FN 4329: INVESTMENTS AND PORTFOLIO MANAGEMENT

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1 Question 1

Decide on the allocation of your budget to each of the above instruments. Rationalize your choice based on your own investment philosophy. You should fill out a risk tolerance questionnaire and supply both the questionnaire and its suggested allocation (in the project's Appendix). In your discussion, please explain the choice of your investment vehicles and their significance in your portfolio. Label this as your investment policy statement (IPS).

2 Question 2

Outline your investment objectives, constraints and strategies and discuss the relative importance of risk and return in your investment decision making.

3 Question 3

Provide a preliminary discussion of the effects of the short investment horizon in your decision(s) and diversification strategy

4 Question 4

Compute the descriptive statistics for each instrument. Explain each metric you computed from the perspective of the investor. Provide graphs as well.

Descriptive statistics are brief descriptive coefficients that summarize a given data set. In our case, this dataset is the returns of each instrument. These coefficients enable the reader to understand the features of the dataset and derive quantitative insights on its nature.

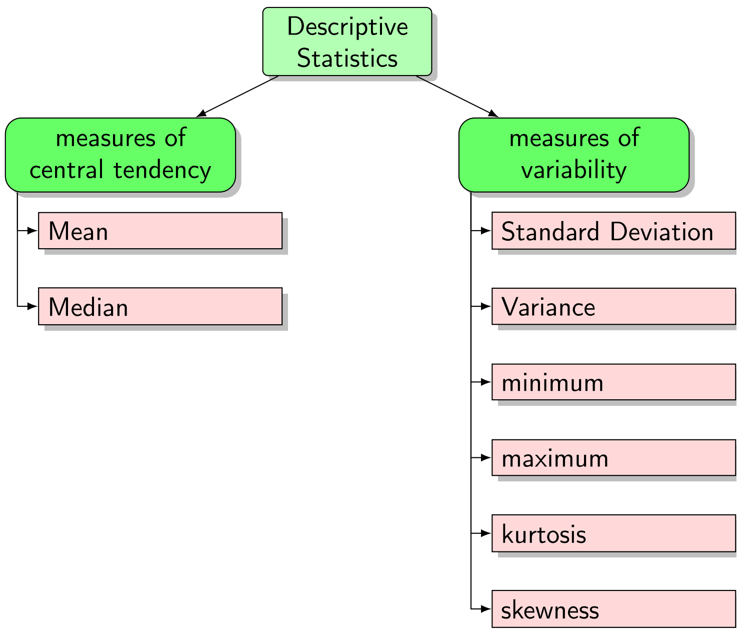


Figure 1: Taxonomy of descriptive statistics

Let $x_i, i \in [n]$ be the sample, i.e. the returns of an instrument.

Measures of central tendency

Measures of central tendency describe the center position a distribution for a dataset.

Mean The mean, denoted as $\bar{x}$ is the sum of the sampled values divided by the number of items in the sample:

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n} \tag{4.1}$$

From the investor's perspective, the mean describes the average performance of the instrument. If $\bar{x} > 0$ then the instrument increases in value on average.

Median The median of data sample can be thought as the middle value of dataset, separating the higher from the lower half. It is a more robust measure than the mean, since it is not affected by outliers. The median can inform the investor on whether the returns are positive or negative on most time instances.

Measures of Variability

Measures of variability describe how the data is distributed within the set.

Standard Deviation The Standard deviation expresses the variability of a population. It indicates the extent to which the values tend to be close to the mean. It is commonly used to measure confidence in statistics. For this reason, it is a measure of risk in economic terms. It is given by the following formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{4.2}$$

From the perspective of the investor, an instrument with high standard deviation carries more risk and should have higher returns to accomodate this variability of returns.

Minimum & Maximum Minimum and maximum describe the range of values. In economic terms, the maximum and minimum of returns reflect events that shape the price of the instrument. An example would be the recent economic crisis.

