Context Free Grammar

CFG → We establish a recursive relationship

$0^m 1^m$ Here, number of 0's and 1's same.

0000 1111 .

$\overbrace{0\,\underbrace{0\,0\,\underbrace{0\,1}_{S}\,1\,1}_{S}\,1}^{S}$

So, we can write it as,

S
⇒ 0S1
⇒ 00S11
⇒ 000S111
⇒ 000111

Thus the number of 0 and 1 remains same

$A \to 0A$
means 0^n

At a moment that S will become ∈ ; $S \to \epsilon$

CFG is a set of production rules,
→ (i) $S \to 0S1$
(ii) $S \to \epsilon$

Ex # $0^n 1^{2n}$

Some strings can be 00 1111
000 111111

So, $S \to 0S11$
$S \to \epsilon$

or, $S \to 0S11 \mid \epsilon$

$0^{3n} 1^{5n}$

000 11111
000000 1111111111

$S \to 000S11111$
$S \to \epsilon$

$S \to 000S11111 \mid \epsilon$

Non-terminal/Var Terminal Productive Rule

CFG → 4 Tuple (V, T, P, S) → Starting Var

~~# S → 0B1~~

Starting

A → 0A1 | B1

B → 00

Derivation

A
⇒ B1
⇒ 001

A
⇒ 0A1
⇒ 00A11
⇒ 00B111
⇒ 0000111

S → 0S1

S → A

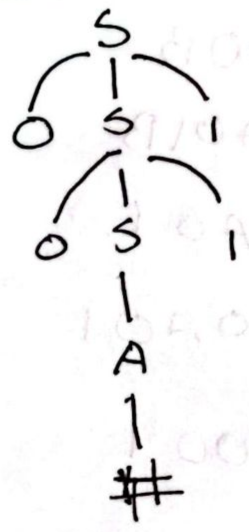
A → #

Here, V = {A, A}

T = {0, 1, #}

S → Starting

Persed Tree



Ans: 00#11

$0^n \# 1^n$

S
⇒ 0S1
⇒ 00S11
⇒ 00A11 ; Recursion
⇒ 00#11

Ans: $0^n \# 1^n$

$S \to A0B$

$A \to 0A \mid \epsilon$

$B \to 1B \mid \epsilon$

Using Leftmost

S

$\Rightarrow A0B$

$\Rightarrow 0A0B$

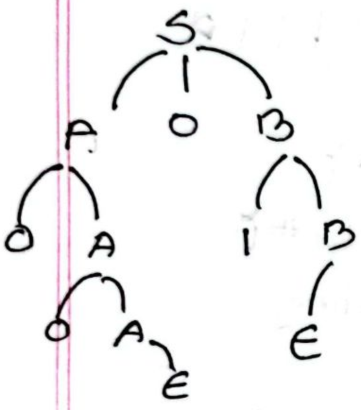
$\Rightarrow 00A0B$

$\Rightarrow 000B$

$\Rightarrow 0001B$

$\Rightarrow 0001$

Persed Tree



$0 0 \epsilon 0 1 \epsilon$

$\Rightarrow 0001$

Without Using leftmost

S

$\Rightarrow A0B$

$\Rightarrow 0A0B$

$\Rightarrow 0A01B$

$\Rightarrow 0A01$

$\Rightarrow 00A01$

$\Rightarrow 0001$

Property of Grammar

$\downarrow$

$\boxed{\text{Ambiguity}}$ Ambigious Grammar $\rightarrow$ More than 1 parsed tree / Left most / Right most

$A \rightarrow A1 \mid 0A1 \mid 01$; "001111" derive this string

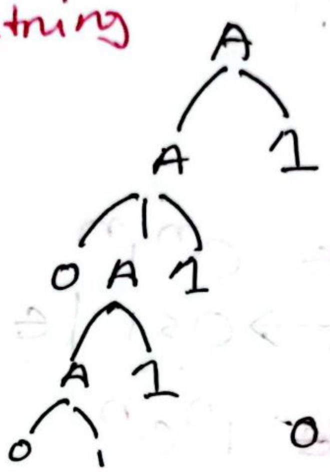
A

$\Rightarrow$ A1 [A1]

$\Rightarrow$ 0A11 [0A1]

$\Rightarrow$ 0 A111 [A1]

$\Rightarrow$ 001111 [01]



