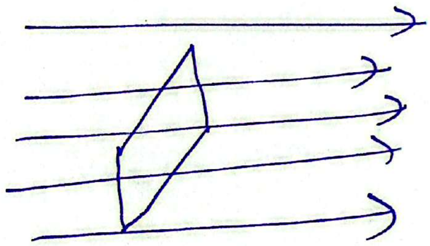
Gauss's Law

* In the previous chapter we derive electric field from charge distribution. Now, our next question is if the electric field is known in a given region, what can we determine about the charge distribution ~~at~~ in ~~that~~ region?

=> Gauss's law can answer that question.

=> To ~~undest~~ understand gauss's law we must know what ~~is~~ flux. is.

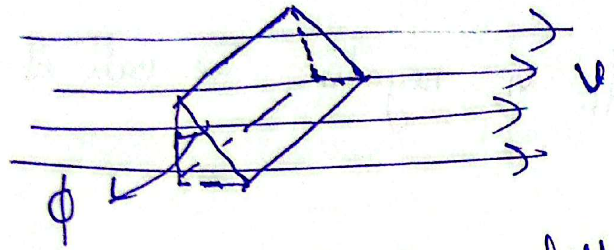
Flux: Fluid - Flow Analogy:



This figure shows a fluid flowing steadily from left to right. And there is a wire rectangle in the fluid which area is perpendicular to the flow velocity $\vec{v}$.

Then the volume flow rate dV/dt through the ~~air~~ wire is given by:

$$\frac{dV}{dt} = vA.$$



When the rectangle is tilted at an ~~ag~~ angle ϕ, then,

$$\frac{dV}{dt} = vA\cos\phi$$
$$= \vec{v}.\vec{A}$$

* This dot product of a vector field with a area is called flux.

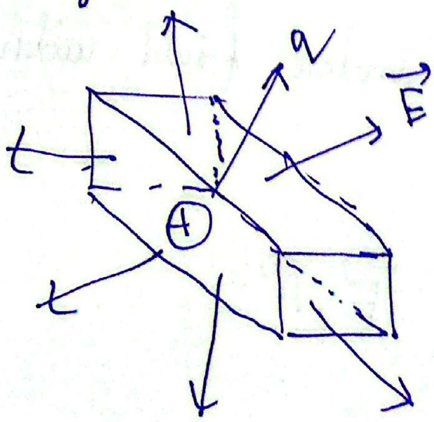
* Electric Flux: $\phi_E = \vec{E}.\vec{A}$

We can represent $\vec{A}$ as $\vec{A} = A\hat{n}$. Where $\hat{n}$ is ~~the~~ a unit vector perpendicular to the area. $\hat{n}$ stands for normal.

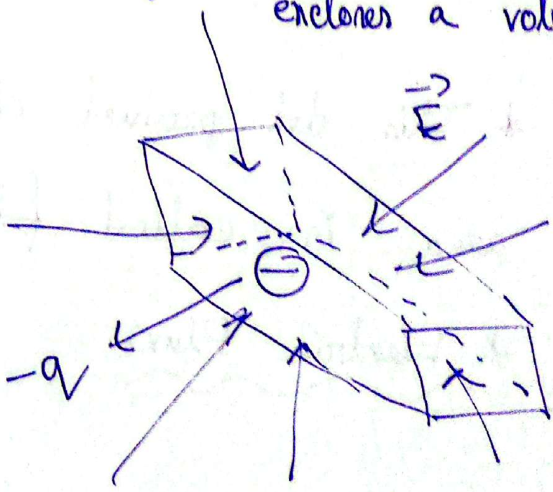
* A surface has two sides, so there are two possible directions for $\hat{n}$ and $\vec{A}$. We must specify which direction we choose.

* If ϕ_E is positive we call it outward electric flux. And if ϕ_E is negative we call it inward electric flux.

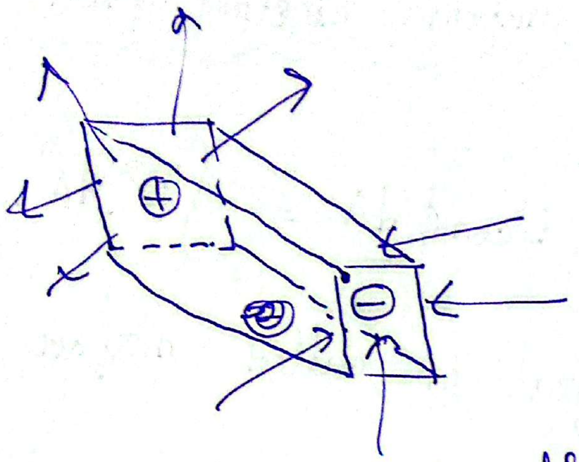
* To develop a relation bet^n charge and electric flux let us imagine a box around various charges. The box is called closed surface because it completely encloses a volume.



