

# Portfolio Management

## FN 4329

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American College of Greece

Spring Semester 2020

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- 1 Portfolio Construction
  - Asset Allocation
  - Security Selection
  - Diversification strategy
- 2 Theory
  - Descriptive Statistics
  - Financial Metrics
- 3 Building the Portfolio
  - Naive allocation
  - Portfolio Optimization
  - Optimal Overall Portfolio

## Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

# Questionnaire

1/30/2020

Vanguard - Investor Questionnaire

PERSONAL INVESTORS



## Investor questionnaire

1. I plan to begin taking money from my investments in ...

- ☐ 1 year or less
- ☐ 1 - 2 years
- ☐ 3 - 5 years
- ☐ 6 - 10 years
- ☐ 11 - 15 years
- ☒ More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- ☒ 2 years or less
- ☐ 3 - 5 years
- ☐ 6 - 10 years
- ☐ 11 - 15 years
- ☐ More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- ☐ 1 - 2 years
- ☐ 3 - 4 years
- ☐ 5 - 6 years
- ☐ 7 - 8 years
- ☒ More than 8 years

4. From September 2008 through November 2008, stocks lost more than 21%. If I owned a stock investment that lost about 21% in 3 months, I would ... (If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)

- ☐ Sell all of the remaining investment.
- ☐ Sell a portion of the remaining investment.
- ☒ Hold onto the investment and sell nothing.
- ☐ Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- ☐ Strongly disagree
- ☒ Disagree
- ☐ Somewhat agree
- ☐ Agree
- ☐ Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

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- ☐ Disagree
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7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... (If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)

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<https://personal.vanguard.com/us/FundInvQuestionnaire>

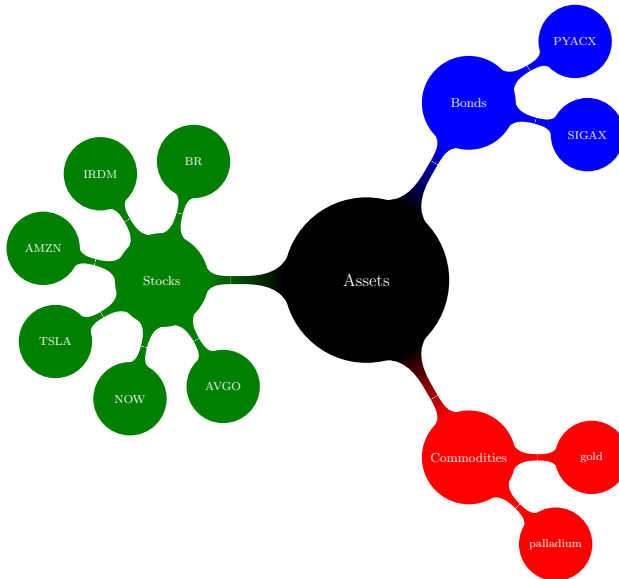
1/2



## Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

# Security Selection



# Stocks

AVGO - Broadcom Inc

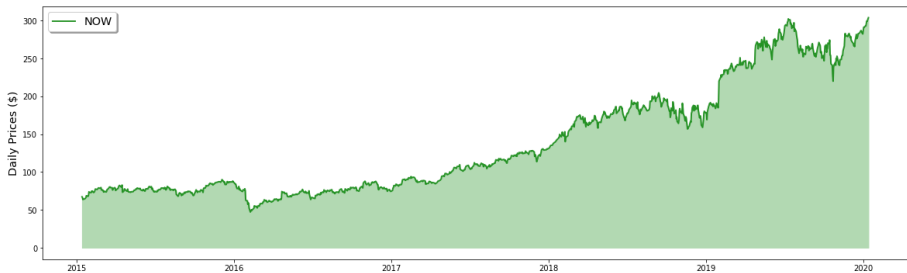
## add - DO THIS FOR EVERY SECURITY

- category
- few words
- price chart



# Stocks

NOW - ServiceNow Inc.





