Autonomous and Mobile Robotics

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Wheeled Mobile Robots 5

Motion Control of WMRs: Regulation

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regulation

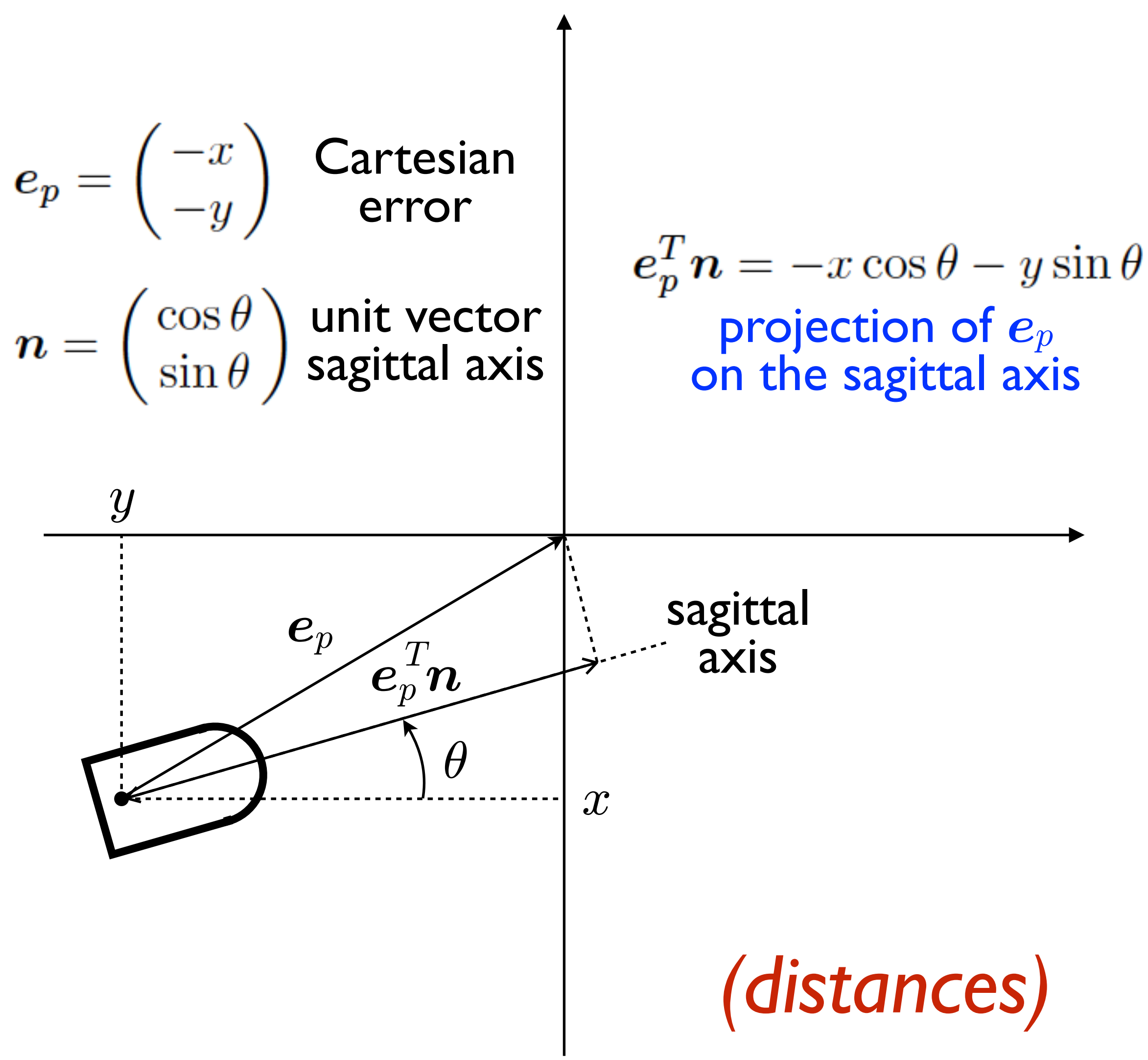
- drive the unicycle to a desired configuration $\boldsymbol{q}_d$

- the obvious approach (choose a path/trajectory that stops in $\boldsymbol{q}_d$, then track it via feedback) does not work:
  - linear/nonlinear controllers based on the error dynamics require persistent trajectories
  - i/o linearization leads point B to the destination rather than the representative point of the unicycle

