# Portfolio Management
## FN 4329

Dimitriadis Nikolaos
Kyratzis Apostolos

American College of Greece

Spring Semester 2020

# Contents

# todo

## Todo

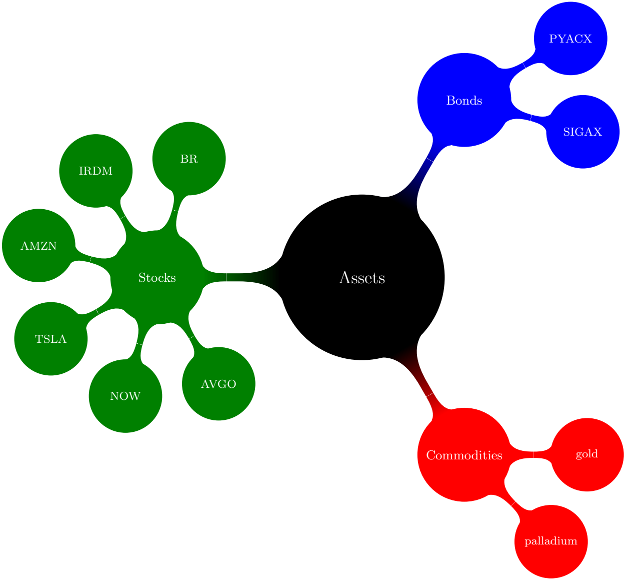
- fix contents (example: Intro, Theory, Results)

# Asset allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

# Contents

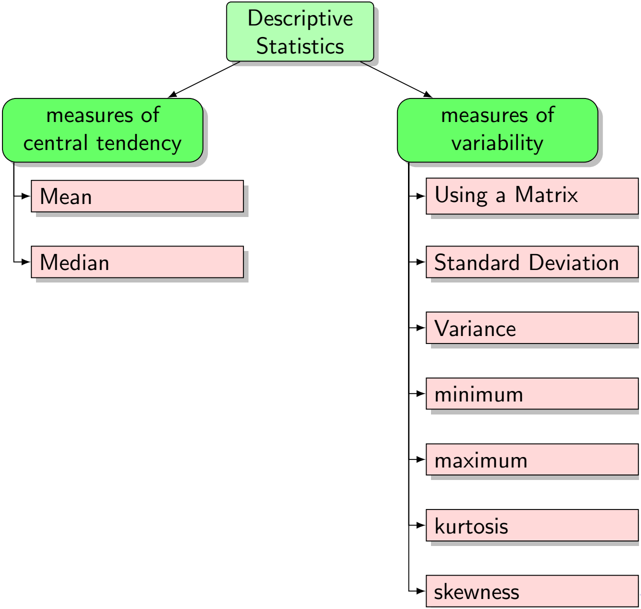
# Security Selection



# Contents

# Contents

# Descriptive statistics Taxonomy



# Measures of central tendency

## Mean

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

## Median

The median of data sample can be thought as the middle value of dataset, separating the higher from the lower half. It is a more robust measure than the mean, since it is not affected by outliers. The median can inform the investor on whether the returns are positive or negative on most time instances.

