Electric Field

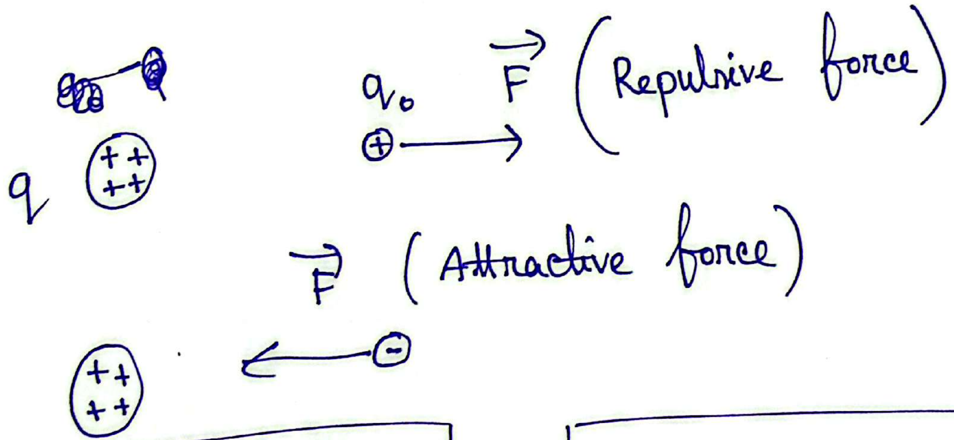
* How charged particles or objects exerts force on each other?

⇒ One way to think about force between charged objects is as an "action at a distance" force that is as a force that acts across empty space without needing any physical contact between the objects. A more fruitful way to visualize the force between two objects is as a two-way two-stage process. We first envision that ~~b~~ one object as a result of the charge that it carries, somehow modifies the properties of the space around it. Then the second ~~body~~ object, as a result of the charge that it carries, senses how space has been modified at its position. The respon of the second body is to experience the force.

⇒ We say a charged object or particle creates a field in space. This field is called electric field.

⇒ The electric force on a charged body is exerted by the electric field created by other charged bodies.

⇒ We define the electric field $\vec{E}$ at a point as the electric force $\vec{F}$ experienced by a test charge q_0 at the point.



$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}} \Rightarrow \boxed{\vec{F} = q_0 \vec{E}}$$

⇒ Unit N/C.

⇒ As force is a vector $\vec{E}$ is also a vector.

⇒ From Cuolomb's Law,

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r}$$

$\hat{r} \rightarrow$ points in the direction ~~of~~ from q to q_0

$$\therefore \vec{E} = \frac{\vec{F}}{q_0}$$

Elec field formulaa

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

This is the vector form of electric field created by a point charge q at a distance r from the charge. Here, ~~$\hat{r}$~~ ~~always~~ $\boxed{\hat{r} = \text{Unit vector from point charge toward where field is measured}}$.

⇒ Magnitude of the electric field of a point charge at a distance r is $\frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$.

* Like all the vectors if there are multiple electric fields at a point, the net electric field at that point will be the vector sum of all the fields.

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots}$$

CamScanner

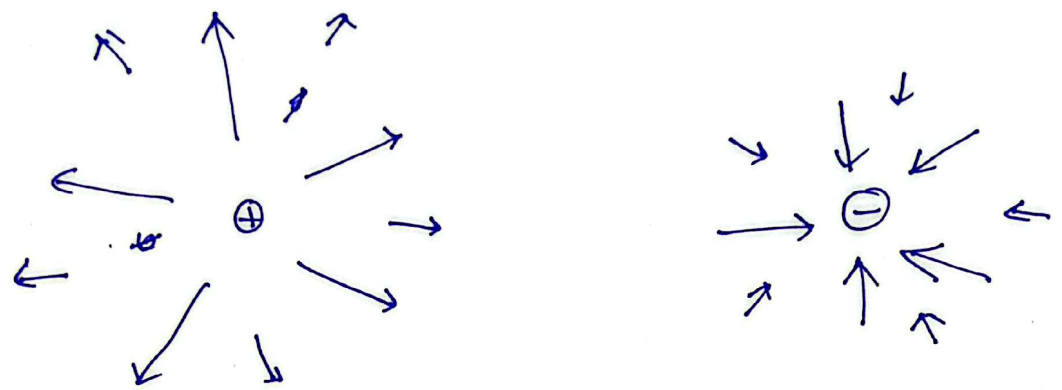


Figure: Visual representation of Electric field.

