

Problem 1

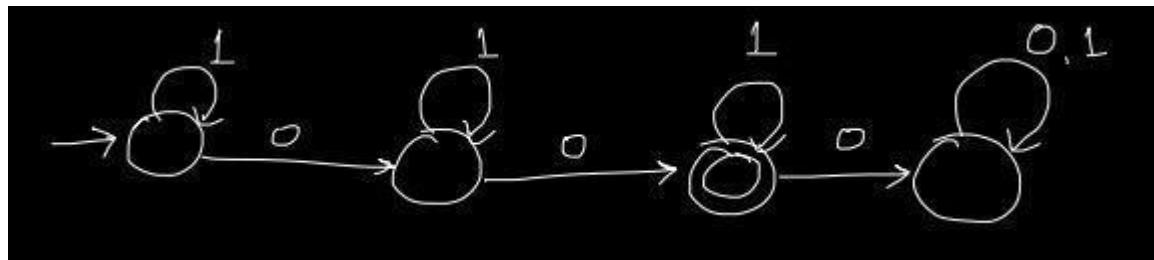
- a) $(0 \mid 1)^* (0 \mid 11 \mid 001) \mid \epsilon \mid 1 \mid 01$
- b) abba $(a \mid b \mid c)^*$ bac | abbac
- c) $(0 \mid 1)(0 \mid 1) 0 (0 \mid 1)^*$
- d) $(1^* 01^* 01^*)^* \mid 0^* 10^* 10^*$
- e) $(b^* ab^* ab^* ab^*)^* b^* ab^*$

Problem 2

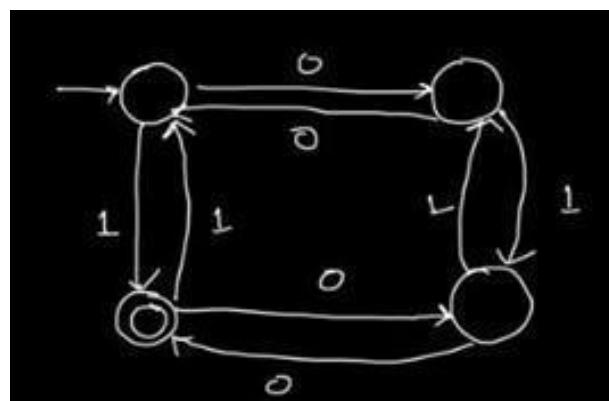
- a) Strings over $\{0,1\}$ that start with 01
- b) Strings over $\{0,1\}$ that start with 0 or end with 1
- c) Strings over $\{0,1\}$ that do not contain 00 as a substring
- d) Strings over $\{0,1\}$ that have neither consecutive 0's, nor consecutive 1's
- e) Strings over $\{0,1\}$ that may have consecutive 0's, or consecutive 1's, but not both

Problem 3

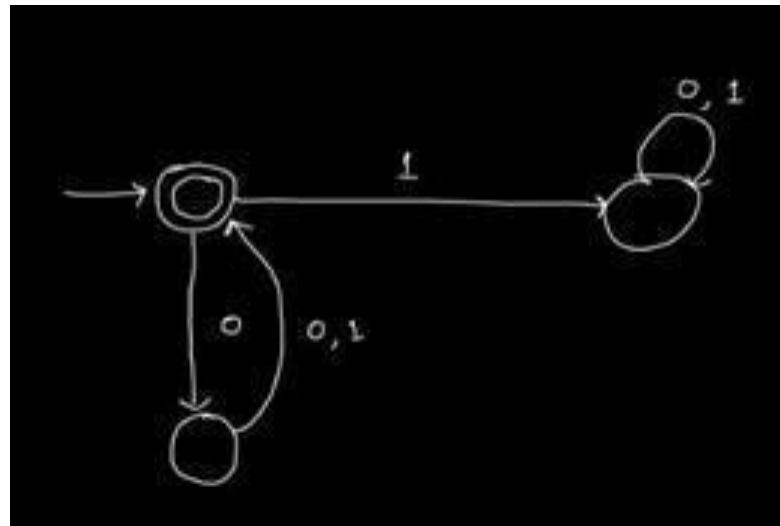
a)



