Autonomous and Mobile Robotics

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Humanoid Robots 3:
Balance

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recap

- sufficient condition for balance: **ZMP** inside the **support polygon**
- ZMP dynamics modeled from Newton-Euler equations
- approximate model: **Linear Inverted Pendulum (LIP)**

$$\boxed{\ddot{\boldsymbol{c}}^{\boldsymbol{x},\boldsymbol{y}} = \frac{g^z}{c^z}\left(\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}\right)}$$

Linear Inverted Pendulum: basic scope

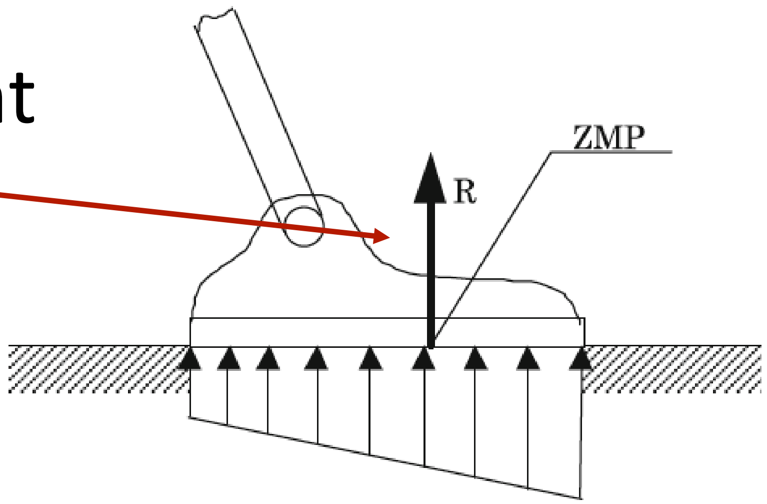
$$\boxed{\ddot{\boldsymbol{c}}^{x,y} = \frac{g^z}{c^z}\left(\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}\right)}$$

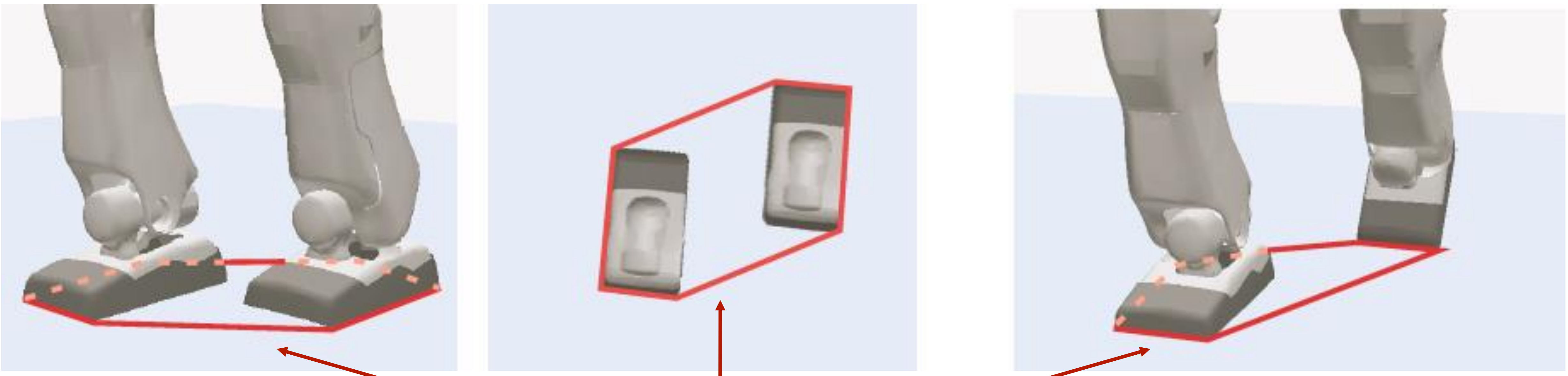
Although extremely simplified, the LIP equation describes in first approximation the time evolution of the CoM trajectory. Moreover

- it defines a differential relationship between the CoM trajectory and the ZMP (or CMP) time evolution
- it is easier to design a controller which makes the actual CoM follow a desired behaviour
- dynamic balancing will be characterized in terms of the ZMP
- the problem will then be to understand which CoM trajectory, solution of the LIP equation, guarantees that dynamic balancing is achieved

ZMP

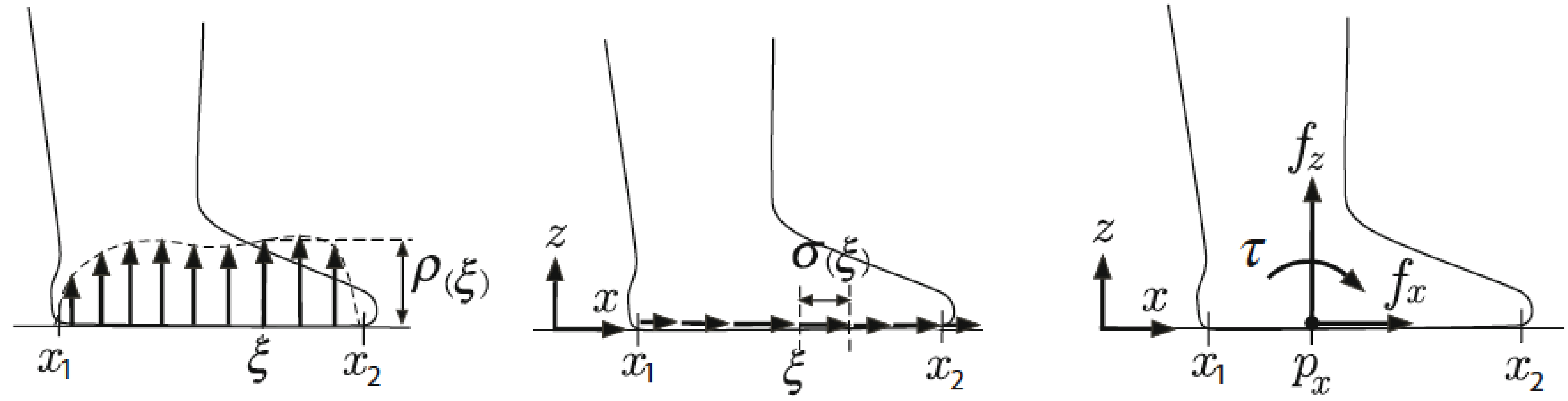
vertical component
of the GRF





Support Polygons during
Double Support

ZMP - 2D case



(a) Vertical force (b) Horizontal force

components of the GRF

$$f_x = \int_{x_1}^{x_2} \sigma(\xi)d\xi$$

equivalent force/torque

$$f_z = \int_{x_1}^{x_2} \rho(\xi)d\xi$$

$$\tau(p_x) = -\int_{x_1}^{x_2} (\xi - p_x)\rho(\xi)d\xi$$

generic p_x

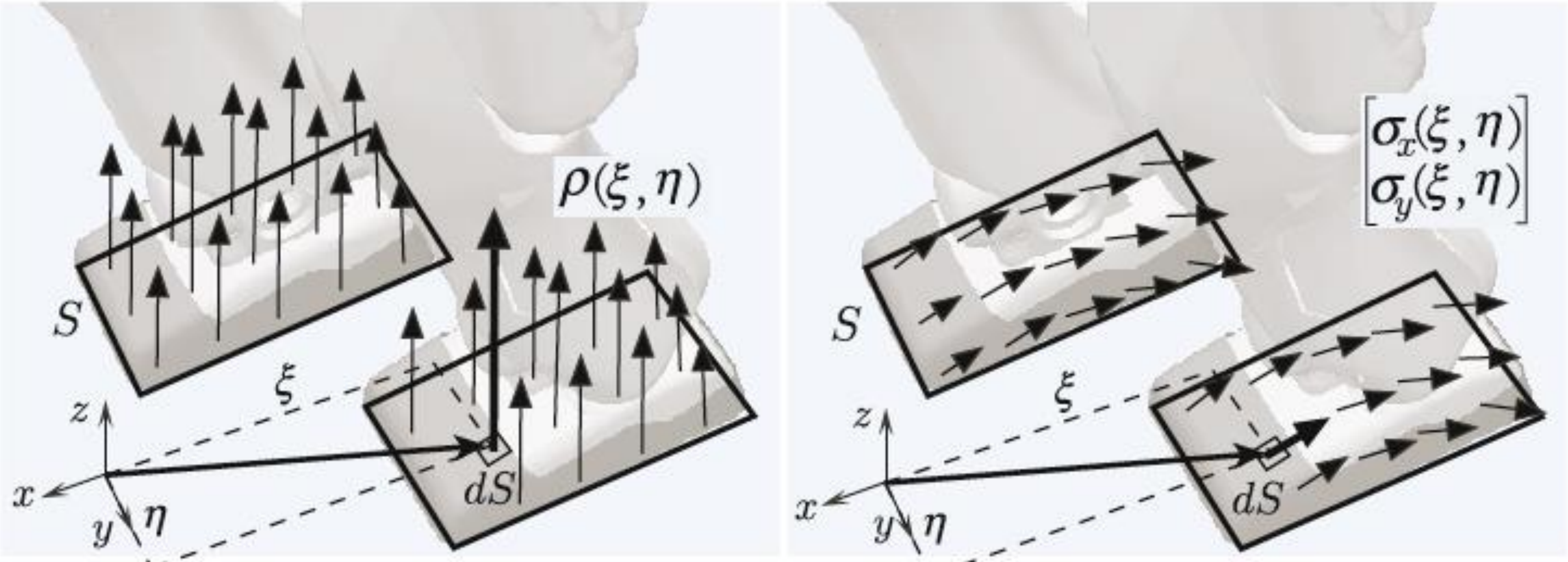
$$\tau(p_x) = 0$$

specific p_x

CoP/ZMP

$$p_x = \frac{\int_{x_1}^{x_2} \xi\rho(\xi)d\xi}{\int_{x_1}^{x_2} \rho(\xi)d\xi}$$

