4. Making Simple Decisions

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Constraints on rational preferences
Utility

4.3 Utility Functions

Utility of money
Utility scales and utility assessment

4.4 Multi-attribute Utility Functions

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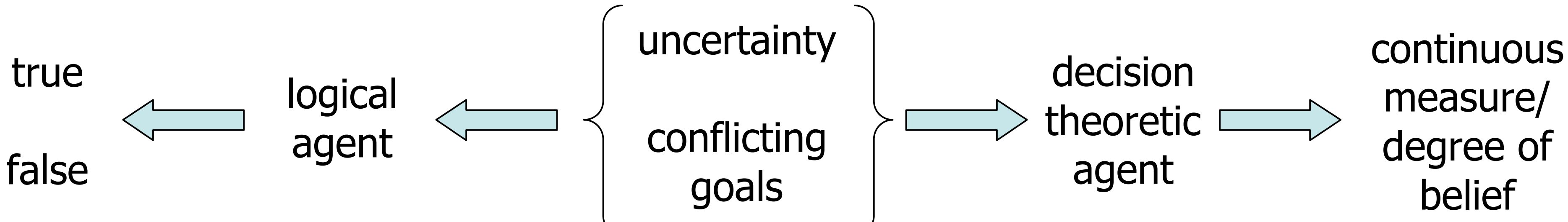
Preference structure and multi-attribute utility
Preference without uncertainty
Preference with uncertainty

4.5 Decision Networks

Representing a decision problem with a decision network
Evaluating decision networks

4.6 The Value of Information

Introduction



utility theory + probability theory = decision theory

basic principle of decision theory: maximization of expected utility

4.1 Combining Beliefs and Desires under Uncertainty

Definition

utility function assigns a single number to express desirability of state

expected utility = utility + outcome probabilities

$U(S)$: utility of state S according to decision making agent
where state is complete snapshot of the whole world

Remark

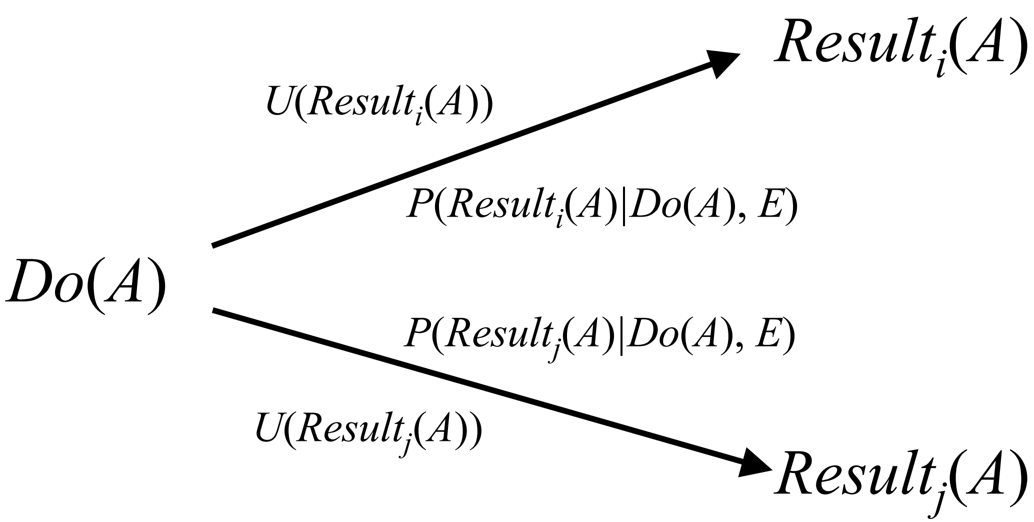
It can become rather cumbersome to specify utility of each possible state separately ($\rightarrow$ decomposition of states)

Expected Utility

- Nondeterministic action A will have possible outcome states $Result_i(A)$, where the index i ranges over the different outcomes.

- Prior to execution of A, the agent assigns probability $P(Result_i(A)|Do(A), E)$ to each outcome, where

  E summarizes the agent's available evidence about the world and
  $Do(A)$ is the proposition that action A is executed in the current state.



nondeterministic action

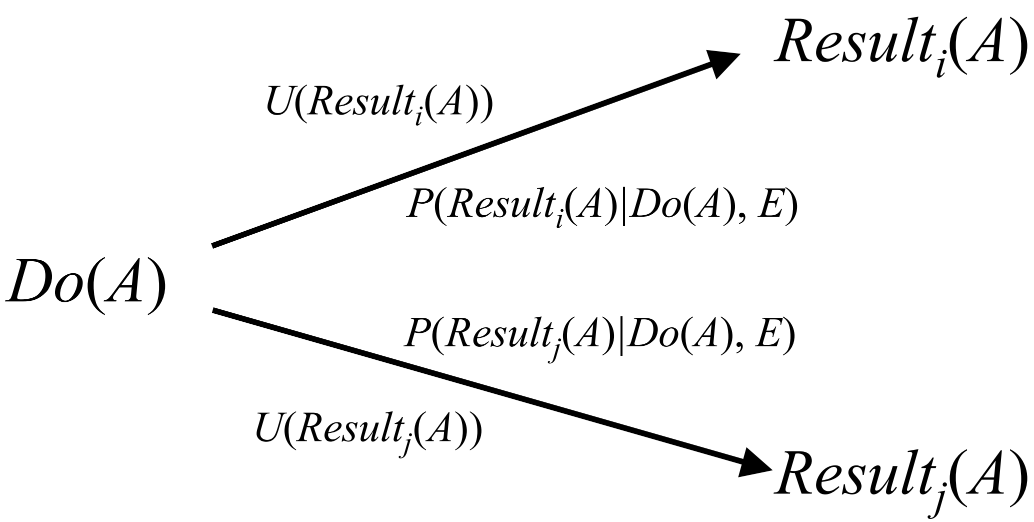
Expected Utility

- Expected utility of action A given evidence E

$$EU(A \mid E) = \sum_i P(Result_i(A) \mid Do(A), E)\,U(Result_i(A)) \qquad (4.1)$$

- Principle of maximum expected utility (MEU):

A rational agent should choose an action that maximizes the agent's expected utility



nondeterministic action

Maximum Expected Utility

- Problem:

  If we wanted to choose the best sequence of actions using this equation we would have to enumerate all action sequences and choose the best. This is clearly infeasible for long sequences!!!!