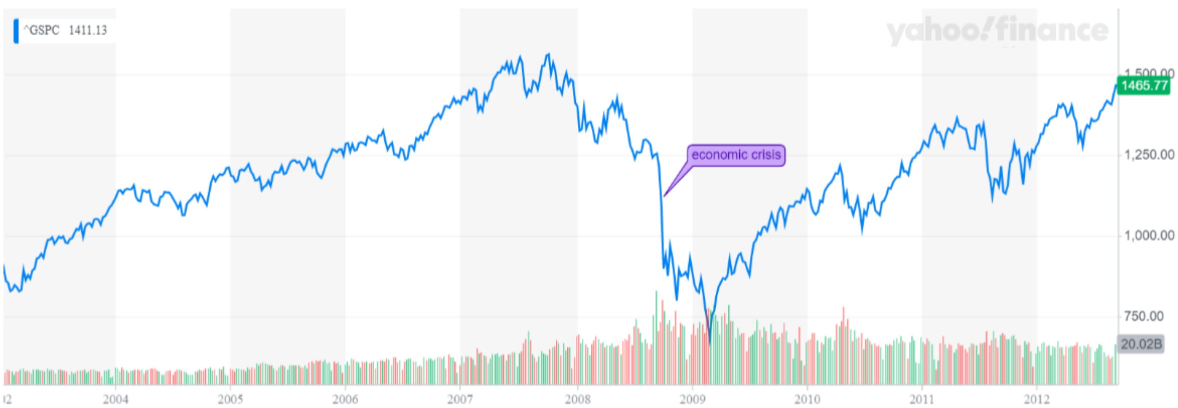


Figure 2: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

Kurtosis Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution. Data sets with high kurtosis tend to have heavy tails, or outliers, whilst data sets with low kurtosis lack outliers. The kurtosis is given by the following formula:

$$\mathrm{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^4\right] \tag{4.3}$$

For investors, high kurtosis of the return distribution implies that the investor will experience occasional extreme returns.

Skewness Skewness is a measure of asymmetry. To be more precise, this coefficient indicates if the tail of the distribution is on the left or the right. Let us consider a simple example:

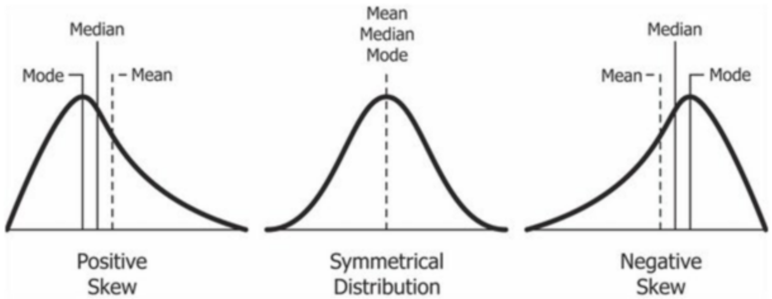


Figure 3: Source: Wikipedia

The skewness is given by the following formula:

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X-\bar{x}}{\sigma}\right)^3\right] \tag{4.4}$$

Skewness is a vary important measure for investors. Standard Deviation is commonly associated with the estimation of an instrument's risk, but it has a major flaw in assming a normal distribution. Since few return distributions resemble a normal distribution, skewness is a better measure for predicting performance.

Most asset returns are skewed, either left or right. An investor can utilize this infomrmation to better predict future returns. A positively-skewed investment return means that there were frequent small losses and a few large gains. The opposite is true for negatively-skewed distributions.