0011.11

Another way

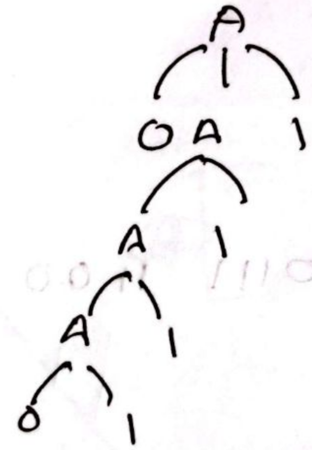
A

$\Rightarrow$ 0 A 1 [0A1]

$\Rightarrow$ 0A11 [A1]

$\Rightarrow$ 0A111 [A1]

$\Rightarrow$ 001111 [01]



001111

Regular Operation

OR (+) | Concatanation (.) | Kleene Closure

OR

$w_1 \in 0^n 1^n$

$S_1 \rightarrow 0S1 \mid \epsilon$

$w_2 \in 1^n 0^n$

$S_2 \rightarrow 1S0 \mid \epsilon$

$S_0,\ S_1 \rightarrow S_a \mid S_b$

$S_a \rightarrow 0 S_a 1 \mid \epsilon$

$S_b \rightarrow 1 S_b 0 \mid \epsilon$

Conc.

000111 1100

$S \rightarrow S_a . S_b$

$\downarrow$ $\downarrow$

000111 1100

*

$0^* = \{\epsilon, 0, 00, 000, 0000, \dots\}$

We can write 0^* as,

$\underbrace{\overbrace{0\,0\,0\,0\,0\,0}^{A}}_{A}$

$S_0,\ A \rightarrow 0A \mid \epsilon$

$(01001)^* = \{\epsilon, 01001, 0100101001, \dots\}$

$\underbrace{01000 \overbrace{1010010100 \underbrace{10100101001 01001}_{B}}^{B}}_{B}$

