•4 Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electric force of 3.9×10^{-15} N acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?

4. (a) $E = F/e = \left(3.9\times10^{-15}\text{ N}\right)/\left(1.60\times10^{-19}\text{ C}\right) = 2.4\times10^{4}\text{ N/C} = 2.4\times10^{4}\text{ V/m.}$

(b) $\Delta V = E\Delta s = \left(2.4\times10^{4}\text{ N/C}\right)\left(0.12\text{ m}\right) = 2.9\times10^{3}\text{ V.}$

•6 When an electron moves from A to B along an electric field line in Fig. 24-34, the electric field does 3.94×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?

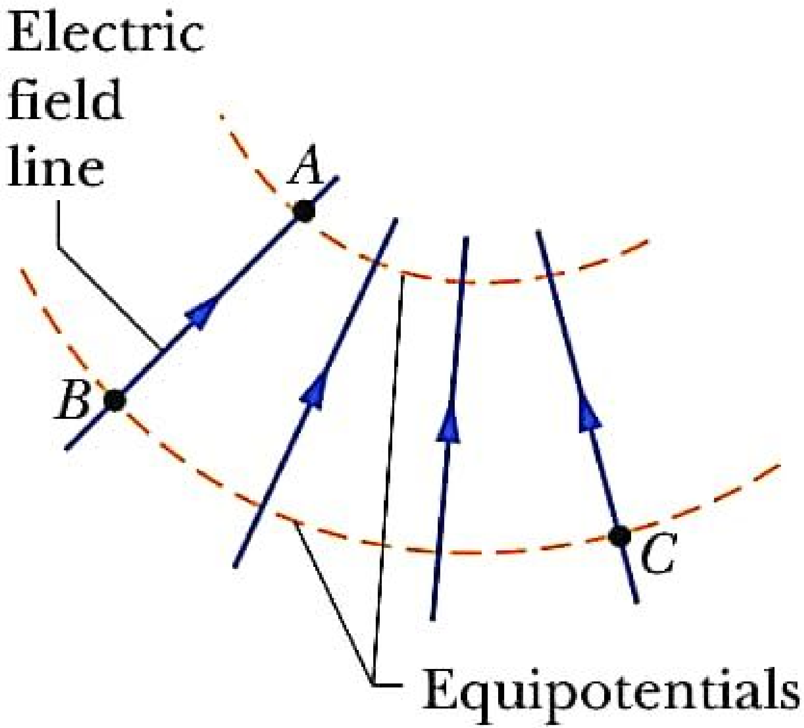

Figure 24-34 Problem 6.

6. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -(3.94 \times 10^{-19}\ \text{J})/(-1.60 \times 10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (since C and B are on the same equipotential line).

••9 An infinite nonconducting sheet has a surface charge density $\sigma = +5.80\ \mathrm{pC/m^2}$. (a) How much work is done by the electric field due to the sheet if a particle of charge $q = +1.60 \times 10^{-19}$ C is moved from the sheet to a point P at distance $d = 3.56$ cm from the sheet? (b) If the electric potential V is defined to be zero on the sheet, what is V at P?

9. (a) The work done by the electric field is

$$W = \int_i^f q_0 \vec{E} \cdot d\vec{s} = \frac{q_0 \sigma}{2\varepsilon_0} \int_0^d dz = \frac{q_0 \sigma d}{2\varepsilon_0} = \frac{(1.60\times10^{-19}\,\mathrm{C})(5.80\times10^{-12}\ \mathrm{C/m^2})(0.0356\ \mathrm{m})}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}$$

$$= 1.87\times10^{-21}\,\mathrm{J}.$$

(b) Since

$$V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0,$$

with V_0 set to be zero on the sheet, the electric potential at P is

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80\times10^{-12}\ \mathrm{C/m^2})(0.0356\ \mathrm{m})}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} = -1.17\times10^{-2}\ \mathrm{V}.$$

•**14** Consider a particle with charge $q = 1.0\ \mu C$, point A at distance $d_1 = 2.0$ m from q, and point B at distance $d_2 = 1.0$ m. (a) If A and B are diametrically opposite each other, as in Fig. 24-36*a*, what is the electric potential difference $V_A - V_B$? (b) What is that electric potential difference if A and B are located as in Fig. 24-36*b*?

