••2 An electric field given by $\vec{E} = 4.0\hat{\text{i}} - 3.0(y^2 + 2.0)\hat{\text{j}}$ pierces a Gaussian cube of edge length 2.0 m and positioned as shown in Fig. 23-7. (The magnitude E is in newtons per coulomb and the position x is in meters.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?

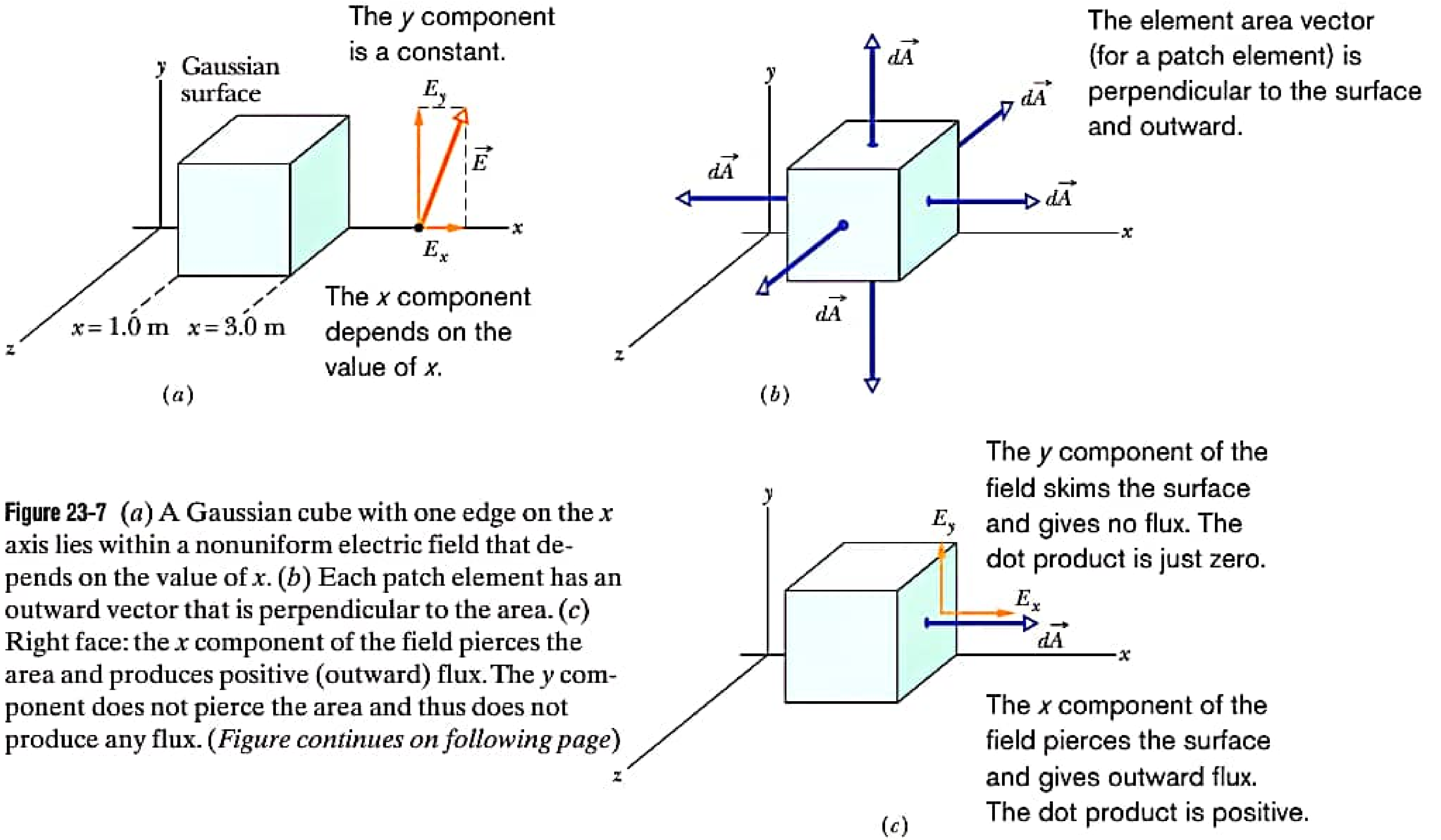


Figure 23-7 (*a*) A Gaussian cube with one edge on the x axis lies within a nonuniform electric field that depends on the value of x. (*b*) Each patch element has an outward vector that is perpendicular to the area. (*c*) Right face: the x component of the field pierces the area and produces positive (outward) flux. The y component does not pierce the area and thus does not produce any flux. (*Figure continues on following page*)

