Searching for a Moving Target: Optimal Path Planning

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***Abstract*— We consider the problem of constrained path planning for one or two agents in search of a single randomly moving target such that we maximize the probability of intercepting the target at some time in its trajectory. We assume the agents operate in a receding-horizon optimization framework with some finite planning horizon. We present and compare several search path planning methods. This problem is particularly applicable in the case of wide area search munitions searching and engaging moving ground targets.**[1]

I. Introduction

Target search is a pervasive problem in both civilian and military applications such as search and rescue, weapon targeting, etc. Generally, in target search we are forced to make decisions in an uncertain, and often dynamic, environment where the target may or may not take actions to evade detection.

The search problem explored in this paper is described as follows. Time and space are discrete. The search area is described by an n by n grid. A search agent has limited endurance, E, and an agent has a constrained search path. That is an agent's movement is limited such that an agent's next available search location is a function of the agent's current search location. The initial target location (state) is known with certainty at time $t = 0$ but then the target moves among the set N of n^2 location states according to a homogenous Markov chain. The target's motion is independent of the agents' motions (in other words there are no evasive actions). A finite planning horizon, T, is used to plan each agent j path, $Path_j = (x_j^0,\ x_j^1,\ x_j^2, \ldots, x_j^T)$, where $x_j^i \in N$ is the position of the agent j at time i. For convenience we let $Path = Path_1$ when considering single-agent search models. We assume a probability of overlook, $p_O \geq 0$, which means we do not necessarily assume positive detection if the agent and target occupy the same grid location at the same time epoch. Of course the probability of detection of a target at some time t is always zero if the agent and target are not collocated in the same cell at time t. The objective is to plan the agents' paths to maximize the probability of target intercept.

An example of an application of this search model is that of friendly forces searching for an enemy tank. Based on known tank dynamics, enemy concept of operations and terrain features, a Markov chain of location states for the tank may be constructed for the area of operations. Due to a rough and varied terrain, the tank's movements are somewhat limited (i.e., it will not drive into a lake or climb a mountain). Suppose an enemy tank is detected and classified by friendly remote overhead sensors at time $t = 0$. One or more wide area search munitions (WASMs) with limited endurance is launched some time later. Using onboard sensors, each WASM is able to search, detect, and classify targets as it flies. However, there is a positive probability of overlook due to obscurants, weather, foliage, etc. The WASMs' initial search paths and updated search paths are constructed with some finite planning horizon and with the goal to maximize the probability that a WASM intercepts the tank.

[1]

A. Background

Several similar search problems have been investigated in the past. Koopman [6] acknowledged there are situations where a target is recognized and localized but some amount of time must pass before action may be taken. If the target is mobile, then a large area may have to be considered in reacquiring the target. Examples given by Koopman are a hostile submarine that must be attacked and a life raft that must be saved. Washburn [14] considered randomly moving targets such as lost hikers. He pointed out that a lost hiker may move considerable distances from the point of loss. Pollock [8] investigated a simplified model of searching for a target that moves among two states according to a Markov process. He considered two fundamental problems of minimizing the time to search and maximizing the probability of detection given some fixed amount of time to search. Brown [1], [2] considered an optimal search for a moving target in discrete space where a limited amount of search effort is available in each of a fixed number of time intervals. Compounding the problem is the assumption there is an exponential target detection function. The exponential target detection function is a common assumption found in the works such as by Stone [11] and Washburn [13], [15]. Stewart [9], [10] extended the results of Brown [1] by constraining the agent's search path.

For solution approaches, Eagle and Yee [5] considered a branch and bound procedure to find the optimal constrained path for a moving target search. They point out that the problem is as at least as difficult as the path constrained search problem for a stationary target which is NP-complete [12]. Besides the branch and bound solution algorithm, other approaches have included dynamic programming [3], [4] and iterative heuristics [2], [11].

B. Contributions of This Paper

Our model of constrained search for a moving target most closely follows the assumptions by Eagle [3]. We extend the model by considering more than one search agent and show a cooperative approach greatly enhances search effectiveness. A new approach to developing the Markov model for target movement is discussed. We introduce some solution methods and compare them to some simple approaches. Additionally, we provide some comments concerning Eagle's results.