ZMP - 3D case



(a) Vertical reaction forces (b) Horizontal reaction forces

vertical component of the GRF

$$f_z = \int_S \rho(\xi,\eta)dS$$

$$\boldsymbol{\tau}_n(\boldsymbol{p}) \equiv [\tau_{nx}\ \tau_{ny}\ \tau_{nz}]^T$$

$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi,\eta)dS$$

$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi,\eta)dS$$

$$\tau_{nz} = 0.$$

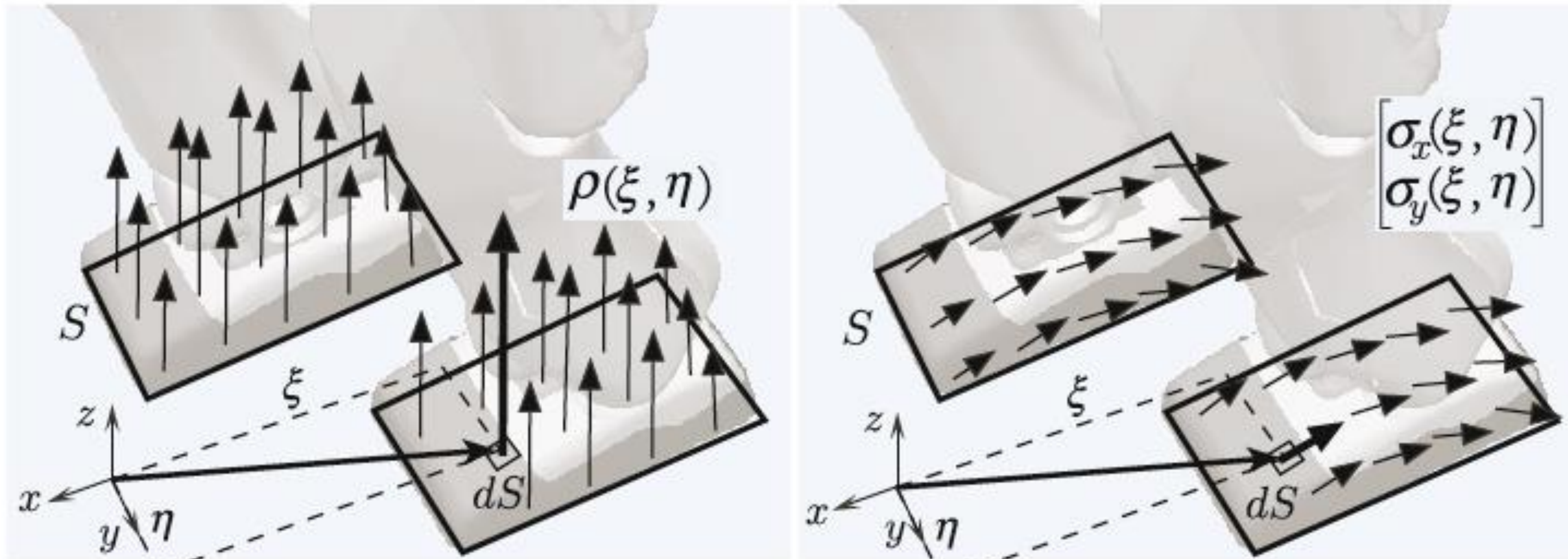
$$\tau_{nx} = 0$$

$$\tau_{ny} = 0$$

$$p_x = \frac{\int_S \xi\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}$$

$$p_y = \frac{\int_S \eta\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}.$$

ZMP - 3D case



(a) Vertical reaction forces (b) Horizontal reaction forces

horizontal component of the GRF

$$f_x = \int_S \sigma_x(\xi,\eta)dS$$

$$f_y = \int_S \sigma_y(\xi,\eta)dS.$$

$$\boldsymbol{\tau}_t(\boldsymbol{p}) \equiv [\tau_{tx}\ \tau_{ty}\ \tau_{tz}]^T$$

$$\tau_{tx} = 0$$

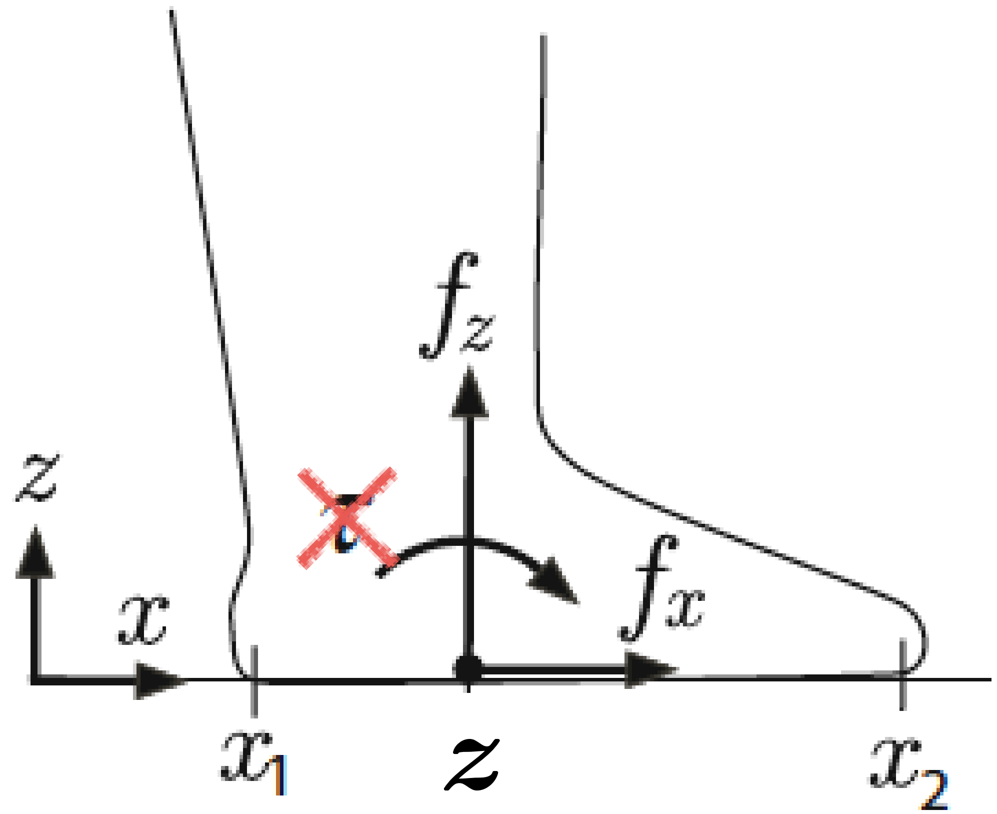
$$\tau_{ty} = 0$$

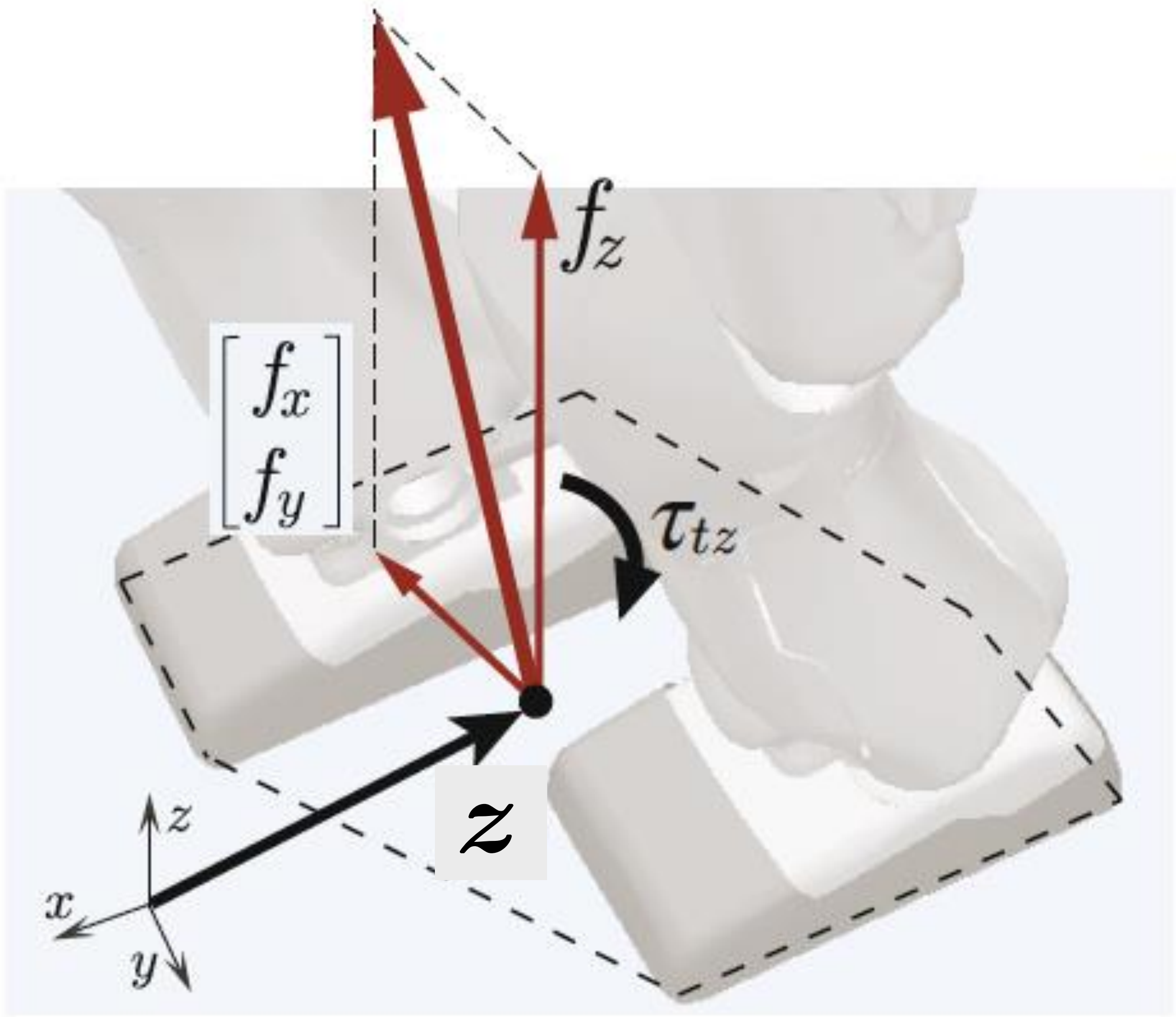
$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi,\eta) - (\eta - p_y)\sigma_x(\xi,\eta)\}dS$$

