Proceedings of the IEEE
International Conference on Automation and Logistics
Shenyang, China August 2009

Multi-Objective Path Planning for Unrestricted Mobile *

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***Abstract* - Problem of multi-objective path planning is investigated in this paper for the ball and plate system. The purpose of multi-objective path planning is to obtain the safe and shortest path for the ball to track. Workspace is represented by distance map and hazard map. Weights for multi-objectives are calculated by entropy method for each grid node. Dijkstra algorithm is employed to solve the multi-objective path planning problem finally. As illustrated by simulation results, the path obtained by multi-objective method proposed in this paper is much safer compared with single-objective A* algorithm.**

***Index Terms* - multi objective, path planning, entropy weight, ball and plate system**

I. INTRODUCTION

Path planning problem is to determine a collision free path from an initial location to a destination through a workspace populated with obstacles. Path planning problem for ball and plate system[1] is investigated in this paper. Ball and plate system is an extension of ball and beam. Plate inclinations are changed by motors, and then ball moves freely on the plate. Path planning problem for ball and plate system is studied on the ball and plate experimental platform BPVS-JLU II[2,3]. The experimental platform BPVS-JLU II was developed by Control Theory and Intelligent System Laboratory of Jilin University. The ball is an unconstrained object. And some obstacles are placed on the plate. The ball is expected to move from an initial position to a destination as fast as possible. Travelling time of the ball along the planned path should be optimal. Once a ball is provided with a map of the working environment and its position estimate is available, path planning system should generate a collision-free path for the ball to follow. And travelling time of the ball from start to the goal point is expected as small as possible. As a result, multiple objectives such as the minimum distance, minimum time, and minimum risk should be introduced to the path planning problem. Path planning problem for the ball and plate system is a multi-objective path planning problem.

Path planning problem has been studied and applied in many research fields, such as robot manipulators, autonomous robots, autonomous underwater vehicles, very large scale integration design problem, construction automation, road routing for traffic and etc. A number of approaches have been presented for the path planning problem. They could be categorized into cell decomposition, potential field and roadmap solutions [4]. Most of the methods mentioned above search a collision free path in the discretized configuration space. The cell decomposition approaches divide the set of free configurations into simple non-overlapping regions. Roadmap or skeleton methods attempt to transform a set of feasible motions into a network of one-dimensional lines, called roadmap or skeleton. Potential field solutions obtain the optimal path by introducing a potential function. Potential functions are generally solutions of a partial differential equation with bounded conditions [5].

These methods discussed above generally solved the path planning problem as a single objective problem. However, several objectives are inherently involved at the same time for the design of engineering system rather than single objective. This is known as multi-objective optimization (MOP) problems. Multiple objectives are usually conflicting with each other. Multi-objective optimization algorithm is required for the path planning problem to satisfy several objectives simultaneously. A few approaches have been presented to find the candidate solutions of the Multi-objective optimization problem. In order to solve the multi-objective problems, the methods such as min-max optimization [6], goal programming [7] have been proposed. These methods are suitable for the simple multi-objective problems, and usually fall into local optimum while dealing with the complex multi-objective problems. Dynamic method is usually employed to deal with the multistage mathematical programming problem. And the dynamic method can only deal with small scaling problems for the problem of "the course of dimensionality" [8]. When the network is complex, it is usually hard to obtain the optimal solution with dynamic method. Concept of fuzzy set was used to represent the uncertain situations or the subjective judgments. And fuzzy sets have been incorporate into the conventional statistical or mathematical programming methods to reflect the ambiguity and uncertainty in practice [9]. Evolutionary algorithms have been employed to deal with the complex and large scaling problems [10].

Multi-objective path planning problem for the unconstrained ball on the plate is investigated in this paper. Workspace is of the ball on the plate is decomposed into overlapping grid nodes. Distance map and hazard map is introduced to represent the work space. Weights for multi-objective cost functions are obtained by entropy method for each grid node. Dijkstra algorithm is employed to solve the

* The work is supported by Chinese College Doctor Special Scientific Research Fund under Grant 20060183006 and Postgraduate 985 Project Creation Plan in Jilin University under contract No. 20080212.
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978-1-4244-4795-4/09/$25.00 © 2009 IEEE

multi-objective path planning problem finally. The proposed path planning scheme is evaluated by simulation, and its performance is is compared with single-objective A* algorithm.