# Stocks

TSLA - Tesla Inc.



# Stocks

AMZN - Amazon Inc.



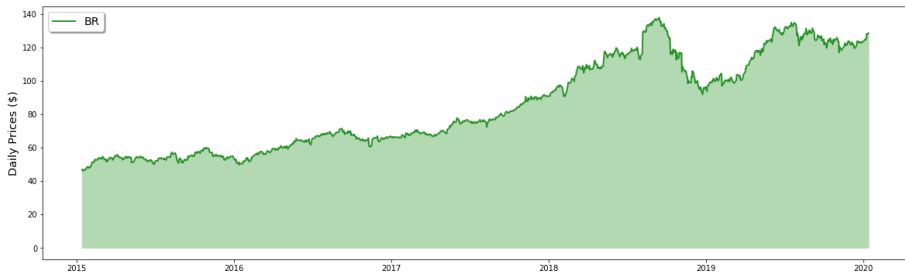
# Stocks

IRDM - Iridium Communications Inc.



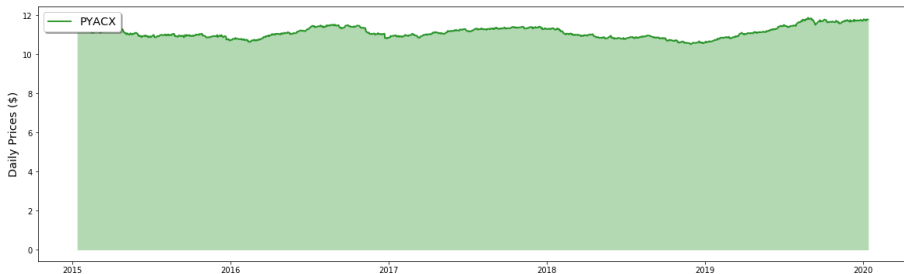
# Stocks

BR - Broadridge Financial Solutions Inc.



