

Portfolio Management

FN 4329

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American College of Greece

Spring Semester 2020

Contents

1 Portfolio Construction

- Asset Allocation
- Security Selection
- Diversification strategy

2 Theory

- Descriptive Statistics
- Financial Metrics

3 Building the Portfolio

- Naive allocation
- Portfolio Optimization
- Optimal Overall Portfolio

Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

Questionnaire

1/30/2020

Vanguard - Investor Questionnaire

PERSONAL INVESTORS



Investor questionnaire

1. I plan to begin taking money from my investments in ...

- 1 year or less
- 1 - 2 years
- 3 - 5 years
- 6 - 10 years
- 11 - 15 years
- More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- 2 years or less
- 3 - 5 years
- 6 - 10 years
- 11 - 15 years
- More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- 1 - 2 years
- 3 - 4 years
- 5 - 6 years
- 7 - 8 years
- More than 8 years

4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would ... (If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)

- Sell all of the remaining investment.
- Sell a portion of the remaining investment.
- Hold onto the investment and sell nothing.
- Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- Strongly disagree
- Disagree
- Somewhat agree
- Agree
- Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

- Strongly disagree
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- Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

- Strongly disagree
- Disagree
- Somewhat agree
- Agree
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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost about 4% in 2 months, I would ... (If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)

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1/30/2020

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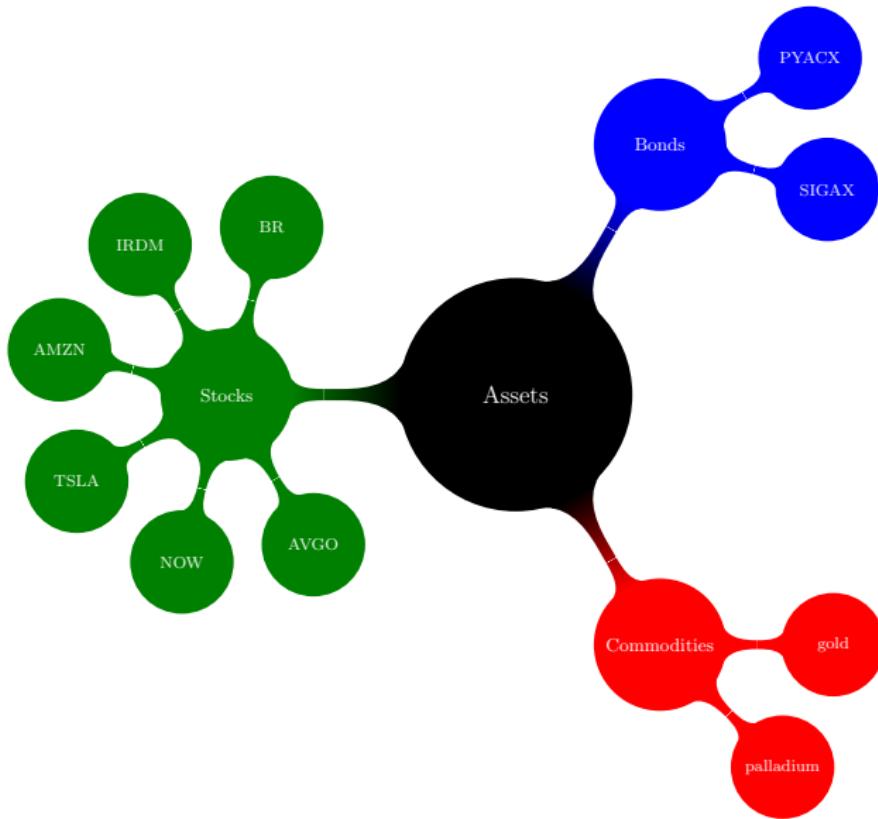
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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost about 4% in 2 months, I would ... (If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)

Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

Security Selection



Stocks

AVGO - Broadcom Inc

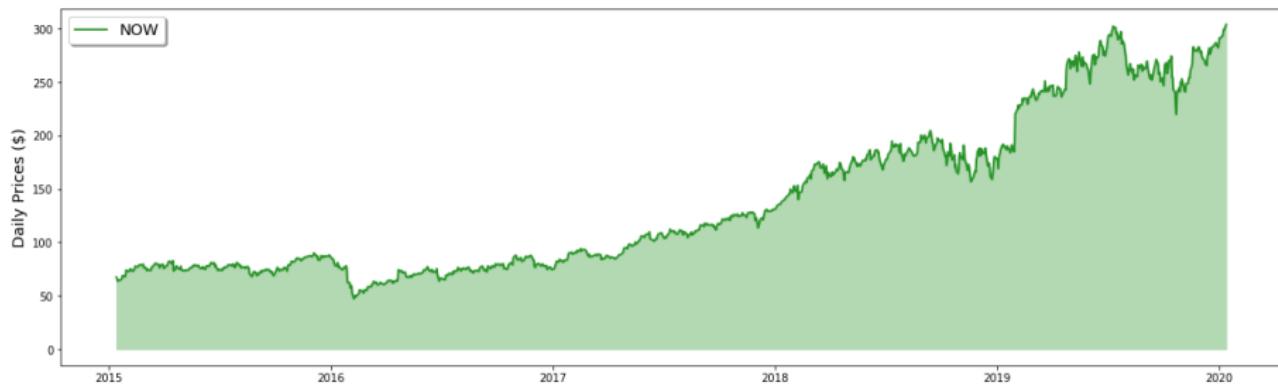
add - DO THIS FOR EVERY SECURITY

- category
- few words
- price chart



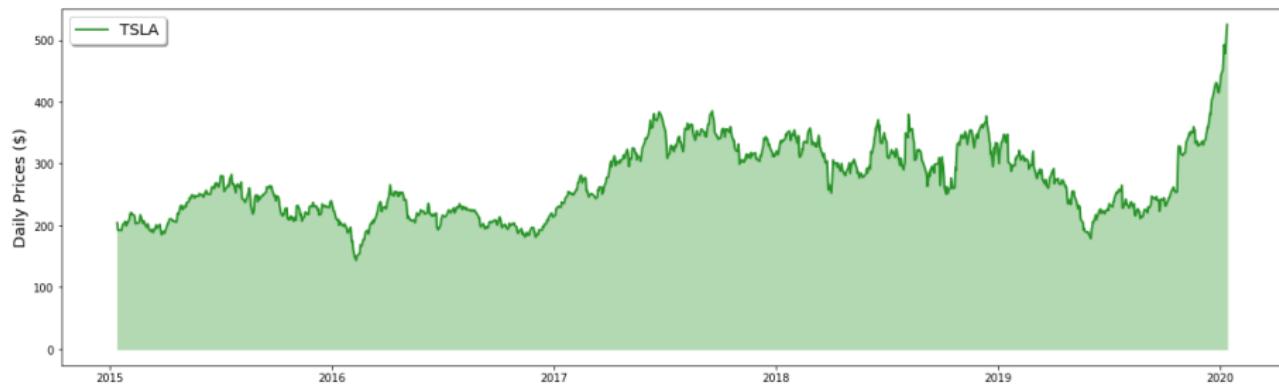
Stocks

NOW - ServiceNow Inc.



