

# FN 4329: INVESTMENTS AND PORTFOLIO MANAGEMENT

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### TODO 1: general

- copy-paste final tables and optimal weights

## 1 Question 1

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Decide on the allocation of your budget to each of the above instruments. Rationalize your choice based on your own investment philosophy. You should fill out a risk tolerance questionnaire and supply both the questionnaire and its suggested allocation (in the project's Appendix). In your discussion, please explain the choice of your investment vehicles and their significance in your portfolio. Label this as your investment policy statement (IPS).

**Investment Policy Statement** We are classified as young investors, with a small initial capital available. A summary of our investment philosophy is as follows:

### Objectives

- Long-term growth and capital appreciation
- Risk-profile: Aggressive
- Time horizon: Greater than 10 years
- Short-term liquidity needs: Minimal

**Portfolio Selection Guidelines** Long-term investment performance is generally determined by the underlying asset's performance. From a historical point of view, stock assets tend to achieve higher rates of return, along with greater volatility. Fixed income assets (i.e bonds) yield lower rates of return and bear less risk.

Based on our risk tolerance profile (see Appendix) and aim for long-term capital growth, the portfolio asset allocation will be:

- 70% Stocks
- 20% Bonds
- 10% Commodities

The selected assets are traded on US exchanges. The individual composition of holdings will be selected from the following asset classes:

**Equity** Equities are the main investment instrument of our portfolio, which will implement the required capital growth. A total of 6 stocks are selected, with the following characteristics:

- Company Size: 4 Large, 1 Mid, 1 Small
- Stock Type: 4 Growth, 1 Aggressive Growth, 1 High-Dividend

**Bonds** Fixed income assets are of secondary importance, because of our minimal needs for generation of current income. Instead of two individual corporate bonds, 2 bond mutual funds are selected, which invest at least 85% of their assets in corporate bonds.

**Commodities** For a portfolio that has a long-term horizon, allocating a small amount to commodities offers some protection against inflation and reduces the overall portfolio risk, as commodities historically move opposite from the market. A total of 2 precious metals are selected.

A more elaborate analysis of the final selected assets is performed in Section 2.

**Risk-free Rate** The 10-Year U.S. Treasury Bond is chosen as the risk-free asset, as it matches the time horizon of our investment.

**Benchmark Index** The S&P 500 index is used as the benchmark index of the portfolio.

## 2 Question 2

Outline your investment objectives, constraints and strategies and discuss the relative importance of risk and return in your investment decision making.

**Objectives and Constraints** The portfolio's main objective is long-term growth and capital appreciation, which is defined as a rise in an investment's market price. Given our young age, the limited initial capital available is the main constraint. Because of our risk-tolerance profile, we do not explicitly specify a loss limit. As a general rule, if a holding asset presents a one-year loss of its value greater than 20%, then it should be considered for exclusion from the portfolio.

**Investment Strategy** The main investment vehicles are stocks highlighted for their growth potential. We mainly focus on the technology sector or sectors that directly benefit from advancements in technology and innovations. A small amount is also allocated to corporate bond funds of medium credit quality (A and/or BBB) and to precious metals, for diversification purposes.

The final selected assets from each category are presented and analysed below:

### 2.1 Stocks

A total of six stocks are selected for the portfolio:

**Broadcom Inc. (Large/High-Dividend)** Broadcom Inc. is a global leader in designing and developing semiconductor and software infrastructure solutions. Broadcom's product portfolio serves data centres, networking & telecom hardware and high-end smartphone components. In January 2020, Broadcom announced a \$15 billion deal with Apple to supply upcoming iPhone products with wireless chips. With new 5G technology well under-way towards global scale deployment, Broadcom will gain the opportunity to expand its business and develop 5G compatible components. Even though currently is categorised as a "value" type stock, we believe it has a good growth potential in the future.

**ServiceNow Inc. (Large/Growth)** ServiceNow Inc. is a provider of cloud-based software solutions to structure, manage and automate business processes for global enterprises. The company delivers services based on a Software as a Service (SaaS) model, and its main focus is IT service management. However, more recently it expanded the available workflow solutions to other departments, such as Customer Support, Human Resources and Security Operations. It targets a wide range of industries including financial services, healthcare, government and education.

**Tesla Inc. (Large/Growth)** Tesla Inc. is a leading auto manufacturer company for next-generation electric vehicles, while also a designer and operator of sustainable energy systems. Tesla's global vehicle deliveries were 367,656 units in 2019 and we expect this figure to grow in the upcoming years, as more advancements in battery technology will allow electric vehicles to increase their autonomy and become a wider commercial product.

**Amazon Inc. (Large/Growth)** Amazon Inc. is one of the biggest online retailers, recently reaching the \$1 trillion market cap for the second time in the last 3 years. Its core business are online product and digital media sales, however we believe a higher potential lies in its Amazon Web Services Department. AWS offers solutions for machine learning, big data, IoT, cloud-computing and storage application. All of these technological fields are expected to greatly advance in the upcoming years and therefore further boosting the company growth.

**Iridium Communications Inc. (Small/Growth)** Iridium Communications Inc. is a leader in providing global voice and data services with satellite communications. It operates and maintains a vast network with over 70 satellites in orbit and related ground infrastructure, achieving true global coverage. In 2018 a partnership was announced with Amazon Web Services, aiming to develop a satellite network for future IoT applications. The company's commercial end user base includes business and organisations of various sectors, including maritime, aviation, government & military, emergency services, oil & gas, mining and transportation.

**Broadridge Financial Solutions Inc. (Mid/Aggressive Growth)** Broadridge Financial Solutions Inc. is a provider of communications and technology-driven solutions to investors, banks, brokerage offices, mutual funds and corporate issuers. The company delivers a range of solutions, including communication platforms, securities processing and data analytics that aid its clients in improving their quality of customer services, along the whole life cycle of an investment.

## 2.2 Bonds

A total of two corporate bond mutual funds are selected for the portfolio:

**Payden Corporate Bond Mutual Fund** The fund invests in a variety of debt instruments and income generation securities. It invests at least 85% of its assets in corporate bonds of mainly A and BBB credit rating. It may also use derivative products, such as CDS in order to gain exposure to the corporate debt market.

**Western Asset Corporate Bond Mutual Fund** The fund invests at least 80% of its assets in corporate debt securities, while 10% of the assets might be non-U.S dollar nominated fixed income securities of foreign issuers. The majority of holding assets are of A and BBB credit rating.

## 2.3 Commodities

A total of two precious metals are selected as commodities:

**Gold** After reaching historical lows in 2015, gold has since then seen a significant rise in value. The precious metal is traditionally considered a safe investment during rough economic times and a hedge against inflation. Recently investors are turning to gold as safe houses instead of government bonds, as the latter had negative inflation-adjusted returns in 2019. Due to this fact investors have shifted their focus to gold, because even though it does not generate any return, it also does not have any holding costs and thus is more favourable for the time being.

**Palladium** Palladium is now more expensive than the four major precious metals, as its value has almost doubled over the last year. Palladium's main commercial use is in catalytic converters of car's exhaust systems, that help turn toxic gas emissions into less harmful ones. This impressive rise in price can be solely explained by the fact that demand outstrips supply. Palladium is a secondary product from mining operations of other metals, such as platinum or nickel and thus miners have less control over the extracted quantities in response to prices rising. Palladium production is projected to fall short of demand for the 8<sup>th</sup> straight year and therefore the metal is expected to maintain its current market value.

## 2.4 Overview

The portfolio aims in capital appreciation and it focuses on assets, such as stocks, that traditionally are more volatile and thus bear more risk. However they may also generate higher returns compared to less risky assets, hence their selection as the main investment instrument of the present portfolio.

