

Portfolio Management

FN 4329

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Spring Semester 2020

Contents

- 1 Portfolio Construction
 - Asset Allocation
 - Security Selection
 - Diversification strategy
- 2 Theory
 - Descriptive Statistics
 - Financial Metrics
- 3 Building the Portfolio
 - Naive allocation
 - Portfolio Optimization
 - Optimal Overall Portfolio

Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

Questionnaire

1/30/2020

Vanguard - Investor Questionnaire

PERSONAL INVESTORS



Investor questionnaire

1. I plan to begin taking money from my investments in ...

- ☐ 1 year or less
- ☐ 1 - 2 years
- ☐ 3 - 5 years
- ☐ 6 - 10 years
- ☐ 11 - 15 years
- ☒ More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- ☒ 2 years or less
- ☐ 3 - 5 years
- ☐ 6 - 10 years
- ☐ 11 - 15 years
- ☐ More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- ☐ 1 - 2 years
- ☐ 3 - 4 years
- ☐ 5 - 6 years
- ☐ 7 - 8 years
- ☒ More than 8 years

4. From September 2008 through November 2008, stocks lost more than 21%. If I owned a stock investment that lost about 21% in 3 months, I would ... (If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)

- ☐ Sell all of the remaining investment.
- ☐ Sell a portion of the remaining investment.
- ☒ Hold onto the investment and sell nothing.
- ☐ Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- ☐ Strongly disagree
- ☒ Disagree
- ☐ Somewhat agree
- ☐ Agree
- ☐ Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

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- ☐ Somewhat agree
- ☐ Agree
- ☐ Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... (If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)

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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... (If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)

<https://personal.vanguard.com/us/FundInvQuestionnaire>

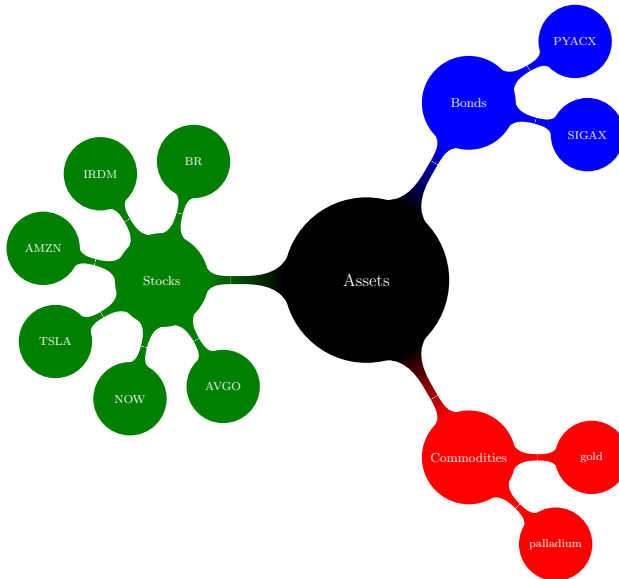
1/2



Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

Security Selection



Stocks

AVGO - Broadcom Inc

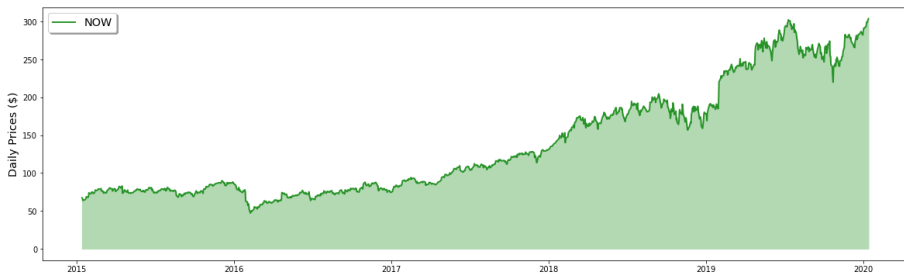
add - DO THIS FOR EVERY SECURITY

- category
- few words
- price chart



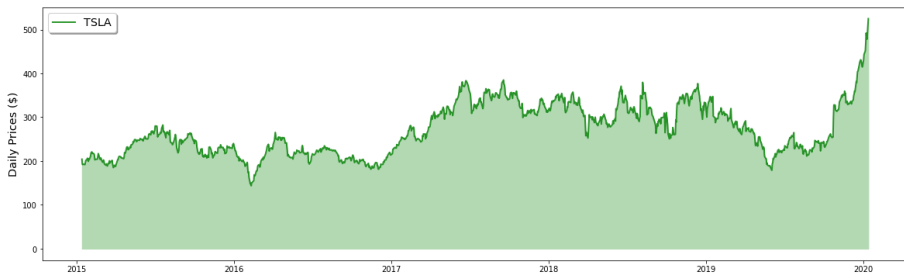
Stocks

NOW - ServiceNow Inc.