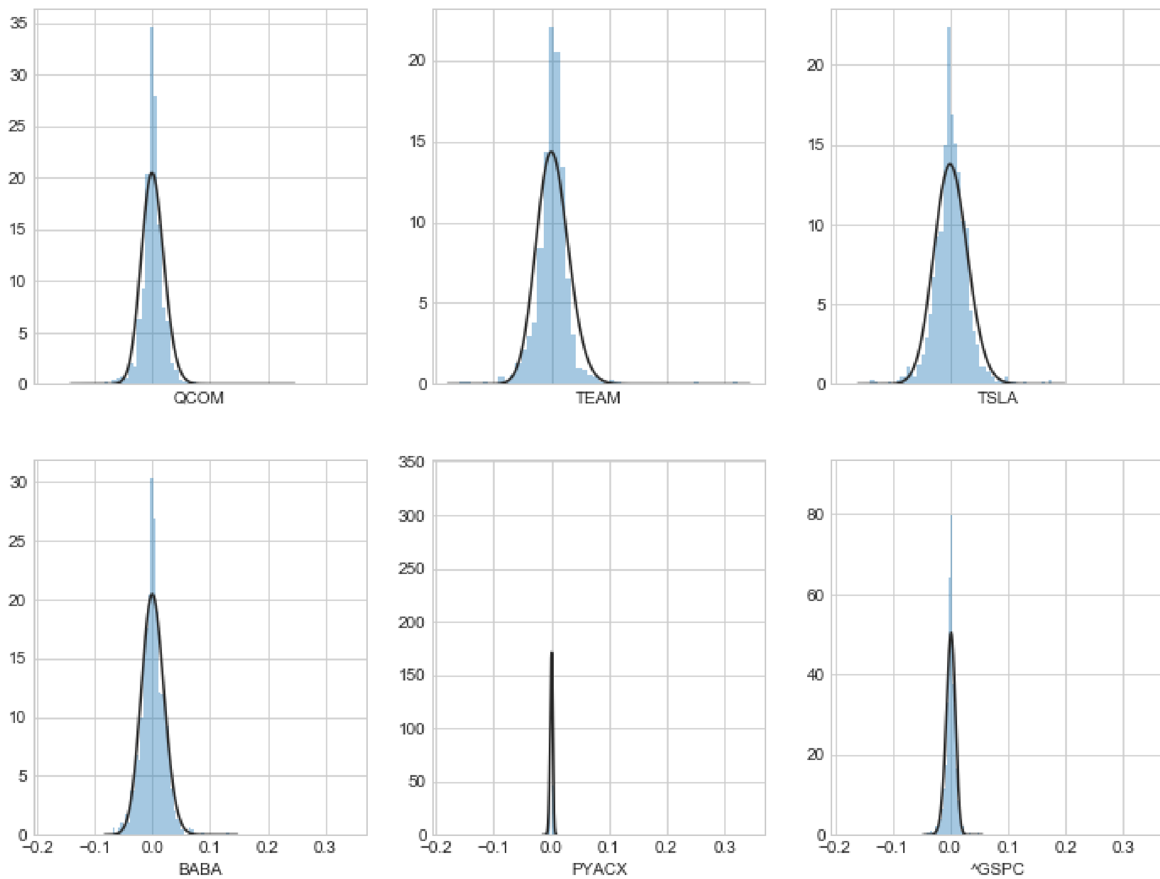


Figure 4: Histogram of the returns

	QCOM	TEAM	TSLA	BABA	^GSPC
mean	0.000902	0.002253	0.001306	0.001065	0.000477
median	0.000957	0.002923	0.000855	0.001053	0.000600
std	0.019919	0.028439	0.029307	0.019771	0.008154
var	0.000397	0.000809	0.000859	0.000391	0.000066
min	-0.127205	-0.158750	-0.139015	-0.075999	-0.040979
max	0.232162	0.322857	0.176692	0.132919	0.049594
kurtosis	23.313450	22.733758	5.155017	2.897280	4.356755
skewness	1.332546	1.605961	0.317190	0.193228	-0.571087

5 Question 5

Compute additional metrics for the assets such as the correlation and covariance matrices, for the entire and two subperiods (of your own choosing), if needed. Interpret your findings. Also, compute each fund's alpha, beta, R-square. Interpret your findings from the perspective of the investor

5.1 correlation and covariance

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X,Y) = \mathbb{E}[(X-\bar{x})(Y-\bar{y})] \tag{5.1}$$

In our case these random variables are the returns of the various instruments. The elememnt (i,j) of the covariance matrix is the correlation coefficient between intruments i and j:

	QCOM	TEAM	TSLA	BABA	^GSPC
QCOM	0.000397	0.000133	0.000116	0.000144	0.000081
TEAM	0.000133	0.000809	0.000166	0.000170	0.000091
TSLA	0.000116	0.000166	0.000859	0.000170	0.000087
BABA	0.000144	0.000170	0.000170	0.000391	0.000094
^GSPC	0.000081	0.000091	0.000087	0.000094	0.000066

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y} \tag{5.2}$$

The correlation ranges between -1 and 1 and it measures the linear dependence between two variables.

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1,1), & \text{indicating the degree of linear dependence between the variables} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases} \tag{5.3}$$

the correlation matrix is presented below:

	QCOM	TEAM	TSLA	BABA	^GSPC
QCOM	1.000000	0.235646	0.199471	0.364991	0.499449
TEAM	0.235646	1.000000	0.199616	0.301822	0.392658
TSLA	0.199471	0.199616	1.000000	0.293921	0.365000
BABA	0.364991	0.301822	0.293921	1.000000	0.580228
^GSPC	0.499449	0.392658	0.365000	0.580228	1.000000

Both correlation and covariance matrices are symmetric.

The correlation matrix is also shown in Figure 5 and in Figure 6 another visualitization of the effect of the various values of correlation coefficients is presented:

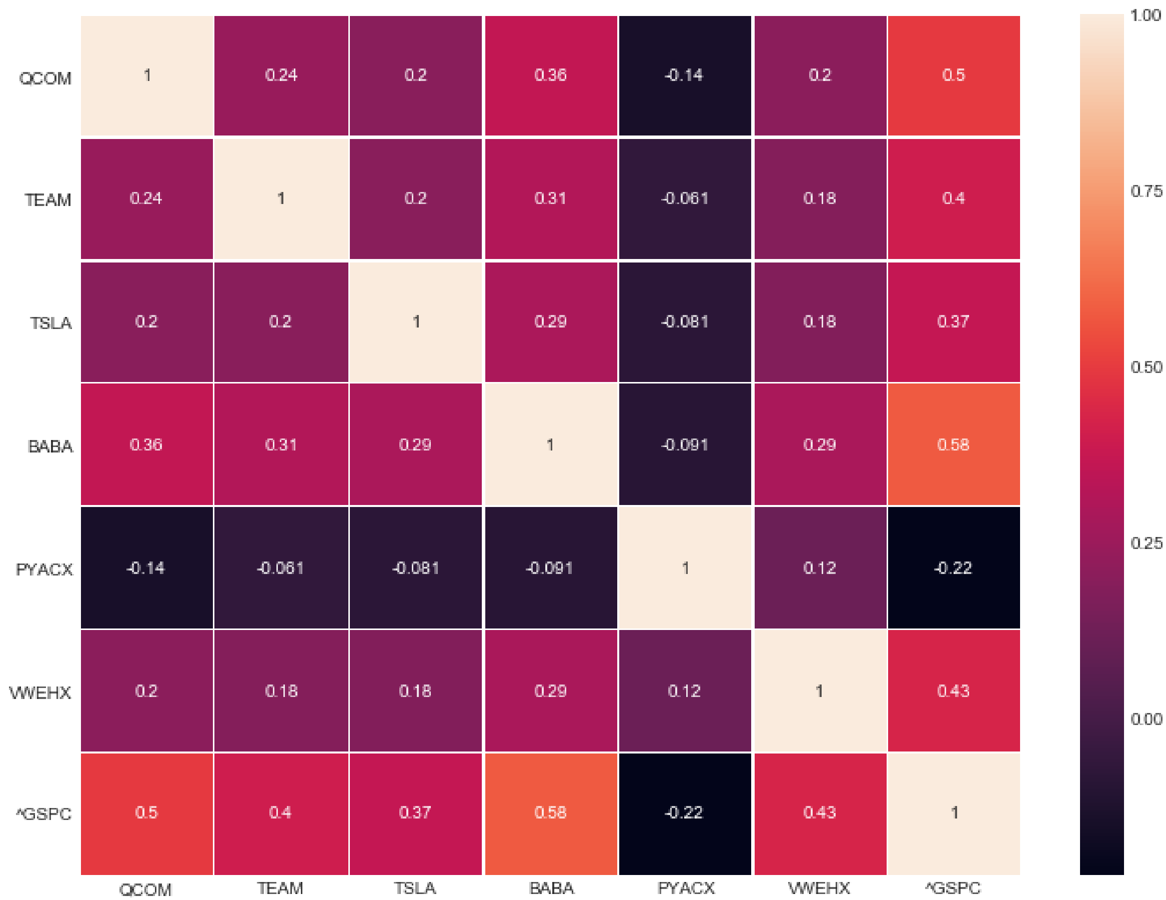


Figure 5: The correlation matrix for our instruments.