### 3 Question 3

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**Provide a preliminary discussion of the effects of the short investment horizon in your decision(s) and diversification strategy**

The investor's time horizon plays a crucial role in choosing the instruments of a portfolio. As stated previously, our main objective is long-term capital appreciation and the primary selected asset are stocks. Stocks are widely considered as risky assets, but they also tend to generate higher returns over long periods of time compared to other less risky assets, such as bonds.

If an investor wants to make an investment of short-term horizon, then stocks probably should not be the preferred asset, as their prices usually have substantial fluctuations over short periods of time, affected by temporary financial, social as well as geopolitical news and events. Therefore the desired return might not be realised, leaving the investor short on his target. Bonds should be a more favourable choice in this occasion, as they are less risky (in terms of variability) and provide a relatively steady generation of income over the short period.

However, this does not mean that a long-term portfolio should not include bonds. One of their important roles is that they reduce overall volatility, as their returns present very low or negative correlation values with stock returns. Additionally, they offer some protection against any emergency short-term capital or liquidity needs that may arise, in the form of regular interest payments.

In conclusion, our diversification strategy can be summarized as follows:

- Even though we focused on growth style stocks, our choices included companies of different market capitalizations (Large, Mid, Small) and industry sectors (i.e Semiconductors, Communication Services, Online Retail and Cloud Services).
- Two corporate bond mutual funds were included in the portfolio, for the reasons analysed above.
- Two commodities were also included, as a hedge against inflation and for risk reduction, since they exhibit low or negative correlation values with stock assets.

## 4 Question 4

Compute the descriptive statistics for each instrument. Explain each metric you computed from the perspective of the investor. Provide graphs as well.

Descriptive statistics are brief descriptive coefficients that summarize a given data set. In our case, this dataset is daily returns of each instrument, over the last 5 years. These coefficients enable the reader to understand the features of the dataset and derive quantitative insights on its nature.

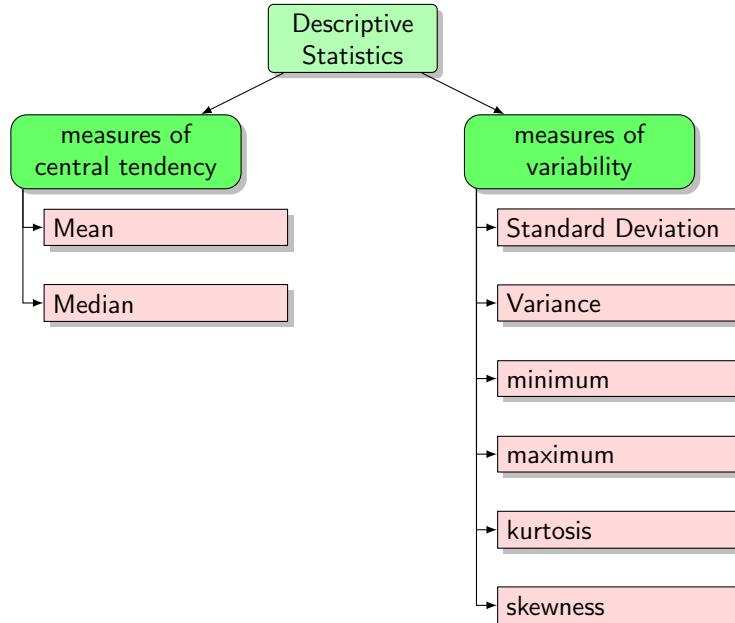


Figure 1: Taxonomy of descriptive statistics

Let  $x_i, i \in [n]$  be the sample, i.e. the returns of an instrument.

### 4.1 Measures of central tendency

Measures of central tendency describe the centre position of a distribution for a dataset.

**Mean** The mean, denoted as  $\bar{x}$  is the sum of the sampled values divided by the number of items in the sample:

$$\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (4.1)$$

From the investor's perspective, the mean describes the average performance of the instrument. If  $\bar{x} > 0$  then the instrument increases in value on average.

**Median** The median of data sample can be thought as the middle value of dataset, separating the higher from the lower half. It is a more robust measure than the mean, since it is not affected by outliers. The median can inform the investor on whether the returns are positive or negative on most time instances.

### 4.2 Measures of Variability

Measures of variability describe how the data is distributed within the set.

**Standard Deviation** The Standard deviation expresses the variability of a population. It indicates the extent to which the values tend to be close to the mean. It is commonly used to measure confidence in statistics. For this reason, it is a measure of risk in economic terms. It is given by the following formula:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2} \quad (4.2)$$

From the perspective of the investor, an instrument with high standard deviation carries more risk and should have higher returns to accommodate for this variability.

**Minimum & Maximum** Minimum and maximum describe the range of values. In economic terms, the maximum and minimum of returns reflect events that shape the price of an instrument. An example would be the economic crisis of 2008, or more recently the announcement of Tesla's (TSLA) new vehicle:

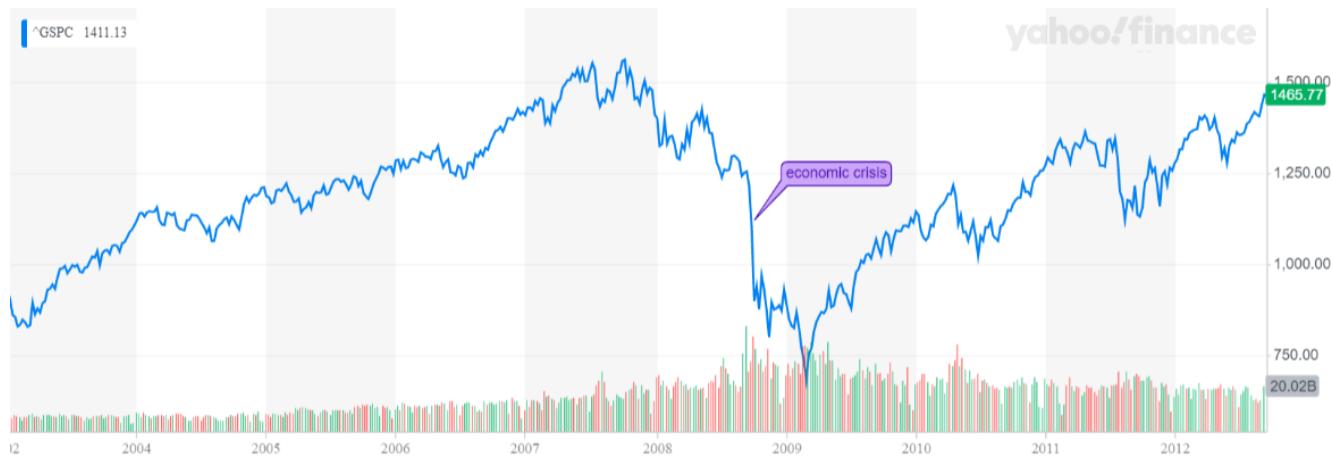


Figure 2: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

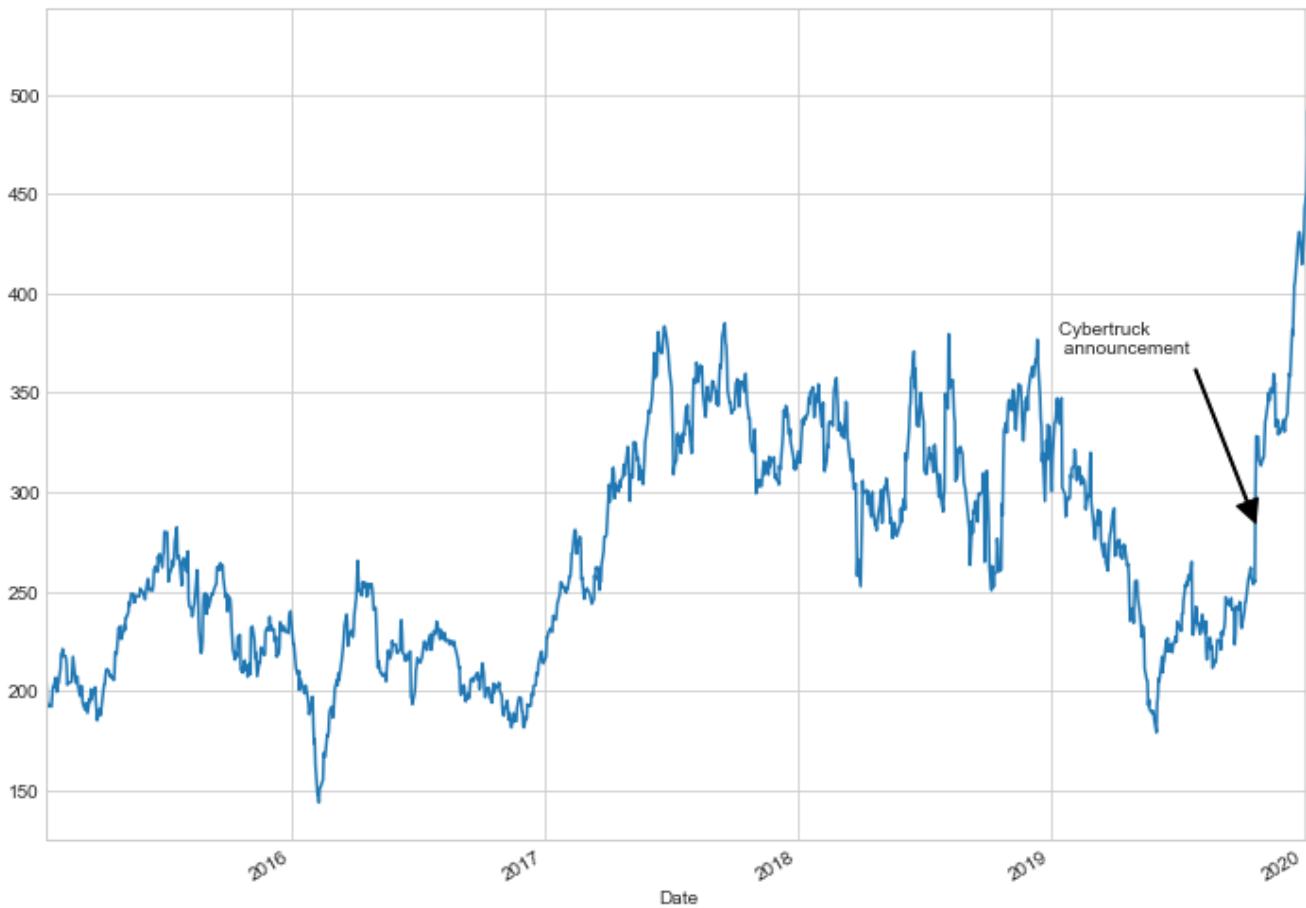


Figure 3: Tesla's announcement of the Cybertruck resulted in a steep price increase.

**Kurtosis** Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution. Data sets with high kurtosis tend to have heavy tails, or outliers, whilst data sets with low kurtosis lack outliers. The kurtosis is given by the following formula:

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E} \left[ \left( \frac{X - \bar{x}}{\sigma} \right)^4 \right] \quad (4.3)$$

For investors, high kurtosis of the returns distribution implies that the investor might experience occasional extreme returns.

**Skewness** Skewness is a measure of asymmetry. To be more precise, this coefficient indicates whether the tail of the distribution is on the left or the right of the mean/median value. Let us consider a simple example:

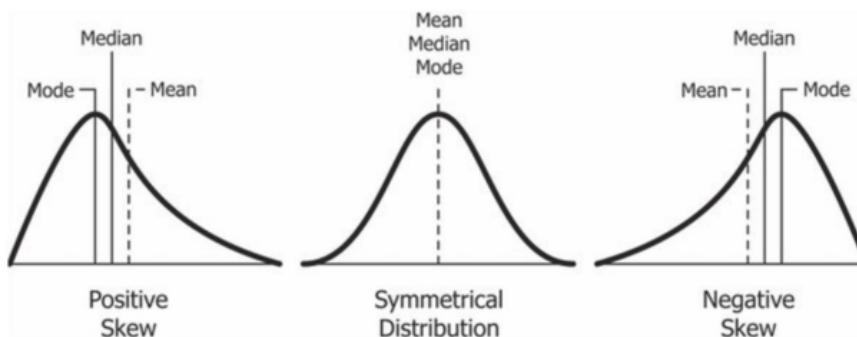


Figure 4: Distribution diagrams for three different skew values (Source: Wikipedia)

The skewness is given by the following formula:

$$\tilde{\mu}_3 = \mathbb{E} \left[ \left( \frac{X - \bar{x}}{\sigma} \right)^3 \right] \quad (4.4)$$

Skewness is a very important measure for investors. Standard Deviation is commonly associated with the estimation of an instrument's risk, but it has a major flaw in assuming a normal distribution. Since few return distributions resemble a normal distribution, skewness is a better measure for predicting performance.

Most asset returns are skewed, either left or right. An investor can utilize this information to better predict future returns. A positively-skewed investment return means that there were frequent small losses and a few large gains. The opposite is true for negatively-skewed distributions.

A summary of the descriptive statistics of the selected asset's daily returns is given in the table below (in percentage values). A histogram for these statistics is presented in Figure 5.

	AVGO	NOW	TSLA	AMZN	IRDM	BR	PYACX	SIGAX	gold	palladium	S&P 500
mean	0.1072	0.1459	0.1162	0.1660	0.1150	0.0891	0.0043	0.0034	0.0235	0.0899	0.0425
median	0.1113	0.2066	0.0604	0.1374	0.1185	0.0910	0.0000	0.0000	0.0000	0.1505	0.0504
std	2.0410	2.2267	2.8508	1.8480	2.6336	1.2677	0.2672	0.2282	0.9882	1.7089	0.8450
var	4.1656	4.9583	8.1271	3.4151	6.9358	1.6071	0.0714	0.0521	0.9765	2.9202	0.7140
min	-13.7447	-15.6561	-13.9015	-7.8197	-11.1297	-9.7015	-2.0777	-1.2893	-4.3193	-7.4046	-4.0979
max	14.7054	14.0685	17.6692	14.1311	22.2393	11.1618	0.7712	0.6683	5.0975	7.2825	4.9594
kurtosis	5.3069	6.5043	5.0603	9.8273	6.4736	9.2869	3.4009	1.2845	4.5577	1.4909	3.8721
skewness	0.2343	-0.3319	0.3027	1.0068	0.6237	-0.1879	-0.7082	-0.4345	0.3295	-0.1472	-0.4688