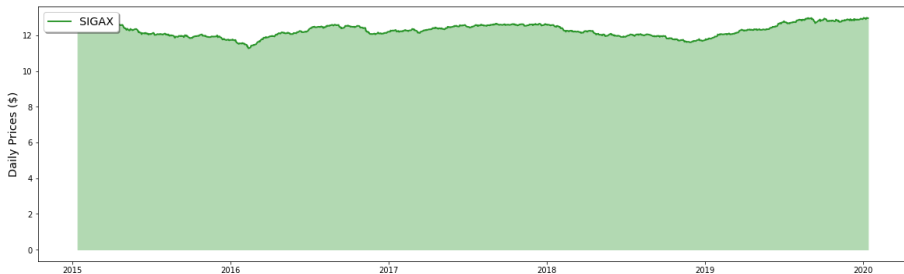
# Bonds

## PYACX - Payden Corporate Bond Mutual Fund



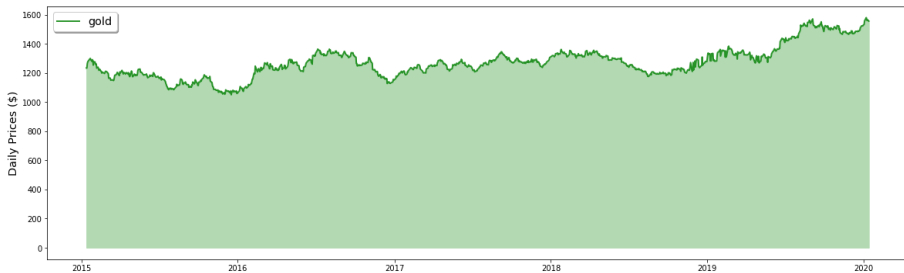
# Bonds

## SIGAX - Western Asset Corporate Bond Mutual Fund



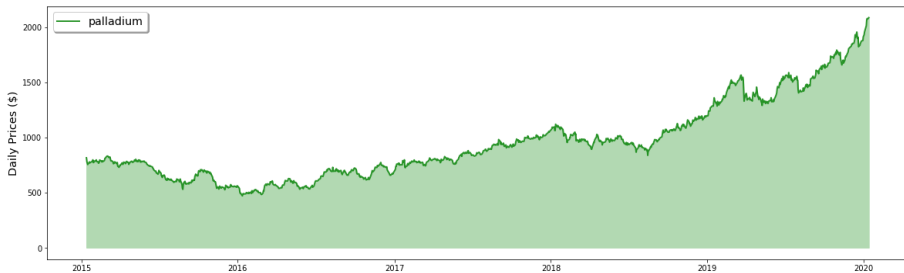
# Commodities

gold



# Commodities

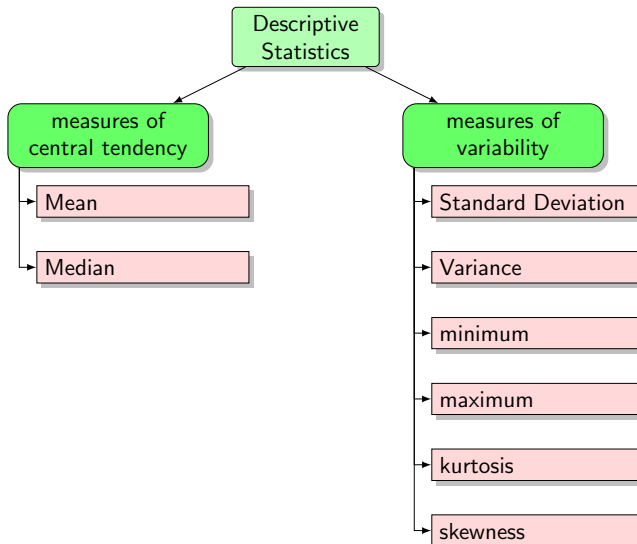
## palladium





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# Descriptive statistics Taxonomy



# Measures of central tendency

## Mean

### Mean

$$\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

# Measures of central tendency

## Median

- i.e. the middle value

# Measures of central tendency

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- i.e. the middle value
- Why?

# Measures of central tendency

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- i.e. the middle value
- Why? robust w.r.t. outliers

# Measures of central tendency

## Median

- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

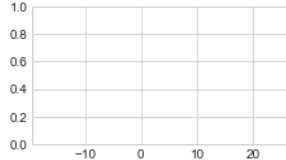
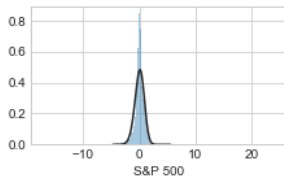
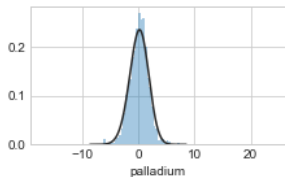
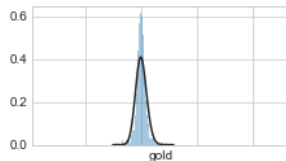
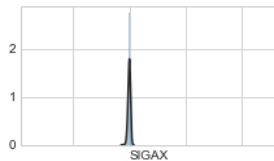
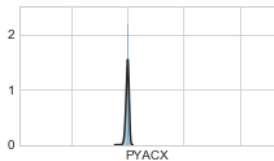
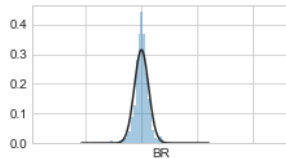
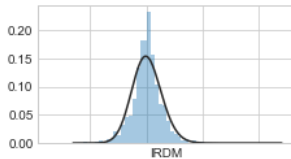
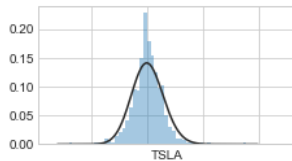
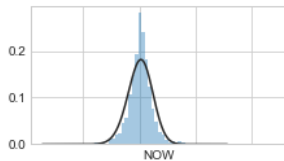
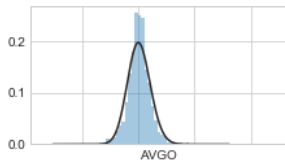
# Measures of Variability