# Measures of Variability

## Standard Deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics $\implies$ risk in finance

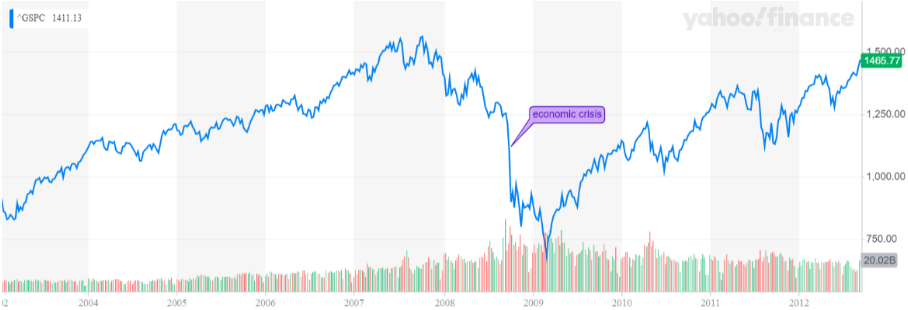



Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

# Measures of Variability
## Minimum & Maximum

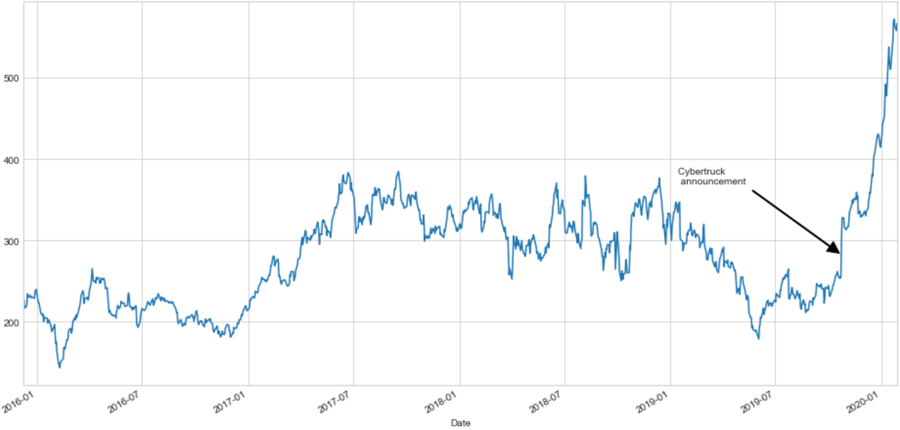



Figure: Tesla's announcement of the Cybertruck resulted in a steep price increase.

### Definition

$$\mathrm{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X-\bar{x}}{\sigma}\right)^4\right]$$

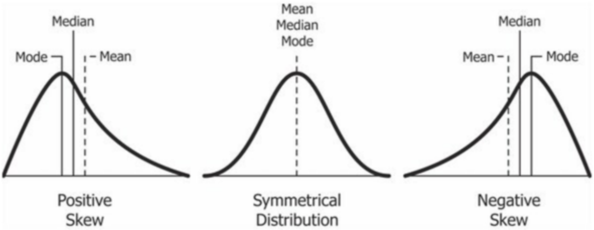
Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

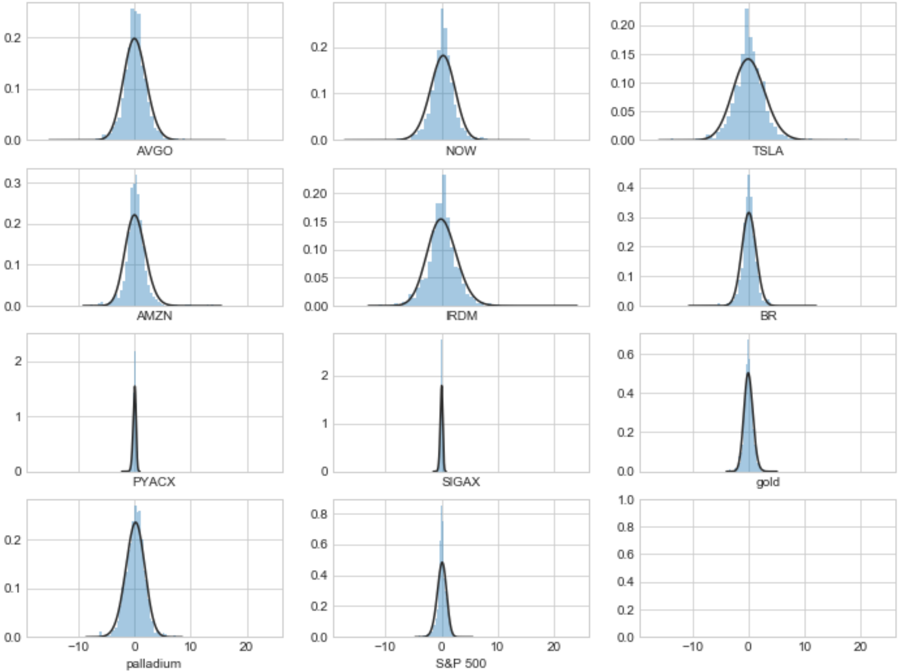
- high kurtosis $\rightarrow$ heavy tails **(outliers)**
- low kurtosis $\rightarrow$ no outliers

## Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.