Following this introduction, rest of the paper is organised as follows. The path planning problem of ball and plate system is presented in Section II. Model of the workspace and the multi-objective evaluation and search method are detailed in section III. Simulation results of the path planning for the ball and plate system is shown in section IV.

II. PROBLEM STATEMENT

Ball and plate system BPVS-JLU II is used for the research of path planning. The task of path planning on BPVS-JLU II is shown in Fig.1. Position of the ball is measured by the machine vision system. And ball position is regulated by changing the plate inclinations. Purpose of this paper is to generate a collision free path for the ball from its initial location to a destination position through a workspace populated with obstacles.

A match of path planning can be held on BPVS-JLU II, the match participants must be give a path based on match rules. For instance, if the rules required a path through time as short as possible, and permit collisions to obstacles. The path planning method must pay more attention to the time factor. If the rules required a path safe enough, and permit a long distance and long time to through the path. The path planning must pay more attention to the risk of collision to obstacles. So it must be given a method change the attention to different factors according to the rules.

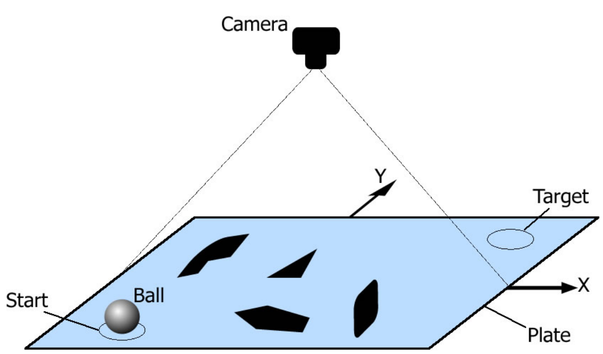


Fig.1 Task of multi-objective path planning for ball and plate system BPS-JLU II

Several criterions could be considered for path planning system design of the ball and plate system. These criterions may be length of the path, time to travel from the initial position to the destination position, risk of colliding with obstacles in workspace, smoothness of the planned trajectory and etc. Safety and path length is selected as the multi-objective in this path planning problem to make the ball move from the initial position to the destination position smoothly and safely. The planned path should be as smooth as possible. Path length and hazard of colliding with obstacles are considered in the multi-objective optimization.

The start point is denoted as s and the destination point is denoted as g. Once the cell map for the workspace is available, a candidate route for the multi-objective path planning problem is denoted as

$$P = [g_1, g_2, \cdots g_n] \tag{1}$$

where g_i denotes the index for ith grid along the planned route P.

$$g_1 = s, g_n = g \tag{2}$$

Multi-objective optimization problem for the path planning is simplified as

$$f(P) = \min[f_1(P), f_2(P)] \tag{3}$$

where f_1 is used to denote the total length along the planned path P, f_2 is used to denote hazard of colliding with obstacles along the planned path P.

A multi-objective optimization problem is usually more than one Pareto optimal solution. These solutions all constitute to the problem of Pareto optimal solution set. Optimization task is to produce a valid set of Pareto optimal path solution [6].

III. MULTI-OBJECTIVE OPTIMIZAITON FOR PATH PLANNIGN

Multi-objective optimization method for the path planning is studied in this section. Possible paths are generated based on two criteria, path length and hazard of colliding with obstacles for the ball on plate. The path planning problem is divided into 4 different sub-problems as below.

A Map building
B Establishing the multi-objective
C Weight calculation
D Search algorithm

Schematic representation of the path planning problem is shown in Fig. 2. Evaluation methods for safety and path length are required for quantitative analysis of the workplace. And the workplace or environment is transformed into two-dimensional grid containing information of distance and collision hazard.

