Review of Probability Theory

Friday, April 24, 2020 2:33 AM

Probability measure: A function $P: \mathcal{F} \to \mathbb{R}$ that satisfies the following properties:

- $P(A) \geq 0$, for all $A \in \mathcal{F}$ ← event space (set of event)
- $P(\Omega) = 1$
- If $A_1, A_2, \ldots$ are disjoint events (i.e. $A_i \cap A_j = \emptyset$ whenever $i \neq j$), then $P(\cup_i A_i) = \sum_i P(A_i)$

Properties

- if $A \subseteq B \Rightarrow P(A) \leq P(B)$
- $P(A \cap B) \leq \min(P(A), P(B))$
- (Union Bound) $P(A \cup B) \leq P(A) + P(B)$
- $P(\Omega \setminus A) = 1 - P(A)$
- (Law of Total Probability) If $A_1, \ldots, A_k$ are a set of disjoint events such that $\cup_{i=1}^{k} A_i = \Omega$, then $\sum_{i=1}^{k} P(A_k) = 1$

Conditional probability and independence

Let B be an event with non-zero probability

The conditional probability of any event A given B is

$$P(A|B) \triangleq \frac{P(A \cap B)}{P(B)}$$

Two events are called independent if and only if $P(A \cap B) = P(A)P(B)$ or $P(A|B) = P(A)$

in other word, we say that observing B has no effect on the probability of A

Random variables

Let $w_0 = \{H, H, T, H, T, H, H, T, T, T\} \in \Omega$

in practice, we genally don't care about obtaining a particular sequences of heads and tails. What we care is real-valued function of outcomes such as: # of heads appear among 10 tosses or the length of the longest run of tail.

These functions are what we called random variables.

Formally, a random variable X is a function $X: \Omega \longrightarrow \mathbb{R}^2$

Discrete random variables

X only takes on a finite number of values

example:

let $X(w)$ = # of heads with occur in the sequence of tosses w

$$P(X = k) := P(\{w : X(w) = k\})$$

Continuous random variables

X only takes on a infinite number of possible values.

example:

let $X(w)$ = amount of time takes for a radioactive particle to decay.

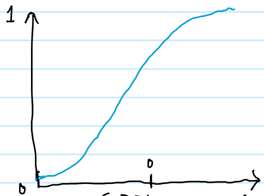
$$P(a \leq X \leq b) := P(\{w : a \leq X(w) \leq b\})$$

Cumulative distribution functions (CDF)

CDF is a function $F_X : \mathbb{R} \to [0,1]$ which specifies a probability measure as, $F_X(x) \triangleq P(X \leq x)$

Properties:

- $0 \leq F_X(x) \leq 1$
- $\lim_{x \to -\infty} F_X(x) = 0$
- $\lim_{x \to \infty} F_X(x) = 1$
- $x \leq y \Rightarrow F_X(x) \leq F_X(y)$



CDF example

Probability mass function (PMF)