Language → "set"

Regular Expression

(i) Union (OR) [A ∪ B] (|) (+)

(ii) Concatenation (.) [A • B]

(iii) Kleene Closure/ Star (*) [A*B]

let

A = {good, bad} B = {boy, girl}

* A ∪ B/A | B = {good, bad, boy, girl}

* A * B = {good boy, good girl, bad boy, badgirl}

* A* = {ϵ, good, bad, good good, goodbad, badgood, bad bad, goodgoodgood, goodgoodbad ... }

$0^* = $ 0's for any number of times.

$= \{\epsilon, 0, 00, 000, 0000, 00000 \ldots\}$

$01^* = \{0, 01, 011, 0111, 01111 \ldots\}$

$(01)^* = \{\epsilon, 01, 0101, 010101, 01010101, \ldots\}$

$(0 \cup 1)^*$ ~~or~~ $(0|1)^*$ ~~or~~ or $(0+1)$

So, $(0|1)^0 = \epsilon$

$(0|1)^1 = \{0, 1\}$

$(0|1)(0|1) = \{00, 01, 10, 11\}$

$(0|1)(0|1)(0|1) = \{000, 001, 010, 011, 100, 101, 110, 111\}$

So, $\Sigma^* = (0|1)^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111 \ldots\}$

↳ All possible binary strings

$\cancel{\square}$ $(0+1)^* = \Sigma^*$

Strings starting with '01'

$01\Sigma^*$ $\boxed{01(0+1)^*}$

Strings ~~endit~~ ending with "01"

$\Sigma^* 01$ $\boxed{(0+1)^* 01}$

Strings having "100" as a substring.

$\Sigma^* 100 \Sigma^*$ $\boxed{(0+1)^* 100 (0+1)^*}$

Strings having "100" ~~as~~ as a subsequence.

$\Sigma^* 1 \Sigma^* 0 \Sigma^* 0 \Sigma^*$

$0^* 1 1^* 0 \phi 0 \Sigma^*$

$\Sigma^* 1 0^* 0 1^* 0 1^*$

$(0+1)^* 1 (0+1)^* 0 (0+1)^* 0 (0+1)^*$

String having even ~~no~~ number of 1's

$(0^* 1 0^* 1)^* 0^*$ or $0^* (1 0^* 1 0^*)^*$

$(0+\epsilon)(10)^*(1+\epsilon)$

Strings that end in three consecutive 1's

$(0+1)^*111$

At least one 1

$(0+1)^* 1 (0+1)^*$

* At most one 1

$(0^*10^*)+0^*$ অথবা, $0^*(1+\epsilon)0^*$

Contains the substring "101"

$(1+0)^*101(1+0)^*$

Do not have consecutive 1's $0(100^*)1+\epsilon$

$0^*(100^*)(1+\epsilon)$ অথবা, $(0+10)^*(1+\epsilon)$

* Do not contain the substring "11"

$0^*10(0+1)^*(1+\epsilon)$ অথবা Same ans

* Neither consecutive 1's, nor consecutive 0s.

$(0+\epsilon)(10)^*(1+\epsilon)$ 10101010

May have consecutive 1's or consecutive 0's but not both

$$(0+10)^*(1+\epsilon)+(1+01)^*(0+\epsilon)$$

The number of 0's is even.

$$(1^*01^*01)^*$$

The number of 0's is odd.

$$(1^*01^*01^*)^*1^*01^*$$

The number of 0's is divisible by 4

$$(0+(1+01^*01^*01^*0)^*$$

$\Sigma = \{0, 1\}$

(a) The language containing strings

* where 0's and 1's are alternate.

101010 ~~$0?(10)^* 1?$ ϕ / $(0+1)(10)^*$ (~~

$0?(10)^* 1^*$ | $1?(01)^* 0^*$ } $1?(01)^* 0!$ { $0?(10)^* 1!$

* which starts and ends with same character

$0(0+1)^* 0 \mid 1(0+1)^* 1 \mid 0 \mid 1$

* In which the number of 1's between every pair of consecutive 0's is odd.