$$\begin{aligned}\boldsymbol{\tau}_p &= \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p})\\ &= [0\ 0\ \tau_{tz}]^T,\end{aligned}$$

if robot moves, the z component will be different from 0

ZMP

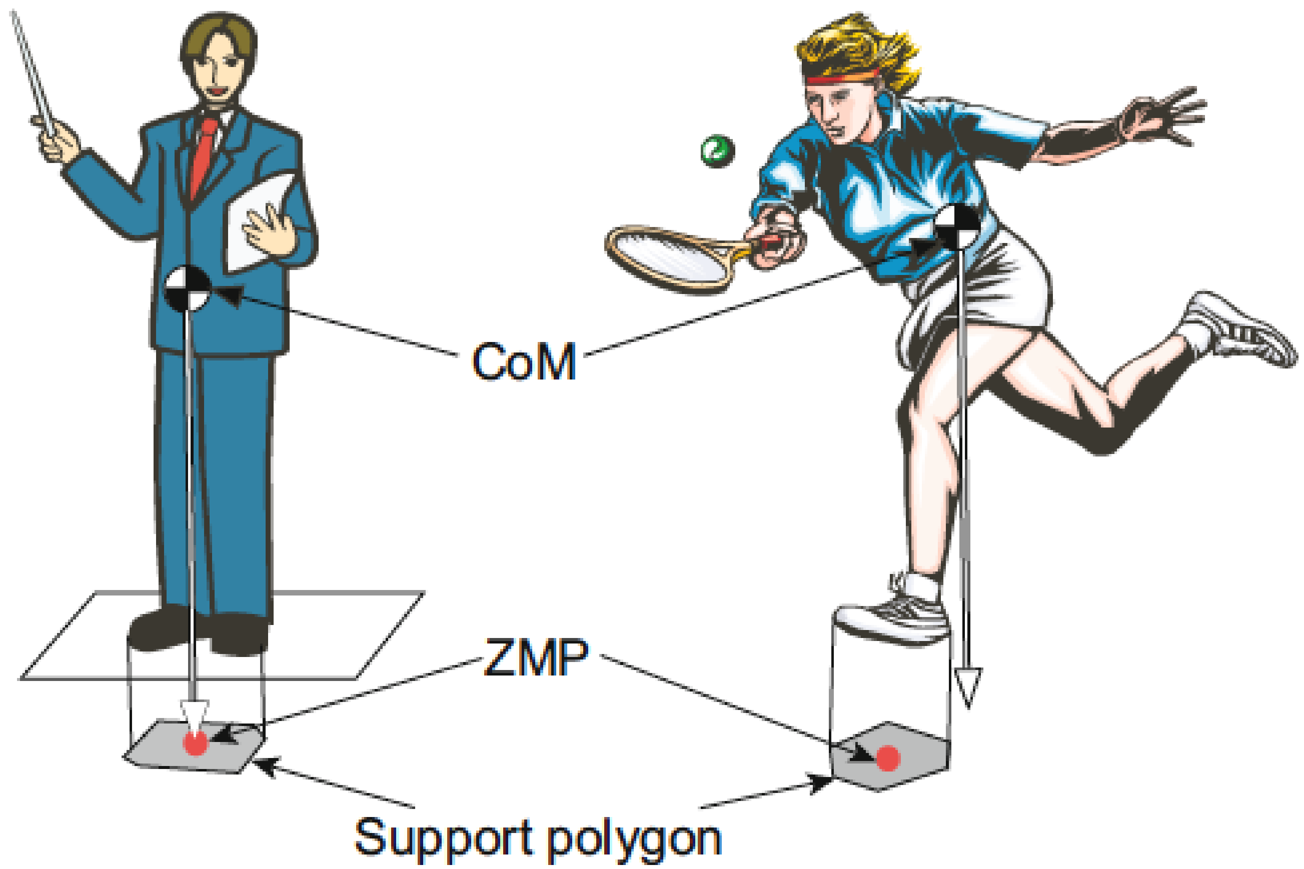




as long as the ZMP is in the Support Polygon,

the support foot will not rotate

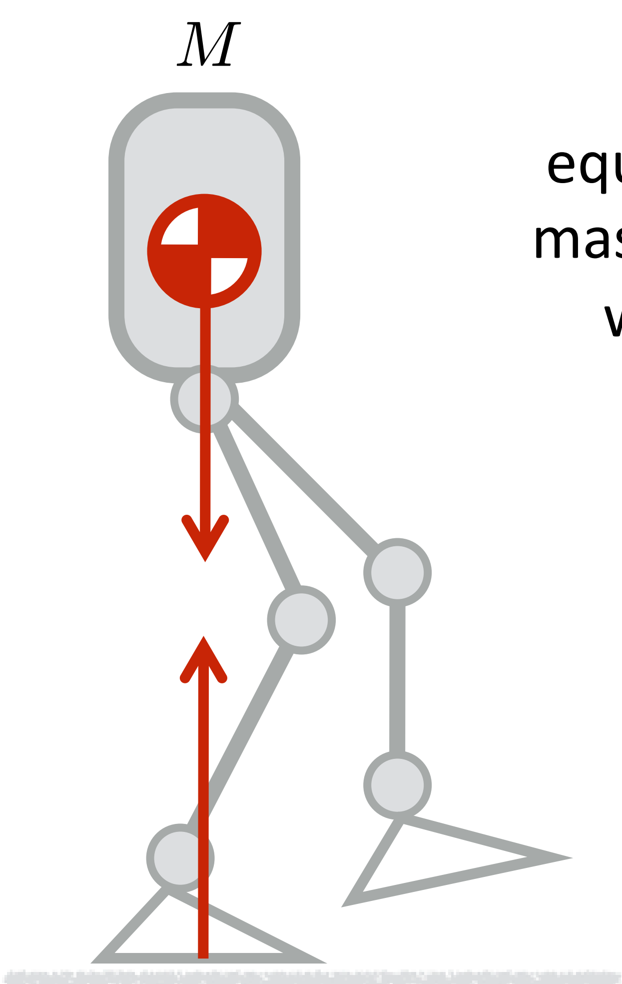
ZMP



static balance

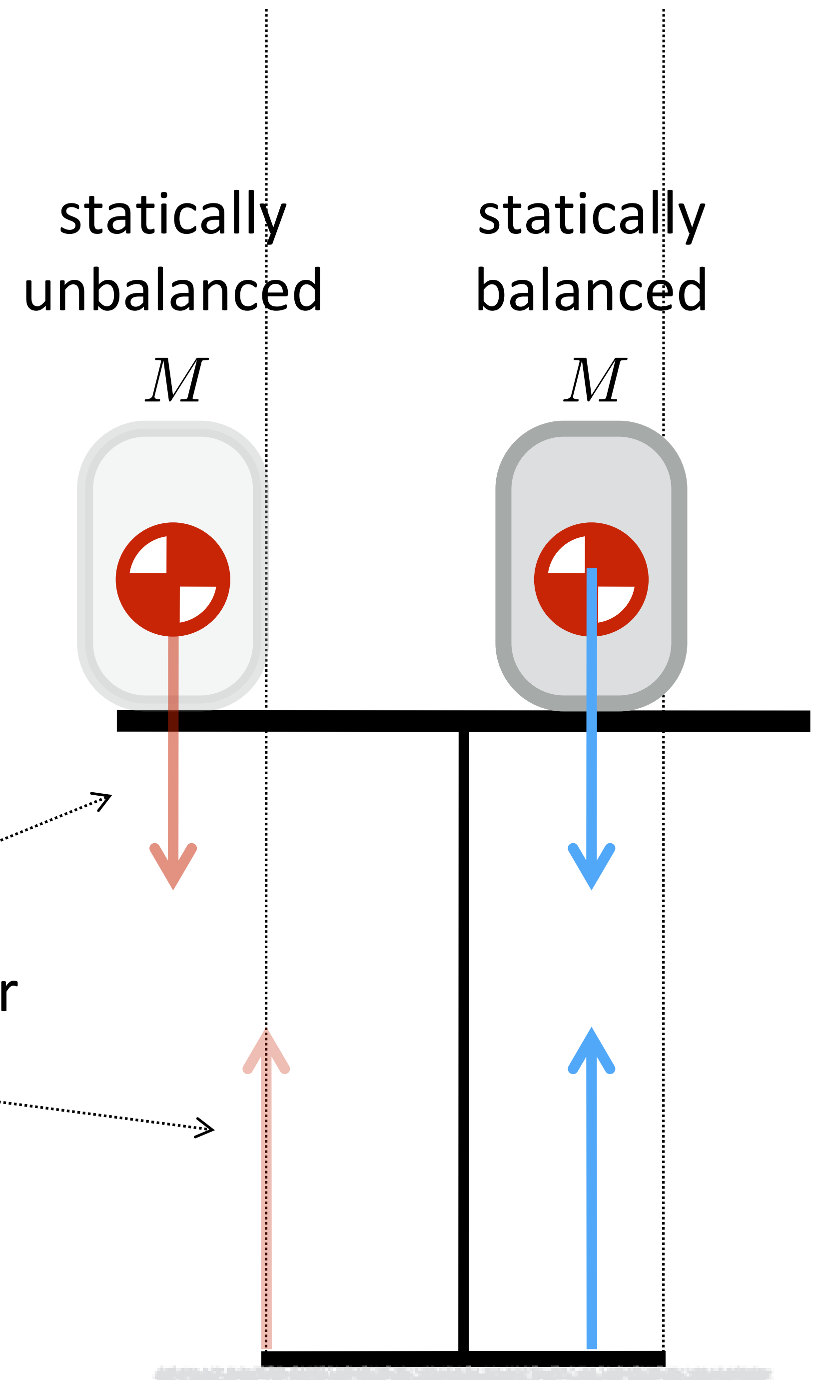
humanoid motionless:
statically balanced robots keep the center of mass within the polygon of support in order to maintain postural stability (sufficient when the robot moves slow enough so all the inertial forces are negligible)

if the CoM stays within these boundaries no tipping over occurs



equivalent representation:
mass M on a massless table with finite length base

the table starts tipping over

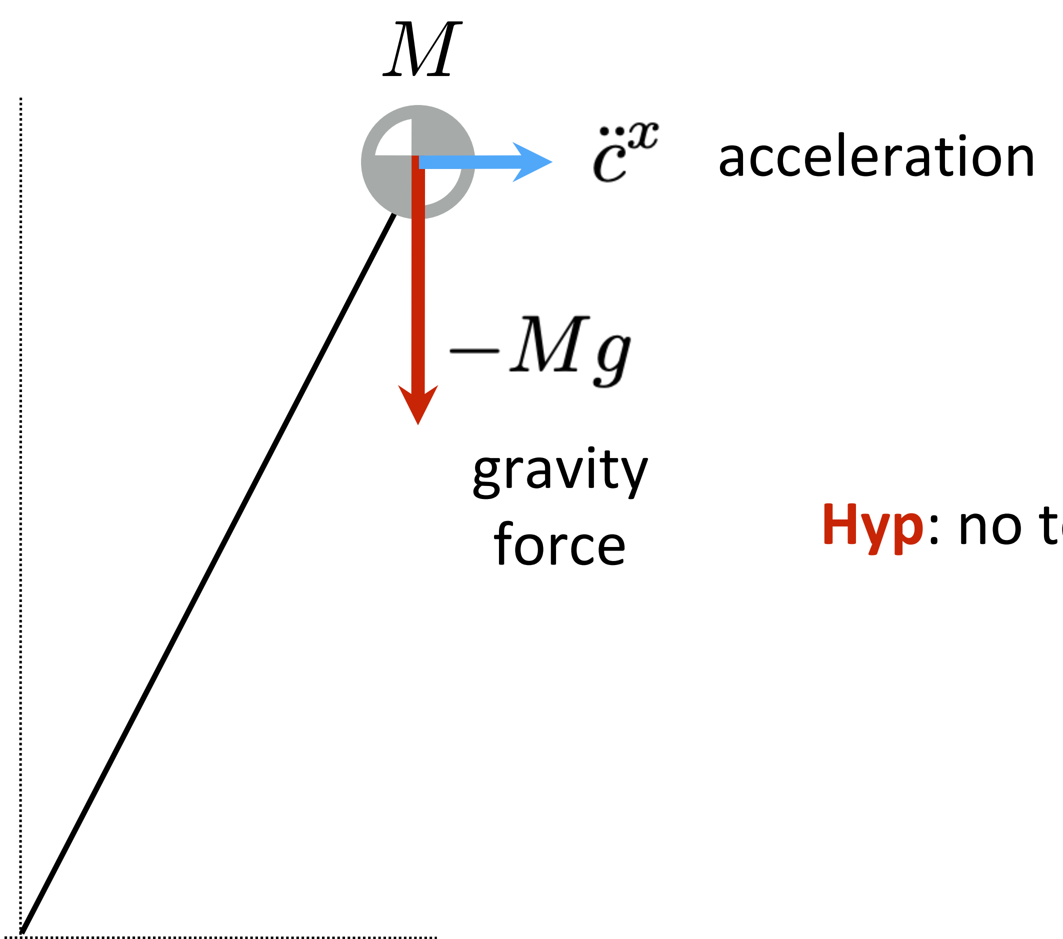


dynamic balance

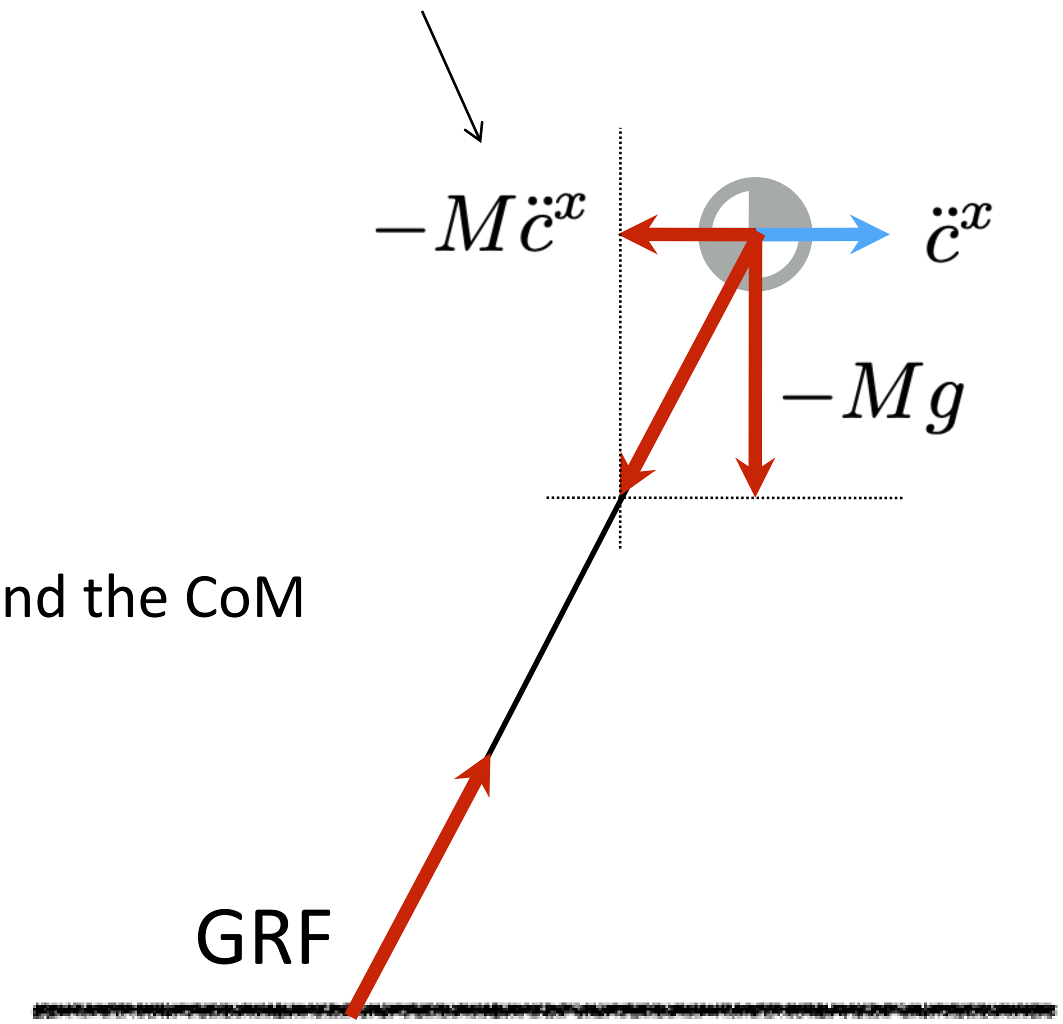
Question:
how do you keep a pendulum
in a non-vertical position?