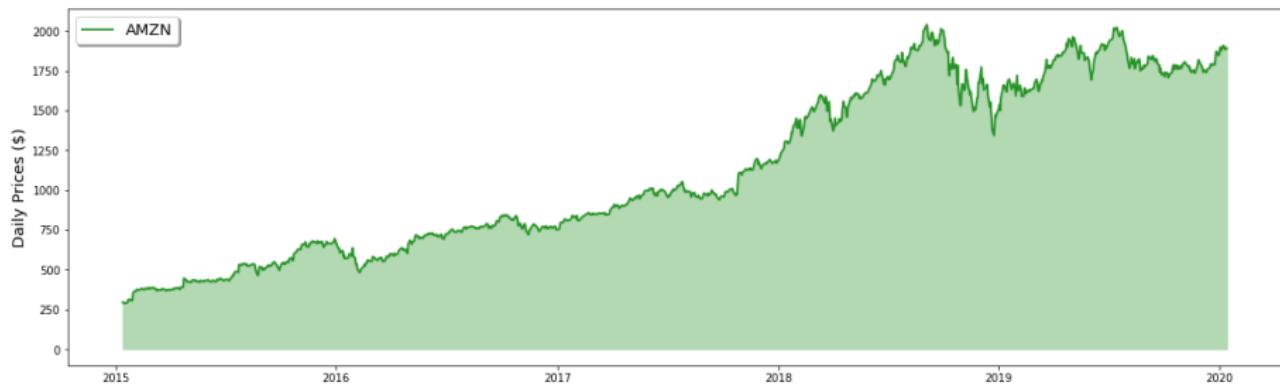
Stocks

TSLA - Tesla Inc.



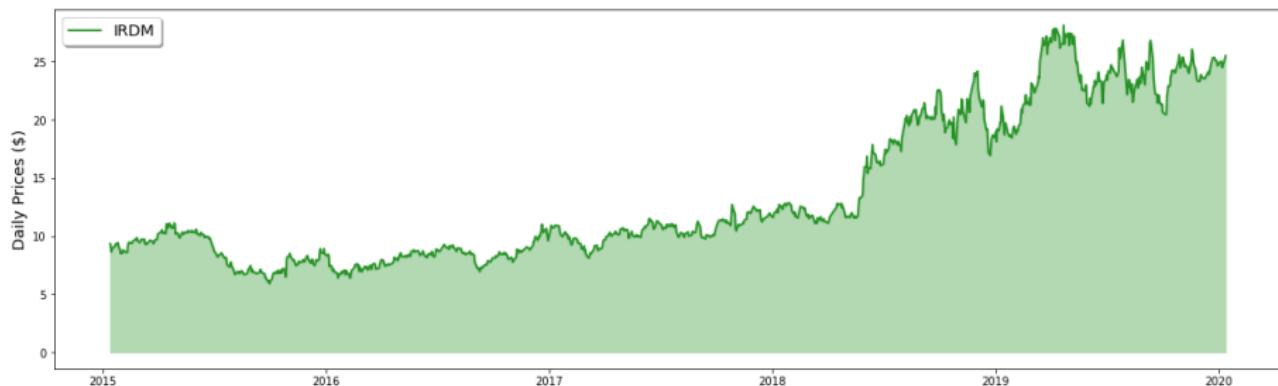
Stocks

AMZN - Amazon Inc.



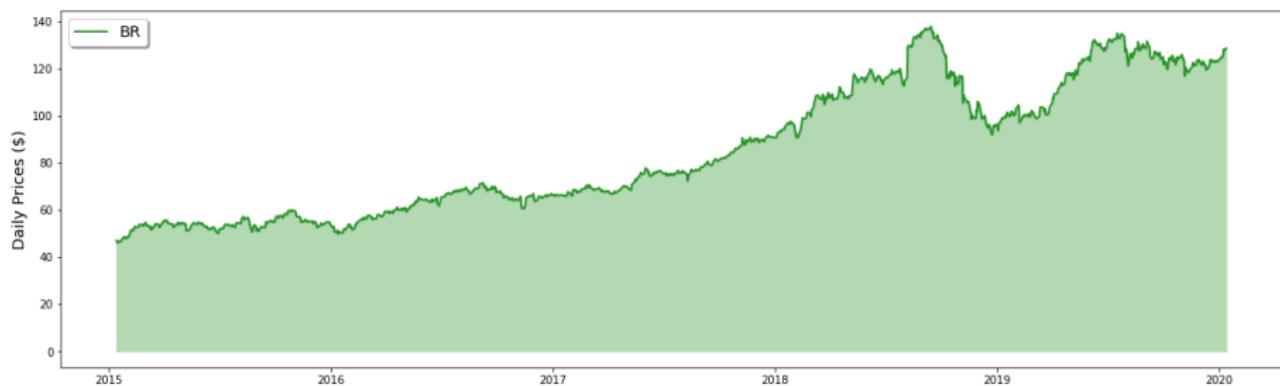
Stocks

IRDM - Iridium Communications Inc.



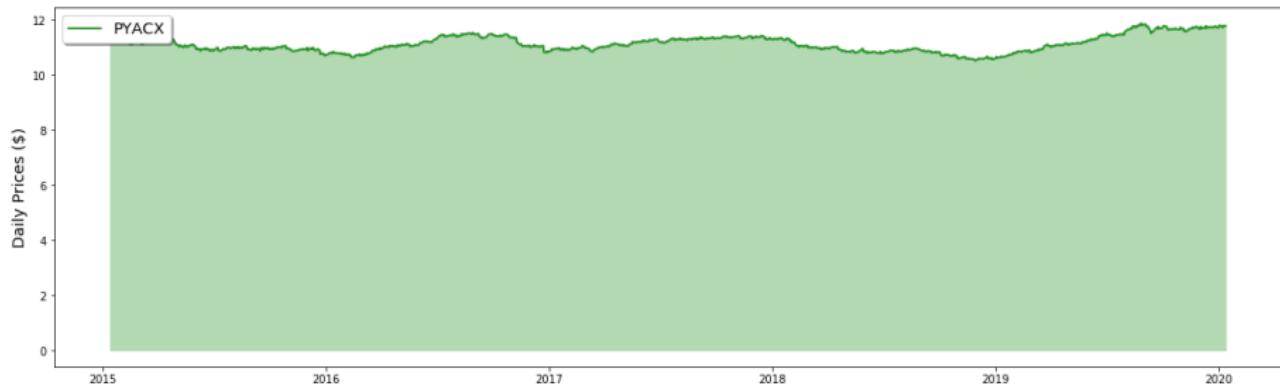
Stocks

BR - Broadridge Financial Solutions Inc.



