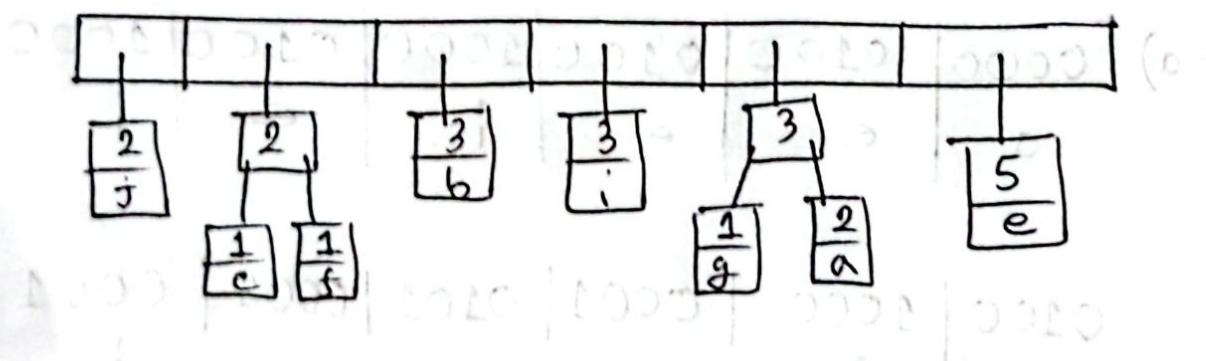
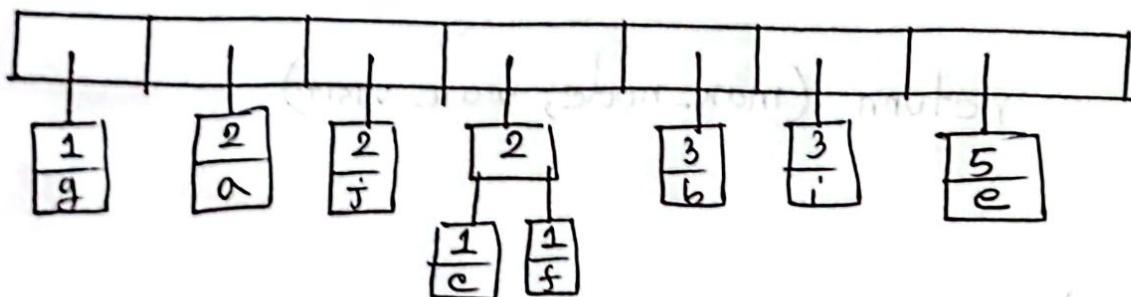
Message = aeeieieibfbbgaejcg

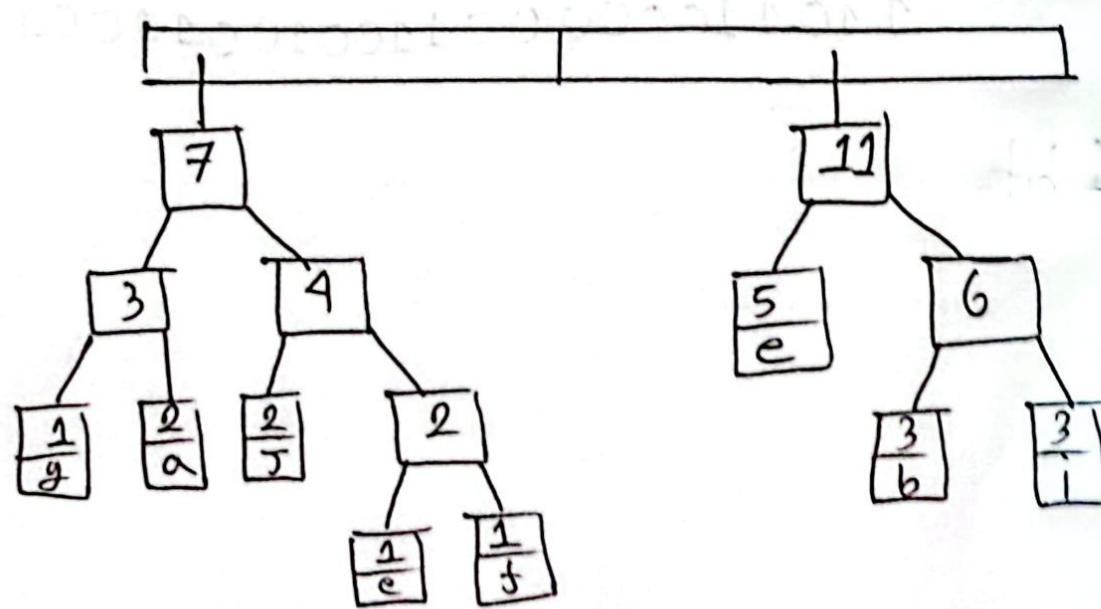
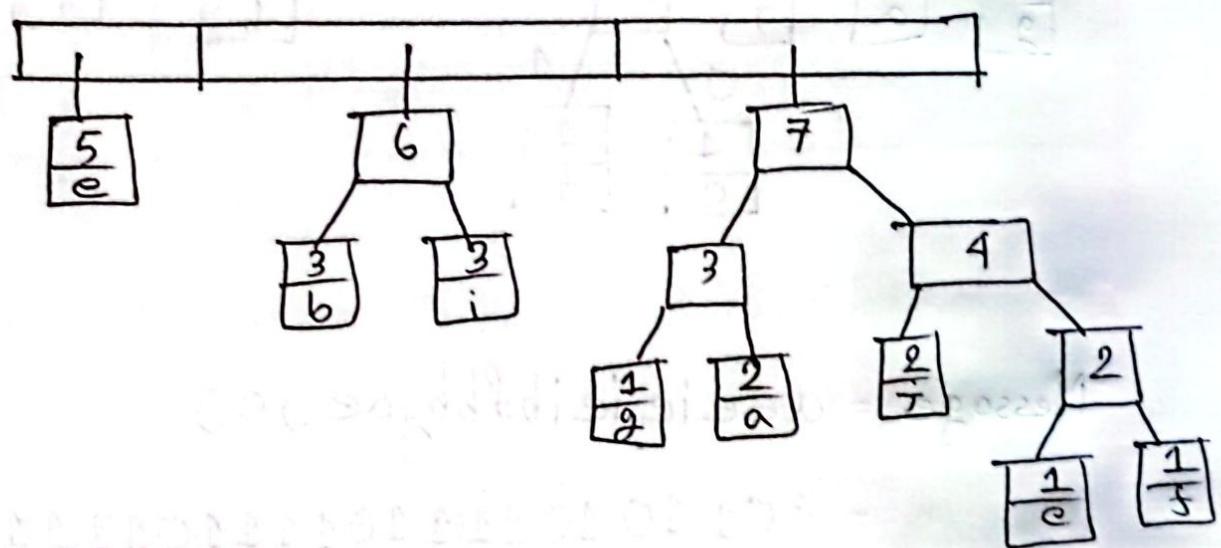
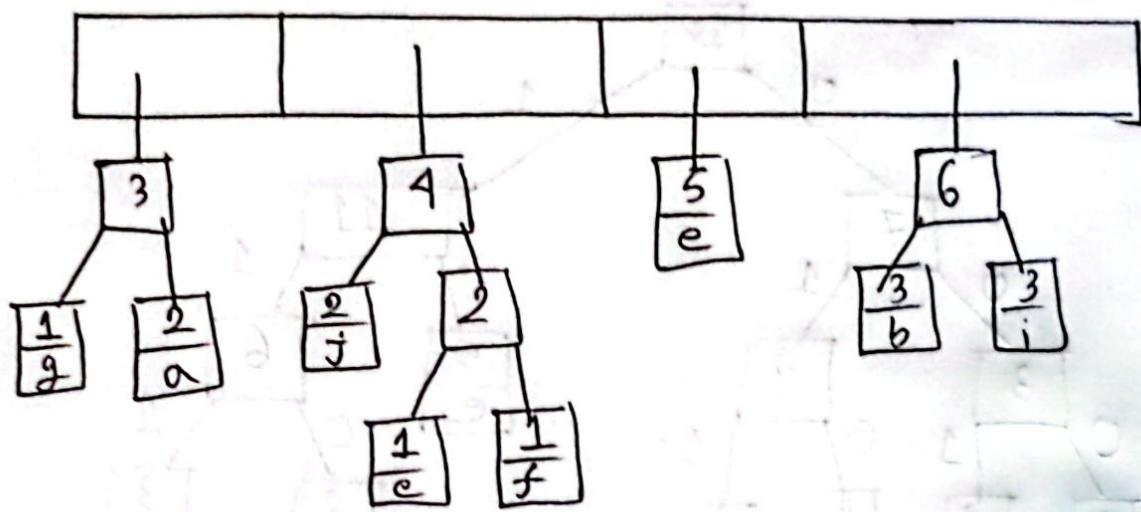
b) Frequency Table

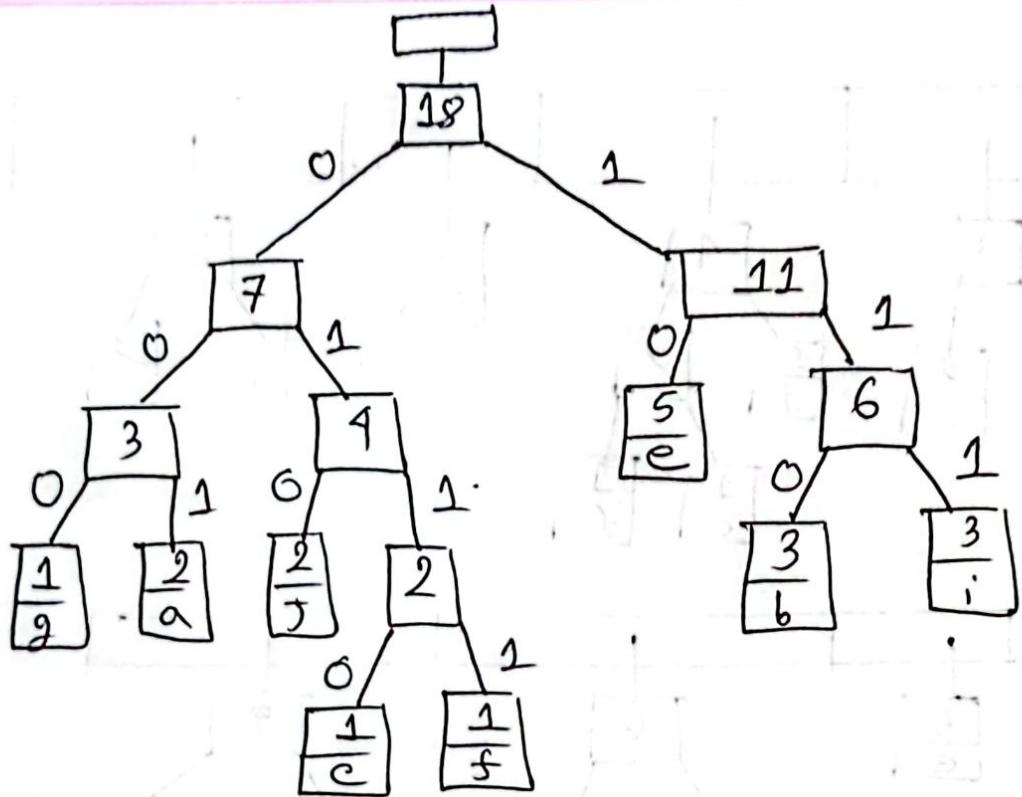
Character	c	f	g	a	j	b	i	e
Frequency	1	1	1	2	2	3	3	5



0-[0] Note







Message = aeeieieibfbbaejej

= 0011010111101111101111100111
110110000001100100110010

c) 51 bits

2) a) Leo's end early

b)

Faculty	Start	End	Picked	Reason
MZU	2	3	✓	
FGZ	3	6	✓	
RIM	5	7	✗	
AGD	6	8	✓	
SBD	7	10	✗	
MIBA	1	10	✗	
MNR	10	11	✓	

Maximum 4 Class — MZU, FGZ, AGD, MNR

c)

Time	Start / End	People Needed	Count
1	+ Miba	{Miba}	1
2	+ MZU	{Miba, MZU}	2
3	+ FGZ, - MZU	{Miba, FGZ}	2
5.	+ RIM	{Miba, FGZ, RIM}	3
6.	+ AGD, - FGZ	{Miba, AGD, RIM}	3
7	+ SBD, - RIM	{Miba, SBD, AGD}	3
8	- AGD	{Miba, SBD}	2
10	- Miba, + MNR	{MNR}	1
11	- MNR	0	0

Minimum number of students = 3

3)

a) $\text{knapSack}(n, w) = \begin{cases} \text{knapSack}(n-1, w) & \text{if } w_j > w \\ \max(V_n + \text{knapSack}(n-1, w - w_j), \\ \text{knapSack}(n-1, w)) \end{cases}$

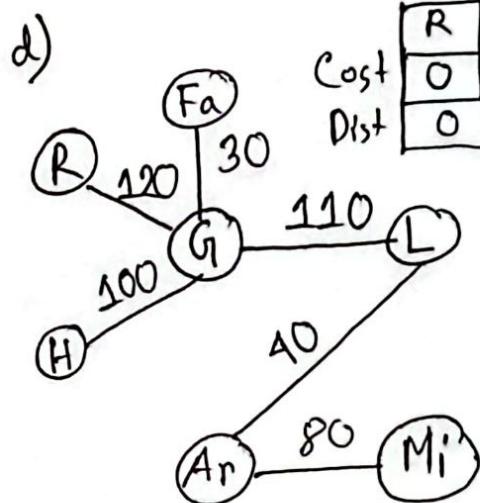
b)

Value	Size	Item	Maximum Available Space					
			0	1	2	3	4	5
	0	0	0	0	0	0	0	0
5	1	1	0	5	5	5	5	5
7	2	2	0	5	7	12	12	12
8	3	3	0	5	8	13	15	20
9	4	4	0	5	8	13	15	20
10	5	5	0	5	8	13	15	20

$$\begin{aligned}
 20 - 8 &= 12 \rightarrow (8, 2) & \text{Max profit} = 20 \\
 12 - 7 &= 5 \rightarrow (7, 2) \\
 5 - 5 &= 0 \rightarrow (5, 1)
 \end{aligned}$$

$$\textcircled{C} \quad \begin{aligned} \mathcal{D}(n, w, k) = \max & (v_n + \mathcal{D}(n-1, w-w_j, k), \\ & v_n + \mathcal{D}(n-1, \frac{w-w_j}{2}, k-1), \\ & \mathcal{D}(n-1, w, k)) \end{aligned}$$