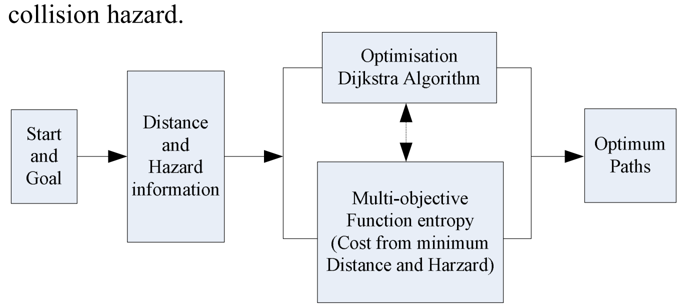


Fig.2 Structure of path planning

A. Map building

Working environment of the ball on plate is represented approximately by a two dimensional grid discretized into cells. And the two dimensional grid is usually called grid map. The grid map represents the world as a number of small cells. Each cell is defined by a pair of integer numbered coordinates. The grid size should be determined according to an appropriate resolution of the working environment. The cells that are known to contain obstacles can be marked as occupied. If the objective for path planning is to find the shortest path, only distance information should be attributed to each cell in the grid map. An example of grid map including distance information only is presented in Fig.3. Black cells in Fig.3 represent obstacles of the working environment. However, the optimal path in terms of distance may be not smooth enough for the ball to follow. If the planned path is too steep, it may be not possible for controllers to regulate the ball on the plate to track the shortest path. When the planned path is not smooth enough, ball may collide with obstacles on the plate or become unstable. Hazard information is needed to obtain the safe path. Both distance information and hazard information should be considered in the grid maps. Two grid maps are constructed here to represent the workspace of the ball on plate. They are denoted as hazard map and distance map. Examples of hazard map and distance map are shown in Fig.4. In Fig.4, (a) denotes the distance map and (b) denotes the hazard map.

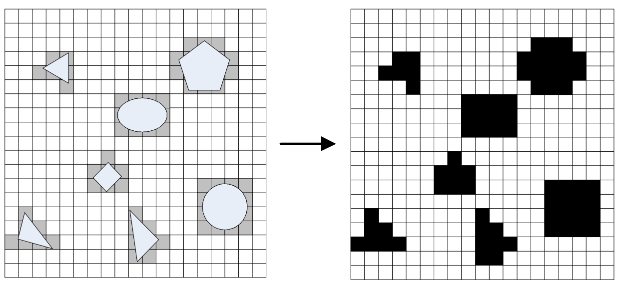
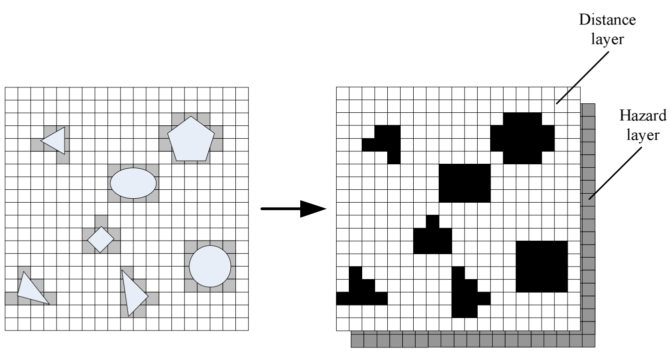


Fig.3 Grid map including distance information



(a) (b)

Fig.4 Examples of distance map and hazard map.

Distance map and hazard map are constructed as below

(1) Distance Map

Path length is represented in the distance map for the grid. Euclidean distance between the grid nodes is considered in this map. The distance between two adjacent grid nodes is 1 unit for both horizontally and vertically aligned grid nodes. And the distance for diagonally aligned grid nodes is $\sqrt{2}$ unit. This objective represents the travelling cost or path length.

(2) Hazard Map

The objective of hazard map is to represent the collision between the moving ball and the obstacles on the plate. Information of obstacles on the plate is considered in the hazard map. The hazard map is introduced to find a safe path instead of a shortest path. Hazard modelling for the workspace is associated with the presence of obstacles. The hazard can be expressed as a probability *P*. The probability of obstacles grid is 1.For a plate with one obstacle, more close the ball is to the obstacle, more probability the current location is. In normal consider dangerous and distance inverse proportion relationship. This rule for hazard calculation is illustrated in Fig. 5. Hazard value for cells in the first layer is bigger than those in the second layer.

Algorithm to obtain the hazard for the cells are given as below

```
1 set status in the M by N matrix for all cell nodes R(x,y);
2 set status in the matrix for cell nodes of Obstacles;
3 set a value of the first layer, for example the value is
   ValueH=10;
4 while (all the element of R(x,y) is not zero) do {
5  for (i∈ [2,N-1]) do {
6   if (R(i,j) == ValueH OR R(i,j) == Obstacle) then{
7    for (i_num∈ [-1,1])
       // i_num is the eight neighbourhoods row of R(i,j)
       do {
8      for (j_num∈ [-1,1])
        // j_num is the eight neighbourhoods row of R(i,j)
        do {
9        if(R((i+i_num),(j+j_num)) < ValueH) then {
10         R((i+i_num),(j+j_num)) = ValueH – 1
11       }end if
12      }end for
13    }end for
14   }end if
15  }end for
16  ValueH = ValueH-1;
17 }end while
```