C. Organization of the Paper

The paper is organized as follows. In section II, we clarify agent and target movement constraints. In section III we consider the case of a single search agent and introduce an exact solution method and compare three heuristic approaches. We consider the two-agent case in section IV. We provide some concluding remarks in the last section.

II. Model Clarification

In this section, we clarify the agent and target movements. Time and space are discrete with the search area described by an n by n grid. An agent and target move among the set N of n^2 location states.

A. Agent Movement

A search agent has a constrained search path which means agent's next available search location is a function of the agent's current search location. In our model, at time $t+1$ the agent may again search the cell previously searched at time t or search a cell that shares a side with the previously searched cell. Although not discussed here, the model can easily be extended to consider obstacles or cells that cannot be searched. Consider a search area that is $n \times n$ where $n = 3$ (see [3] and Figure 1). An agent at cell 1 at time t may next search cells 1, 2, or 4. An agent at cell 5 may search cells 2, 4, 5, 6 or 8 in the next time period. This model of agent movement may be consistent with a searcher on foot looking for a lost hiker or a low flying WASM searching for a ground target. The agent may begin its search in any cell of the search area and it is assumed the agent has a limited endurance E.

1	2	3
4	5	6
7	8	9

Fig. 1. 3 by 3 grid of search area.

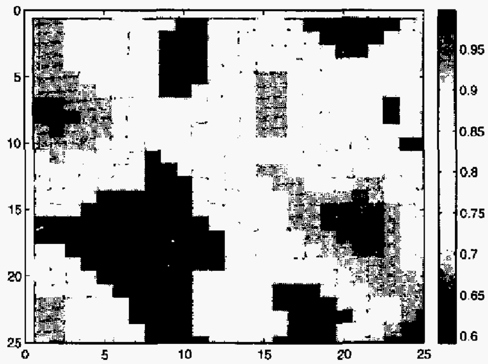

Fig. 2. Hospitibility map of a 25x25 area. Cells with high weights indicate areas that are easy to traverse such as level, hard ground.

B. Target Movement

The initial target location (state) is known with certainty at time $t = 0$ but then the target moves among the set N of n^2 location states according to a homogenous Markov chain. That is when the target is in state i at time t, there is a fixed probability p_{ij} that it will be in state j at time $t+1$.

The Markov transition matrix, $P = (p_{ij})$, for target movement may be derived from target dynamics, target concept of operations and terrain features. More specifically, a Markov transition matrix can be derived from a "hospitability map." A hospitability map provides a weight for each cell proportional to the ability of a target to move or maneuver in that cell (see [7]). Using the tank example, a low weight is provided in an cell where the cell coincides with a swamp, whereas, a high weight is provided if the cell coincides with level, hard ground like a roadway. Figure 2 shows a typical hospitability map.

In our analysis, the Markov transition matrix is derived by considering the target may stay in the current cell or move to one of the adjacent cells in the next time step. The probability of staying or moving to one of the adjacent cells is 'proportional' to the hospitability map weights in those cells. Let H be the set of h cells that are feasible to the current target cell, i. That is any one of the set $\{H_1, H_2, \ldots, H_h\}$ can be reached from the current target location i in the next time step. Associated with H_j is a weight, w_{H_j} taken from the hospitability map. If we let $j = H_j$, then

$$p_{ij} = \frac{w_j}{\sum_{k=1}^{h} w_k}.$$

This can be easily modified to consider various vehicle speeds, a generalized direction of travel, etc.

III. Single Agent Constrained Search

There are many available search methods to consider for a single agent. We consider and compare three heuristic methods.

Random Walk This simply allows the agent to randomly search cells with no regard for the target's location or potential movements.

Serpentine The agent searches the area in a serpentine manner (e.g. lawnmower fashion). Again, there is no regard for the target's location or potential movements. This method is commonly used when searching a tactical area of interest with no *a priori* target location information.

Myopic The myopic approach [14] is a greedy approach based on searching the next available cell with the highest probability of the target occupying the cell.

An exact probability of detection for a given search path is given below. From this, we can use a branch and bound method to implicitly enumerate all feasible solutions and find the optimal solution that maximizes the probability of target intercept.