<u>D</u>	Edge	Weight
✓	(G-L)	110
✓	(Ar-Mi)	80
✓	(Fa-G)	30
✓	(Ar-L)	40
X	(G-Mi)	50
✓	(Fa-Mi)	60
X	(Ar-Fa)	70
X	(Ar-Fa-L)	90
✓	(G-H)	100
✓	(G-R)	120
X	(H-Mi)	150
X	(Mi-R)	180
$ V =7$		
$ E =12$		



R	G	Fa	H	L	Ar
0	120	150	240	230	270
0	40	50	65	85	105

Mi
350
135

- b) 480
c) 460

Vertex Set = $\{\underline{G}\}, \{\underline{L}\}, \{\underline{Ar}\}, \{\underline{Mi}\},$
 $\{\underline{Fa}\}, \{\underline{R}\}, \{\underline{H}\}$

$$= \{\cancel{G}, \cancel{L}\}, \{\cancel{Ar}, \cancel{Mi}\} \quad \{\cancel{Fa}, \cancel{G}, \cancel{L}\}$$

$$= \{\cancel{Ar}, \cancel{Mi}, \cancel{Fa}, \cancel{G}, \cancel{L}\}$$

$$= \{\cancel{Ar}, \cancel{Mi}, \cancel{Fa}, \cancel{G}, H, L\}$$

$$= \{\cancel{Ar}, \cancel{Mi}, \cancel{Fa}, \cancel{G}, H, L, R\}$$

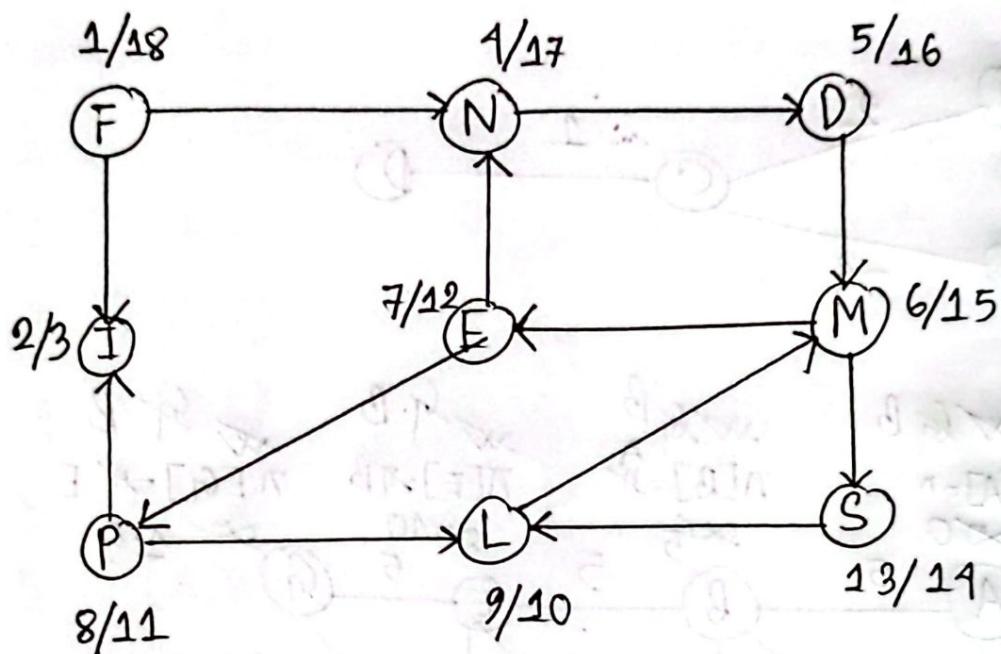
a)

$T = \emptyset \vee \emptyset \neq \emptyset$

Fall 2023

(B)

1) a)

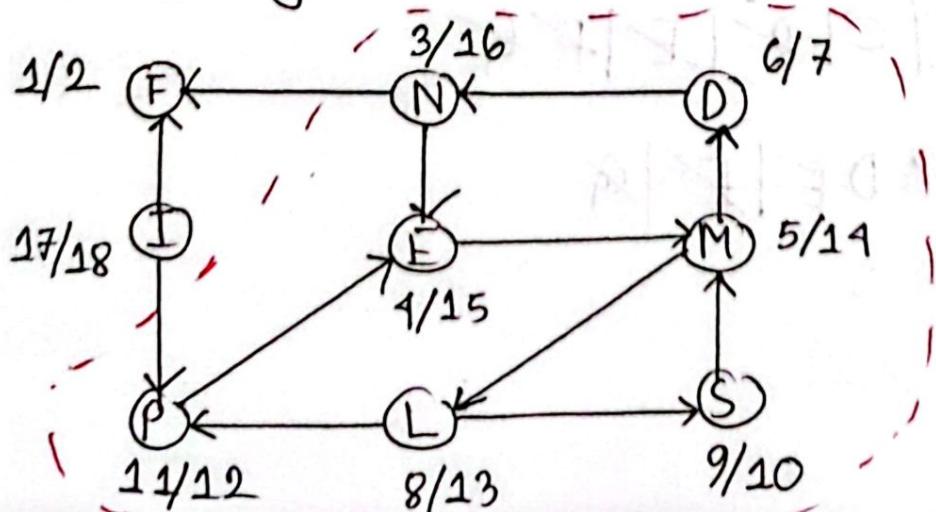


Node	D	E	F	I	M	S	L	P	N
Start	5	7	1	2	6	13	9	8	9
End	16	12	18	3	15	14	10	11	17

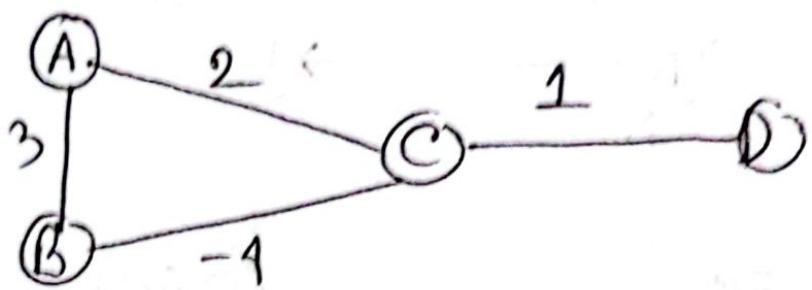
b) i) Korasaju's Strongly Connected Component Algorithm

ii)

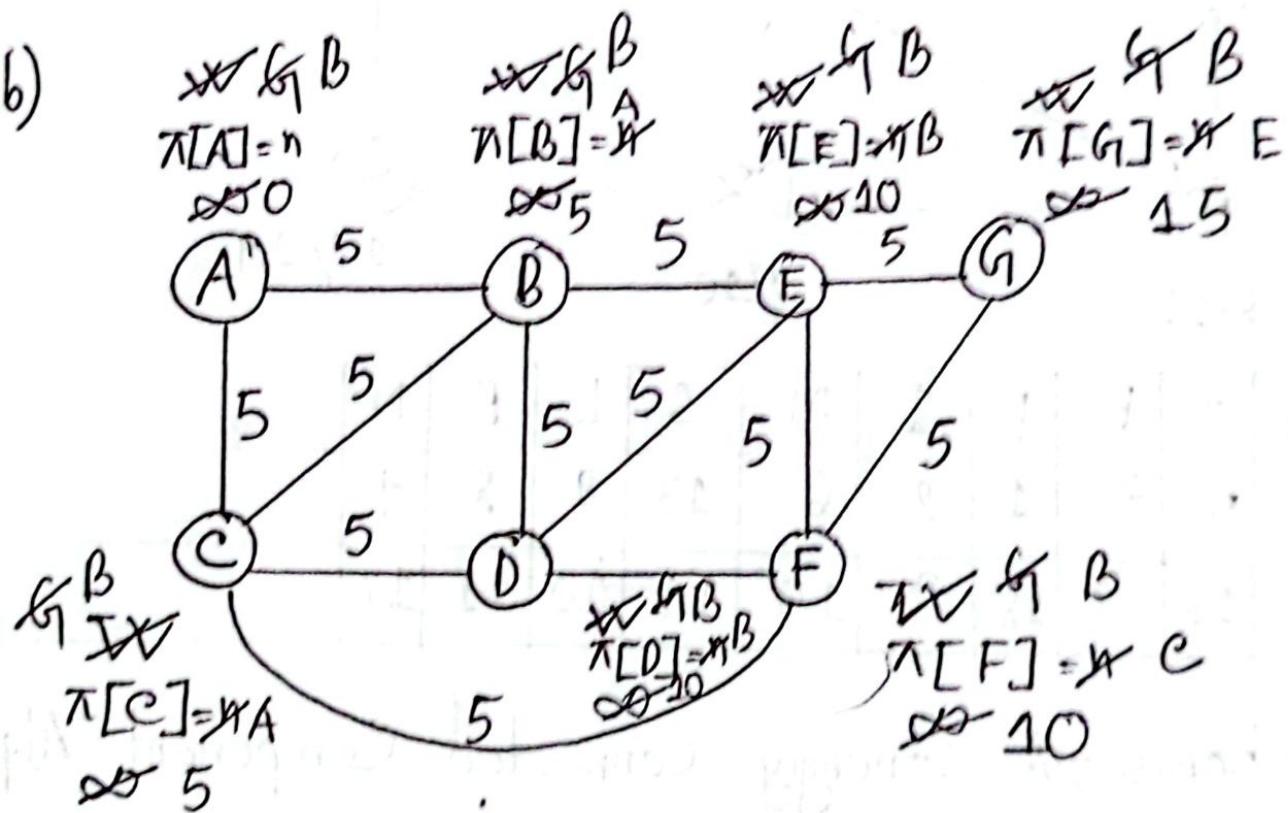
Decreasing order: $F \rightarrow N \rightarrow D \rightarrow M \rightarrow S \rightarrow E \rightarrow P \rightarrow L \downarrow I$



2) a) False



b)



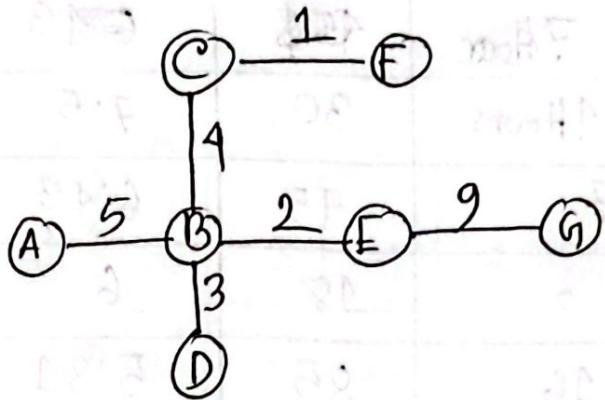
$$Q = \emptyset \text{ A } B \text{ C } D \text{ E } F \text{ G}$$

$$U = A | B | C | D | E | F | G$$

$$V = B, C | D, E | F | G$$

(C)

Weight	Edge
✓ 1	(C, F)
✓ 2	(B, E)
✓ 3	(B, D)
✓ 4	(B, C)
✗ 5	(C, D)
✓ 5	(A, B)
✗ 6	(D, E)
✗ 6	(D, F)
✓ 9	(E, G)
✗ 13	(A, C)



- i) Explorer will choose ticked bridges
ii) 24

$$|V|=7$$

$$|E|=10$$

$$T = \emptyset \times \times \times \times \times \times$$

$$\text{Vertex Set} = \{\{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{F\}, \{G\}\}$$

$$\begin{aligned} &= \{\{C, F\}, \{\{B, E\}, \{D, E\}\}\} \\ &= \{\{B, C, D, E, F\}, \{\{A, B, C, D, E, F\}\}\} \\ &= \{A, B, C, D, E, F, G\} \end{aligned}$$

3) a) Fractional Knapsack

Time Limit = 20 Hours

Game	Time	Prize	Value Index	Chosen	Prize
HOB	7 Hours	45\$	6.43		
VC	4 Hours	30	7.5	4	30
HOD	7	95	6.43	7	95
FC	3	18	6	3	18
RR	16	85	5.31	6	31.86
GTA	20	95	4.75	0	0
DMC	11	50	4.55	0	0
					121.86\$

b) i) Encoding table

Character	Encoded	Character	Encoded	Character	Encoded
U	000	r	1001		
t	001	e	101		
B	0100	sp	110		
I	0101	a	1110		
O	0110	c	1111		
h	0111				
S	1000				

ii) 01111101110110011011000111110001101
 01001001111011111000010101110110100101101101001

Bacula to the rescue

3) a) seq(6, 4)