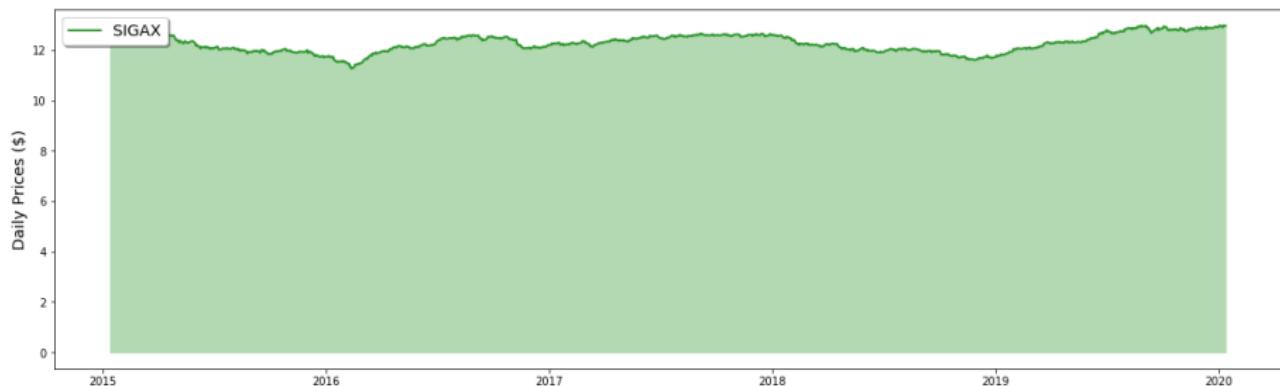
Bonds

PYACX - Payden Corporate Bond Mutual Fund



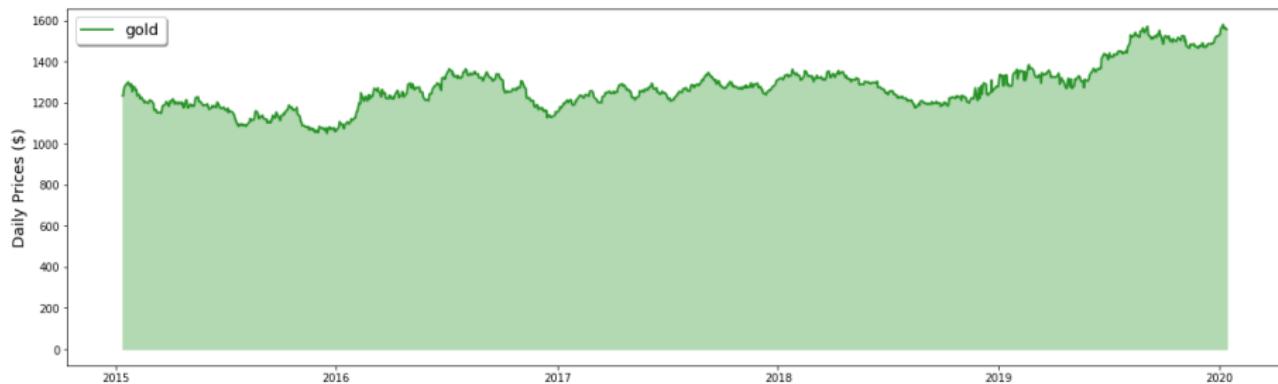
Bonds

SIGAX - Western Asset Corporate Bond Mutual Fund



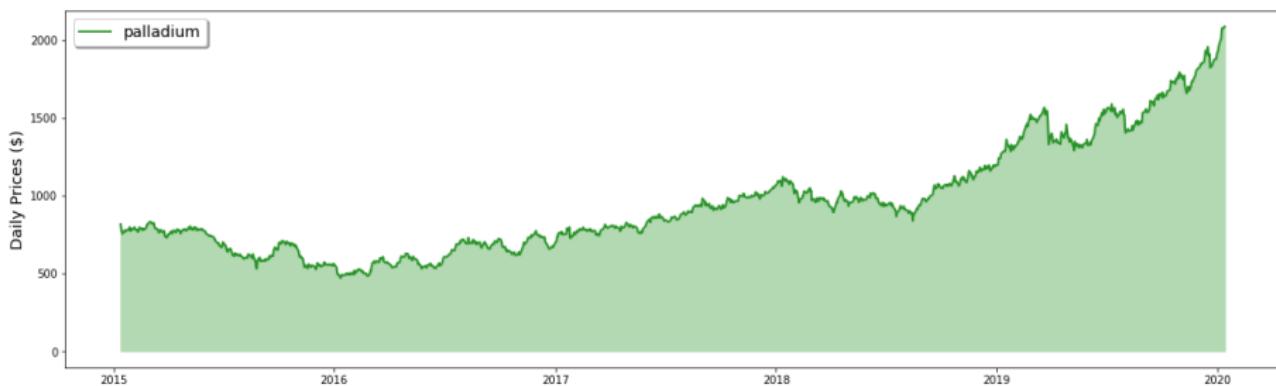
Commodities

gold



Commodities

palladium



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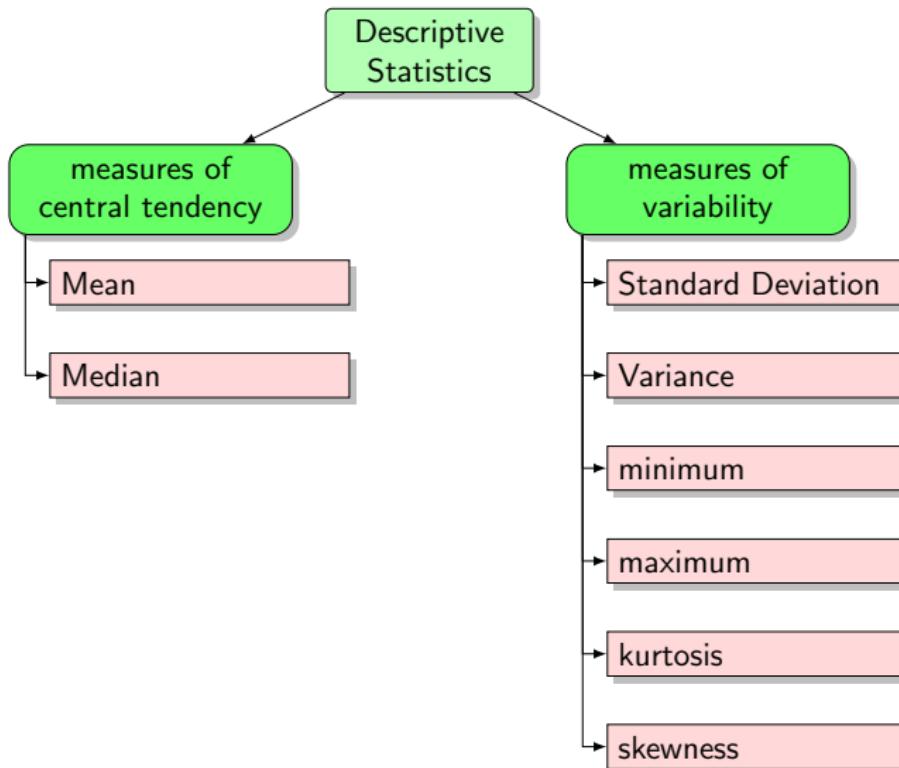
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Descriptive statistics Taxonomy



Measures of central tendency

Mean

Mean

$$\bar{x} = \frac{1}{n} \left(\sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Measures of central tendency

Median

- i.e. the middle value

Measures of central tendency

Median

- i.e. the middle value
- Why?