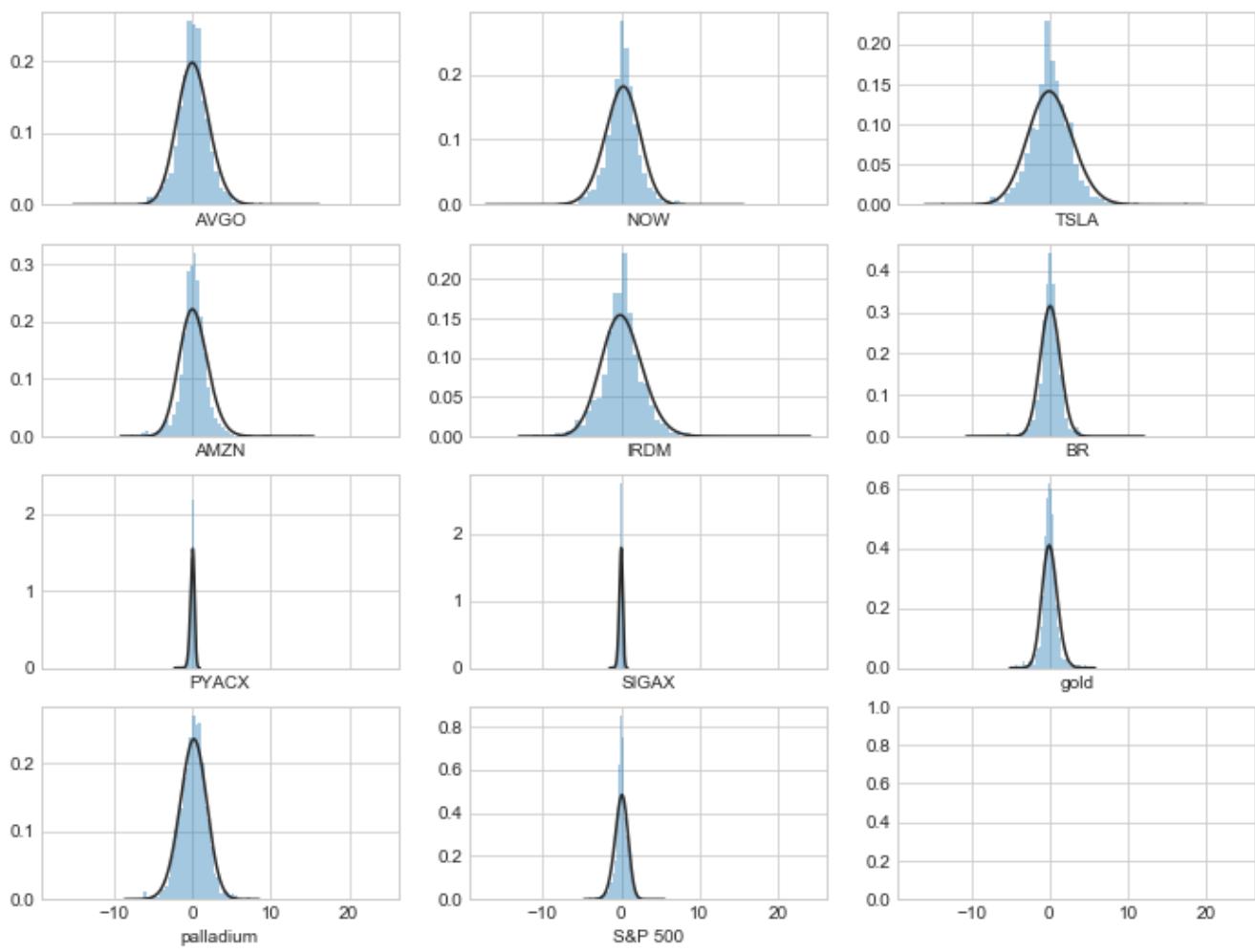


Figure 5: Histogram of the returns. A bell curve approximation is fitted. Note that the x-axis is common among the histograms. This allows us to grasp the "spread" or variability of each asset. It is easy to see that bonds are less risky than commodities and especially stocks.

## 5 Question 5

Compute additional metrics for the assets such as the correlation and covariance matrices, for the entire and two subperiods (of your own choosing), if needed. Interpret your findings. Also, compute each fund's alpha, beta, R-square. Interpret your findings from the perspective of the investor

### 5.1 correlation and covariance

Let  $X$  and  $Y$  be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$\text{cov}(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})] \quad (5.1)$$

In our case these random variables are the returns of the various instruments. The element  $(i, j)$  of the covariance matrix is the covariance between instruments  $i$  and  $j$ :

	QCOM	TEAM	TSLA	BABA	BLK	PYACX	VWEHX	gold	palladium	^GSPC
QCOM	0.000394	0.000138	0.000116	0.000141	0.000121	-6.761679e-06	1.027417e-05	-0.000020	0.000042	0.000000
TEAM	0.000138	0.000811	0.000167	0.000175	0.000111	-4.098970e-06	1.331404e-05	-0.000011	0.000038	0.000000
TSLA	0.000116	0.000167	0.000857	0.000169	0.000106	-5.648866e-06	1.360635e-05	-0.000013	0.000057	0.000000
BABA	0.000141	0.000175	0.000169	0.000387	0.000122	-4.224175e-06	1.450409e-05	-0.000012	0.000067	0.000000
BLK	0.000121	0.000111	0.000106	0.000122	0.000196	-8.851623e-06	1.374404e-05	-0.000019	0.000050	0.000000
PYACX	-0.000007	-0.000004	-0.000006	-0.000004	-0.000009	5.610073e-06	7.027645e-07	0.000007	-0.000001	-0.000000
VWEHX	0.000010	0.000013	0.000014	0.000015	0.000014	7.027645e-07	6.491153e-06	-0.000002	0.000007	0.000000
gold	-0.000020	-0.000011	-0.000013	-0.000012	-0.000019	6.612944e-06	-1.918765e-06	0.000058	0.000029	-0.000000
palladium	0.000042	0.000038	0.000057	0.000067	0.000050	-1.173603e-06	7.206858e-06	0.000029	0.000278	0.000000
^GSPC	0.000080	0.000093	0.000087	0.000093	0.000090	-4.288050e-06	8.913414e-06	-0.000011	0.000030	0.000000

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \cdot \sigma_Y} \quad (5.2)$$

The correlation ranges between  $-1$  and  $1$  and it measures the linear dependence between two variables.

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence between the variables} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases} \quad (5.3)$$

the correlation matrix is presented below:

	QCOM	TEAM	TSLA	BABA	BLK	PYACX	VWEHX	gold	palladium	^GSPC
QCOM	1.000000	0.243419	0.199395	0.361220	0.436478	-0.143858	0.203212	-0.135153	0.126725	0.497517
TEAM	0.243419	1.000000	0.200639	0.312442	0.279119	-0.060784	0.183548	-0.050210	0.081051	0.400008
TSLA	0.199395	0.200639	1.000000	0.294151	0.259546	-0.081488	0.182472	-0.058540	0.116378	0.365220
BABA	0.361220	0.312442	0.294151	1.000000	0.441629	-0.090667	0.289416	-0.077828	0.204843	0.578349
BLK	0.436478	0.279119	0.259546	0.441629	1.000000	-0.266900	0.385268	-0.180232	0.213814	0.787960
PYACX	-0.143858	-0.060784	-0.081488	-0.090667	-0.266900	1.000000	0.116457	0.367191	-0.029720	-0.222572
VWEHX	0.203212	0.183548	0.182472	0.289416	0.385268	0.116457	1.000000	-0.099047	0.169668	0.430108
gold	-0.135153	-0.050210	-0.058540	-0.077828	-0.180232	0.367191	-0.099047	1.000000	0.228830	-0.179643
palladium	0.126725	0.081051	0.116378	0.204843	0.213814	-0.029720	0.169668	0.228830	1.000000	0.218967
^GSPC	0.497517	0.400008	0.365220	0.578349	0.787960	-0.222572	0.430108	-0.179643	0.218967	1.000000

Both correlation and covariance matrices are symmetric.