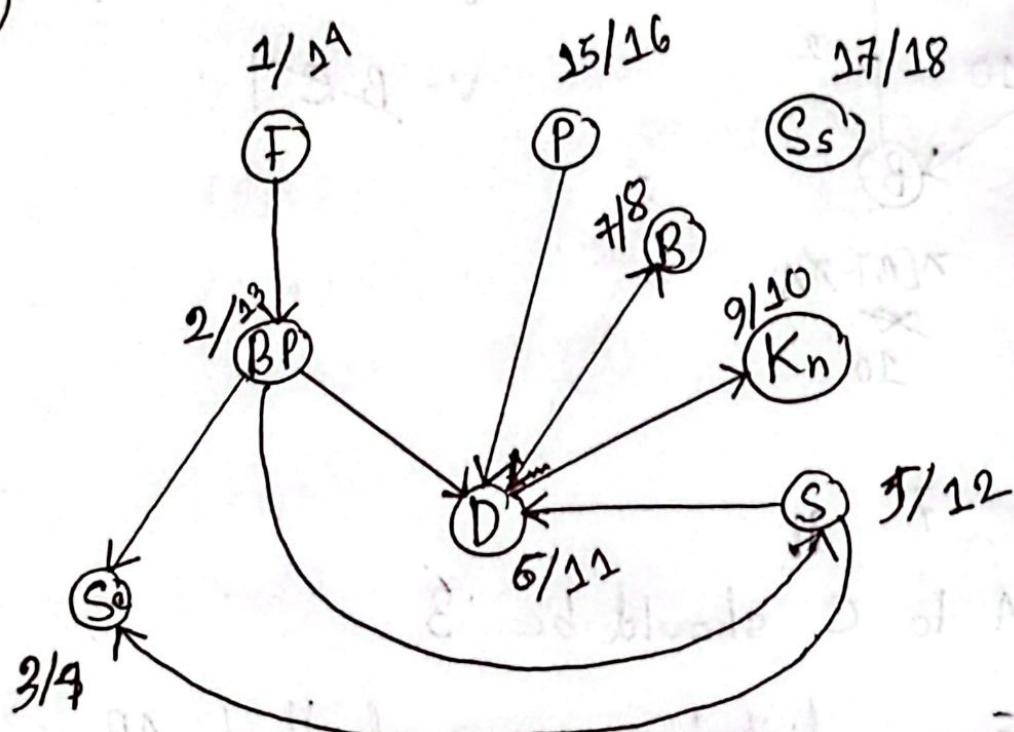
	G	A	T	C	
b)	0	1	2	3	4
	0	0	0	0	0
c	1	0	-1	-1	-1
T	2	0	-1	-2	0
G	3	0	0	-1	-1
A	1	0	-1	0	-1
G	5	0	0	-1	-2
C	6	0	-1	-2	-3

c) $O(NM)$

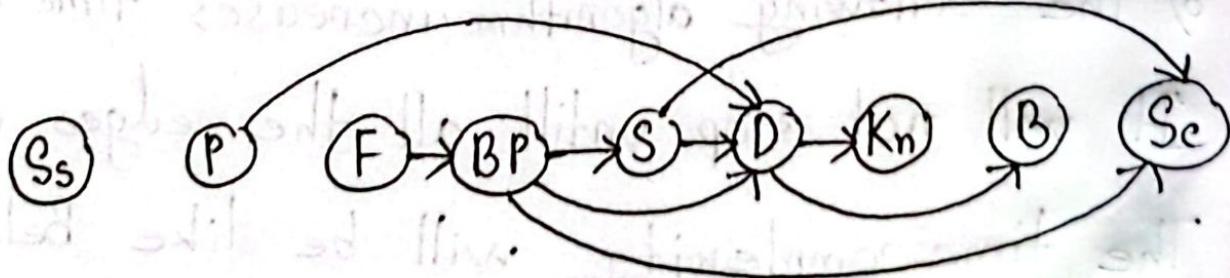
d) To reduce time complexity we just need to track the previous row. So, it can be done in $O(\min(N, M))$

1) a) Topological Sort Algorithm

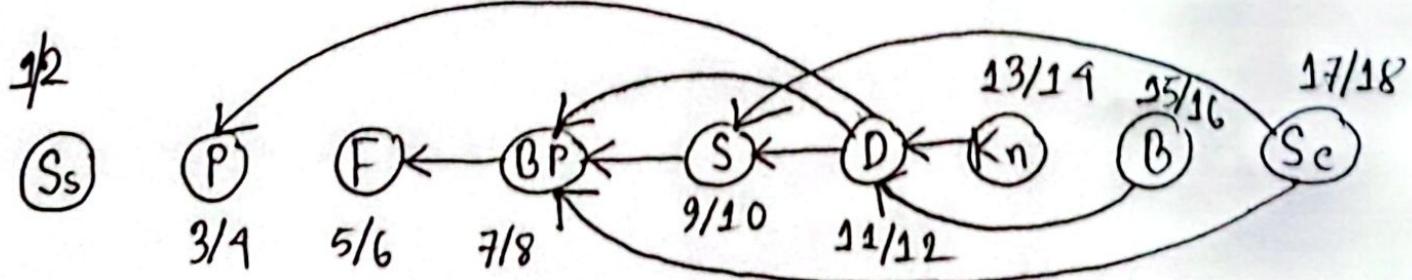
b)



end	18	16	14	13	12	11	10	8	9
Node	Ss	P	F	BP	S	D	Kn	B	Sc

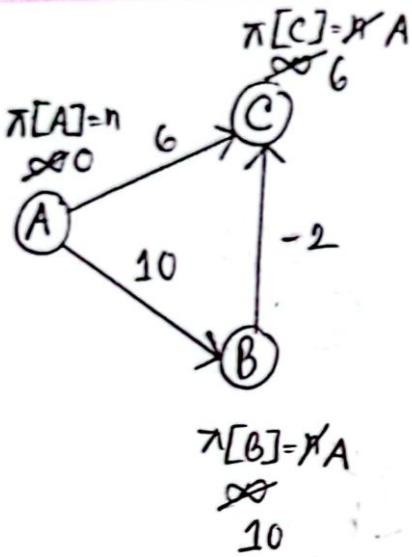


(C)



All are SCC

2) a)



$$PQ = A \ B \ C$$
$$U = A \ C \ B$$
$$V = \cancel{B} \ C \ |$$

Let's, $BC = -7$

So, A to C should be 3

But since Dijkstra assumed that AB is 10.
and AC is 6. Dijkstra fails.

b) The following algorithm increases time complexity.
It will not stop until all the edges are visited.
The time complexity will be like Bellman Ford
 $O(V * E)$

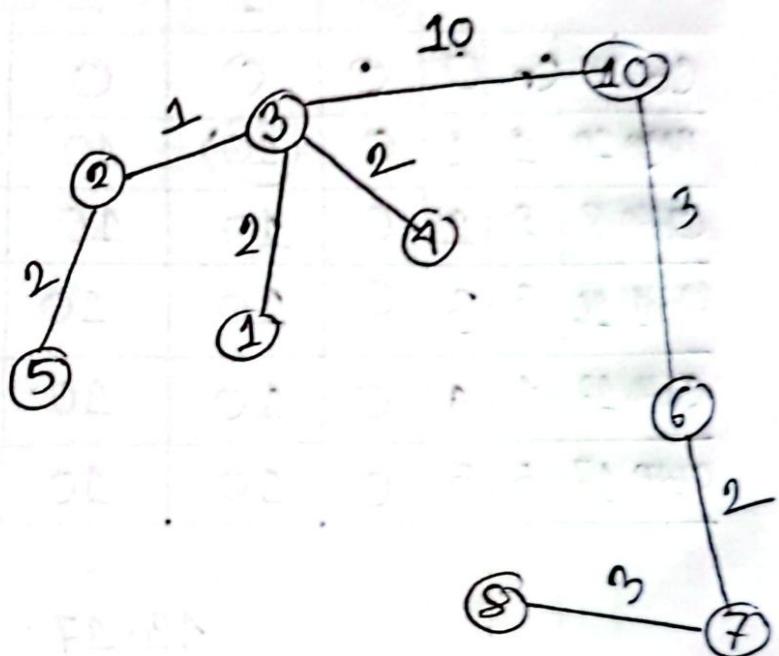
c)

<u>Weight</u>	<u>Edge</u>
✓ 1	(2,3)
✓ 2	(2,5)
✓ 2	(3,1)
✓ 2	(3,4)
✓ 2	(6,7)
✗ 3	(2,1)
✗ 3	(4,1)
✓ 3	(10,6)
✓ 3	(8,7)
✗ 4	(5,1)
✗ 7	(6,8)
✓ 9	(3,10)

$$|V|=10$$

$$|E|=12$$

$$T = \emptyset \times \cancel{1} \times \cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \\ \quad \quad \quad \times 8$$



$$\text{Vertex Set} = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}\} \\ = \{\{2,3\}, \{2,3,5\}, \{1,2,3,5\}, \{1,2,3,4,5\}, \{6,7\}, \{6,7,10\}, \{6,7,8,10\}, \{1,2,3,4,5,6,7,8,10\}\}$$

3) a)

		0	1	2	3	4	5	6	7	8	9
Co	V	Cr	0	Q	0	0	0	0	0	0	0
PHY112	10	1	1	0	(10)	10	10	10	10	10	10
CSE330	8	2	2	0	10	10	18	18	18	18	18
CSE331	16	3	3	0	10	10	18	(26)	26	39	39
CSE370	13	1	1	0	10	10	18	26	26	39	39
CSE499	17	5	5	0	10	10	18	26	26	39	39
											43

$$43 - 17 = 26 \quad \text{CSE 499}$$

$$26 - 16 = 10 \quad \text{CSE 331}$$

$$10 - 10 = 0 \quad \text{PHY 112}$$

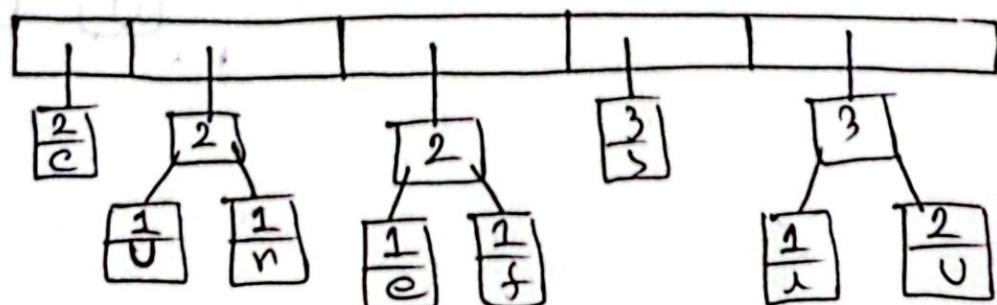
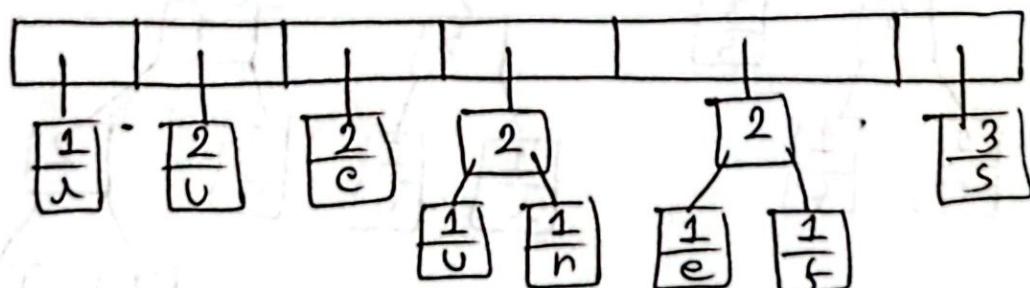
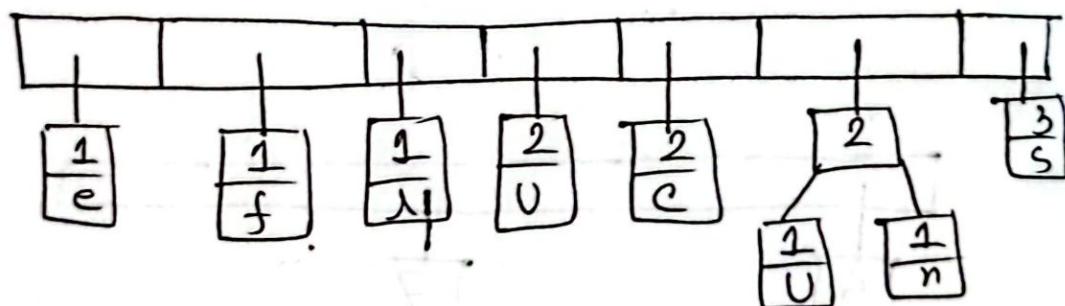
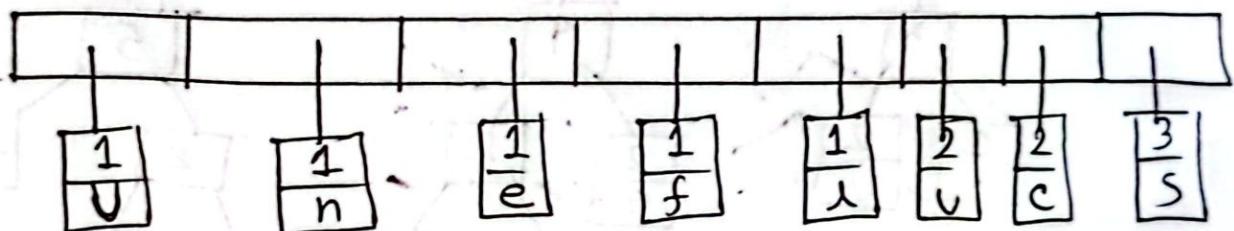
b)