  $\longrightarrow$ discussion of simple decisions in this and next lecture,

  for more on complex decisions see Russel & Norvig

Maximum Expected Utility

- Remark

  In principle MEU is all what AI is about! MEU principle defines the right action to take in any decision problem.

- Question

  Does this mean, the AI problem is solved??

- Answer

  Not really!

Maximum Expected Utility

- Answer

  Not really!

- Why
  - knowing the initial state of the world requires perception, learning, knowledge representation, and inference
  - computations involved can be prohibitive, and it is sometimes difficult even to formulate the problem completely;
  - computing $P(Result_i(A) | Do(A), E)$ requires a complete causal model of the world and, as we saw in Chapter 14, NP-hard inference in Bayesian networks
  - computing the utility of each state, $U(Result_i(A))$, often requires searching or planning, because an agent does not know how good a state is until it knows where it can get to from that state.
  - So, decision theory is not a panacea that solves the AI problem.

4.2 Basis of Utility Theory

Questions

- Is Maximum Expected Utility (MEU) the only rational, reasonable way to make decisions?
- Why not try to maximize the sum of the cubes of the possible utilities?
- Or try to minimize the worst possible loss?
- Also, couldn't an agent act rationally just by expressing preferences between states, without giving them numeric values?
- Finally, why should a utility function with the required properties exist at all?

Answer

Questions can be answered by writing down some constraints on the preferences that a rational agent should have and then showing that the MEU principle can be derived from the constraints

Constraints on rational preference

- Are there constraints on the preferences that a rational agent should have?
- Is it possible to derive MEU principle from these constraints?
- Notation:

$A \succ B$	A is preferred to B
$A \sim B$	agent is indifferent between A and B
$A \succsim B$	agent prefers A to B or is indifferent between them

Constraints on rational preference

- What are A and B ?
- If agent's actions are deterministic, A and B will typically be the concrete, fully specified outcome states of those actions.
- In the more general, nondeterministic case, A and B will be lotteries.

Definition: Lottery

- A lottery is essentially a probability distribution over a set of actual outcomes.

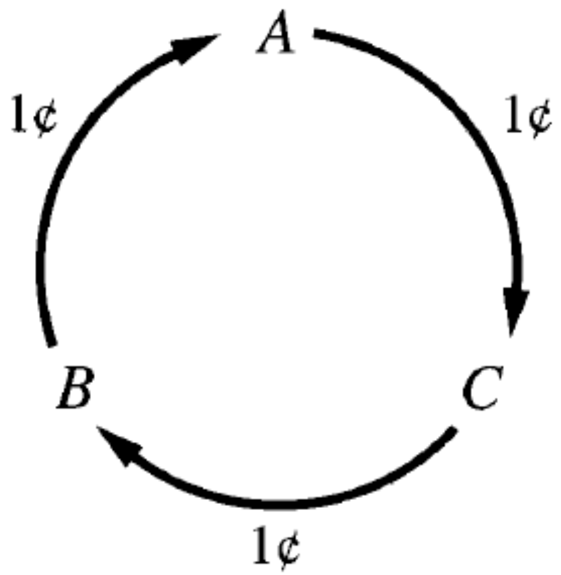
  A lottery L with possible outcomes $C_1, \ldots, C_n$ that can occur with probabilities $p_1, \ldots, p_n$ is written

$$L = [p_1, C_1; p_2, C_2; \ldots; p_n; C_n]$$

- A lottery with only one outcome can be written either as A or as [1, A].
- In general, each outcome of a lottery can be either an atomic state or another lottery.
- What is a reasonable constraint on preference?

Constraints on rational preference

- One reasonable constraint is that preference should be transitive:
  that is, if $A \succ B$ and $B \succ C$, then we would expect that $A \succ C$



Assume an agent has the non transitive preferences

$$A \succ B \succ C \succ A$$

where A, B, and C are goods that can be freely exchanged.

If agent currently has A, then we could offer to trade C for A and some cash. Agent prefers C and would be willing to give up some amount of cash for this trade. Next we could offer to trade B for C, extracting more cash, and finally trade A for B. This brings us back where we started, except that the agent has lost quite a bit of money.

Assuming non-transitivity would imply irrational behavior!!

The axioms of utility theory

◇ **Orderability:** Given any two states, a rational agent must either prefer one to the other or else rate the two as equally preferable. That is, the agent cannot avoid deciding.

$$(A \prec B) \vee (B \prec A) \vee (A \sim B)$$

◇ **Transitivity:** Given any three states, if an agent prefers A to B and prefers B to C, then the agent must prefer A to C.

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

◇ **Continuity:** If some state B is between A and C in preference, then there is some probability p for which the rational agent will be indifferent between getting B for sure and the lottery that yields A with probability p and C with probability 1 - p.

$$(A \succ B \succ C) \Rightarrow \exists p\ [p, A; 1-p, C] \sim B$$

The axioms of utility theory

- **Substitutability**: If an agent is indifferent between two lotteries, A and B, then the agent is indifferent between two more complex lotteries that are the same except that B is substituted for A in one of them. This holds regardless of the probabilities and the other outcome(s) in the lotteries.

$$A \sim B \;\Rightarrow\; [p, A; 1-p, C] \sim [p, B; 1-p, C]$$

- **Monotonicity:** Suppose there are two lotteries that have the same two outcomes, A and B. If an agent prefers A to B, then the agent must prefer the lottery that has a higher probability for A (and vice versa).

$$A \succ B \;\Rightarrow\; (p \geq q \;\Leftrightarrow\; [p, A;\; 1-p, B] \succsim [q, A;\; 1-q, B]$$

- **Decomposability:** Compound lotteries can be reduced to simpler ones using the laws of probability. This has been called the "no fun in gambling" rule because it says that two consecutive lotteries can be compressed into a single equivalent lottery

$$[p, A;\; 1-p, [q, B;\; 1-q, C]] \;\sim\; [p, A;\; (1-p)q,\; B;\; (1-p)(1-q), C]$$

