

Autonomous and Mobile Robotics

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Localization I

Odometric Localization

DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI



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- planning and feedback control require the knowledge of the **robot configuration q**
- in **robot manipulators**, joint encoders provide a direct measure of q
- **WMRs** are equipped with incremental encoders that measure only the rotation of the wheels, **not** the position and orientation of the vehicle
- **localization** is a procedure for estimating the robot configuration q , typically in real time

- consider a unicycle under **constant** velocity inputs v_k, ω_k in $[t_k, t_{k+1}]$, as in a digital control implementation; in each sampling interval, the robot moves along an arc of circle of radius v_k/ω_k (a line segment if $\omega_k=0$)
- assume q_k, v_k and ω_k are known; compute q_{k+1} by **integration** of the kinematic model over $[t_k, t_{k+1}]$
- first possibility: **Euler integration**

$$x_{k+1} = x_k + v_k T_s \cos \theta_k$$

$$y_{k+1} = y_k + v_k T_s \sin \theta_k \quad T_s = t_{k+1} - t_k$$

$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- x_{k+1} and y_{k+1} are **approximate**; θ_{k+1} is **exact**

- second possibility: 2nd order **Runge-Kutta integration**

$$x_{k+1} = x_k + v_k T_s \cos \left(\theta_k + \frac{\omega_k T_s}{2} \right)$$

$$y_{k+1} = y_k + v_k T_s \sin \left(\theta_k + \frac{\omega_k T_s}{2} \right)$$

$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- the **average orientation** during $[t_k, t_{k+1}]$ is used
- as a consequence, x_{k+1} and y_{k+1} are still approximate, but **more accurate**

- third possibility: **exact integration**

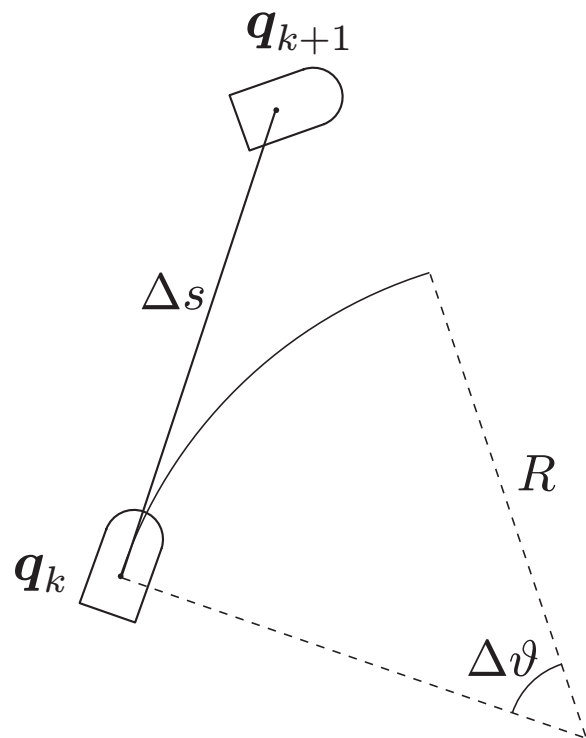
$$x_{k+1} = x_k + \frac{v_k}{\omega_k} (\sin \theta_{k+1} - \sin \theta_k)$$

$$y_{k+1} = y_k - \frac{v_k}{\omega_k} (\cos \theta_{k+1} - \cos \theta_k)$$

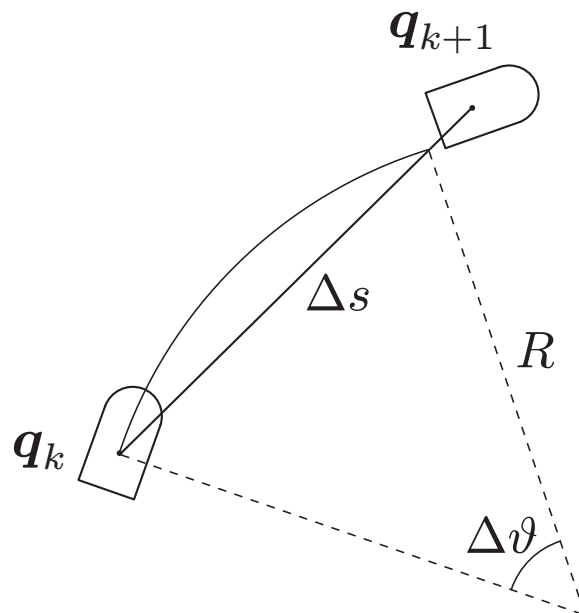
$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- for $\omega_k=0$, x_{k+1} and y_{k+1} are still **defined** and coincide with the solution by Euler and Runge-Kutta
- for $\omega_k \approx 0$, a **conditional instruction** may be used in the implementation

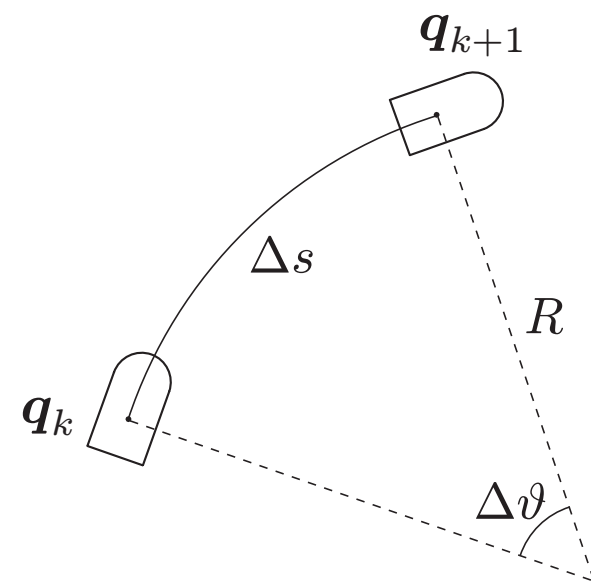
geometric comparison



Euler



Runge-Kutta



exact

- in practice, due to the non-ideality of any actuation system, the commanded inputs v_k and ω_k are **not** used
- instead, measure the effect of the actual inputs:

$$v_k T_s = \Delta s \quad \omega_k T_s = \Delta \theta \quad \frac{v_k}{\omega_k} = \frac{\Delta s}{\Delta \theta}$$

Δs (traveled length) and $\Delta \theta$ (total orientation change) are reconstructed **via proprioceptive sensors**

- for example, for a **differential-drive** robot

$$\Delta s = \frac{r}{2} (\Delta \phi_R + \Delta \phi_L) \quad \Delta \theta = \frac{r}{d} (\Delta \phi_R - \Delta \phi_L)$$

where $\Delta \phi_R$ and $\Delta \phi_L$ are the total rotations measured by the **wheel encoders**

- maintaining an estimate of the robot configuration by iterative integration of the kinematic model is called **odometric localization** or **dead reckoning**
- subject to an error (odometric **drift**) that grows over time, becoming significant over sufficiently long paths
- causes include **wheel slippage** (model perturbation), **inaccurate calibration** of, e.g., wheel radius (model uncertainty) or **numerical integration error**
- **effective** localization methods use proprioceptive as well as **exteroceptive** sensors