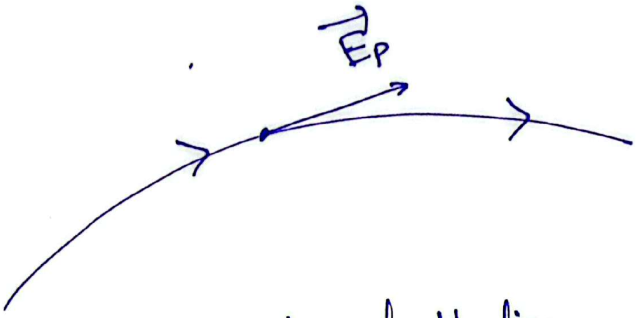
Electric Field lines:

=> A way to visualise electric field.

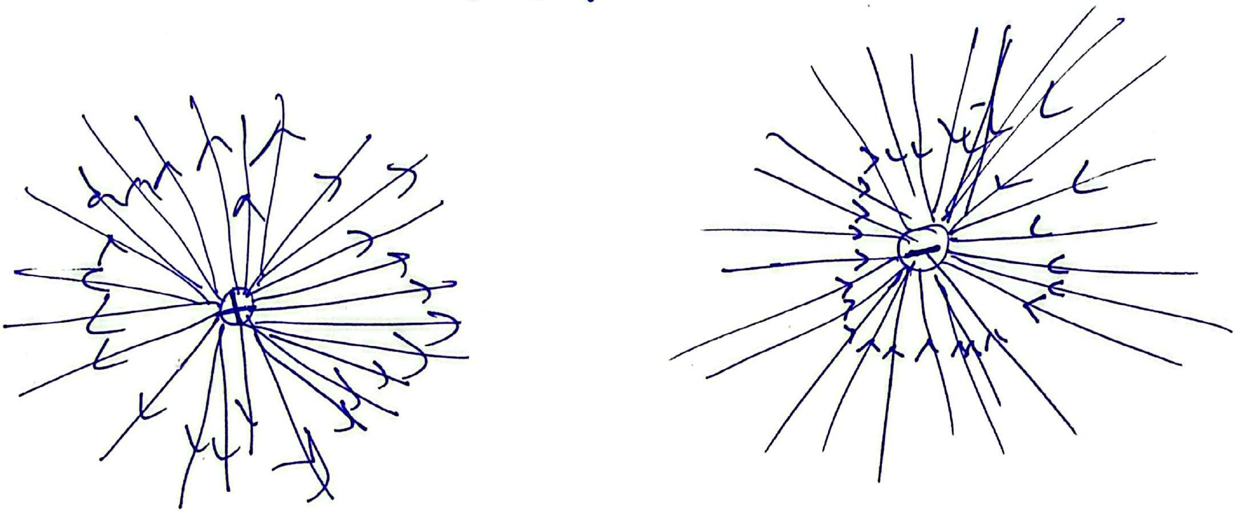
Rules:

1) At any point, the electric field vector must be tangent to the electrical field line through that point and in same direction.

2) In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.



Electric field line



Visual representation of electric field line.

Electric field due to a dipole:

⇒ If same amount of opposite charges are kept at a distant, then the arrangement is called a dipole.