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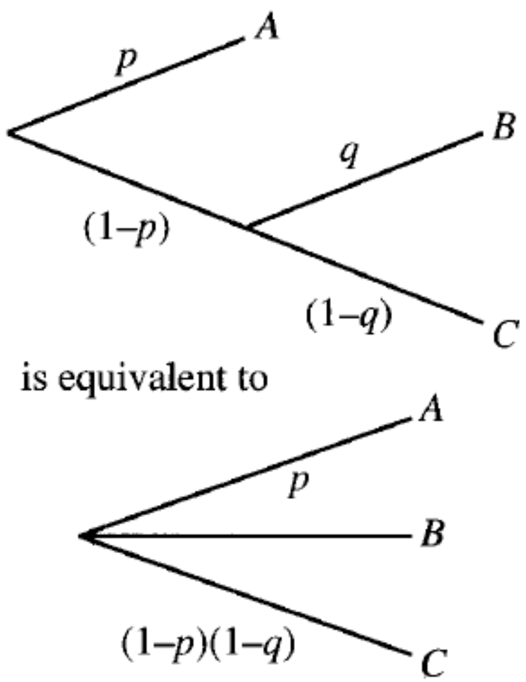

Principles of Utility

Remark

Axioms of utility theory do not say anything about utility: They talk only about preferences. Preference is assumed to be a basic property of rational agents.

1. **Utility principle**

   If an agent's preferences obey the axioms of utility, then there exists a real-valued function U that operates on states such that $U(A) > U(B)$ if and only if A is preferred to B, and $U(A) = U(B)$ if and only if the agent is indifferent between A and B.

$$U(A) > U(B) \iff A \succ B$$

$$U(A) = U(B) \iff A \sim B$$

Principles of Utility

2. Maximum Expected Utility principle

The utility of a lottery is the sum of the probability of each outcome times the utility of that outcome.

$$U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i).$$

Once probabilities and utilities of possible outcome states are specified, utility of a compound lottery involving those states is completely determined.

Because outcome of a nondeterministic action is a lottery, this gives us the MEU decision rule from Equation 4.1

4.3 Utility Functions

- Utility is a function that maps from states to real numbers.
- Is that it? That is it!
- An agent can have any preferences it likes even irrational one.
- However, if utility functions were arbitrary, then utility theory would not be of much help because we would have to observe the agent's preferences in every possible combination of circumstances before being able to make any predictions about its behavior.
- Fortunately, the preferences of real agents are usually more systematic.
- There are systematic ways of designing utility functions generating certain behaviors

The utility of money

- Economics provides one obvious candidate for a utility measure: money
- Almost universal exchangeability of money for all kinds of goods and services suggests that money plays a significant role in human utility functions.
- If we restrict our attention to actions that only affect the amount of money that an agent has, then agent usually prefers more money to less.
- Agent exhibits a monotonic preference for definite amounts of money.
- Is this sufficient to guarantee that money behaves as a utility??

The utility of money

- No, it says nothing about preferences between lotteries involving money.
- Example:
  - You won $1,000,000 in a television game show.
  - You can gamble it on the flip of a coin: triple the prize or loose everything.
  - If the coin comes up heads, you end up with nothing, but if it comes up tails, you get $3,000,000.
  - How do you decide??

The utility of money

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  - If the coin comes up heads, you end up with nothing, but if it comes up tails, you get $3,000,000.
  - How do you decide??
  - If you're like most people, you would decline the gamble and pocket the million.
  - Assuming you believe that the coin is fair, the expected monetary value (EMV) of the gamble is

$$1/2(\$0) + 1/2(\$3.000.000) = \$1.500.000$$

and the EMV of taking the original prize is of course $1.000.000, which is less.

The utility of money

- Does this mean that accepting the gamble is a better decision??
- Suppose we use S_n to denote the state of possessing total wealth $\$n$, and that your current wealth is $\$k$.
- Then the expected utilities of the two actions of accepting and declining the gamble are

$$\begin{aligned} EU(Accept) &= \tfrac{1}{2}U(S_k) + \tfrac{1}{2}U(S_{k+3.000.000}) \\ EU(Decline) &= U(S_{k+1.000.000}) \end{aligned}$$

- To determine what to do, we need to assign utilities to the outcome states.
- Utility is not directly proportional to monetary value, because the utility for your first million is very high, whereas the utility for an additional million is much smaller.

The utility of money

- Utility is not directly proportional to monetary value, because the utility for your first million is very high, whereas the utility for an additional million is much smaller.
- Suppose you assign a utility of 5 to your current financial status (S_k), a 10 to the state $S_{k+3.000.000}$ and an 8 to the state $S_{k+1.000.000}$
- Rational action would be to decline, because the expected utility of accepting is only 7.5 (less than the 8 for declining).
- What if you have $500.000.000 in the bank already???

The utility of money

- In a pioneering study of actual utility functions, Grayson (1960) found that the utility of money was almost exactly proportional to the logarithm of the amount.
- One particular curve, for a certain Mr. Beard, is shown in Figure 16.2(a). Data obtained for Mr. Beard's preferences are consistent with a utility function

$$U(S_{k+n}) = -263.31 + 22.09 \log(n + 150.000)$$

for the range between n = -\$150.000 and n = \$800.000.