Measures of central tendency

Median

- i.e. the middle value
- Why? robust w.r.t. outliers

Measures of central tendency

Median

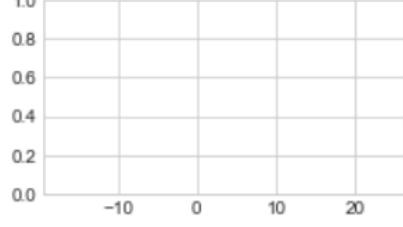
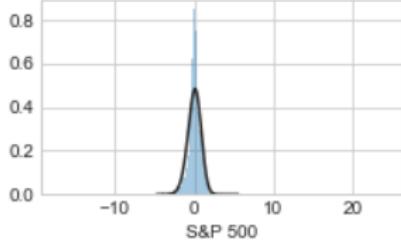
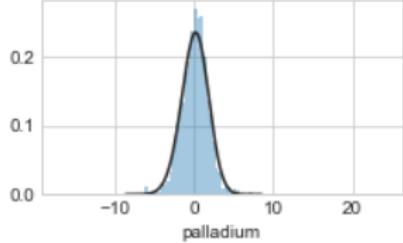
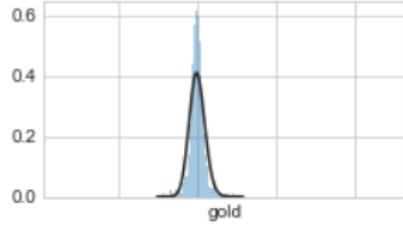
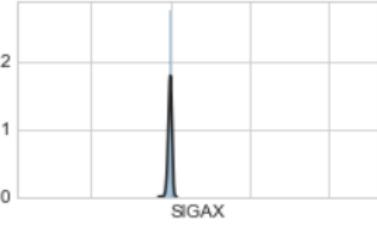
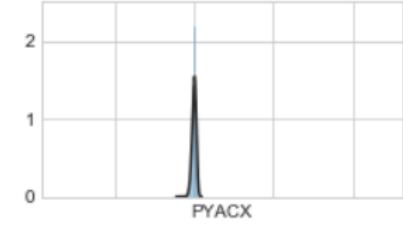
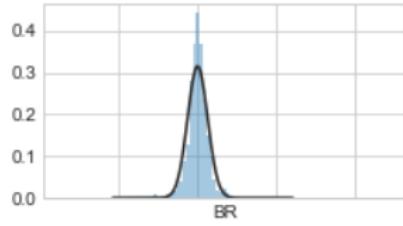
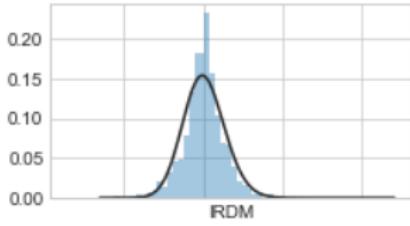
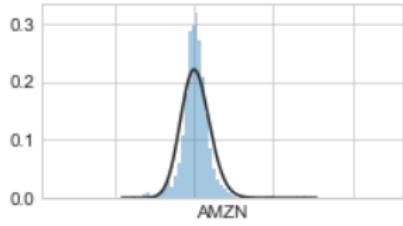
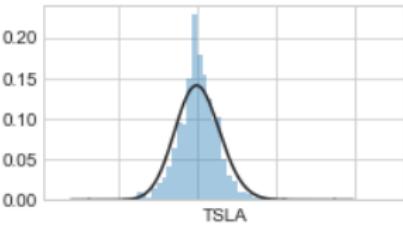
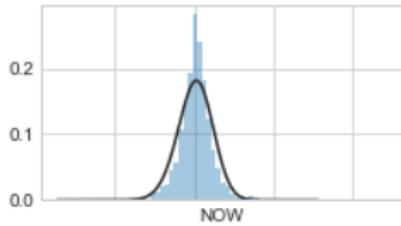
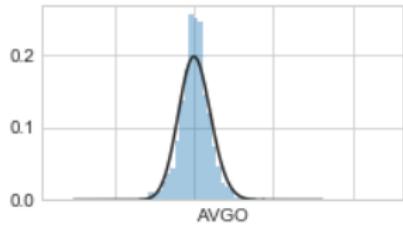
- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

Measures of Variability

Standard Deviation

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics \implies risk in finance