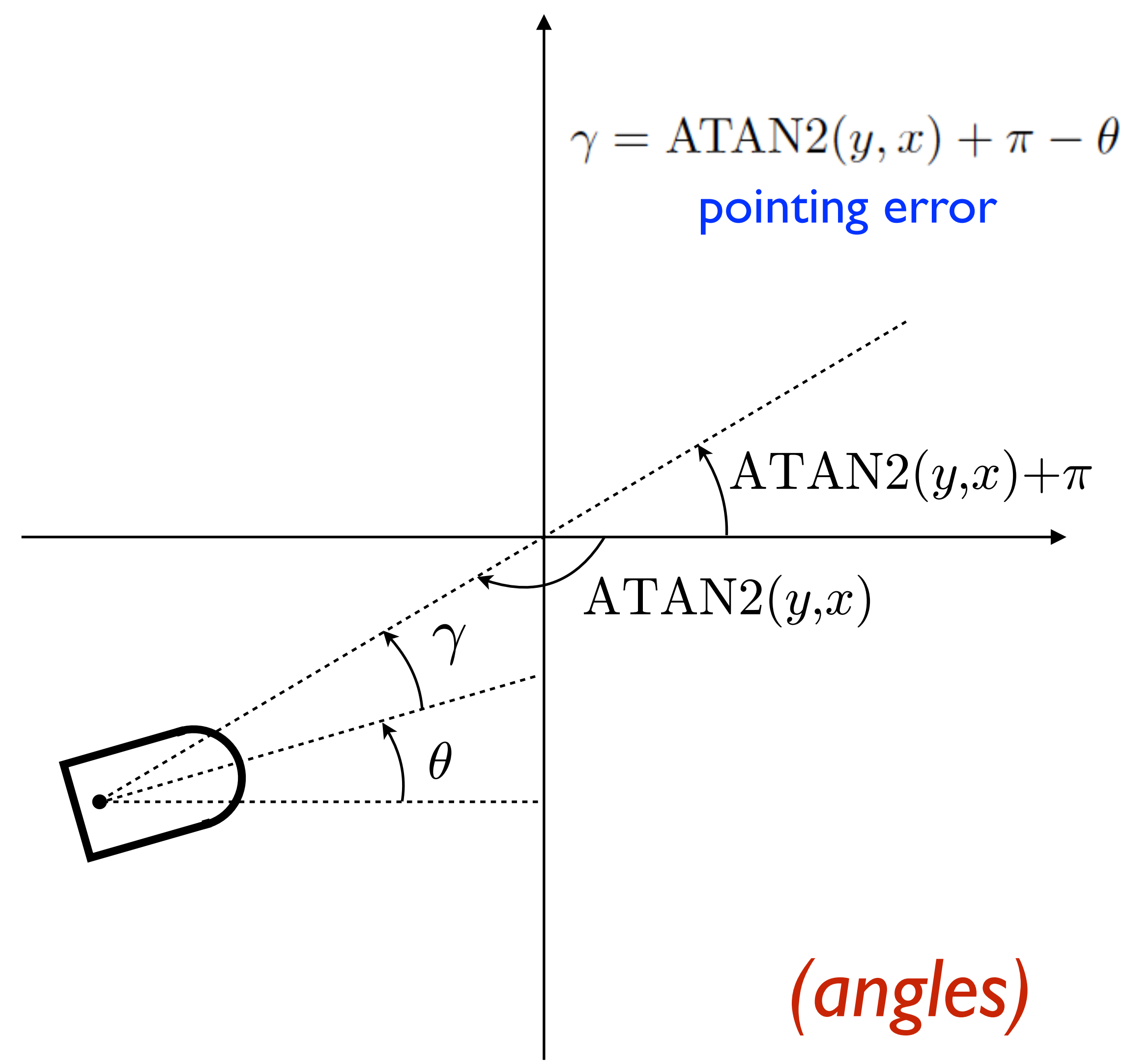
- being nonholonomic, WMRs (unlike manipulators) do not admit universal controllers, i.e., controllers that can stabilize arbitrary trajectories, persistent or not

Cartesian regulation

- drive the unicycle to a given Cartesian position (w.l.o.g., the origin $(0\ 0)^T$), regardless of orientation
- geometry:



(distances)



(angles)