A. Random Walk Search

Search by random walk is simple enough to require no explanation except to express a certain assumption of movement. At each time step, the agent searches one of the available cells with equal probability.

B. Serpentine Search

The serpentine search is a simple, yet logical, approach to searching an area when target location and potential movements are unknown. As an example of a serpentine search, consider the search grid in Figure 1. An agent beginning in cell 1 would search cells in the following sequence: 2, 3, 6, 5, 4, 7, 8, 9, 6, 5,..., until target detection or $t = E$, which ever comes first.

C. Myopic Search

Given a target location at time $t = 0$ and Markov target motion, a probability density function (p.d.f.) for the target location may be developed for each discrete time step up to E. Let P_i^t be the probability the target is located in cell i at time t given the target is not intercepted in time periods $1, 2, ..., t-1$. Using this method, we create probability maps of the search grid for each time epoch. In a myopic approach, an agent at time t that has not intercepted the target will search the available cell with the maximum P_i^{t+1}. If $P_i^{t+1} = 0$ for all available cells, then the agent moves towards to the "center of gravity" of the probability map. Myopic search is a greedy heuristic.

D. Probability of Intercept

We can evaluate the relative effectiveness of each single-agent search method by comparing probability of intercept. The probability of intercept can be obtained by an exact formula or through simulation. For a single-agent search path, $Path$, let us define the indicator random variable I_{x^i}, $i = 0, 1, \ldots, E$ by

$$I_{x^i} = \begin{cases} 1 & \text{if } x^i \text{ is a cell where the agent} \\ & \text{and target are co-located at time } i, \\ 0 & \text{otherwise.} \end{cases}$$

Let $J(x) = P(I_{x^0} = 1 \bigcup I_{x^1} = 1 \bigcup I_{x^E} = 1)$ be the probability the agent intercepts the target at least one time during the path that is E long.

$$\begin{aligned} P\left(\bigcup_{i=0}^{E} I_{x^i} = 1\right) &= \sum_{i=0}^{E} P(I_{x^i} = 1) \\ &- \sum\sum_{i<j} P(I_{x^i} = 1, I_{x^j} = 1) \\ &+ \sum\sum\sum_{i<j<k} P(I_{x^i} = 1, I_{x^j} = 1, I_{x^k} = 1) \\ &- \cdots + (-1)^{E+1} P(I_{x^0} = 1, I_{x^1} = 1, ..., I_{x^E} = 1) \end{aligned}$$

The above equation is solved by taking advantage of the Markov properties. Recall the *n-step* transition probability p_{ij}^n for a Markov chain is the probability that a process in state i will be in state j after n additional transitions. Also from Markov properties

Searcher Starting Cell	Optimal Search Paths	Probability of Detection
1	1 2 5 6 9 8 5 6 5 4 5, 1 2 5 8 9 6 5 8 5 2 5, 1 4 5 6 9 8 5 6 5 4 5, and 1 4 5 8 9 6 5 8 5 2 5	0.7871

TABLE I

THE ABOVE PATHS REPRESENT THE OPTIMAL PATHS THAT MAXIMIZE THE PROBABILITY OF DETECTION FOR THE MODEL STUDIED BY EAGLE [3].

we know in general

$$\begin{aligned} P(I_{x^i} = 1, I_{x^j} = 1, I_{x^k} = 1) &= P(I_{x^k} = 1 | I_{x^j} = 1) \\ &\cdot P(I_{x^j} = 1 | I_{x^i} = 1) \\ &\cdot P(I_{x^i} = 1) \end{aligned}$$

where $P(I_{x^j} = 1 | I_{x^i} = 1) = p_{x^i, x^j}^{(j-i)}$. Due to the combinatorial nature of this exact equation, it is only relevant to relatively small problems.

A simulation approach can be used that runs many trials of a particular model and records the number of "hits." The proportion of hits to the number of trials gives an estimate of the probability of detection.

We used the exact formula with branch and bound to evaluate the findings by Eagle, ([3], Table 1). The evaluation revealed that the solutions reported are not optimal. Our results are summarized in Table I for the case when the searcher starts in cell 1 and the target starts in cell 9 (refer to Figure 1). For brevity we direct you to [3] for model details. We have verified the probability of detection is correct for the paths identified by Eagle as being optimal; however, better search paths are available.