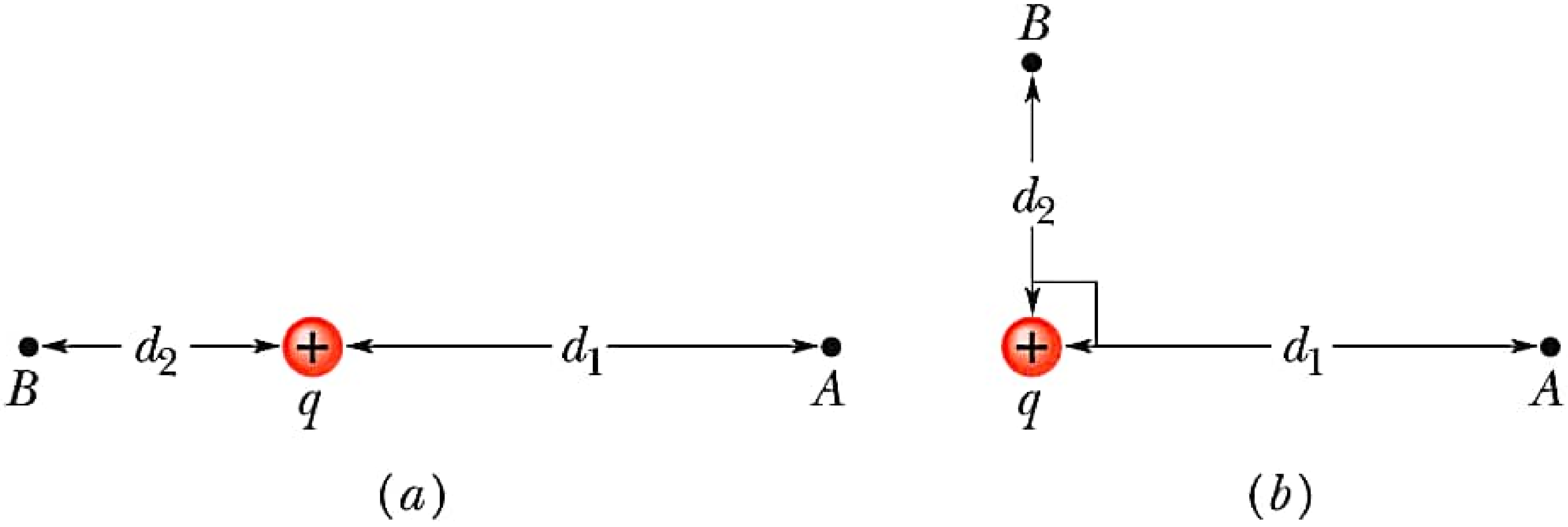

Figure 24-36 Problem 14.

14. (a) The potential difference is

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} = \left(1.0\times10^{-6}\ \mathrm{C}\right)\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{2.0\ \mathrm{m}} - \frac{1}{1.0\ \mathrm{m}}\right)$$
$$= -4.5\times10^{3}\ \mathrm{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

••16 Figure 24-37 shows a rectangular array of charged particles fixed in place, with distance $a = 39.0$ cm and the charges shown as integer multiples of $q_1 = 3.40$ pC and $q_2 = 6.00$ pC. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center? (*Hint:* Thoughtful examination of the arrangement can reduce the calculation.)

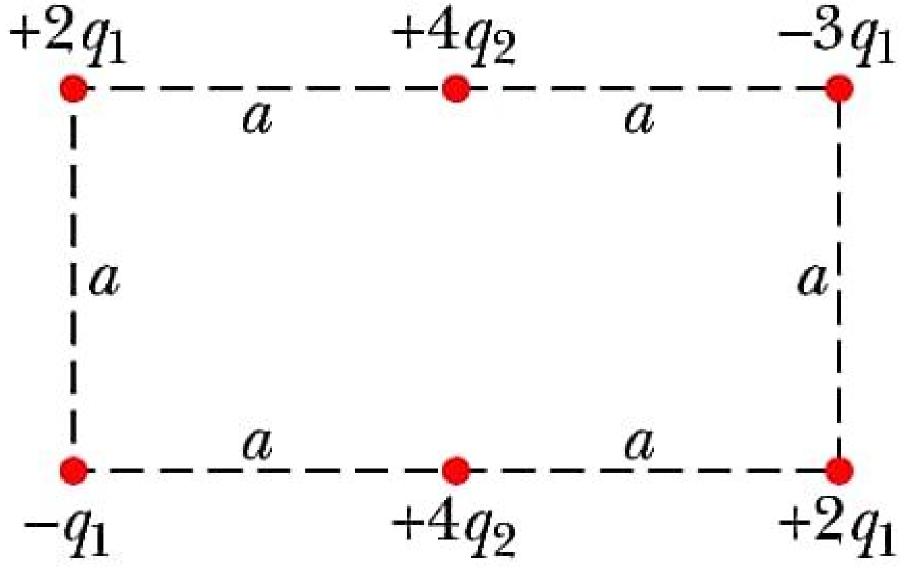

Figure 24-37 Problem 16.

16. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-12}\,\text{C})}{0.39\ \text{m}}$$

$$= 2.21\ \text{V}.$$

••19 In Fig. 24-40, particles with the charges $q_1 = +5e$ and $q_2 = -15e$ are fixed in place with a separation of $d = 24.0$ cm. With electric potential defined to be $V = 0$ at infinity, what are the finite (a) positive and (b) negative values of x at which the net electric potential on the x axis is zero?

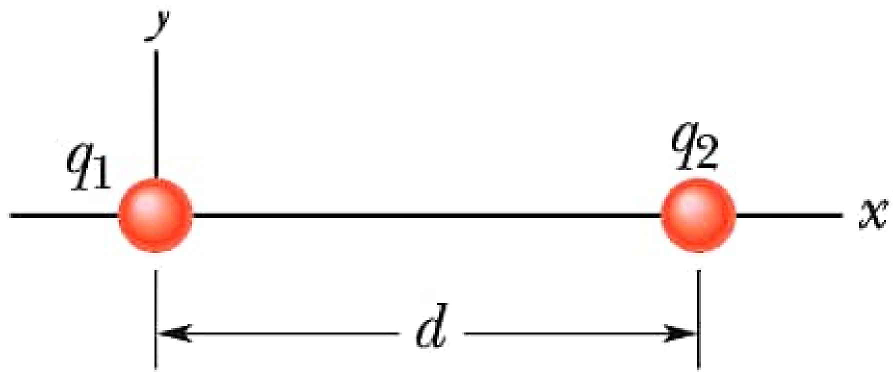

Figure 24-40 Problems 19 and 20.

19. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

Module 24-5 Potential Due to a Continuous Charge Distribution

•23 (a) Figure 24-42*a* shows a nonconducting rod of length $L =$ 6.00 cm and uniform linear charge density $\lambda = +3.68$ pC/m. Assume that the electric potential is defined to be $V = 0$ at infinity. What is V at point P at distance $d = 8.00$ cm along the rod's perpendicular bisector? (b) Figure 24-42*b* shows an identical rod except that one half is now negatively charged. Both halves have a linear charge density of magnitude 3.68 pC/m. With $V = 0$ at infinity, what is V at P?