2. We use $\Phi = \oint \vec{E} \cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube $y = 2.0$ m and $d\vec{A} = (dA)\hat{\mathrm{j}}$. Therefore, we have $\vec{E} = 4\hat{\mathrm{i}} - 3\left((2.0)^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}\right) \cdot (dA)\hat{\mathrm{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2\,\mathrm{N \cdot m^2/C} = -72\,\mathrm{N \cdot m^2/C}.$$

(b) On the bottom face of the cube $y = 0$ and $d\vec{A} = (dA)\left(-\hat{\mathrm{j}}\right)$. Therefore, we have $E = 4\hat{\mathrm{i}} - 3\left(0^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2\,\mathrm{N \cdot m^2/C} = +24\,\mathrm{N \cdot m^2/C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{i}}\right)$. So

$$\Phi = \int_{\text{left}} \hat{E} \cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\mathrm{i}} + E_y\hat{\mathrm{j}}\right) \cdot (dA)\left(-\hat{\mathrm{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2\,\mathrm{N \cdot m^2/C} = -16\,\mathrm{N \cdot m^2/C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{k}}\right)$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 $\mathrm{N \cdot m^2/C}$. Thus the net flux through the cube is

$$\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\,\mathrm{N \cdot m^2/C} = -48\,\mathrm{N \cdot m^2/C}.$$

••3 The cube in Fig. 23-31 has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$, and (c) $-3.00\hat{i} + 4.00\hat{k}$. (d) What is the total flux through the cube for each field?

