AMR
14.10.2020

Mechanics of mobile Robots

- Wheels & Kinematic structures
- Constraints and mobility
- Nonholonomic constraints
- Rolling coh (wheels)

Ground locomotion

contact and ground via → wheels / → legs

wheels → wheeled mobile robot (WMR) →

→ 1 rigid body (base, chassis) + wheels

legs → legged (mobile) robot (LR) →

→ several rigid bodies (arms, torso)

some of them: feet

Balance (not falling) → statical / → dynamical

- The CoM (centre of mass) of the robot falls (once projected) within support polygon.

Support polygon → the convex hill of the conduct surfaces.

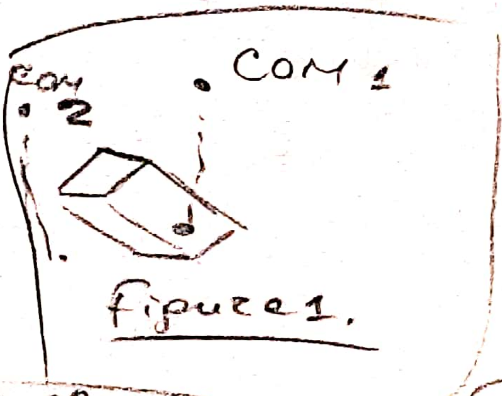


Figure 1.

COM 1 → doesn't fall, COM 2 → falls

Note: Dynamical balance will be analysed later.

- Balance in WMR's

each wheel → 1 part contacts with ground

To get an actual support polygon, need 3 wheels.

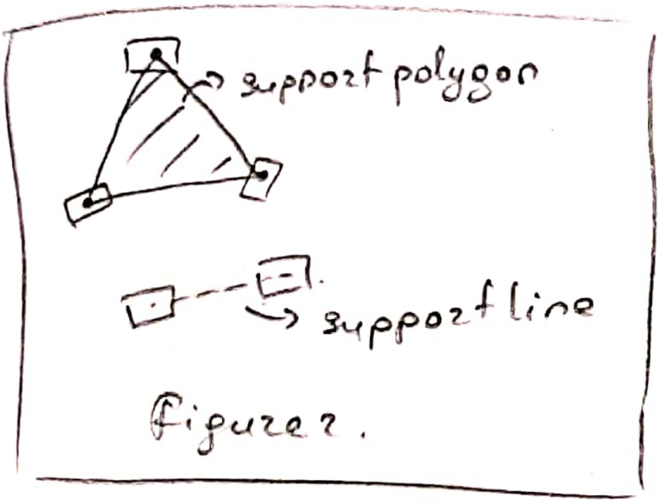


figure 2.

Whenever we have 3 wheels, we need suspensions (which make robot be able to move in vertical direction).

Wheels.

