1. MCD y MCM

Teorema 1.1. $a, b \in \mathbb{Z}^+ \implies ab = \operatorname{mcd}(a, b) \cdot \operatorname{mcm}(a, b)$

Prueba: Realizamos la descomposición de a y b en factores primos:

$a = P_1^{a_1} \cdot P_2^{a_2} \cdots P_n^{a_n}$
$b = P_1^{b_1} \cdot P_2^{b_2} \cdots P_n^{b_n}$

Donde $P_1, P_2, \ldots, P_n$ son primos y $a_1, a_2, \ldots, a_n$ y $b_1, b_2, \ldots, b_n$ son enteros positivos. Ahora

$\operatorname{mcd}(a, b) = P_1^{\min(a_1, b_1)} \cdot P_2^{\min(a_2, b_2)} \cdots P_n^{\min(a_n, b_n)}$
$\operatorname{mcm}(a, b) = P_1^{\max(a_1, b_1)} \cdot P_2^{\max(a_2, b_2)} \cdots P_n^{\max(a_n, b_n)}$

Luego

$$
\begin{aligned}
\operatorname{mcd}(a, b) \cdot \operatorname{mcm}(a, b) &= \left[P_1^{\min(a_1, b_1)} \cdot P_2^{\min(a_2, b_2)} \cdots P_n^{\min(a_n, b_n)}\right] \\
&\quad \cdot \left[P_1^{\max(a_1, b_1)} \cdot P_2^{\max(a_2, b_2)} \cdots P_n^{\max(a_n, b_n)}\right] \\
&= P_1^{a_1 + b_1} \cdot P_2^{a_2 + b_2} \cdots P_n^{a_n + b_n} \\
&= P_1^{a_1} P_1^{b_1} \cdot P_2^{a_2} P_2^{b_2} \cdots P_n^{a_n} P_n^{b_n} \\
&= \left(P_1^{a_1} \cdot P_2^{a_2} \cdots P_n^{a_n}\right) \cdot \left(P_1^{b_1} \cdot P_2^{b_2} \cdots P_n^{b_n}\right) \\
&= a \cdot b
\end{aligned}
$$

$\square$

Ejemplo: Verifique teorema con $a = 18$ y $b = 24$

$$
\begin{aligned}
18 &= 2^1 \cdot 3^2 \\
24 &= 2^3 \cdot 3 \\
\operatorname{mcd}(18, 24) &= 2^1 \cdot 3^1 = 6 \\
\operatorname{mcm}(18, 24) &= 2^3 \cdot 32 = 72
\end{aligned}
$$

$\therefore \operatorname{mcd}(18, 24) \cdot \operatorname{mcm}(18, 24) = 6 \cdot 72 = 432 = 18 \cdot 24 = a \cdot b$