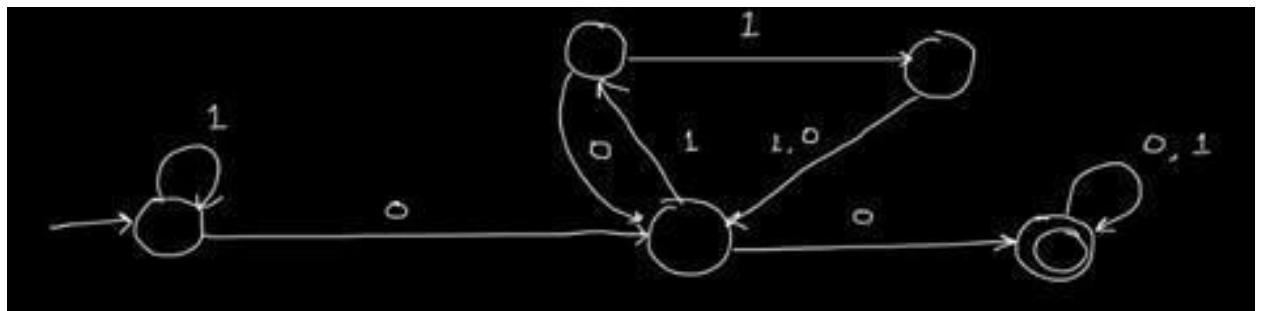
b)



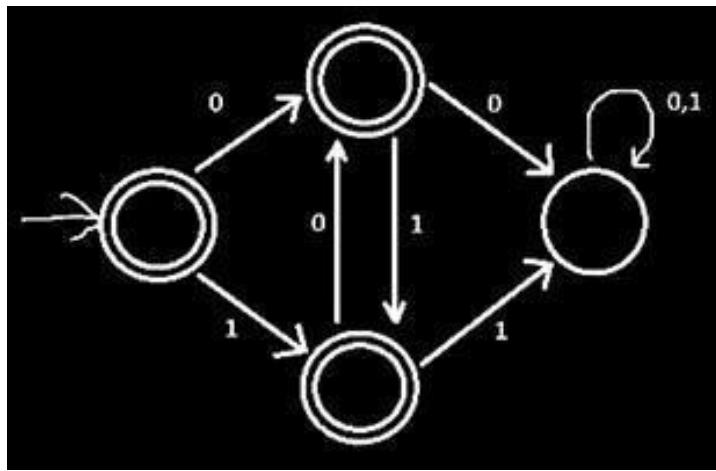
c)



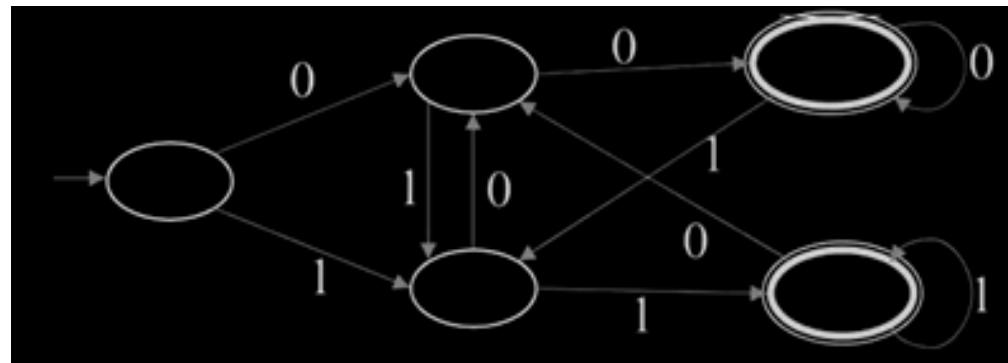
d)



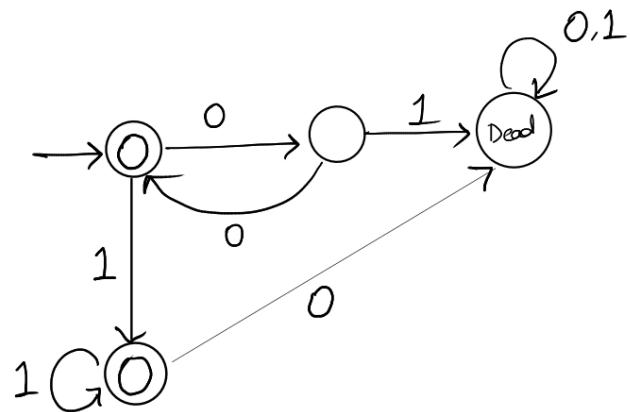
e)