Measures of Variability

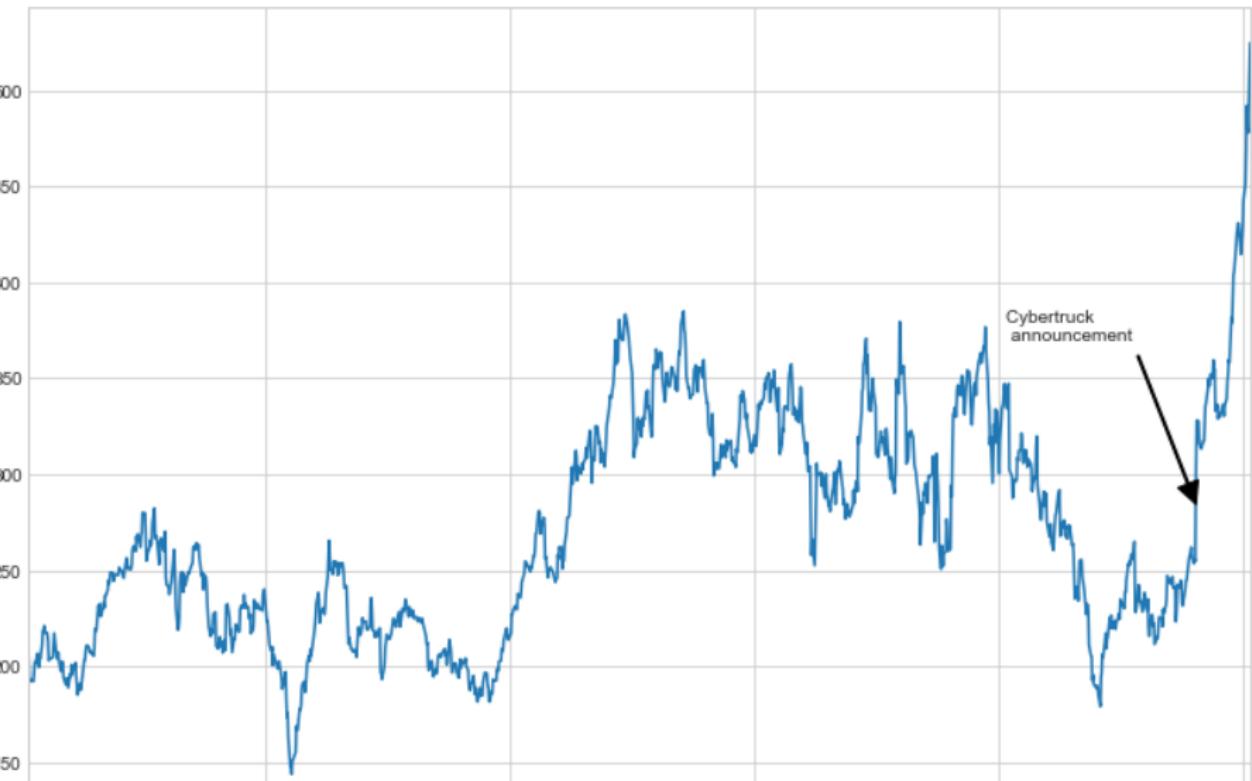
Minimum & Maximum



Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

Minimum & Maximum



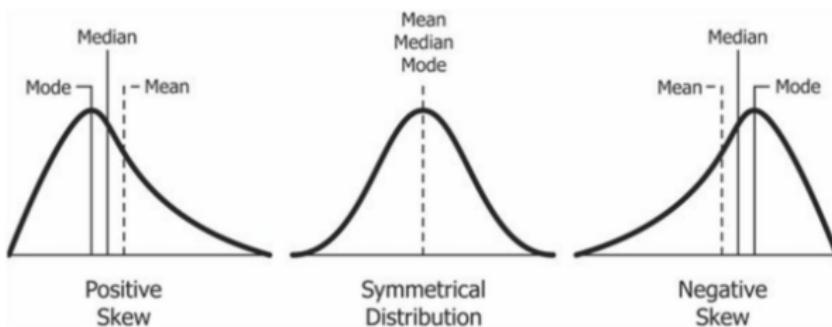
Measures of Variability

Skewness

Definition

$$\tilde{\mu}_3 = \mathbb{E} \left[\left(\frac{X - \bar{x}}{\sigma} \right)^3 \right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



Measures of Variability

Kurtosis

Definition

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E} \left[\left(\frac{X - \bar{x}}{\sigma} \right)^4 \right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis → heavy tails (**outliers**)
- low kurtosis → no outliers

Measures of Variability

An overview

Moments from an investor's perspective

- $\tilde{\mu}_2$ standard deviation σ
- $\tilde{\mu}_3$ skewness
- $\tilde{\mu}_4$ kurtosis

TODO

add what each moments means to understanding security performance

Descriptive Statistics

For our assets

	mean%	median%	std%	var%	min%	max%	kurtosis	skewness
AVGO	0.11	0.11	2.04	4.17	-13.74	14.71	5.31	0.23
NOW	0.15	0.21	2.23	4.96	-15.66	14.07	6.50	-0.33
TSLA	0.12	0.06	2.85	8.13	-13.90	17.67	5.06	0.30
AMZN	0.17	0.14	1.85	3.42	-7.82	14.13	9.83	1.01
IRDM	0.12	0.12	2.63	6.94	-11.13	22.24	6.47	0.62
BR	0.09	0.09	1.27	1.61	-9.70	11.16	9.29	-0.19
PYACX	0.00	0.00	0.27	0.07	-2.08	0.77	3.40	-0.71
SIGAX	0.00	0.00	0.23	0.05	-1.29	0.67	1.28	-0.43
gold	0.02	0.00	0.99	0.98	-4.32	5.10	4.56	0.33
palladium	0.09	0.15	1.71	2.92	-7.40	7.28	1.49	-0.15
S&P 500	0.04	0.05	0.84	0.71	-4.10	4.96	3.87	-0.47

Covariance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$\text{cov}(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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- not so helpful!

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- not so helpful! \rightarrow correlation

Correlation

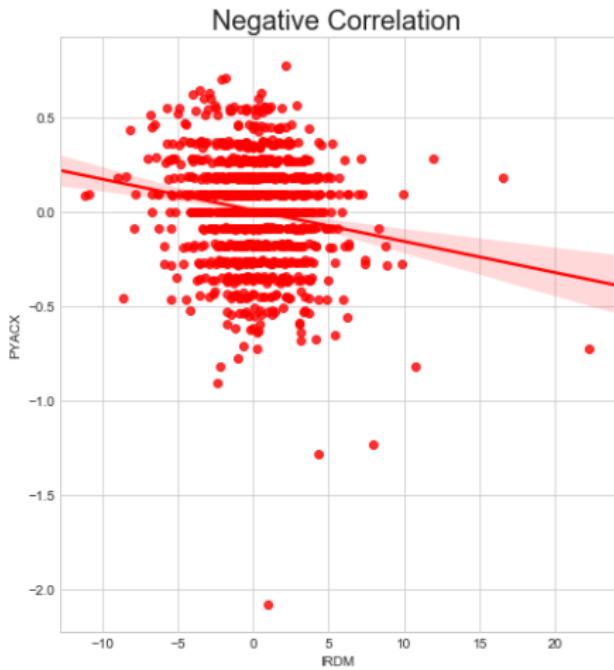
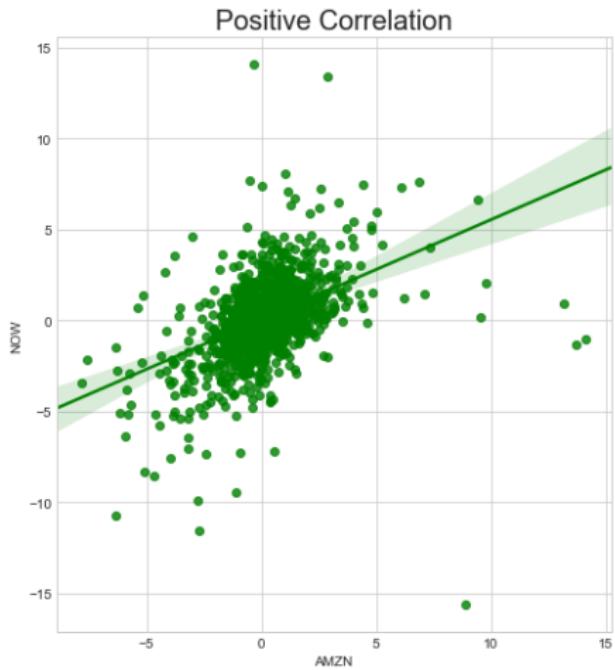
Definition

The correlation is the normalization of the covariance.