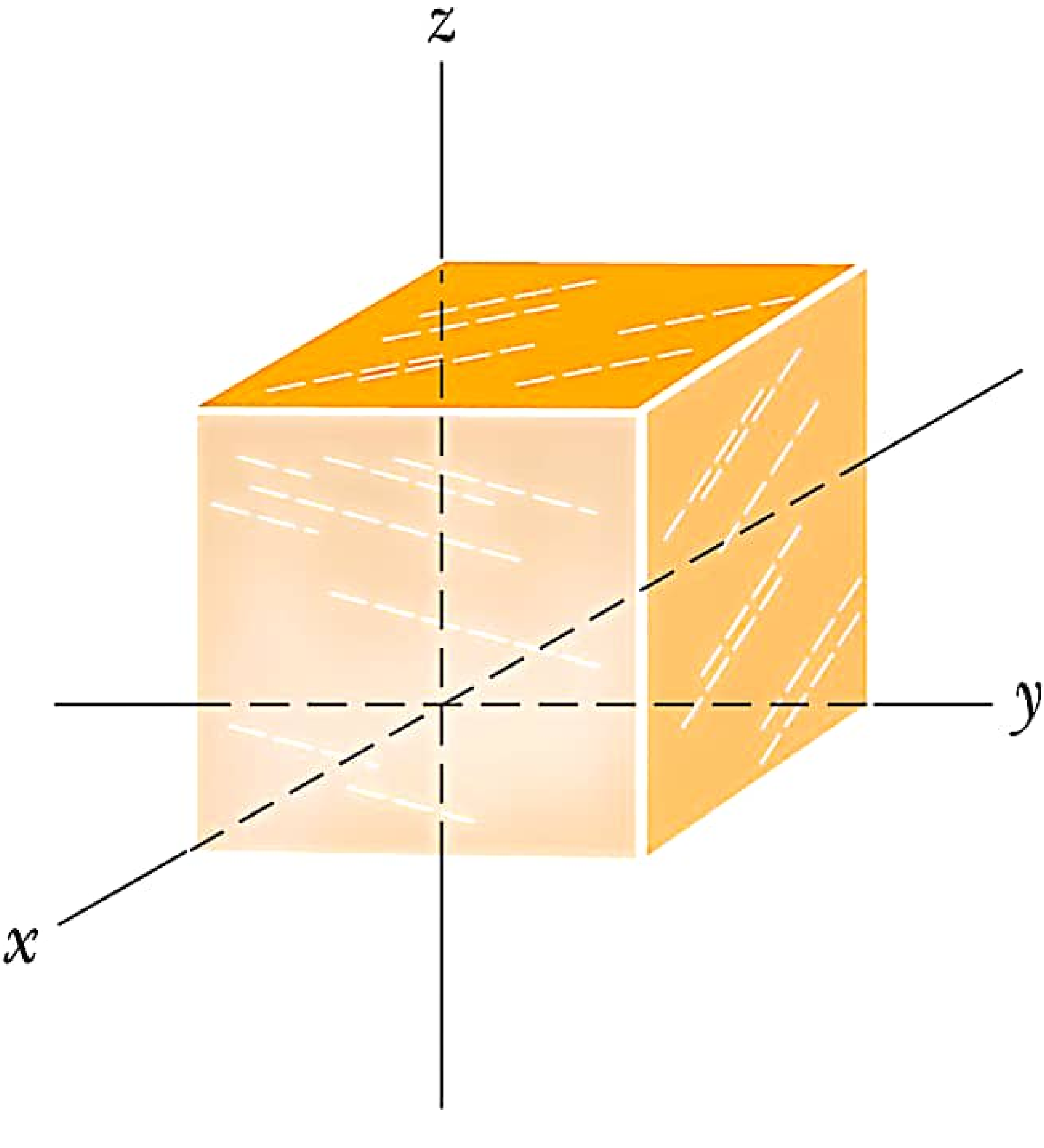


Figure 23-31 Problems 3, 6, and 9.

3. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\mathrm{j}} = \left(1.40\mathrm{m}\right)^2 \hat{\mathrm{j}}$.

(a) $\Phi = \left(6.00 \ \mathrm{N/C}\right)\hat{\mathrm{i}} \cdot \left(1.40 \ \mathrm{m}\right)^2 \hat{\mathrm{j}} = 0.$

(b) $\Phi = \left(-2.00 \ \mathrm{N/C}\right)\hat{\mathrm{j}} \cdot \left(1.40 \ \mathrm{m}\right)^2 \hat{\mathrm{j}} = -3.92 \ \mathrm{N \cdot m^2/C}.$

(c) $\Phi = \left[\left(-3.00 \ \mathrm{N/C}\right)\hat{\mathrm{i}} + \left(400 \, \mathrm{N/C}\right)\hat{\mathrm{k}}\right] \cdot \left(1.40 \ \mathrm{m}\right)^2 \hat{\mathrm{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

•6 At each point on the surface of the cube shown in Fig. 23-31, the electric field is parallel to the z axis. The length of each edge of the cube is 3.0 m. On the top face of the cube the field is

$\vec{E} = -34\hat{k}$ N/C, and on the bottom face it is $\vec{E} = +20\hat{k}$ N/C. Determine the net charge contained within the cube.

6. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude

$(20)(3.0)^2$). With "inward" flux being negative, the result is $\Phi = -\ 486\ \text{N}\cdot\text{m}^2/\text{C}$. Gauss' law then leads to

$$q_{\text{enc}} = \varepsilon_0 \Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-486\ \ \text{N}\cdot\text{m}^2/\text{C}) = -4.3\times10^{-9}\,\text{C}.$$

••9 ILW Fig. 23-31 shows a Gaussian surface in the shape of a cube with edge length 1.40 m. What are (a) the net flux Φ through the surface and (b) the net charge q_{enc} enclosed by the surface if $\vec{E} = (3.00y\hat{j})$ N/C, with y in meters? What are (c) Φ and (d) q_{enc} if $\vec{E} = [-4.00\hat{i} + (6.00 + 3.00y)\hat{j}]$ N/C?

9. (a) Let $A = (1.40\text{ m})^2$. Then

$$\Phi = \left(3.00y\,\hat{\mathrm{j}}\right)\cdot\left(-A\,\hat{\mathrm{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\mathrm{j}}\right)\cdot\left(A\,\hat{\mathrm{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \mathrm{N\cdot m^2/C}.$$

(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(8.23\ \mathrm{N\cdot m^2/C}\right) = 7.29\times10^{-11}\ \mathrm{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\mathrm{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\mathrm{i}} + 6.00\hat{\mathrm{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still $8.23\ \mathrm{N\cdot m^2/C}$.

[redacted]

(d) The charge is again given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(8.23\ \mathrm{N\cdot m^2/C}\right) = 7.29\times10^{-11}\ \mathrm{C}.$$

••10 Figure 23-34 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by $\vec{E} = (3.00x + 4.00)\hat{\mathrm{i}} + 6.00\hat{\mathrm{j}} + 7.00\hat{\mathrm{k}}$ N/C, with x in meters. What is the net charge contained by the cube?

