Pumping Lemma

|S| means length of S.

Pumping Lemma is used to prove that a Language is NOT REGULAR

It can't be used to prove that a Language is Regular

- If L is a Regular Language, then L has a Pumping Length 'p' such that any string 'w' where $|w| \geq P$ may be divided into 3 parts $w = x\,y\,z$ such that the following conditions must be true.

(i) $x y^i z \in L$ for every $i \geq 0$; $y^i = \underbrace{y\,y \cdots y}_{i}$

(ii) $|y| > 0$ অথবা, $y \neq \varepsilon$

(iii) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, Follow the below steps:

(We prove using Contradiction)

(i) Assume that A is regular

(ii) A has to have a Pumping Length (say P)

(iii) All strings longer than P can be Pumped $|s| \geq P$

(iv) Now find a string 'S' in A such that $|s| \geq P$

(v) Divide S into $x\,y\,z$.

(vi) Show that $x y^i z \notin A$ for some i.

(vii) Then consider all ways that S can be divided into $x\,y\,z$.

(viii) Show that none of these can satisfy all the 3 pumping conditions at the same time.

(ix) S can not be Pumped == CONTRADICTION

KKP

Regular Language are those, যাদের DFA/NFA/RegEx বানানো যায়।

Pumping → কোনো string এর কোনো একটা portion বারবার Repeate করা

Ex → 000 10101 10101 10101 ___ ...

pumping → 10101

Pumping Lemma

(i) Every Regular language has a special property.

(ii) Special value → Pumping length (P)

L → if L is regular,

for all long strings ω, → $|\omega| \geq P$

$\omega \in L$, $\boxed{xy^iz \in L \text{ for all } i \geq 0}$

$x \mid y \mid z$

$xy^iz \in L$

$xy^2z = xyyz$

$xy^3z = xyyyz$

$L = \{0^n 1^n \; n \geq 0\}$ Prove L is non regular.

Assume L is ~~regu~~ regular.

Let P be the pumping length of L.

I have to clearly pick a long string $\omega \in L$, $|\omega| \geq P$

$\boxed{00001111}$ (P)

Let, $\omega = 0^P 1^P \in L$; ~~2P~~ $|\omega| = 2P$; $2P \geq P$

$P = 3$ হলে, $\omega = 000111 = 3(0) + 3(1) = 6 = 2 \times 3$

$P = 4$ হলে, $\omega = 00001111 = 4(0) + 4(1) = 8 = 2 \times 4$

So, $\omega = 2 \times P = 2P$

Let $P = 6$,

$$\underbrace{00}_{x} \,|\, \underbrace{000}_{y} \,|\, 0 \; \underbrace{111111}_{z}$$

$xyyz = 00\;000000\;0111111$

Language থেকে বের হয়ে গেছে।

Pumping Lemma says we can divide $w = xyz$

If I repeat Y as many as times, It should stay in the language. However if we do $xyyz$ then number of 0's increases than number of 1's

So, $xyyz \notin L$; So Non-Regular

More Example
last page of CFG

1. Ans

(a) a $L = \{w \in \{0,1\}^* \; 0^i 1^j \text{ where } i > j\}$

Assume, L is regular, and p be the pumping length.

So, let, $w = 0^{p+1} 1^{p}$; $|w| \geq p$

Here, $i = p+1$; $j = p$; so, $i > j$

let, $p = 2$, So, $w = 0^{2+1} 1^{2} = 0^3 1^2$

$= 00011$

Here, $xy = 00$

So on, $x = 0$, $y = 0$, $z = 011$

Now, we need to prove, for all $i \geq 0$; $xy^iz \in L$

let, $i = 1$; $xy^1z = 00011 \in L$

$i = 2$; $xy^2z = 000011 \in L$

$i = 0$; $xy^0z = 0011 \notin L$

So, given L is not regular as contradiction happens

[Proved] (Ans.)

$\boxed{b}$ $L = \{ \omega \in \{1\}^*, \ \omega = 1^n$, where n is a prime number$\}$

Assume, L is regular and p is the pumping length

let, $\omega = 1^p$; $|\omega| \geq p$

let, $p = 3$; so, $\omega = 1^3 = 111$

$\omega = 111$ where,

$xy = 11$ so, $x = 1, \ y = 1, \ z = 1$

~~for~~ Now, we need to prove for all $i \geq 0$; $xy^iz \in L$,

for, $k = 0$; $xy^0z = 11 \in L$

$k = 1$; $xy^1z = 111 \in L$

$k = 2$; $xy^2z = 1111 \notin L$

So, given L is not regular, as contradiction happens.

[Proved]

$\boxed{c}$ $L = \{w \in \{0,1\}^*, \ w \text{ is a palindrome}\}$

Assume, L is regular and p is the pumping length.

let, $w = 0^p 1 0^p$; $|w| \geq p$

let, $p = 2$ so, $w = 0^2 1 0^2$

$= 00100$

$xy = 00$

So, $x = 0$, $y = 0$, $z = 100$

Now, proving $xy^iz \in L$ for all $i \geq 0$.

For, $i = 1$; $xy^1z = 00100 \in L$

$i = 2$; $xy^2z = 000100 \notin L$

So, given L is not regular as contradiction happens.

[Proved]

d

$L = \{ w \in \{0,1\}^*$ length of w is odd and w has 0 in the exact middle position $\}$

Assume, L is regular, and p is the pumping length

let, $w = 1^p 0 1^p$; $|w| \geq p$

let, $p = 2$; so, $w = 1^2 0 1^2$

$= 11011$

$xy = 11$

so, $x = 1$, $y = 1$, $z = 011$

Proving $xy^iz \in L$ for all $i \geq 0$.

For, $k = 1$; $xy^1z = 11011 \in L$

$k = 2$; $xy^2z = 111011 \notin L$

$\therefore$ L is not regular as contradiction happens.

[Proved]

$\boxed{e}$ $L = \{ w \in \{0,1\}^*$, number of 0s in $w >$ number of 1s in $w \}$

~~Let $w = 0^{p+1}$~~ Assume L is regular and p is the pumping length

let, $w = 0^{p+1} 1^p$; $|w| \geq p$

let, $p = 2$; so, $w = 0^{2+1} 1^2$

$= 00011$

so, $xy = 00$

so, $x = 0$, $y = 0$, $z = 011$

Proving $xy^iz \in L$ for all $i \geq 0$

for, $i = 1$; $xy^1z = 00011 \in L$

$i = 2$; $xy^2z = 000011 \in L$

$i = 0$; $xy^0z = 0011 \notin L$

so, given L is not regular as contradiction happens. [proved]

$S \rightarrow 000S11111$

$S \rightarrow \epsilon$

$S \rightarrow 000S11111$ K

$\boxed{f}$ $L = \{w \in \{a,b\}^*, \ w = a^n b^n, \text{ where } n \neq 1\}$

Assume L is regular and p is the pumping length.

let, $w = a^p b^{p+1}$; $|w| \geq p$

let, $p = 2$; so, $w = a^2 b^{2+1}$

$= aa\,bbb$

so, $xy = aa$

so, $x = a$, $y = a$, $z = bbb$

Now, proving $xy^iz \in L$ for all $i \geq 0$.

for, $i = 1$; $xy^1z = aa\,bbb \in L$

$i = 2$; $xy^2z = aaa\,bbb \notin L$

So, L is not regular as contradiction happens.

[proved]