Cartesian regulation

- consider the feedback control law

$$v = -k_1(x\cos\theta + y\sin\theta)$$
$$\omega = k_2(\mathrm{Atan2}(y, x) - \theta + \pi)$$

- geometrical interpretation:
  - v is proportional to the orthogonal projection of the Cartesian error e_p on the sagittal axis
  - ω is proportional to the pointing error (i.e., the difference between the orientation of e_p and that of the unicycle)

- Lyapunov-like function

$$V = \frac{1}{2}(x^2 + y^2) \qquad \text{positive semidefinite}$$

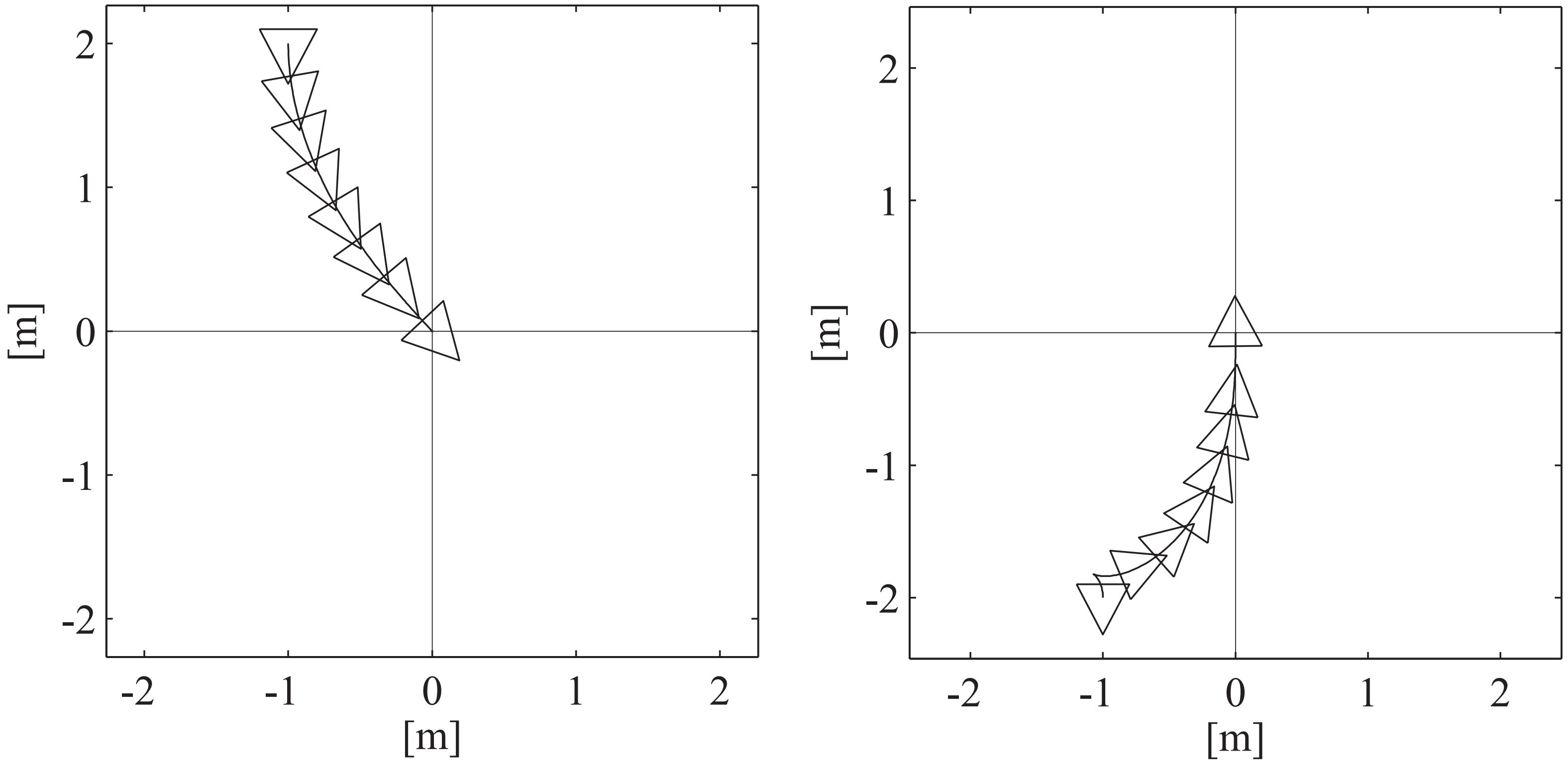
$$\dot{V} = -k_1(x\cos\theta + y\sin\theta)^2 \qquad \text{negative semidefinite}$$