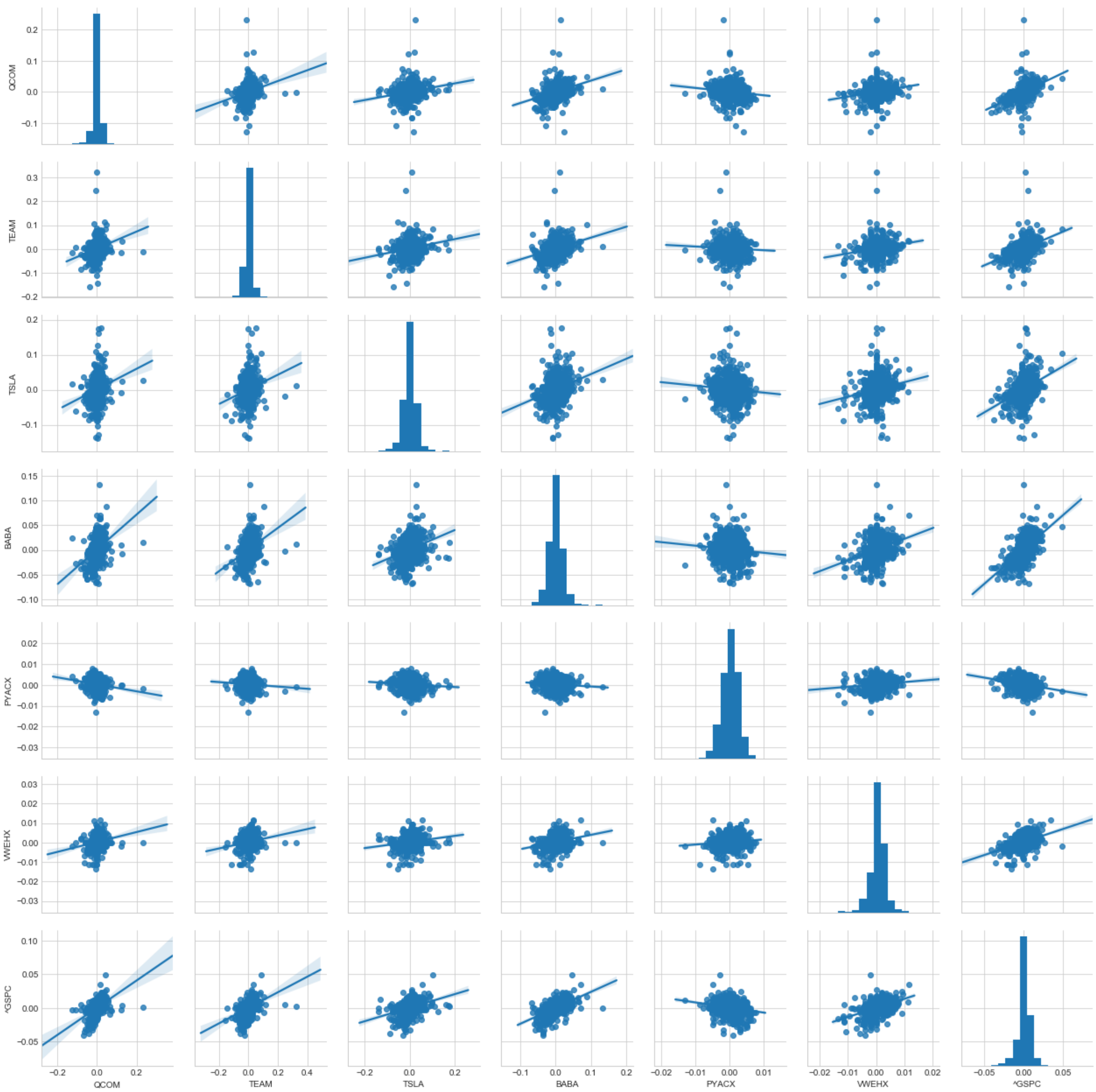


Figure 6: An illustration of the correlation matrix. When the correlation coefficient is negative the slope of the regression line is negative.

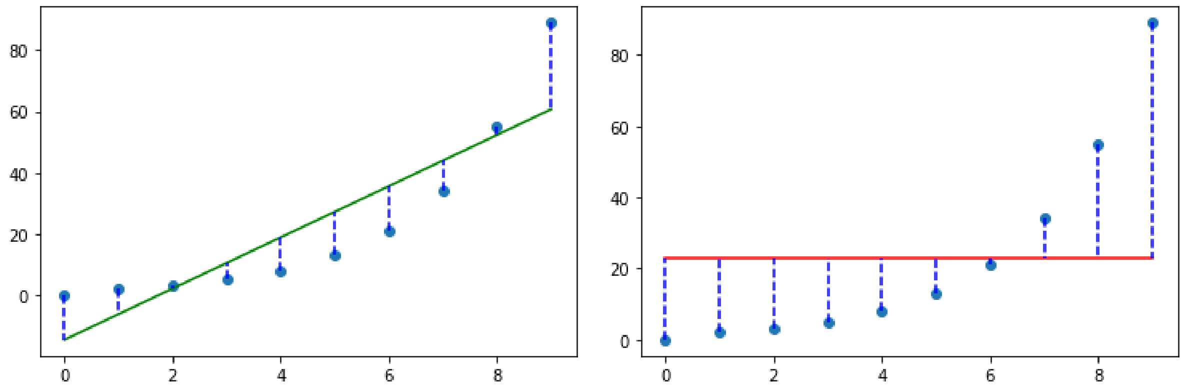
5.2 beta

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk. The beta formula is:

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)} \tag{5.4}$$

where R_e is the return of the individual stock and R_m is the return of the overall market. In order to calculate β, a regression model has to be fitted on the data points from an individual stock's returns against those of the market. Then β is the slope of the aforementioned line.

Even though the formula is straightforward, the data selection is not. The result depends on the time frame and frequency of historical data selected. Hence, many different β values can be found online. We



(a) Explained Variation

(b) Total Variation

Figure 7: R-squared calculation

use the `Yahoo Finance` model: 3 years of monthly data. To be more specific, let $\mathcal{D} = \{d_0, d_1, \ldots, d_{36}\}$ be the closing prices of the individual stock. Then, $\mathcal{P} = \{p_1, \ldots, p_{36}\}$ is the set of the percent changes of said closing prices where $p_i = \frac{d_i - d_{i-1}}{d_{i-1}} \times 100\%$. Note that $\mathcal{P} = R_e$ (see (5.4)). By the same token, R_m is calculated using the historical closing prices of the market index (`GSPC`). Using these data points and equation (5.4), β is calculated.

