••7 **SSM ILW WWW** In Fig. 22-35, the four particles form a square of edge length $a = 5.00$ cm and have charges $q_1 = +10.0$ nC, $q_2 = -20.0$ nC, $q_3 = +20.0$ nC, and $q_4 = -10.0$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center?

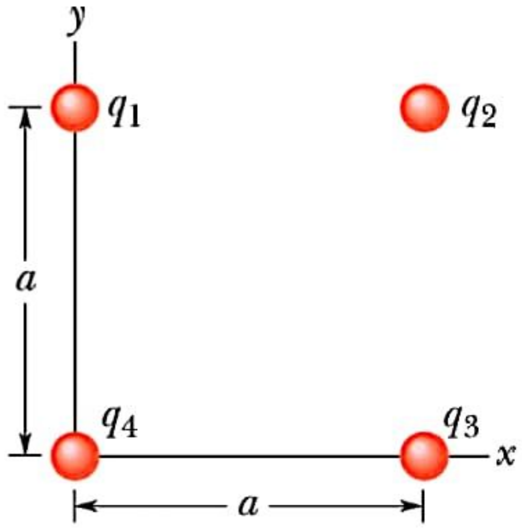


Figure 22-35 Problem 7.

••8 **GO** In Fig. 22-36, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu$m. What is the magnitude of the net electric field at point P due to the particles?

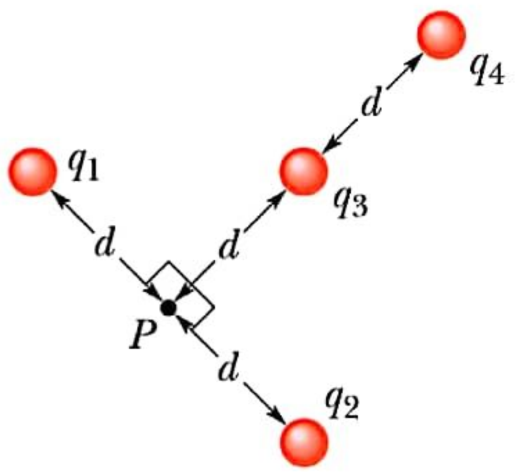


Figure 22-36 Problem 8.

••9 **GO** Figure 22-37 shows two charged particles on an x axis: $-q = -3.20 \times 10^{-19}$ C at $x = -3.00$ m and $q = 3.20 \times 10^{-19}$ C at $x = +3.00$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = 4.00$ m?

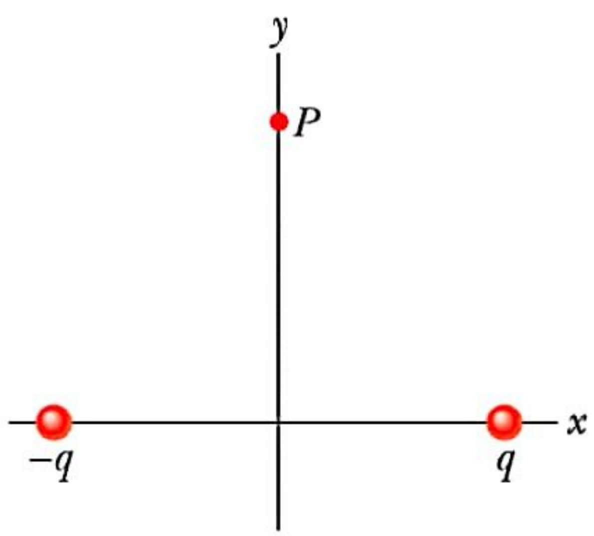


Figure 22-37 Problem 9.

7. **THINK** Our system consists of four point charges that are placed at the corner of a square. The total electric field at a point is the vector sum of the electric fields of individual charges.