## Standard Deviation

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics  $\implies$  risk in finance





# Measures of Variability

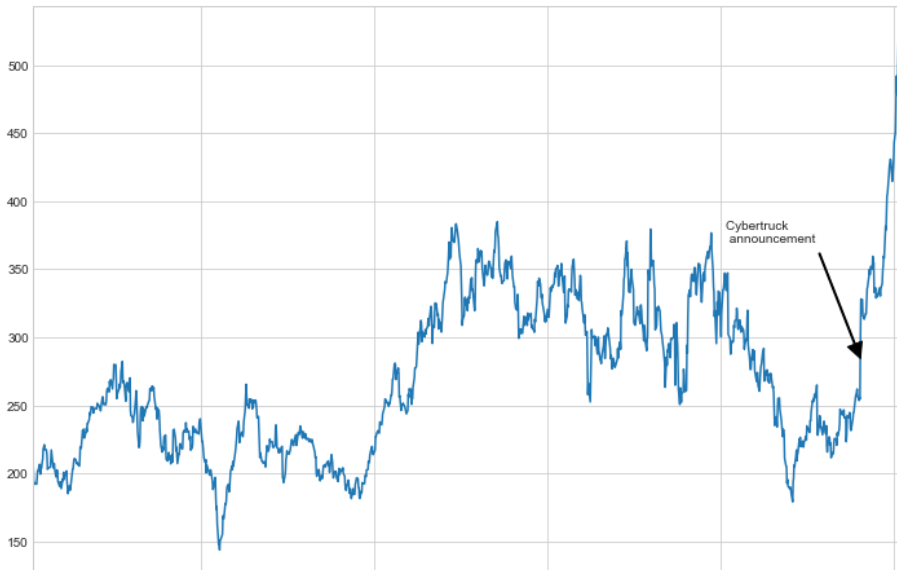
## Minimum & Maximum



**Figure:** The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

# Measures of Variability

## Minimum & Maximum



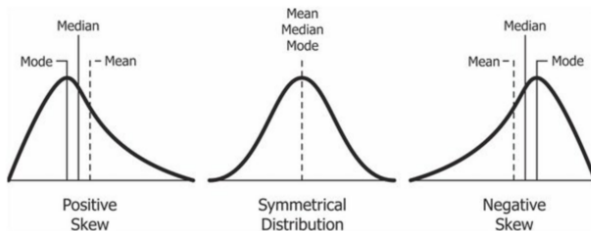
# Measures of Variability

## Skewness

### Definition

$$\tilde{\mu}_3 = \mathbb{E} \left[ \left( \frac{X - \bar{x}}{\sigma} \right)^3 \right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



# Measures of Variability

## Kurtosis

### Definition

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E} \left[ \left( \frac{X - \bar{x}}{\sigma} \right)^4 \right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis  $\rightarrow$  heavy tails (**outliers**)
- low kurtosis  $\rightarrow$  no outliers

# Measures of Variability

## An overview

### Moments from an investor's perspective

- $\tilde{\mu}_2$  standard deviation  $\sigma$
- $\tilde{\mu}_3$  skewness
- $\tilde{\mu}_4$  kurtosis