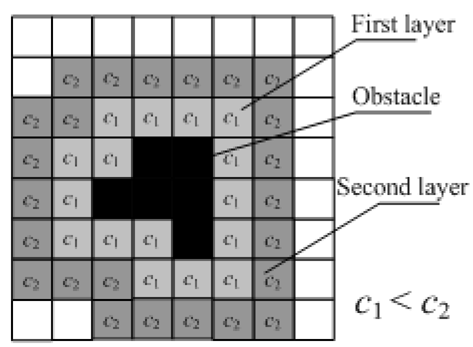


Fig.5 Illustration of hazard calculation

B. Establish the multi-objective

For a planned path covering *m* grids, objective to obtain the shortest path from the initial location to the destination is

$$\text{Min} \sum_{j=1}^{m-1} d(g_j, g_{j+1}) \tag{4}$$

where *d* is distance between the two adjacent grid

$$d(g_i, g_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{5}$$

where (x_i, y_i) denotes coordinate of grid g_i.

Objective to obtain the safest path is established as

$$\text{Min} \sum_{j=1}^{m} h(g_j) \tag{6}$$

where *h* denotes hazard for the cells in the grid map.

C. Weight calculation

Entropy weight method is employed to determine corresponding weights for several objectives [11]. Concept of entropy is initially studied in thermodynamics. It is used to describe an irreversible phenomenon. Later it is employed to measure the uncertainty in information theory. It is introduced to represent the state of disorder.

Grid node is allocated to each cell as shown in Fig.6. Node is used to store the information for the cell in the grid map. Distance and hazard values are attributed to each parent node as shown in Table I. These distance and hazard values above are further written as

$$D=\begin{bmatrix} d(i-1,j+1) & h(i-1,j+1) \\ d(i,j+1) & h(i,j+1) \\ d(i+1,j+1) & h(i+1,j+1) \\ d(i-1,j) & h(i-1,j) \\ d(i+1,j) & h(i+1,j) \\ d(i-1,j-1) & h(i-1,j-1) \\ d(i,j-1) & h(i,j-1) \\ d(i+1,j-1) & h(i+1,j-1) \end{bmatrix} \tag{7}$$

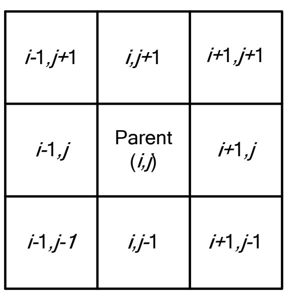


Fig.6 Grid nodes for cells

TABLE I
DISTANCE AND HAZARD VALUES FOR NODES

Grid(node)	Distance	Hazard
1	$d(i\text{-}1,j+1)$	$h(i\text{-}1,j+1)$
2	$d(i,j+1)$	$h(i,j+1)$
3	$d(i+1,j+1)$	$h(i+1,j+1)$
4	$d(i\text{-}1,j)$	$h(i\text{-}1,j)$
5	$d(i+1,j)$	$h(i+1,j)$
6	$d(i\text{-}1,j\text{-}1)$	$h(i\text{-}1,j\text{-}1)$
7	$d(i,j\text{-}1)$	$h(i,j\text{-}1)$
8	$d(i+1,j\text{-}1)$	$h(i+1,j\text{-}1)$

Distance and hazard elements in matrix $D_{m\times n}$ (m=8,n=2) corresponding to the parent node are normalized. The purpose of the normalization is to get the cost needed in the subsequent search process. The normalized element is

$$r_{ij}=\frac{D_{ij}}{\sum_{k=1}^{8}D_{kj}} \quad (i=1,2;\, j=1,...,8) \tag{8}$$

The normalized matrix is

$$R=(r_{ij})_{m\times n} \tag{9}$$

where $r_{ij}\in[0,1]$.