The utility of money

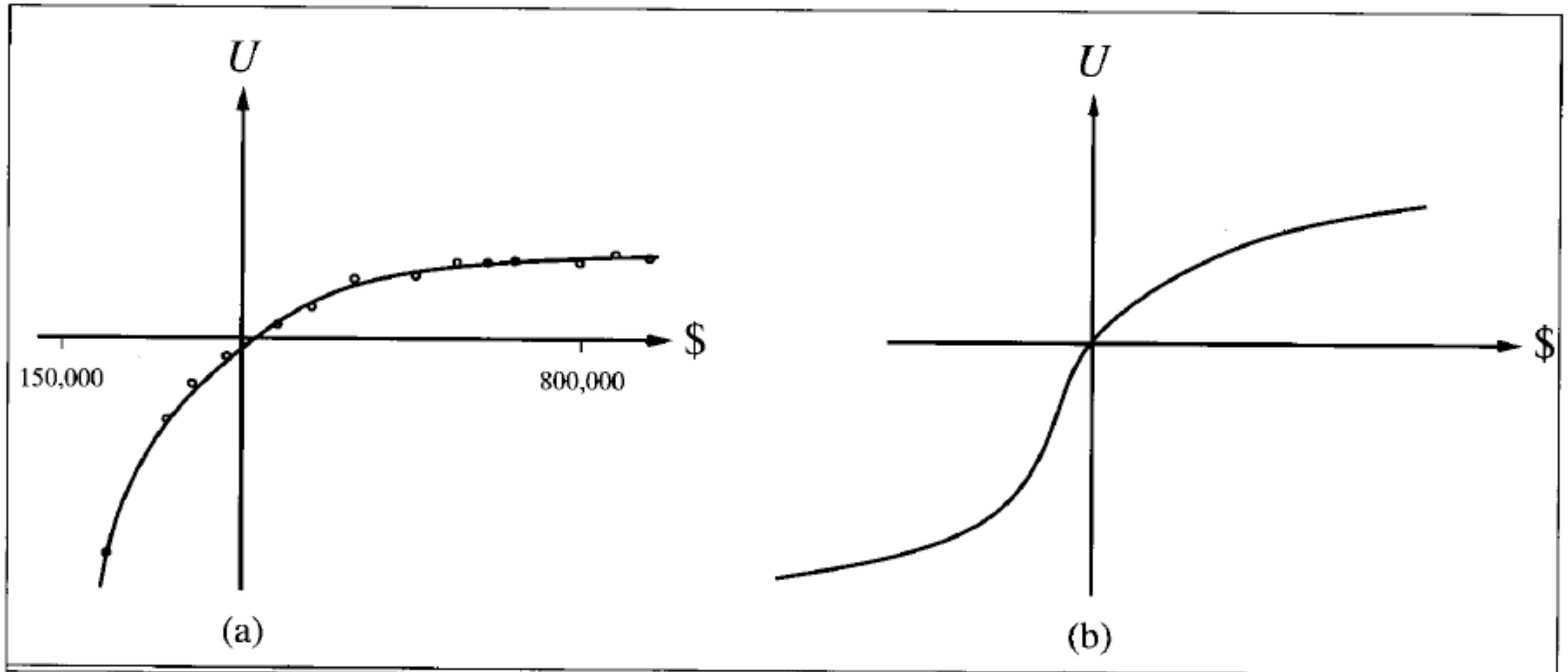

Figure 16.2 The utility of money. (a) Empirical data for Mr. Beard over a limited range. (b) A typical curve for the full range.

- Is this the definitive utility function for monetary value?

The utility of money

- Is this the definitive utility function for monetary?
- Not always, but it is likely that most people have a utility function that is concave for positive wealth.
- Going into debt is usually considered disastrous, but preferences between different levels of debt can display a reversal of the concavity associated with positive wealth.
- For example, someone already $\$10.000.000$ in debt might well accept a gamble on a fair coin with a gain of $\$10.000.000$ for heads and a loss of $\$20.000.000$ for tails.

The utility of money

- People operating in the region of the curve with decreasing slope are risk-averse, in other words they prefer a sure thing with a payoff that is less than the expected monetary value of a gamble.
- In the "desperate" region at large negative wealth in Figure 16.2(b), the behavior is risk-seeking.
- The value an agent will accept in lieu of a lottery is called the certainty equivalent of the lottery.
- Most people will accept about $\$400$ in lieu of a gamble that gives $\$1000$ half the time and $\$0$ the other half - that is, the certainty equivalent of the lottery is $\$400$.
- The difference between the expected monetary value of a lottery and its certainty equivalent is called the insurance premium.
- Risk aversion is the basis for the insurance industry, because it means that insurance premiums are positive.

The utility of money

- People would rather pay a small insurance premium than gamble the price of their house against the chance of a fire.
- From insurance company's point of view, price of house is very small compared with the firm's total reserves.
- Insurer's utility curve is approximately linear over such a small region, and the gamble costs the company almost nothing.
- Notice that for small changes in wealth relative to the current wealth, almost any curve will be approximately linear.
- An agent that has a linear curve is said to be risk-neutral.
- For gambles with small sums, therefore, we expect risk neutrality.

Utility scales and utility assessment

- The axioms of utility do not specify a unique utility function for an agent, given its preference behavior. For example, we can transform a utility function $U(S)$ into

$$U'(S) = k_1 + k_2\, U(S)\,,$$

  where k_1 is a constant and k_2 is any positive constant.
- Clearly, this linear transformation leaves the agent's behavior unchanged.
- An agent in a deterministic environment is said to have a value function or ordinal utility function; the function really provides just rankings of states rather than meaningful numerical values.

Normalized utilities

- One procedure for assessing utilities is to establish a scale with a "best possible prize" at $U(S) = u_\top$ and a "worst possible catastrophe" at $U(S) = u_\perp$
- Normalized utilities use a scale with $u_\perp = 0$ and $u^\top = 1$.
- Utilities of intermediate outcomes are assessed by asking the agent to indicate a preference between the given outcome state S and a standard lottery [$p, u_\top; (1- p), u_\perp$].
- The probability p is adjusted until the agent is indifferent between S and the standard lottery.
- Assuming normalized utilities, the utility of S is given by p.

4.3 Multiattribute Utility Functions

- Example: Placing a new airport requires consideration of:
  - the disruption caused by construction;
  - the cost of land;
  - the distance from centers of population;
  - the noise of flight operations;
  - safety issues arising from local topography and weather conditions;
  - and so on.
- Problems in which outcomes are characterized by two or more attributes, are handled by multiattribute utility theory.
- We will call the attributes $\mathbf{X} = X_1, \ldots, X_n$ a complete vector of assignments will be $\mathbf{x} = \langle x_1, \ldots, x_n \rangle$.
- Each attribute is generally assumed to have discrete or continuous scalar values.

- We will assume that each attribute is defined in such a way that, all other things being equal, higher values of the attribute correspond to higher utilities.
- Example: *AbsenceOfNoise*

  if we choose *AbsenceOfNoise* as an attribute in the airport problem then the greater its value, the better the solution.
- In some cases, it may be necessary to subdivide the range of values so that utility varies monotonically within each range.

- We begin by examining cases in which decisions can be made without combining the attribute values into a single utility value.
- Then we look at cases where the utilities of attribute combinations can be specified very concisely.

Dominance

- Example:

  Suppose that airport site S_1 costs less, generates less noise pollution, and is safer than site S_2.

  Which would would you choose?

  There is strict dominance of S_1 over S_2.