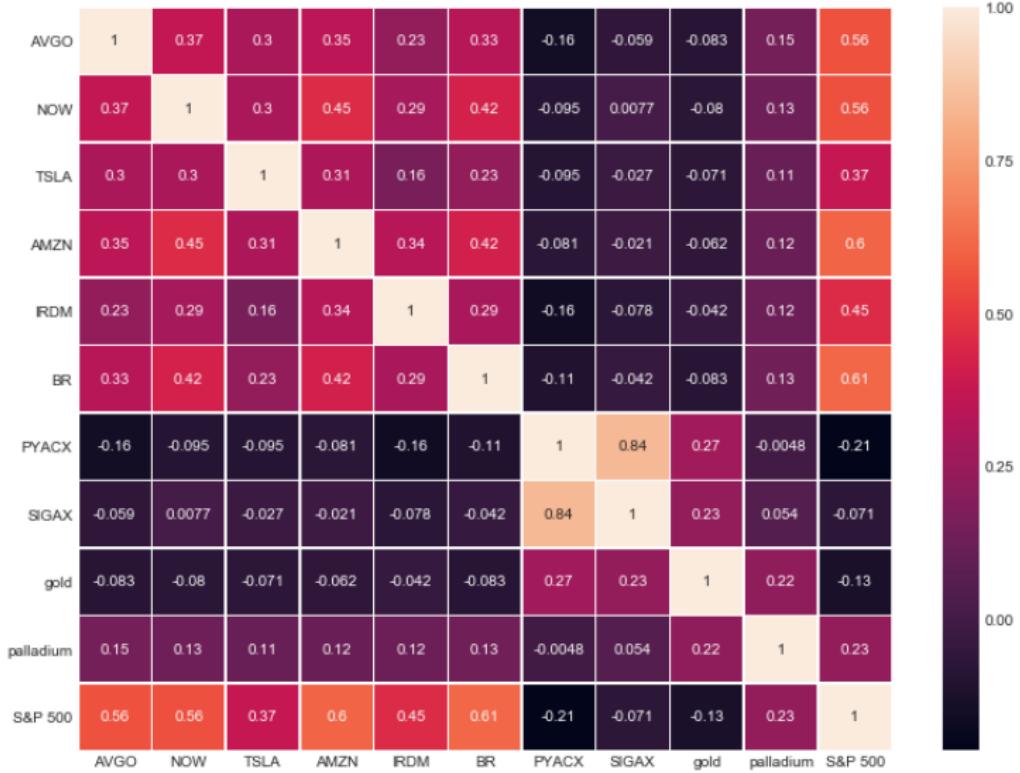
$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \cdot \sigma_Y}$$

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation



Correlation Matrix



Beta

Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

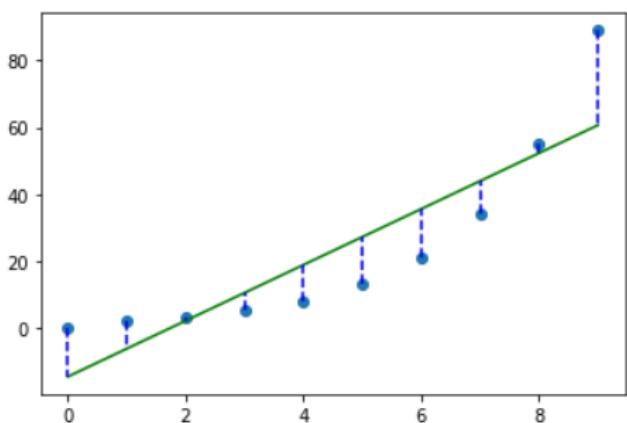
Definition

Alpha is the difference between the realised returns and the expected returns:

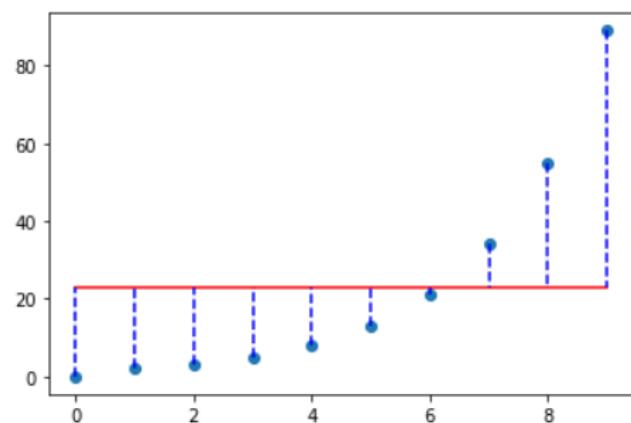
$$\begin{aligned}\alpha &= \bar{R} - \mathbb{E}(R) \\ \xrightarrow{\text{CAPM}} \alpha &= \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}\end{aligned}$$

R-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation



(b) Total Variation

Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- R_p = return of mutual fund
- R_f = risk-free rate
- σ_p = standard variation of the portfolio's excess return

Financial Metrics

For our assets

	alpha	beta	R-squared
AVGO	0.14213	1.35333	0.31393
NOW	0.24718	1.46614	0.30955
TSLA	0.17711	1.26445	0.14046
AMZN	0.32815	1.32199	0.36539
IRDM	0.15867	1.41795	0.20698
BR	0.13206	0.92064	0.37655
PYACX	-0.00576	-0.06792	0.04613
SIGAX	-0.01215	-0.01911	0.00501
gold	0.05087	-0.15174	0.01684
palladium	0.17817	0.47398	0.05493

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Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk

$$\sigma_p = \sqrt{\vec{w}^\top K \vec{w}}$$

$$\begin{aligned} &= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \text{cov}_{1,2} & \dots & \text{cov}_{1,n} \\ \text{cov}_{2,1} & \sigma_2^2 & \dots & \text{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}_{n,1} & \text{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\ &= \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{cov}_{i,j}} \end{aligned}$$