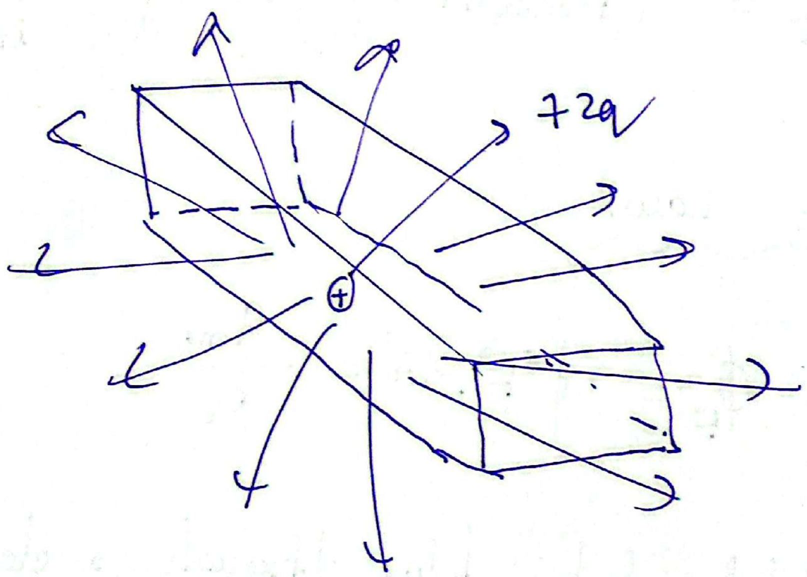
Positive charge inside the box, outward flux.



Negative charge inside the box, inward flux.



Zero net charge inside the box, inward flux cancels outward flux.



Doubling the enclosed charge also doubles the magnitude of the electric field on the surface so the electric flux through the surface is twice as great as if +q charge was enclosed.

Flux of a non-uniform electric field:

$$\phi_E = \int E\cos\phi \, dA = \int \vec{E}.\vec{dA}.$$

This integral is called surface integral.

If we evaluate this integral over closed surface,

$$\phi_E = \oint E\cos\phi \, dA = \oint \vec{E}.\vec{dA} \left(\text{Closed surface integral}\right).$$

Gauss's Law:

$$\phi_E = \oint \vec{E}.\vec{dA} = \frac{q_{enc}}{\epsilon_0}.$$

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0.

* We imagine various surfaces to apply gauss's law. These surfaces are called gaussian surfaces.

Gauss's Law and Coulomb's law:

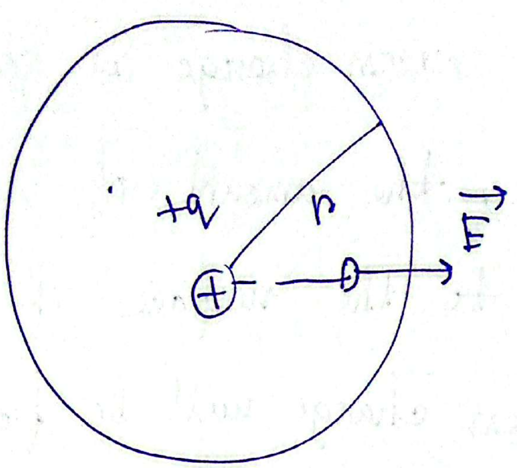


Fig: A spherical gaussian surface centered on a particle with charge q.