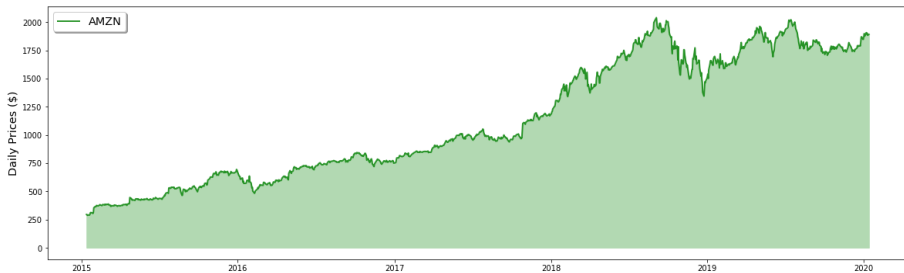
Stocks

TSLA - Tesla Inc.



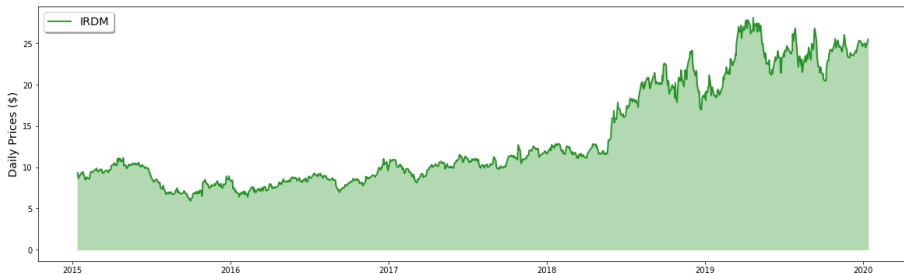
Stocks

AMZN - Amazon Inc.



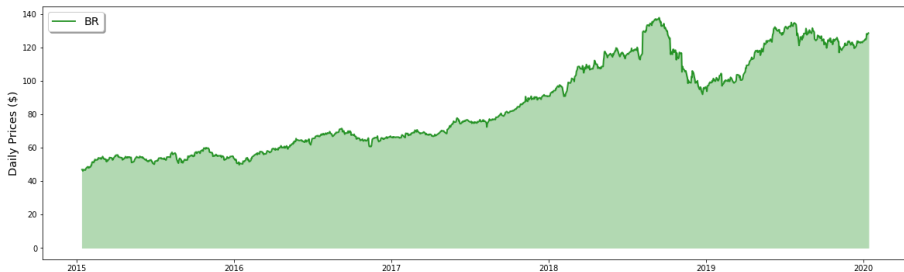
Stocks

IRDM - Iridium Communications Inc.



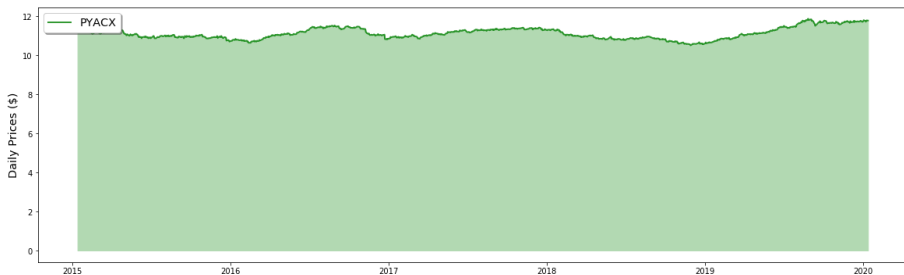
Stocks

BR - Broadridge Financial Solutions Inc.