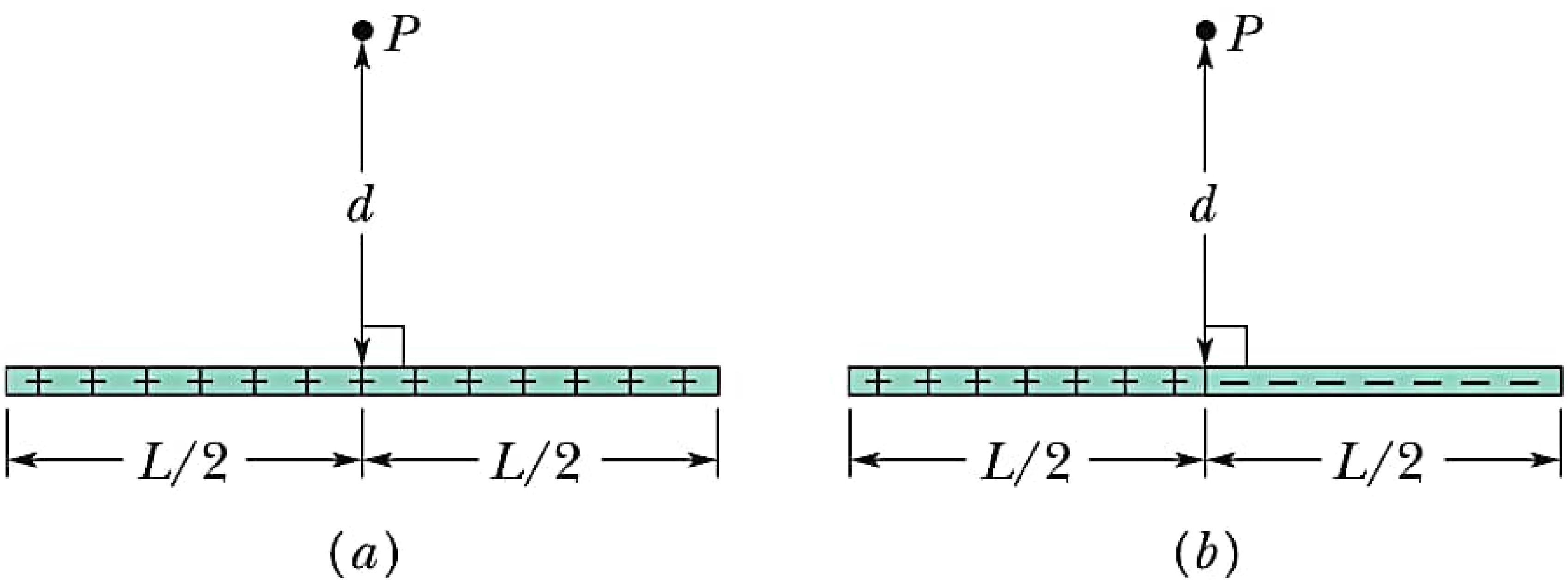

Figure 24-42 Problem 23.

23. (a) From Eq. 24-35, we find the potential to be

$$V = 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2+\sqrt{(L^2/4)+d^2}}{d}\right]$$

$$= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.68\times10^{-12}\,\mathrm{C/m})\ln\left[\frac{(0.06\ \mathrm{m}/2)+\sqrt{(0.06\ \mathrm{m})^2/4+(0.08\ \mathrm{m})^2}}{0.08\ \mathrm{m}}\right]$$

$$= 2.43\times10^{-2}\ \mathrm{V}.$$

(b) The potential at P is $V = 0$ due to superposition.

•24 In Fig. 24-43, a plastic rod having a uniformly distributed charge $Q = -25.6$ pC has been bent into a circular arc of radius $R = 3.71$ cm and central angle $\phi = 120°$. With $V = 0$ at infinity, what is the electric potential at P, the center of curvature of the rod?

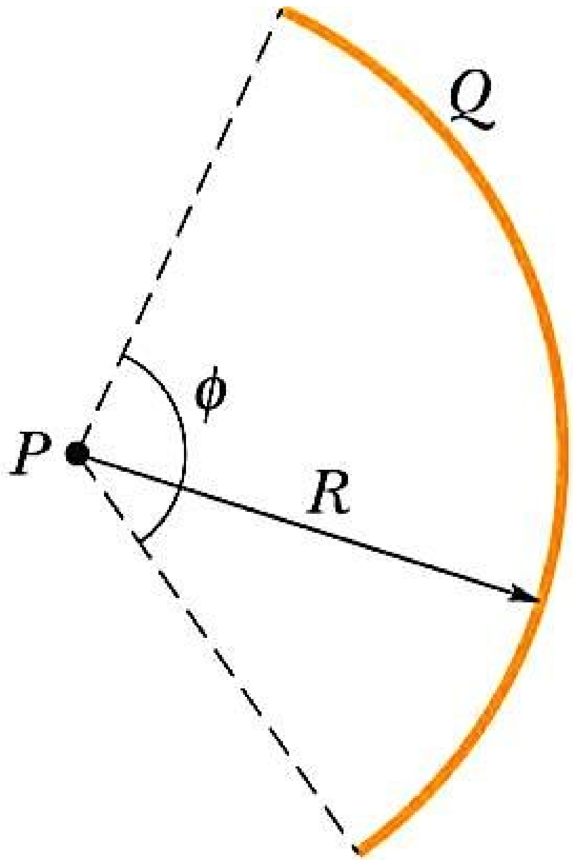

Figure 24-43 Problem 24.

24. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(25.6\times10^{-12}\ \text{C})}{3.71\times10^{-2}\ \text{m}}$$
$$= -6.20\ \text{V}.$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

••26 GO Figure 24-45 shows a thin rod with a uniform charge density of 2.00 μC/m. Evaluate the electric potential at point P if $d = D = L/4.00$. Assume that the potential is zero at infinity.

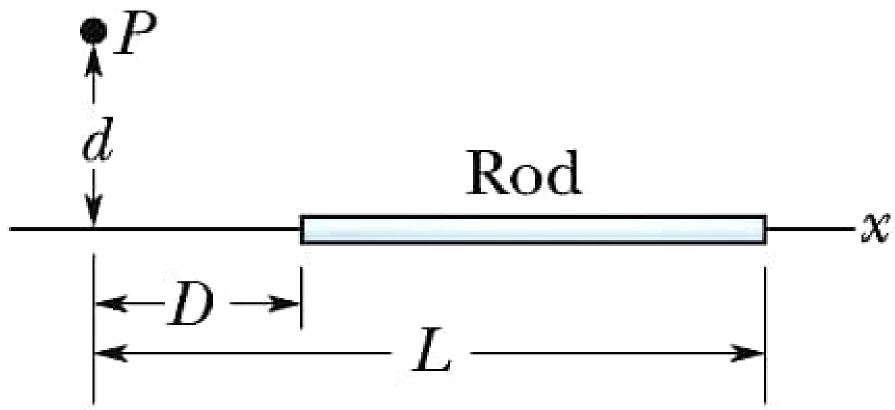

Figure 24-45 Problem 26.

26. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+\sqrt{L^2+d^2}}{D+\sqrt{D^2+d^2}}\right) = \frac{2.0\times10^{-6}}{4\pi\varepsilon_0}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 2.18\times10^4\ \text{V}.$$

••28 GO Figure 24-47 shows a thin plastic rod of length $L = 12.0$ cm and uniform positive charge $Q = 56.1$ fC lying on an x axis. With $V = 0$ at infinity, find the electric potential at point P_1 on the axis, at distance $d = 2.50$ cm from the rod.

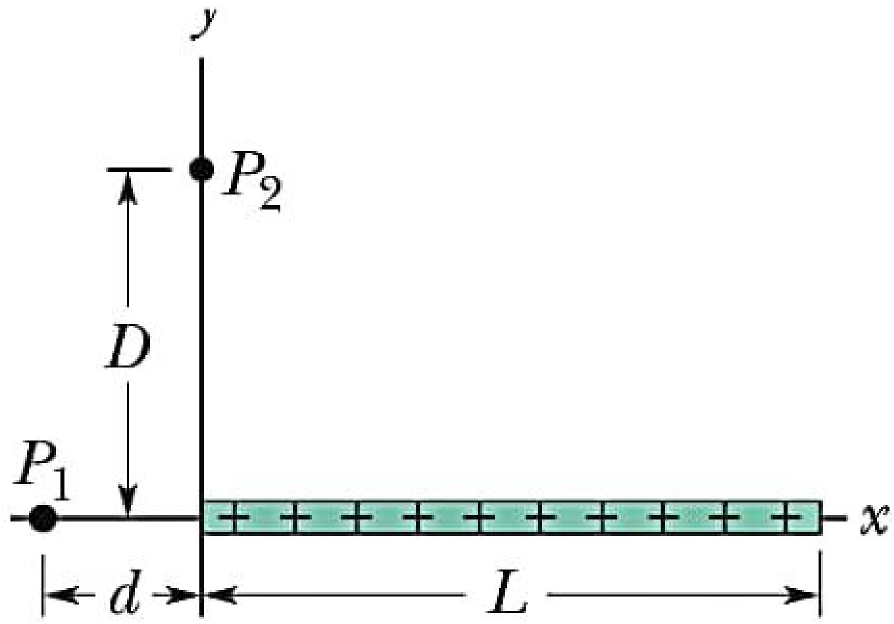

Figure 24-47 Problems 28, 33, 38, and 40.

28. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$

$$= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(56.1\times10^{-15}\ \mathrm{C})}{0.12\ \mathrm{m}}\ln\left(1+\frac{0.12\ \mathrm{m}}{0.025\ \mathrm{m}}\right)$$

$$= 7.39\times10^{-3}\ \mathrm{V}.$$

•43 SSM ILW WWW How much work is required to set up the arrangement of Fig. 24-52 if $q = 2.30$ pC, $a = 64.0$ cm, and the particles are initially infinitely far apart and at rest?

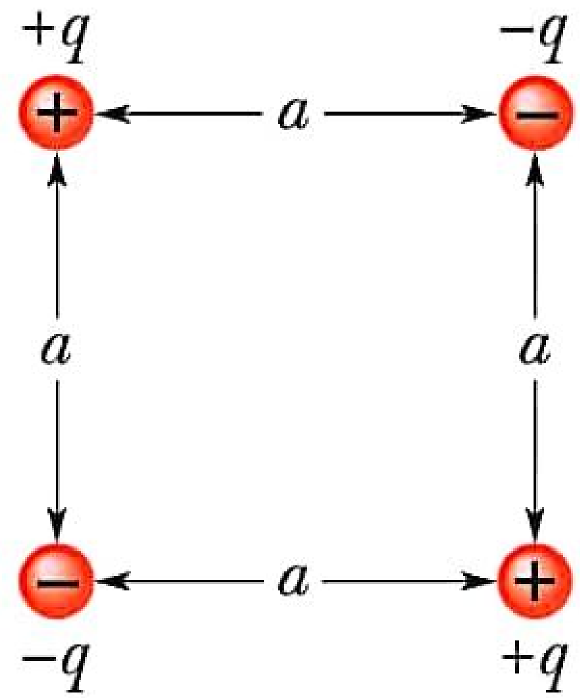

Figure 24-52 Problem 43.