Now,

$$\phi_E = \oint \vec{E}.\vec{dA} = \oint E\,dA\cos 0^\circ = \oint E\,dA$$

$$= E\oint dA = E\,4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\cancel{\therefore E = \frac{1}{4\pi r^2}\frac{q}{\epsilon_0}}$$

$$\therefore E = \frac{1}{4\pi r^2}\frac{q}{\epsilon_0}.$$

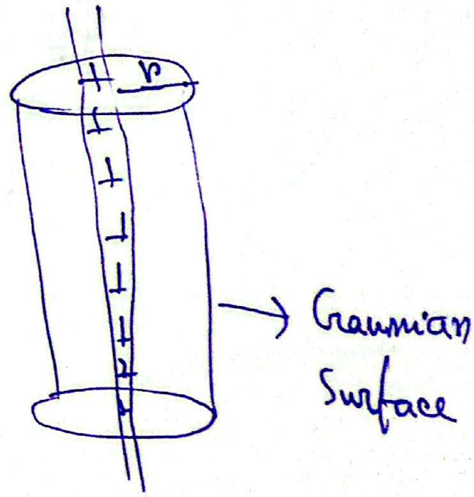
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}.$$

A charged isolated conductor:

⇒ If an excess charge is placed on an isolated conductor, the amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Applying Gauss' Law:

Cylindrical Symmetry:

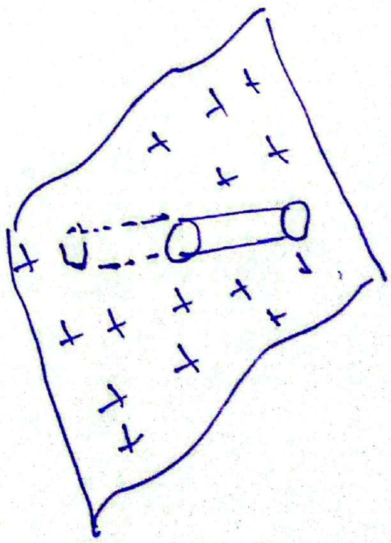


$$\phi = EA\cos\theta = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\,2\pi r h = \frac{\lambda h}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \epsilon_0}$$

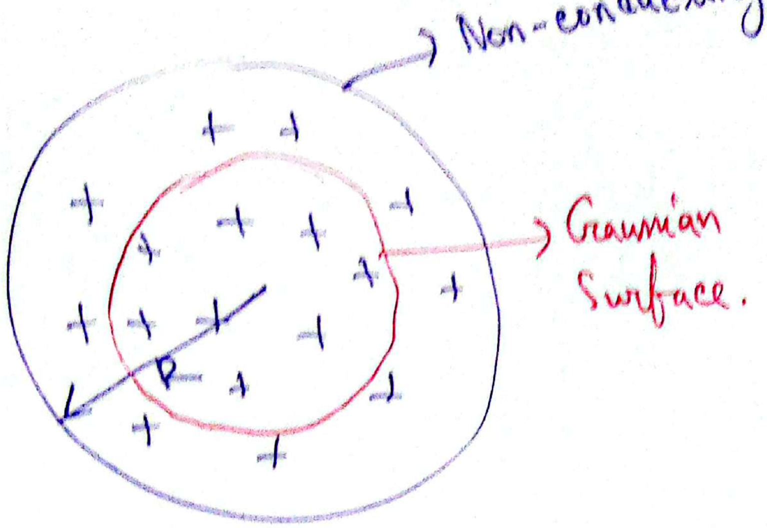
Planar Symmetry:



$$\phi = EA + EA = \frac{q_{enc}}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{2\epsilon_0}$$

Spherical Symmetry:



$$E_{outside} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$q' \longrightarrow$ charge inside the Gaussian Surface.

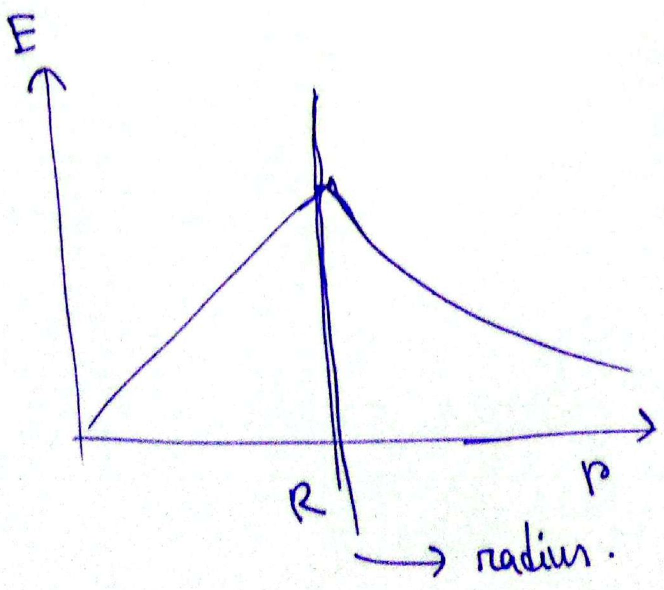
$$E_{inside} = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}$$