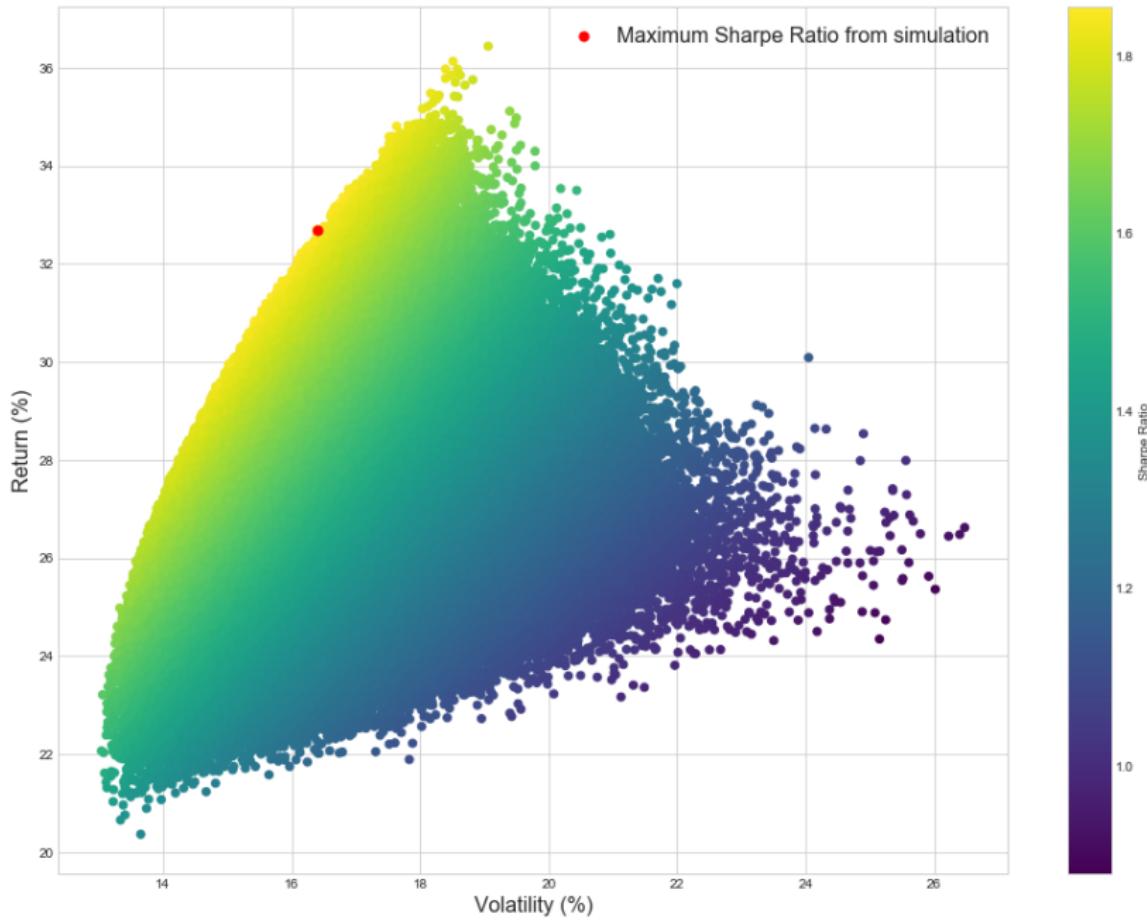
Naive weight allocation

Remember!

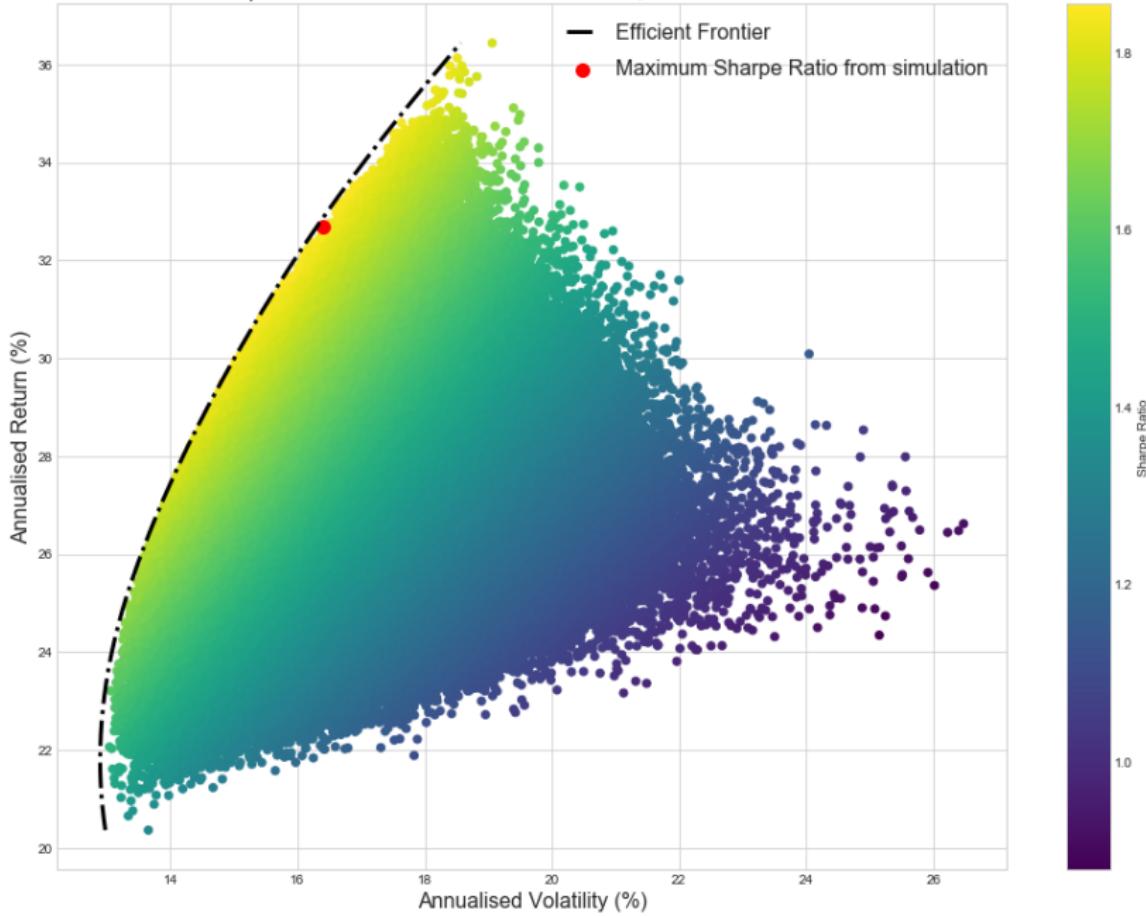
- 70% Stocks
- 20% Bonds
- 10% Commodities

What about \bar{w} ?

- 70% for 6 Stocks $\rightarrow \frac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow 10\%$ each
- 10% for 2 Commodities $\rightarrow 5\%$ each



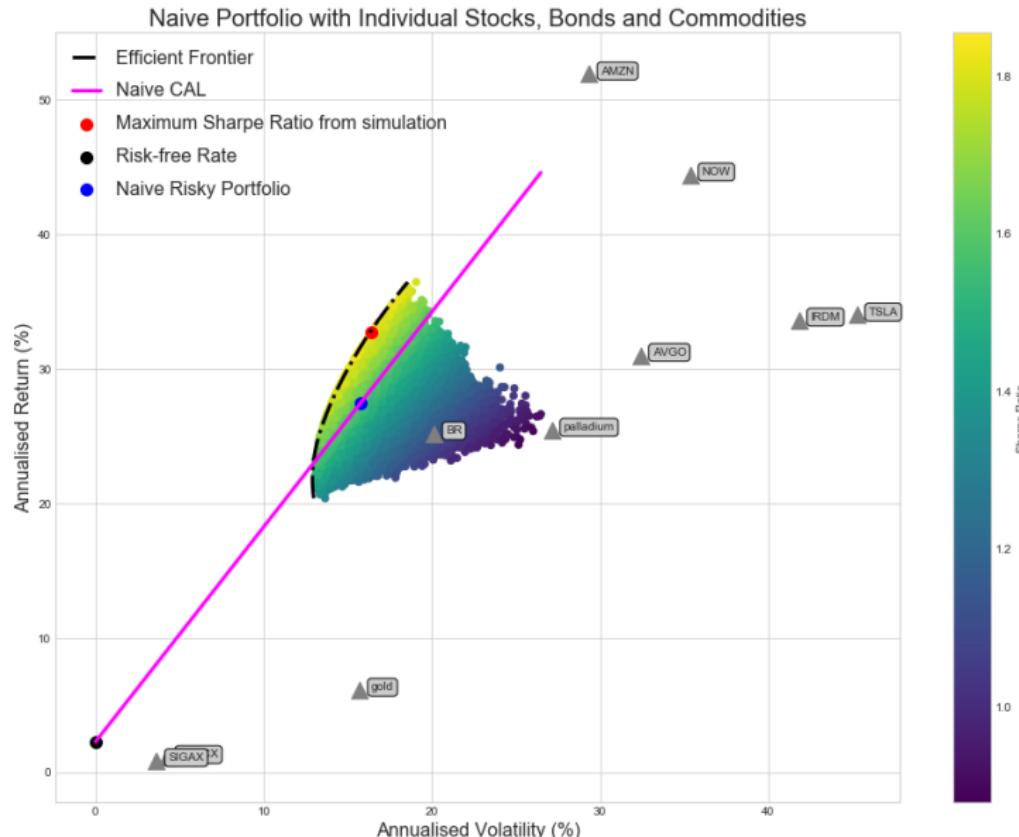
Portfolio Optimization with Individual Stocks, Bonds and Commodities