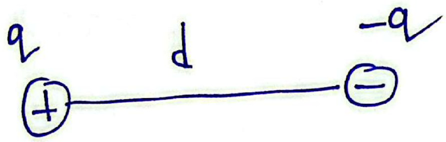
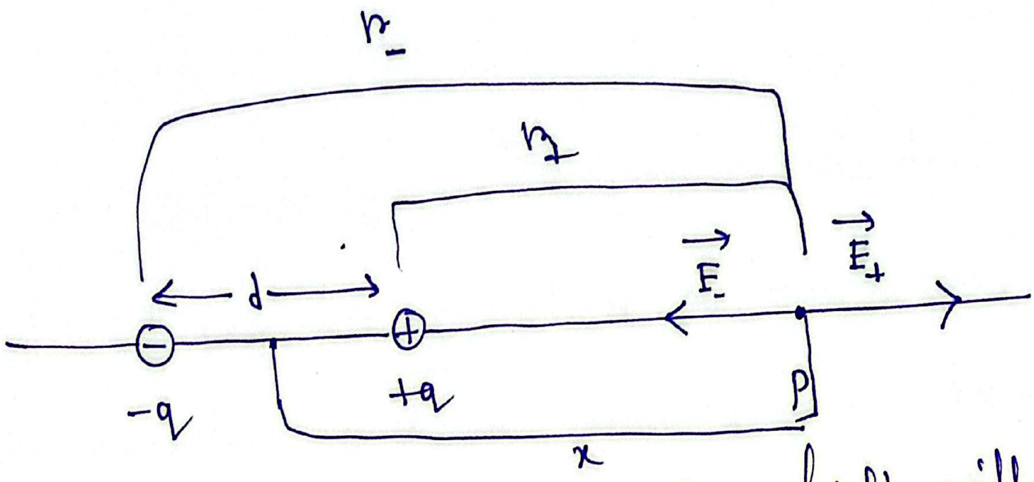


Fig: A dipole.



The magnitude of the net electric field will be,

$$E = E_+ - E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{r_+^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_-^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{\left(x - \frac{d}{2}\right)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{\left(x + \frac{1}{2}d\right)^2}$$

$$= \frac{q}{4\pi\epsilon_0 x^2}\left(\frac{1}{\left(1 - \frac{d}{2x}\right)^2} - \frac{1}{\left(1 + \frac{d}{2x}\right)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0 x^2}\frac{2d/x}{\left(1 - \left(\frac{d}{2x}\right)^2\right)^2}$$

$$= \frac{q}{2\pi\epsilon_0 x^3}\frac{d}{\left(1 - \left(\frac{d}{2x}\right)^2\right)^2}$$