EXPRESS Applying the superposition principle, the net electric field at the center of the square is

$$\vec{E} = \sum_{i=1}^{4} \vec{E}_i = \sum_{i=1}^{4} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{\mathrm{r}}_i .$$

With $q_1 = +10$ nC, $q_2 = -20$ nC, $q_3 = +20$ nC, and $q_4 = -10$ nC, the x component of the electric field at the center of the square is given by, taking the signs of the charges into consideration,

$$\begin{aligned} E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} - \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}(|q_1| + |q_2| - |q_3| - |q_4|)\frac{1}{\sqrt{2}}. \end{aligned}$$

Similarly, the y component of the electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} + \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}(-|q_1| + |q_2| + |q_3| - |q_4|)\frac{1}{\sqrt{2}}. \end{aligned}$$

The magnitude of the net electric field is $E = \sqrt{E_x^2 + E_y^2}$.

ANALYZE Substituting the values given, we obtain

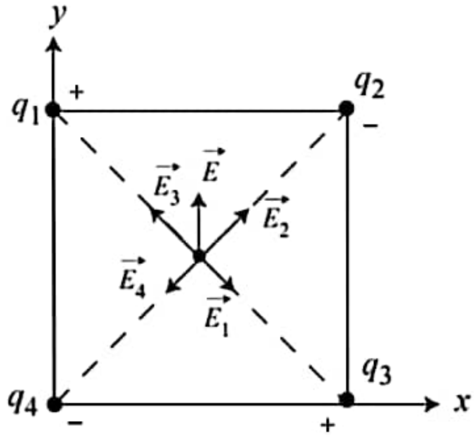
$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}(|q_1|+|q_2|-|q_3|-|q_4|) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}(10\text{ nC}+20\text{ nC}-20\text{ nC}-10\text{ nC}) = 0$$

and

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}(-|q_1|+|q_2|+|q_3|-|q_4|) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}(-10\text{ nC}+20\text{ nC}+20\text{ nC}-10\text{ nC}) \\ &= \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-8}\text{ C})\sqrt{2}}{(0.050\text{ m})^2} \\ &= 1.02\times10^5\text{ N/C}.\end{aligned}$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\text{ N/C})\hat{\text{j}}$.

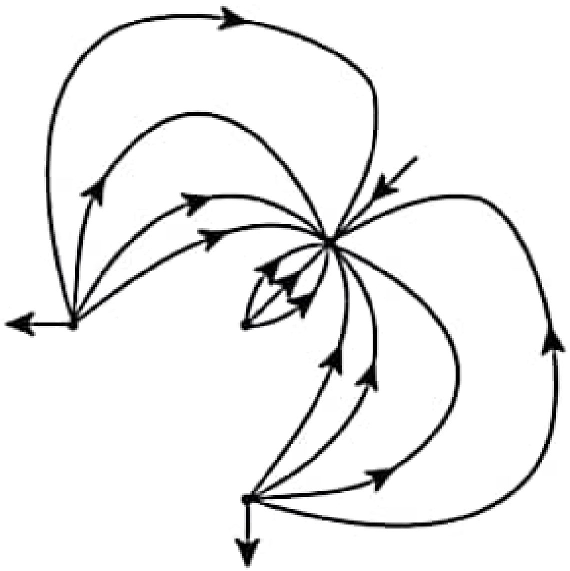
LEARN The net electric field at the center of the square is depicted in the figure below (not to scale). The field, pointing to the $+y$ direction, is the vector sum of the electric fields of individual charges.



8. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\text{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\text{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown next:



9. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using $d = 3.00$ m and $y = 4.00$ m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.00\ \text{m})}{[(3.00\ \text{m})^2+(4.00\ \text{m})^2]^{3/2}}$$

$$= 1.38\times10^{-10}\ \text{N/C}\ .$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

••15 In Fig. 22-42, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\ \mu\text{m}$. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?