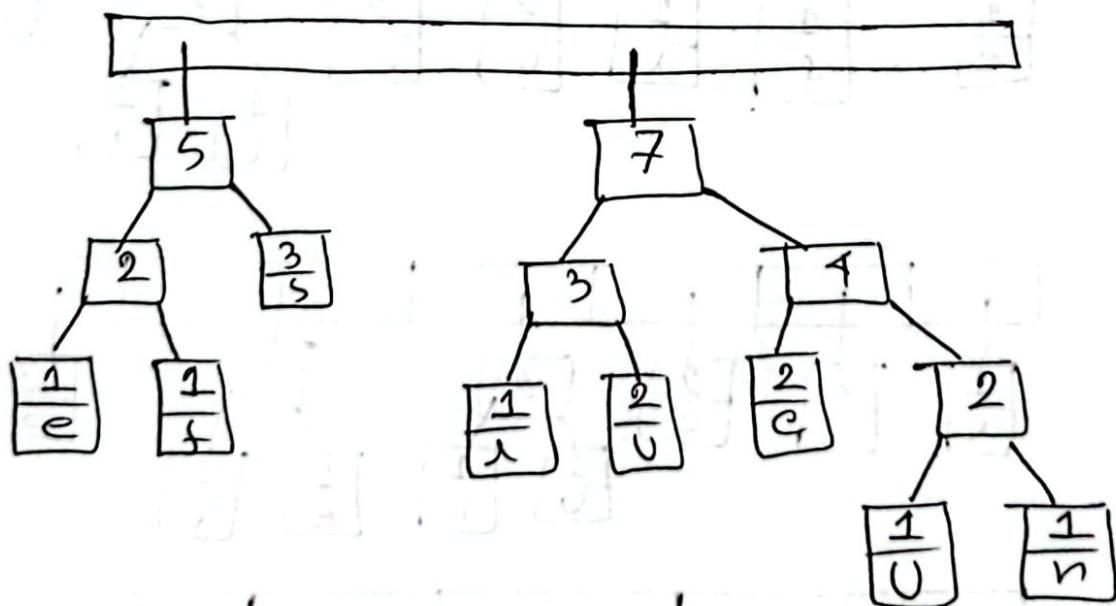
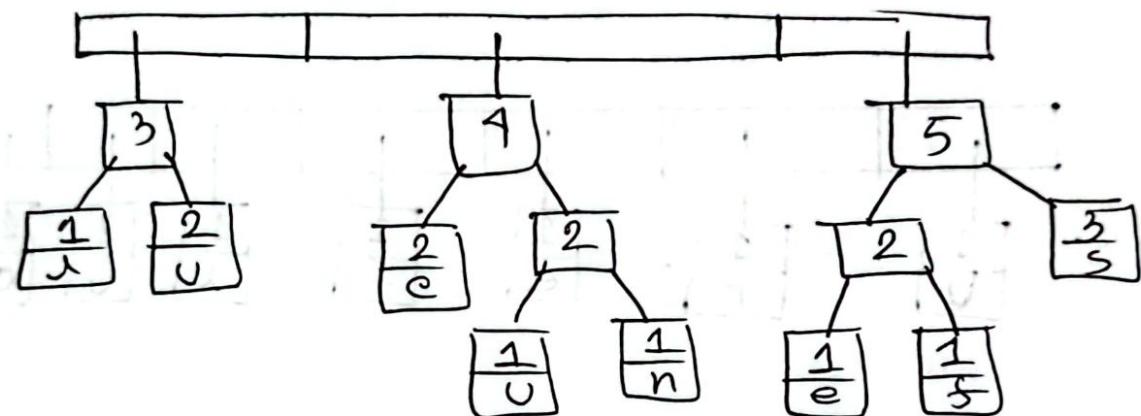
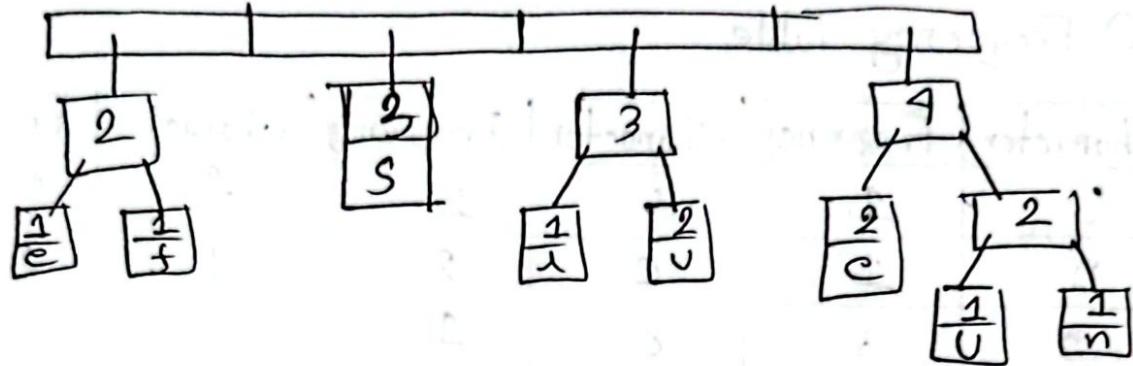
Course	Value	Credit	Value Index	Choosen	
PHY112	10	1	10	1	
CSE331	16	3	5.3	3	
CSE330	8	2	1	2	
CSE499	17	5	3.4	0	
CSE370	13	1	3.25	0	

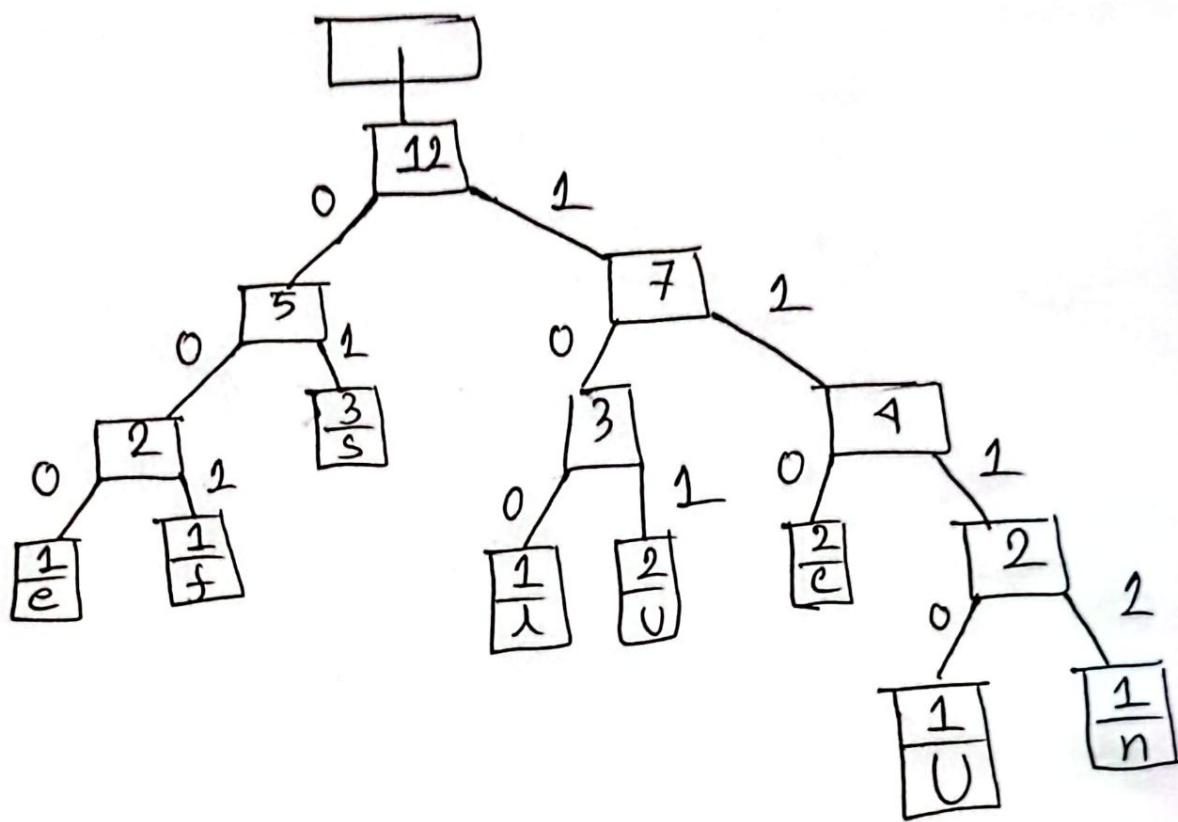
6 < 9 Unsuccessful

© Frequency Table

Character	Frequency	Character	Frequency	Character	Frequency
U	1	v	2	f	1
n	1	c	2	l	1
s	3	e	1		





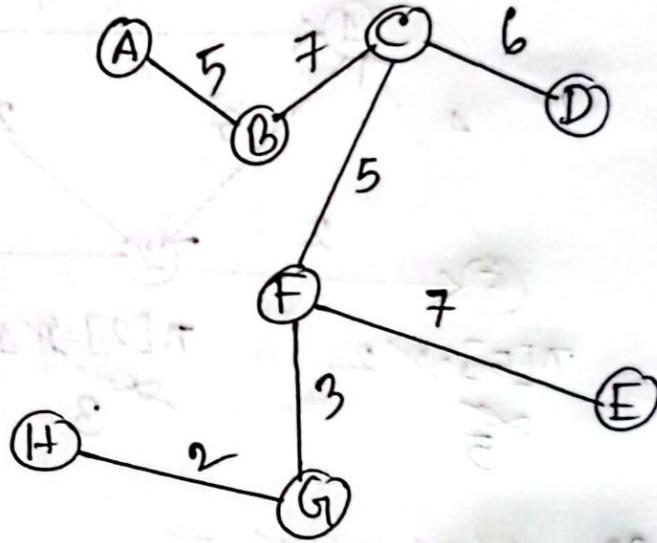


Encoded Message

111011110110111011000000101
001101100

a) Kruskal's MST

<u>Weight</u>	<u>Edge</u>
✓ 2	(H, G)
✓ 3	(F, G)
✓ 5	(A, B)
✓ 5	(F, C)
✓ 6	(C, D)
✓ 7	(B, C)
✓ 7	{E, F}
✗ 8	(D, E)
✗ 9	(A, H)
✗ 10	(B, H)
✗ 12	(F, D)



- b) Minimal cost = 35. Greedy.
 c) If we add (A, H) 9. Total cost will be = 41

$$M = 8$$

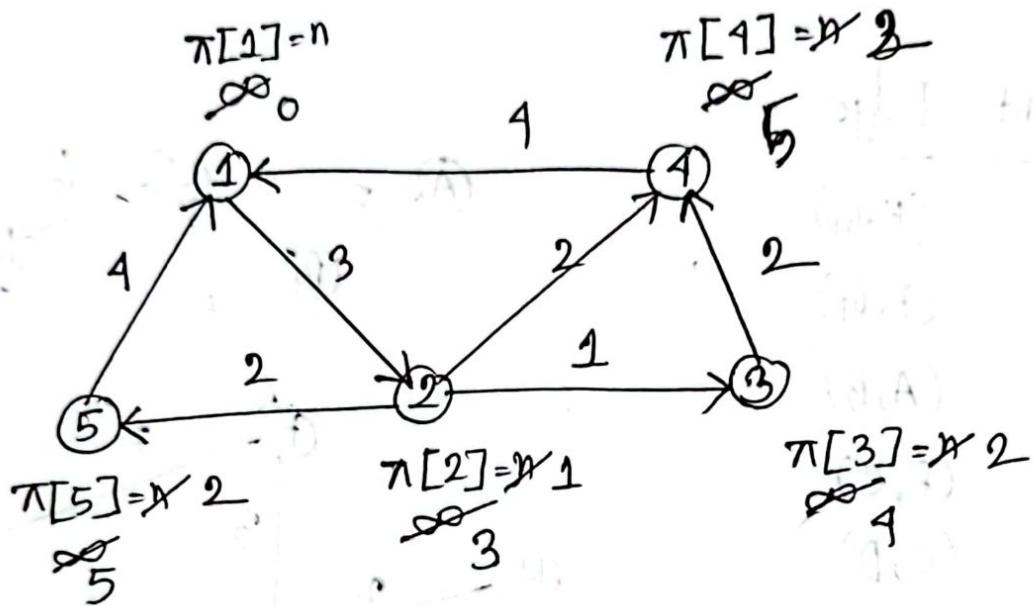
$$|E| = 11$$

$$T = \emptyset \times \times \times \times \times \times \times$$

$$\text{Vertex Set} = \{\{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{F\}, \{G\}, \{H\}\}$$

$$\begin{aligned}
 &= \{\{H, G\}, \{E, G, H\}, \{A, B\}, \\
 &\quad \{C, F, G, H\}, \{C, D, F, G, H\}, \\
 &\quad \{A, B, C, D, F, G, H\}, \\
 &\quad \{A, B, C, D, E, F, G, H\}
 \end{aligned}$$

⑥

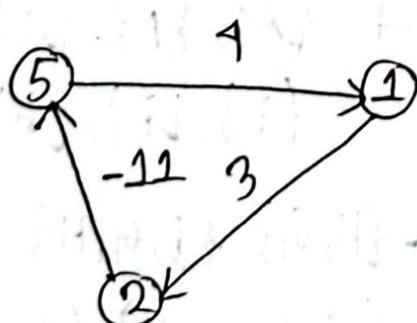


$$PQ = 1 2 3 4 5$$

$$U = 1 2 3 5 4$$

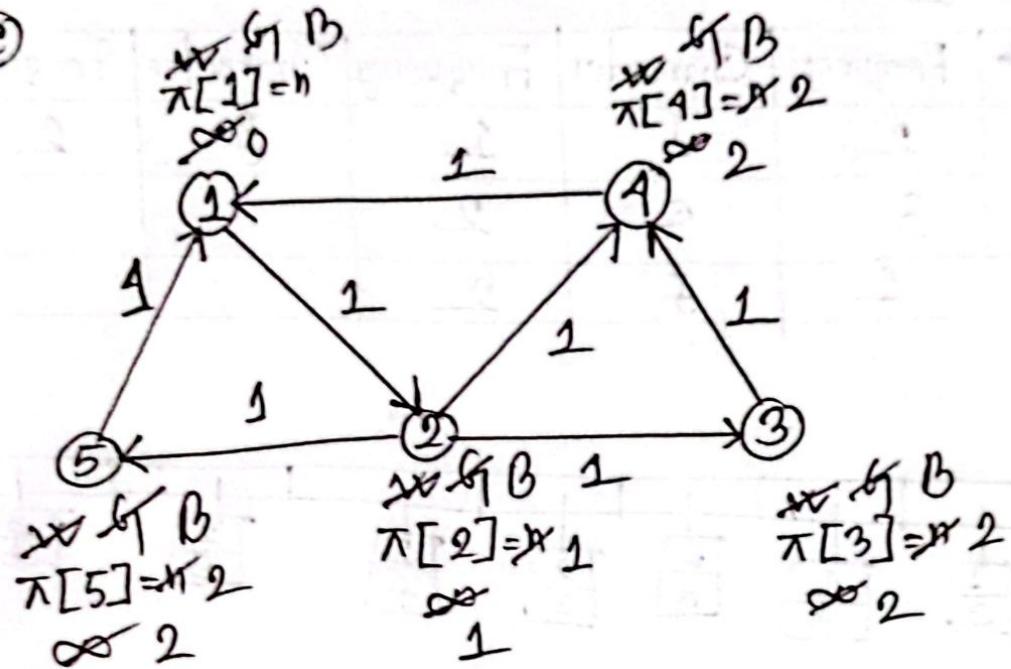
$$V = 2 1 5 4 3 1 4 1$$

⑥



Negative weighted cycle
dijkstra won't work

(c)



$$Q = \emptyset 2 2 A 3 5$$

$$U = A 2 A 3 5$$

$$V = 2 \underline{1} \underline{3} 5 1$$

(3)