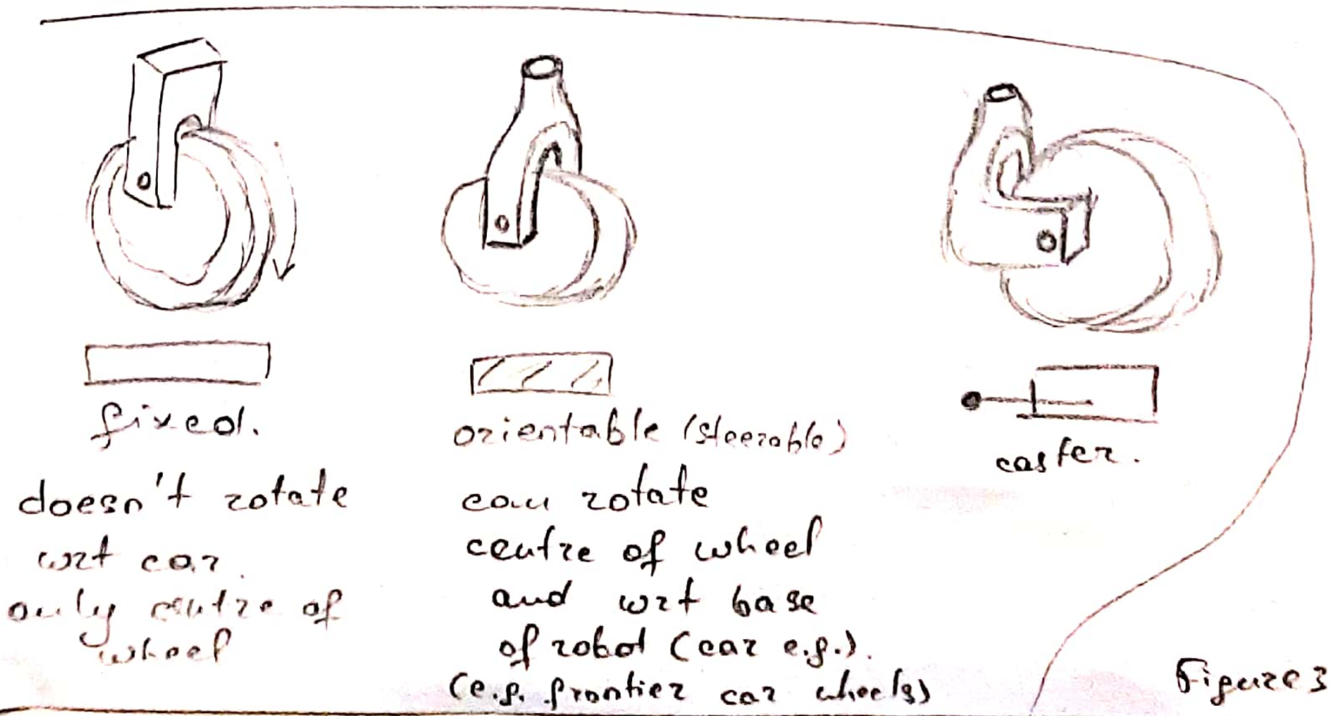


figure 3

Kinematic structures.

Differential Drive mobile robot:

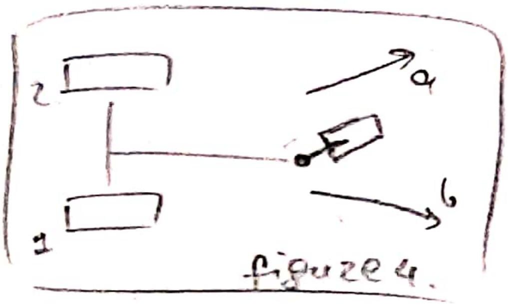


figure 4.

2 fixed → motion
1 - caster → balance

DD means : it has 2 motors : one per each fixed wheel. They are driven independently.

For example, wheel 1's velocity is bigger than wheel2's velocity → it will go in the direction a, in reverse to b.

Synchro-Drive mobile robot

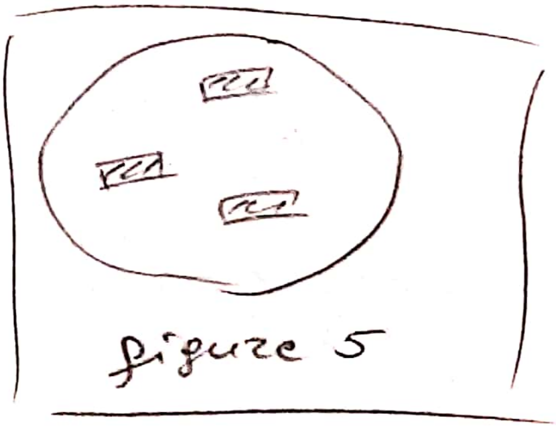


figure 5

Tricycle.

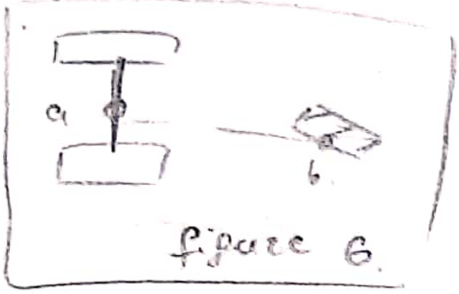


figure 6.

2 motors a and b.

- Constraints and mobility

robot configuration $q \in C$; $C \simeq \mathbb{R}^n$

locally looks like

constraints:

- geometrical $h(q) = 0$ (GC)
- Kinematic $a(q, \dot{q}) = 0$. (KC)

- Geometric Constraints

$h_i(q) = 0$ $i = 1, \ldots, k$ → k $(<n)$ constraints

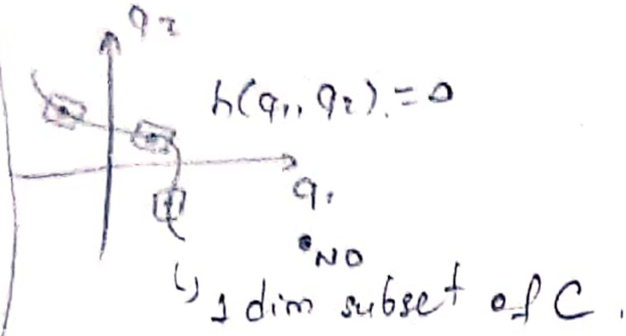
↳ depends on only values q.

