1 | $a, a+2, a+4, a+6, a+8$ are odd

$a + a+2+a+4+a+6+a+8 = K$ is even/50

$5a + 20 = K$

$5a = K - 20$

$a = \frac{K-20}{5}$: An even divided by an odd is either even or not an integer hence proven

2 | 1. True $1^2 = 1 + 2^2 = 4$, no integer between

2. True, for every x^2 a y exists that's larger

3. False, $2+4=6$ $4-2 \neq 1$ $3+3=6$ $3-3 \neq 1$ $5+1=6$ $5-1 \neq 1$

4 True, each square has a smaller integer

5. false, what if $x = y$

3. | 1. $\exists a \in \mathbb{Z} : \frac{a}{5} = b \in \mathbb{Z}$

2. $\forall a \in \mathbb{R} : a^2 = |b \in \mathbb{R}|$

3. $\exists a \in \mathbb{Z} : |a| = 0$

4. $\forall a \in \mathbb{R}, \forall b \in \mathbb{R} : (-a)(-b) = |ab|$

5. $\forall a \in \mathbb{R}, \forall b \in \mathbb{R} : |a+b| \leq |a| + |b|$

6 | $n=1 \quad 2(1) = 1(1+1)$ ✓

$n = k \in \mathbb{Z}$

$n = k+1 \quad 2(k+1) = k(k+1) + 2(k+1)$

$= (k+1)(k+2)$

$\Rightarrow 2+4+6+\dots+2k+2(k+1) = (k+1)(k+2)$

which is true

7 $n \in Z$ is even, $5n^2+n+4 = 5(2x)^2+(2x)+4$

$= 10x^2+2x+4$

$= 2(5x^2+x+2)$ is even

$n \in Z$ is odd $5n^2+n+4 = 5(2x+1)^2+(2x+1)+4$

$= (10x+5)^2+2x+1+4$

$= 100x^2+75+50x+2x+1+4$

$= 100x^2+52x+30$

$= 2(100x^2+26+15)$ is even

Proved

8 $x \in Z, y \in Z$ if $x+y < 100 : x < 30 \vee y < 70$

$x < 30, x = 29 + y < 100$

$y < 100-29$

$y < 71$

$y < 70$

$y = 69$ $69+x < 100$

$x < 100-69$

$x < 31$

All cases should be proved new.

9 $x = \frac{p}{r}, y = \frac{q}{b}$ $x+y = \frac{p}{r} + \frac{q}{b}$

$= \frac{pb+qr}{rb} = \frac{m}{n}$ = definition of rational number ✓

10 $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z} : (x+1)^2 - x^2 = 4y - 2$

$x = 2k$ ↑ $(2k+1)^2 - (2k)^2 = 4y - 2$

$4k^2 + 4k + 1 - 4k^2 = 4y - 2$

x is even $4k + 1 = 4y - 2$

$4k = 4y - 3$ comes out odd but integer

$x = 2k+1$

$(2k+1+1)^2 - (2k+1)^2 = 4y - 2$

$(2k+2)^2 - (2k+1)^2 = 4y - 2$

$4k^2 + 4 + 8k - 4k^2 - 1 - 4k = 4y - 2$

$8k + 4 - 1 - 4k = 4y - 2$

$4k + 3 = 4y - 2$

$4k = 4y + 1$ again odd but exists