- In general, if an option is of lower value on all attributes than some other option, it need not be considered further.
- Strict dominance is often very useful in narrowing down the field of choices to the real contenders, but rarely yields a unique choice.

Strict Dominance

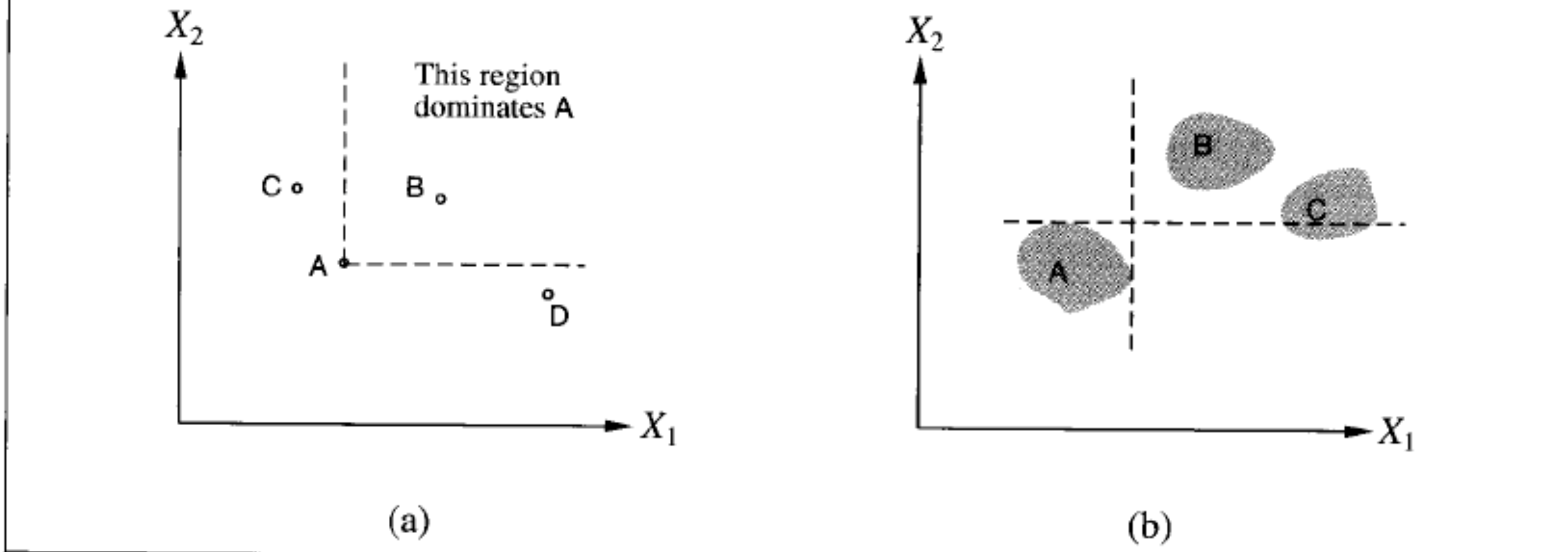

Figure 16.3 Strict dominance. (a) Deterministic: Option A is strictly dominated by B but not by C or D. (b) Uncertain: A is strictly dominated by B but not by C.

Strict Dominance

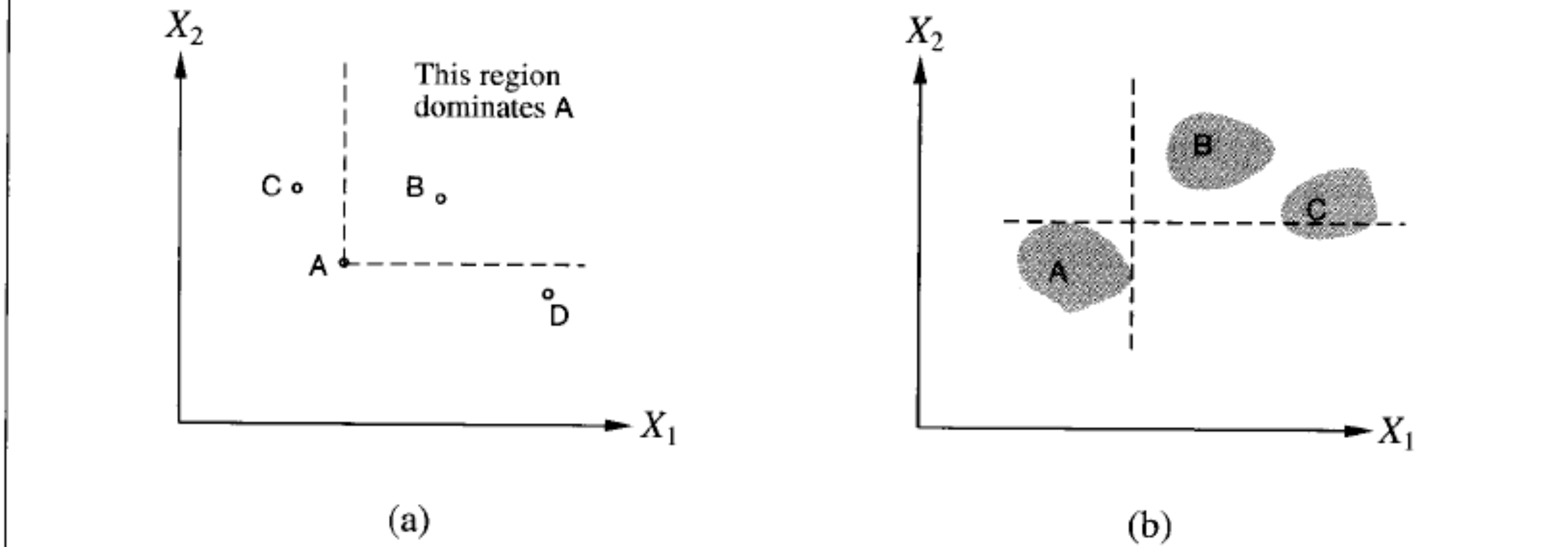

Figure 16.3 Strict dominance. (a) Deterministic: Option A is strictly dominated by B but not by C or D. (b) Uncertain: A is strictly dominated by B but not by C.

- What about cases, where action outcome are uncertain?
- A direct analog of strict dominance can be constructed, where, despite the uncertainty, all possible concrete outcomes for S_1 strictly dominate all possible outcomes for S_2. (see Figure 16.3(b)); will not occur very often.

