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1 The Koch Snowflake

The *Koch snowflake* one of the first fractals, is based on work by the Swedish mathematician Helge von Koch . It is what we get if we start with an The initial equilateral triangle and the refinement of the Koch snowflake after one, two, and ther iterations.

equilateral triangle and repeat the following an infinite number of times:

Divide all line segments into three segments of equal length. Then draw, for each middle line segment an equilateral triangle that has the middle segment as its base and points outward. Finally, remove all middle segments.

shows the firstiterations in the construction. (Original)

1.1 Two properties

Theorem 1. *The Koch snowflake has infinite length.* *Proof.* Let Δ denote a triangle, with side length s, on which we base the construction of a snowflake. Len N_i denote the number of line segments, and L_i the lenth of the segments, in iteration i of the construction. Then

$$N_n = \{$$

which solves to

$$N_n = 3 \cdot 4^n, \tag{1}$$

while

$$L_n = \frac{L_{n-1}}{3} = \frac{L_{n-2}}{3^2} = \frac{L_{n-3}}{3^3} = \ldots = \frac{L_0}{3^n} = \frac{s}{3^n} \tag{2}$$

The total length

$$N_n L_n = 3 \cdot 4^n \frac{s}{3^n} = 3s\frac{4^n}{3^n} = 3s\left(\frac{4}{3}\right)^n.$$

Since $4/3 > 1$, it follows that $N_n L_n$ tends to infinity as $n \to \infty$, i.e. the Koch snowflake has infinite length. $\square$