5.3 alpha

By finding β we can proceed to calculate α. To be more precise α denotes the excess return. The Capital Asset Pricing Model (CAPM) is given by:

$$\mathbb{E}(R_i) = R_f + \beta_i(\mathbb{E}(R_m) - R_f) \tag{5.5}$$

where $\mathbb{E}(R_i)$ is the expected return of the indvidual asset, R_f is the risk-free rate. Then, α is calculated by substracting the expected returns from the actual mean portfolio returns $\bar{R}$:

$$\alpha = \bar{R} - \mathbb{E}(R) \tag{5.6}$$

5.4 R-squared

R-squared or coefficient of variation is calculated using the following formula:

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}} \tag{5.7}$$

R^2 is a statistical measure that indicates the proportion of the variation of dependent variable that can be explained using the independent variables of a simple regression model. An illustrative explanation is adduced using the following figures. In 7a a simple regression model is fitted using the data provided. The explained variation is the sum of the squared distances from the regression line divided by the number of points. Said distances are denoted with a blue color in the figures below. The total variation is calculated by the same token but with a crucial difference: the line is horizontal and denotes the mean of the data.

In economics, the R^2 measure denotes the percentage of a fund's movements that can be explained away by the historical changes of the benchmark index.

5.5 Shapre ratio

The Sharpe ratio is named after Nobel Laureate William Sharpe. It is a measure that relates the return of an investment to its risk; it is the risk-adjusted return.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{5.8}$$

where

- R_p = return of mutual fund
- R_f = risk-free rate
- σ_p = standard variation of the portfolio's excess return

The calculation of the Sharpe Ratio using `Excel` uses "the trailing three-year period by dividing a fund's annualized excess returns over the risk-free rate by its annualized standard deviation"[1].

Using the historical data for each fund and the analysis presented above, we are able to calculate α, β, R^2 and the Sparpe Ratio using `Excel`. The results are presented below:

	VGPMX	VHGEX
alpha	-0.78 %	-1.07 %
beta	0.76 %	1.13 %
R-squared	0.10 %	0.88 %
Sharpe Ratio	0.06 %	0.52 %

Methodology Historical data of a 5-year time frame with monthly intervals were used for the above calculations. The index fund used for stock mutual funds is ACWI, as stated in MorningStar.

Risk-free Rate To compute the Risk-Free rate, 1-Year Treasury Constant Maturity Rate was used (5 year average with monthly intervals).

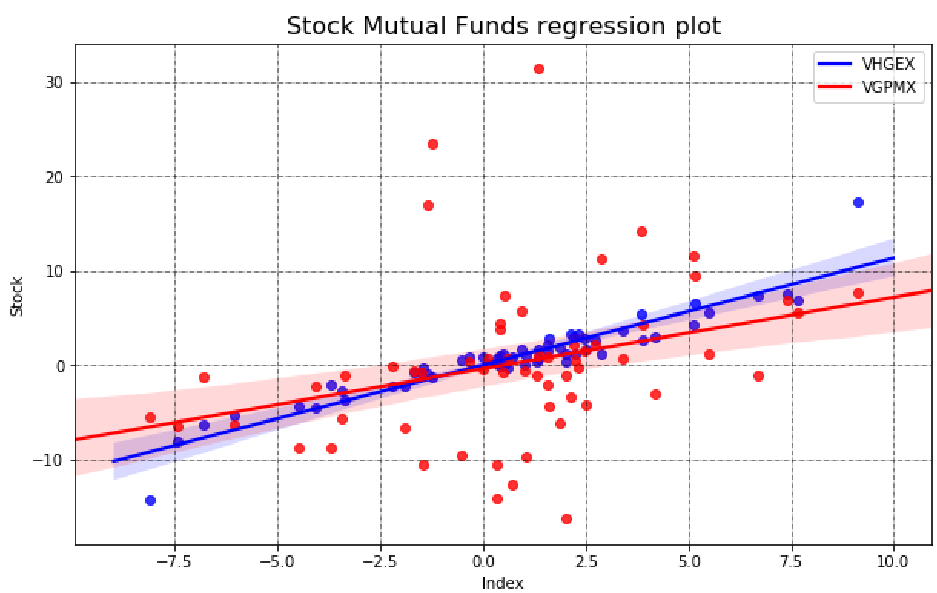


Figure 8: Stocks Regression Plot

[1] link

6 Question 6

Calculate the return/risk of your risky portfolio. Explain each step in your analysis. You must use EXCEL's mmult functions for this part of the analysis.

Let R_i be the returns data sample for the instrument i and $\mathcal{R} = (R_1, R_2, \cdots, R_n)$ be the collection of those random variables. Let $\vec{w}$ be the vector of weights that represents the allocation of the various instruments in our risky portfolio. Obvisouly, $\|\vec{w}\| = 1$. Finally, let K be the associated covariance matrix.