$B \rightarrow 01001B \mid \epsilon$

অথবা এভাবেও লিখতে পারি,

$B \rightarrow CB \mid \epsilon$

$C \rightarrow 01001$

$\boxed{\text{RegEx} \rightarrow \text{CFG}}$

$\underbrace{(0 \mid 10)^*}_{A} \mid \underbrace{10^*}_{B}$

$S \rightarrow A \mid B$

$B \rightarrow 1C$

$C \rightarrow 0C \mid \epsilon$

$A \rightarrow$

উপরের অংশের জন্য

$A = (0 \mid 10)^*$

$B = 10^*$

$C = 0^*$

$D = 10$

$1^* = 1S$
$0^* = 0S$

RegEx → CFG

$\underbrace{(0|10)^*}_{A} \mid \underbrace{10^*}_{B}$

$S \rightarrow A \mid B$

$B \rightarrow 1C$

$C \rightarrow 0C \mid \epsilon$

$A \rightarrow DA \mid \epsilon$

$D \rightarrow (0 \mid 10)$

বরাদ

$A = (0|10)^*$

$B = 10^*$

$C = 0^*$ { $B = 1C$

$D = 0|10$

$A = D^*$

Some Examples

$0^n 1 2^n$; 0000122222, 0001222, 1

Ans: $S \rightarrow 0S2 \mid 1$

$0^n 1^n 2$; 000111,12 ; 000011112 ; 00112

000111

Ans: $A \rightarrow S2$
$S \rightarrow 0S1 \mid \epsilon$

$0^n 1^{n+1}$ → $0^n 1 1^n$, $0^n 1^n 1$ } 2টাই connect

$0^n 1 \, 0 \, 1^n$

$S \rightarrow 0S1 \mid \epsilon$

$0^m 1^n 2^k$ where $k = m+n$

Here $0^m 1^n 2^k$

$$0^m 1^n 2^{m+n}$$

$$\underbrace{0^m \underbrace{1^n 2^n}_{X} 2^m}_{S}$$

So, $S \rightarrow 0S2 \mid X$

$X \rightarrow 1X2 \mid \epsilon$

$S \Rightarrow 0S2$

$\Rightarrow 00S22$

$\Rightarrow 000S222$

$\Rightarrow 000X222$

$\Rightarrow 0001X2222$

$\Rightarrow 0001\,2222$

0S1

$0^m 1^n 2^k$ where, $n = m+k$

$$0^m 1^{m+k} 2^k$$

$$\underbrace{0^m 1^m}_{A} \underbrace{1^k 2^k}_{B}$$

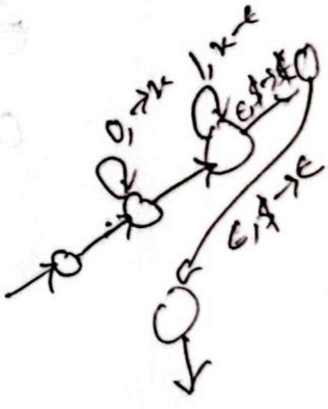
A → 0S2

S → 1

$S \longrightarrow AB$

$A \longrightarrow 0A1 \mid \epsilon$

$B \longrightarrow 1B2 \mid \epsilon$



$0011 \rightarrow 00\epsilon 11$

$000111 \rightarrow 00A11$

$0^m 1^m \quad m \geq 2$

$S \rightarrow 00A11$

$A \rightarrow 0A1 \mid \epsilon$

$0^m 1^n \quad m > n$

$S \rightarrow 0A1 \mid A$ $\quad S \rightarrow 0S1 \mid A$

$A \rightarrow 0A \mid 0$

$0^m 1^n 2^k 3^j \; ; \; k = m+n+j$

~~$0^m 1^n 2^m$~~

$0^m \underbrace{1^n 2^n}_{A} 2^m \underbrace{2^j 3^j}_{B}$

$A \rightarrow 1A2$

$B \rightarrow 2B3$

$S \rightarrow 0S2B \mid A$

$A \rightarrow 1A2 \mid \epsilon$

$B \rightarrow 2B3 \mid \epsilon$

$S[0^n 1^n]$; $S \to 0S1$
$S[0^*]$; $S \to 0S|\epsilon$

$S[(AB)^*]$ $S \to AAB$
$A \to 011$

{w|w contains at least three 1's}

We can ans in 2way

$\Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$

$S \to A1A1A1A$

$A \to 0A|1A|\epsilon$

$0^* 1 0^* 1 0^* 1 \Sigma^*$

$S \to B1B1B1A$

$A \to 0A|1A|\epsilon$

$B \to 0B|\epsilon$

{w|w ~~contains~~ starts and ends with same symbol}

$0\Sigma^*0 | 1\Sigma^*1$

$S \to 0A0|1A1$

$A \to 0A|1A|\epsilon$

{w|w the length of w is odd}

$\underset{A}{\Sigma}\underset{B}{(\Sigma\Sigma)^*}$

$S \to AB$

$A \to 0|1$

$B \to AAB|\epsilon$

$\{w \mid w$ The length of w is odd and it's middle symbol is $0\}$

10001 $\underbrace{1001}\,0\,\underbrace{0011}$

S

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$

De Derivation, let, ~~1~~~~0~~~~1~~~~1~~ 0 ~~0~~~~1~~~~0~~~~1~~

S — Both side থেকে কাটব আর S এর পাশে add করব।

