Question 2. Proof by Induction

Let F(n) denote the nth Fibonacci number. Prove $F(n) > (4/3)^n$ for $n > 4$.

Hints:

(1) Use the strong induction

(2) Use the fact F(n) = F(n-1) + F(n-2)

(3) Since you are using two values, you must prove the two base cases: n = 5 and n = 6

Answer:

1. Prove the statement is true with n = 5 and n = 6
   - N = 5
     $F(5) = 5 > (4/3)^5 = 1024/243$
   - N = 6
     $F(6) = 8 > (4/3)^6 = 4096/729$

2. F(n+1) = F(n) + F(n-1)
   $F(n) > (4/3)^n$ for n is from [4..k)

   If $F(k) > (4/3)^k$ then F(k-1) is also $> (4/3)^{(k-1)}$

   Then we will prove it is true with F(k+1)

   $$\begin{aligned} F(k+1) &= F(k) + F(k - 1) \\ &= (4/3)^k + (4/3)^{(k-1)} \\ &= (4/3)^k * (1 + (4/3)^{-1}) \\ &= (4/3)^k * (1 + (3/4)) \\ &= (4/3)^k * (7/4) > (4/3)^{k+1} = (4/3)^k * (4/3) \\ &= (7/4) > (4/3) \end{aligned}$$