The correlation matrix is also shown in Figure 6 and in Figure 7 another visualization of the effect of the various values of correlation coefficients is presented:

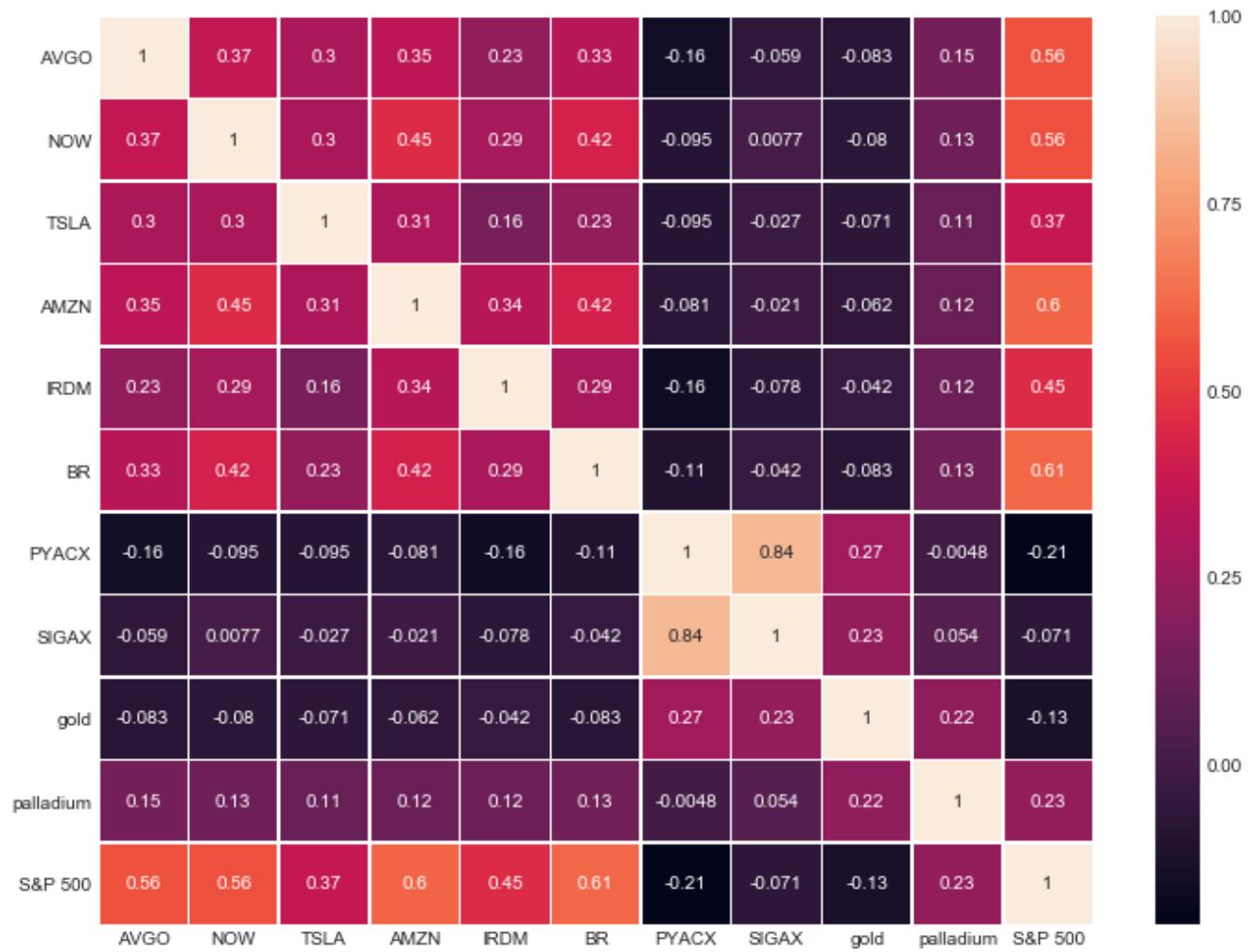


Figure 6: The correlation matrix for our instruments.

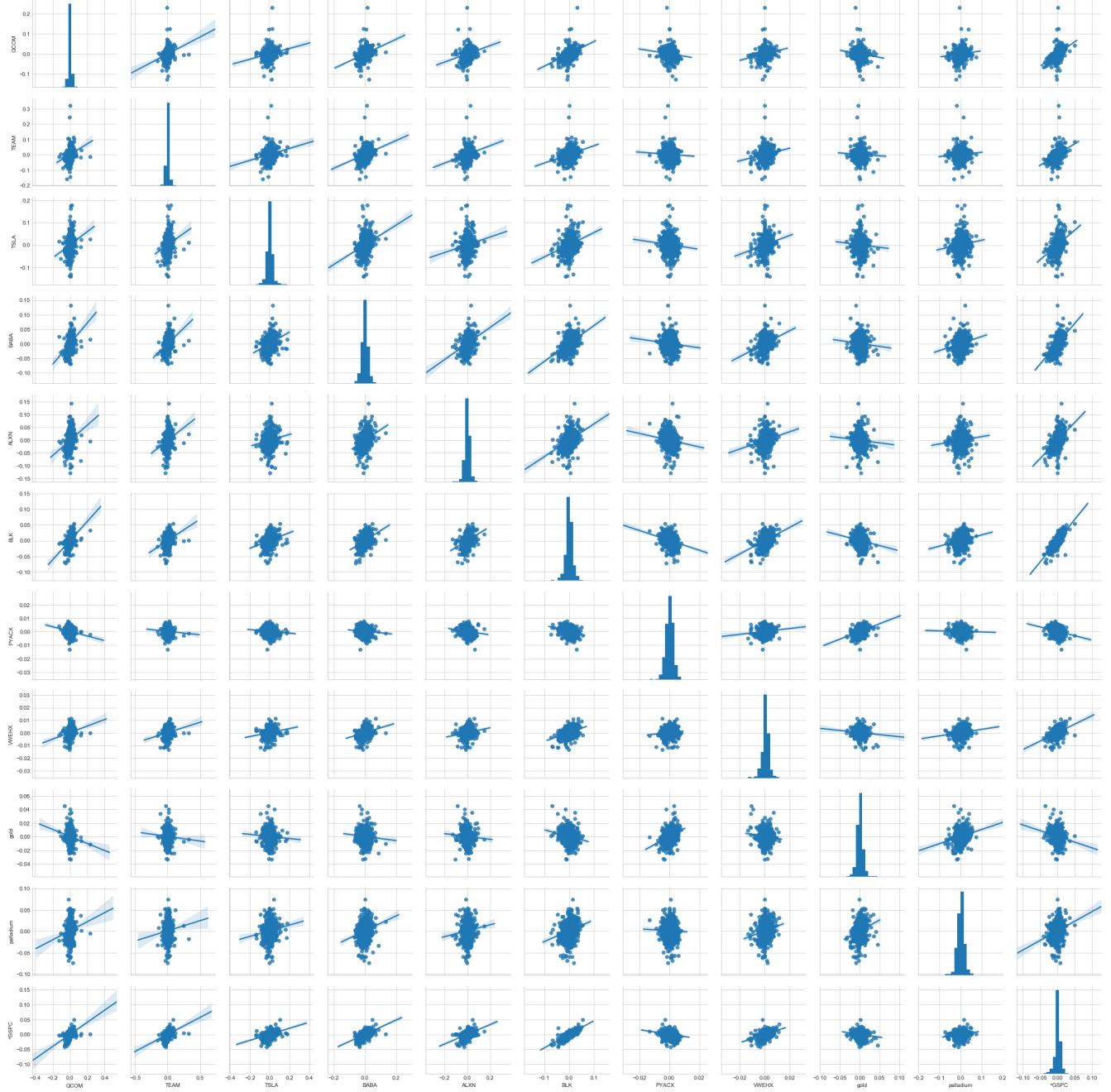


Figure 7: An illustration of the correlation matrix. When the correlation coefficient is negative the slope of the regression line is negative.

## 5.2 beta

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk. The beta formula is:

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)} \quad (5.4)$$

where  $R_e$  is the return of the individual stock and  $R_m$  is the return of the overall market. In order to calculate  $\beta$ , a regression model has to be fitted on the data points from an individual stock's returns against those of the market. Then  $\beta$  is the slope of the aforementioned line.