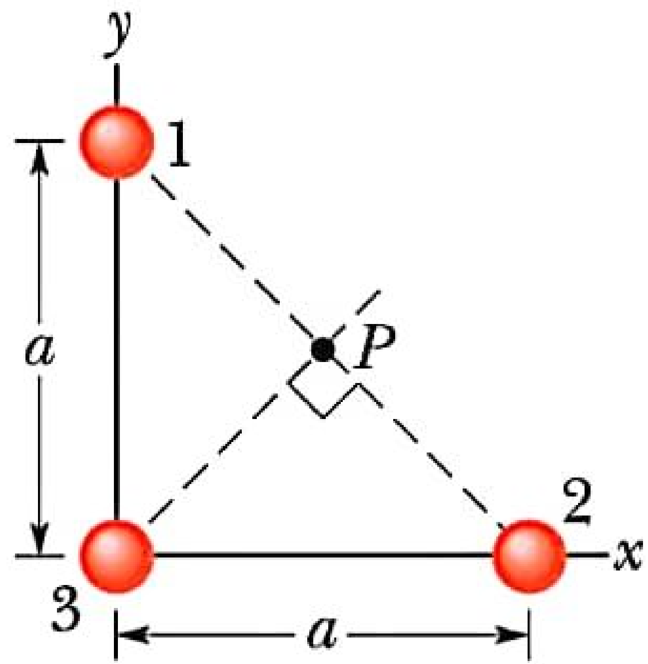


Figure 22-42 Problem 15.

15. By symmetry we see that the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\,\text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

[redacted]

(b) This field points at 45.0°, counterclockwise from the x axis.

••24 A thin nonconducting rod with a uniform distribution of positive charge Q is bent into a complete circle of radius R

(Fig. 22-48). The central perpendicular axis through the ring is a z axis, with the origin at the center of the ring. What is the magnitude of the electric field due to the rod at (a) $z = 0$ and (b) $z = \infty$? (c) In terms of R, at what positive value of z is that magnitude maximum? (d) If $R = 2.00$ cm and $Q = 4.00\ \mu\text{C}$, what is the maximum magnitude?

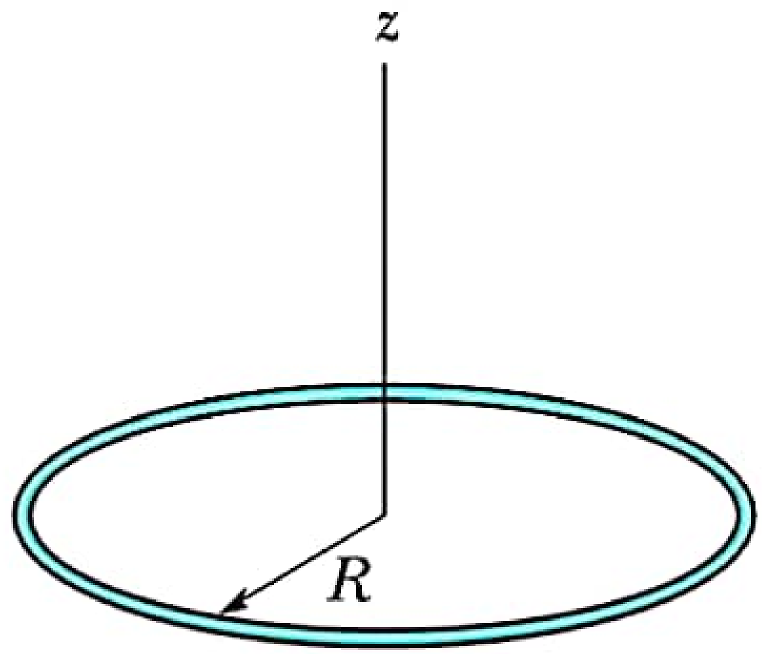


Figure 22-48 Problem 24.

y

24. (a) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center.

(b) The result ($E = 0$) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z^2$ as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r^2$ at distant points).

(c) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right)=\frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}}=0 \;\Rightarrow\; z=+\frac{R}{\sqrt{2}}=0.707R\,.$$

(d) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{\max} = 3.46 \times 10^7$ N/C.

•••32 GO In Fig. 22-55, positive charge $q = 7.81$ pC is spread uniformly along a thin nonconducting rod of length $L = 14.5$ cm. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at point P, at distance $R = 6.00$ cm from the rod along its perpendicular bisector?