1S1

10S01

101S101

1011S0101

1011 0 0101

$\{w \mid w = w^R$ As, w is a pallindrome$\}$

1011 0 01101

$S \rightarrow 0S0 \mid 1S1 \mid 1 \mid 0 \mid \epsilon$

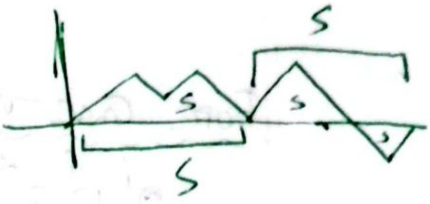
1 0

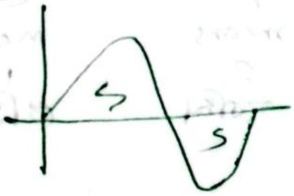
Num of 0 = Num of 1   001011

~~S → 0S1S | 1S0S | ε~~

$S \rightarrow S0S1 \mid S1S0 \mid \epsilon$

$S \rightarrow 0S1S \mid 1S0S \mid \epsilon$





$S \rightarrow 0S1 \mid 1S0$

$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \epsilon$

We may use

$S \rightarrow S0S1 \mid S1S0 \mid \epsilon$

$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \epsilon$

$S \rightarrow 0AS \mid 1BS \mid \epsilon$

$A \rightarrow 1 \mid 0AA$

$B \rightarrow 0 \mid 1BB$

$0^i 1^j 2^k$; where $\overbrace{i = j}^{A}$ or $\overbrace{j \neq k}^{B}$

For Case-A

A $0^i 1^j 2^k$ অথবা, $\underbrace{0^i 1^j}_{P} \underbrace{2^k}_{Q}$

$A \rightarrow PQ$

$Q \rightarrow 2Q \mid \epsilon$

$P \rightarrow 0P1 \mid \epsilon$

For case-B

$\underbrace{0^i}_{W} \underbrace{1^j 2^k}_{X}$

$0^i 1^j 2^k$ where, $\underbrace{i=j}_{A}$ or $\underbrace{j \neq k}_{B}$

For Case-A

$\underbrace{0^i 1^j}_{P} \underbrace{2^k}_{Q}$

$A \rightarrow PQ$

$Q \rightarrow 2Q \mid \epsilon$

$P \rightarrow 0P1 \mid \epsilon$

For Case-B

$\underbrace{0^i}_{W} \underbrace{1^j 2^k}_{X}$

$B \rightarrow WX$

$W \rightarrow 0W \mid \epsilon$

$X \rightarrow Y \mid Z$

$Y \rightarrow 1Y2 \mid Y'$

$Y' \rightarrow 2Y' \mid \epsilon$

Here, $j \neq k$ means $\underbrace{j < k}_{Y}$ or $\underbrace{j > k}_{Z}$

Y means 2 বেশি

Z means 1 বেশি

$Z \rightarrow 1Z2 \mid Z'$

$Z' \rightarrow 1Z' \mid \epsilon$

So, Main Ans

$S \rightarrow A \mid B$

$A \rightarrow PQ$

$Q \rightarrow 2Q \mid \epsilon$

$P \rightarrow 0P1 \mid \epsilon$

$B \rightarrow WX$

$W \rightarrow 0W \mid \epsilon$

$X \rightarrow Y \mid Z$

$Y \rightarrow 1Y2 \mid Y'$

$Y' \rightarrow 2Y' \mid \epsilon$

$Z \rightarrow 1Z2 \mid Z'$

$Z' \rightarrow 1Z' \mid \epsilon$

(#) $L = w_1 \# w_2$ where,

num. of 0's in w_1 = no. of 1's in w_2

$S \rightarrow 1S \mid S0 \mid 0S1 \mid \#$

\# (a) Give a context-free grammar for the following language: For, $\{w \in \{0,1\}^* \}$

~~(a) (A)~~ =

(a) w contains odd number of 1's

(b) w starts and ends with same characters.

(c) $w = u 0^{2i} v 1^{3i}$ where $u \in B, v \in A$ and $i \geq 0\}$

(a) RE $\rightarrow$ $\underbrace{(\overbrace{0^* 1 0^* 1}^{D})^*}_{B} 0^* 1 \underbrace{0^*}_{C}$

$A \rightarrow B \mid C1C$

$C \rightarrow 0C \mid \epsilon$

$B \rightarrow DB \mid \epsilon$

$D \rightarrow C1C1$

(b) $0\Sigma^{*}0 \mid 1\Sigma^{*}1 \mid 0 \mid 1$

$W \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1$

$P \rightarrow QP \mid \epsilon$

$Q \rightarrow 0 \mid 1$

(c) $W = \overbrace{u}^{w} \overbrace{0^{2i}\ v\ 1^{3i}}^{x}$

$S \rightarrow Wx$

$x \rightarrow 00x111 \mid A$

~~A→BC W→0P0~~ উপরের ~~B and~~ A, B যেমন,

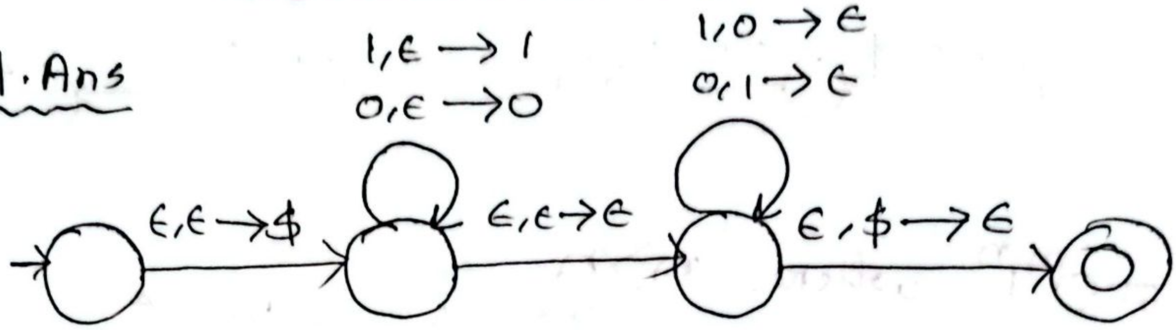
(d) $\overbrace{u}^{W} \underbrace{0^{2i}\ 1^{3i}}_{x} \overbrace{V}^{A}$