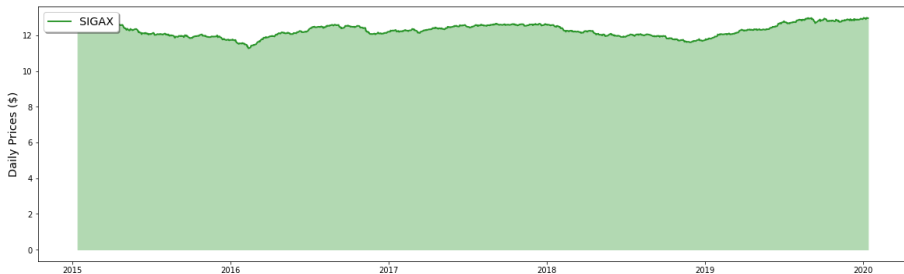
Bonds

PYACX - Payden Corporate Bond Mutual Fund



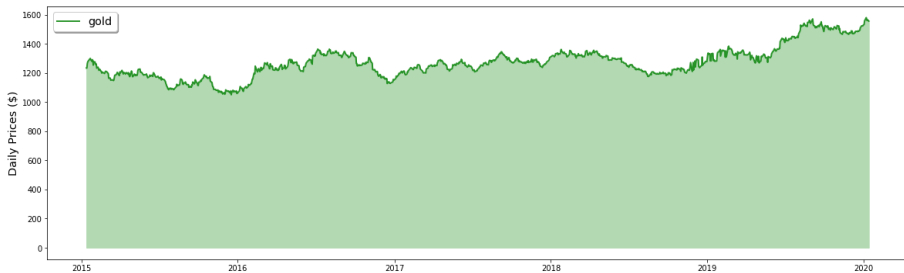
Bonds

SIGAX - Western Asset Corporate Bond Mutual Fund



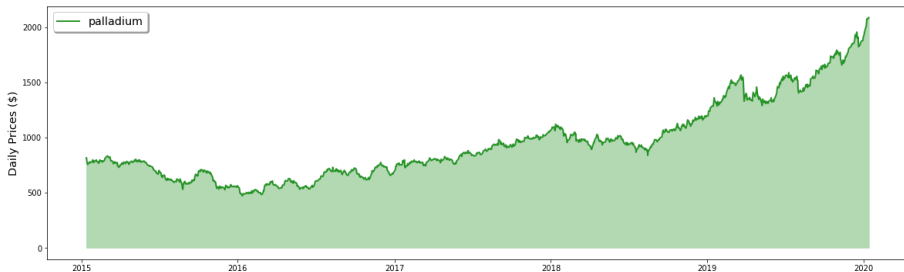
Commodities

gold



Commodities

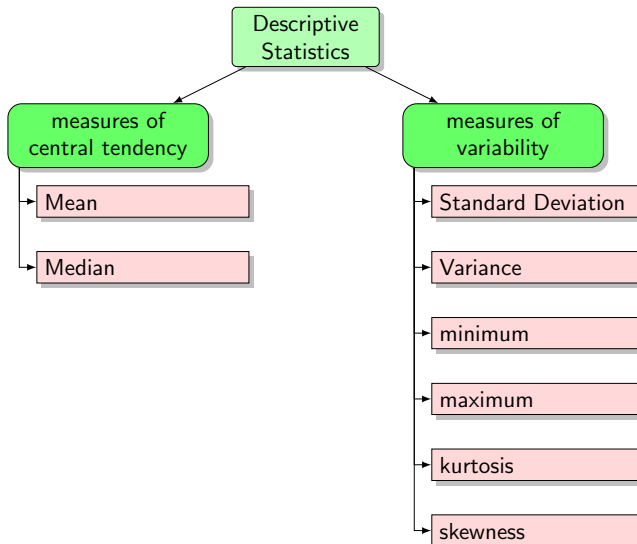
palladium



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Descriptive statistics Taxonomy



Measures of central tendency

Mean

Mean

$$\bar{x} = \frac{1}{n} \left(\sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Measures of central tendency

Median

- i.e. the middle value

Measures of central tendency

Median

- i.e. the middle value
- Why?