- cannot use LaSalle theorem, but Barbalat lemma implies that $\dot{V}$ tends to zero, i.e.

$$\lim_{t \to \infty} (x\cos\theta + y\sin\theta) = 0$$

- under the proposed controller, this implies that the Cartesian error goes to zero

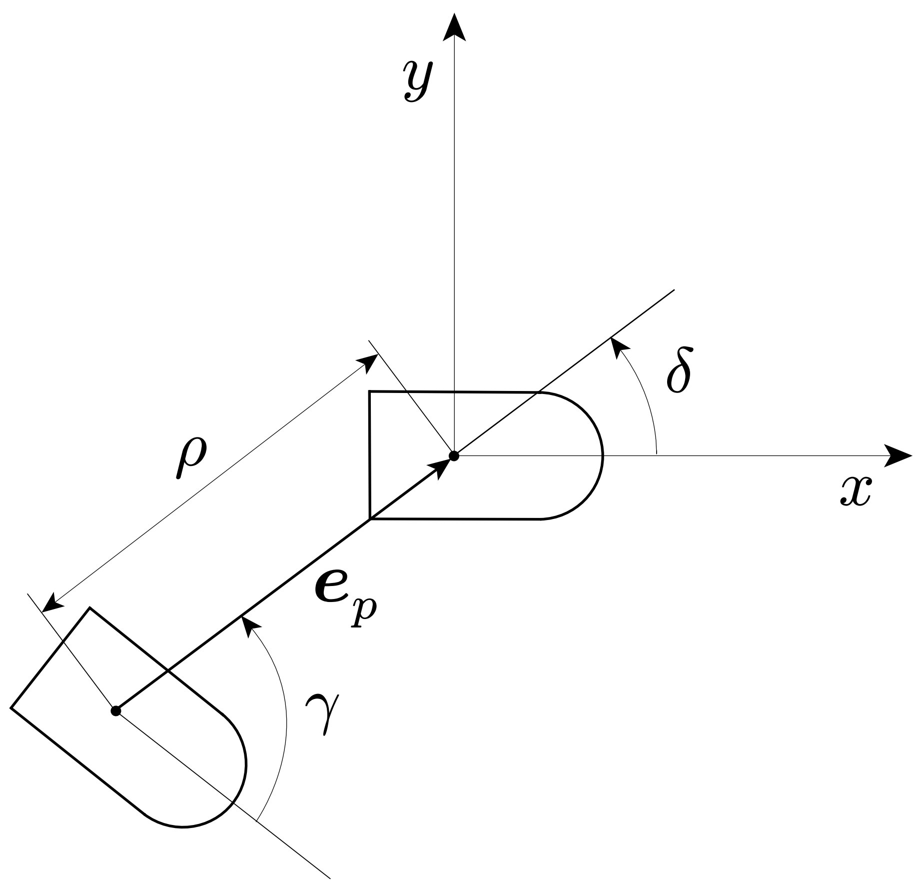
simulation



- final orientation is not controlled
- at most one backup maneuver

posture regulation

- drive the unicycle to a given configuration (w.l.o.g., the origin $(0\ 0\ 0)^T$)
- convert to polar coordinates



$$\rho = \sqrt{x^2 + y^2}$$
$$\gamma = \mathrm{Atan2}(y, x) - \theta + \pi$$
$$\delta = \gamma + \theta$$

- kinematic model in polar coordinates

$$\dot{\rho} = -v \cos\gamma$$
$$\dot{\gamma} = \frac{\sin\gamma}{\rho} v - \omega$$
$$\dot{\delta} = \frac{\sin\gamma}{\rho} v$$

note the singularity at the origin

- consider the control law (compare with previous)

$$v = k_1 \rho \cos\gamma$$

new term

$$\omega = k_2 \gamma + k_1 \frac{\sin\gamma \cos\gamma}{\gamma} (\gamma + k_3 \delta)$$

- Lyapunov candidate

$$V = \frac{1}{2}\left(\rho^2 + \gamma^2 + k_3\,\delta^2\right) \qquad \text{positive definite}$$