$S \rightarrow W\,x\,A$

$x \rightarrow 00x111 \mid \epsilon$

Spring - 2022 } Final

1. Ans



2. Ans

(a) $A = \{ w \in \{0,1\}^* : \text{at least two 0's} \}$

$1^* 0 1^* 0 \Sigma^*$

$S \longrightarrow P0P0Y$
$P \longrightarrow 1P \mid \epsilon$
$Y \longrightarrow 0Y \mid 1Y \mid \epsilon$

(b) $L = \{ w \in \{0,1\}^* : w = 0^{3i} v 1^{2i} \text{ where } v \in A \text{ and } i \geq 0 \}$

$S' \longrightarrow 000S'11$

$S' \longrightarrow S$

$S \longrightarrow P0P0Y$ $\quad$ $Y \rightarrow 0Y \mid 1Y \mid \epsilon$

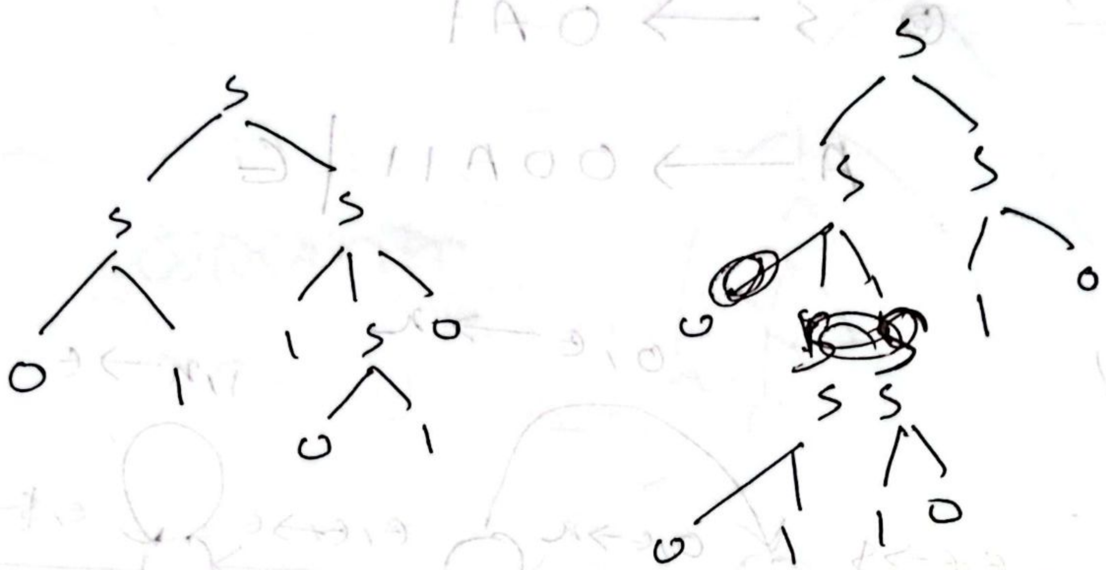
$P \rightarrow 1P \mid \epsilon$

3. Ans

$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10$

(a)

011010



(b)

111000 ; $S \Rightarrow 1S0$

$\Rightarrow 11S00$

$\Rightarrow 111000$

Summer 22

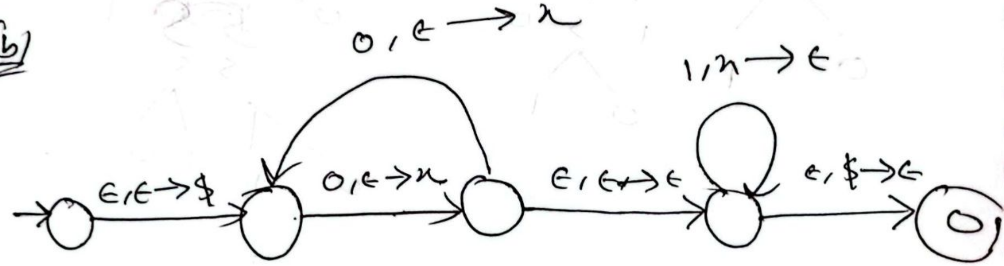
1. Ans

$L = \{ w \in \Sigma^* ;\ w = 0^n 1^n ;\ n \text{ is odd} \}$

(a) $S \longrightarrow 0A1$

$A \longrightarrow 00A11 \mid \epsilon$

(b)