assumption

It has effect on mobility

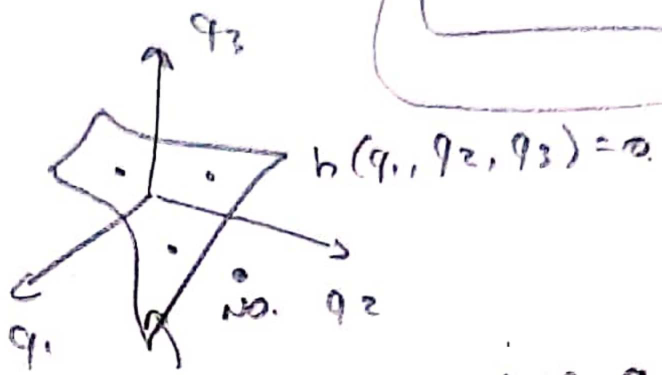
a mobility limitation → the robot can only configurations in C that satisfy (GC)

i.e. a subset of C of dimension $n-k$



train on tracks.

figure 1



2 dim subset of C

figure 2

Note. Robot can't move in given C to any point. It can move only to points of given subset (thus there're some points as NO).

Can we redefine C so that we only use n-k coordinates?

Implicit function theorem

$h(q) = 0$
it is locally possible to make it explicit as

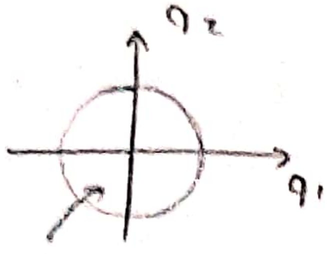
$$q_1 = \varphi(q_2, \ldots, q_n)$$

provided that

some nonsingularity conditions

• example:

$q \in \mathbb{R}^2$ $\qquad q_1^2 + q_2^2 = 1$ $\rightarrow$ it is a GC.



unitcircle.

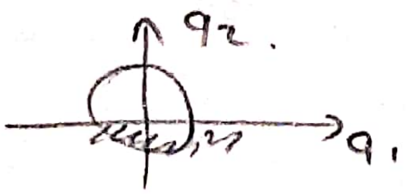
We can define q_2 in terms of q_1:

$$q_2 = \pm\sqrt{1 - q_1^2}$$

When we know q_1 we can define location of robot. Because if we know q_1, we'll also know q_2.

But it isn't global solution it is local solution.

If we eliminate negative part of q_2:



$\rightarrow$ only + in the uppon half circle

How can we solve it?

Better way is to change variables;

In this case: S (arc length) or. φ (phase angle)

$\rightarrow$ as we did for the manipulators.

Kinematic Constraints

$$a_i(q, \dot{q}) = 0 \quad ; \quad i = 1, \dots, k$$

- involve q, $\dot{q}$.

- main common behaviour of this kind of robots is typically linear in $\dot{q}$.

$$a_i^T(q)\,\dot{q} = 0$$

$$(a_1(q) \;\dots\; a_n(q))^T\,\dot{q} = 0$$

arrange them like this: (matrix)

$$\begin{array}{c} \uparrow \\ k \\ \downarrow \end{array} \begin{bmatrix} \text{---} & a_1^T(q) & \text{---} \\ \text{---} & a_2^T(q) & \text{---} \\ & \vdots & \\ \text{---} & a_k^T(q) & \text{---} \end{bmatrix} \dot{q} = 0 \implies A^T(q)\,\dot{q} = 0$$

$\longleftarrow n \longrightarrow$

$\downarrow$

Pfaffen form of KC

(linear in $\dot{q}$)

- mobility limitation: depends on q and $\dot{q}$, together
  at each q, the admissible $\dot{q}$ must belong to $N(A^T(q))$

  a linear space of dimension $n-k$.
  ↳ hyperplane

→ This is local mobility limitation
It limits motion form of ~~and~~ configuration not configuration ~~...~~ themselves)

Relationship GC / KC.

a GC => a KC.

(always impacts)

$h(q) = 0$ → continuous satisfaction $\frac{dh(q)}{dt} = 0$.

$$\frac{dh}{dt} = \frac{\partial h}{\partial q} \dot{q} = 0.$$

$$\underbrace{\left(\frac{\partial h}{\partial q_1} \cdots \frac{\partial h}{\partial q_n} \right)}_{\text{the transpose of the gradient.}} = \nabla_q^T h$$

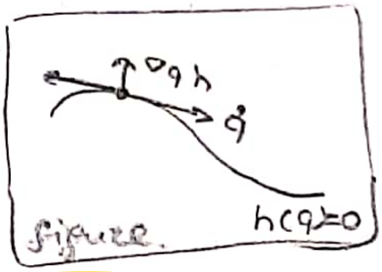
$$(\underbrace{\nabla_q^T h}_{a(q)\ \leftarrow\ \text{function of } q.}) \dot{q} = 0 \quad \Rightarrow \quad a(q)\dot{q} = 0$$

→ Pfaffian constraint.