Answer:
by continuously accelerating it

non-inertial frame
(pendulum stands still in an accelerating frame) → inertial force (fictitious force)



Hyp: no torque around the CoM

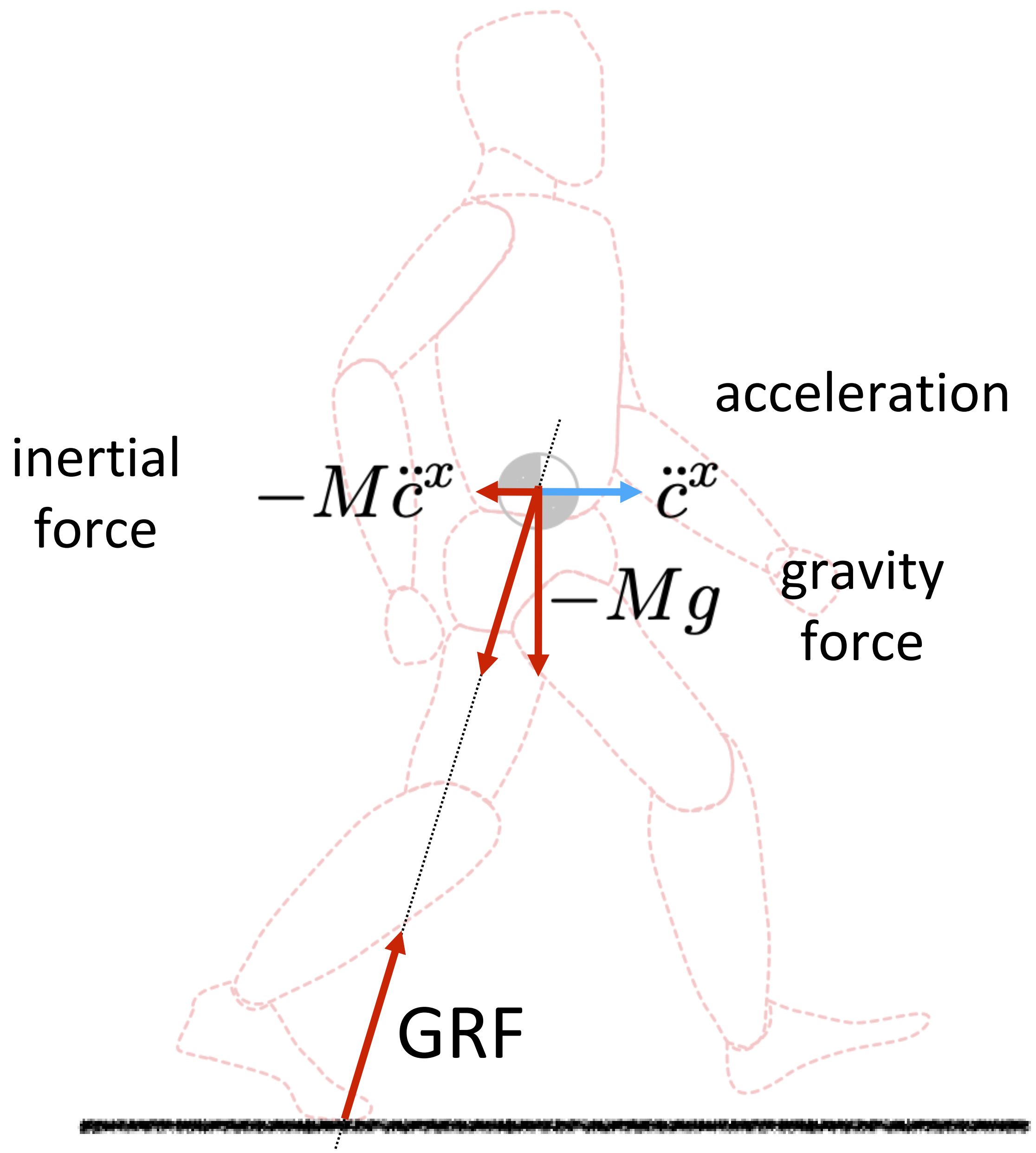


dynamic balance

humanoid walking:

the GRF will also have a component parallel to the ground;

the motion requires the exchange of horizontal frictional force with the ground



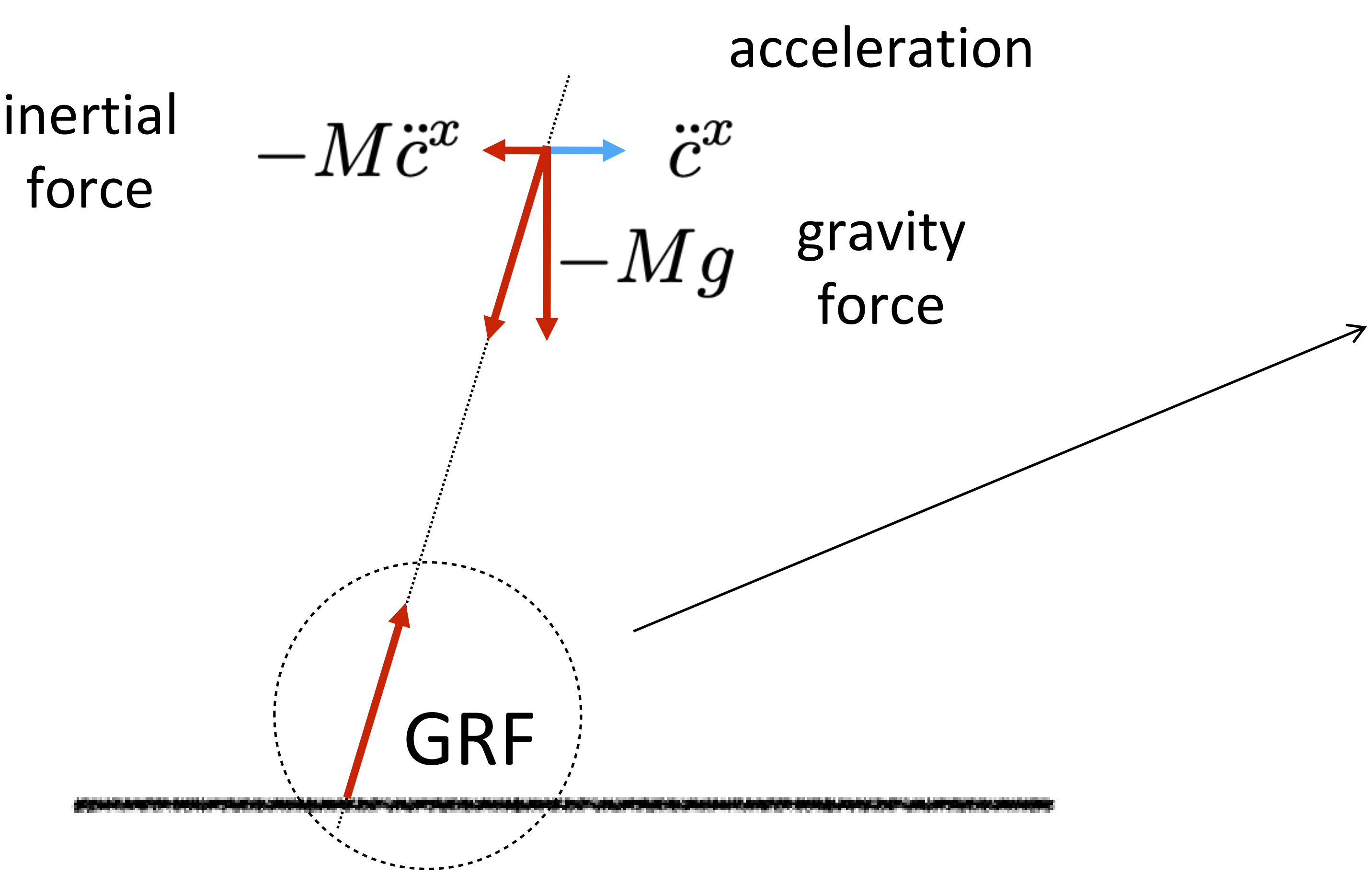
hyp: no torque around the CoM

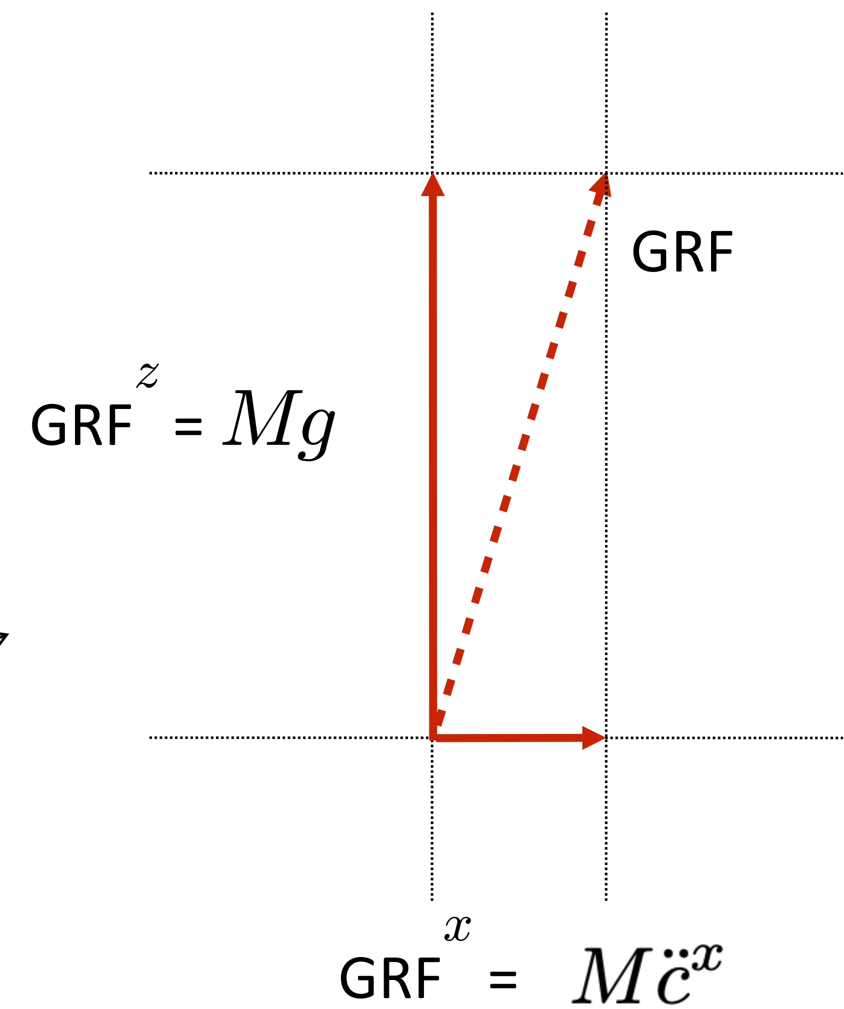
dynamic balance

Ground Reaction Force (GRF): 2D components (x,z)