3. Ans

001111

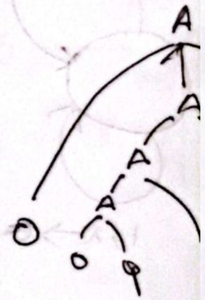
(a) $A \Rightarrow A1$

$\Rightarrow A1$

$\Rightarrow 0A1$

$\Rightarrow 00$

(c)



(d) 01111

3. Ans $A \to A1 \mid 0A1 \mid 01$

001111

(a) $A \Rightarrow A1$

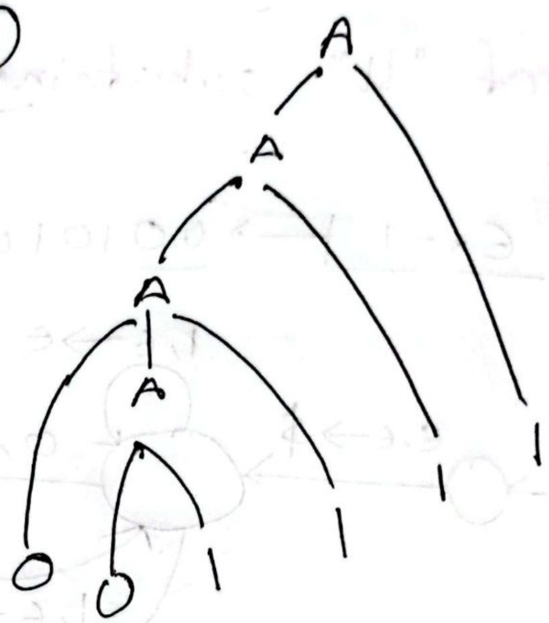
$\Rightarrow A11$

$\Rightarrow 0A111$

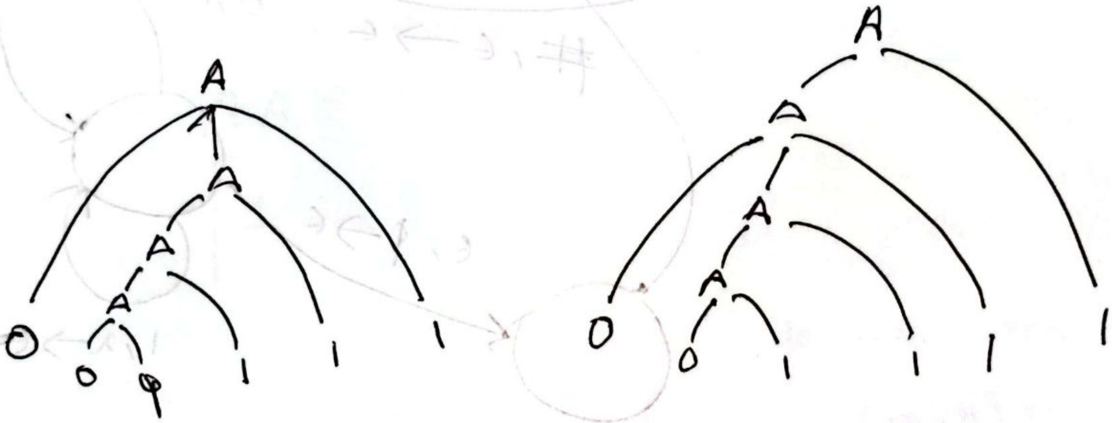
~~$\Rightarrow 00A1111$~~

$\Rightarrow 001111$

(b)



(c)