Stochastic Dominance

- Example (single attribute):

  Suppose we believe that the cost of siting the airport at S_1 is uniformly distributed between \$2.8 billion and \$4.8 billion and that the cost at S_2 is uniformly distributed between \$3 billion and \$5.2 billion.

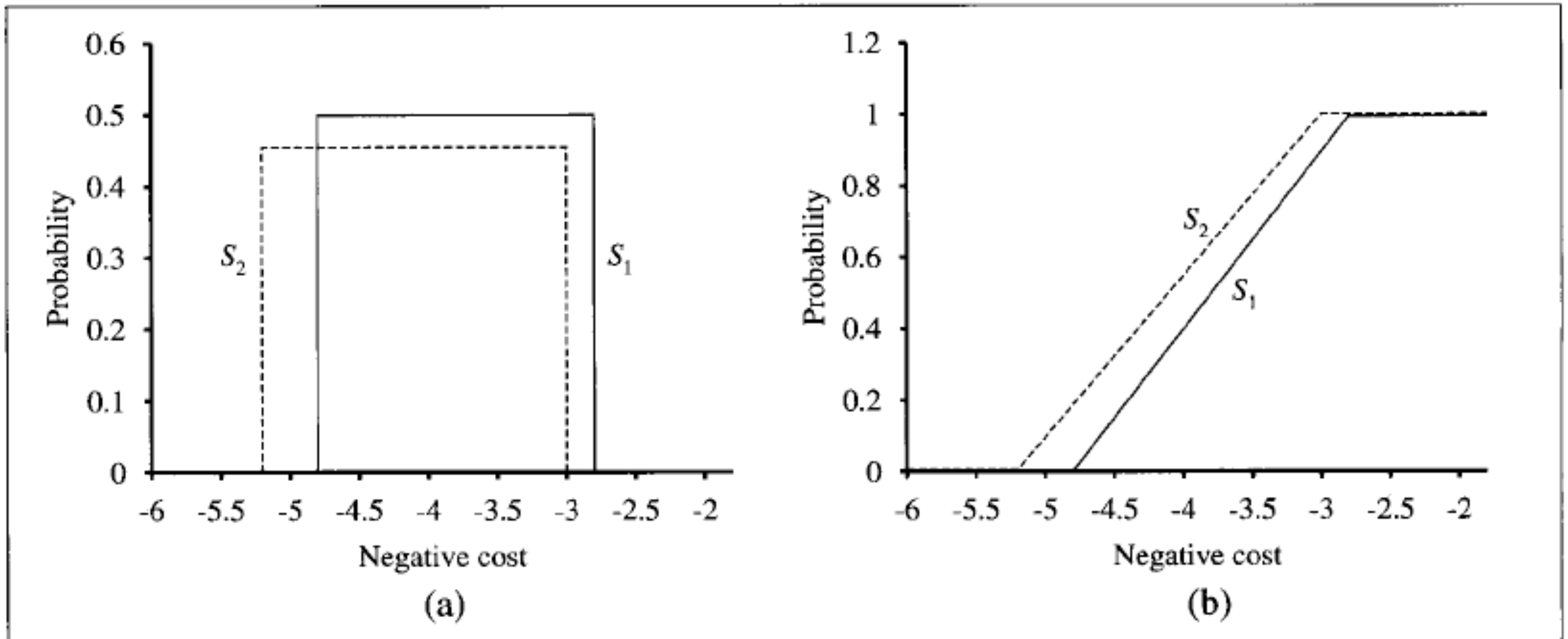

Figure 16.4 Stochastic dominance. (a) S_1 stochastically dominates S_2 on cost. (b) Cumulative distributions for the negative cost of S_1 and S_2.

Stochastic Dominance

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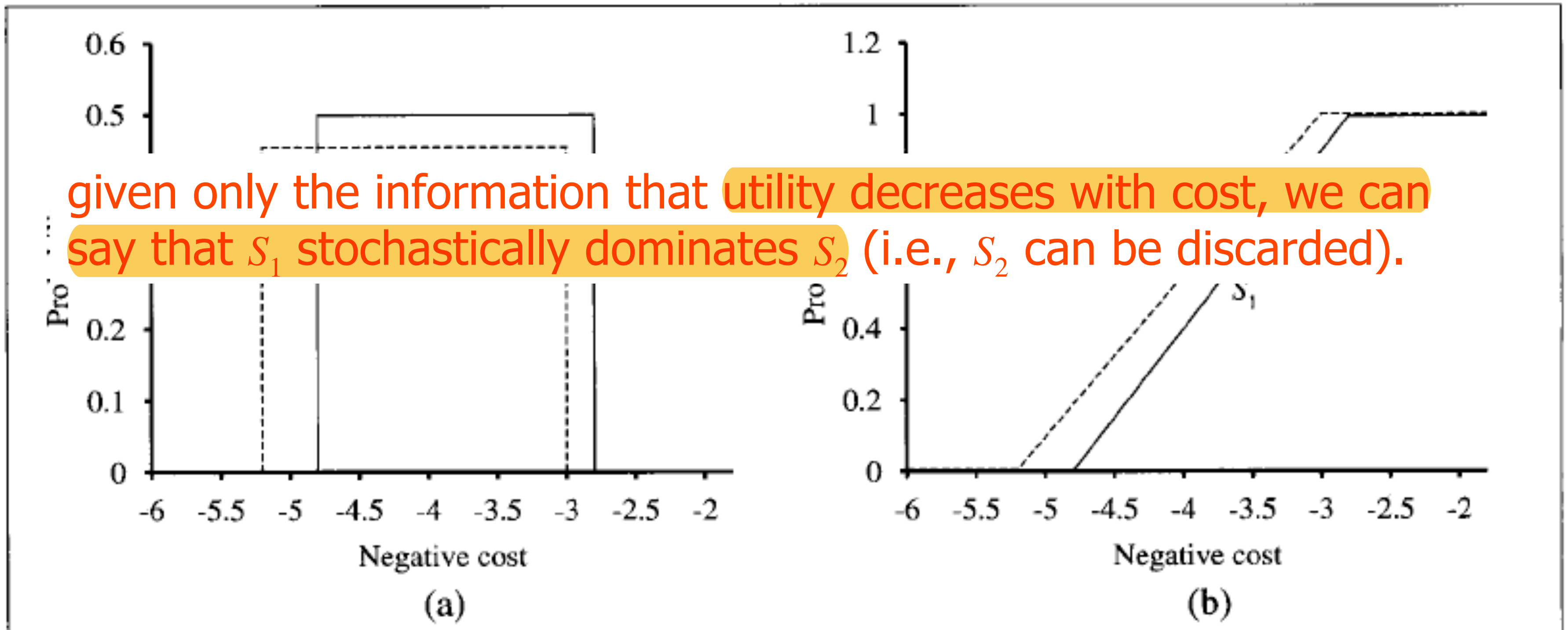

given only the information that utility decreases with cost, we can say that S_1 stochastically dominates S_2 (i.e., S_2 can be discarded).