If $x >> d$,

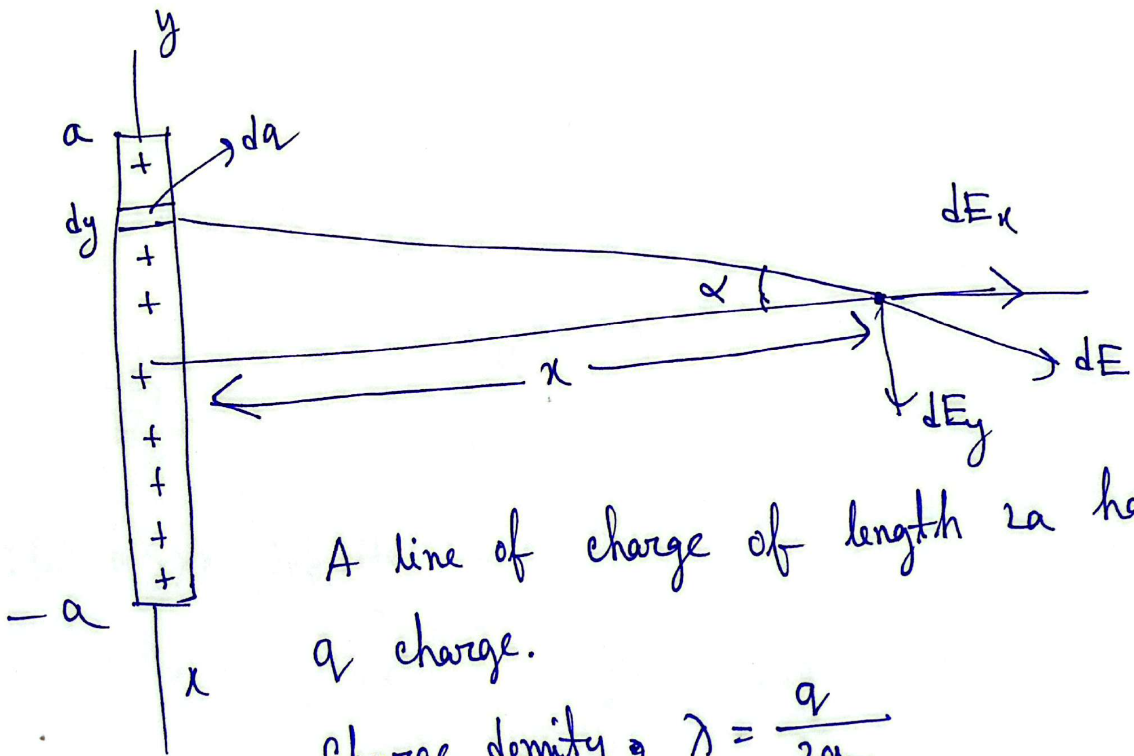
$$E = \frac{1}{2\pi\epsilon_0}\frac{qd}{x^3}$$

⇒ The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the electric dipole moment. $\vec{p}$ of the dipole.

$p = qd$ ∴ $E = \frac{1}{2\pi\epsilon_0} \frac{p}{x^3}$.

* The direction of $\vec{p}$ is taken to be from the negative to the positive end of the dipole.

Electric field of charged line segment:



A line of charge of length 2a has q charge.

Charge density, $\lambda = \frac{q}{2a}$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+y^2}.$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+y^2}\cos\alpha = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{x\,dq}{(x^2+y^2)^{3/2}}$$

Similarly,

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+y^2}\sin\alpha = \frac{1}{4\pi\epsilon_0}\frac{y\,dq}{(x^2+y^2)^{3/2}}.$$

$$\therefore E_x = \int dE_x = \int_{-a}^{a}\frac{1}{4\pi\epsilon_0}\frac{x\,dq}{(x^2+y^2)^{3/2}} = \int_{-a}^{a}\frac{1}{4\pi\epsilon_0}\frac{x\lambda\,dy}{(x^2+y^2)^{3/2}}$$

$$= \cancel{\frac{K_e q}{x\sqrt{x^2+a^2}}}\ \frac{1}{4\pi\epsilon_0}\frac{q}{x\sqrt{x^2+a^2}}$$

$$\therefore E_y = \int dE_y = 0.$$

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{x\sqrt{x^2+a^2}}\hat{i}.$$

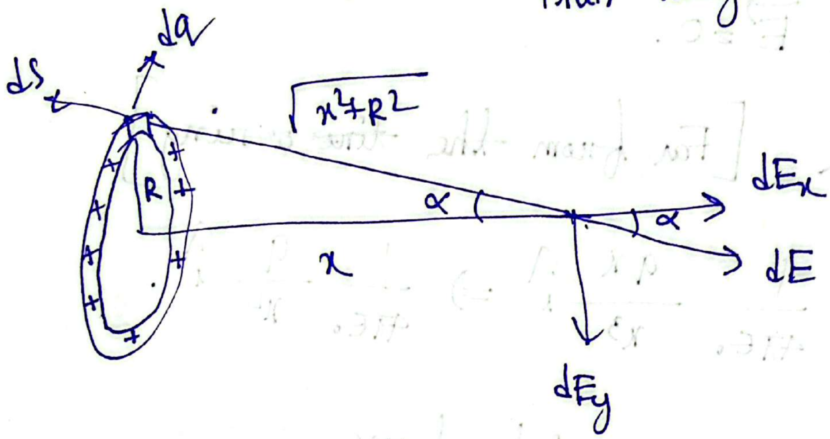
If $a \gg x$ (Infinite line of charge),

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{ax}\hat{i} = \frac{1}{2\pi\epsilon_0}\frac{q}{2ax}\hat{i} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{i}.$$

* For a infinite line of charge, at any point p at a perpendicular distance r from the line in any direction $\vec{E}$ has magnitude $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

Field of a ring of charge:

Thin ring has radius R.