Measures of central tendency

Median

- i.e. the middle value
- Why? robust w.r.t. outliers

Measures of central tendency

Median

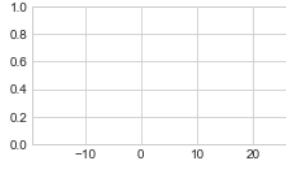
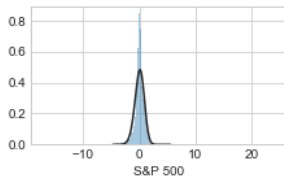
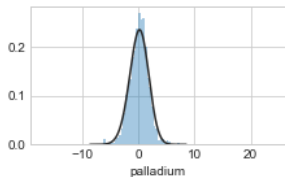
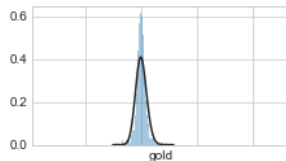
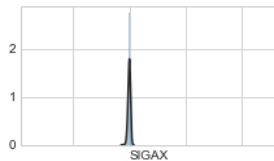
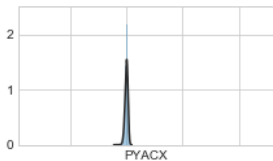
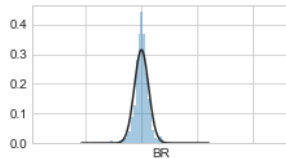
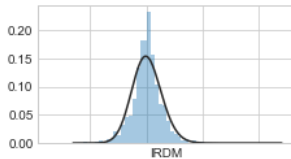
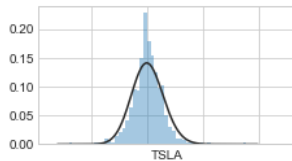
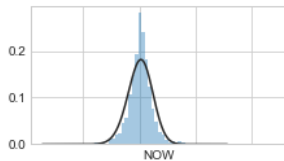
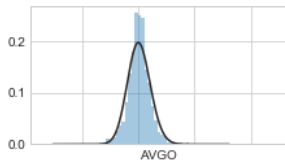
- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

Measures of Variability

Standard Deviation

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics \implies risk in finance



Measures of Variability

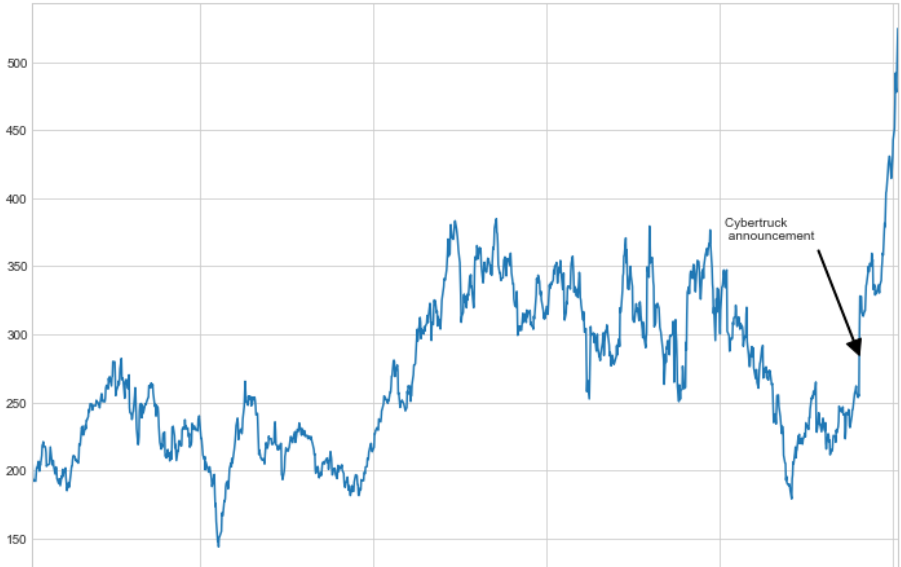
Minimum & Maximum



Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

Minimum & Maximum



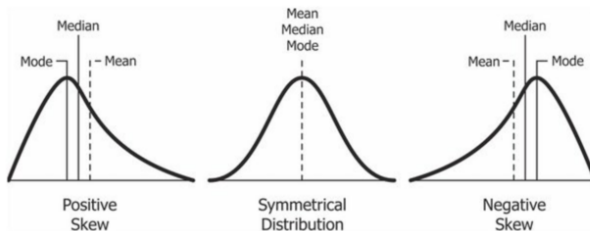
Measures of Variability

Skewness

Definition

$$\tilde{\mu}_3 = \mathbb{E} \left[\left(\frac{X - \bar{x}}{\sigma} \right)^3 \right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



