Machine Learning – February 12, 2018

Time limit: **2 hours**.

Last Name ..... First Name ..... Matricola .....

Note: if you are not doing the exam ML 2017/18, write below name of exam, CFU, and academic year (when you were supposed to attend the course). Please specify also if you are an Erasmus student.

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EXERCISE 0 (points $[0, 1]$ mulitplied to the overall score of the test)

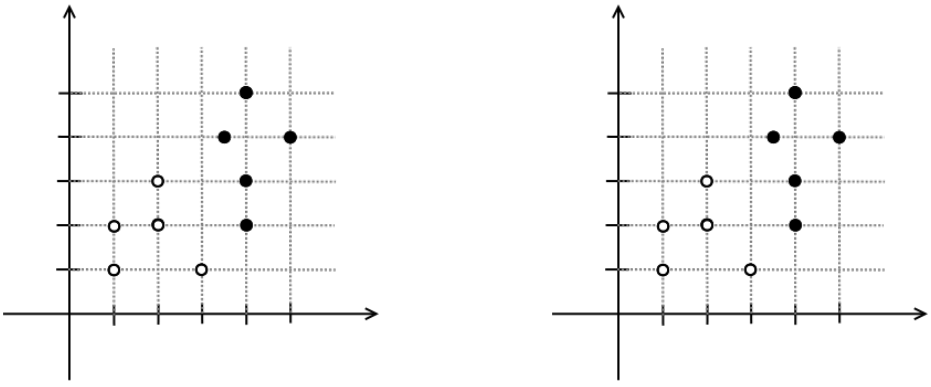
1. Write your name and matricola code in each paper you deliver.
2. Write all the answers of exercises **A** on one sheet marked as **A**, and all the answers of exercises **B** on another sheet marked as **B**. Do not mix answers of exercises **A** and **B** on the same sheet.
3. Do not use text books, slides, notes, mobile phone, laptop, etc.

EXERCISE A1

Machine learning problems can be categorized in supervised and unsupervised. Explain the difference between them providing a precise formal definition (not only explanatory text) in terms of input and output of the two categories of problems.

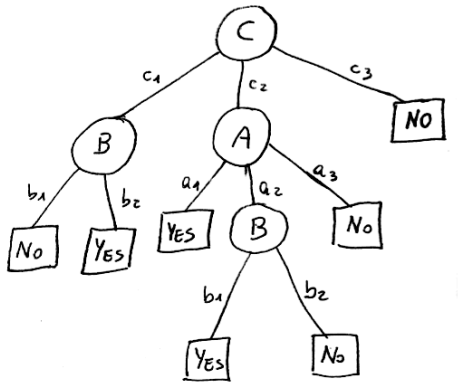
EXERCISE A2

Consider the following data set for binary classification, where the two classes are represented with white and black circles. Draw in each of the diagrams a possible solution for a method based on Perceptron with very small learning rate and a method based on SVM. Describe the difference between the two solutions and explain how these are obtained with the two methods. Discuss which solution would you prefer and why.



EXERCISE A3

Given a classification problem for the function $f : A \times B \times C \to \{YES, NO\}$, with $A = \{a_1, a_2, a_3\}, B = \{b_1, b_2\}, C = \{c_1, c_2, c_3\}$ and the following decision tree T that is the result of training on a given data set:



1. Provide a rule based representation of the tree T.
2. Determine if the tree T is consistent with the following set of samples

   $S \equiv \{s_1 = \langle a_1, b_1, c_1, NO\rangle, s_2 = \langle a_2, b_1, c_2, YES\rangle,$
   $s_3 = \langle a_1, b_2, c_3, NO\rangle, s_4 = \langle a_3, b_2, c_1, YES\rangle\}.$

   Motivate your answer.

EXERCISE B1

1. Provide the main features about boosting.
2. Write the error function whose minimization leads to a formulation equivalent to the AdaBoost algorithm.

EXERCISE B2

Consider the problem of finding a function which describes how the salary of a person (in hundreds of euros) depens on his/her age (in years), the months in higher education and average grades in higher education. A dataset in the form $\mathcal{D} = \{(\mathbf{x}_1^T, t_1), \ldots, (\mathbf{x}_N^T, t_N)\}$ is provided, with $\mathbf{x} \in \mathbb{R}^3$ denoting the input values and t the target values (salary).

Assuming that one tries to identify this function with a deep feed-forward network:

1. Explain how the problem is formalized by writing the parametric form of the function to be learned highlighting the parameters $\boldsymbol{\theta}$.
2. Explain what are suitable choices for the activation functions of the hidden and output units of the network.
3. Explain what is a suitable choice for the loss function used for training the network and write the corresponding mathematical expression.

EXERCISE B3

Given input values $\mathbf{x}_i$ and the corresponding target values t_i with $i = 1, \ldots, N$, the solution of regularized linear regression can be written as:

$$y(\mathbf{x}) = \sum_i^N \alpha_i \mathbf{x}_i^T \mathbf{x},$$

with $\boldsymbol{\alpha} = (XX^T + \lambda I)^{-1}\mathbf{t}$, $X = [\mathbf{x}_1, \ldots, \mathbf{x}_N]^T$ and λ the regularization weight.

Considering a kernel function $k(\mathbf{x}, \mathbf{x}')$:

1. Provide a definition of the Gram matrix.
2. Explain how a kernelized version for regression can be obtained based on the equations provided above.