$E_y = 0$ for symmetry.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+R^2}$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+R^2}\cos\alpha$$

$$= \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+R^2}\frac{x}{\sqrt{x^2+R^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{x\,\lambda\, ds}{(x^2+R^2)^{3/2}}$$

$$\therefore E_x = \int dE_x = \int_0^{2\pi R} \frac{\lambda x \, ds}{4\pi\epsilon_0 (x^2+R^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{qx}{(x^2+R^2)^{3/2}}$$

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qx}{(x^2+R^2)^{3/2}} \hat{i} .$$

At center $\vec{E} = 0$.

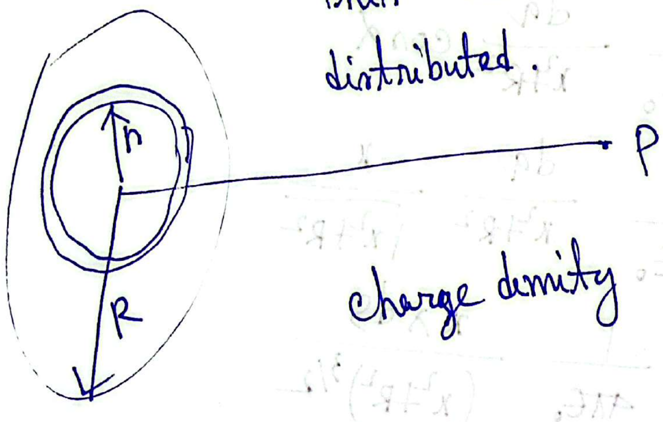
If $x >> R$ [Far from the ~~line~~ or ring]

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qx}{x^3} \hat{i} \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \hat{i} .$$

Act like a point charge.

Field of a charged disk:

This disk has q charge uniformly distributed.



charge density $\sigma = \frac{q}{\pi R^2}$.

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dq\, x}{(x^2+r^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(2\pi r\, dr\, \sigma)\, x}{(x^2+r^2)^{3/2}}.$$

$$\therefore E_x = \int dE_x = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{2\pi r\, dr\, \sigma\, x}{(x^2+r^2)^{3/2}}.$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

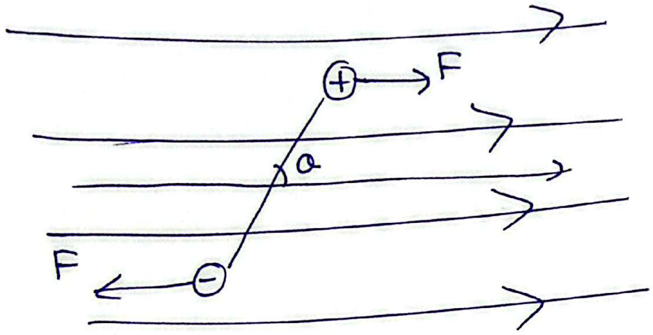
if $R \to \infty$ [~~Unifor~~ Infinite disk]

or, $\cancel{r\gg}$ $R \gg x$ [Very close to the disk]

$$E = \frac{\sigma}{2\epsilon_0}$$

[Uniform electric field].

