Portfolio Management
FN 4329

Dimitriadis Nikolaos   Kyratzis Apostolos

American College of Greece

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Contents

todo

Todo

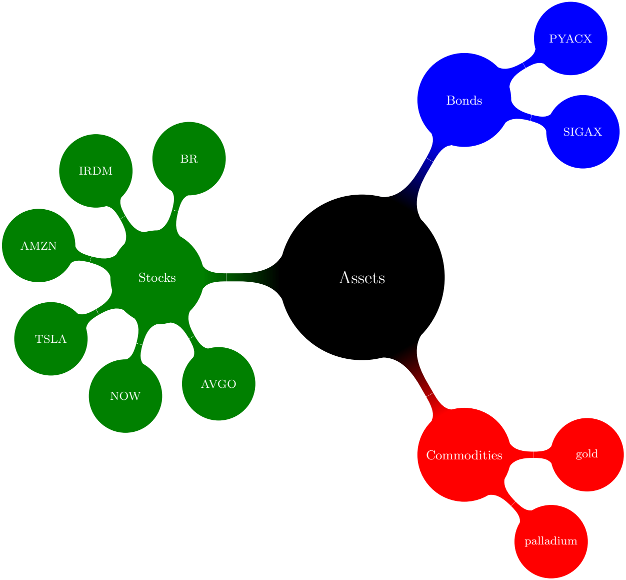
- fix contents (example: Intro, Theory, Results)

Asset allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

Contents

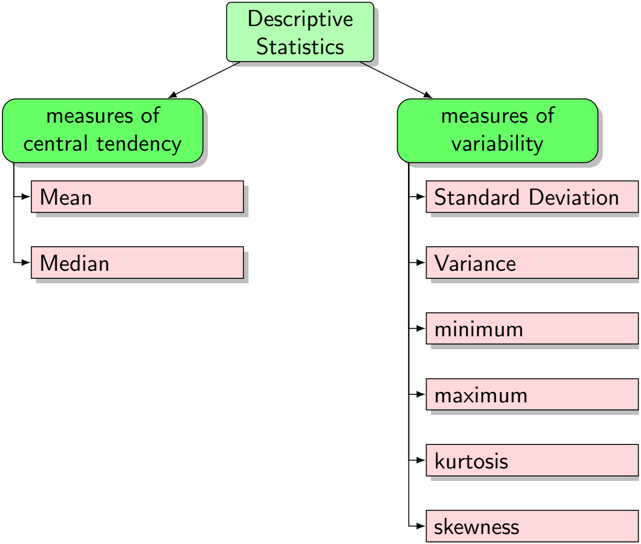
Security Selection



Contents

Contents

Descriptive statistics Taxonomy



Mean

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Measures of central tendency

Median

- i.e. the middle value

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- i.e. the middle value
- Why?

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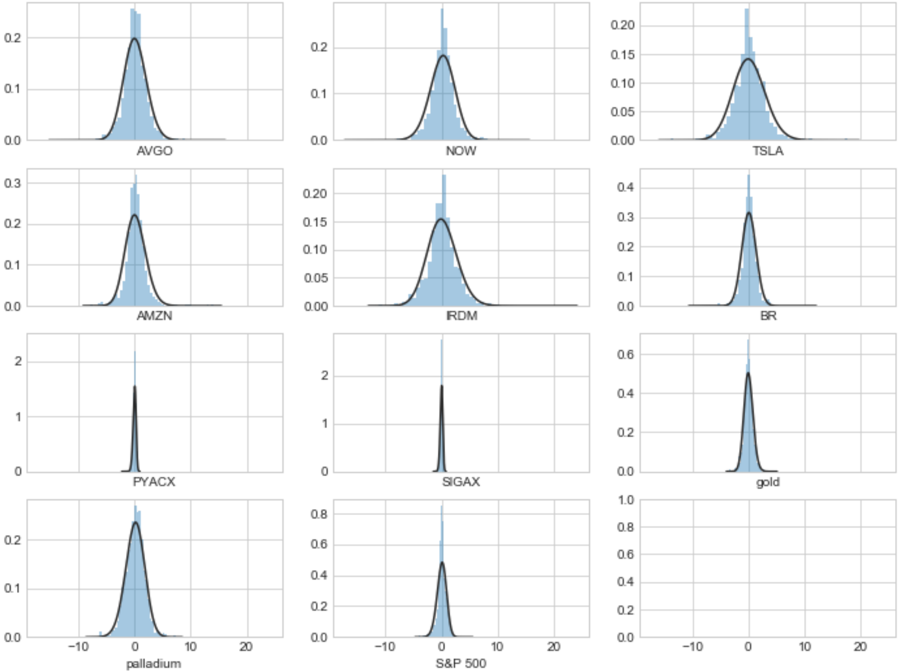
- Why? robust w.r.t. outliers

- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics $\implies$ risk in finance


AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500
0.0
0.1
0.2
0.3
0.4
0.6
0.8
1.0
0.00
0.05
0.10
0.15
0.20
0
1
2
−10
0
10
20

Measures of Variability

Minimum & Maximum

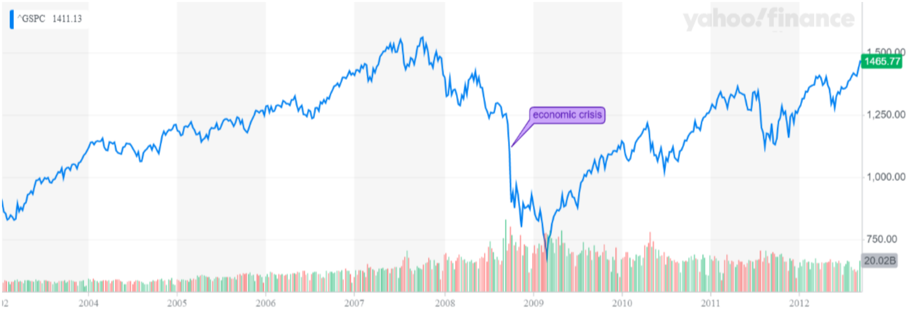


Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

Minimum & Maximum

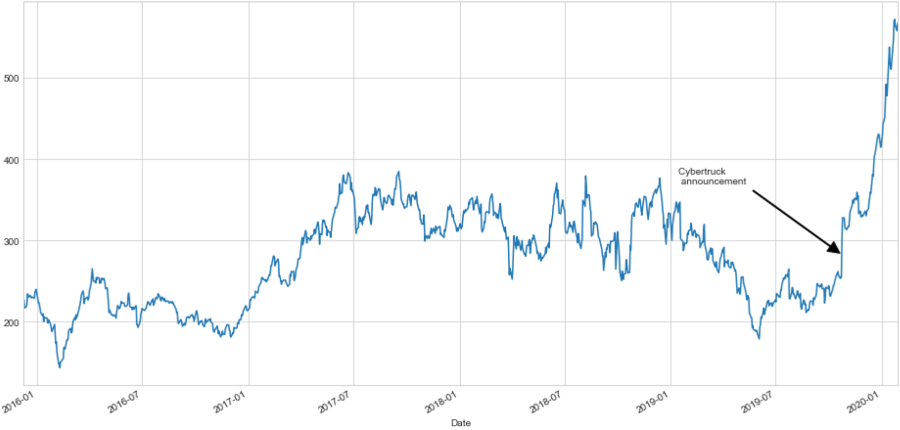


Figure: Tesla's announcement of the Cybertruck resulted in a steep price increase.

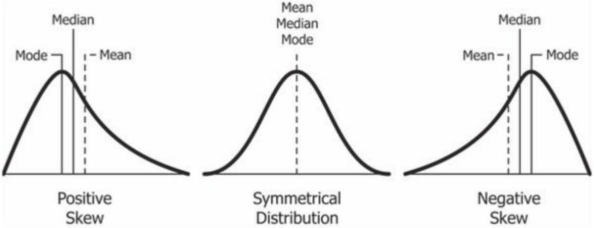
Measures of Variability

Skewness

Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



Definition

$$\mathrm{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X-\bar{x}}{\sigma}\right)^4\right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis $\rightarrow$ heavy tails **(outliers)**
- low kurtosis $\rightarrow$ no outliers

Measures of Variability

An overview

Moments

- $\tilde{\mu}_2$ standard deviation σ
- $\tilde{\mu}_3$ skewness
- $\tilde{\mu}_4$ kurtosis

Contents

Covariance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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- not so helpful! $\rightarrow$ correlation