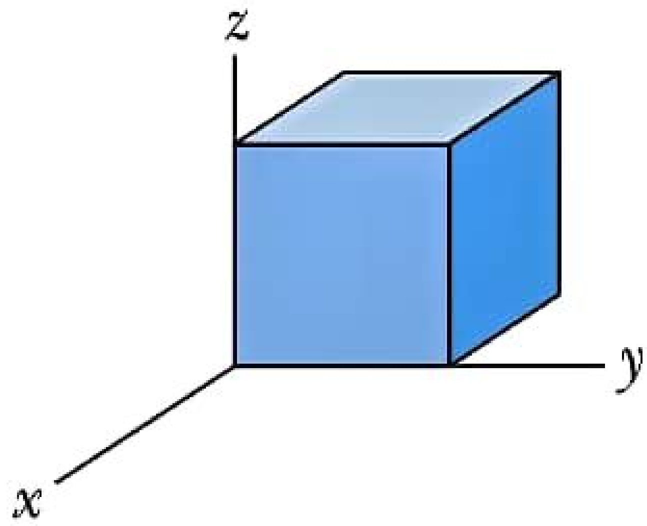


Figure 23-34 Problem 10.

10. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only. In Si units, we have

$$E_{\text{nonconstant}} = 3x\,\hat{\mathrm{i}}\ .$$

The face of the cube located at $x = 0$ (in the yz plane) has area $A = 4\ \mathrm{m}^2$ (and it "faces" the $+\hat{\mathrm{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{nonconstant}}A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area A (and this one "faces" the $-\hat{\mathrm{i}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}}A = -(3)(\,-2)(4) = 24\ \mathrm{N{\cdot}m/C^2}.$$

Thus, the net flux is $\Phi = 0 + 24 = 24\ \mathrm{N{\cdot}m/C^2}$. According to Gauss' law, we therefore have $q_{enc} = \varepsilon_o\,\Phi = 2.13 \times 10^{-10}$ C.

••11 GO Figure 23-35 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vector is given by $\vec{E} = -3.00\hat{i} - 4.00y^2\hat{j} + 3.00\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube?

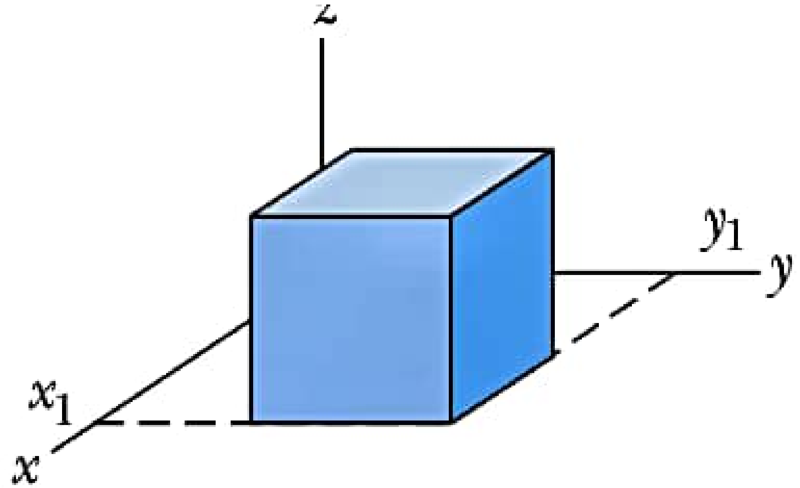


Figure 23-35 Problem 11.

11. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only:

$$E_{\text{nonconstant}} = (-4.00y^2)\ \hat{\text{i}}\ \text{(in SI units)}\ .$$

The face of the cube located at $y = 4.00$ has area $A = 4.00\ \text{m}^2$ (and it "faces" the $+\hat{\text{j}}$ direction) and has a "contribution" to the flux equal to

$$E_{\text{nonconstant}} A = (-4)(4^2)(4) = -256\ \text{N}\cdot\text{m/C}^2.$$

The face of the cube located at $y = 2.00$ m has the same area A (however, this one "faces" the $-\hat{\text{j}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}} A = -\ (-4)(2^2)(4) = 64\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = (-256 + 64)\ \text{N}\cdot\text{m/C}^2 = -192\ \text{N}\cdot\text{m/C}^2$. According to Gauss's law, we therefore have

$$q_{\text{enc}} = \varepsilon_0 \Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-192\ \ \text{N}\cdot\text{m}^2/\text{C}) = -1.70\times10^{-9}\,\text{C}.$$

•24 Figure 23-40 shows a section of a long, thin-walled metal tube of radius $R = 3.00$ cm, with a charge per unit length of $\lambda = 2.00 \times 10^{-8}$ C/m. What is the magnitude E of the electric field at radial distance (a) $r = R/2.00$ and (b) $r = 2.00R$? (c) Graph E versus r for the range $r = 0$ to $2.00R$.