A dipole in an uniform electric field:



Individual torque:

$$\vec{\tau}_{ind} = \vec{r} \times \vec{F} = \frac{d}{2} q E \sin\theta$$

Total torque:

$$\tau = dq E \sin\theta$$

$$= pE \sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Potential energy of an electric dipole:

$$dW = \vec{F}.\vec{ds} = \vec{F}.\vec{r}d\theta = rF d\theta \cos 0^\circ$$

$$= rF d\theta$$

$$= \tau \, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$= PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= PE \left[-\cos\theta \right]_{\theta_1}^{\theta_2}$$

$$= -PE\cos\theta_2 + PE\cos\theta_1$$

$$= -\left(PE\cos\theta_2 - PE\cos\theta_1 \right)$$

$$= -\left(U_2 - U_1 \right)$$

$$W = -\Delta U$$

$$\therefore U = -PE\cos\theta.$$

$$= -\vec{P} \cdot \vec{E}.$$

Electric Field Problem Sol^n

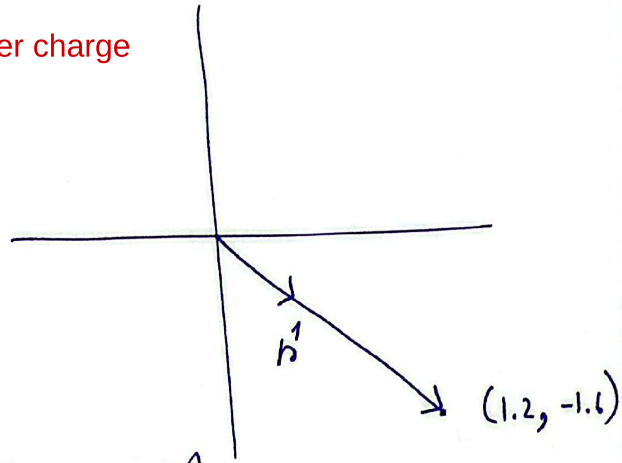
minus hishab kora hbe

q= onno point er charge not nijer charge

1)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{-8\times 10^{-9}}{(1.2)^2+(1.6)^2}\hat{r}$$



$$= \frac{1}{4\pi\epsilon_0}\frac{-8\times 10^{-9}}{(1.2)^2+(1.6)^2}\frac{1.2\hat{i}-1.6\hat{j}}{\sqrt{(1.2)^2+(1.6)^2}}.$$

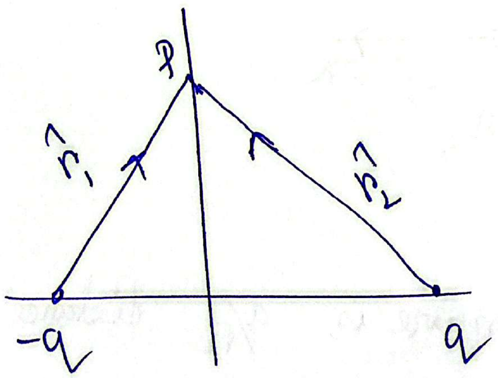
$$= \left(-8\;11\ N/c\right)\hat{i} + \left(14\ N/c\right)\hat{j}.$$

2)

$$\vec{E}_{q_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 = 9\times 10^{9}\times\frac{-4.80\times 10^{-19}}{\left(\sqrt{3^2+4^2}\right)^2}\frac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}}$$

$$= -\left(3.456\times 10^{-11}\right)\left(3\hat{i}+4\hat{j}\right)$$

$$= -\left(1.0368\times 10^{-10}\ N/c\right)\hat{i} - \left(1.3824\times 10^{-10}\ N/c\right)\hat{j}.$$



TC

$$\vec{E_2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2} \hat{r_2} = 9\times10^9 \times \frac{3.20\times10^{-19}}{3^2+4^2} \times \frac{-3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}}$$