Measures of Variability

Kurtosis

Definition

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E} \left[\left(\frac{X - \bar{x}}{\sigma} \right)^4 \right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis \rightarrow heavy tails (**outliers**)
- low kurtosis \rightarrow no outliers

Measures of Variability

An overview

Moments from an investor's perspective

- $\tilde{\mu}_2$ standard deviation σ
- $\tilde{\mu}_3$ skewness
- $\tilde{\mu}_4$ kurtosis

TODO

add what each moments means to understanding security performance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$\text{cov}(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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- not so helpful! \rightarrow correlation

Correlation

Definition

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

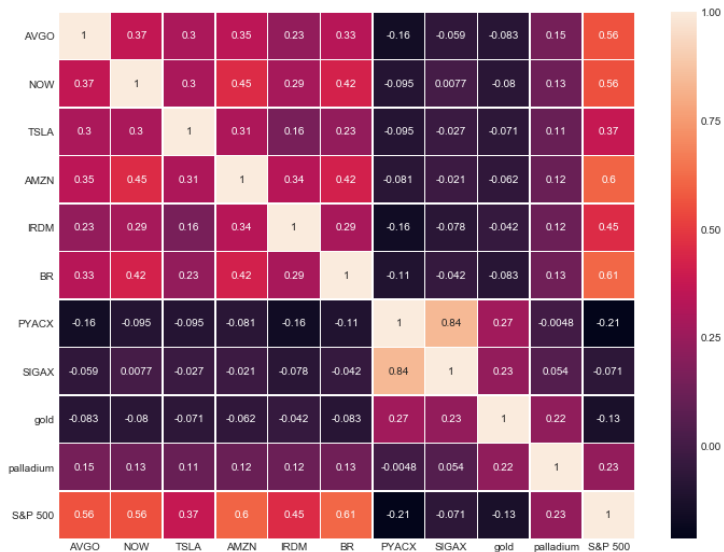
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation

todo

add regression plots to show the difference

Correlation Matrix



Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

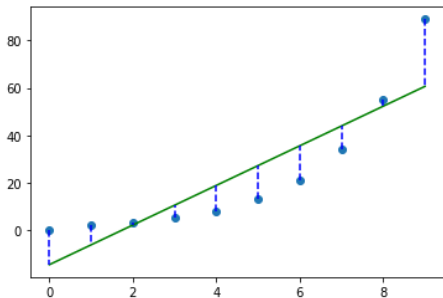
Definition

Alpha is the difference between the realised returns and the expected returns:

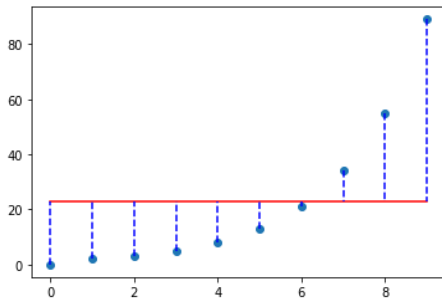
$$\alpha = \bar{R} - \mathbb{E}(R)$$
$$\stackrel{\text{CAPM}}{\implies} \alpha = \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}$$

R-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation



(b) Total Variation

Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- R_p = return of mutual fund
- R_f = risk-free rate
- σ_p = standard variation of the portfolio's excess return

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Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk

$$\sigma_p = \sqrt{\vec{w}^\top K \vec{w}}$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \text{COV}_{1,2} & \dots & \text{COV}_{1,n} \\ \text{COV}_{2,1} & \sigma_2^2 & \dots & \text{COV}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{COV}_{n,1} & \text{COV}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}}$$
$$= \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{COV}_{ij}}$$