Figure 16.4 Stochastic dominance. (a) S_1 stochastically dominates S_2 on cost. (b) Cumulative distributions for the negative cost of S_1 and S_2.

Stochastic Dominance

- The cumulative distribution measures the probability that the cost is less than or equal to any given amount - that is, it integrates the original distribution.
- If the cumulative distribution for S_1 is always to the right of the cumulative distribution for S_2, then, stochastically speaking, S_1 is cheaper than S_2.
- Formally, if two actions A_1 and A_2 lead to probability distributions $p_1(X)$ and $p_2(X)$ on attribute X, then A_1 stochastically dominates A_2 on X

$$\forall x \quad \int_{-\infty}^{x} p_1(x')dx' \quad \leq \quad \int_{-\infty}^{x} p_2(x')dx'$$

Stochastic Dominance

- Stochastic dominance condition might seem rather technical and perhaps not so easy to evaluate without extensive probability calculations.
- In fact, it can be decided very easily in many cases.

  Example: construction cost depends on the distance to centers of population. The cost itself is uncertain, but the greater the distance, the greater the cost. If S_1 is less remote than S_2, then S_1 will dominate S_2 on cost.

Preference structure and multi-attribute utility

- Suppose we have n attributes, each of which has d distinct possible values. To specify the complete utility function $U(x_1, . . . , x_n)$, we need d^n values in the worst case.
- Worst case corresponds to situation in which the agent's preferences have no regularity at all.
- Multi-attribute utility theory is based on the supposition that the preferences of typical agents have much more structure than that.
- Basic approach is to identify regularities in the preference behavior we would expect to see and to use what are called representation theorems to show that an agent with a certain kind of preference structure has a utility function

$$U(x_1,\ldots,x_n) = f[f_1(x_1),\ldots,f_n(x_n)]$$

where f is, we hope, a simple function such as addition.

Preference without uncertainty

- basic regularity that arises in deterministic preference structures is called preference independence.
- Two attributes X_1 and X_2 are preferentially independent of a third attribute X_3 if the preference between outcomes $\langle x_1, x_2, x_3 \rangle$ and $\langle x'_1, x'_2, x'_3 \rangle$ does not depend on the particular value x_3 for attribute X_3.
- Example: airport
  - attributes to consider (among others): $Noise$, $Cost$, and $Deaths$
  - one may propose that $Noise$ and $Cost$ are preferentially independent of $Deaths$.
  - if we prefer a state with 20.000 people residing in the flight path and a construction cost of \$4 billion to a state with 70.000 people residing in the flight path and a cost of \$3,7 billion when the safety level is 0.06 deaths per million passenger miles in both cases, then we would have the same preference when the safety level is 0,13 or 0,01;
  - and the same independence would hold for preferences between any other pair
  - It is also apparent that $Cost$ and $Deaths$ are preferentially independent of $Noise$ and that $Noise$ and $Deaths$ are preferentially independent of $Cost$.

Preference without uncertainty

- We say that the set of attributes $\{Noise, Cost, Deaths\}$ exhibits mutual preferential independence (MPI).
- MPI says that, whereas each attribute may be important, it does not affect the way in which one trades off the other attributes against each other.

Preference without uncertainty

- Thanks to a remarkable theorem due to the economist Debreu (1960), we can derive from it a very simple form for the agent's value function:

  If attributes $X_1, \ldots, X_n$ are mutually preferentially independent, then the agent's preference behavior can be described as minimizing the function

  $$V(x_1, \ldots, x_n) = \sum_i V_i(x_i)$$

  where each V_i is a value function referring only to the attribute X_i.

- Example:

  it might be the case that the airport decision can be made using a value function

  $V(noise, cost, deaths) = \text{-noise} \times 10^4 - cost - deaths \times 10^{12}$.

- A value function of this type is called an additive value function.
- Additive functions are an extremely natural way to describe an agent's value function and are valid in many real-world situations.

Preference with uncertainty

- When uncertainty is present in the domain, we will also need to consider the structure of preferences between lotteries and to understand the resulting properties of utility functions, rather than just value functions.
- The mathematics of this problem can become quite complicated!
- The basic notion of utility independence extends preference independence to cover lotteries:

  A set of attributes $\mathbf{X}$ is utility-independent of a set of attributes $\mathbf{Y}$ if preferences between lotteries on the attributes in $\mathbf{X}$ are independent of the particular values of the attributes in $\mathbf{Y}$.

4.5 Decision Networks

- Decision networks combine Bayesian networks with additional node types for actions and utilities and represents information about
  - –the agent's current state,
  - –its possible actions,
  - –the state that will result from the agent's action, and
  - –the utility of that state.