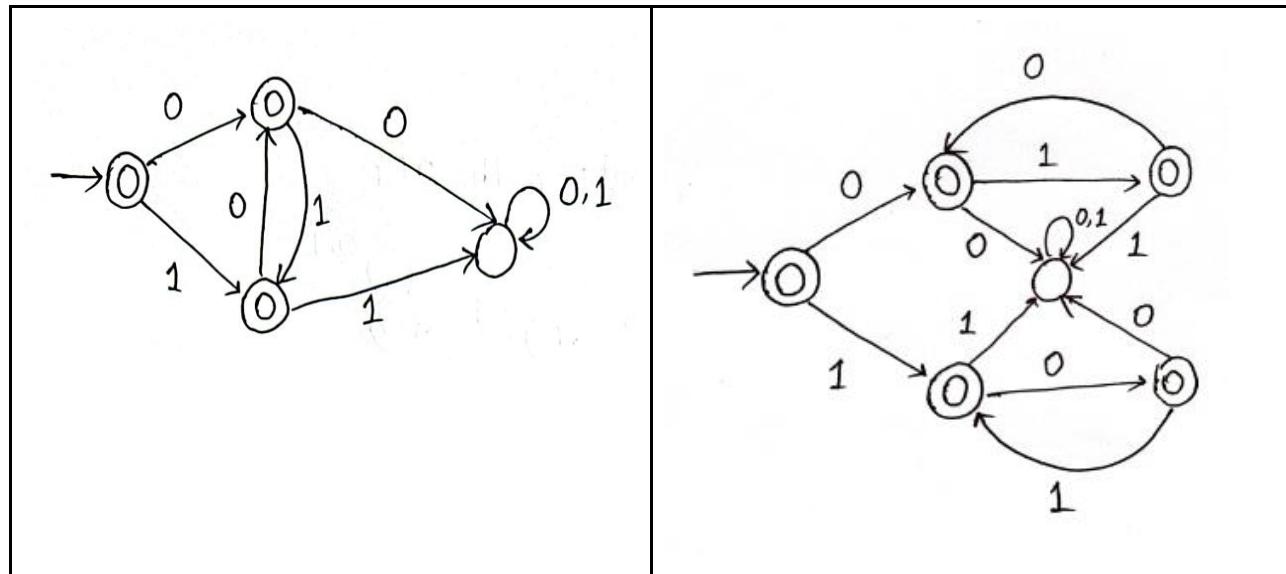
f)



g)

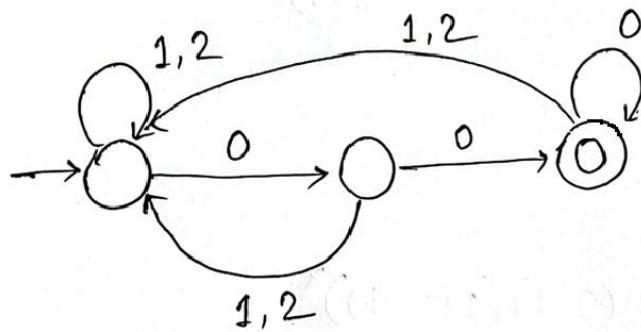


h)



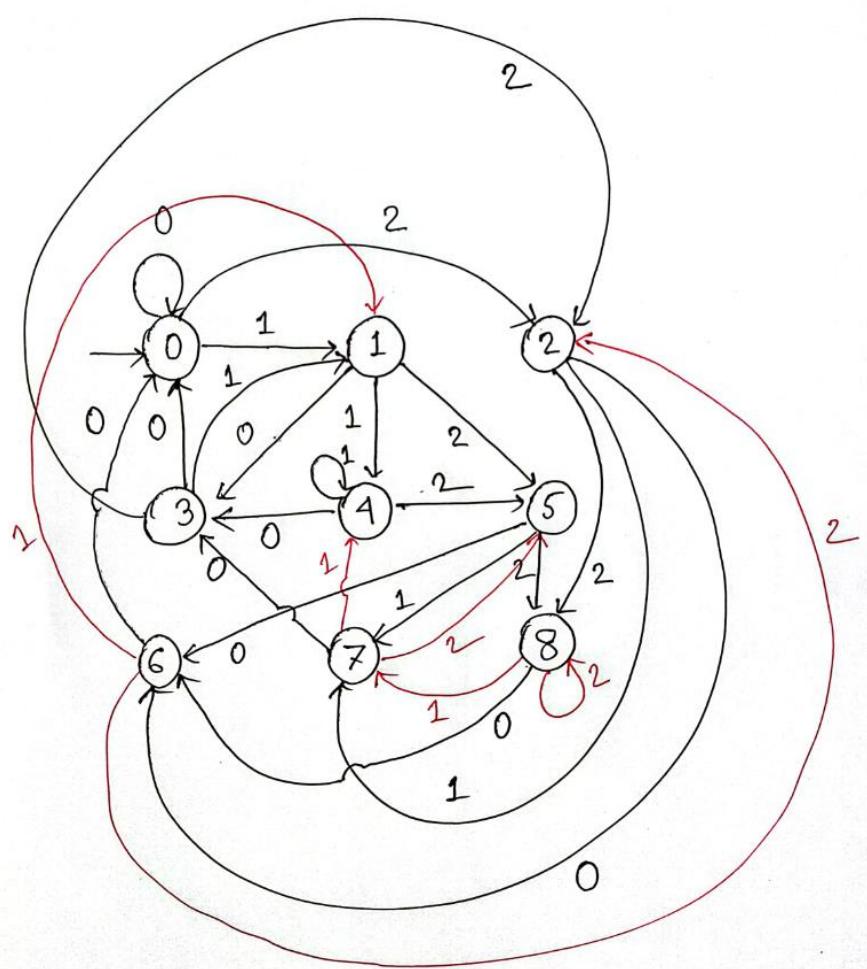
i)

In three base number system, the numbers which ends with 00, only those numbers are divisible by 9.