$\boxed{0\ 1^{odd}\ 0}$ 000 $1^*(0(11)^* 1)^* 0 1^* \mid 1^*$

$00\ (111)\ 00$

$0^*((11)^* 1)0)^*$

$(0+1) = \Sigma^*$

* Write RegEx for the language. $L_1 \cap L_2$; $\{0,1\}$

$L_1 = \{w :$ the length of w is divisible by 3$\}$

$L_2 = \{w :$ every second letter in w is a 0$\}$

$L_1 = [(0+1)(0+1)(0+1)]^*$

$L_2 = [(0+1)0]^*((0+1)+\varepsilon)$
$= (\Sigma 0)^* \Sigma ?$

So, $L_1 \cap L_2 = [(0+1)0(0+1)0(0+1)0]^* + ((0+1)0(0+1)+\epsilon)$
$= (\Sigma 0 \Sigma 0 \Sigma 0)^* (\Sigma 0 \Sigma | \epsilon$

* $L = \{w \in \{0,1,2\}^*$: the last letter of w appears at least twice in $w\}$

$\Sigma^* 0 \Sigma^* 0 + \Sigma^* 1 \Sigma^* 1 + \Sigma^* 2 \Sigma^* 2$

* $L = \{w \in \{0,1,2\}^*$: w contains at least one 1 and one 0$\}$

$\Sigma^* 1 \Sigma^* 0 \Sigma^* \mid \Sigma^* 0 \Sigma^* 1 \Sigma^*$

* $L = \{w \in \{0,1\}^*$: exactly one occurrence of 00 appears in $w\}$

$1^*(01^+)^* 00 (1^+0)^* 1^*$

$\{0, 1\}$

The language containing strings,

where 0s and 1s always appear in pairs.

for example - 001100, 1100110011001100 etc

=> $(00+11)^*$

which starts and ends with different characters.

=> $(0\Sigma^*1 + 1\Sigma^*0)$.

In which the number of 1's between every pair of consecutive 0s is even.

=> $1^*(0(11)^*)^*1^*$

Having equal number of "01" and "10" substrings

=> $0\Sigma^*0 + 1\Sigma^*1 + 0 + 1$

whose parity of 0 and 1 are different.

=> 0 even 1 odd | if same
1 even 0 odd | 0 even 1 even
if different | 0 odd 1 odd

Parity case

L_1 = Where every 0 is followed by at least three 1's.

$1^*(01111^*)^*$ | $L_1' = \Sigma^* 0(\epsilon+1+11)(\epsilon+0\Sigma^*)$

whose parity of 0 and 1 are different.

$(\Sigma\Sigma)^* \; (\Sigma\Sigma)^*\Sigma$

Parity cases →

Different Parity	Same Parity
0 even, 1 odd	0 even 1 even
1 even, 0 odd	0 odd 1 odd
length = Odd	length = even

The set of strings that end with 11

$\Rightarrow \quad \Sigma^* 11$

That do not end with 11

$\Rightarrow \quad \Sigma^* (00 + 10 + 01) + (0 + 1 + \epsilon)$

L_1 = That contains 01 as a substring

$\Rightarrow \quad \Sigma^* 01 \Sigma^*$

* $\overline{L_1}$ = ?

$\Rightarrow \quad 1^* 0^*$

The set of strings having 0 at every odd position.

$\Rightarrow \quad \left(0(0+1)\right)^* (0+\epsilon)$ অথবা $(00+01)^* (0+\epsilon)$

* having 0 at every third position.

$\Rightarrow \quad (\cancel{\Sigma\Sigma 0})^* (0.$ অথবা $((0+1)(0+1)0)^*$

$$\boxed{(\Sigma \Sigma 0)^* (0+1+\epsilon)(0+1+\epsilon)}$$

Having 0 at 3rd position.

$\Rightarrow \Sigma\Sigma 0 \Sigma^*$

Having 0 at the third last position.

$\Rightarrow \Sigma^* 0 \Sigma\Sigma$

Assume $\Sigma = \{0,1\}$

$L_1 = \{ w \mid$ the length of w is even $\}$

$L_2 = \{ w \mid w$ starts and ends with different characters $\}$

$L_3 = \{ w \mid w$ have 0's at all odd positions $\}$

(a) $L_1 = ?$ (b) $L_2 = ?$ (c) $L_3 = ?$

(d) ~~L_2~~ $L_2 \setminus L_1$ (e) $L_3 \setminus L_2$

(a)

$L_1 = (\Sigma\Sigma)^*$

(b)

$(0\Sigma^*1) + (1\Sigma^*0)$

(c)

$L_3 = (0\Sigma)^*(0+\epsilon)$ অথবা, $(0\Sigma)^* 0$?