### TODO

add what each moments means to understanding security performance

# Descriptive Statistics

For our assets

	mean%	median%	std%	var%	min%	max%	kurtosis	skewness
AVGO	0.11	0.11	2.04	4.17	-13.74	14.71	5.31	0.23
NOW	0.15	0.21	2.23	4.96	-15.66	14.07	6.50	-0.33
TSLA	0.12	0.06	2.85	8.13	-13.90	17.67	5.06	0.30
AMZN	0.17	0.14	1.85	3.42	-7.82	14.13	9.83	1.01
IRDM	0.12	0.12	2.63	6.94	-11.13	22.24	6.47	0.62
BR	0.09	0.09	1.27	1.61	-9.70	11.16	9.29	-0.19
PYACX	0.00	0.00	0.27	0.07	-2.08	0.77	3.40	-0.71
SIGAX	0.00	0.00	0.23	0.05	-1.29	0.67	1.28	-0.43
gold	0.02	0.00	0.99	0.98	-4.32	5.10	4.56	0.33
palladium	0.09	0.15	1.71	2.92	-7.40	7.28	1.49	-0.15
S&P 500	0.04	0.05	0.84	0.71	-4.10	4.96	3.87	-0.47

## Covariance Definition

Let  $X$  and  $Y$  be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$\text{cov}(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$



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- not so helpful!

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- not so helpful!  $\rightarrow$  correlation

# Correlation

## Definition

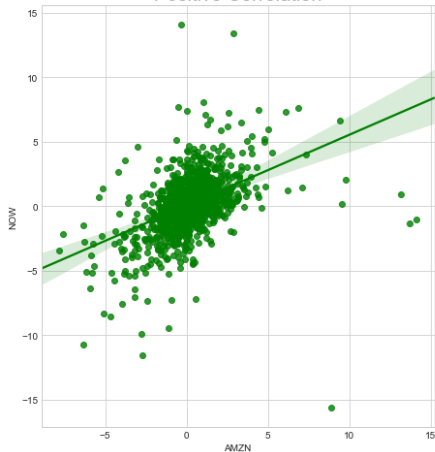
The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

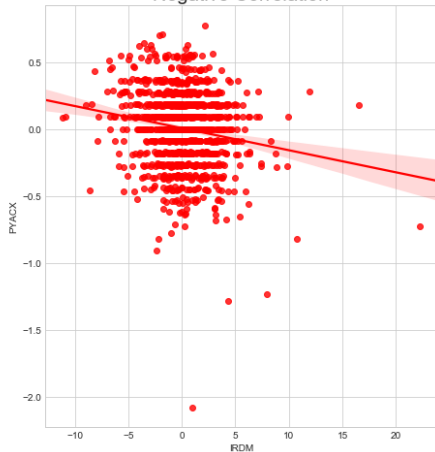
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

# A closer look at correlation

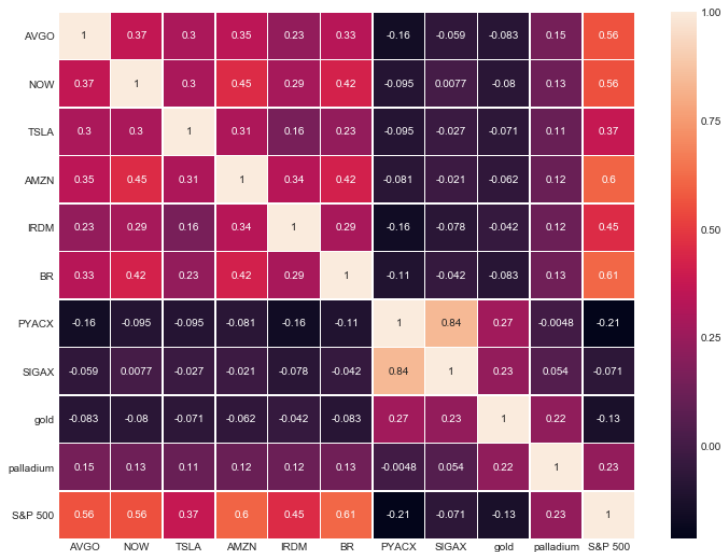
Positive Correlation



Negative Correlation



# Correlation Matrix



## Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

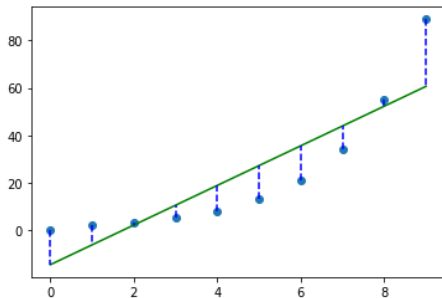
## Definition

Alpha is the difference between the realised returns and the expected returns:

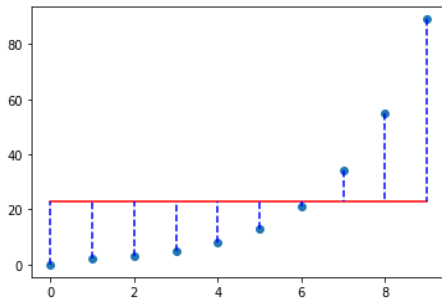
$$\alpha = \bar{R} - \mathbb{E}(R)$$
$$\stackrel{\text{CAPM}}{\implies} \alpha = \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}$$

# R-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation



(b) Total Variation



# Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- $R_p$  = return of mutual fund
- $R_f$  = risk-free rate
- $\sigma_p$  = standard variation of the portfolio's excess return

# Financial Metrics

For our assets

	alpha	beta	R-squared
AVGO	0.14213	1.35333	0.31393
NOW	0.24718	1.46614	0.30955
TSLA	0.17711	1.26445	0.14046
AMZN	0.32815	1.32199	0.36539
IRDM	0.15867	1.41795	0.20698
BR	0.13206	0.92064	0.37655
PYACX	-0.00576	-0.06792	0.04613
SIGAX	-0.01215	-0.01911	0.00501
gold	0.05087	-0.15174	0.01684
palladium	0.17817	0.47398	0.05493

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# Risk and return

## Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

## Risk

$$\sigma_p = \sqrt{\vec{w}^\top K \vec{w}}$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \text{COV}_{1,2} & \dots & \text{COV}_{1,n} \\ \text{COV}_{2,1} & \sigma_2^2 & \dots & \text{COV}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{COV}_{n,1} & \text{COV}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}}$$
$$= \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{COV}_{ij}}$$