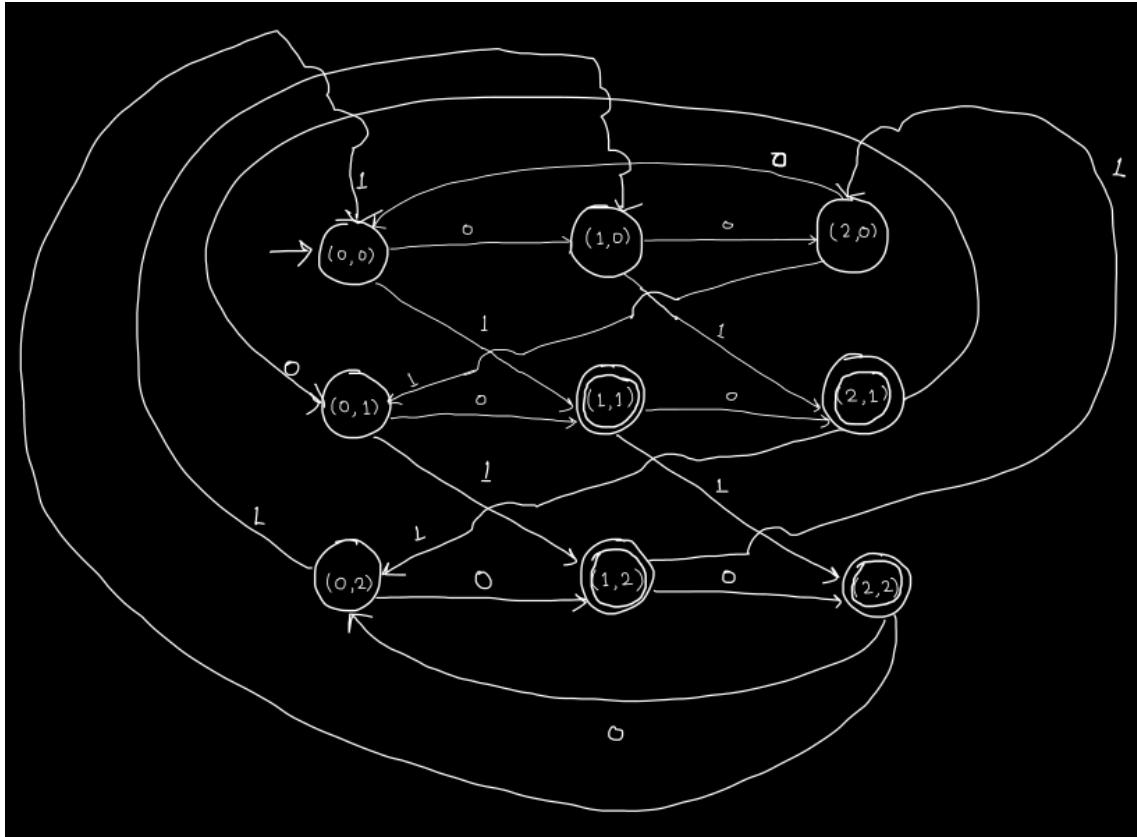


There is also a DFA with 9 states and 27 transitions - which is also correct.

The logic is - if there is a number x , in ternary representation, adding a 0 at the end will make the number $3x$, adding 1 at the end will make it $3x+1$ and for 2 it will become $3x+2$.



j)



A correct solution is anything equivalent to the above. The DFA diagram is a bit clunky due to there being a lot of states and transitions. However, the idea is simple: the states are indexed by a pair (i, j) where $0 \leq i, j \leq 2$, and i, j are the length and the number of 1s seen so far modulo 3. Upon seeing a new letter i goes up by 1, while j goes up by 1 only if the new letter is a 1.

The main observation is that $i^2 + j^2 \equiv 2 \pmod{3}$ iff $i = 1, 2$ and $j = 1, 2 \pmod{3}$. This gives us four accepting states.

Any DFA for this language necessarily requires 9 states. So, any DFA with fewer states is wrong.

Problem 4

- a) Yes
- b) 1100
- c) 0100
- d) Yes
- e) No