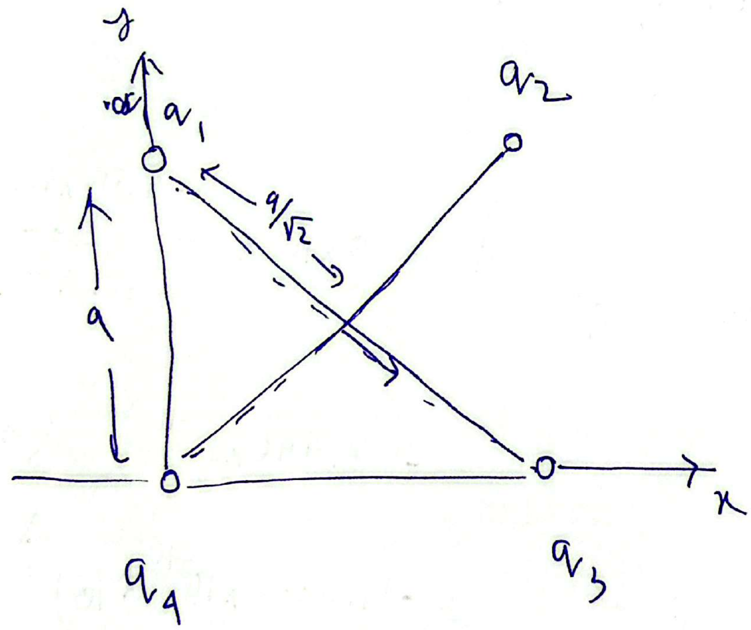
$$= \cancel{8.}\left(2.304\times10^{-11}\right)\left(-3\hat{i}+4\hat{j}\right)$$

$$= \left(-6.912\times10^{-11}\,N/C\right)\hat{i} + \left(9.216\times10^{-11}\,N/C\right)\hat{j}$$

$$\therefore \vec{E} = \vec{E_1} + \vec{E_2} = \cancel{\left(-7.9488\,N/C\right)\hat{i}}$$

$$= \left(-1.728\times10^{-10}\,N/C\right)\hat{i} - \left(4.608\times10^{-11}\,N/C\right)\hat{j}.$$

3)



The center of the ~~dect~~ square is $a/\sqrt{2}$ distance away away from the charges.

$$\vec{E_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}\hat{r_1}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{10\times10^{-9}}{a^2/2}\left(+\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E_2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2}\hat{r_2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{-20\times10^{-9}}{a^2/2}\left(-\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E_3} = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3^2}\hat{r_3} = \frac{1}{4\pi\epsilon_0}\frac{20\times10^{-9}}{a^2/2}\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E_4} = \frac{1}{4\pi\epsilon_0}\frac{q_4}{r_4^2}\hat{r_4} = \frac{1}{4\pi\epsilon_0}\frac{-10\times10^{-9}}{a^2/2}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right).$$

$$\therefore \vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4}.$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{2\pi\epsilon_0 a^2}\left(\cancel{\frac{10\times10^{-9}}{\sqrt{2}}\hat{i}} - \frac{10\times10^{-9}}{\sqrt{2}}\hat{j} + \cancel{\frac{20\times10^{-9}}{\sqrt{2}}\hat{i}}\right.$$

$$+ \frac{20\times10^{-9}}{\sqrt{2}}\hat{j} - \cancel{\frac{20\times10^{-9}}{\sqrt{2}}\hat{i}} + \frac{20\times10^{-9}}{\sqrt{2}}\hat{j}$$

$$\left. - \cancel{\frac{10\times10^{-9}}{\sqrt{2}}\hat{i}} - \frac{10\times10^{-9}}{\sqrt{2}}\hat{j}\right)$$

$$= \frac{1}{2\pi\epsilon_0 (5\times10^{-2})^2} \times \frac{20\times10^{-9}}{\sqrt{2}}\hat{j}$$

$$= \left(1.02\times10^{5}\ N/C\right)\hat{j}.$$

4)