Then the return of the risky portfolio is a weighted average of the expected returns:

$$R_{\text{risky portfolio}} = \vec{w}^{\top} \cdot \mathbb{E}(\mathcal{R}) \tag{6.1}$$

and the risk of the risky portfolio is measured by its standard deviation:

$$\begin{aligned}\sigma_{\text{risky portfolio}} &= \sqrt{\vec{w}^{\top} K \vec{w}} &(6.2)\\ &= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j} &(6.3)\end{aligned}$$

7 Question 7

Derive and graph the Capital Allocation Line. Graph the Efficient Frontier with your available investment instruments (assets) and superimpose your CAL. Discuss the various options you may have and finalize your optimal point.

By taking various random weight allocations we are able to create portfolios with different risks and returns. The optimality criterion lies in the Sharpe Ratio associated with each risky portfolio. The optimal risky portfolio is characterized by the highest Sharpe ratio.

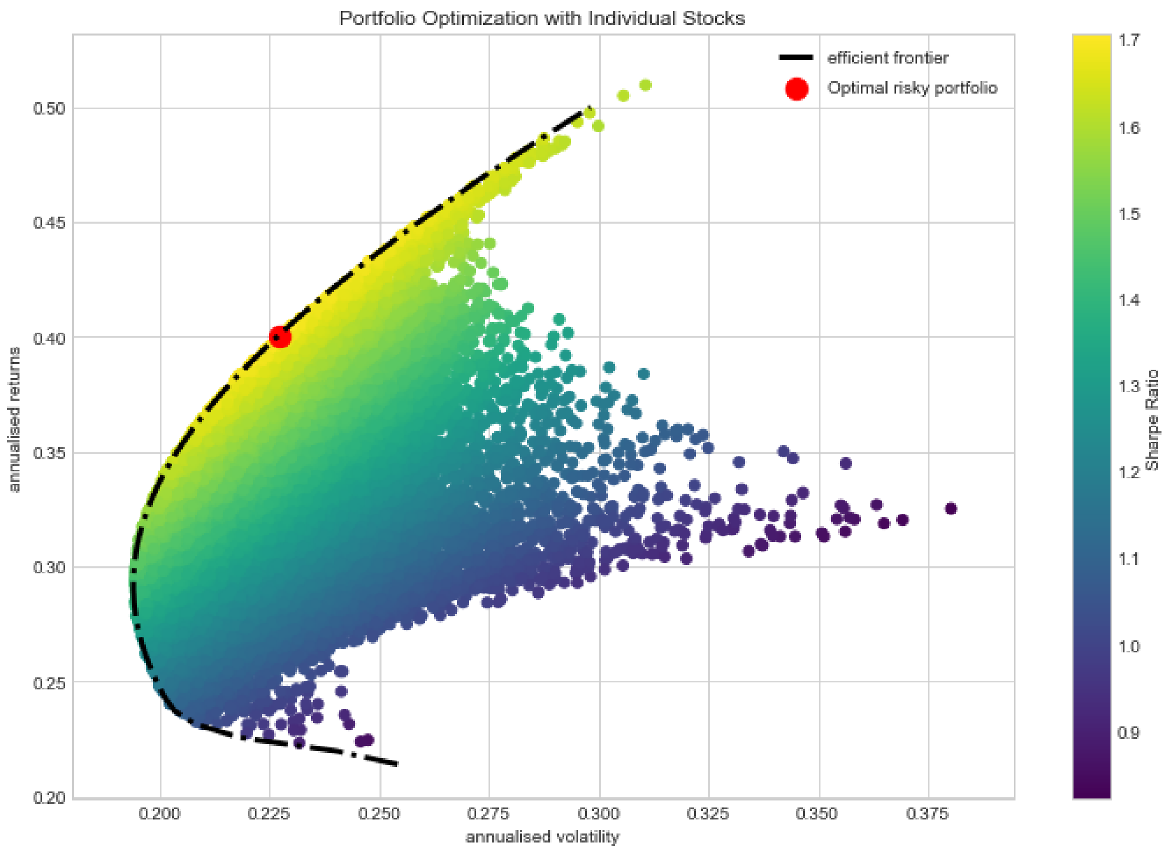


Figure 9: $n = 50,000$ random portfolios are created and plotted in a common graph. The optimal risky portfolio and the Efficient Frontier also appear.

The above graph is conducive with the theoretical one with respect to its shape. By creating the random portfolios with a large enough sample size ($n = 50,000$), the range of annualized returns and volatilities is known. Yo find the Efficient Frontier an optimization problem must be solved. To be more specific, we seek to maximize the Sharpe Ratio given a fixed annualized portfolio return. By solving numerous such optimization problems, points on the Efficient Frontier are obtained and graphed.