→ KC.

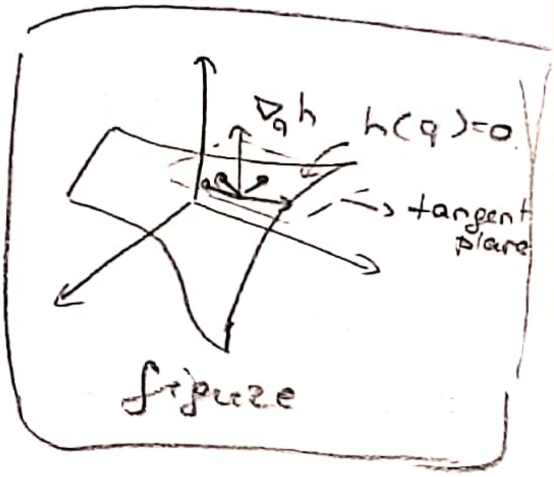
Geometrical interpretation

If we have GC $h(q)=0$, then $\dot{q}$ must be orthogonal to $\nabla_q^T h$



$\nabla_q h$ is orthogonal to surface. Velocity is orthogonal to gradient's transpose. To sum up, velocity must be tangent to the surface

In $\mathbb{R}^3$:



Tangent plane → is a plane that tangent to $h(q)=0$ ~~plane~~ surface. Velocity will be on that plane. Other vectors can't be velocity.

Note
a KC $\not\Rightarrow$ a GC → does not always imply (it may, it may not)

Now we will try to get GC by using KC.

$$q_1\dot{q}_1 + q_2\dot{q}_2 = 0 \quad \text{in } \mathbb{R}^2$$

$$\underbrace{(q_1 \;\; q_2)}_{A^T(q)} \underbrace{\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}}_{\dot{q}} = 0 \Rightarrow A^T(q)\,\dot{q} = 0.$$

↳ Pfaffian KC

Can the given constraint be written as $h(q)=0$, for some h (a GC)

i.e. is it integrate?

Yes: $q_1^2 + q_2^2 = C_1$

$\rightarrow 2\dot{q}_1 q_1 + 2\dot{q}_2 q_2 = 0$.

$\dot{q}_1 q_1 + \dot{q}_2 q_2 = 0$ can be integrated as $q_1^2+q_2^2=C$

C is integration constant → we can't put value to that.

It's seen that ~~the~~ resulting GC is a circle. It says that, robot will always move along the circle which has radius $\sqrt{C}$.

$\dot{q}_1 q_1 + \dot{q}_2 q_2 = 0$ → a local mobility limitation

$q_1^2 + q_2^2 = C$ → global mobility limitation

Initial condition affects result of integration.

Thus, if robot is on the centre, it can't move. Because, it must satisfy constraints.

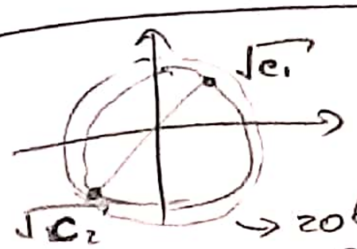


figure.

• example .

$q_1 \dot{q}_2 + q_2 \dot{q}_1 = 0$

$(q_2 \ \ q_1) \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = 0 \rightarrow$ integrable ? $\rightarrow$ YES,

$\boxed{q_1 q_2 = C}$ $\rightarrow$ global mobility limitation

$\boxed{q_1 \dot{q}_2 + \dot{q}_1 q_2 = 0}$ $\rightarrow$ local mobility limitation