Acceleration $a_y = \frac{F}{m} = \frac{QE}{m}$

Vertical and horizontal displacement are,

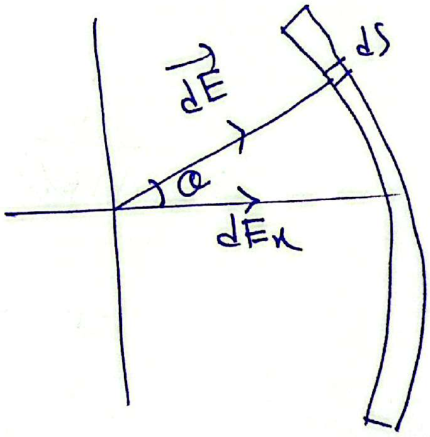
$$y = \frac{1}{2} a_y t^2 \quad , \quad L = V_x t$$

Solving for t we get, $t = \frac{L}{V_x}$

$$\therefore y = \frac{1}{2} \frac{QE}{m} \left(\frac{L}{V_x}\right)^2 = \frac{QEL^2}{2mV_x^2}$$

$$= 0.64 \text{ mm}.$$

5) $E_y = 0$ because of symmetry.



$$dE_x = dE \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda\, ds}{r^2} \cos\theta$$

λ = charge density.

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda\, r d\theta}{r^2} \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda \cos\theta\, d\theta}{r}.$$

$$\therefore E = \int_{-60}^{60} \frac{1}{4\pi\epsilon_0} \frac{\lambda \cos\theta d\theta}{r}$$

$$= \frac{\lambda}{4\pi\epsilon_0 r} \left[-\sin\theta \right]_{-60^\circ}^{60^\circ}$$

$$= \frac{1.73\lambda}{4\pi\epsilon_0 r}$$

$$\therefore \lambda = \frac{Q}{2\pi r/3}$$

$$\therefore E = \frac{0.83\,Q}{4\pi\epsilon_0 r^2} .$$

6]

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{x\sqrt{x^2+a^2}} \hat{i}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{R^2+(L/2)^2}} \hat{i}$$

$$= 12.4 \text{ N/C } \hat{i}$$

$\therefore$

7) $\frac{1}{4\pi\epsilon_0} \frac{q x}{(x^2+R^2)^{3/2}} = E$

In this problem,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q z}{(z^2+R^2)^{3/2}}$$

a) $z = 0$, $E = 0$

b) $E = \frac{1}{4\pi\epsilon_0} \frac{q z}{z^3(1+R^2/z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2(1+R^2/z^2)^{3/2}}$

$z = \infty$ $\quad E = 0.$ as $0 = \frac{1}{\infty}$.

c) $\frac{d}{dz}\left(\frac{q z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}\right) = 0$ [Condition for maximum].

$\Rightarrow \frac{q}{4\pi\epsilon_0} \frac{d}{dz}\left(\frac{z}{(z^2+R^2)^{3/2}}\right) = 0$

$\Rightarrow \frac{d}{dz}\left(\frac{z}{(z^2+R^2)^{3/2}}\right) = 0 \Rightarrow \frac{(z^2+R^2)^{3/2} \cdot 1 - z \cdot \frac{3}{\cancel{2}}(z^2+R^2)^{3/2-1} \cdot \cancel{2}z.}{\left((z^2+R^2)^{3/2}\right)^2} = 0$

$$\Rightarrow \frac{(z^2+R^2)^{3/2}\left\{1 - \frac{3z^2}{(z^2+R^2)}\right\}}{\left((z^2+R^2)^{3/2}\right)^2} = 0$$

$\Rightarrow$

$$\Rightarrow \quad 1 - \frac{z^2}{z^2+R^2} = 0$$

$$\Rightarrow \quad \frac{z^2+R^2-3z^2}{z^2+R^2} = 0$$

$$\Rightarrow \quad R^2 - 2z^2 = 0$$

$$\Rightarrow \quad z = \frac{R}{\sqrt{2}}.$$

d) $E_{max} = \dfrac{1}{4\pi\epsilon_0} \dfrac{qz}{(z^2+R^2)^{3/2}} \quad \left[z = R/\sqrt{2} \right]$

$$= 3.46 \times 10^7 \ N/c.$$