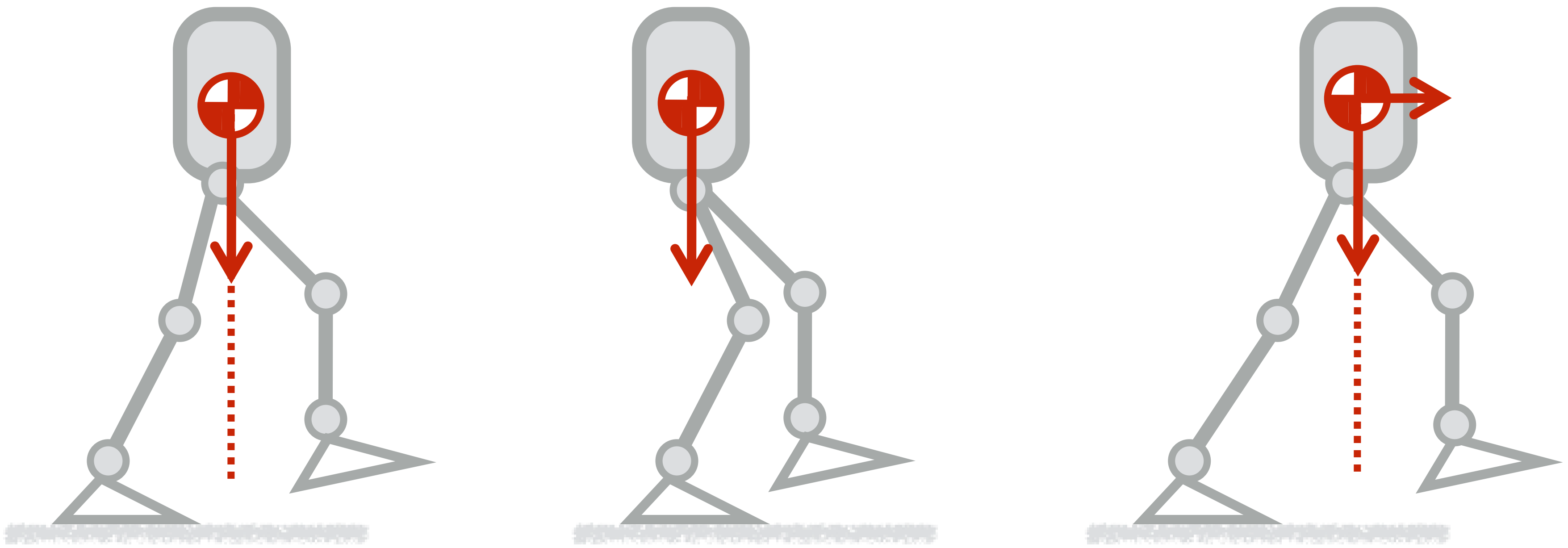
hyp: no torque around the CoM

from the previous derivation:

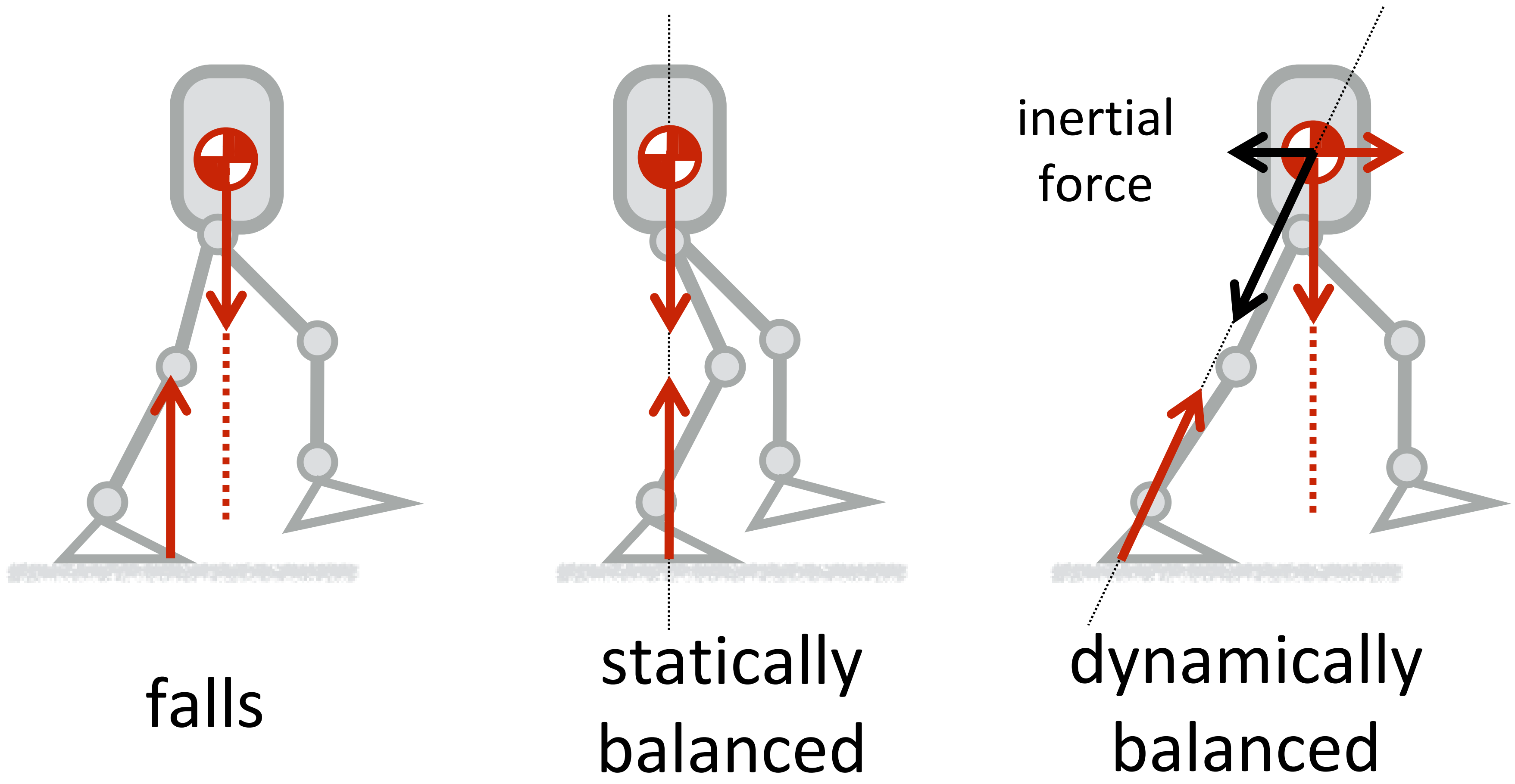




which robot falls down?



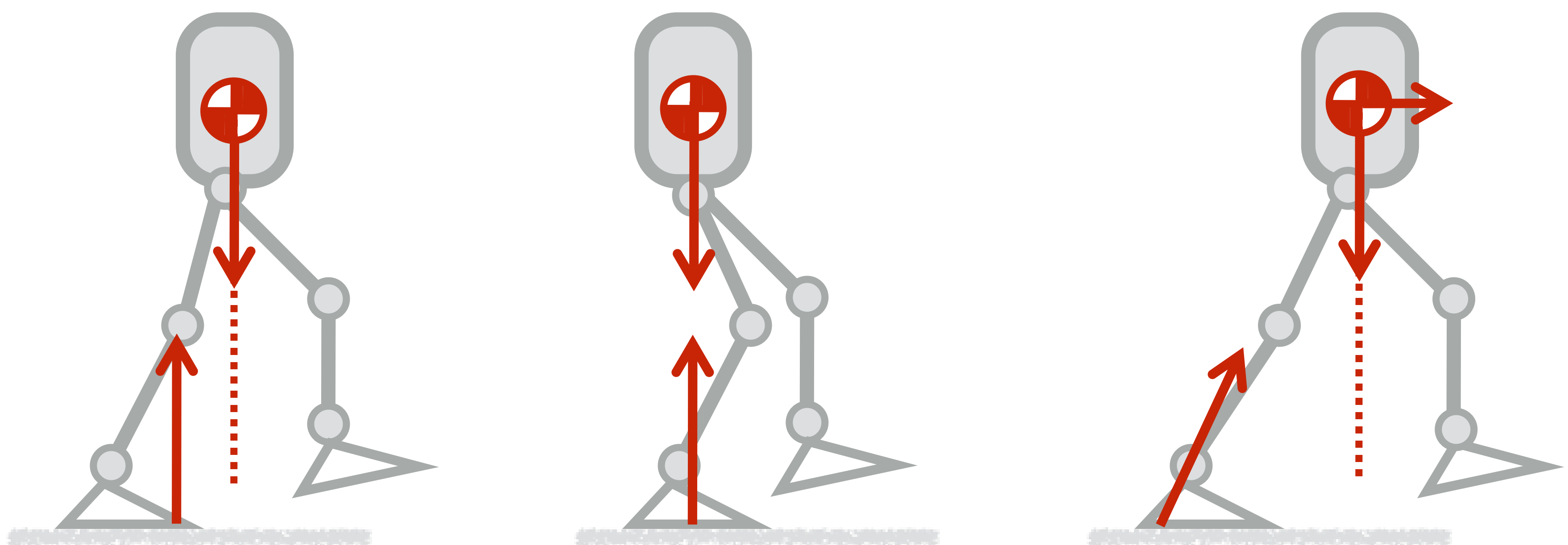
which robot falls down?