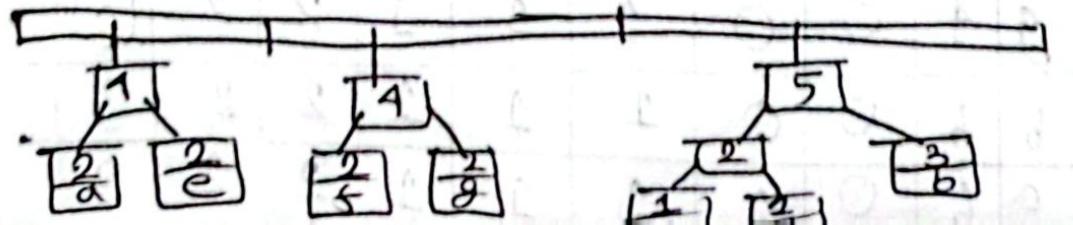
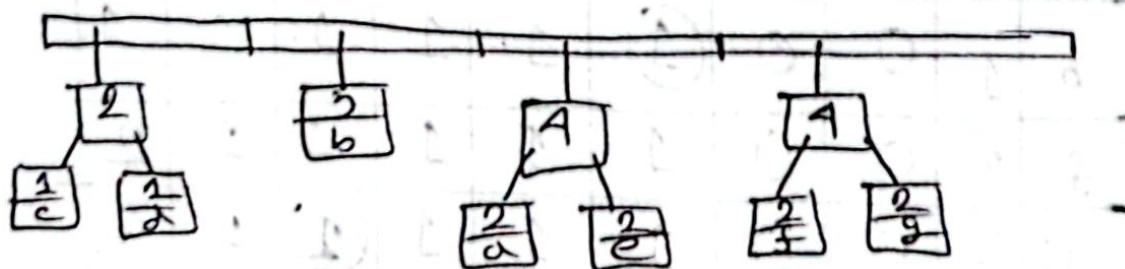
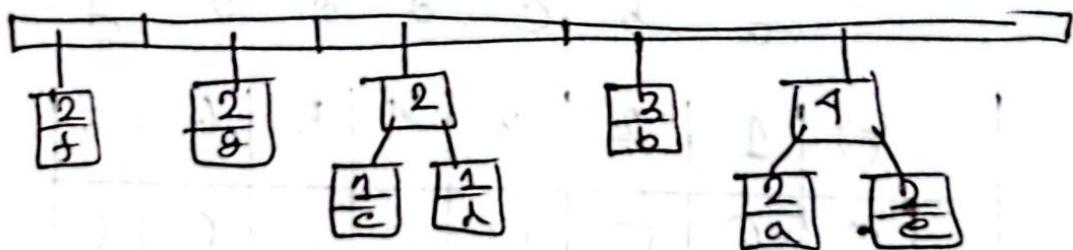
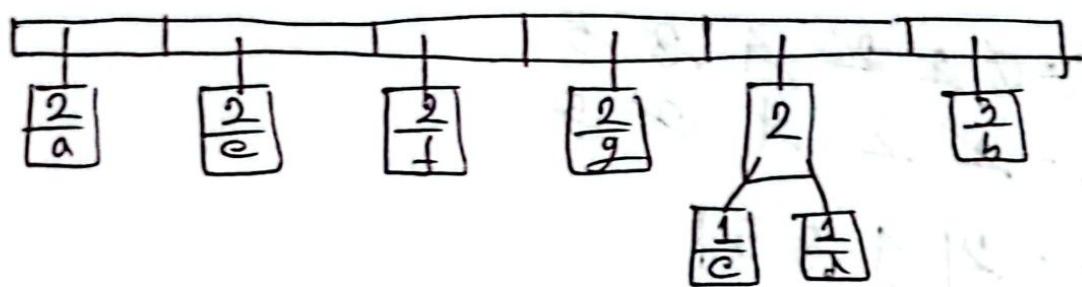
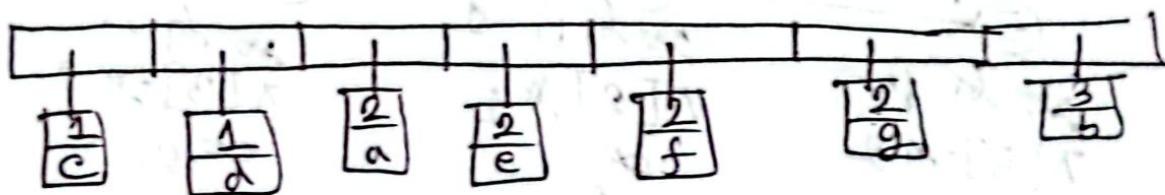
a b c d e f g

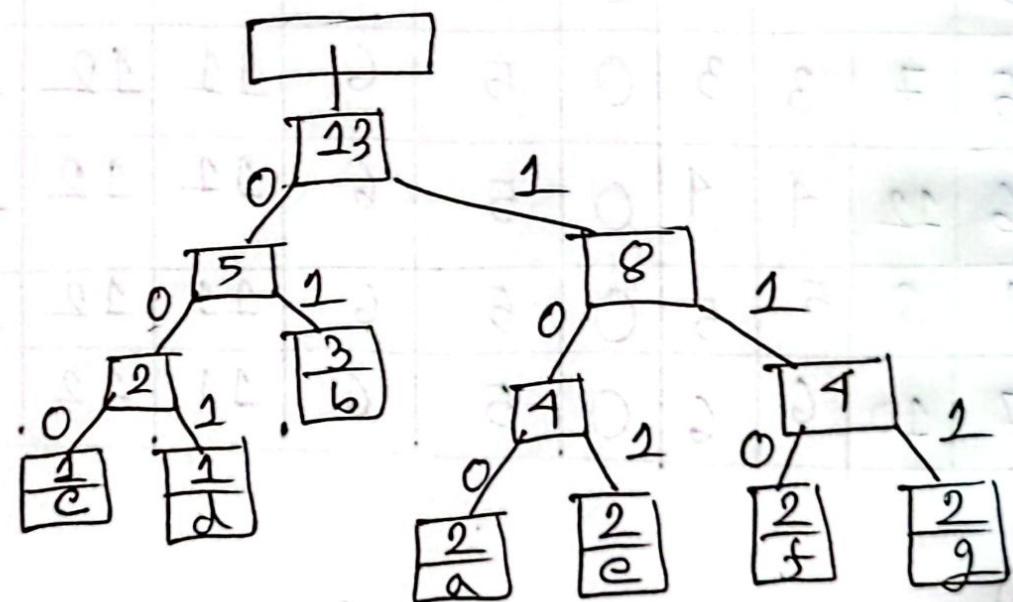
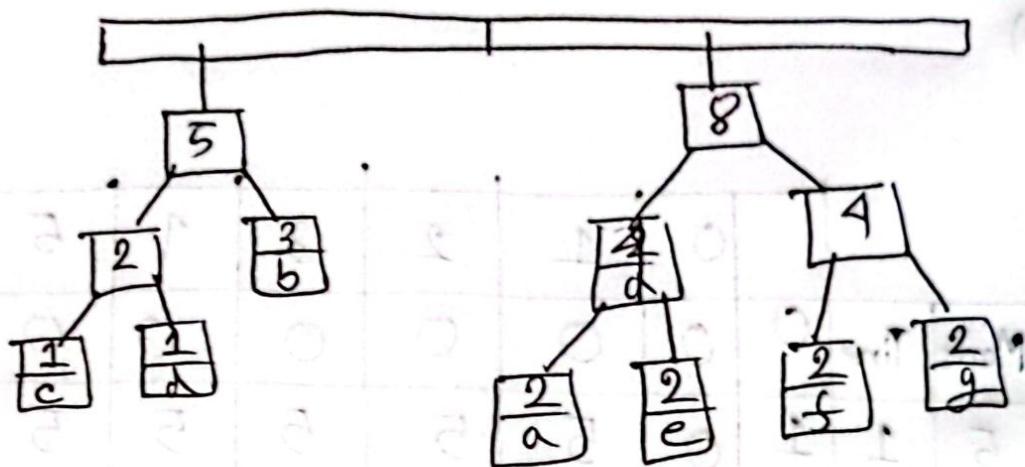
	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
b	1	0	0	①	1	1	1	1
f	2	0	0	1	1	1	2	2
e	3	0	0	1	1	1	②	2
g	1	0	0	1	1	1	2	3
b	5	0	0	1	1	1	2	3
a	6	0	1	1	1	1	2	3

beg

⑥ Frequency Table

Character	Frequency	Character	Frequency	Character	Frequency
a	2	d	1	g	2
b	3	e	2		
c	1	f	2		





Encoded message

10001000001101110111010111101100

Total Bits = 36

a) a)

		0	1	2	3	4	5	6
Question	Marks	Time	0	0	0	0	0	0
9	5	1 1	0	5	5	5	5	5
8	6	2 2	0	5	6	11	11	11
5	7	3 3	0	5	6	11	12	13
6	12	4 1	0	5	6	11	12	17
1	8	5 5	0	5	6	11	12	17
7	10	6 6	0	5	6	11	12	17

b)

Question	Marks	Time	Value Index	Choosen	Profit
9	5	1	5	1	5
8	6	2	3	2	6
6	12	4	3	3	9
5	7	3	2.3	0	
7	10	6	1.67	0	
4	8	5	1.6	0	

Yes scored more

20

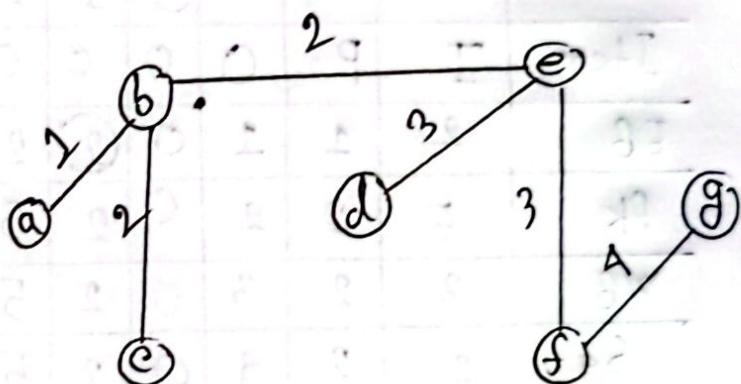
c) Memoization

Spring - 23

(B)

1) a)

<u>Weight</u>	<u>Edge</u>
✓ 1	(a, b)
✓ 2	(b, c)
✓ 2	(b, e)
✗ 3	(a, c)
✓ 3	(d, e)
✓ 3	(e, f)
✓ 4	(f, g)
✗ 5	(e, g)
✗ 6	(b, d)
✗ 6	(d, f)
✗ 6	(c, f)
✗ 7	(c, d)



Cost = 15

$$|V| = 7$$

$$|E| = 12$$

$$T = \{a, b, c, d, e, f, g\}$$

$$\text{Vertex Set} = \{\underline{a}, \{b\}, \{e\}, \{d\}, \{f\}, \{g\}\}$$

$$= \{\underline{a, b}\} \{a, \underline{b, c}\} \{a, b, \underline{c, e}\} \\ \{a, b, c, \underline{d, e}\} \{a, b, c, d, \underline{e, f}\} \\ \{a, b, c, d, e, \underline{f, g}\}$$

2)

a)

				0	1	2	3	4	5	6	7	8	9
Item	I	P	O	0	0	0	0	0	0	0	0	0	0
EB	2	1	1	0	②	2	2	2	2	2	2	2	2
PK	5	2	2	0	2	5	⑦	7	7	7	7	7	7
WB	3	2	3	0	2	5	7	8	⑩	10	10	10	10
Sg	2	2	1	0	2	5	7	8	10	10	12	12	12
NV	4	3	5	0	2	5	7	8	10	11	12	14	19 ✓
GB	1	8	6	0	2	5	7	8	10	11	12	14	19

$$14 - 4 = 10 \quad \text{Night Vision}$$

$$10 - 3 = 7 \quad \text{Waterproof Boots}$$

$$7 - 5 = 2 \quad \text{Painkiller}$$

$$2 - 2 = 0 \quad \text{Energy bar}$$

b) Exponential $O(2^N)$

3) a) m

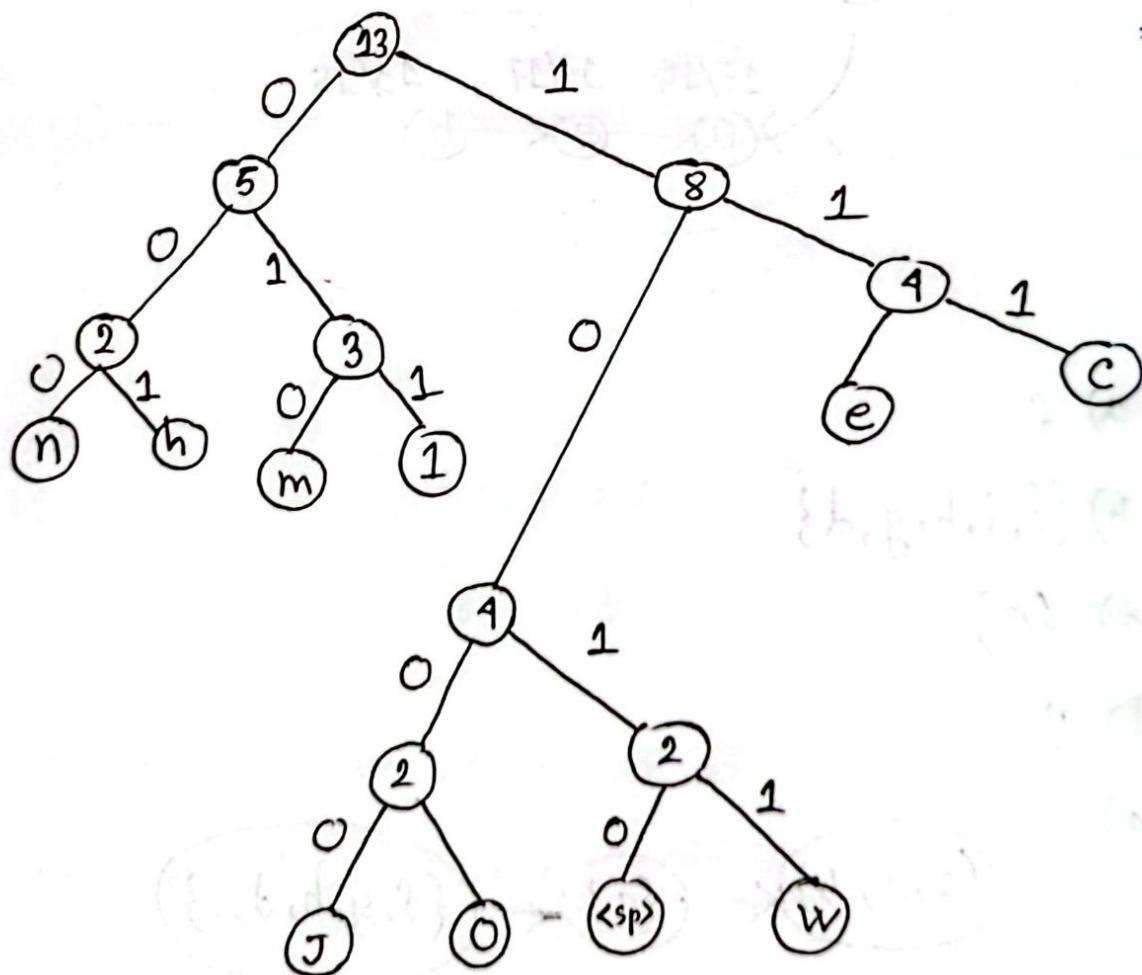
c) e - 2
c - 2

d) Welcome John.