Correlation

Definition

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

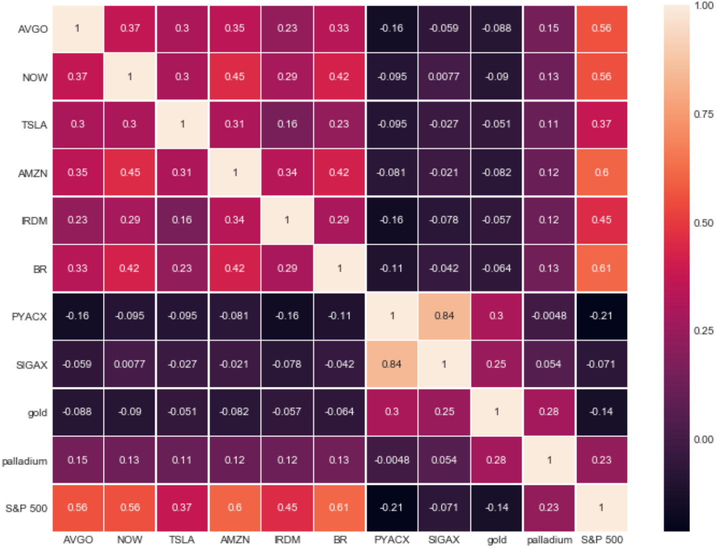
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation

todo

add regression plots to show the difference

Correlation Matrix



Beta

Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\mathrm{cov}(R_e, R_m)}{\mathrm{var}(R_m)}$$

alpha

Definition

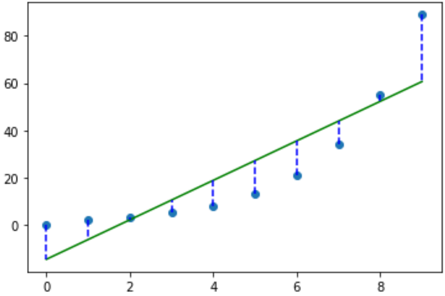
Alpha is the difference between the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

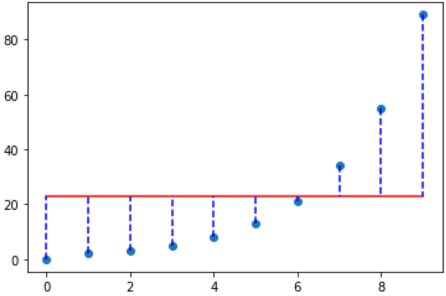
$$\overset{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{R_f + \beta(\mathbb{E}(R_m) - R_f)\right\}$$

R-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation



(b) Total Variation

Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- R_p = return of mutual fund
- R_f = risk-free rate
- σ_p = standard variation of the portfolio's excess return