where is the ZMP?

z^x (ZMP): point on the ground where the GRF is applied

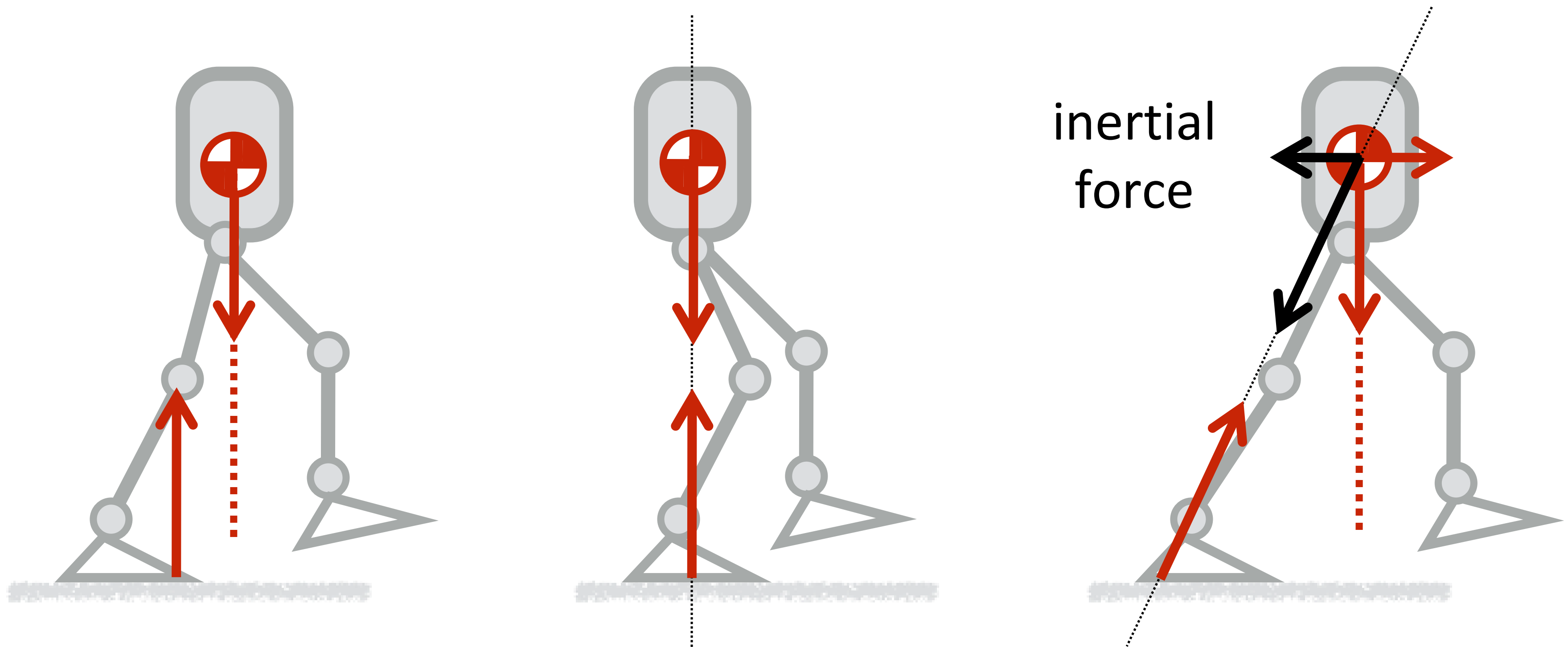
use the dynamics equation on **horizontal** flat ground and neglect $\dot{\boldsymbol{L}}^{x,y}$



$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{\boldsymbol{c}}^{x,y} + \cancel{\boldsymbol{g}^{x,y}}) = (\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}) + \cancel{\frac{\boldsymbol{S}\dot{\boldsymbol{L}}^{x,y}}{M(\ddot{c}^z + g^z)}}$$

where is the ZMP?

hyp CoM at **constant height** $c^z = \text{constant}$ $\longrightarrow$ LIP equation in the sagittal plane



falls

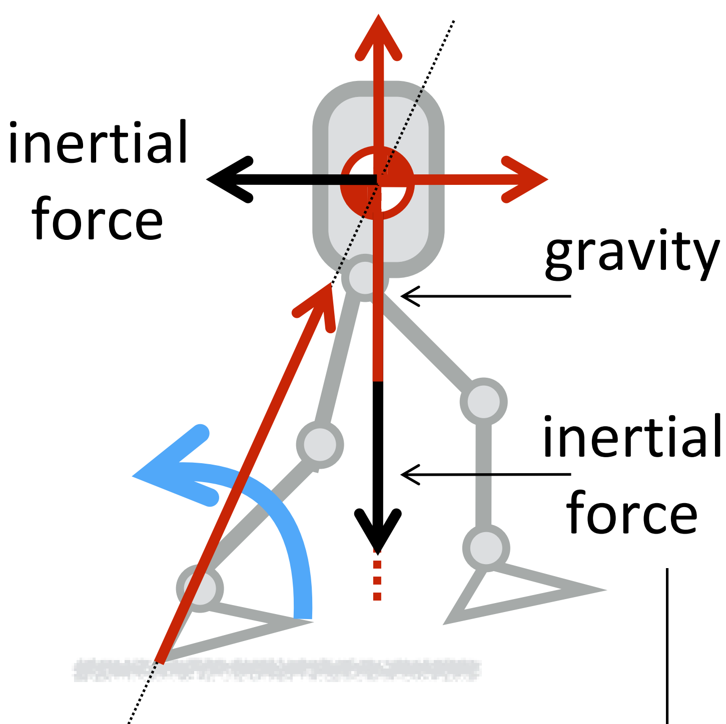
statically balanced $\quad z^x = c^x$

dynamically balanced $\quad z^x = c^x - \dfrac{c^z}{g^z}\ddot{c}^x$

what if CoM acceleration is increased?

from the previous analysis one could think that the ZMP, increasing the CoM acceleration, would leave the support foot support, but it doesn't

- once the ZMP has reached the foot border, a rotation starts around that point
- with the rotation of the foot, the center of mass starts accelerating vertically
- with a vertical acceleration of the CoM, its height does not remain constant
- model changes, ZMP remains constant