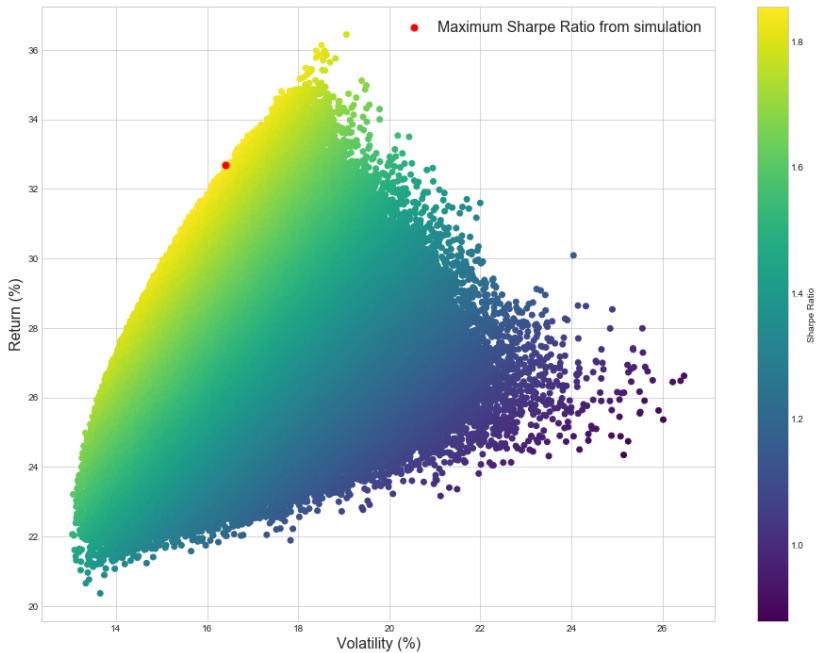
# Naive weight allocation

## Remember!

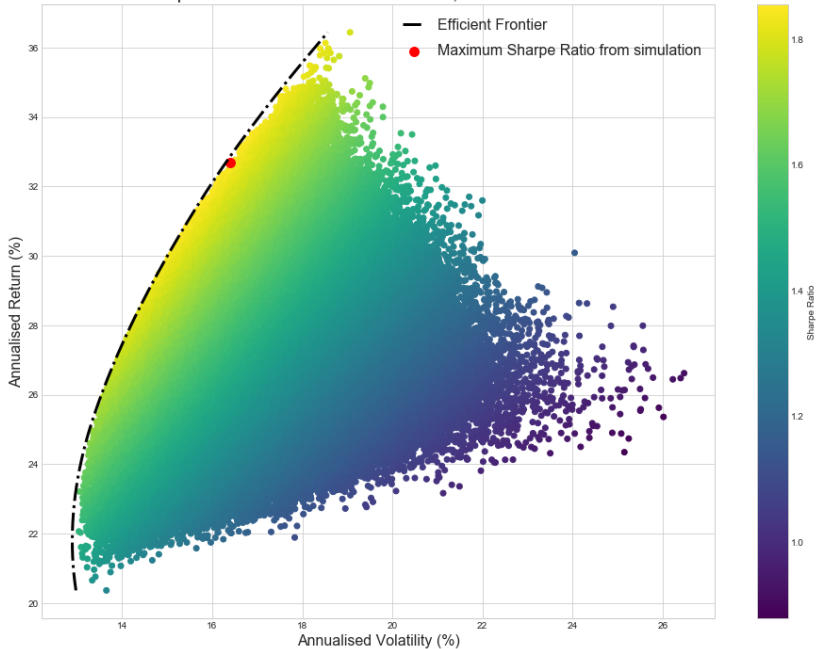
- 70% Stocks
- 20% Bonds
- 10% Commodities

## What about $\vec{w}$ ?

- 70% for 6 Stocks  $\rightarrow \frac{70\%}{6} = 11.67\%$  each
- 20% for 2 Bonds  $\rightarrow 10\%$  each
- 10% for 2 Commodities  $\rightarrow 5\%$  each



## Portfolio Optimization with Individual Stocks, Bonds and Commodities

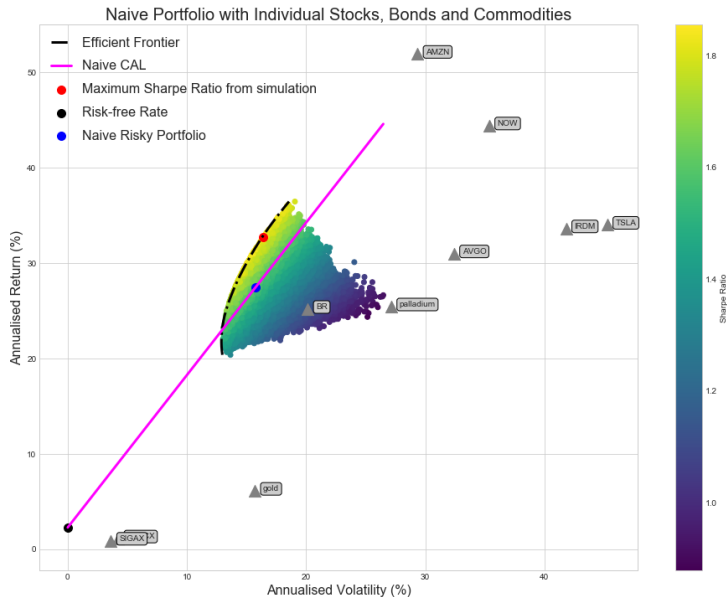


# CAL construction

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \Rightarrow \epsilon_{\text{naive}} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$



# Naive CAL



But...

