Practical No 6
Implement Arithmetic circuits.

Aim:

a) Design and implement a 2-bit by 2-bit multiplier.
b) Design and implement a 2-bit comparator..

Theory:

a) Design and implement a 2-bit by 2-bit multiplier.

Multiplier:

A 2-bit by 2-bit multiplier is a digital circuit that performs the multiplication of two 2-bit binary numbers. This type of multiplier takes two binary numbers, each consisting of 2 bits, and produces a result of up to 4 bits, as multiplying two 2-bit numbers can result in a value ranging from 0 (0000) to 9 (1001) in decimal (which is 4 bits in binary).

Binary Multiplication Basics

Binary multiplication is similar to decimal multiplication, except that it only involves two digits, 0 and 1. The rules for binary multiplication are simple:

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Inputs and Outputs of a 2-bit by 2-bit Multiplier

Inputs:

a) Let the first 2-bit binary number be A with bits A_1 and A_0
b) Let the second 2-bit binary number be B with bits B_1 and B_0

So the two binary numbers can be represented as:

$$A = A_1A_0 \quad \text{and} \quad B = B_1B_0$$

Output: The result of the multiplication will be a 4-bit binary number, with bits P_3,P_2,P_1, and P_0. The product can be written as:

$$P= P_3P_2P_1P_0$$

Truth Table:

Inputs				Outputs			
A_1	A_0	B_1	B_0	P_3	P_2	P_1	P_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

To design the given circuit we need to draw 4 k-maps and solve them.

K-map for P_3:

	$\bar{A}_1\bar{A}_0$	$\bar{A}_1A_0$	A_1A_0	$A_1\bar{A}_0$
$\bar{B}_1\bar{B}_0$	0	0	0	0
$\bar{B}_1B_0$	0	0	0	0
B_1B_0	0	0	$\boxed{1}$	0
$B_1\bar{B}_0$	0	0	0	0

The equation for the k-map is

$$P_3 = A_1.A_0.B_1.B_0$$

K-map for P_2:

	$\bar{A}_1\bar{A}_0$	$\bar{A}_1A_0$	A_1A_0	$A_1\bar{A}_0$
$\bar{B}_1\bar{B}_0$	0	0	0	0
$\bar{B}_1B_0$	0	0	0	0
B_1B_0	0	0	0	1
$B_1\bar{B}_0$	0	0	1	1

The equation for the k-map is

$$P_2 = A_1.\bar{A}_0.B_1 + A_1.B_1.\bar{B}_0$$

K-map for P_1:

	$\bar{A}_1\bar{A}_0$	$\bar{A}_1A_0$	A_1A_0	$A_1\bar{A}_0$
$\bar{B}_1\bar{B}_0$	0	0	0	0
$\bar{B}_1B_0$	0	0	1	1
B_1B_0	0	1	0	1
$B_1\bar{B}_0$	0	1	1	0

The equation for the k-map is

$$P_1 = A_1.\bar{A}_0.B_0 + A_1.\bar{B}_1.B_0 + \bar{A}_1.A_0.B_1 + A_0.B_1.\bar{B}_0$$

