Vision and Perception

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Abstract

This document is used to illustrate all the homework exercises that has been given during Vision and Perception course in 2020/21. There are overall 10 exercises that needed to be done during this time.

1 Homework 1 - Degenerate Conic

As mentioned during the video of Geometric Parameters, we needed to compute the M_2 as we did for M_1 and then computing the null space of M_1 and verifying that is 2, then computing the cross product of null space vectors and after normalizing to obtain the $x_3 = 1$, we needed to think about the result.

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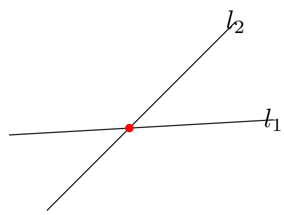

Thus, we need to consider the two intersecting lines that we have seen in our lecture:

$l_1 = (0.2500, 3.200, 1.0000)^T$
$l_2 = (-2.0000, 0.5000, 1.0000)^T$
$x = l_1 x l_2 = (-0.4138, 0.3448, 1.0000)^T$

$$M_1 = l_1 l_1^T = \begin{pmatrix} 0.0625 & 0.8000 & 0.2500 \\ 0.8000 & 10.2400 & 3.2000 \\ 0.2500 & 3.2000 & 1.0000 \end{pmatrix}$$

Thus, by computing the degenerate conic for l_2, we could get something following:

$$M_2 = l_2 l_2^T = \begin{pmatrix} 4.0000 & -1.0000 & -2.0000 \\ -1.0000 & 0.2500 & 0.5000 \\ -2.0000 & 0.5000 & 1.0000 \end{pmatrix}$$

As we see from the book, the null space can be computed by span as following:

$$Null(M_1) = Span\{\begin{pmatrix} 0.2210 \\ 0.2751 \\ -0.9357 \end{pmatrix}, \begin{pmatrix} -0.9724 \\ 0.1353 \\ -0.1899 \end{pmatrix}\}$$

and

$$Null(M_2) = Span\{\begin{pmatrix} 0 \\ 0.8944 \\ -0.4472 \end{pmatrix}, \begin{pmatrix} -0.4880 \\ -0.3904 \\ -0.7807 \end{pmatrix}\}$$

Finally, we need to calculate the cross-product of the null-space components of M_1 as:

$M_1 = (0.2210, 0.2751, -0.9357)x(-0.9724, 0.1353, -0.18, 99) = (0.0744, 0.9518, 0.2974)^T$
then, for M_2 it will be like:

$M_2 = (0, 0.8944 - 0.4472)x(-0.4880, -0.3904, -0.7807) = (-.8728, 0.2182, 0.4365)^T$

We come up to the result that these two results are no more than the original lines, l_1 and l_2 with unitary norm.

2 Homework 2 - Five Points Define a Conic

For each point the conic passes through

$$ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f = 0$$

As we see from the equation, we need to determine the parameters of the equation with at least five points. So, for this homework we just chose 4 points in xy plane with different values, thus we have the following set of equations:

$$\begin{bmatrix} 0 & 0 & 16 & 0 & 4 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 9 & 24 & 64 & 3 & 8 & 1 \\ 36 & 36 & 36 & 6 & 6 & 1 \\ 49 & 21 & 9 & 7 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = 0$$

from which we obtain the following values:

$a = 0.0876$, $b = 0.0258$, $c = 0.0532$, $d = -0.7038$, $e = -0.4629$, $f = 1$,

thus giving us following result:

3 Homework 3 - Compute the DLT algorithm

In order to solve this problem, we needed to take one picture and apply the algorithm to check the coordinates of the shape. The idea is to pick 4 points on two different images by hand and apply the DLT algorithm showing the calculations. Additionally, the point selection is arbitrary.

4 Homework 4 - Affine Transformations

The task for this exercise is to show that an affine transformation preserves both parallel lines and area.

5 Homework 5 - Homography keeps lines tangent to conics

6 Homework 6 - SVD for given matrix

As we have seen from the example, the matrix A which we out to decompose is the following:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & -2 \end{bmatrix}$$

