# Portfolio Management
## FN 4329

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# Contents

1. Question 1
2. Question 2
3. Question 3
4. Question 4
   - Measures of central tendency
   - Measures of Variability
5. Question 5
   - correlation and covariance
   - beta
   - alpha
   - R-squared
   - Sharpe ratio
6. Question 6
7. Question 7

# todo

## Todo

- fix contents (example: Intro, Theory, Results)

# Asset allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

# Contents

1. Question 1
2. Question 2
3. Question 3
4. Question 4
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   - Measures of Variability
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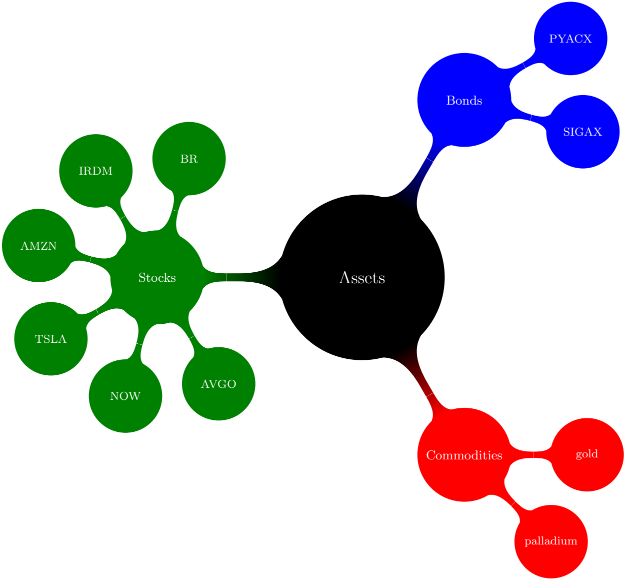
   - beta
   - alpha
   - R-squared
   - Sharpe ratio
6. Question 6
7. Question 7

# Security Selection



# Contents

1 Question 1
2 Question 2
3 Question 3
4 Question 4
- Measures of central tendency
- Measures of Variability

5 Question 5
- correlation and covariance
- beta
- alpha
- R-squared
- Sharpe ratio

6 Question 6
7 Question 7

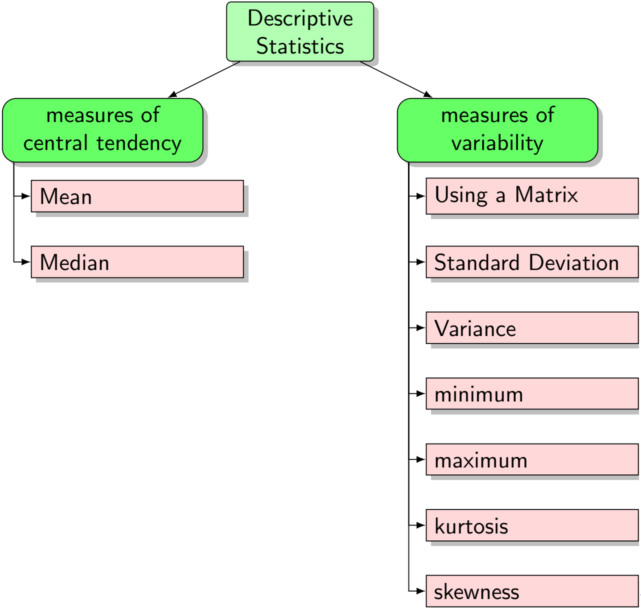
# Contents

1 Question 1
2 Question 2
3 Question 3
4 Question 4
- Measures of central tendency
- Measures of Variability

5 Question 5
- correlation and covariance
- beta
- alpha
- R-squared
- Sharpe ratio

6 Question 6
7 Question 7

# Descriptive statistics Taxonomy



# Measures of central tendency

## Mean

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

## Median

The median of data sample can be thought as the middle value of dataset, separating the higher from the lower half. It is a more robust measure than the mean, since it is not affected by outliers. The median can inform the investor on whether the returns are positive or negative on most time instances.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measure confidence in statistics $\Longrightarrow$ risk in finance


Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

# Measures of Variability

## Minimum & Maximum

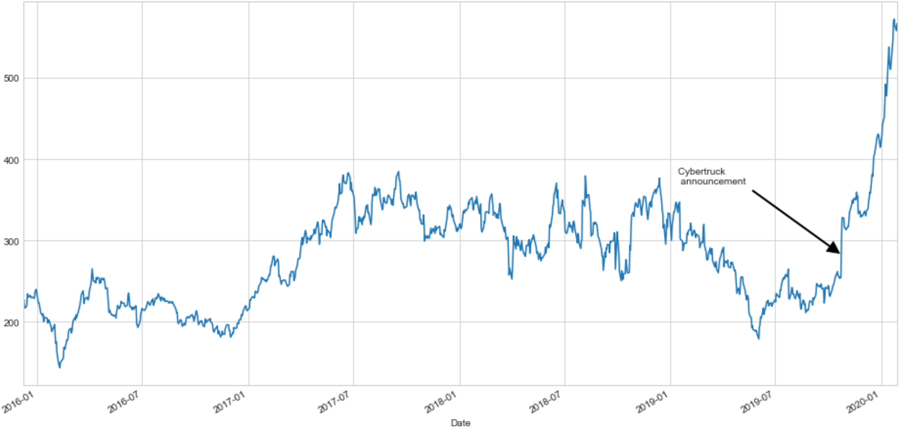



Figure: Tesla's announcement of the Cybertruck resulted in a steep price increase.

**Definition**

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^4\right]$$

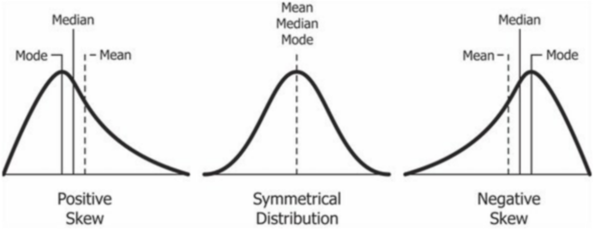
Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

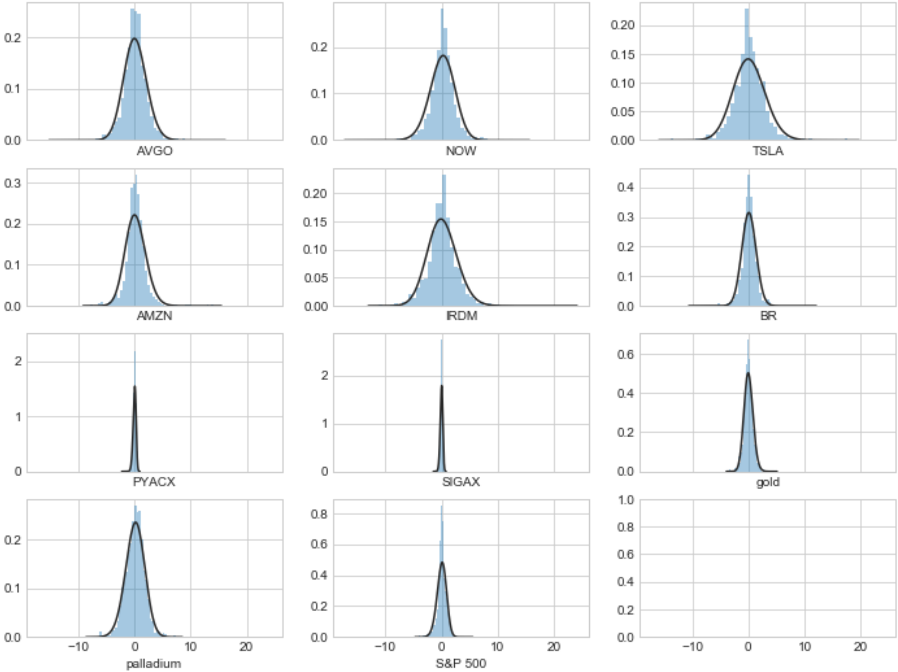
- high kurtosis $\rightarrow$ heavy tails **(outliers)**
- low kurtosis $\rightarrow$ no outliers

## Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.




AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500

# Contents

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2 Question 2
3 Question 3
4 Question 4
- Measures of central tendency
- Measures of Variability

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- alpha
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6 Question 6
7 Question 7

# Correlation and covariance

## Covariance Definition

Let $X$ and $Y$ be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

## Correlation Definition

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X, Y)}{\sigma_X \cdot \sigma_Y}$$

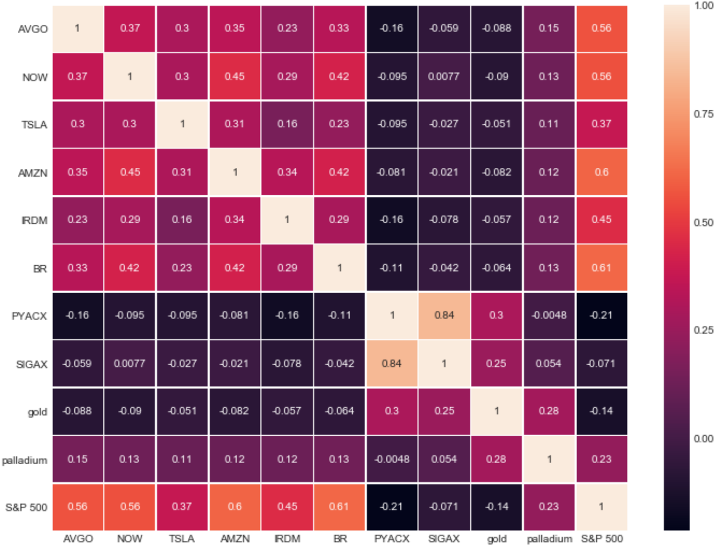
# A closer look at correlation

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

**todo**

add regression plots to show the difference

# Correlation Matrix



# Beta

**Definition**

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

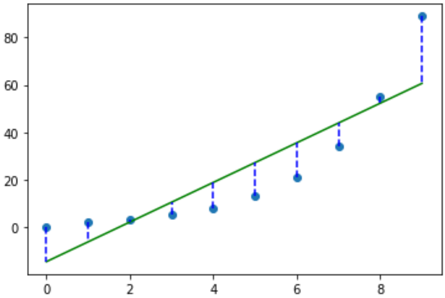
# alpha

$$\mathbb{E}(R_i) = R_f + \beta_i(\mathbb{E}(R_m) - R_f) \tag{1}$$

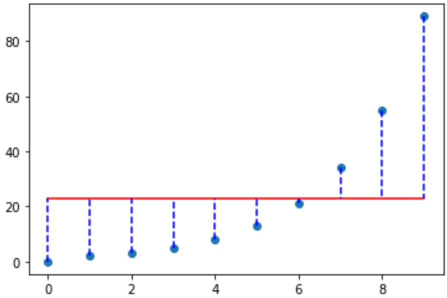
$$\alpha = \bar{R} - \mathbb{E}(R) \tag{2}$$

# $R$-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$




(a) Explained Variation




(b) Total Variation

# Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{3}$$

where

- $R_p$ = return of mutual fund
- $R_f$ = risk-free rate
- $\sigma_p$ = standard variation of the portfolio's excess return

# Contents


1. Question 1
2. Question 2
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# Risk and return

## Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

## Risk

$$\begin{aligned}
\sigma_p &= \sqrt{\vec{w}^\top K \vec{w}} \\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}
\end{aligned}$$