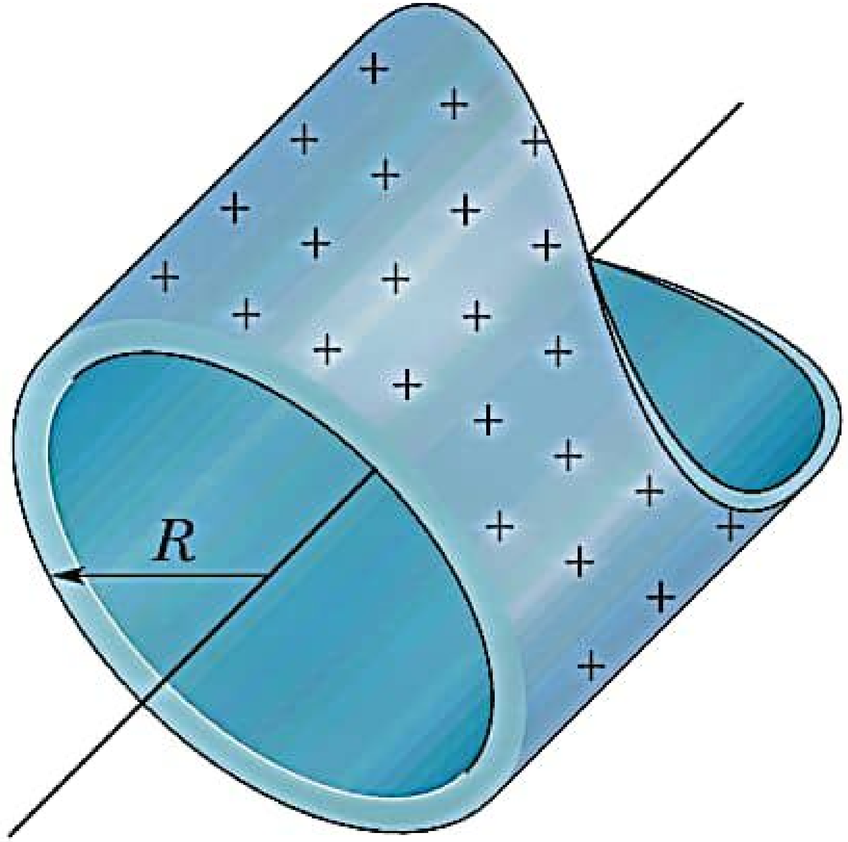


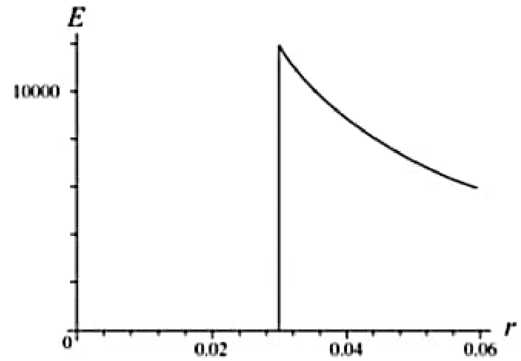
Figure 23-40 Problem 24.

24. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E} \cdot d\vec{A} = 2\pi rE = \frac{q_{enc}}{\varepsilon_0}$.

(a) For $r < R$, $q_{enc} = 0$, so $E = 0$.

(b) For $r > R$, $q_{enc} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R = 0.0600$ m, we obtain

$$E = \frac{(2.0\times10^{-8}\ \text{C/m})}{2\pi(0.0600\ \text{m})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.99\times10^3\ \text{N/C}.$$



(c) The plot of E vs. r is shown to the right. Here, the maximum value is

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$$E_{max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{(2.0\times10^{-8}\ \text{C/m})}{2\pi(0.030\ \text{m})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.2\times10^4\ \text{N/C}.$$

•25 SSM An infinite line of charge produces a field of magnitude 4.5×10^4 N/C at distance 2.0 m. Find the linear charge density.

25. **THINK** Our system is an infinitely long line of charge. Since the system possesses cylindrical symmetry, we may apply Gauss' law and take the Gaussian surface to be in the form of a closed cylinder.