(d)

$L_2 \setminus L_1 = L_2 \cap \overline{L_1}$

$= (0(\Sigma\Sigma)^*\Sigma 1) + (1(\Sigma\Sigma)^*\Sigma 0)$

(e)

$L_3 \setminus L_2 = L_3 \cap \overline{L_2}$

= 0 is at all odd position and (starts, end similar)

$= ((0\Sigma)^* 0) + ((0\Sigma)^* 00)$

$= (0\Sigma)^* (0 + 00)$

$\Sigma = \{0,1\}$

$L_1 = \{w \mid$ the length of w is divisible by $4\}$

$\Rightarrow$ $L_1 = (\Sigma\Sigma\Sigma\Sigma)^*$

$\overline{L_1} = (\Sigma\Sigma\Sigma\Sigma)^* (\Sigma + \Sigma\Sigma + \Sigma\Sigma\Sigma)$

$L_2 = \{w \mid w$ has 0's at all even positions$\}$

$\Rightarrow$ $L_2 = (\Sigma 0)^*(\Sigma + \epsilon)$ $\quad \overline{L_2} = (\Sigma\Sigma)^* \Sigma 1 \Sigma$

$L_3 = \{w \mid w$ has 1's at all odd positions$\}$

$\Rightarrow$ $L_3 = (1\Sigma)^* (1 + \epsilon)$ $\quad \overline{L_3} = (\Sigma\Sigma)^* 0 \Sigma^*$

$L_4 = \{w \mid w$ has a 1 at every third position$\}$

$\Rightarrow$ $L_4 = (\Sigma\Sigma 1)^* (\epsilon + 0)(\epsilon + 0)$

$\overline{L_4} = (\Sigma\Sigma\Sigma)^* (\Sigma\Sigma 0 \Sigma^*)$

$\Sigma = \{0,1,2$

$L_5 = \{w \mid w$ ha

$L_5 : (0\Sigma^* 0)$

$\overline{L_5} : 0\Sigma^*(1+2$

$L_6 = \{w \mid w$ st

with a

$L_6 : (0\Sigma^* 1) + ($

অথবা, $0\Sigma^*$

$\overline{L_6} : (1+2)\Sigma^* \; +$

$\Sigma = \{0, 1, 2\}$

$L_5 = \{w \mid w$ has the same first and last character$\}$

$$L_5 : (0\Sigma^*0) + (1\Sigma^*1) + (2\Sigma^*2) + \underbrace{(0+1+2)}_{\Sigma}$$

$$\overline{L_5} : 0\Sigma^*(1+2) + 1\Sigma^*(0+2) + 2\Sigma^*(0+1) + \Sigma$$

$L_6 = \{w \mid w$ starts with $\underbrace{0}_{p}$ but $\underbrace{\text{does not ends with a } 0}_{q}\}$

$$L_6 : (0\Sigma^*1) + (0\Sigma^*2) + (1+2)$$

অথবা, $0\Sigma^*(1+2)$

$$\overline{L_6} : (1+2)\Sigma^* + (\Sigma^*0)$$

for $\overline{L_6}$
$\neg(p \wedge q)$
$\neg p \vee \neg q$
And (or Not হলে দিলে OR হয়।

$\Sigma = \{0, 1\}$

\# $L_1 = \{ w \mid w$ does not contain $11 \}$

$L_2 = \{ w \mid$ every 1 in w is followed by at leatst one $0 \}$

$L_3 = \{ w \mid$ the number of times 1 appears in w is even $\}$

(a) Reg ex for L_1.

(b) Your frien claims that $L_1 = L_2$. Prove her wrong by writting down a 5-letter string in $L_1 \setminus L_2$. Recall that $L_1 \setminus L_2$ contains all strings in L_1 but not in L_2

(c) $L_1 \setminus L_2$

(d) L_3 (e) $L_2 \setminus L_3$

Solution

(a) $L_1 : (0+10)^*(1+\epsilon)$

~~(b) $L_2 : 1(1+0)\,(1(0+\epsilon))^*$~~

(b) $L_2 : 0^*(10^+)^*$ Ex $\to$ $10101 \in L$

$\notin L$

(c) $L_1 \setminus L_2 : 0^*(10^+)^*1$

(d) $L_3 : (0^*10^*1)^*0^*$

(e) $L_2 \setminus L_3 : \underbrace{0^*(10^+10^+)^*10^+}_{\text{odd number of 1's}}$