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2. Question 2
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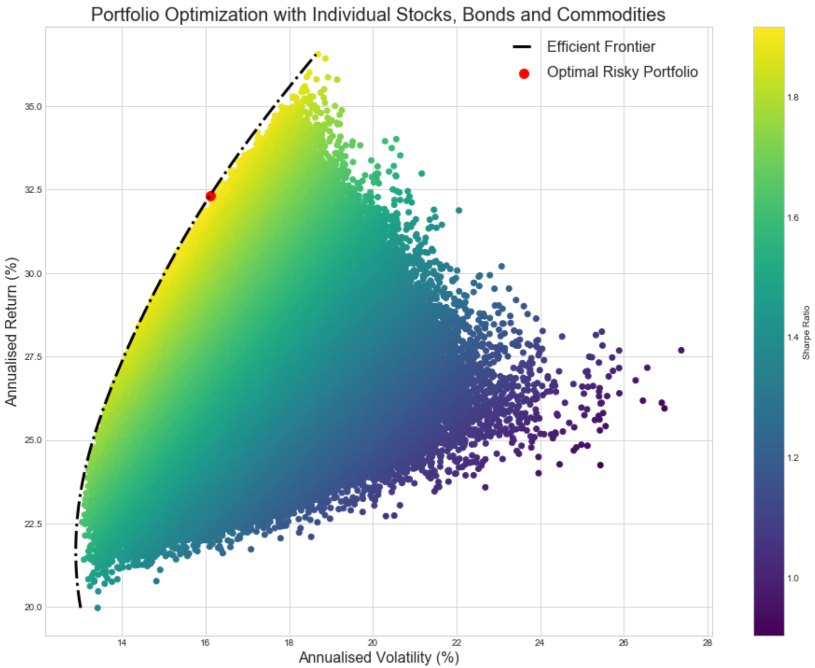
7. Question 7


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal Risky Portfolio
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
35.0
32.5
30.0
27.5
25.0
22.5
20.0
14
16
18
20
22
24
26
28
1.8
1.6
1.4
1.2
1.0

# Optimal portfolio
## Optimization problem formulation

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Sharpe Ratio

$$\max \quad \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}}$$

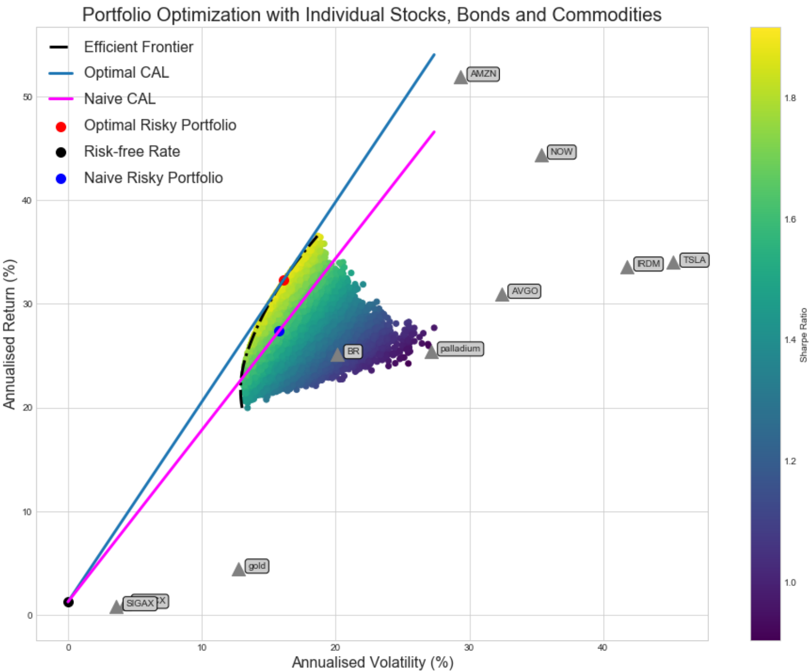
$$\begin{aligned}
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}$$

# Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\mathrm{CAL}} \\ (\sigma_{\mathrm{OPT}}, r_{\mathrm{OPT}}) \in \epsilon_{\mathrm{CAL}} \end{array} \right\}$$

# Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{OPT}} - r_f}{\sigma_{\text{OPT}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal CAL
Naive CAL
Optimal Risky Portfolio
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

# References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The end!