W-1	m-1	<sp>-1
e-2	J-1	
l-1	h-1	
c-1	n-1	
o-2	.-1	

$$\begin{aligned} \text{Tom's required bit} &= \frac{1+1}{2} \\ &= \log_2^{(11)} \\ &= 4 \end{aligned}$$

b)



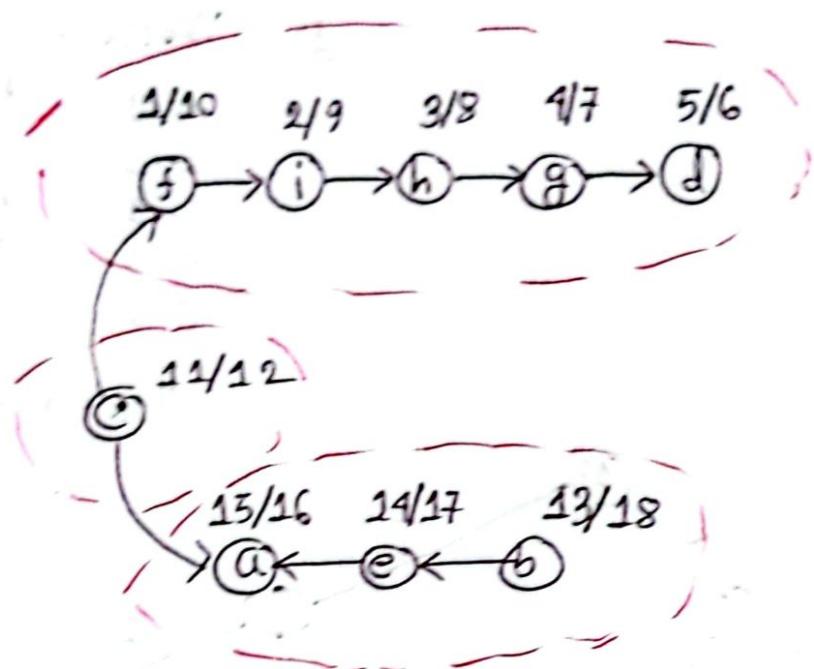
e) 101111001111100101011010101010001001001
000

13 bits

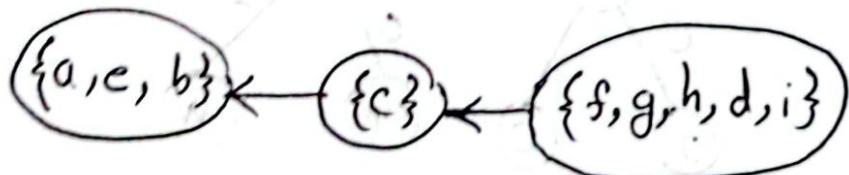
Mine is better

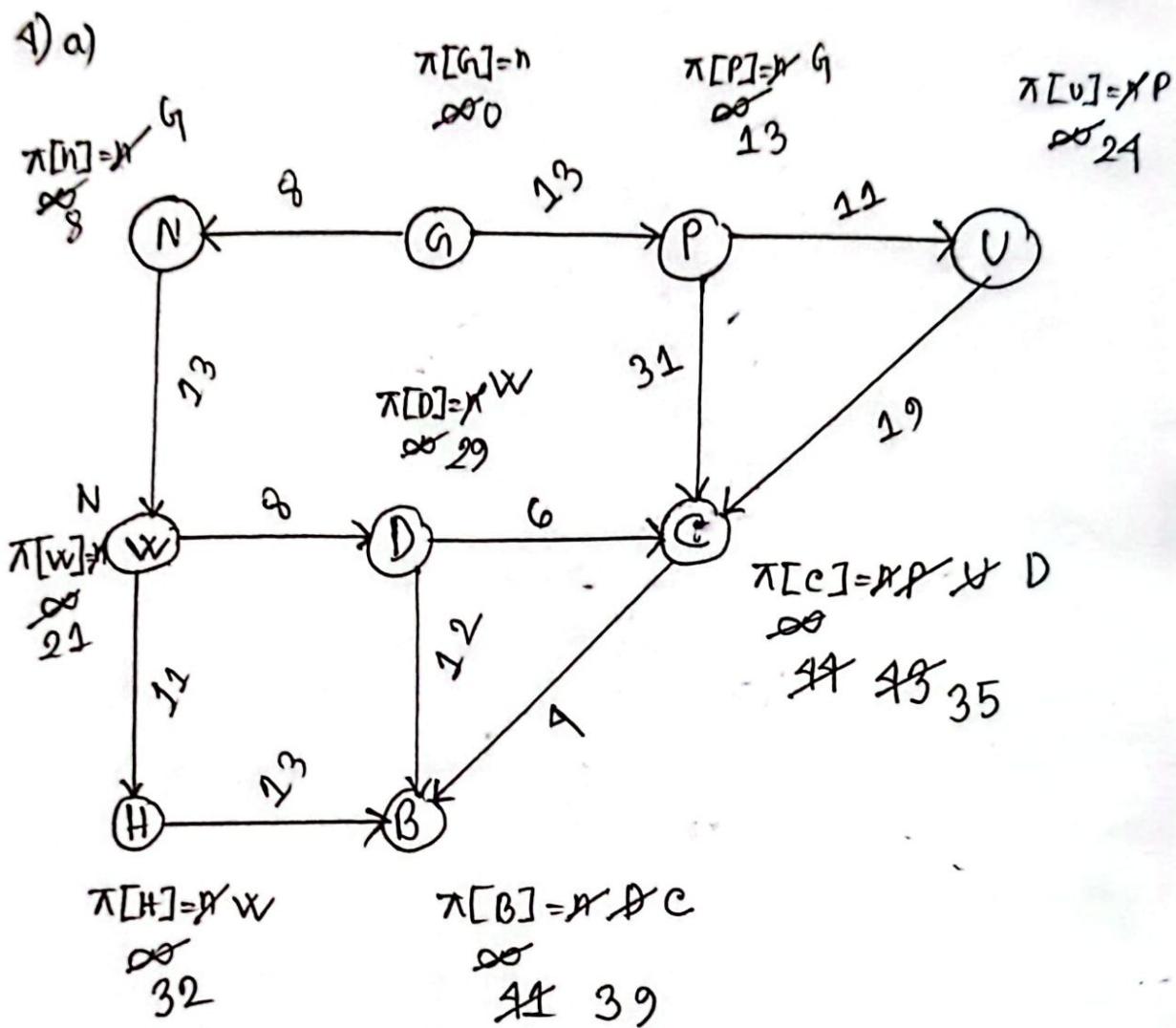
Tom's = $12 * 4 = 48$ bits

4)



- a) f
- b) {f, i, h, g, d}
- c) {c}
- d) 3
- e)





$$PQ = G | N | P | G | W | D | C | H | B$$

$$U = G | N | P | W | D | C | H | B$$

$$V = N, P | W | C | D | H | C | B | C | B$$

b) Run dijkstra from G in graph

Run " " B in reverse graph

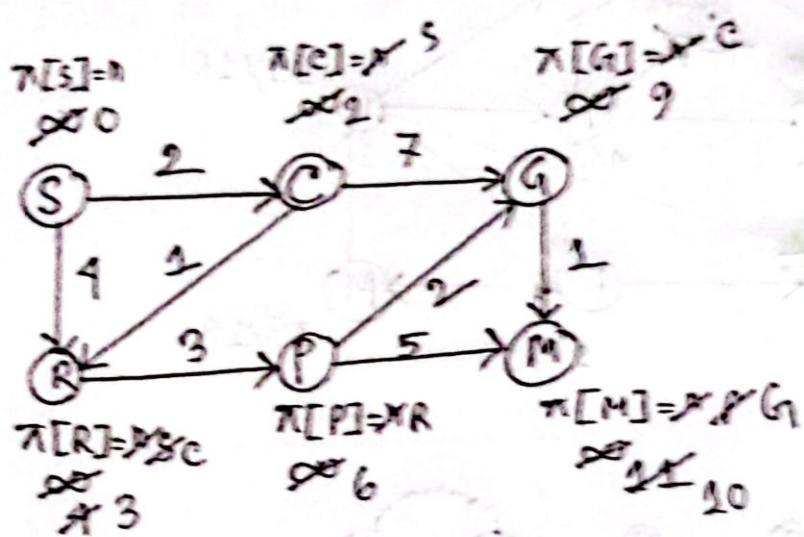
Let, $w(u, v)$ is new road with

$$\min(d_G(u) + w(u, v) + d_B(v))$$

Fall 22

(B)

1) a)

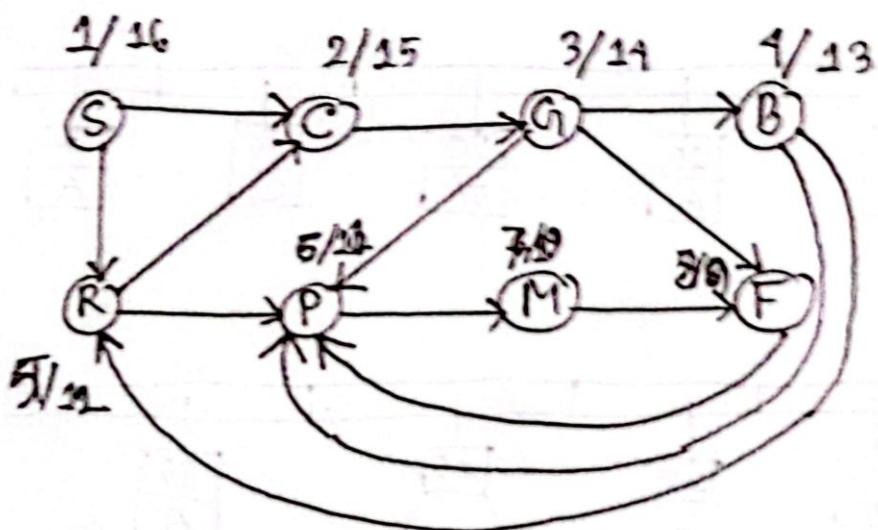


$$PQ = S \ R \ C \ P \ G \ M$$

$$U = S \ R \ P \ G \ M$$

$$V = C R | G R | P | M G |$$

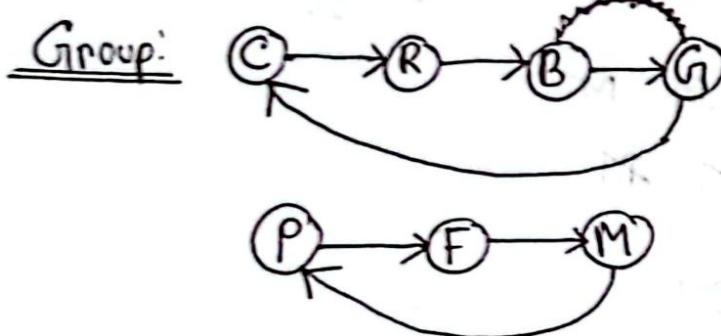
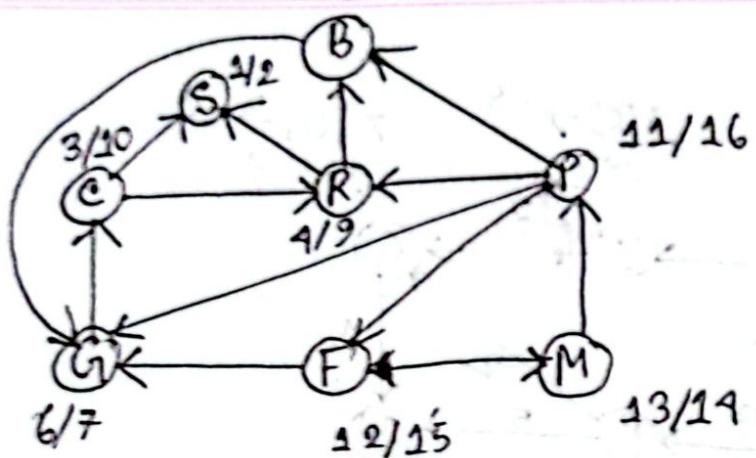
(b)