EXPRESS We imagine a cylindrical Gaussian surface A of radius r and length h concentric with the metal tube. Then by symmetry,

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi rhE = \frac{q}{\varepsilon_0},$$

where q is the amount of charge enclosed by the Gaussian cylinder. Thus, the magnitude of the electric field produced by a uniformly charged infinite line is

$$E = \frac{q/h}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ is the linear charge density and r is the distance from the line to the point where the field is measured.

ANALYZE Substituting the values given, we have

$$\begin{aligned}\lambda &= 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.5\times10^{4}\ \mathrm{N/C}\right)(2.0\ \mathrm{m})\\ &= 5.0\times10^{-6}\ \mathrm{C/m}.\end{aligned}$$

LEARN Since $\lambda > 0$, the direction of $\vec{E}$ is radially outward from the line of charge. Note that the field varies with r as $E \sim 1/r$, in contrast to the $1/r^2$ dependence due to a point charge.

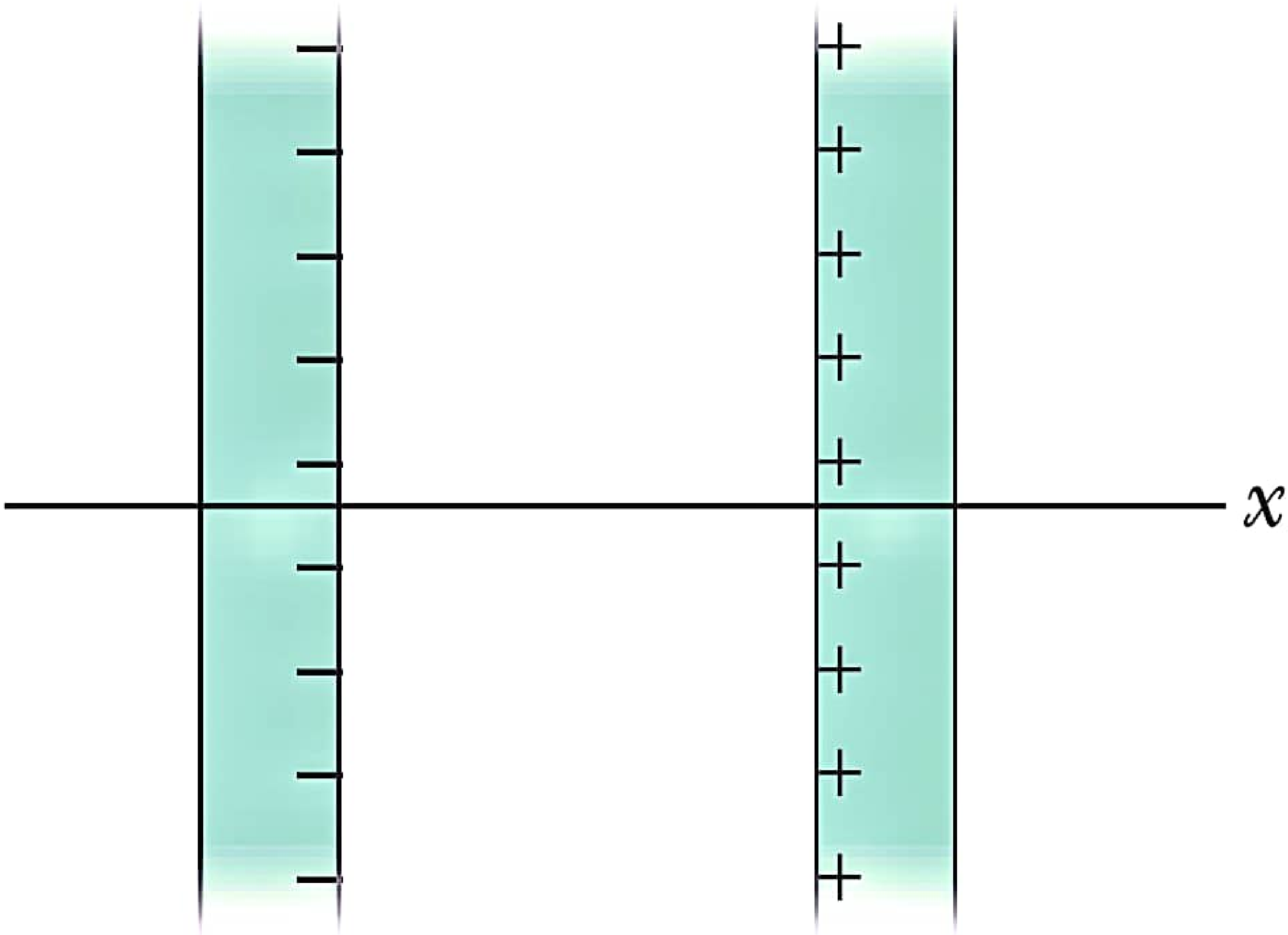


Figure 23-44 Problem 33.