Contents

Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

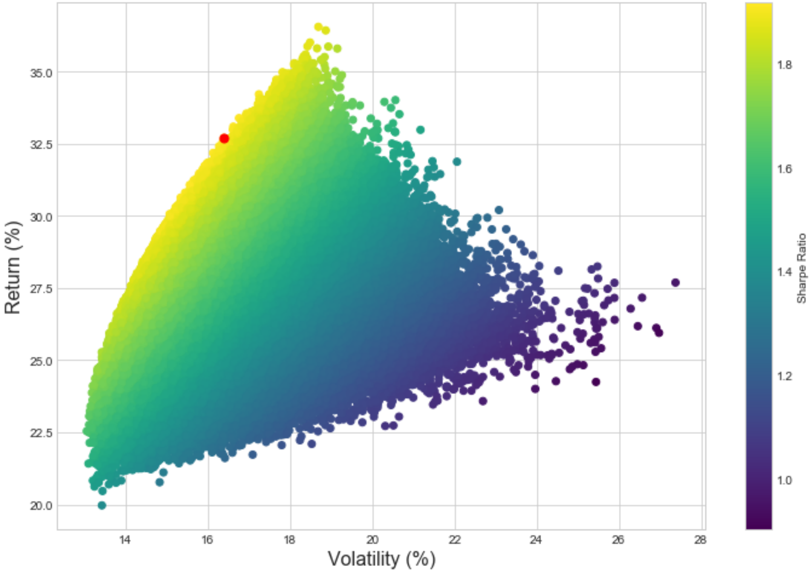
Risk

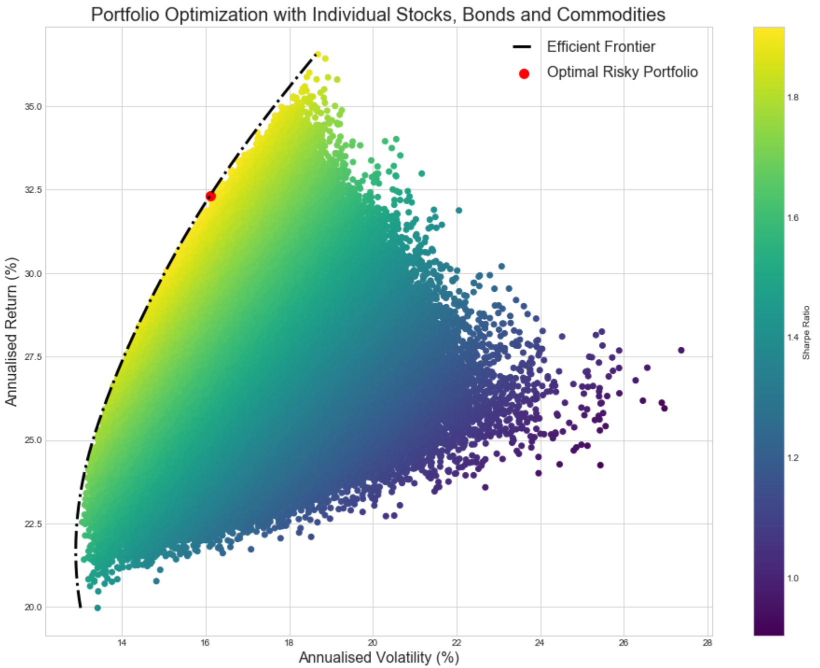
$$\sigma_p = \sqrt{\vec{w}^\top K \vec{w}}$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}}$$

$$= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}$$

Contents


35.0
32.5
30.0
27.5
25.0
22.5
20.0
Return (%)
14
16
18
20
22
24
26
28
Volatility (%)
1.8
1.6
1.4
1.2
1.0
Sharpe Ratio



Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal Risky Portfolio
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
20.0
22.5
25.0
27.5
30.0
32.5
35.0
14
16
18
20
22
24
26
28
1.0
1.2
1.4
1.6
1.8

$$\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}$$

Sharpe Ratio

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
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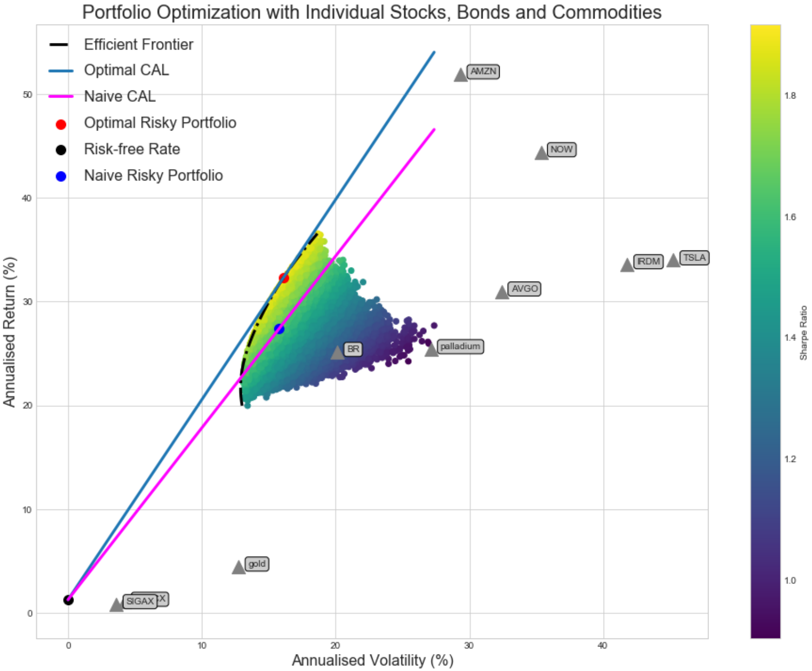
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{OPT}} - r_f}{\sigma_{\text{OPT}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal CAL
Naive CAL
Optimal Risky Portfolio
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
0
10
20
30
40
50
Sharpe Ratio
1.0
1.2
1.4
1.6
1.8

References

Nikiforos Laopodis.
Understanding investments: Theories and strategies.
Routledge, 2012.

Yahoo finance.

Morning star.

Us news money.

Fidelity.

Charles schwab.

Vanguard advisors.

Morgan stanley investment management.

The end!