$$z^x = c^x - \frac{c^z}{\ddot{c}^z + g^z}\ddot{c}^x$$

the vertical CoM acceleration generates a vertical inertial force

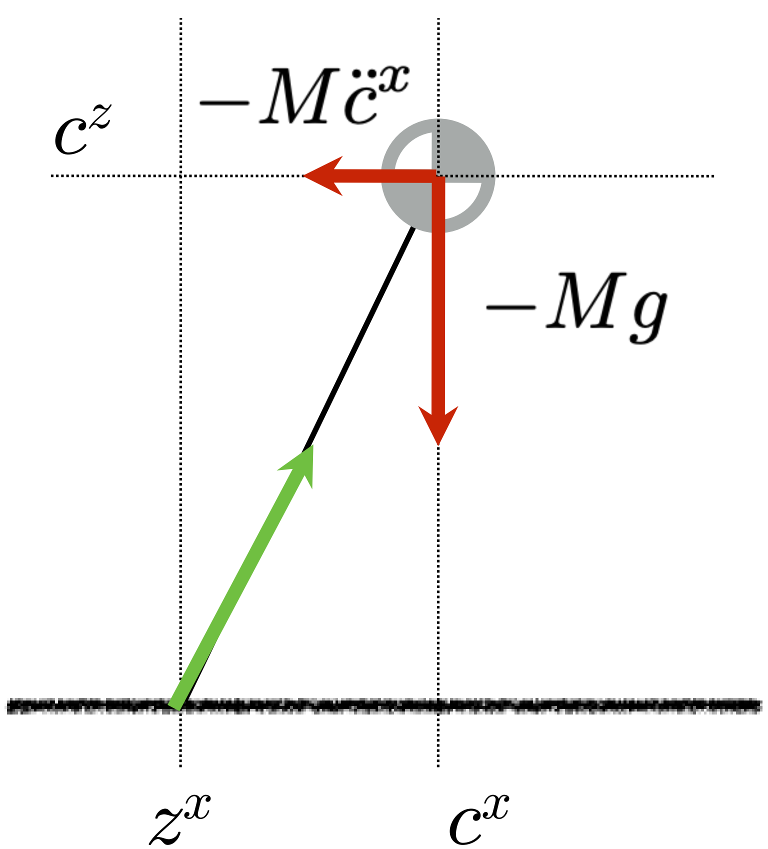
dynamically balanced

Hyp: no torque around the CoM

sum of moments around z^x

$$-Mg(c^x - z^x) + M\ddot{c}^x c^z = 0$$

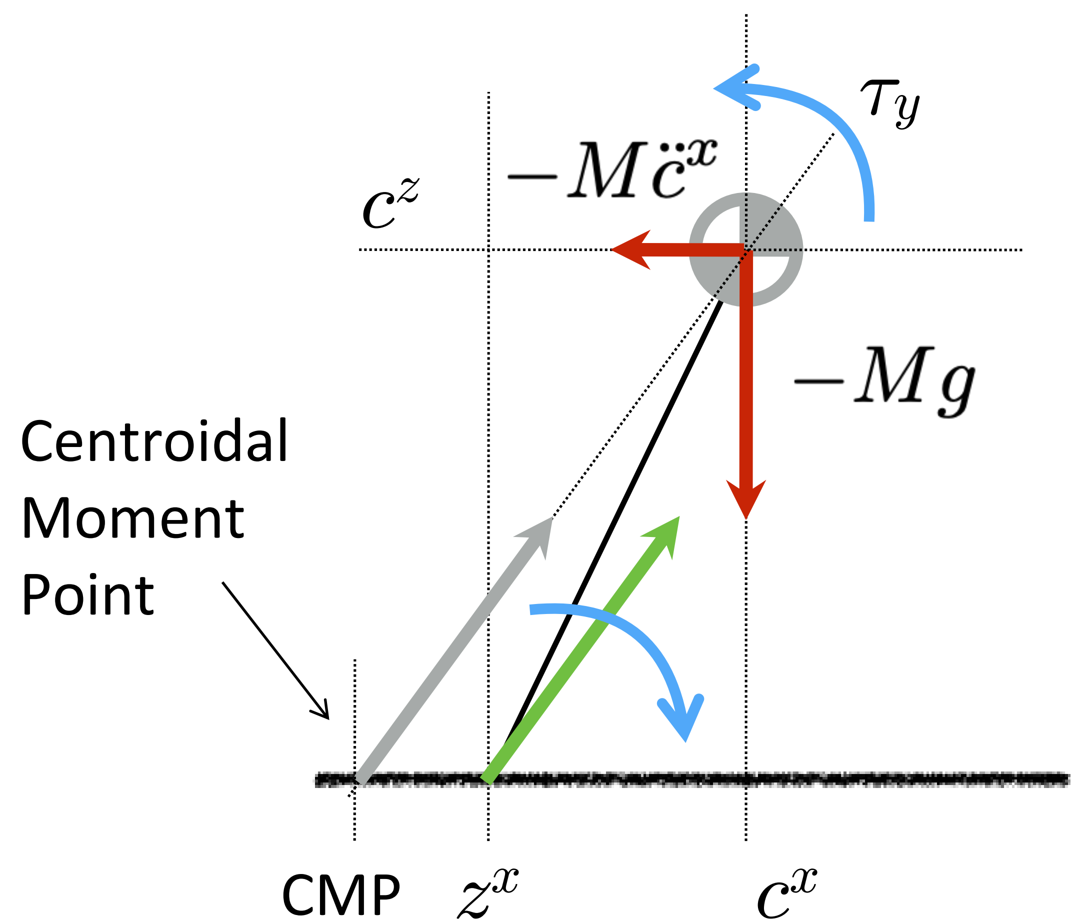
$$\longrightarrow \qquad z^x = c^x - \frac{c^z}{g^z}\ddot{c}^x$$



+ torque τ_y around the CoM

$$-Mg(c^x - z^x) + M\ddot{c}^x c^z + \tau_y = 0$$

$$\longrightarrow \qquad z^x + \frac{\tau_y}{Mg} = c^x - \frac{c^z}{g^z}\ddot{c}^x$$



dynamically balanced

+ torque τ_y around the CoM (or equivalently an ankle torque τ_y)

$$-Mg(c^x - z^x) + M\ddot{c}^x c^z + \tau_y = 0$$

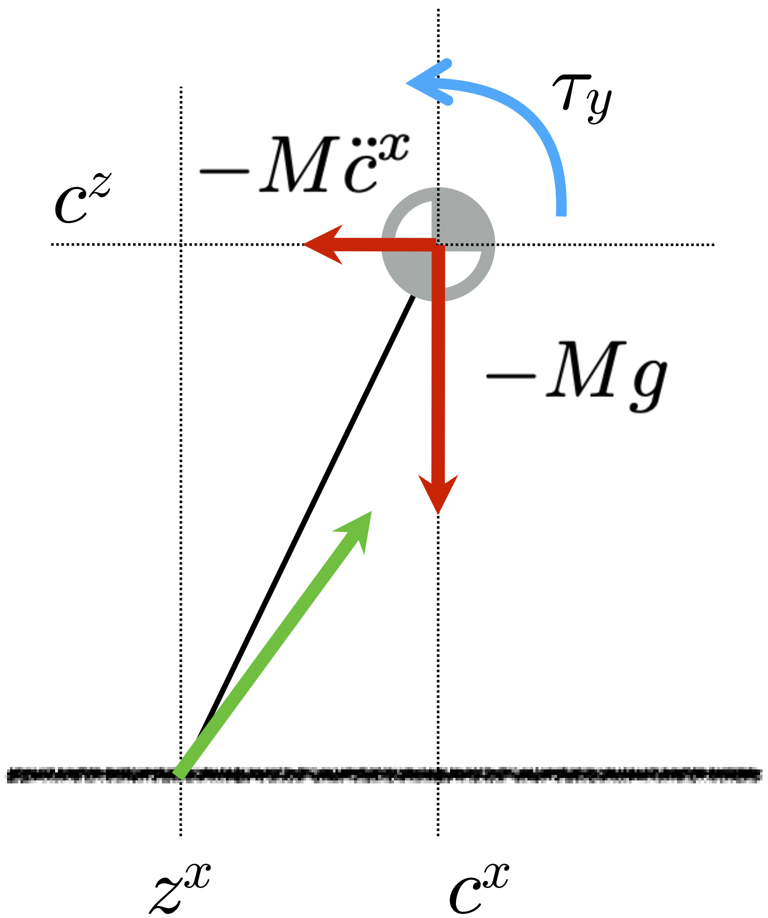
$$\longrightarrow \quad z^x + \frac{\tau_y}{Mg} = c^x - \frac{c^z}{g^z}\ddot{c}^x$$

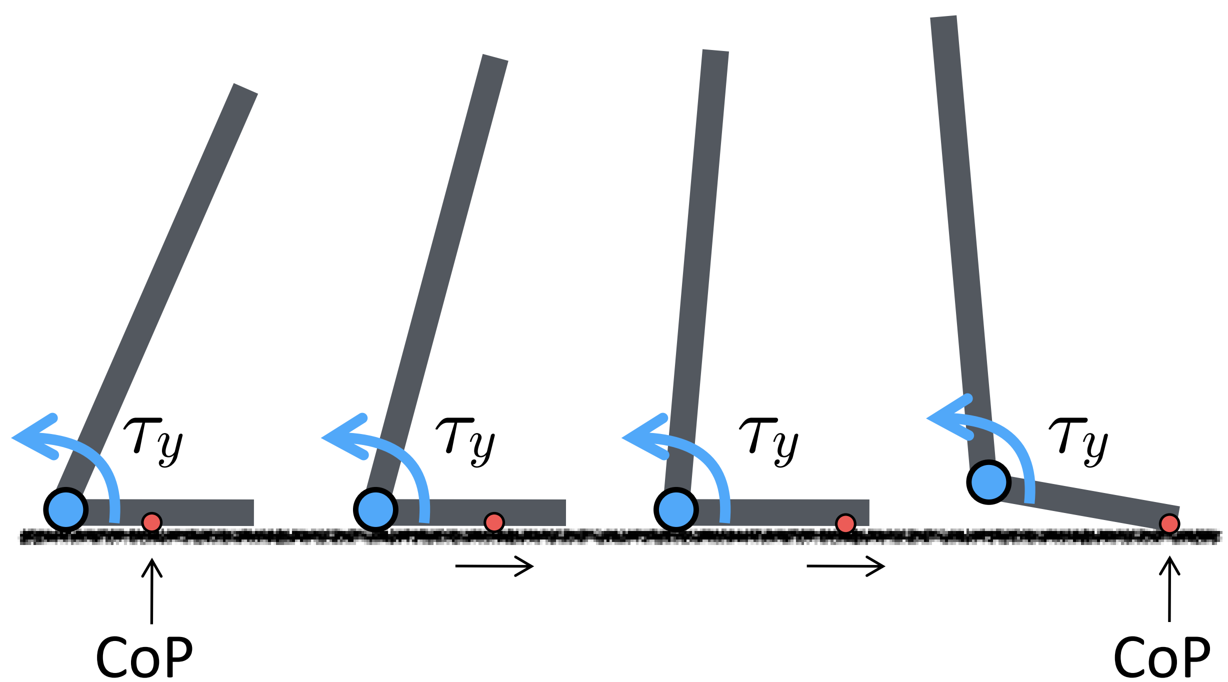
positive torque τ_y (counter-clockwise)

moves the Center of Pressure CoP to the right

z^x is not the CoP anymore

$$\text{CoP} = z^x + \frac{\tau_y}{Mg}$$





dynamically balanced locomotion

generate a gait for walking while maintaining balance

maintaining balance assumed to be equivalent to **not tipping over the support foot**

↓

ZMP-based criterion

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ZMP needs to stay inside the Support Polygon