43. **THINK** The work required to set up the arrangement is equal to the potential energy of the system.

EXPRESS We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\text{p}\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\text{p}\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.30\times10^{-12}\ \text{C})^2}{0.640\ \text{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$

$$= -1.92\times10^{-13}\ \text{J}.$$

LEARN The work done in assembling the system is negative. This means that an external agent would have to supply $W_{\text{ext}} = +1.92\times10^{-13}$ J in order to take apart the arrangement completely.

••50 In Fig. 24-54, how much work must we do to bring a particle, of charge $Q = +16e$ and initially at rest, along the dashed line from infinity to the indicated point near two fixed particles of charges $q_1 = +4e$ and $q_2 = -q_1/2$? Distance $d = 1.40$ cm, $\theta_1 = 43°$, and $\theta_2 = 60°$.

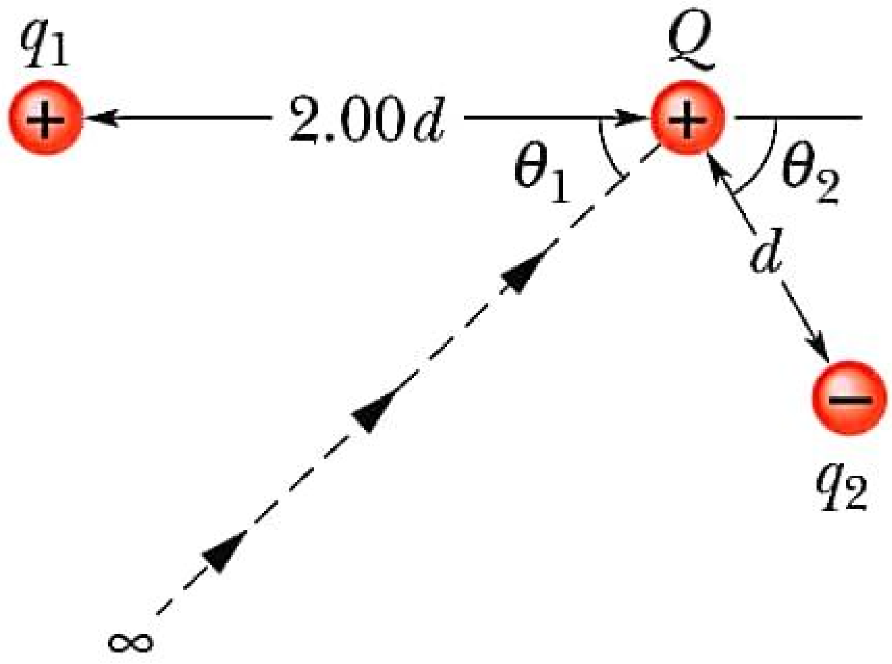

Figure 24-54 Problem 50.

50. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{q_2 Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$

••59 In Fig. 24-60, a charged particle (either an electron or a proton) is moving rightward between two parallel charged plates separated by distance $d = 2.00$ mm. The plate potentials are $V_1 = -70.0$ V and $V_2 = -50.0$ V. The particle is slowing from an initial speed of 90.0 km/s at the left plate. (a) Is the particle an electron or a proton? (b) What is its speed just as it reaches plate 2?

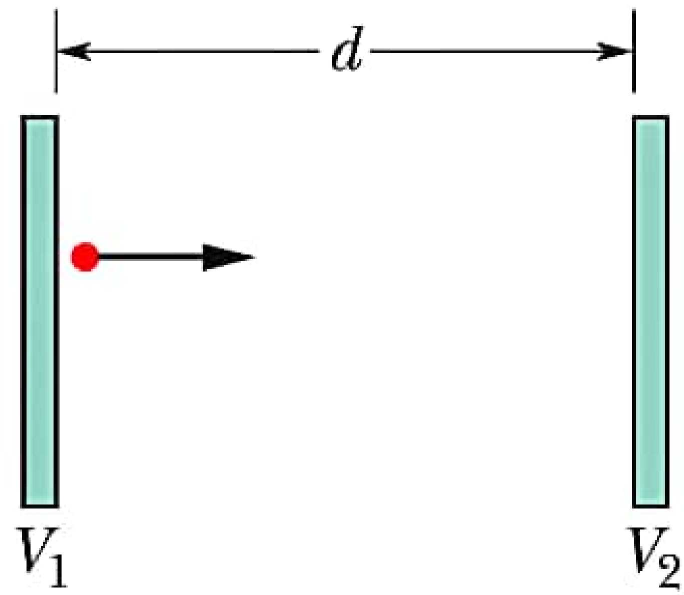

Figure 24-60 Problem 59.