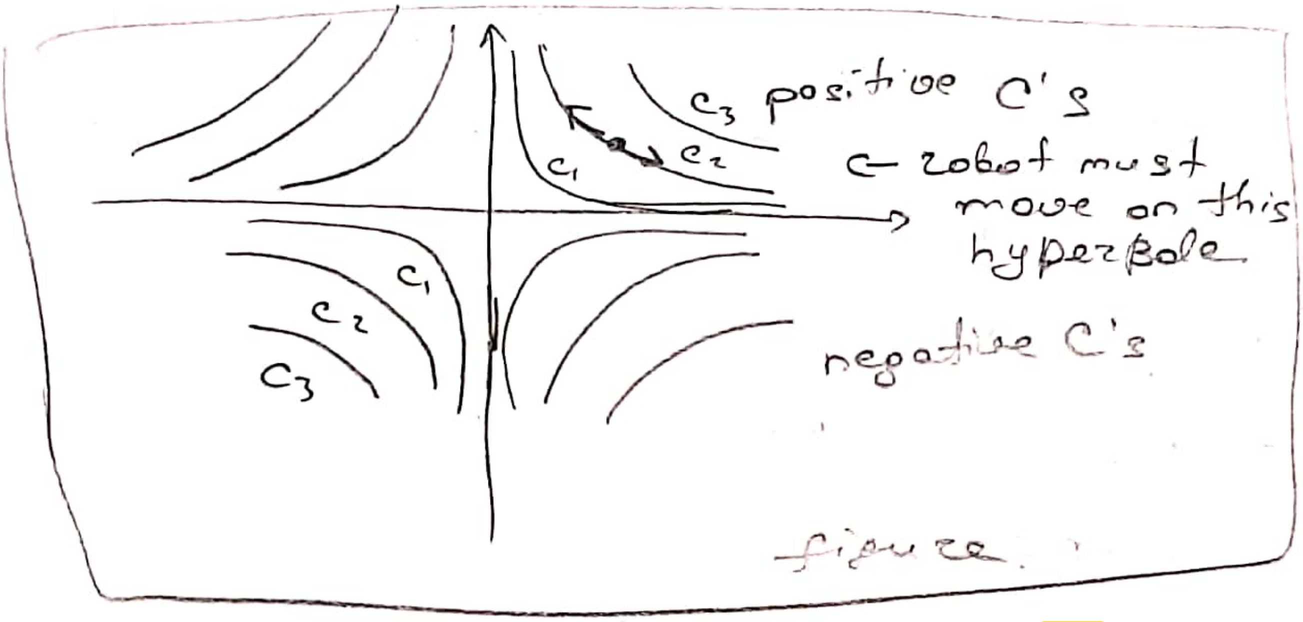


figure.

These cases are called as **FOLIATION of $\mathbb{R}^2$**.

Example :

constraint that can't be integrable:

in $\mathbb{R}^3$

$$\underbrace{(\sin q_3 \ \ -\cos q_3 \ \ 0)}_{A^T(q)} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = 0$$

$\sin q_3 \dot{q}_1 - \cos q_3 \dot{q}_2 = 0$

is it integrable ? $\rightarrow$ NO !

In this case we have to find non-integrable constraint recapitulate

$a^T(q)\dot{q} = 0 \rightarrow$ a KC (pfaffian)

• if there exists a $h(q)$ such that

$$\frac{\partial h^T}{\partial q} = \gamma(q)\, a^T(q) = 0 \; (\gamma(q) \neq 0)$$

Then KC can be integrated as $h(q) = c$.

Why? If $\gamma(q) \neq 0$, in order to satisfy constraint $a^T(q)$ will have some combinations. ⟸ we put constraint on $a^T(q)$.

In this case KC is called

INTEGRABLE OR HOLONOMIC

* Global mobility limitation.

• if there exists no such function, then KC can't be interpreted. KC is called

NON-INTEGRABLE or NON-HOLONOMIC.

↳ THE KC remains a purely local mobility limitation

How to tell if

$$a_i^T(q)\,\dot{q} = 0 \quad \text{is} \begin{cases} H \\ NH \end{cases} ?$$

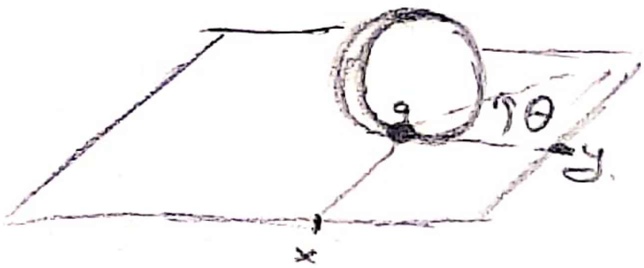
- One mathematical tool for that is Frebenious Theorem

~~interp~~

- a different newpoint.
  let's look at mobility (global). ask ourselves can the ~~blost~~ robot go anywhere?

Example

Rolling coin



We need to define configuration of robot.

$a \to$ ground point

$\theta \to$ ~~rot~~ rot wrt ~~y~~ x.

generalized coordinates: $q = [x \; y \; \theta]^T$

pure rolling (or rolling without slipping)

$$\dot{x}\sin\theta - \dot{y}\cos\theta = [\sin\theta \quad -\cos\theta \quad 0]\,\dot{q} = 0$$