According to specific requirements, different criterion is generally not of equal importance. It is necessary to obtain the weights for each criterion in the evaluation. Some methods could be used to get the weights for each criterion. Entropy method is employed to determine the weights of the evaluation of several criteria.

$$E_j=-k\sum_{j=1}^{m}f_{ij}\ln f_{ij} \qquad (j=1,\cdots,n) \tag{10}$$

where

$$f_{ij}=\frac{r_{ij}}{\sum_{i=1}^{m}r_{ij}},\ k=\frac{1}{\ln m} \tag{11}$$

and assume that when f_{ij}=0，$f_{ij}\ln f_{ij}$=0.

Weight for each criterion is selected as

$$\omega_j=\frac{1-E_j}{n-\sum_{j=1}^{n}E_j} \tag{12}$$

Weight ω_j depends on the various inherent information. It is called objective weight.

A composite objective function for a grid node P_i is obtained as below

$$Z_i=\sum_{j=1}^{n}\omega_j r_{ij} \tag{13}$$

The grids in Fig.9 can be shown as nodes tree in Fig.7. Z_i is the cost between parent nodes and children nodes.

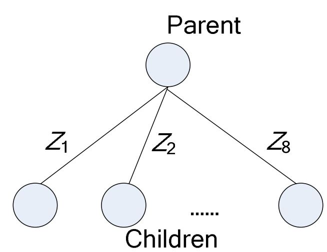


Fig.7 Nodes tree

Cost function J is selected as

$$J=\sum_{k=1}^{l-1}Z_{P_k} \tag{14}$$

where l is the number of grid nodes residing between the start and the goal nodes. P_k is the arbitrary grid nodes. The path cost function J is the summation of the values of the grid nodes associated with the grey cells, as shown in Fig.8.

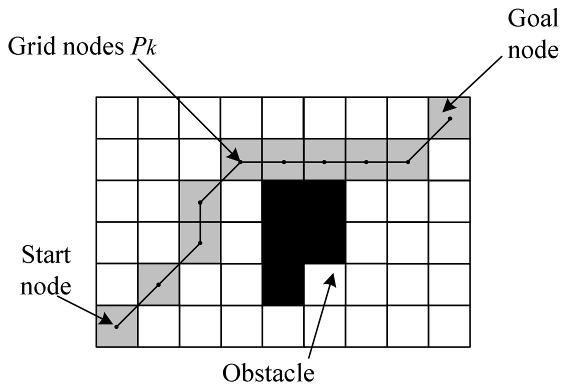


Fig.8 Path cost evaluation function

D. Solve with the search algorithm

In this paper, Dijkstra algorithm is used to solve the multi-objective problem. Dijkstra's algorithm is an often cited and well-known algorithm to solve the shortest paths for a given graph with nonnegative edge weights. The priority queue is assumed to be implemented by a simple binary heap, and the priority of a vertex is its current best path cost[12]. Dijkstra algorithm modifies the breadth-first strategy by always expanding the lowest-cost leaf node first as measured by the path cost g(n), rather than the lowest-depth node, where g(n) gives the path cost from a start node to a goal node n. Dijkstra algorithm is complete and its time complexity is the same as for breath-first search[13][14].

Search the whole tree that from the initial location to the destination position by Dijkstra algorithm, and then the multi-objective path is obtained.

IV. Simulation

The path planning approach discussed above is evaluated on the ball and plate system. An image collected by the digital camera is shown in Fig.9. This image is used to generate the distance map and hazard map. The objective here is to obtain a safe and short path for the ball. Single-objective path planning is also performed for comparison. The path planning approach is the A* algorithm.

The simulation was performed in Matlab 6.5. The grid size used for map building is in 100 by 100 cells for both multi-objective and single-objective path planning approaches in simulation.

Path planning results using the two approaches are shown in Fig.10. The path obtained by A* algorithm is shorter than the path obtained by multi-objective path planning method. However, the ball may collide with obstacles on the plate when it follows the distance optimal path. As a result, the shortest path or the distance optimal path may be not safe for the system.

Hazard of the two paths is further presented in Fig.11 and Fig.12. Hazard value varies in the section [0.3, 0.75] along the multi-objective path. And hazard value varies in the section [0.4, 1] along the single-objective path. Hazard value of a few grids along the shortest path is 1. Performance of the two path planning methods could be further compared in Table II. Average hazard of the multi-objective path is smaller compared with the single-objective path.

However, one shortcoming of the multi-objective path planning is the time consuming. It is due to the Dijkstra algorithm. The Dijkstra algorithm needs more time to search more grids in the workspace.