Module 23-5 Applying Gauss' Law: Planar Symmetry

•33 In Fig. 23-44, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge densities of opposite signs and magnitude 7.00×10^{-22} C/m^2. In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

•36 Figure 23-47 shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density $\sigma = 1.77 \times 10^{-22}\ \text{C/m}^2$. In unit-vector notation, what is $\vec{E}$ at points (a) above the sheets, (b) between them, and (c) below them?

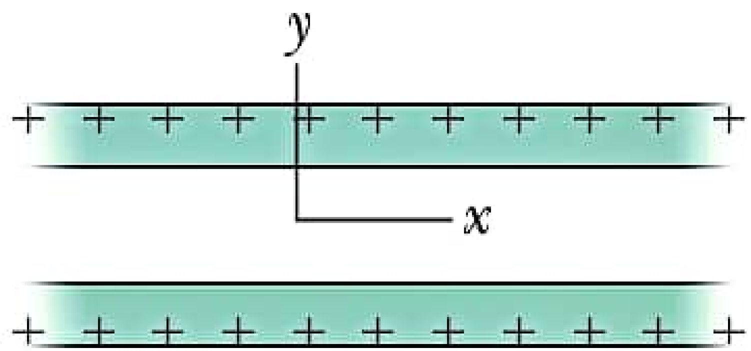


Figure 23-47 Problem 36.

36. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22}$ C/m^2)/(8.85 $\times 10^{-12}$ C^2/N$\cdot$m^2) = 2.00×10^{-11} N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$.

••39 SSM In Fig. 23-49, a small, nonconducting ball of mass $m = 1.0$ mg and charge $q = 2.0 \times 10^{-8}$ C (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30°$ with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

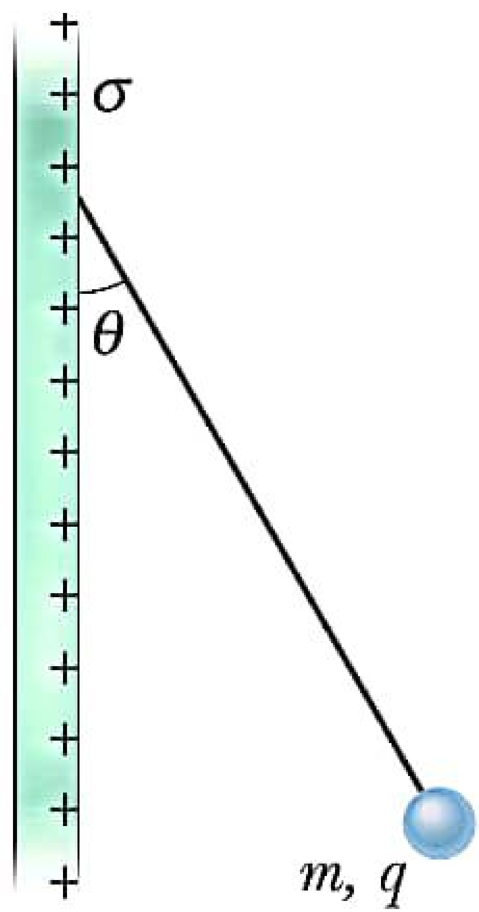
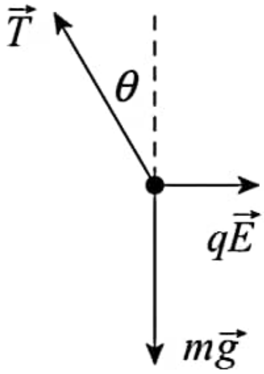


Figure 23-49 Problem 39.

39. **THINK** Since the non-conducting charged ball is in equilibrium with the non-conducting charged sheet (see Fig. 23-49), both the vertical and horizontal components of the net force on the ball must be zero.



EXPRESS The forces acting on the ball are shown in the diagram to the right. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical. Since the ball is in

equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T\sin\theta = 0$$

and the sum of the vertical components yields

$$T\cos\theta - mg = 0 .$$

We solve for the electric field E and deduce σ, the charge density of the sheet, from $E = \sigma/2\varepsilon_0$ (see Eq. 23-13).