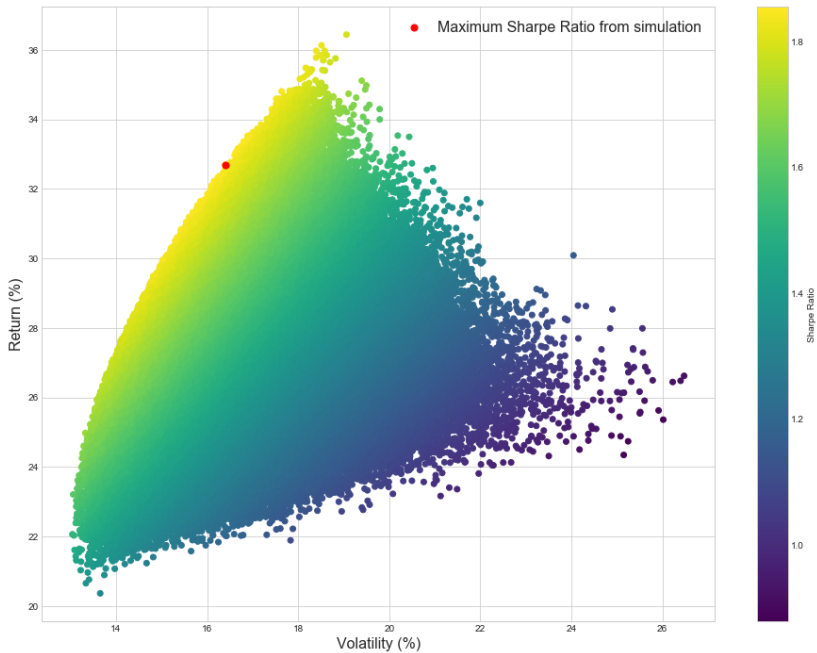
Naive weight allocation

Remember!

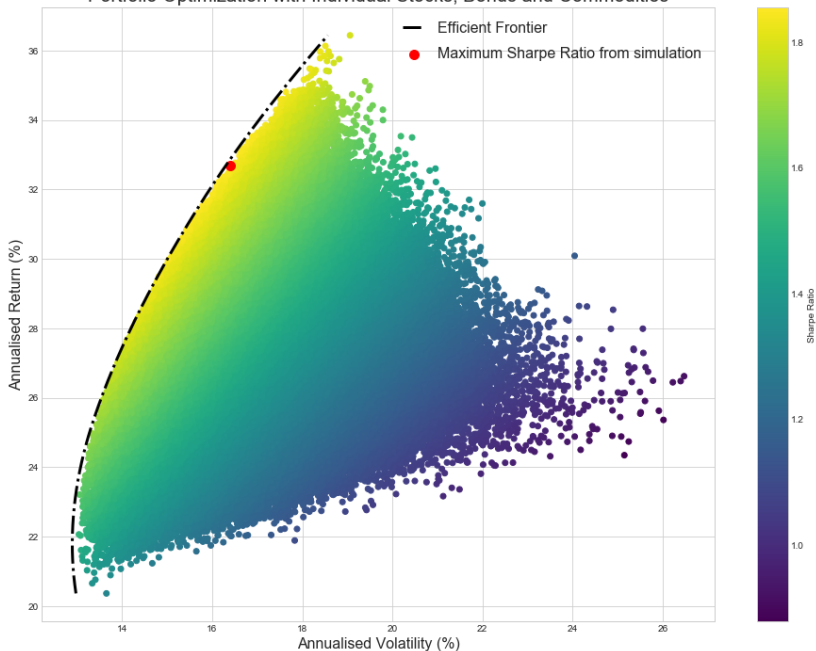
- 70% Stocks
- 20% Bonds
- 10% Commodities

What about \vec{w} ?

- 70% for 6 Stocks $\rightarrow \frac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow 10\%$ each
- 10% for 2 Commodities $\rightarrow 5\%$ each



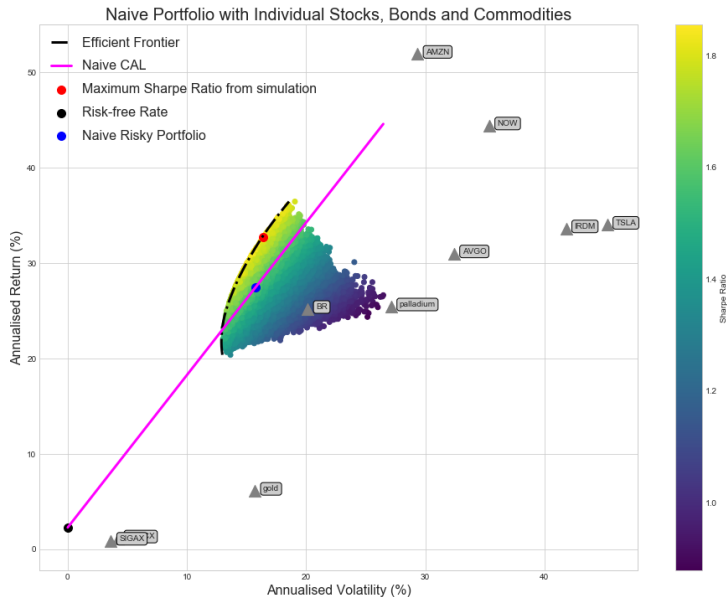
Portfolio Optimization with Individual Stocks, Bonds and Commodities



CAL construction

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \Rightarrow \epsilon_{\text{naive}} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$

Naive CAL



But...