The path obtained by multi-objective method proposed in this paper is much safer than that obtained by single-objective A* algorithm. The safe path obtained by the multi-objective solution is more feasible for the ball to track. As the ball and plate system is open-loop unstable, the path should be carefully chosen to guarantee the ball to track the planned path. Path planning system should also be designed to avoid collision with obstacles.

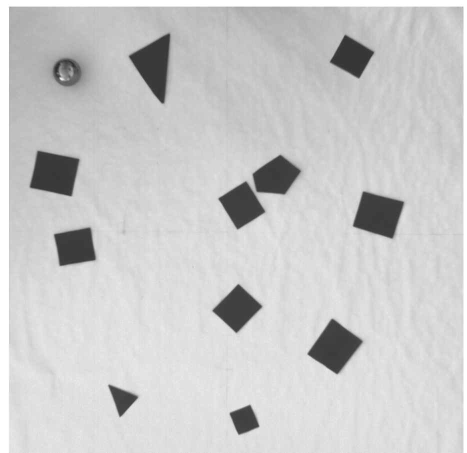


Fig.9 An image collected by the digital camera on BPVS-JLU II

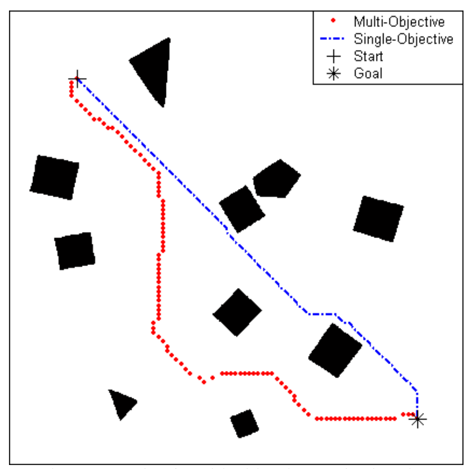


Fig.10 Result of multi-objective path planning using entropy weight method and A*method

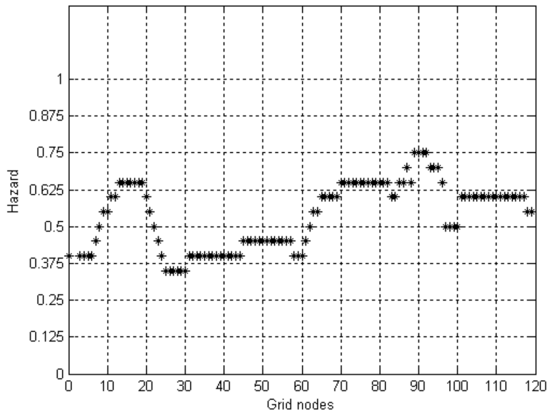


Fig.11 Hazard of Grid nodes with entropy method

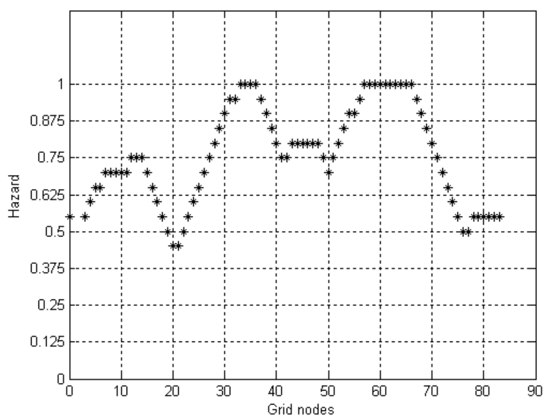


Fig 12 Hazard of Grid nodes with A* method

Table II
DISTANCE AND HAZARD FOR TWO METHODS

Method	Total Distance	Average Hazard	time-consuming(s)
A*	110.980	0.7598	0.0160
Multi-objective	137.485	0.5377	1.7820

V. CONCLUSION

Multi-objective path planning problem is studied in this paper for the ball and plate system. Path length and hazard of colliding with obstacles are considered for the multi-objective path optimization problem. Workspace of the ball on plate is represented by distance map and hazard map. And then entropy method is used to obtain the weights for the two objectives respectively for each grid node. Dijkstra algorithm is used to solve the multi-objective path planning problem finally. The simulation results had shown that the path obtained by multi-objective proposed in this paper is much safer compared with single-objective A* algorithm. Future research will focus on the engineering implementation for the multi-objective path planning approach, and the multi-objective path planning for the closed-loop system.

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