

PMR, Quiz 01

SUMMER SEMESTER 2017

1. We are given two events A and B , such that $P(A) = 0.7$, $P(B) = 0.5$, and $P(AB) = 0.2$. Are the events independent? Explain why/why not.

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2. Let us assume that we have a robot whose bumper sensor is noisy; in particular, the sensor correctly reports a bump 90% of the time, while it reports a measurement when the robot isn't bumping into anything 20% of the time. For a given environment and a predefined motion of the robot, we are given that the probability of bumping into something is 30%. Suppose that the sensor has returned a measurement. What is the probability that the robot has bumped into something?

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3. Suppose that we are given two discrete random variables X and Y , whose probability mass functions are defined to be

$$f(x) = \begin{cases} \frac{1}{2}, & x = 0, 1 \\ 0, & \text{otherwise} \end{cases} \quad f(y) = \begin{cases} \frac{1}{4}, & y = 0, 1 \\ \frac{1}{2}, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

Assuming that X and Y are independent, calculate:

(a) $P(X = 1, Y = -1)$

(b) $P(X = 1, Y = 1)$

(c) $P(X = 0, Y = 2)$

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4. Calculate the expected value of the discrete random variable X whose probability mass function is given as follows:

$$f(x) = \begin{cases} \frac{1}{8}, & x = 1, 2 \\ \frac{1}{4}, & x = 0, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

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5. Given $P(A) = 0.2$, $P(B) = 0.4$, and $P(A \cap B) = 0.3$, calculate $P(A \cup B)$.

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6. Assuming that X is a continuous random variable whose probability density function is $f(x)$, what is the correct expression for calculating X 's expected value? Please circle the letter in front of your answer.

(a) $E[X] = \int x^2 f(x) dx$

(b) $E[X] = \frac{df(x)}{dx}$

(c) $E[X] = \int x f(x) dx$

(d) $E[X] = \int f(x) dx$

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7. Consider the following joint distribution of two discrete random variables X and Y :

$X \backslash Y$	1	2	3	4
1	$\frac{1}{32}$	0	$\frac{3}{32}$	$\frac{2}{32}$
2	$\frac{2}{32}$	$\frac{5}{32}$	0	$\frac{11}{32}$
3	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{2}{32}$

Calculate:

(a) $P(X = 2)$

(b) $P(Y = 3)$

(c) $P(X = 2|Y = 3)$

(d) $P(Y = 1|X = 1)$

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8. Assuming that X is a discrete random variable whose probability mass function is defined as

$$f(x) = \begin{cases} \frac{3}{8}, & x = -1, 1 \\ \frac{1}{8}, & x = -2, 2 \\ 0, & \text{otherwise} \end{cases}$$

calculate the expected value of X^2 .

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9. Assuming that X , Y , and Z are binary random variables, how many parameters are needed for describing their joint distribution uniquely

(a) if none of the variables are independent of each other?

(b) if we know that X is independent of both Y and Z ?

(c) if all three variables are independent of each other?

Justify your answers.

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10. If two random variables X and Y are correlated, is it necessarily the case that one causes the other? Justify your answer.

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Name: _____ Enrolment number: _____ Points: __ / 20