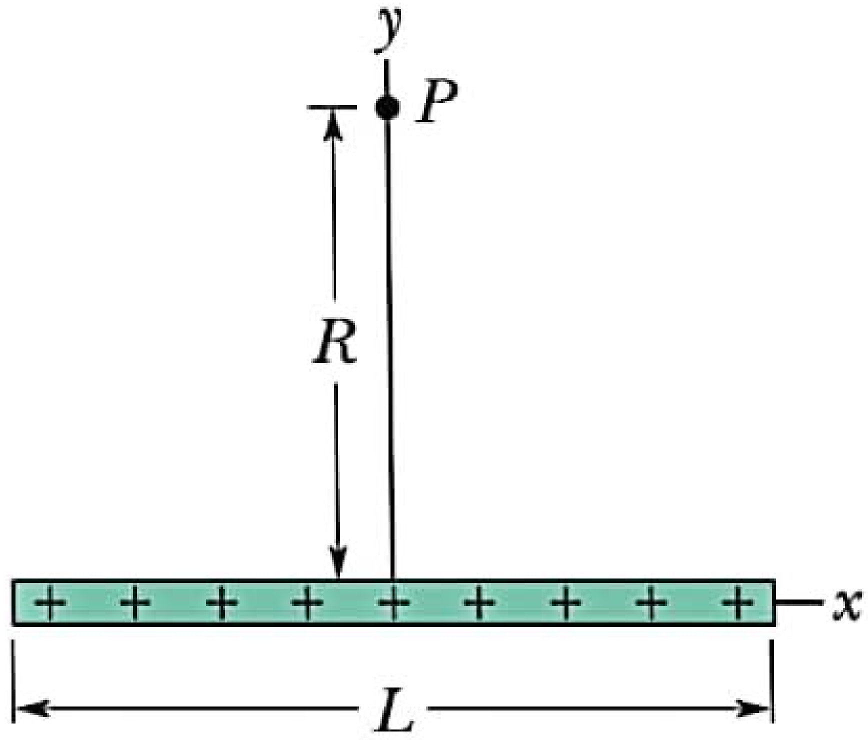


Figure 22-55 Problem 32.

32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

$$|\vec{E}| = 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\, dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right)$$

$$= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2}$$

$$= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C, $L = 0.145$ m, and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C.

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.

•39 In Millikan's experiment, an oil drop of radius 1.64 μm and density 0.851 g/cm^3 is suspended in chamber C (Fig. 22-16) when a downward electric field of 1.92×10^5 N/C is applied. Find the charge on the drop, in terms of e.

39. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$q = -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi\left(1.64\times10^{-6}\,\mathrm{m}\right)^3\left(851\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)}{3\left(1.92\times10^{5}\,\mathrm{N/C}\right)} = -8.0\times10^{-19}\,\mathrm{C}$$

and $q/e = (-8.0 \times 10^{-19}\ \mathrm{C})/(1.60 \times 10^{-19}\ \mathrm{C}) = -5$, or $q = -5e$.

•43 SSM An electron is released from rest in a uniform electric field of magnitude 2.00×10^4 N/C. Calculate the acceleration of the electron. (Ignore gravitation.)

43. **THINK** The acceleration of the electron is given by Newton's second law: $F = ma$, where F is the electrostatic force.

EXPRESS The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. Using Newton's second law, the acceleration of the electron is

$$a = \frac{F}{m} = \frac{eE}{m}.$$

ANALYZE With $e = 1.6 \times 10^{-19}$ C, $E = 2.00 \times 10^{4}$ N/C, and $m = 9.11 \times 10^{-31}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{4}\ \text{N/C}\right)}{9.11 \times 10^{-31}\ \text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2.$$

LEARN In vector notation, $\vec{a} = \vec{F}/m = -e\vec{E}/m$, so $\vec{a}$ is in the opposite direction of $\vec{E}$. The magnitude of electron's acceleration is proportional to the field strength E: the greater the value of E, the greater the acceleration.

••49 A 10.0 g block with a charge of $+8.00 \times 10^{-5}$ C is placed in an electric field $\vec{E} = (3000\hat{\mathrm{i}} - 600\hat{\mathrm{j}})$ N/C. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electrostatic force on the block? If the block is released from rest at the origin at time $t = 0$, what are its (c) x and (d) y coordinates at $t = 3.00$ s?