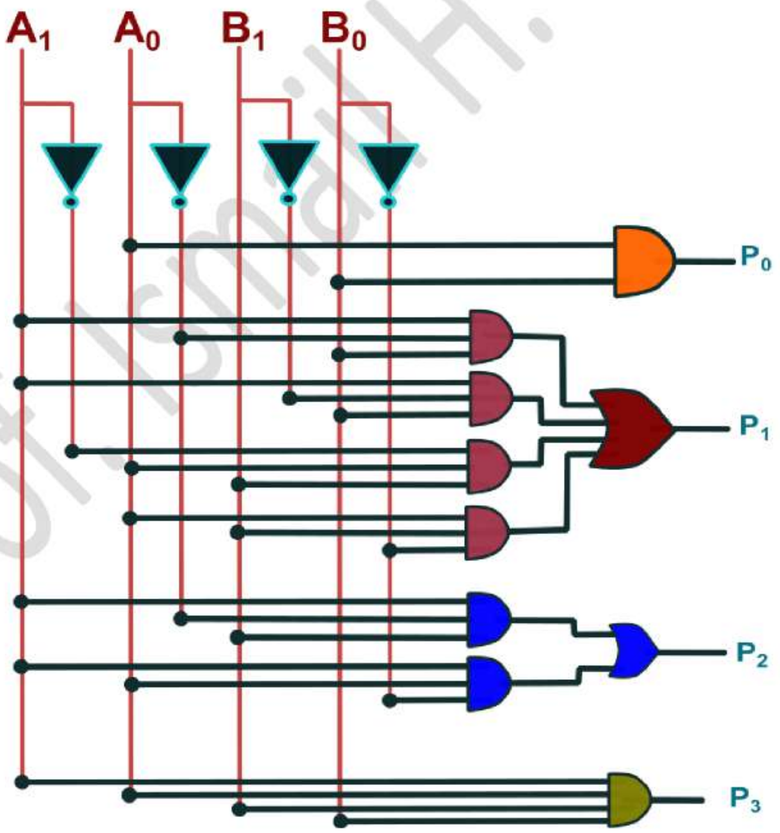
K-map for P_0:

	$\overline{A_1}\,\overline{A_0}$	$\overline{A_1}A_0$	A_1A_0	$A_1\overline{A_0}$
$\overline{B_1}\,\overline{B_0}$	0	0	0	0
$\overline{B_1}B_0$	0	1	1	0
B_1B_0	0	1	1	0
$B_1\overline{B_0}$	0	0	0	0

The equation for the k-map is

$$P_0 = A_0.B_0$$

We implement the circuit as follows



b) Design and implement a 2-bit comparator.

Comparator

A comparator is a combinational circuit that compares two binary numbers and determines their relative magnitudes. It outputs whether one number is greater than, less than, or equal to the other. In this practical, we will design a 2-bit comparator using basic logic gates.

We need to design a 2-bit comparator that compares two 4-bit binary numbers (A and B) and determines the following conditions:

i) A > B
ii) A < B
iii) A = B

A 2-bit magnitude comparator compares two 2-bit numbers, each consisting of four binary digits, A_1A_0 and B_1B_0.
The comparator produces three outputs:

i) Y_0 for A > B: This output is high (1) when the binary number A is greater than B.
ii) Y_1 for A < B: This output is high (1) when the binary number A is less than B.
iii) Y_2 for A = B: This output is high (1) when the binary numbers A and B are equal.

To design this comparator, we follow these steps:

i) Truth Table Construction: We construct a truth table that lists all possible combinations of the 4-bit inputs A) and (B), and the corresponding outputs for A > B, A < B, and A = B.
ii) K-map Simplification: Using the truth table, we create Karnaugh Maps (K-maps) for each output condition to derive simplified Boolean expressions for A > B, A < B, and A = B.
iii) Circuit Implementation: Finally, we implement the simplified Boolean expressions using basic logic gates like AND, OR, and NOT.

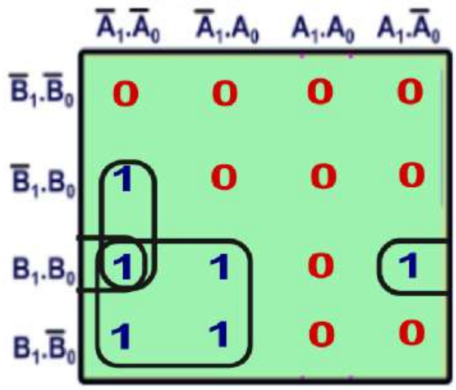
This comparator can effectively determine whether one 4-bit binary number is greater than, less than, or equal to another 2-bit binary number.