The singular value decomposition (SVD) property proposes that any given matrix can be decomposed in three matrices that is:

$$A = U\Sigma V^T$$

where Σ is a rectangular diagonal matrix and, U and V^T are orthogonal matrices. To obtain these matrices, we first compute the product $A^T A$ as following:

$$A^T A = (U\Sigma V^T)^T U\Sigma V^T = V(\Sigma^T \Sigma)V^T$$

which will be equal to

$$A^T A = \begin{bmatrix} 5 & 10 & 0 \\ 10 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix}$$

from this relation, we extract the eigenvalues of $A^T A$ as follows:

$$(A^T A - I\lambda)x = 0$$

in which, in order to obtain a non-trivial solution, the following property must hold:

$$det(A^T A - I\lambda) = 0$$

which gives the following equation:

$$(5-\lambda)(25-\lambda)(5-\lambda) - 25(5-\lambda) - 100(5-\lambda) = 0$$

From this equation, we arrive at the following solutions:

$$\lambda_1 = 30,\ \lambda_2 = 5,\ \lambda_3 = 0$$

To obtain the eingenvectors, we must find the vectors that lie in the null-space of the resulting matrices once each eigenvalue is substituted:

For $\lambda = 30$:

$$\begin{bmatrix} -25 & 10 & 0 \\ 10 & -5 & -5 \\ 0 & -5 & -25 \end{bmatrix} x = 0,$$

Giving us the following expressions:

$$-25x_1 + 10x_2 = 0,$$
$$10x_1 - 5x_2 - 5x_3 = 0,$$
$$-5x_2 - 25x_3 = 0$$

from this linear set of equations, we arrive at the following solution:

$$x = [-1, -2.5, 0.5]^T,$$

which is then normalized:

$$x = [-0.3651, -0.9129, 0.1826]^T.$$

Repeating the same procedure for the other eigenvalues, we arrive at:

$$x = [-0.4472, 0, -0.8944]^T \text{ and } x = [0.8165, -0.4082, -0.4082]^T$$

Like so, we obtain the matrix V, compose of the column vectors:

$$V = \begin{bmatrix} -0.3651 & -0.4472 & 0.8165 \\ -0.9129 & 0 & -0.4082 \\ 0.1826 & -0.8944 & -0.4082 \end{bmatrix}$$

furthemore, we also obtain the matrix λ, which is the square root of the diagonal matrix composed of the calculated eigenvalues:

7 Homework 7 - Projective Transformation

The task here is to find the projective transformation H and define the type of quadric from the quadric equation.

Firstly, let's clarify some points we have learned in our lectures.

3D point in R^3 which is $X = (X, Y, Z)^T$ in $P^3 : (x_1, x_2, x_3, x_4)^T$,

$$\text{Euclidean frame } \pi : ax + by + cz + d = 0, \text{ where } \pi^T X = 0 => (abcd)^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

Thus,

$$X = (X_1, X_2, X_3, X_4) \text{ derives,}$$

$$X^T AX = 4x_1^2 + 4x_1x_2 - 2x_1x_3 + 2x_1x_4 + 5x_2^2 - 2x_2x_4 + 2x_3^2 + 2x_3x_4 + 2x_4^2 = 0$$

Then, we get the following A matrix,

$$A = a_i a_j = \begin{bmatrix} 4 & 2 & -1 & 1 \\ 2 & 5 & 0 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} = 0$$

Then from the following equation we are getting the λ values.

$$|\lambda I - A| = \begin{bmatrix} \lambda - 4 & -2 & 1 & -1 \\ -2 & \lambda - 5 & 0 & 1 \\ 1 & 0 & \lambda - 2 & -1 \\ -1 & 1 & -1 & \lambda - 2 \end{bmatrix} = 0$$

$$\lambda^4 - 13\lambda^3 + 52\lambda^2 - 81\lambda + 25 = 0,$$

$$\lambda_1 = 6.6637,\ \lambda_2 = 3.6360,\ \lambda_3 = 2.7153,\ \lambda_4 = -0.0151$$

Then, from the matrix we get the V values,

$$V_1 = \begin{bmatrix} -9.2387 \\ -11.7071 \\ 2.1953 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 0.9476 \\ -0.6564 \\ 0.0319 \\ 1 \end{bmatrix}, V_3 = \begin{bmatrix} -0.5125 \\ 0.9964 \\ 2.1143 \\ 1 \end{bmatrix}, V_4 = \begin{bmatrix} -0.6963 \\ 0.4770 \\ -0.8417 \\ 1 \end{bmatrix},$$

Then, from the formula of $U_i = AV_i/\sigma_i$, we tried to find the U value and got the following values:

8 Homework 8 - Point Equations

The task here is to define the pairs of point equations for the direction and pair of plane equations for coordinate planes.

9 Homework 9 - Equation of Conic

10 Homework 10 - Proof for $cos(\alpha)$

11 Homework 11 -

12 Homework 12 -

13 Homework 13 -

14 Homework 14 -