We can do better!

Optimal portfolio

Optimization problem formulation

$$\begin{aligned} \max \quad & \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\ \text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\ & \mathbf{1}^\top \vec{w}_S = w_s \\ & \mathbf{1}^\top \vec{w}_B = w_b \\ & \mathbf{1}^\top \vec{w}_C = w_c \\ & w_s + w_b + w_c = 1 \\ & w_i \geq 0 \quad i = 1, \dots, n \end{aligned}$$

Optimal portfolio

Optimization problem formulation

Sharpe Ratio

max

$$\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}$$

s.t.

$$\mathbf{1}^\top \vec{w} = 1$$

$$\mathbf{1}^\top \vec{w}_S = w_s$$

$$\mathbf{1}^\top \vec{w}_B = w_b$$

$$\mathbf{1}^\top \vec{w}_C = w_c$$

$$w_s + w_b + w_c = 1$$

$$w_i \geq 0 \quad i = 1, \dots, n$$

Optimization yields...

optimal weights

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

Optimization yields...

Financial Metrics

| | Optimal Risky Portfolio |
|------------------|-------------------------|
| Return (%) | 32.7164 |
| Risk (%) | 16.3185 |
| Sharpe Ratio | 1.8658 |
| Beta (β) | 0.9067 |

Capital Allocation Line

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

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Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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- i.e. which portfolio along the CAL is the best **for us**?

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Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t. z :

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

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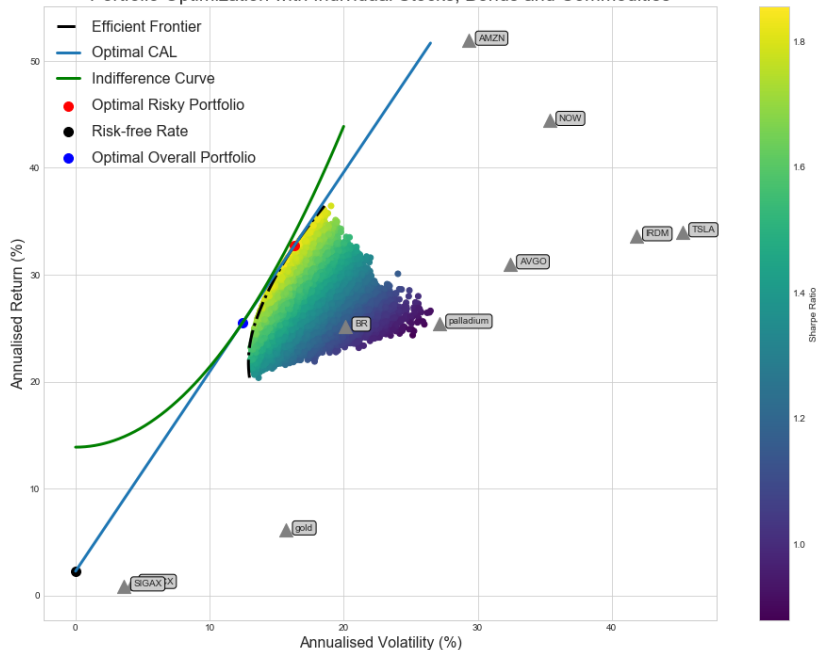
Maximize w.r.t. z :

$$\frac{\partial U}{\partial z} = 0$$

$$\implies r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\implies z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$

Portfolio Optimization with Individual Stocks, Bonds and Commodities



Past Performance

(1/13/2015 - 1/13/2020)

| | Optimal Overall Portfolio | S&P 500 |
|------------------------|---------------------------|---------|
| Return (%) | 25.4769 | 11.3118 |
| Risk (%) | 12.4384 | 13.4138 |
| Sharpe Ratio | 1.8658 | 0.6741 |
| Beta (β) | 0.6911 | 1.0 |
| Alpha (α) (%) | 16.9584 | 0.0 |

References

The end!