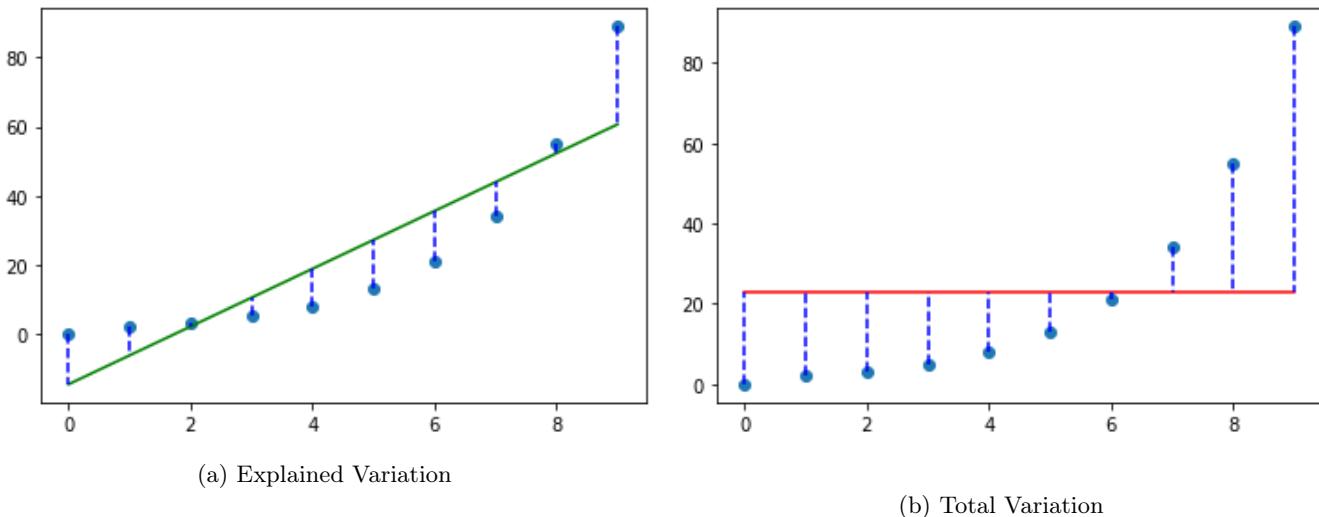


Figure 8:  $R$ -squared calculation

Even though the formula is straightforward, the data selection is not. The result depends on the time frame and frequency of historical data selected. Hence, many different  $\beta$  values can be found online. We use 5 years of daily data. To be more specific, let  $\mathcal{D} = \{d_0, d_1, \dots, d_{36}\}$  be the closing prices of the individual stock. Then,  $\mathcal{P} = \{p_1, \dots, p_{36}\}$  is the set of the percent changes of said closing prices where  $p_i = \frac{d_i - d_{i-1}}{d_{i-1}} \times 100\%$ . Note that  $\mathcal{P} = R_e$  (see (5.4)). By the same token,  $R_m$  is calculated using the historical closing prices of the market index (GSPC). Using these data points and equation (5.4),  $\beta$  is calculated.

### 5.3 alpha

By finding  $\beta$  we can proceed to calculate  $\alpha$ . To be more precise  $\alpha$  denotes the excess return. The Capital Asset Pricing Model (CAPM) is given by:

$$\mathbb{E}(R_i) = R_f + \beta_i(\mathbb{E}(R_m) - R_f) \quad (5.5)$$

where  $\mathbb{E}(R_i)$  is the expected return of the individual asset,  $R_f$  is the risk-free rate. Then,  $\alpha$  is calculated by subtracting the expected returns from the actual mean portfolio returns  $\bar{R}$ :

$$\alpha = \bar{R} - \mathbb{E}(R) \quad (5.6)$$

## 5.4 R-squared

*R*-squared or coefficient of variation is calculated using the following formula:

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}} \quad (5.7)$$

$R^2$  is a statistical measure that indicates the proportion of the variation of dependent variable that can be explained using the independent variables of a simple regression model. An illustrative explanation is adduced using the following figures. In 8a a simple regression model is fitted using the data provided. The explained variation is the sum of the squared distances from the regression line divided by the number of points. Said distances are denoted with a blue color in the figures below. The total variation is calculated by the same token but with a crucial difference: the line is horizontal and denotes the mean of the data.

In economics, the  $R^2$  measure denotes the percentage of a fund's movements that can be explained away by the historical changes of the benchmark index.

## 5.5 Sharpe ratio

The Sharpe ratio is named after Nobel Laureate William Sharpe. It is a measure that relates the return of an investment to its risk; it is the risk-adjusted return.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (5.8)$$

where

- $R_p$  = return of mutual fund
- $R_f$  = risk-free rate
- $\sigma_p$  = standard variation of the portfolio's excess return

To calculate the Sharpe Ratio we use "the trailing *five*-year period by dividing a fund's annualized excess returns over the risk-free rate by its annualized standard deviation"<sup>1</sup>.

Using the historical data for each fund and the analysis presented above, we are able to calculate  $\alpha, \beta, R^2$  and the Sharpe Ratio using Python. The results are presented below:

	QCOM	TEAM	TSLA	BABA	BLK	PYACX	VWEHX	gold	palladium
alpha	0.000332	0.001605	0.000680	0.000403	-0.000046	0.000289	0.000211	0.000438	0.001243
beta	1.213778	1.400108	1.314113	1.398591	1.356401	-0.064811	0.134720	-0.167928	0.448806
r-squared	0.247523	0.160006	0.133386	0.334487	0.620880	0.049538	0.184993	0.032272	0.047946

**Methodology** Historical data of a 5-year time frame with daily intervals were used for the above calculations.

**Risk-free Rate** To compute the Risk-Free rate, 1-Year Treasury Constant Maturity Rate was used (5 year average with monthly intervals).

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<sup>1</sup>[link](#)

## 6 Question 6

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Calculate the return/risk of your risky portfolio. Explain each step in your analysis. You must use EXCEL's mmult functions for this part of the analysis.

Let  $R_i$  be the returns data sample for the instrument  $i$  and  $\mathcal{R} = (R_1, R_2, \dots, R_n)$  be the collection of those random variables. Let  $\vec{w}$  be the vector of weights that represents the allocation of the various instruments in our risky portfolio. Obviously,  $\|\vec{w}\|_1 = 1$ . Finally, let  $K$  be the associated covariance matrix.

Then the return of the risky portfolio is a weighted average of the expected returns:

$$R_{\text{risky portfolio}} = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) \quad (6.1)$$

and the risk of the risky portfolio is measured by its standard deviation:

$$\sigma_{\text{risky portfolio}} = \sqrt{\vec{w}^\top K \vec{w}} \quad (6.2)$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \text{cov}_{1,2} & \dots & \text{cov}_{1,n} \\ \text{cov}_{2,1} & \sigma_2^2 & \dots & \text{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}_{n,1} & \text{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \quad (6.3)$$

$$= \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{cov}_{i,j}} \quad (6.4)$$

## 7 Question 7

Derive and graph the Capital Allocation Line. Graph the Efficient Frontier with your available investment instruments (assets) and superimpose your CAL. Discuss the various options you may have and finalize your optimal point.

By taking various random weight allocations we are able to create portfolios with different risks and returns. The optimality criterion lies in the Sharpe Ratio associated with each risky portfolio. The optimal risky portfolio is characterized by the highest Sharpe ratio.