charge density $\rho = \dfrac{q}{\cancel{\frac{4\pi}{3}}\ \frac{4}{3}\pi R^3}$

$$= \frac{1}{4\pi\epsilon_0}\frac{\frac{q r^3}{R^3}}{r^2}$$

$$\therefore q' = \rho\,\frac{4}{3}\pi r^3 = \frac{q r^3}{R^3}.$$

$$= \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r$$



(Electric field for a non conducting ~~spherical shell~~ sphere.)

Gauss's Law Problems:

1)

$$\phi_E = \oint \vec{E}.\vec{dA}$$

$$= \int_a \vec{E}.\vec{dA} + \int_b \vec{E}.\vec{dA} + \int \vec{E}.\vec{dA}$$

For the area of the left side,

$$\int_a \vec{E}.\vec{dA} = \int E \cos 180^\circ dA = -E \int dA = -EA$$

Similarly for the area of right side,

$$\int_c \vec{E}.\vec{dA} = +EA.$$

For the cylindrical surface,

$$\int_b \vec{E}.\vec{dA} = \int E \cos 90^\circ dA = 0.$$

$$\therefore \phi = -EA + 0 + EA = 0.$$

~~b~~

2)

a) $E = 0$ as $q_{enc} = 0$

b) $E = 0$ as $q_{enc} = 0$

c) $E = 0$, as $q_{enc} = 0$

d) $\oint E\,dA \cos 0^\circ = q_{enc}/\epsilon_0$.

$\Rightarrow E\,4\pi r^2 = \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)\rho/\epsilon_0$

$\Rightarrow E = 7.32\ N/C$ [ρ = charge density]

e) Similarly, $E = 1.21\ N/C$

~~f)~~ Sol: ~~E~~

f) $E\,4\pi r^2 = \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)\rho/\epsilon_0$

$\Rightarrow E = 1.35\ N/C$.

$=$

3] a) $\oint \vec{E}\cdot d\vec{A} = \cancel{\vec{E}\cdot A_x \hat{i}}^{\,0} + \cancel{\vec{E}\cdot A_x(-\hat{i})}^{\,0} + \vec{E}\cdot A_y(\hat{j})$

$+ \vec{E}\cdot A_y(-\hat{j}) + \cancel{\vec{E}\cdot A_z \hat{k}}^{\,0} + \cancel{\vec{E}\cdot A_z(-\hat{k})}^{\,0}.$

$= 3y\hat{j} \times (1.4)^2 \hat{j} \Big|_{y=1.4} + 3y\hat{j} \times (1.4)^2(-\hat{j}) \Big|_{y=0}$

$= 8.23\ Nm^2/C.$

b) $q_{enc} = \epsilon_0 \phi = 7.29 \times 10^{-11}\ C.$

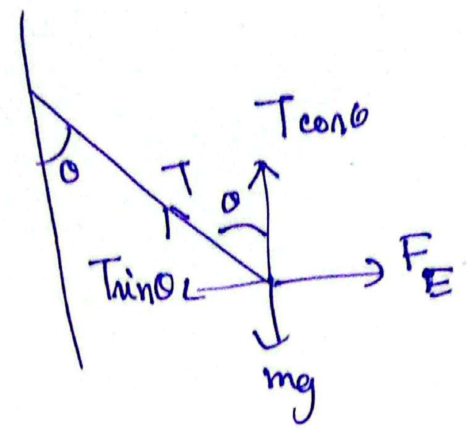
c) Same answer. as a.

d) Same answer as b

4)

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{r^3} = 1.5 \times 10^4 \text{ N/C.}$$

5)



$$mg = T\cos\theta$$

$$\Rightarrow T = \frac{mg}{\cos\theta}$$

$$F_E = T\sin\theta$$

$$\Rightarrow qE = \frac{mg}{\cos\theta} \times \sin\theta$$

$$\Rightarrow E = \frac{mg\tan\theta}{q}$$

$$\Rightarrow \frac{\sigma}{2\epsilon_0} = \frac{mg\tan\theta}{q}$$

$$\Rightarrow \sigma = \frac{mg\tan\theta \times 2\epsilon_0}{q} = 5 \times 10^{-6} \text{ C/m}^2.$$