59. (a) The electric field between the plates is leftward in Fig, 24-59 since it points toward lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \Rightarrow \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \ .$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 90 \times 10^3$ m/s, $V_1 = -70$ V, and $V_2 = -50$ V, we obtain the final speed $v = 6.53 \times 10^4$ m/s. We note that the value of d is not used in the solution.

74 Three particles, charge $q_1 = +10\ \mu C$, $q_2 = -20\ \mu C$, and $q_3 = +30\ \mu C$, are positioned at the vertices of an isosceles triangle as shown in Fig. 24-62. If $a = 10$ cm and $b = 6.0$ cm, how much work must an external agent do to exchange the positions of (a) q_1 and q_3 and, instead, (b) q_1 and q_2?

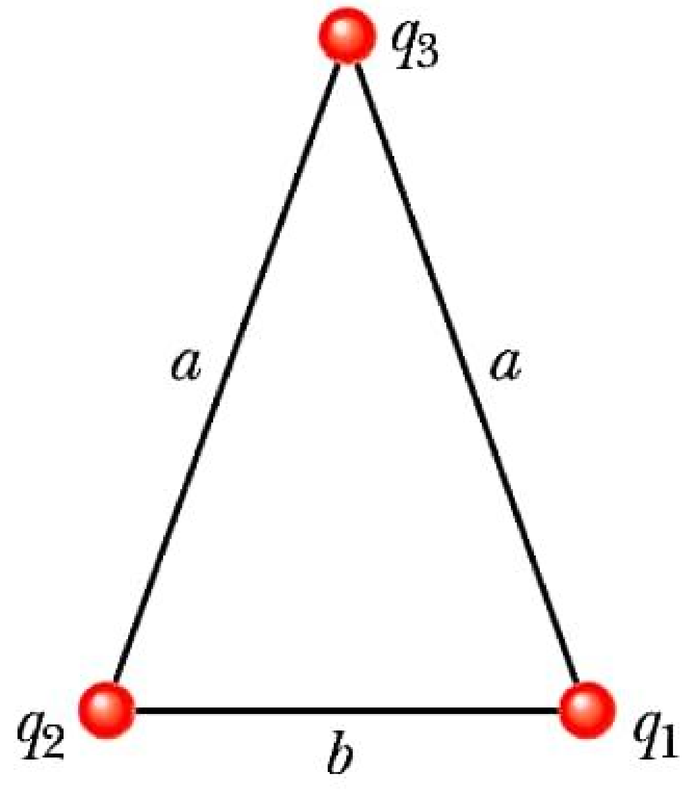

Figure 24-62 Problem 74.

74. The work done is equal to the change in the (total) electric potential energy U of the system, where

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_3 q_2}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

and the notation r_{13} indicates the distance between q_1 and q_3 (similar definitions apply to r_{12} and r_{23}).

(a) We consider the difference in U where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ (r_{13} doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in U where initially $r_{23} = a$ and $r_{13} = a$, and finally r_{23} is again equal to a and r_{13} is also again equal to a (and of course, r_{12} doesn't change in this case). Thus, we obtain $\Delta U = 0$.

83 In Fig. 24-66, point P is at distance $d_1 = 4.00$ m from particle 1 ($q_1 = -2e$) and distance $d_2 = 2.00$ m from particle 2 ($q_2 = +2e$), with both particles fixed in place. (a) With $V = 0$ at infinity, what is V at P? If we bring a particle of charge $q_3 = +2e$ from infinity to P, (b) how much work do we do and (c) what is the potential energy of the three-particle system?

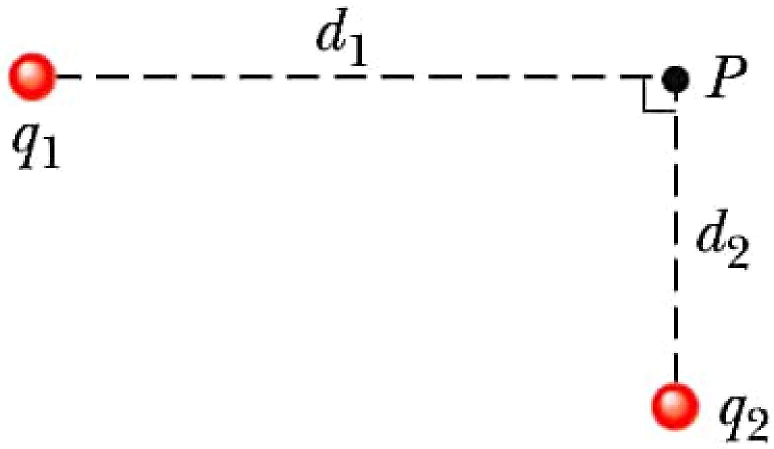

Figure 24-66 Problem 83.