ANALYZE The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform sheet of charge is given by $E = \sigma/2\varepsilon_0$, so

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and we have

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(1.0\times10^{-6}\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)\tan 30^\circ}{2.0\times10^{-8}\ \mathrm{C}}$$
$$= 5.0\times10^{-9}\ \mathrm{C/m^2}.$$

LEARN Since both the sheet and the ball are positively charged, the force between them is repulsive. This is balanced by the horizontal component of the tension in the thread. The angle the thread makes with the vertical direction increases with the charge density of the sheet.

69 Figure 23-59 shows, in cross section, three infinitely large nonconducting sheets on which charge is uniformly spread. The surface charge densities are $\sigma_1 = +2.00\ \mu\text{C/m}^2$, $\sigma_2 = +4.00\ \mu\text{C/m}^2$, and $\sigma_3 = -5.00\ \mu\text{C/m}^2$, and distance $L = 1.50$ cm. In unit-vector notation, what is the net electric field at point P?

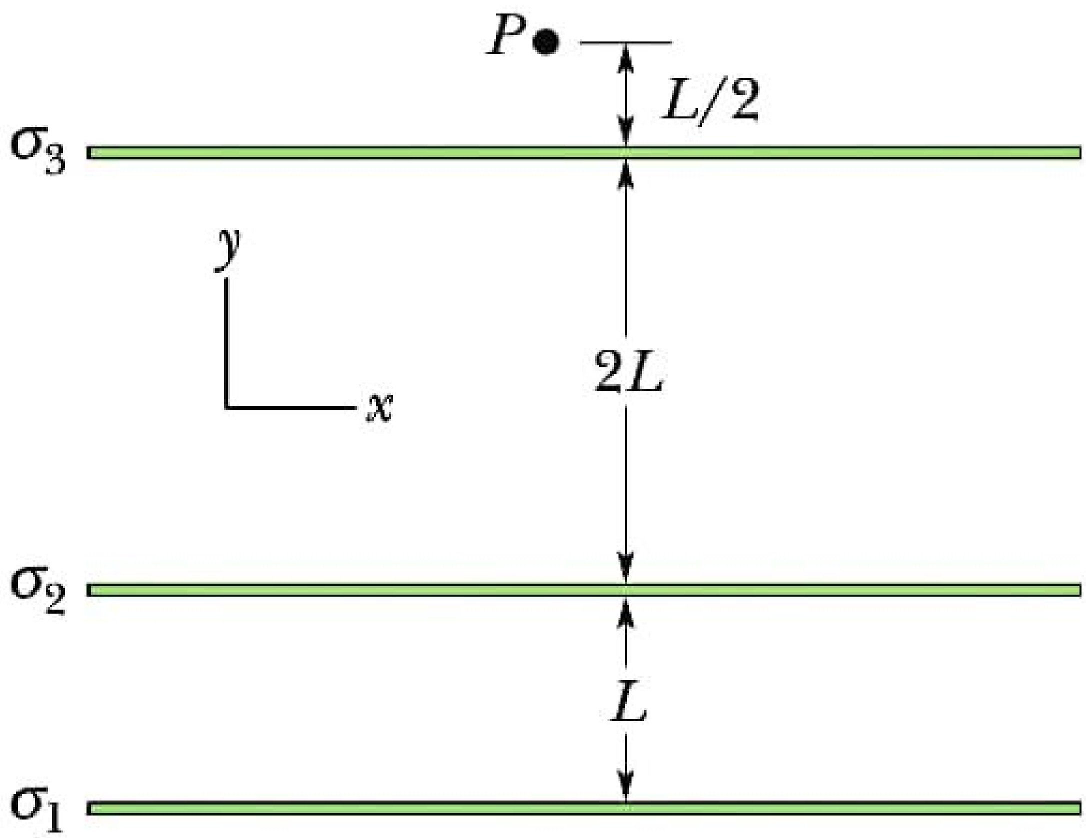


Figure 23-59 Problem 69.

69. Since the fields involved are uniform, the precise location of P is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\mathrm{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}|=\frac{\sigma_1}{2\varepsilon_0}+\frac{\sigma_2}{2\varepsilon_0}+\frac{\sigma_3}{2\varepsilon_0}=\frac{1.0\times10^{-6}\,\mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}=5.65\times10^4\ \mathrm{N/C}.$$

In unit-vector notation, we have $\vec{E}=(5.65\times10^4\ \mathrm{N/C})\hat{\mathrm{j}}$.