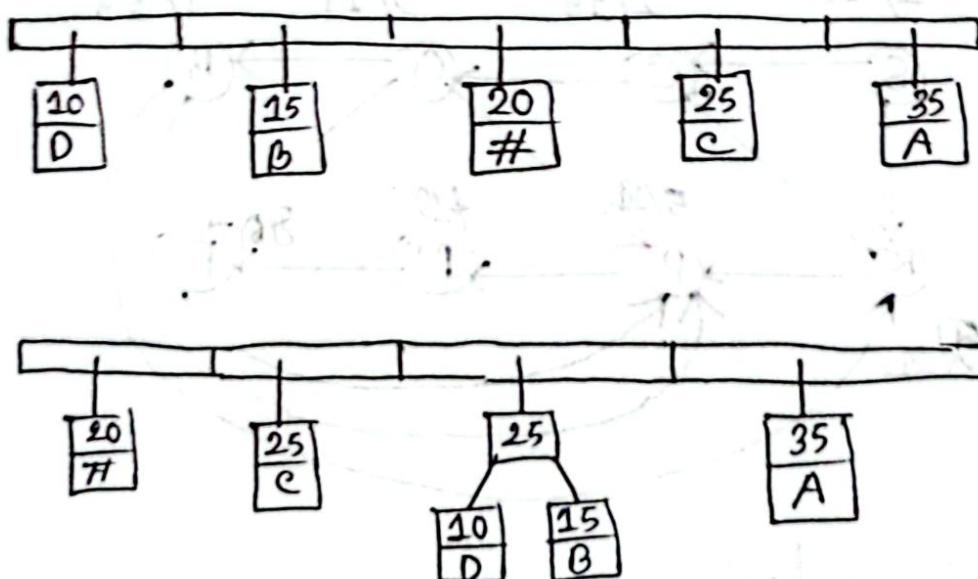
Decreasing order:

$$S \rightarrow C \rightarrow G \rightarrow B \rightarrow R \rightarrow P \rightarrow M \rightarrow F$$

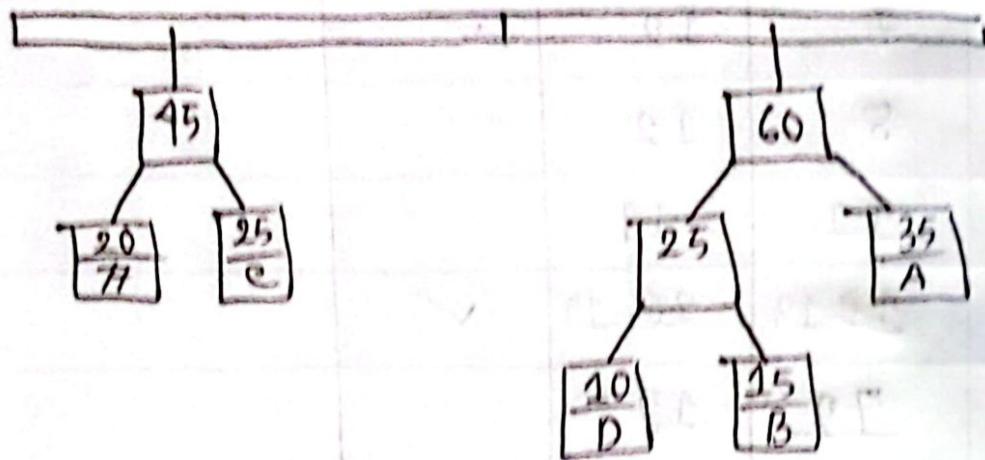
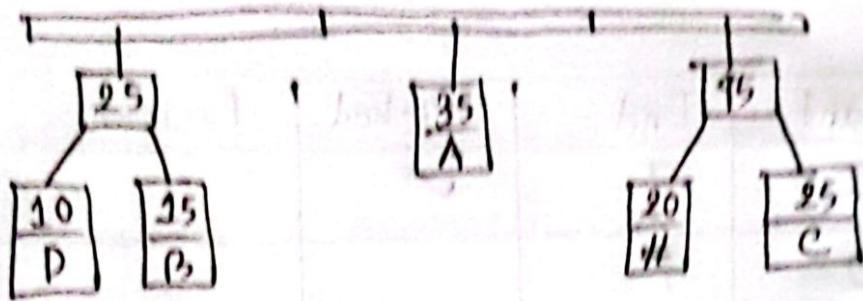
5/8



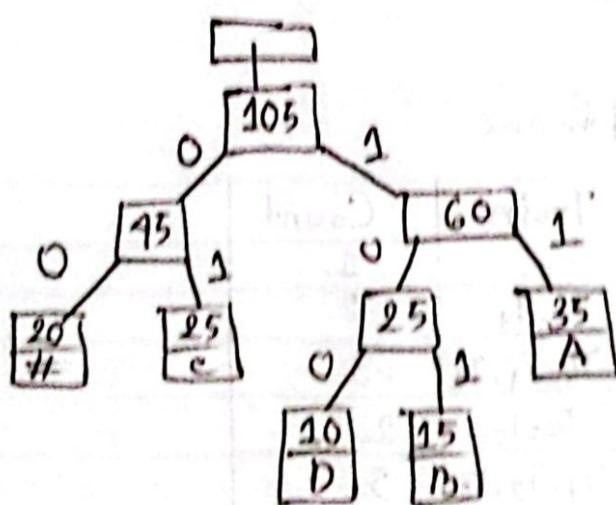
2)(a)



leftmost node = minimum value
rightmost node = maximum value



1)



Symbol	Codeword
A	11
B	101
C	01
D	100
H	00

100010111001010

D C C A H B

b) i)

Trains	Start	End	Picked	Reason
T ₅	1	7	✓	
T ₉	2	7		
T ₂	8	9		
T ₇	7	12	✓	
T ₁	8	12		
T ₃	11	19		
T ₈	12 13	20 19	✓	
T ₆	7 9	19 20		

Maximum 3 trains

ii) Minimum 3 Platforms

Time	Start/End	Train	Count
1	+ T ₅	T ₅	1
2	+ T ₉	T ₅ , T ₉	2
6	+ T ₂	T ₅ , T ₉ , T ₂	3
7	- T ₅ , T ₉ , + T ₇	T ₇ , T ₂	2
8	+ T ₁	T ₇ , T ₂ , T ₁	3
9	- T ₂	T ₇ , T ₁	2
11	+ T ₃	T ₇ , T ₁ , T ₃	3
12	- T ₇ , T ₁ , + T ₆	T ₆ , T ₃	2
13	+ T ₈	T ₆ , T ₃ , T ₈	3
19	- T ₃	T ₆ , T ₈	2
19	- T ₈	T ₆	1
20	- T ₆	0	0

3) i)

				0	1	2	3	4	5	6	7	8
Items	P	W	O	0	0	0	0	0	0	0	0	0
Book	1	1	1	0	4	4	4	4	4	4	4	4
Jewelry	5	3	2	0	4	4	5	9	9	9	9	9
Painting	5	4	3	0	1	4	5	9	9	9	10	14
Sculpture	9	5	4	0	4	4	5	9	9	13	13	14
Mummy	6	12	5	0	9	9	5	9	9	13	13	14

$$14 - 5 = 9$$

Painting

$$9 - 5 = 4$$

Jewelry

$$4 - 4 = 0$$

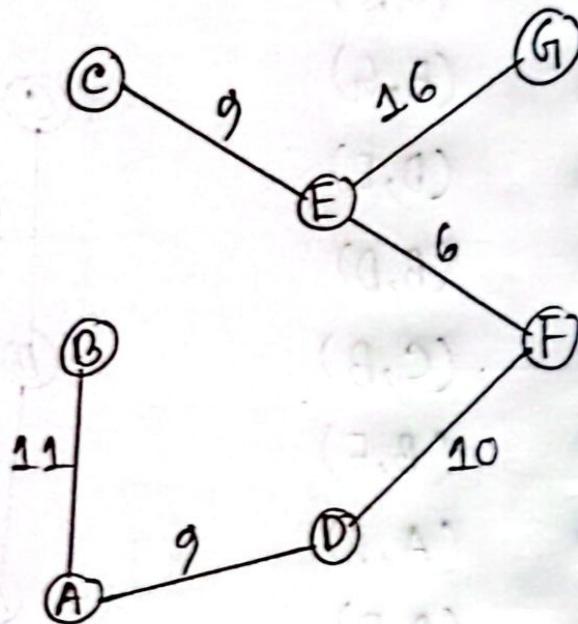
Book

ii) No, fractional knapsack fails.

Items	Profit	Weight	Value Index	Choosen	Profit after Picked
Book	1	1	1	1	1
Sculpture	9	5	1.8	5	9
Jewelry	5	3	1.6		
Painting	5	4	1.25		
Mummy	6	12	0.5		

1)

<u>Weight</u>	<u>Edge</u>
✓ 6	(E, F)
✓ 9	(C, E)
✓ 9	(A, D)
✓ 10	(D, F)
✓ 11	(A, B)
✗ 11	(B, E)
✗ 12	(C, B)
✗ 13	(B, D)
✗ 15	(D, E)
✓ 16	(E, G)
✗ 17	(G, F)



b) MCT = 61

$129 - 61 = 68$

$|V| = 7$

Vertex Set = $\{\underline{A}\}, \{\underline{B}\}, \{\underline{C}\}, \{\underline{D}\}, \{\underline{E}\}, \{\underline{F}\}, \{\underline{G}\}$

$|E| = 11$

T = Ø ∨ X B ∨ A ∨ B ∨ G

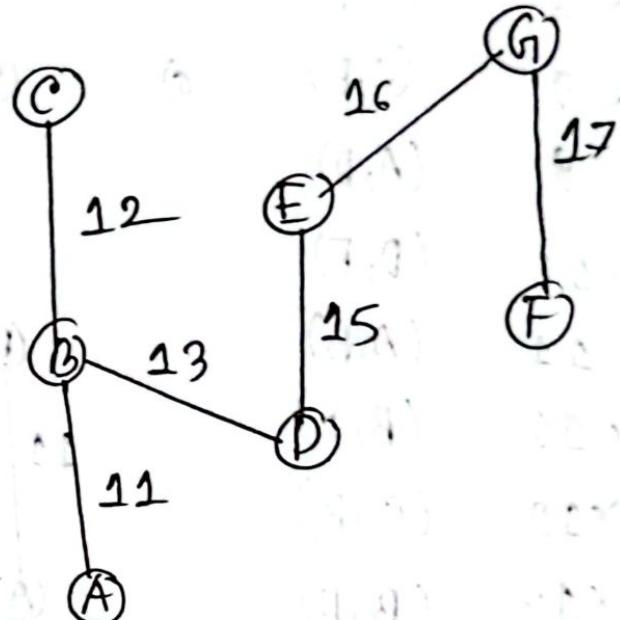
= $\{\underline{E}, \underline{F}\} \quad \{\underline{C}, \underline{E}, \underline{F}\} \quad \{\underline{A}, \underline{D}\}$

$\{\underline{A}, \underline{C}, \underline{D}, \underline{E}, \underline{F}\} \quad \{\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{E}, \underline{F}\}$

$\{\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{E}, \underline{F}, \underline{G}\}$

c)

<u>Weight</u>	<u>Edge</u>
✓ 17	(G, F)
✓ 16	(E, G)
✓ 15	(D, E)
✓ 13	(B, D)
✓ 12	(C, B)
✗ 11	(B, E)
✓ 11	(A, B)
✗ 10	(D, F)
✗ 9	(A, D)
✗ 9	(C, E)
✗ 6	(E, F)



$$|V|=7$$

$$|E|=11$$

$$T=0$$

$$\text{Vertex Set} = \{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{F\}, \{G\}$$

$$= \{\underline{G, F}\} \quad \{\underline{E, G, F}\}$$

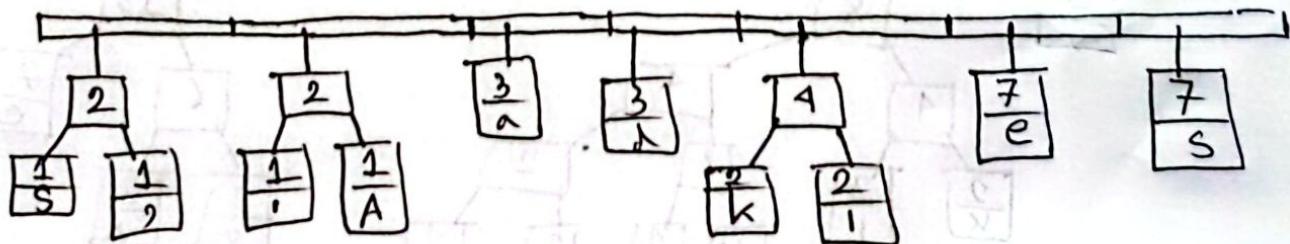
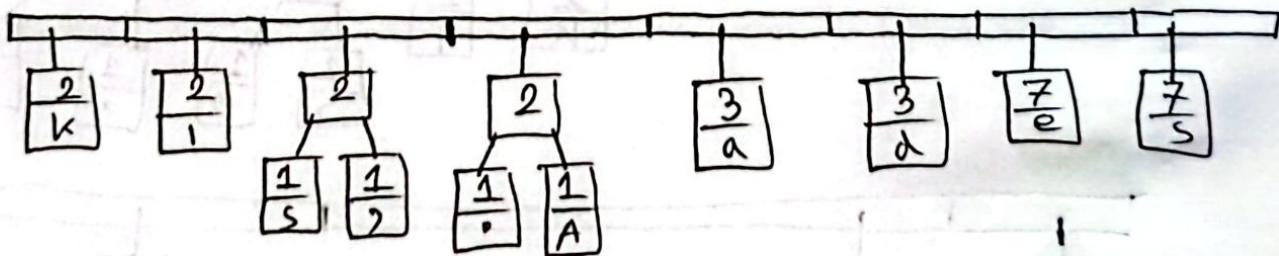
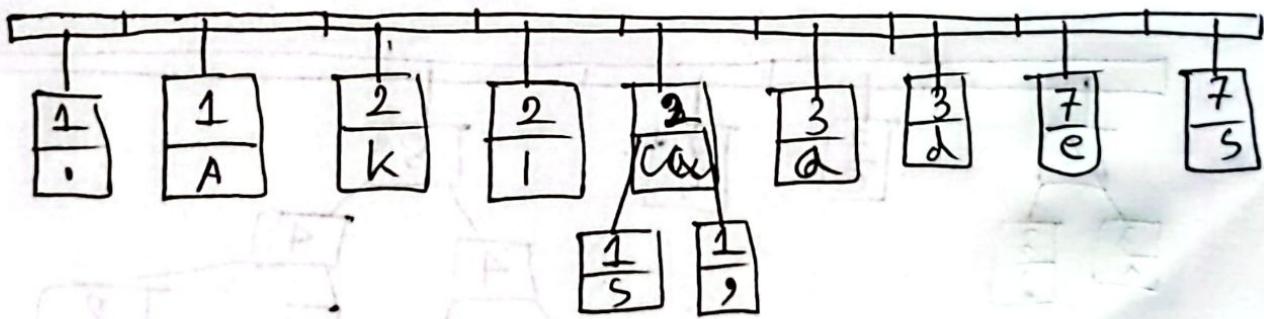
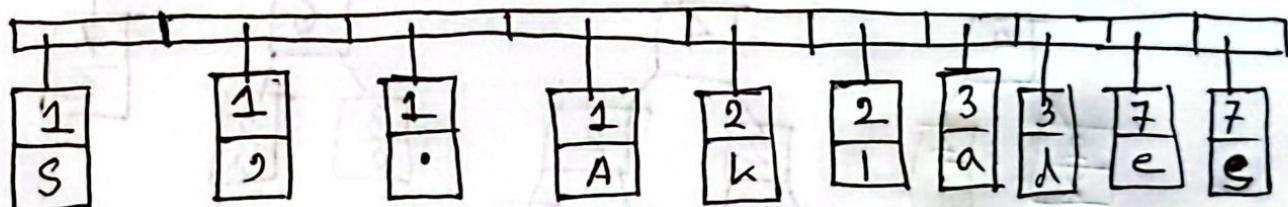
$$\{\underline{E, G, F, D}\} \quad \{\underline{E, G, F, B, D}\}$$