(d) 011111 ; $A \Rightarrow A1$

$\Rightarrow A11$

$\Rightarrow A111$

$\Rightarrow A1111$

$\Rightarrow 011111$

$L = \{0^i 1^j 2^k 3^m \ ; \ j = k+1 \ ; \ i = m-2\}$

$0^i \ 1^j \ 2^k \ 3^m$

$0^i \ \underline{1^{k+1} \ 2^k} \ 3^{i+2}$

$S \longrightarrow 0S3$

$S \longrightarrow P33$

$P \longrightarrow 1P2$

$P \longrightarrow 1$

$L = \{0^i 1^j 2^k 3^m 4^n \ ; \ j = i, \ m = n \ ; \ k > 0\}$

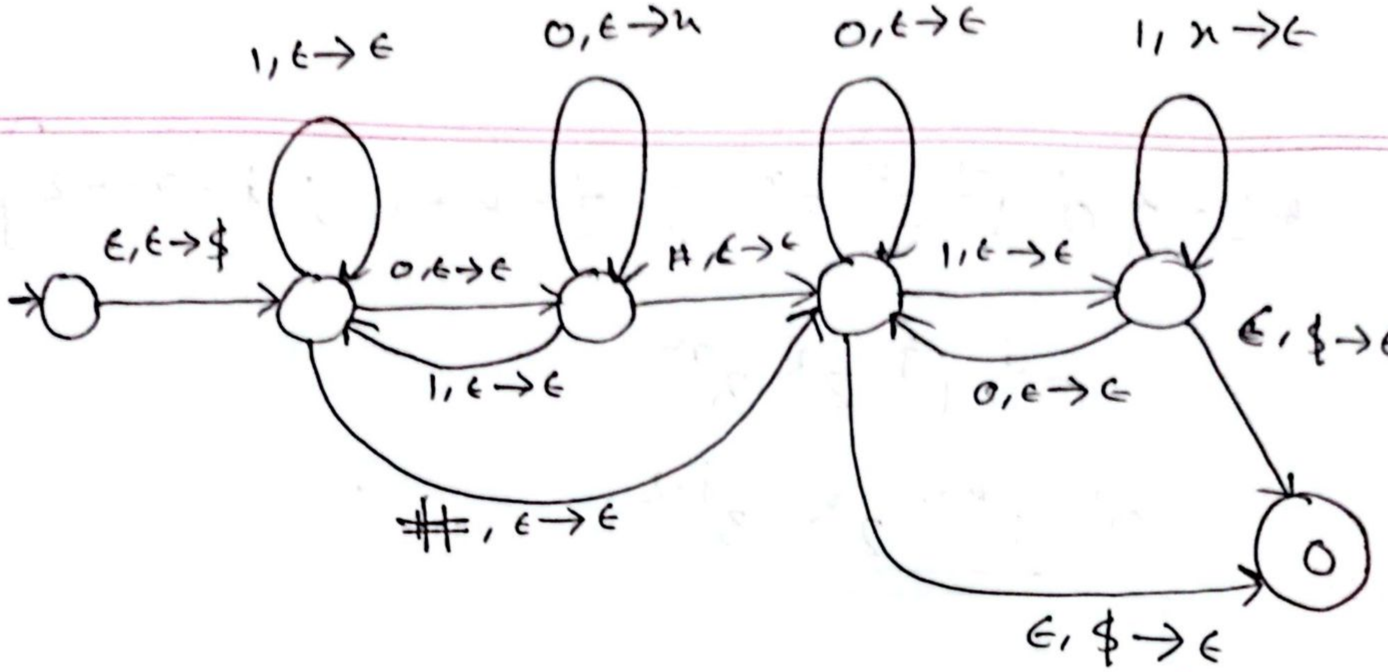
$\underline{0^i \ 1^j} \ 2^k \ \underline{3^m \ 4^n}$

$S \longrightarrow ABC$

$A \longrightarrow 0A1 \mid \epsilon$

$C \longrightarrow 3C4 \mid \epsilon$

$B \longrightarrow 2B \mid \epsilon$



$L = \{ w_1 \# w_2 \mid |w_1|$ is double of $|w_2|$

$\underbrace{\quad\quad}_{w_1} \# \underbrace{\quad\quad}_{w_2}$

$S \Rightarrow AASA$

$\Rightarrow AA\underline{AASA}A$

$S \longrightarrow AASA \mid \#$

$A \longrightarrow 0 \mid 1$

$|w_1| = m$; $|w_2| = n$; $3m = 4n$

$(m, n) = (4, 3)$

$\underbrace{\varepsilon\varepsilon\varepsilon\varepsilon} \# \underbrace{\varepsilon\varepsilon\varepsilon}$

$S \longrightarrow AAAASAAA \mid \#$

$A \longrightarrow 0 \mid 1$