49. (a) Using Eq. 22-28, we find

$$\begin{aligned}\vec{F} &= \left(8.00\times10^{-5}\,\text{C}\right)\left(3.00\times10^{3}\,\text{N/C}\right)\hat{\text{i}} + \left(8.00\times10^{-5}\,\text{C}\right)\left(-600\,\text{N/C}\right)\hat{\text{j}}\\ &= (0.240\text{ N})\hat{\text{i}} - (0.0480\text{ N})\hat{\text{j}}.\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.240\text{ N})^2 + (-0.0480\text{ N})^2} = 0.245\text{ N}.$$

(b) The angle the force $\vec{F}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\text{ N}}{0.240\text{ N}}\right) = -11.3°$$

measured counterclockwise from the $+x$ axis.

(c) With m = 0.0100 kg, the (x, y) coordinates at t = 3.00 s can be found by combining Newton's second law with the kinematics equations of Chapters 2–4. The x coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240\text{ N})(3.00\text{ s})^2}{2(0.0100\text{ kg})} = 108\text{ m}.$$

(d) Similarly, the y coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.0480\text{ N})(3.00\text{ s})^2}{2(0.0100\text{ kg})} = -21.6\text{ m}.$$

64 Three particles, each with positive charge Q, form an equilateral triangle, with each side of length d. What is the magnitude of the electric field produced by the particles at the midpoint of any side?

64. The two closest charges produce fields at the midpoint that cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2} = \frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2}.$$

87 In Fig. 22-69, particle 1 of charge $q_1 = 1.00$ pC and particle 2 of charge $q_2 = -2.00$ pC are fixed at a distance $d = 5.00$ cm apart. In unit-vector notation, what is the net electric field at points (a) A, (b) B, and (c) C? (d) Sketch the electric field lines.

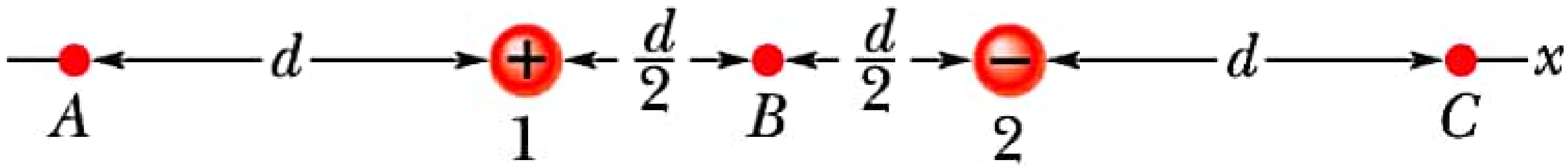


Figure 22-69 Problem 87.

87. (a) For point A, we have (in SI units)

$$\vec{E}_A = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{q_2}{4\pi\varepsilon_0 r_2^2}\right](-\hat{\mathrm{i}}) = \frac{(8.99\times10^9)(1.00\times10^{-12}\,\mathrm{C})}{\left(5.00\times10^{-2}\right)^2}(-\hat{\mathrm{i}}) + \frac{(8.99\times10^9)\,|-2.00\times10^{-12}\,\mathrm{C}|}{\left(2\times5.00\times10^{-2}\right)^2}(+\hat{\mathrm{i}})$$

$$= (-1.80\,\mathrm{N/C})\hat{\mathrm{i}}.$$

(b) Similar considerations leads to

$$\vec{E}_B = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\mathrm{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}} + \frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}$$

$$= (43.2\,\mathrm{N/C})\hat{\mathrm{i}}.$$

(c) For point C, we have

$$\vec{E}_C = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} - \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\mathrm{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}} - \frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}$$

$$= -(6.29\,\mathrm{N/C})\hat{\mathrm{i}}.$$

(d) The field lines are shown to the right. Note that there are twice as many field lines "going into" the negative charge $-2q$ as compared to that flowing out from the positive charge $+q$.