0.0
0.1
0.2
AVGO
0.0
0.1
0.2
NOW
0.00
0.05
0.10
0.15
0.20
TSLA
0.0
0.1
0.2
0.3
AMZN
0.00
0.05
0.10
0.15
0.20
IRDM
0.0
0.1
0.2
0.3
0.4
BR
0
1
2
PYACX
0
1
2
SIGAX
0.0
0.2
0.4
0.6
gold
0.0
0.1
0.2
−10
0
10
20
palladium
0.0
0.2
0.4
0.6
0.8
−10
0
10
20
S&P 500
0.0
0.2
0.4
0.6
0.8
1.0
−10
0
10
20

# Contents

# Correlation and covariance

## Covariance Definition

Let $X$ and $Y$ be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

## Correlation Definition

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X, Y)}{\sigma_X \cdot \sigma_Y}$$

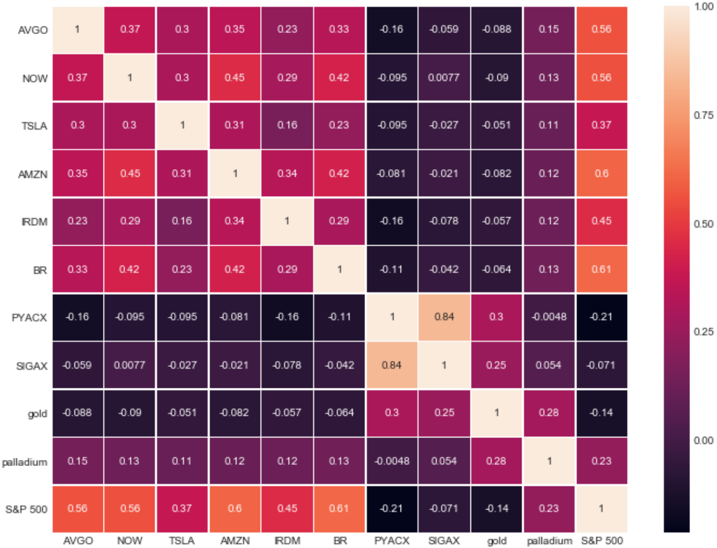
# A closer look at correlation

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

**todo**

add regression plots to show the difference

# Correlation Matrix



# Beta

## Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

# alpha

## Definition

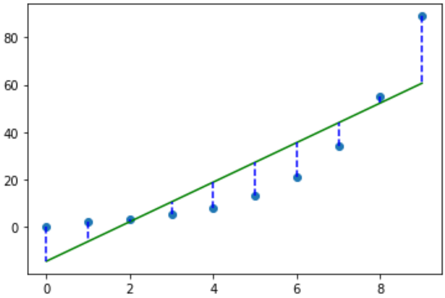
Alpha is the difference betwwen the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

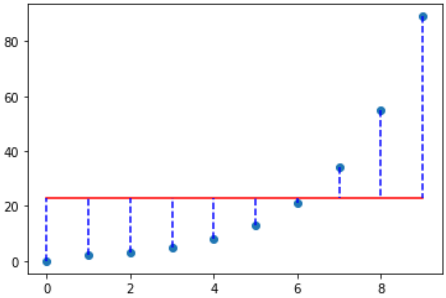
$$\overset{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}$$

# $R$-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$




(a) Explained Variation




(b) Total Variation

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- $R_p$ = return of mutual fund
- $R_f$ = risk-free rate
- $\sigma_p$ = standard variation of the portfolio's excess return