E. Comparative Results for Single Agent Searches

We consider the 25x25 search grid as shown in Figure 2 in which a target is initially located at the SE corner. We assume endurance values of $E = 50$, 100, and 150 time units. For convenience, we assume the agent may begin searching immediately (in reality some period of time would pass before search may begin). Three-agent start locations are considered: NW, NE, and SW corners. This analysis assumes $p_O = 0$. The results shown in Table II show that the myopic approach is superior to both the random and serpentine approaches.

Agent Start Location	NW	NE	SW
		Probability of Detection	
Time Steps		Random Walk	
50	~ 0	~ 0	~ 0
100	0.0011	0.0002	0.0002
150	0.060	0.040	0.030
		Serpentine	
50	0.0008	0.0003	~ 0
100	0.0007	0.0011	0.408
150	0.096	0.118	0.604
		Myopic	
50	0.236	0.492	0.490
100	0.472	0.607	0.605
150	0.537	0.653	0.648

TABLE II

RESULTS COMPARE THE PROBABILITY OF DETECTION USING THREE DIFFERENT SEARCH METHODS FOR A SINGLE AGENT. THREE AGENT START LOCATIONS AND THREE DIFFERENT AGENT-ENDURANCE LEVELS ARE CONSIDERED. THE PROBABILITY OF DETECTION WAS ESTIMATED USING A SIMULATION APPROACH.

IV. MULTIPLE AGENT CONSTRAINED SEARCH

We consider and compare three heuristic methods that are similar to those discussed for a single agent search.

Random Walk This simply allows the agents to randomly search cells with no regard for the target's location or potential movements. There is no cooperation between the search agents.

Serpentine The agents search the area in a serpentine manner (e.g. lawnmower fashion). The serpentine search for each agent is fashioned to heuristically minimize searching the same cell simultaneously. This is done simply by having one agent search horizontally and the other agent search vertically across the area.

Myopic A myopic approach is used as similar for a single agent; however, the agents cooperate by communicating with each other on what cell they are in and whether the target is found. Additionally, agents communicate to ensure a cell is not searched simultaneously.

A. Comparative Results for Multiple Agent Searches

Like for the single agent case, we consider the 25x25 search grid as shown in Figure 2 in which a target is initially located at the SE corner. We assume endurance values of $E = 50$, 100, and 150 time units. Agents begin searching from one of three agent-start locations: NW, NE, and SW corners. This analysis assumes $p_O = 0$. The results shown in Table III show that the myopic approach is superior to both the random and serpentine approaches. It is clear the myopic approach is more robust to different agent-start locations.

Agent Start Location	NW	NE	SW
		Probability of Detection	
Time Steps		Random Walk	
50	~ 0	~ 0	~ 0
100	0.001	0.002	0.0008
150	0.048	0.016	0.085
		Serpentine	
50	0.001	0.305	0.297
100	0.001	0.306	0.408
150	0.01	0.311	0.482
		Myopic	
50	0.406	0.619	0.741
100	0.750	0.826	0.872
150	0.837	0.882	0.913

TABLE III

RESULTS COMPARE THE PROBABILITY OF DETECTION USING THREE DIFFERENT SEARCH METHODS FOR TWO AGENTS. THREE AGENT-START LOCATIONS AND THREE DIFFERENT AGENT-ENDURANCE LEVELS ARE CONSIDERED. THE AGENTS BEGIN THEIR SEARCH FROM THE SAME STARTING LOCATION. THE PROBABILITY OF DETECTION WAS ESTIMATED USING A SIMULATION APPROACH.

V. CONCLUSIONS

We consider the problem of constrained path planning for one or two agents in search of a single randomly moving target such that we maximize the probability of intercepting the target at some time in its trajectory. We find a myopic approach improves probability of intercept over random search approaches. A cooperative strategy for multiple agents greatly increases probability of intercept compared to non-cooperative strategies and is more robust to different agent-starting locations. Future research will consider more complicated path planning approaches such as minimizing entropy, a multi-iterative process to include local-optimal neighborhood search, and solution methods that take further advantage of the Markov process.

Other model extensions are to consider target location updates. This would occur in the case of friendly overhead sensors periodically sending updates to lower altitude WASMs.

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