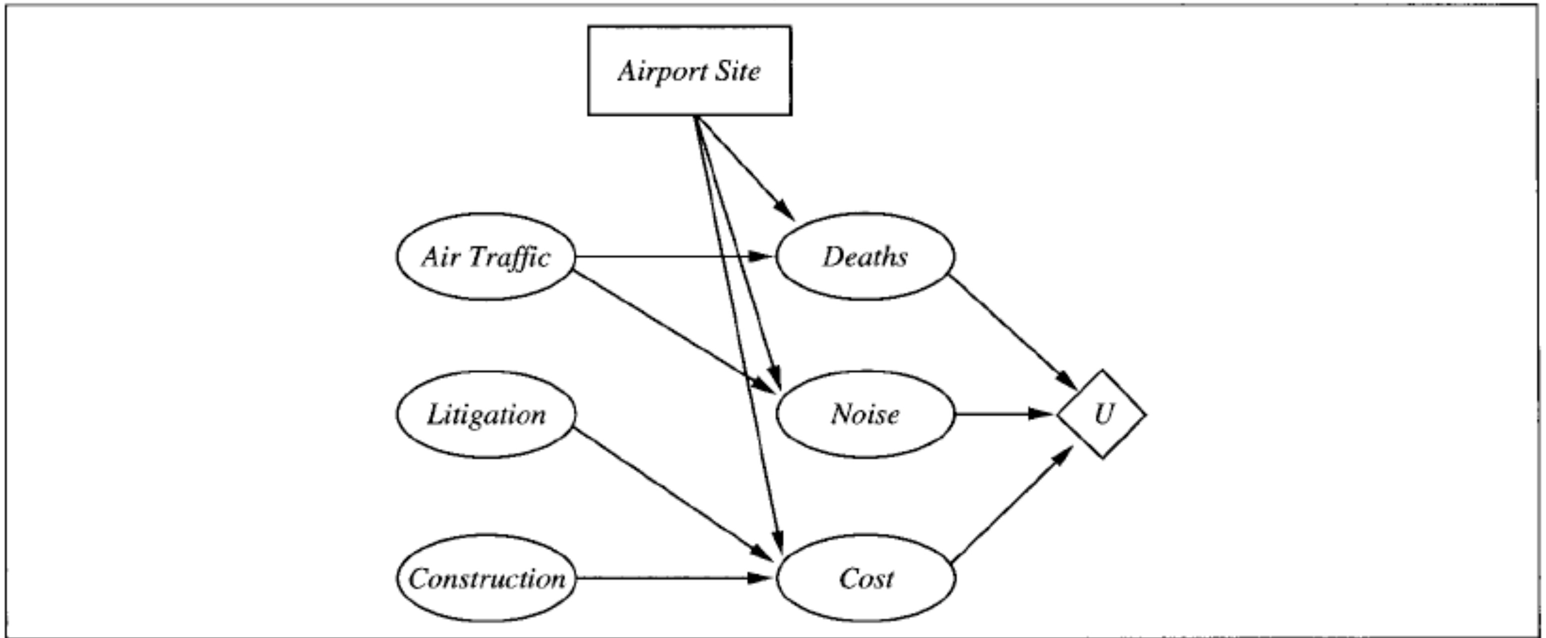

Figure 16.5 A simple decision network for the airport-siting problem.

Representing a decision problem

- Three types of nodes
  - Chance nodes (ovals) represent random variables, just as they do in Bayes nets.

    Each chance node has associated with it a conditional distribution that is indexed by the state of the parent nodes.

    In decision networks, the parent nodes can include decision nodes as well as chance nodes.
  - Decision nodes (rectangles) represent points where the decision-maker has a choice of actions.

    In this case, the *AirportSite* action can take on a different value for each site under consideration. The choice influences the cost, safety, and noise that will result.
  - Utility nodes (diamonds) represent the agent's utility function. The utility node has as parents all variables describing the outcome that directly affect utility.

    Associated with the utility node is a description of the agent's utility as a function of the parent attributes.

Evaluating decision networks

- Actions are selected by evaluating the decision network for each possible setting of the decision node. Once the decision node is set, it behaves exactly like a chance node that has been set as an evidence variable.
- Algorithm for evaluating decision networks:

```
1. set the evidence variables for the current state
2. for each possible value of the decision node
   a) set the decision node to that value.
   b) calculate the posterior probabilities for the parent
      nodes of the utility node, using a standard
      probabilistic inference algorithm.
   c) calculate the resulting utility for the action.
3. return the action with the highest utility.
```

4.6 Value of Information

- Assumption so far: information necessary to make a decision is available and for free
- In practise, this is rarely the case.
- One of the most important parts of decision making is knowing what questions to ask.
- Information value theory enables an agent to choose what information to acquire. The acquisition of information is achieved by sensing actions.

Simple example

- Oil company wants to buy one of n indistinguishable blocks of ocean drilling rights
- Exactly one of the blocks contains oil worth C dollars and that the price of each block is C/n dollars
- If company is risk-neutral, then it will be indifferent between buying a block and not buying one.
- Now suppose seismologist offers the company the results of a survey of block number 3, which indicates definitively whether the block contains oil.

How much should the company be willing to pay for the information?

Simple example

- To answer this question, examine what the company would do if it had the information:
  - With probability $1/n$, the survey will indicate oil in block 3. In this case, the company will buy block 3 for C/n dollars and make a profit of C - C/n = $(n - 1)C/n$ dollars.
  - With probability $(n -1)/n$, the survey will show that the block contains no oil, in which case the company will buy a different block.

    Now the probability of finding oil in one of the other blocks changes from $1/n$ to $1/(n - 1)$, so the company makes an expected profit of $C/(n - 1)$ - C/n = $C/n(n - 1)$
- Now we can calculate the expected profit, given the survey information:

$$\frac{1}{n} \times \frac{(n-1)C}{n} + \frac{n-1}{n} \times \frac{C}{n(n-1)} = C / n$$

Therefore, the company should be willing to pay the seismologist up to C/n dollars for the information: the information is worth as much as the block itself.

In general, the value of a given piece of information is defined to be the difference in expected value between best actions before and after information is obtained.

A general formula

- Usually, we assume that exact evidence is obtained about the value of some random variable E_j, so the phrase value of perfect information (VPI) is used. Let the agent's current knowledge be E. Then the value of the current best action α is defined by

$$EU(\alpha \mid E) = \max_A \sum_i U(Result_i(A)) P(Result_i(A) \mid Do(A), E)$$

and the value of the new best action (after the new evidence E_j is obtained) will be

$$EU(\alpha_{E_j} \mid E, E_j) = \max_A \sum_i U(Result_i(A)) P(Result_i(A) \mid Do(A), E, E_j)$$

- E_j is a random variable whose value is currently unknown, so we must average over all possible values e_{jk} that we might discover for E_j, using our current beliefs about its value. The value of discovering E_j, given current information E, is then defined as

$$VPI_E(E_j) = \left(\sum_k P(E_j = e_{jk} \mid E) EU(\alpha_{e_{jk}} \mid E, E_j = e_{jk}) \right) - EU(\alpha \mid E)$$