CAL construction

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \Rightarrow \epsilon_{\text{naive}} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$

Naive CAL



But...

We can do better!

Optimal portfolio

Optimization problem formulation

$$\begin{aligned} \max \quad & \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\ \text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\ & \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\ & \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\ & \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\ & w_s + w_b + w_c = 1 \\ & w_i \geq 0 \quad i = 1, \dots, n \end{aligned}$$

Optimal portfolio

Optimization problem formulation

Sharpe Ratio

$$\begin{aligned} & \max \quad \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\ \text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\ & \mathbf{1}^\top \vec{w}_S = w_s \\ & \mathbf{1}^\top \vec{w}_B = w_b \\ & \mathbf{1}^\top \vec{w}_C = w_c \\ & w_s + w_b + w_c = 1 \\ & w_i \geq 0 \quad i = 1, \dots, n \end{aligned}$$

Optimization yields...

optimal weights

$$\vec{w} = \begin{bmatrix} w_{AVGO} \\ w_{NOW} \\ w_{TSLA} \\ w_{AMZN} \\ w_{IRDM} \\ w_{BR} \\ w_{PYACX} \\ w_{SIGAX} \\ w_{gold} \\ w_{palladium} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

Optimization yields...

Financial Metrics

Optimal Risky Portfolio	
Return (%)	32.7164
Risk (%)	16.3185
Sharpe Ratio	1.8658
Beta (β)	0.9067

Capital Allocation Line

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{CAL} \\ (\sigma_{OPT}, r_{OPT}) \in \epsilon_{CAL} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{OPT} - r_f}{\sigma_{OPT}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us?**

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t. z :

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us?**

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

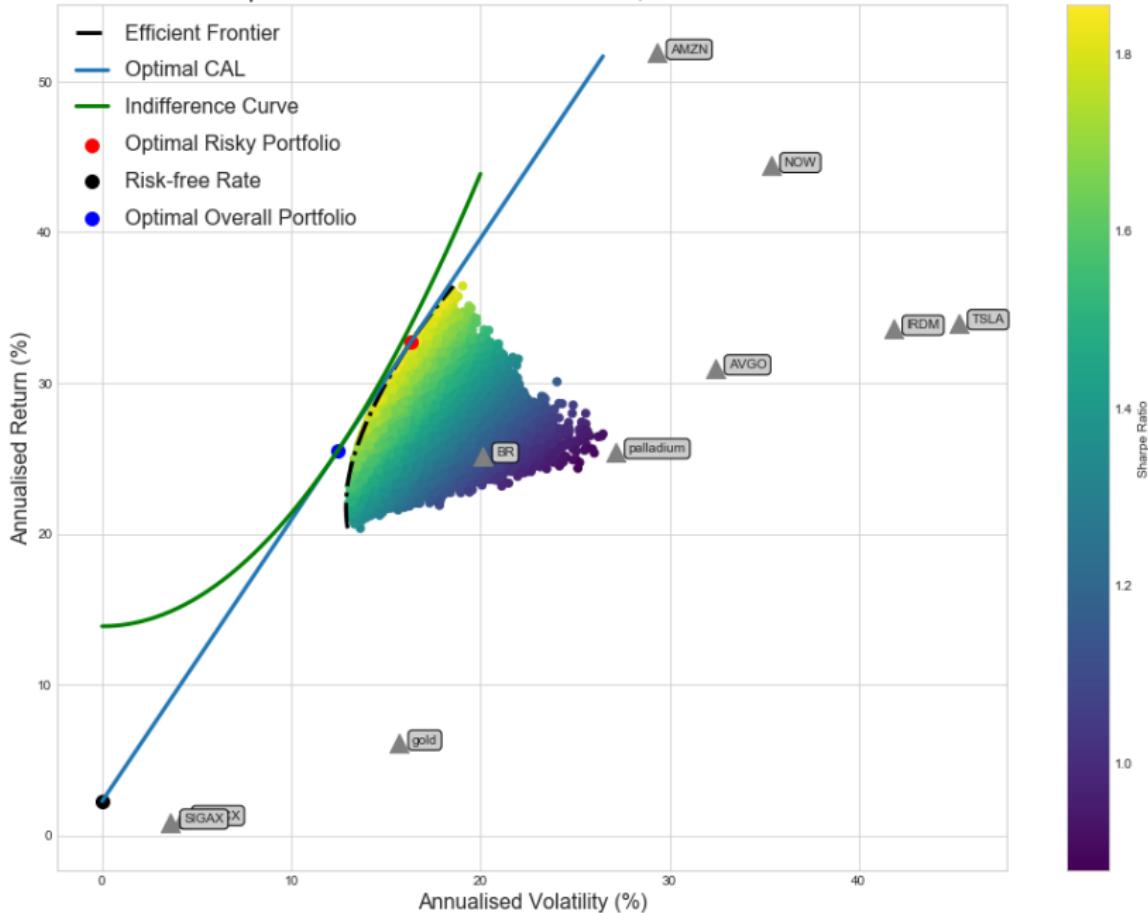
Maximize w.r.t. z :

$$\frac{\partial U}{\partial z} = 0$$

$$\Rightarrow r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\Rightarrow z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$

Portfolio Optimization with Individual Stocks, Bonds and Commodities



Past Performance

(1/13/2015 - 1/13/2020)

	Optimal Overall Portfolio	S&P 500
Return (%)	25.4769	11.3118
Risk (%)	12.4384	13.4138
Sharpe Ratio	1.8658	0.6741
Beta (β)	0.6911	1.0
Alpha (α) (%)	16.9584	0.0

References

The end!