Truth table:

Input				Output		
A		B		Y_0	Y_1	Y_2
A_1	A_0	B_1	B_0	A<B	A>B	A=B
0	0	0	0	0	0	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	1	0
0	1	0	1	0	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	0	0	1
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	0	0	1

In order to design the given logic circuit, we need to draw 3 k-maps, one for each output

K-map for Y_0



The equation for the k-map is

$$Y_0 = \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0 + \bar{A}_1 \bar{A}_0 B_0$$

K-map for Y_1

	$\overline{A_1}.\overline{A_0}$	$\overline{A_1}.A_0$	$A_1.A_0$	$A_1.\overline{A_0}$
$\overline{B_1}.\overline{B_0}$	0	1	1	1
$\overline{B_1}.B_0$	0	0	1	1
$B_1.B_0$	0	0	0	0
$B_1.\overline{B_0}$	0	0	1	0

The equation for the k-map is

$$Y_1 = A_0\overline{B_0}\,\overline{B_1} + A_0A_1\overline{B_0} + A_1\overline{B_1}$$

K-map for Y_2

	$\overline{A_1}.\overline{A_0}$	$\overline{A_1}.A_0$	$A_1.A_0$	$A_1.\overline{A_0}$
$\overline{B_1}.\overline{B_0}$	1	0	0	1
$\overline{B_1}.B_0$	0	1	0	1
$B_1.B_0$	0	0	1	0
$B_1.\overline{B_0}$	0	0	0	1

The equation for the k-map is (We need to solve it until the last equation as given)

$$Y_2 = \overline{A_0}\,\overline{A_1}\,\overline{B_0}\,\overline{B_1} + A_0\overline{A_1}B_0\overline{B_1} + A_0A_1B_0B_1 + \overline{A_0}A_1\overline{B_0}B_1$$

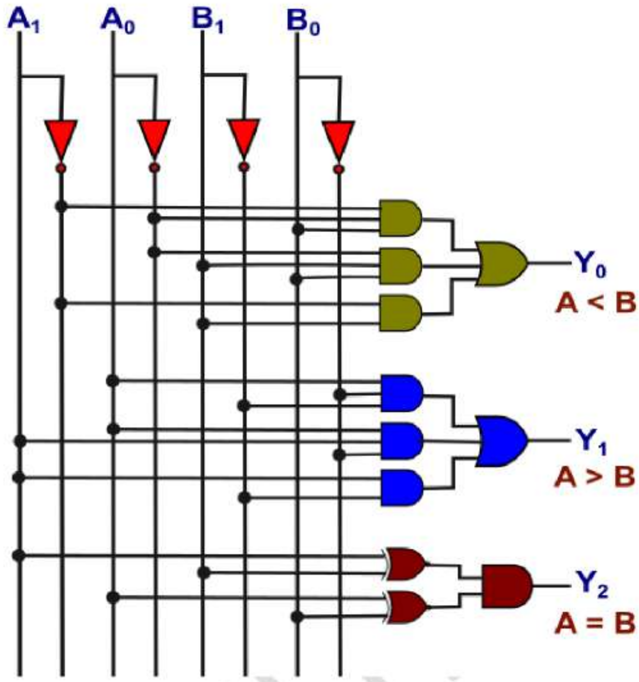
$$Y_2 = \overline{A_0}\,\overline{B_0}(\overline{A_1}\,\overline{B_1} + A_1B_1) + A_0B_0(\overline{A_1}\,\overline{B_1} + A_1B_1)$$

$$Y_2 = \overline{A_0}\,\overline{B_0}(A_1 \odot B_1) + A_0B_0(A_1 \odot B_1)$$

$$Y_2 = (A_1 \odot B_1)(\overline{A_0}\,\overline{B_0} + A_0B_0)$$

$$Y_2 = (A_1 \odot B_1)(A_0 \odot B_0)$$

Logic Diagram:



Result: The following circuits were designed, studied and verified.

i) 2-bit multiplier
ii) 2-bit comparator

For video demonstration of the given practical scan the QR-codes

For 2 – bit Comparator	For 2 – bit Multiplier