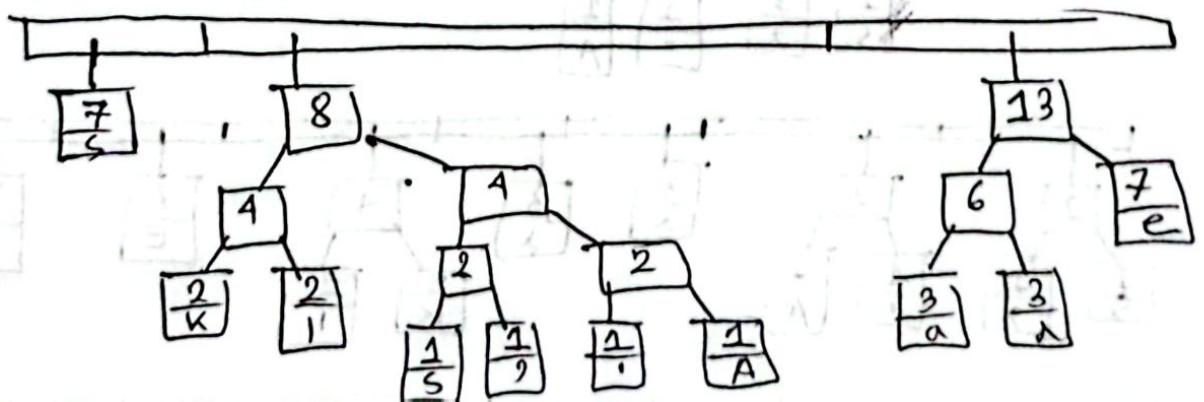
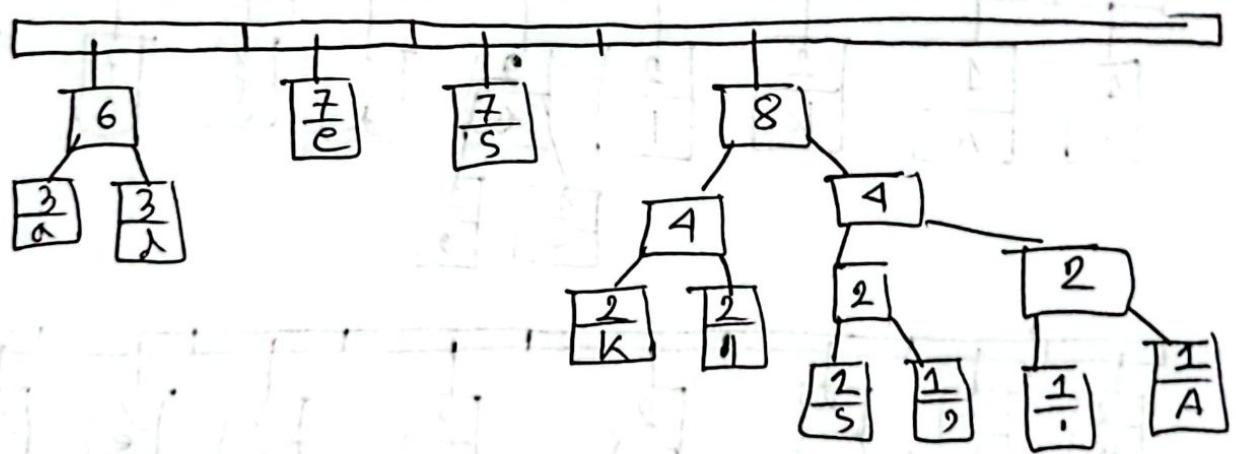
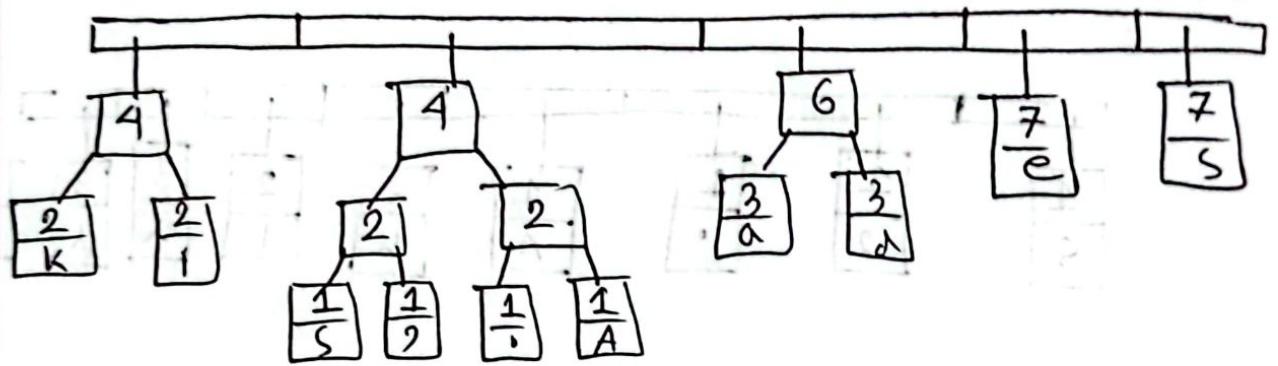
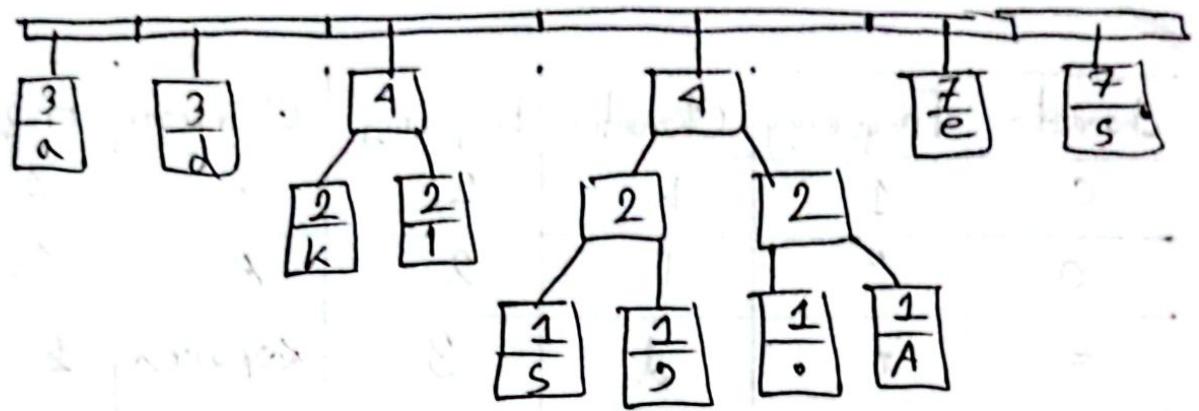
$$\{\underline{C, E, G, F, B, D}\}$$

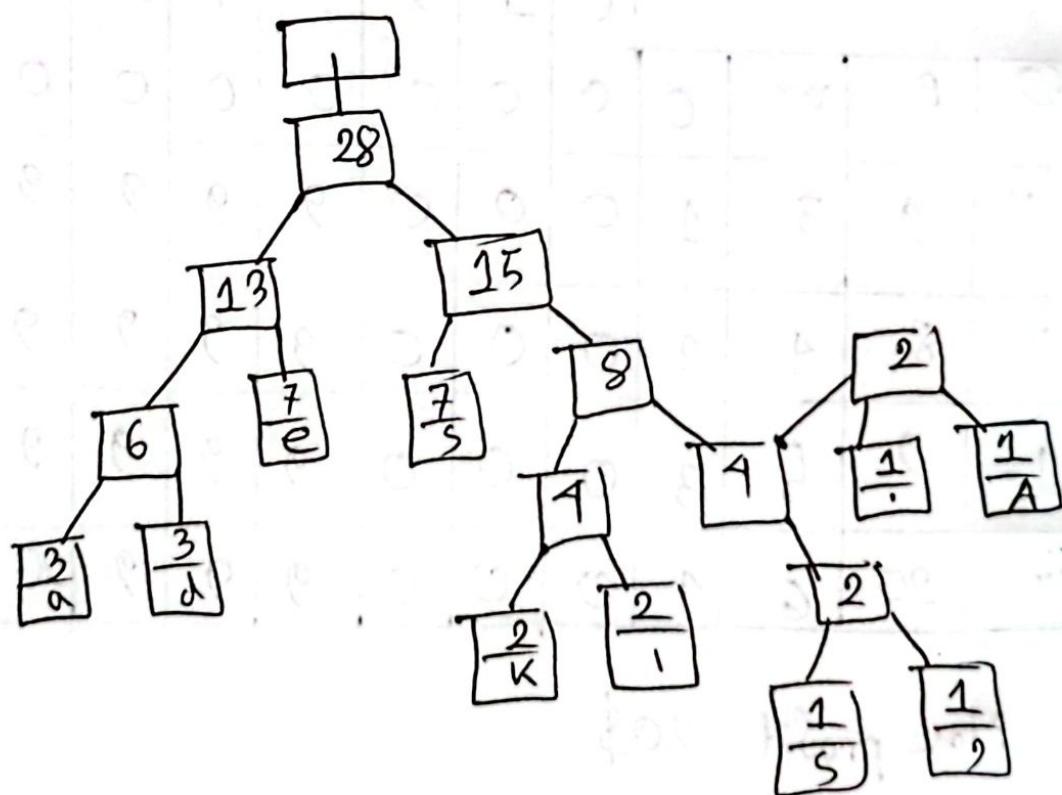
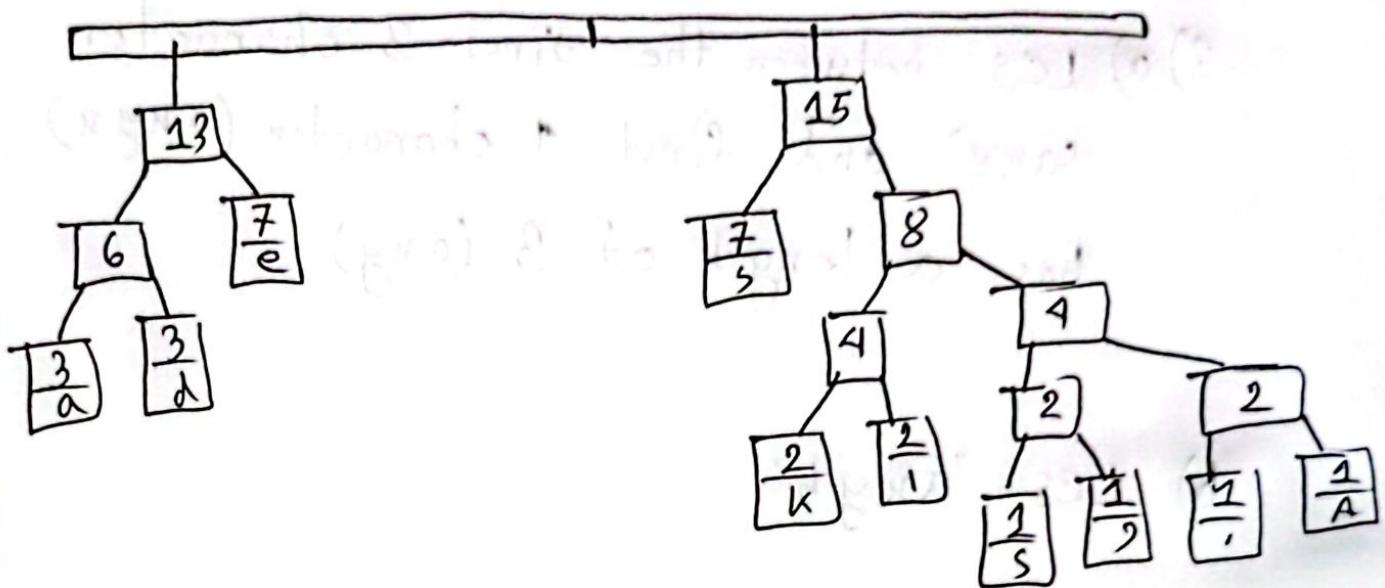
$$\{A, B, C, D, E, F, G\}$$

2)

Character	Frequency	Character	Frequency	Character	Frequency
s	1	k	2	.	1
a	3	i	2	A	1
e	7	d	3	(spaced)	8m
,	7		1		







Couldn't complete miss the space

3) a) LCS between the 'first 3 character
'any' and first 1 character (any)
has a length of 3 (any)

b) LCS = "anyb"

c)

				0	1	2	3	4	5	6	7	8
O	P	w	0	0	0	0	0	0	0	0	0	0
iv	9	3	1	0	0	0	9	9	9	9	9	9
"	8	4	2	0	0	0	9	9	9	9	17	17
i	9	5	3	0	0	0	9	9	9	9	17	18
iii	20	6	4	0	0	0	9	9	9	20	20	20

Max profit = 20\$

Item - III