83. (a) Using $d = 2$ m, we find the potential at P:

$$V_P = \frac{2e}{4\pi\varepsilon_0 d} + \frac{-2e}{4\pi\varepsilon_0 (2d)} = \frac{e}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\,\mathrm{C})}{2.00\ \mathrm{m}}$$
$$= 7.192\times10^{-10}\ \mathrm{V}.$$

Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since $U = qV$, then the movable particle's contribution of the potential energy when it is at $r = \infty$ is zero, and its contribution to U_{system} when it is at P is

$$U = qV_p = 2(1.6\times10^{-19}\,\mathrm{C})(7.192\times10^{-10}\ \mathrm{V}) = 2.30\times10^{-28}\ \mathrm{J}.$$

Thus, the work done is approximately equal to $W_{\text{app}} = 2.30 \times 10^{-28}$ J.

(c) Now, combining the contribution to U_{system} from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{(4.00\ \mathrm{m})^2 + (2.00\ \mathrm{m})^2}} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4)(1.60\times10^{-19}\,\mathrm{C})^2}{\sqrt{20.0}\ \mathrm{m}}$$
$$= -2.058\times10^{-28}\ \mathrm{J}$$

we obtain

$$U_{\text{system}} = W_{\text{app}} + U_{\text{fixed}} = 2.43 \times 10^{-29}\ \mathrm{J}.$$

88 Two charges $q = +2.0\ \mu\text{C}$ are fixed a distance $d = 2.0$ cm apart (Fig. 24-69). (a) With $V = 0$ at infinity, what is the electric potential at point C? (b) You bring a third charge $q = +2.0\ \mu\text{C}$ from infinity to C. How much work must you do? (c) What is the potential energy U of the three-charge configuration when the third charge is in place?

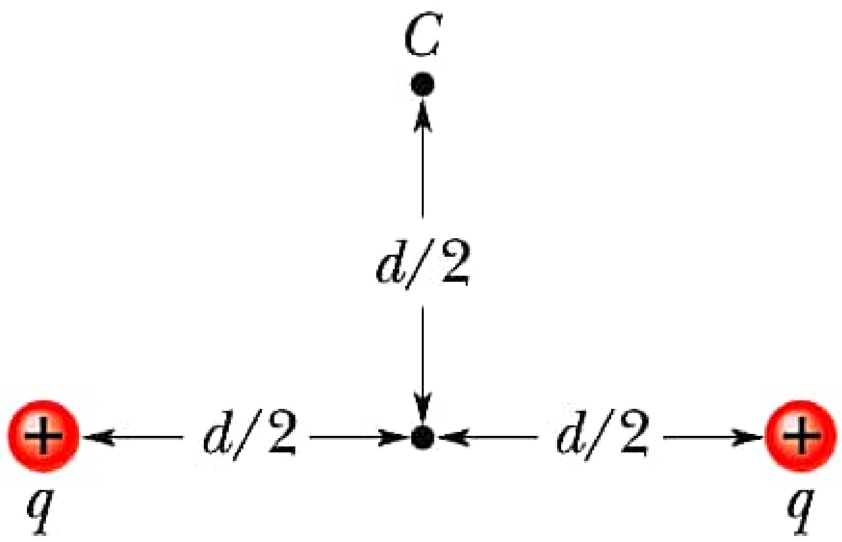

Figure 24-69 Problem 88.

88. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance

$$r=\sqrt{(d/2)^2+(d/2)^2}=d/\sqrt{2}.$$

The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V=\frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d}=\frac{2\sqrt{2}q}{4\pi\varepsilon_0 d}=\frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\mathrm{C}\right)}{0.020\ \mathrm{m}}$$
$$=2.5\times10^{6}\,\mathrm{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W=qV=\left(2.0\times10^{-6}\,\mathrm{C}\right)\left(2.54\times10^{6}\,\mathrm{V}\right)=5.1\ \mathrm{J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U=W+\frac{q^2}{4\pi\varepsilon_0 d}=5.1\ \mathrm{J}+\frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(2.0\times10^{-6}\,\mathrm{C}\right)^2}{0.020\,\mathrm{m}}=6.9\ \mathrm{J}.$$