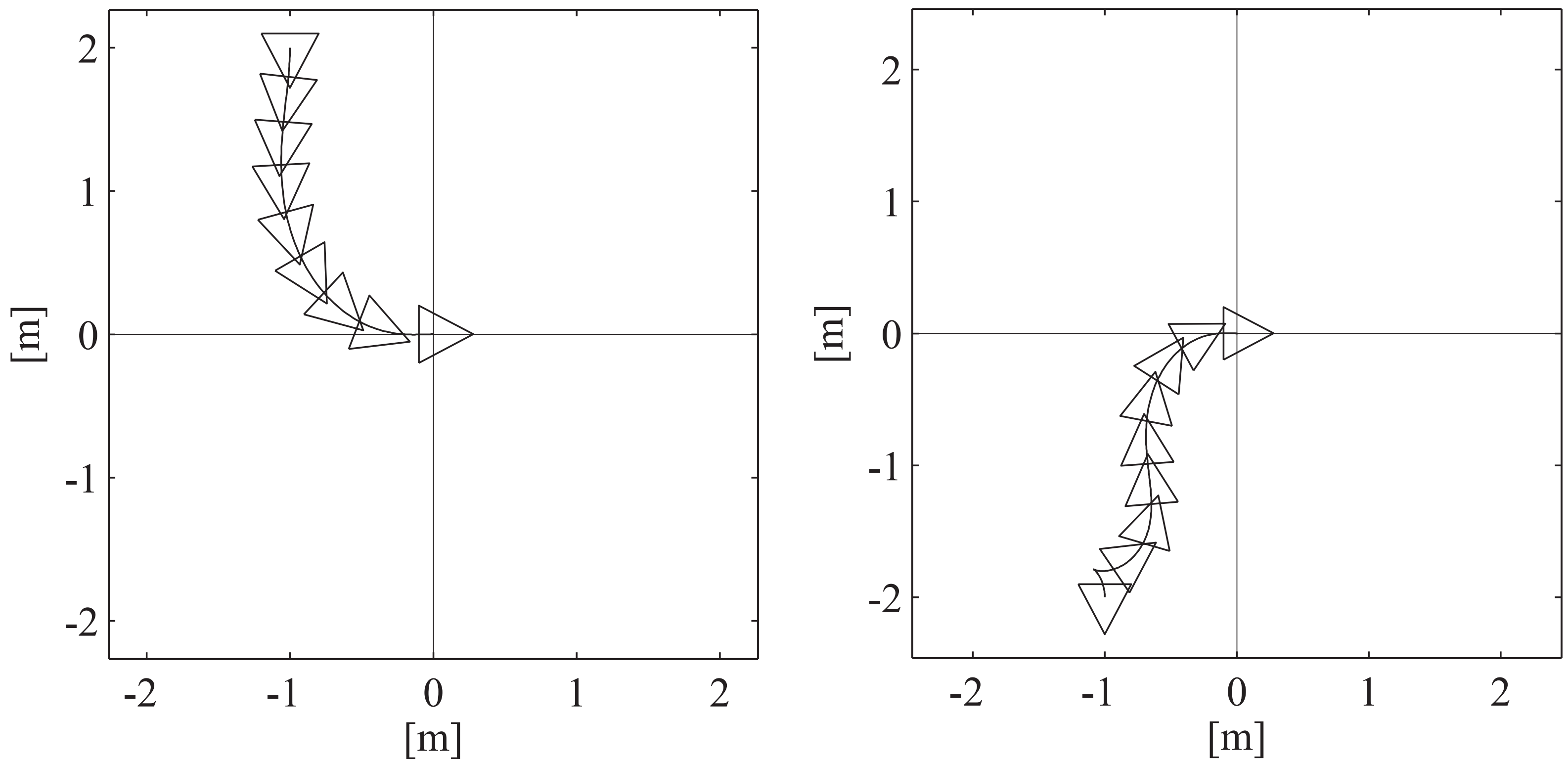
$$\dot{V} = -k_1 \cos^2\!\gamma\,\rho^2 - k_2\,\gamma^2 \qquad \text{negative semidefinite}$$

- Barbalat lemma implies that ρ,γ and δ go to zero

- the above control law, once mapped back to the original coordinates, is discontinuous at the origin

- it can be shown that, due to the nonholonomy, all posture stabilizers must be discontinuous w.r.t. the state or time-varying (Brockett theorem)

simulation



- final orientation is zeroed as well
- at most one backup maneuver