We can do better!

# Optimal portfolio

## Optimization problem formulation

$$\begin{aligned} \max \quad & \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\ \text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\ & \mathbf{1}^\top \vec{w}_S = w_s \\ & \mathbf{1}^\top \vec{w}_B = w_b \\ & \mathbf{1}^\top \vec{w}_C = w_c \\ & w_s + w_b + w_c = 1 \\ & w_i \geq 0 \quad i = 1, \dots, n \end{aligned}$$

# Optimal portfolio

## Optimization problem formulation

Sharpe Ratio

max

$$\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}$$

s.t.

$$\mathbf{1}^\top \vec{w} = 1$$

$$\mathbf{1}^\top \vec{w}_S = w_s$$

$$\mathbf{1}^\top \vec{w}_B = w_b$$

$$\mathbf{1}^\top \vec{w}_C = w_c$$

$$w_s + w_b + w_c = 1$$

$$w_i \geq 0 \quad i = 1, \dots, n$$

# Optimization yields...

optimal weights

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

# Optimization yields...

## Financial Metrics

	Optimal Risky Portfolio
Return (%)	32.7164
Risk (%)	16.3185
Sharpe Ratio	1.8658
Beta ( $\beta$ )	0.9067

# Capital Allocation Line

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

# Capital Allocation Line

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# Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

# Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t.  $z$ :

# Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

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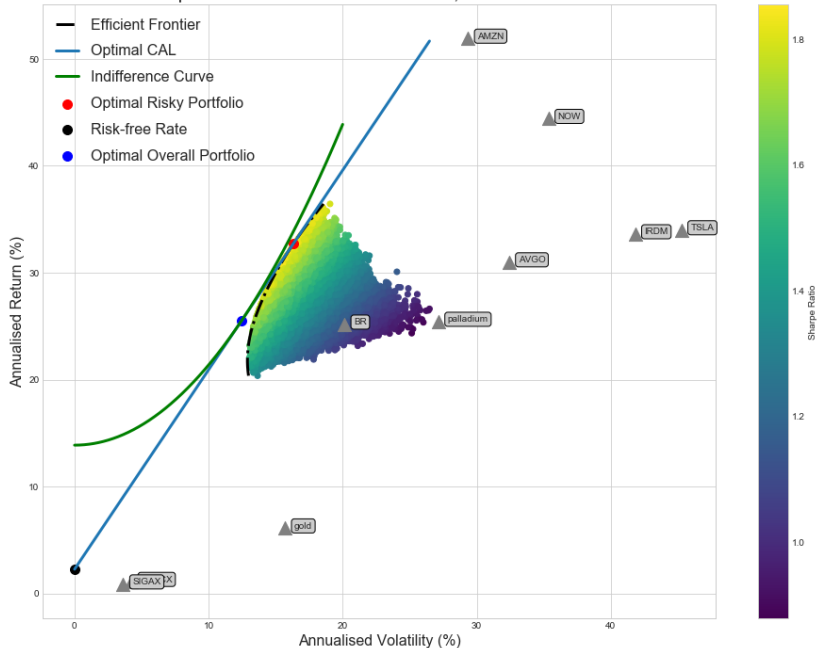
Maximize w.r.t.  $z$ :

$$\frac{\partial U}{\partial z} = 0$$

$$\implies r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\implies z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$

# Portfolio Optimization with Individual Stocks, Bonds and Commodities



# Past Performance

(1/13/2015 - 1/13/2020)

	Optimal Overall Portfolio	S&P 500
Return (%)	25.4769	11.3118
Risk (%)	12.4384	13.4138
Sharpe Ratio	1.8658	0.6741
Beta ( $\beta$ )	0.6911	1.0
Alpha ( $\alpha$ ) (%)	16.9584	0.0

# References



# The end!