# Contents

# Risk and return

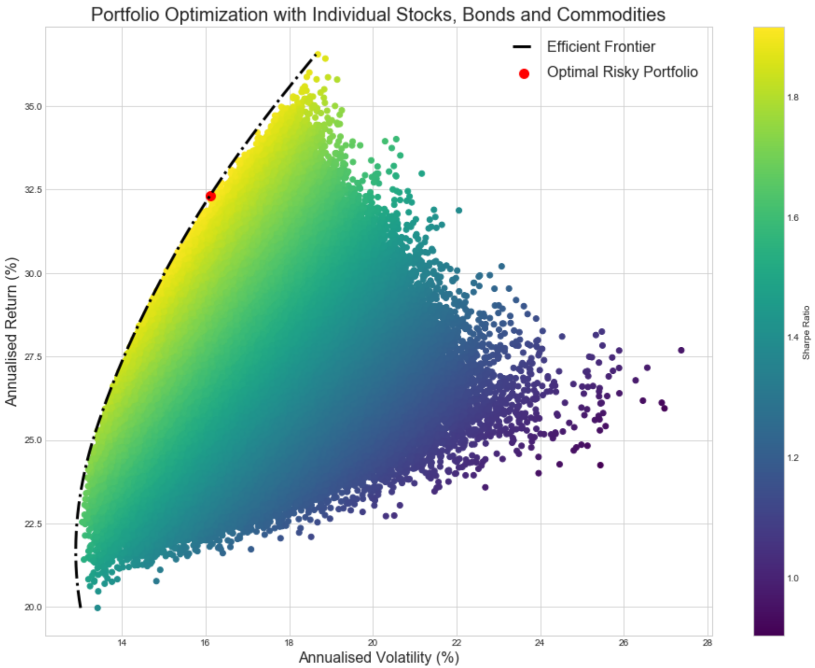
## Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

## Risk

$$\begin{aligned}
\sigma_p &= \sqrt{\vec{w}^\top K \vec{w}} \\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}
\end{aligned}$$

# Contents


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal Risky Portfolio
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
35.0
32.5
30.0
27.5
25.0
22.5
20.0
14
16
18
20
22
24
26
28
1.8
1.6
1.4
1.2
1.0

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Sharpe Ratio

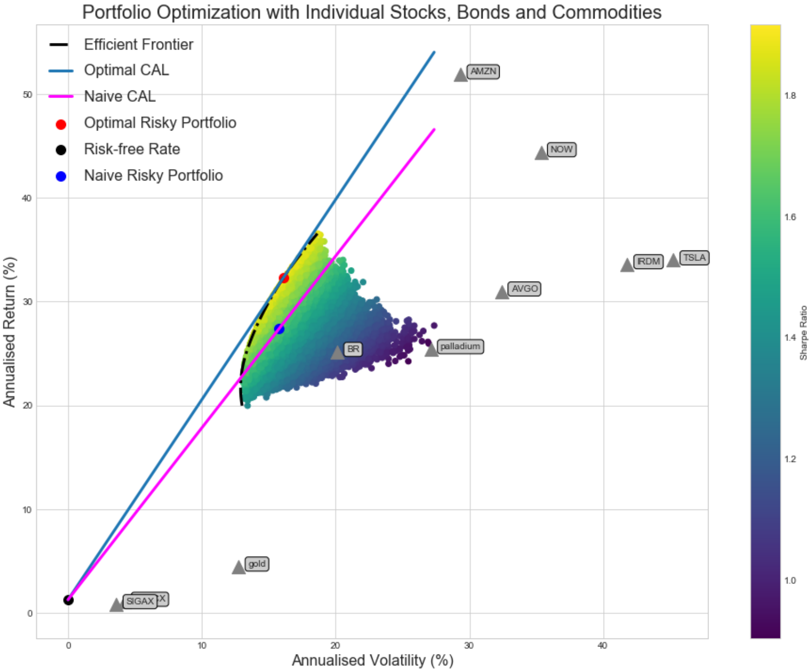
$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \dots, n
\end{aligned}
$$

# Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\mathrm{CAL}} \\ (\sigma_{\mathrm{OPT}}, r_{\mathrm{OPT}}) \in \epsilon_{\mathrm{CAL}} \end{array} \right\}$$

# Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{OPT}} - r_f}{\sigma_{\text{OPT}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal CAL
Naive CAL
Optimal Risky Portfolio
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

# References


Nikiforos Laopodis.
*Understanding investments: Theories and strategies.*
Routledge, 2012.

Yahoo finance.

Morning star.

Us news money.

Fidelity.

Charles schwab.

Vanguard advisors.

Morgan stanley investment management.

# The end!