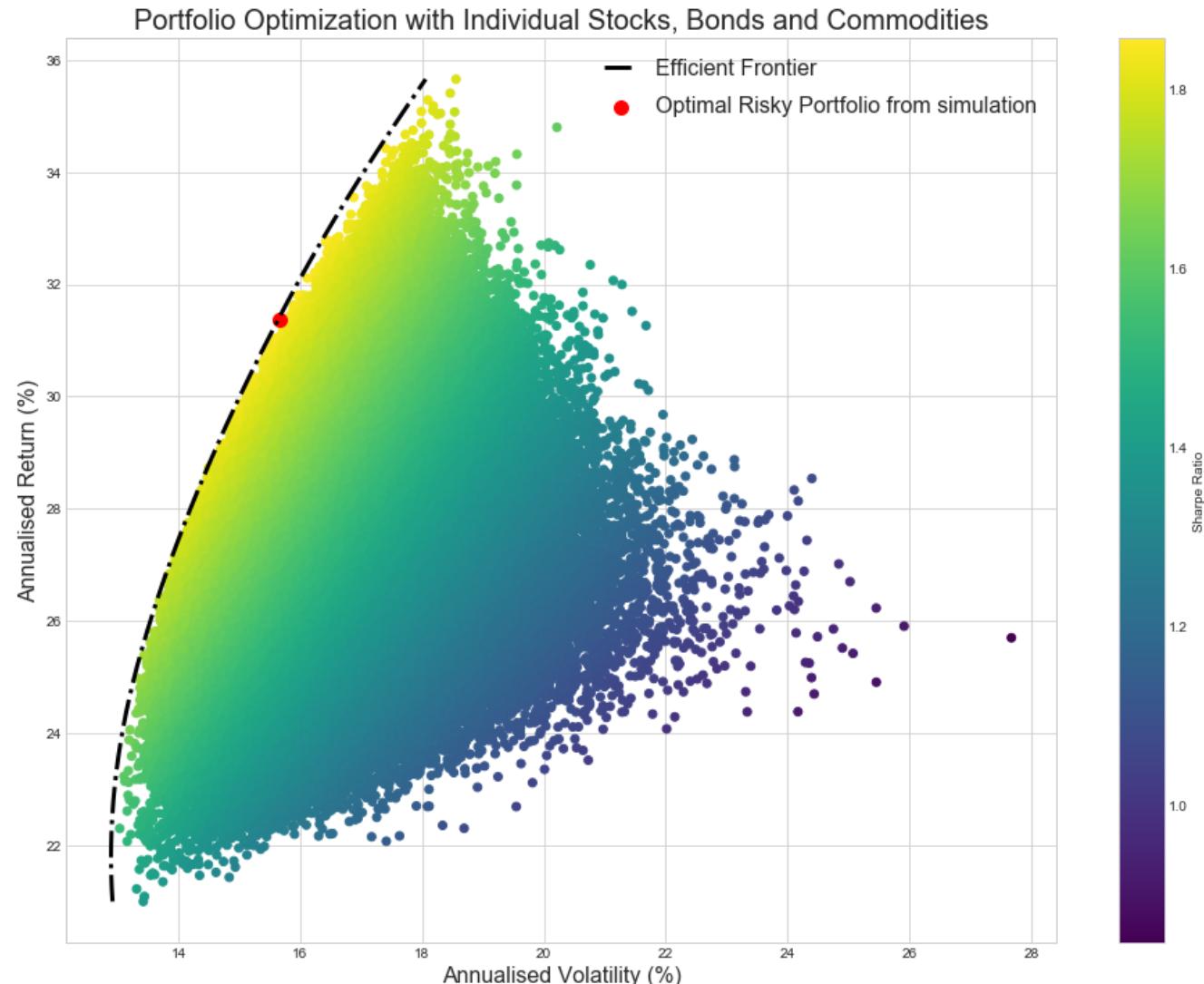


Figure 9:  $n = 50,000$  random portfolios are created and plotted in a common graph. The optimal risky portfolio and the Efficient Frontier also appear.

The above graph is conducive with the theoretical one with respect to its shape. By creating the random portfolios with a large enough sample size ( $n = 50,000$ ), the range of annualized returns and volatilities is known. To find the Efficient Frontier an optimization problem must be solved. To be more specific, we seek to maximize the Sharpe Ratio given a fixed annualized portfolio return. By solving numerous such optimization problems, points on the Efficient Frontier are obtained and graphed.

In order to find the optimal portfolio, we ought to solve the following optimization problem:

$$\begin{aligned}
\max \quad & \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_S = w_s \\
& \mathbf{1}^\top \vec{w}_B = w_b \\
& \mathbf{1}^\top \vec{w}_C = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \dots, n
\end{aligned}$$

where  $\vec{w}$  is the weight vector associated with the  $n$  assets,  $\vec{w}_S$ ,  $\vec{w}_B$  and  $\vec{w}_C$  are the weights associated with the stock, bond and commodity assets and sum up to  $w_s$ ,  $w_b$  and  $w_c$ , respectively. Obviously, the concatenation of these vectors is the original weight vector, i.e.  $\mathcal{S} \sqcup \mathcal{B} \sqcup \mathcal{C} = \{1, \dots, n\}$ . In other words, the objective corresponds to the Sharpe Ratio and the constraints construct the feasibility region of the optimization problem given the demands dictated by the suggested allocation resulting in the weights  $w_s$ ,  $w_b$  and  $w_c$ .

The Capital Allocation Line or CAL is the line that connects the Optimal Portfolio with the risk-free portfolio. These two points suffice in deriving the equation of the CAL  $\epsilon_{CAL}$ :

$$\left\{ \begin{array}{l} (0, r_f) \in \epsilon_{CAL} \\ (\sigma_{OPT}, r_{OPT}) \in \epsilon_{CAL} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{OPT} - r_f}{\sigma_{OPT}}}_{\text{max Sharpe ratio}} \cdot x + r_f \quad (7.1)$$

Thus, we are able to plot the CAL alongside the efficient frontier and the random portfolios in figure 10. Solving this optimization problem yields the following results:

#### TODO 2: EF results

$$\vec{w} = [\dots] \quad (7.2)$$

#### TODO 3: Observations

**Observations** talk about figure 10.

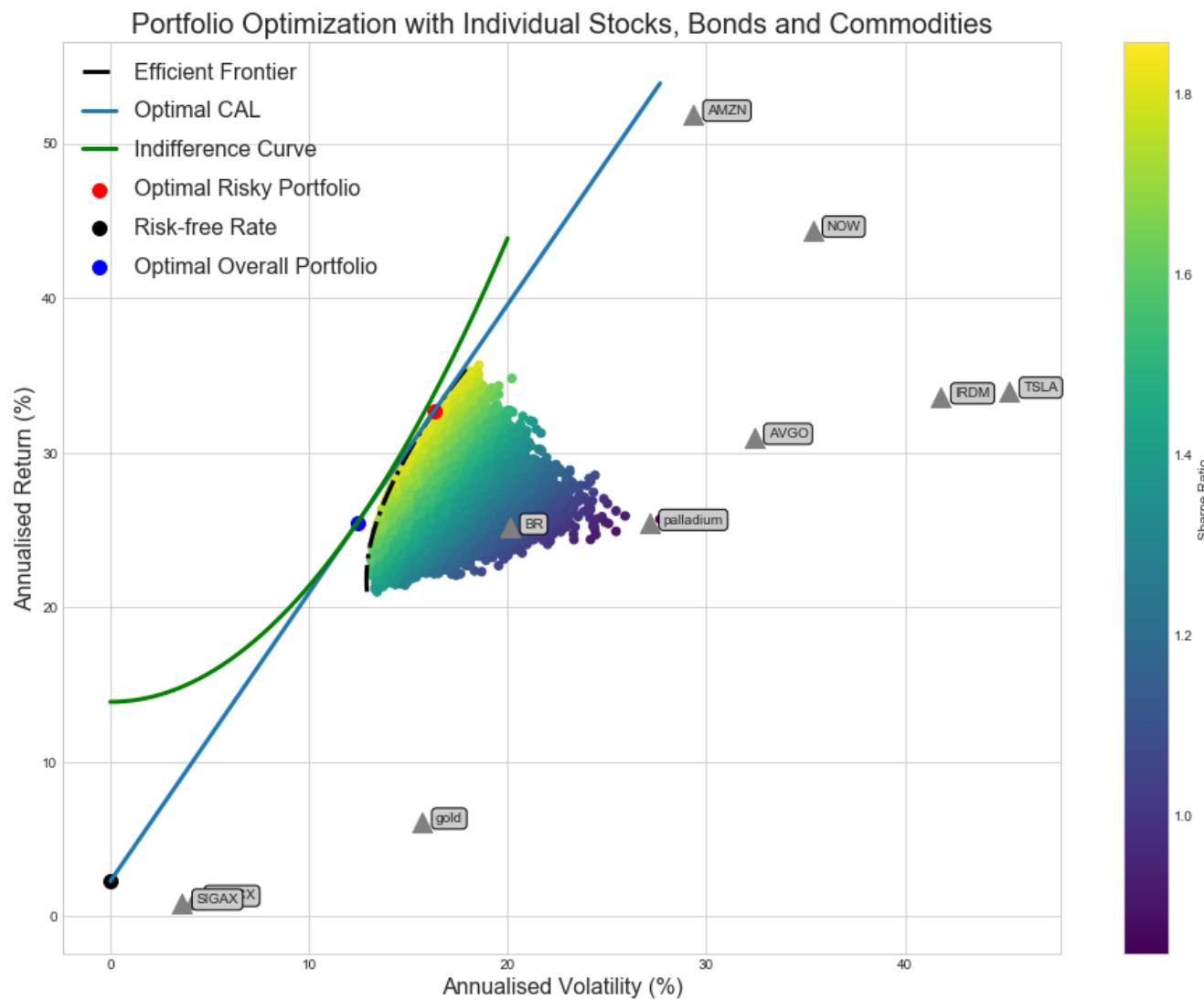


Figure 10: The CAL passes through the risk-free and risky portfolio. The points of the line between these values are a linear combination of the two alternatives. The individual assets are presented. The naive CAL is also depicted. The benefits of the optimization are apparent.

## 8 Question 8

Keep a track record of the macro- and microeconomic events that influenced your assets/portfolio and offer explanations for some of them, if you deem necessary. Your explanations should also be accompanied with some quantitative verification.

## 9 Question 9

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Measure and evaluate your overall portfolio's performance and compare it with the passive investment strategy. In this step, you should apply EXCEL's Solver to evaluate several possible outcomes (in terms of risk and return) and explain each outcome. In that endeavor, compute the various performance measures we have learned. Decide on the best outcome for you. Discuss.

## 10 Question 10

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Finally, perform a critical evaluation of the project. In other words, what did this project accomplish for you regarding the study and (this simple) application of investment theories and strategies? What would be the implications of constructing, managing and evaluating such a portfolio for your portfolio?

## A Questionnaire

<p>1/30/2020</p> <p>PERSONAL INVESTORS</p>  <p><b>Investor questionnaire</b></p> <p>1. I plan to begin taking money from my investments in ...</p> <ul style="list-style-type: none"> <li><input type="radio"/> 1 year or less</li> <li><input type="radio"/> 1 – 2 years</li> <li><input type="radio"/> 3 – 5 years</li> <li><input type="radio"/> 6 – 10 years</li> <li><input type="radio"/> 11 – 15 years</li> <li><input checked="" type="radio"/> More than 15 years</li> </ul> <p>2. As I withdraw money from these investments, I plan to spend it over a period of ...</p> <ul style="list-style-type: none"> <li><input checked="" type="radio"/> 2 years or less</li> <li><input type="radio"/> 3 – 5 years</li> <li><input type="radio"/> 6 – 10 years</li> <li><input type="radio"/> 11 – 15 years</li> <li><input type="radio"/> More than 15 years</li> </ul> <p>3. When making a long-term investment, I plan to keep the money invested for ...</p> <ul style="list-style-type: none"> <li><input type="radio"/> 1 – 2 years</li> <li><input type="radio"/> 3 – 4 years</li> <li><input type="radio"/> 5 – 6 years</li> <li><input type="radio"/> 7 – 8 years</li> <li><input checked="" type="radio"/> More than 8 years</li> </ul> <p>4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would ... (<i>If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.</i>)</p> <ul style="list-style-type: none"> <li><input type="radio"/> Sell all of the remaining investment.</li> <li><input type="radio"/> Sell a portion of the remaining investment.</li> <li><input checked="" type="radio"/> Hold onto the investment and sell nothing.</li> <li><input type="radio"/> Buy more of the investment.</li> </ul> <p>5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.</p> <ul style="list-style-type: none"> <li><input type="radio"/> Strongly disagree</li> <li><input checked="" type="radio"/> Disagree</li> <li><input type="radio"/> Somewhat agree</li> <li><input type="radio"/> Agree</li> <li><input type="radio"/> Strongly agree</li> </ul> <p>6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.</p> <ul style="list-style-type: none"> <li><input type="radio"/> Strongly disagree</li> <li><input type="radio"/> Disagree</li> <li><input type="radio"/> Somewhat agree</li> <li><input type="radio"/> Agree</li> <li><input type="radio"/> Strongly agree</li> </ul> <p>7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.</p> <ul style="list-style-type: none"> <li><input type="radio"/> Strongly disagree</li> <li><input type="radio"/> Disagree</li> <li><input checked="" type="radio"/> Somewhat agree</li> <li><input type="radio"/> Agree</li> <li><input type="radio"/> Strongly agree</li> </ul> <p>8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... (<i>If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.</i>)</p>	<p>Vanguard - Investor Questionnaire</p> <p><a href="https://personal.vanguard.com/us/FundsInvQuestionnaire">https://personal.vanguard.com/us/FundsInvQuestionnaire</a></p> <p style="text-align: right;">1/2</p>
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1/30/2020

Vanguard - Investor Questionnaire

Sell all of the remaining investment.  
 Sell a portion of the remaining investment.  
 Hold onto the investment and sell nothing.  
 Buy more of the investment.

9. The chart below shows the greatest 1-year loss and the highest 1-year gain on 3 different hypothetical investments of \$10,000.\* Given the potential gain or loss in any 1 year, I would invest my money in ...

Investment	Gain (\$)	Loss (\$)
A (loss of \$164, gain of \$593)	\$593	-\$164
B (loss of \$1,020, gain of \$1,921)	\$1,921	-\$1,020
C (loss of \$3,639, gain of \$4,229)	\$4,229	-\$3,639

\*The maximum gain or loss on an investment is impossible to predict. The ranges shown in the chart are hypothetical and are designed solely to gauge an investor's risk tolerance.

10. My current and future income sources (for example, salary, Social Security, pension) are ...

Very unstable  
 Unstable  
 Somewhat stable  
 Stable  
 Very stable

11. When it comes to investing in stock or bond mutual funds or ETFs—or individual stocks or bonds—I would describe myself as ...

Very inexperienced  
 Somewhat inexperienced  
 Somewhat experienced  
 Experienced  
 Very experienced

**My current asset allocation**

Enter the current asset allocation in whole numbers. Your percentages must total 100%. If you don't enter any percentages, the questionnaire will assume that 100% of your assets are in short-term reserves.

Short-term reserves	<input type="text" value="100.0"/> %
Bonds	<input type="text" value="0.0"/> %
Stocks	<input type="text" value="0.0"/> %

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<https://personal.vanguard.com/us/FundsInvQuestionnaire> 2/2

## B Questionnaire Results

1/30/2020

Vanguard - Investor questionnaire results

PERSONAL INVESTORS

Current Allocation

Suggested Allocation ("30% Bonds & 70% Stocks")

Be sure to jot these percentages down so you have them handy when you're selecting specific funds and completing your investments.

**Have questions about your allocation?**

[Compare your percentages with other allocation mixes](#)

[Review certain factors before reallocating your assets](#)

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file:///C:/Users/nikdim/Desktop/Vanguard - Investor questionnaire results.html

1/1

## References

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- [2] Yahoo Finance. URL: <https://finance.yahoo.com/>.
- [3] Morning Star. URL: <https://www.morningstar.com/>.
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- [6] Charles Schwab. URL: <https://www.schwab.com/>.
- [7] Vanguard advisors. URL: <https://advisors.vanguard.com/advisors-home>.
- [8] Morgan Stanley Investment Management. URL: <https://www.morganstanley.com/im/en-us/financial-advisor.html>.