The Capital Allocation Line or CAL is the line that connects the Optimal Portfolio with the risk-free portfolio. These two points suffice in deriving the equation of the CAL ϵ_{CAL}:

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{CAL} \\ (\sigma_{OPT}, r_{OPT}) \in \epsilon_{CAL} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{OPT} - r_f}{\sigma_{opt}}}_{\text{max Sharpe ratio}} \cdot x + r_f \tag{7.1}$$

Thus, we are able to plot the CAL alongside the efficient frontier and the random portfolios.

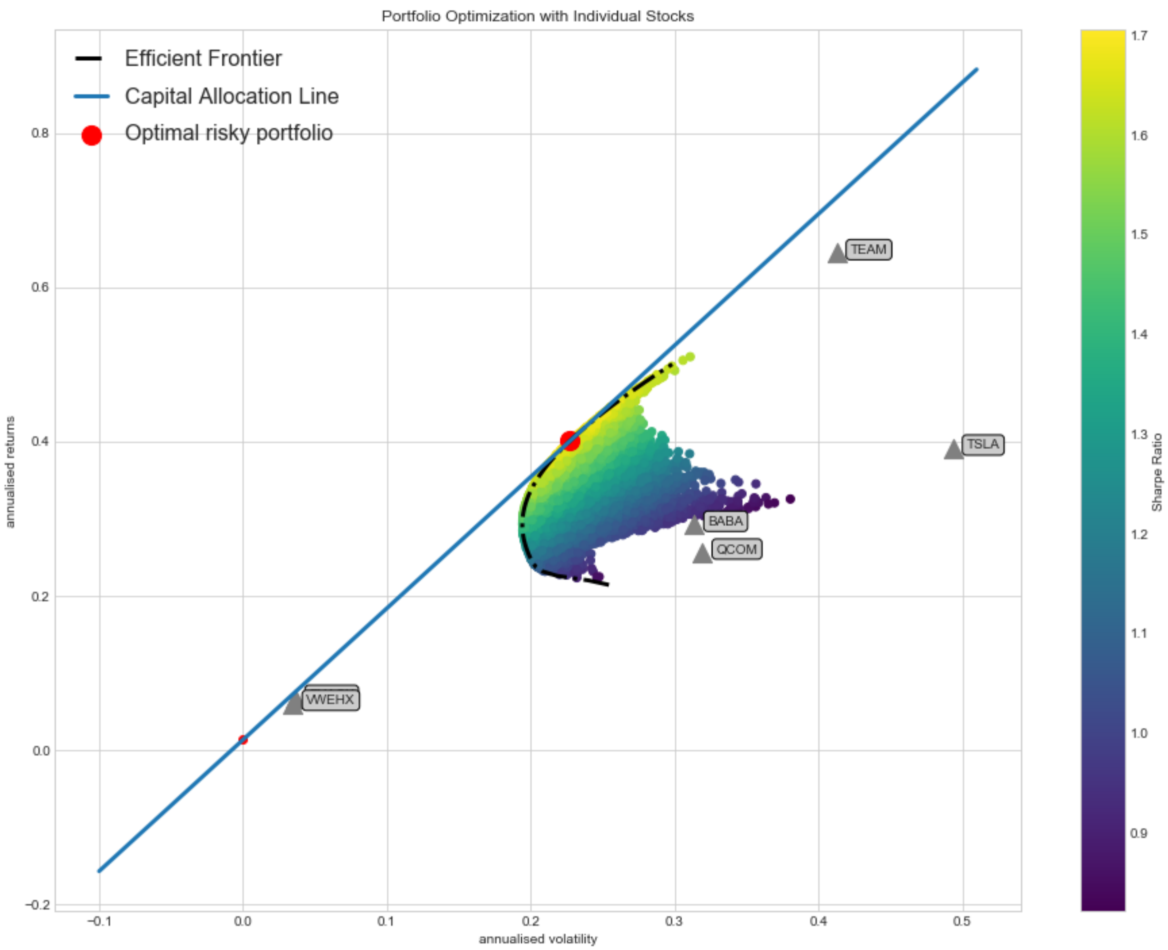


Figure 10: The CAL passes through the risk-free and risky portfolio. The points of the line between these values are a linear combination of the two alternatives.

8 Question 8

Keep a track record of the macro- and microeconomic events that influenced your assets/portfolio and offer explanations for some of them, if you deem necessary. Your explanations should also be accompanied with some quantitative verification.

9 Question 9

Measure and evaluate your overall portfolio's performance and compare it with the passive investment strategy. In this step, you should apply EXCEL's Solver to evaluate several possible outcomes (in terms of risk and return) and explain each outcome. In that endeavor, compute the various performance measures we have learned. Decide on the best outcome for you. Discuss.

10 Question 10

Finally, perform a critical evaluation of the project. In other words, what did this project accomplish for you regarding the study and (this simple) application of investment theories and strategies? What would be the implications of constructing, managing and evaluating such a portfolio for your portfolio?