Portfolio Management

FN 4329

Dimitriadis Nikolaos   Kyratzis Apostolos

American College of Greece

Spring Semester 2020

Contents

todo

Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

Questionnaire

PERSONAL INVESTORS

Vanguard

Investor questionnaire

1. I plan to begin taking money from my investments in ...

- [ ] 1 year or less
- [ ] 1 – 2 years
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [x] More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- [x] 2 years or less
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [ ] More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- [ ] 1 – 2 years
- [ ] 3 – 4 years
- [ ] 5 – 6 years
- [ ] 7 – 8 years
- [x] More than 8 years

4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would ... *(If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)*

- [ ] Sell all of the remaining investment.
- [ ] Sell a portion of the remaining investment.
- [x] Hold onto the investment and sell nothing.
- [ ] Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- [ ] Strongly disagree
- [x] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

- [x] Strongly disagree
- [ ] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

- [ ] Strongly disagree
- [ ] Disagree
- [x] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... *(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)*

Questionnaire

1/30/2020 Vanguard - Investor Questionnaire

PERSONAL INVESTORS

Vanguard

Investor questionnaire

1. I plan to begin taking money from my investments in ...

- [ ] 1 year or less
- [ ] 1 – 2 years
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [x] More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- [x] 2 years or less
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [ ] More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- [ ] 1 – 2 years
- [ ] 3 – 4 years
- [ ] 5 – 6 years
- [ ] 7 – 8 years
- [x] More than 8 years

4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would ... *(If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)*

- [ ] Sell all of the remaining investment.
- [ ] Sell a portion of the remaining investment.
- [x] Hold onto the investment and sell nothing.
- [ ] Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- [ ] Strongly disagree
- [x] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

- [x] Strongly disagree
- [ ] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

- [ ] Strongly disagree
- [ ] Disagree
- [x] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

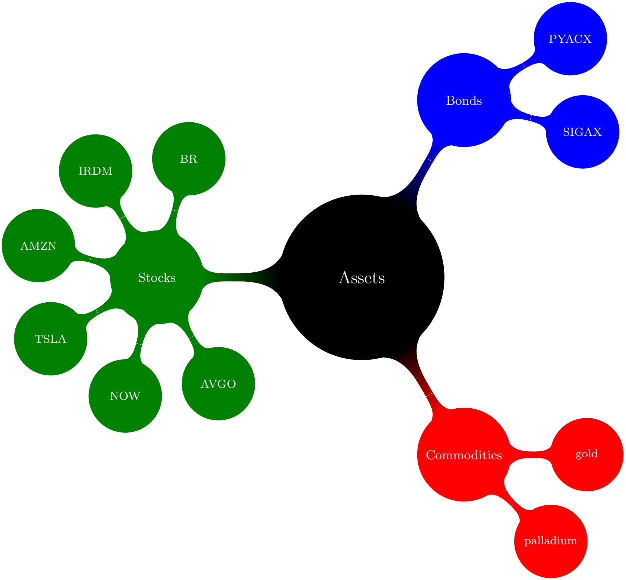
8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... *(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)*

https://personal.vanguard.com/us/FundsInvQuestionnaire 1/2

Asset Allocation

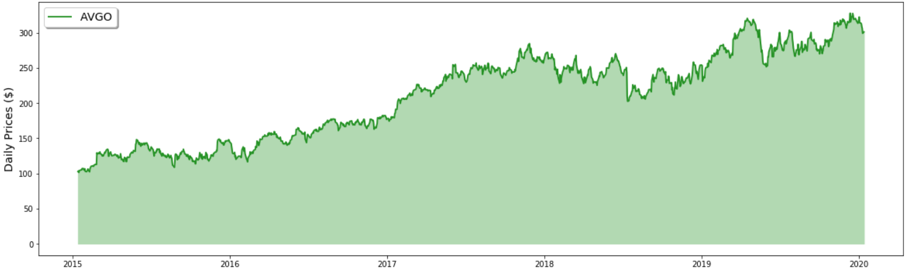
- 70% Stocks
- 20% Bonds
- 10% Commodities

Security Selection



add - DO THIS FOR EVERY SECURITY

- category
- few words
- price chart



Stocks

NOW - ServiceNow Inc.

Stocks

TSLA - Tesla Inc.

Stocks

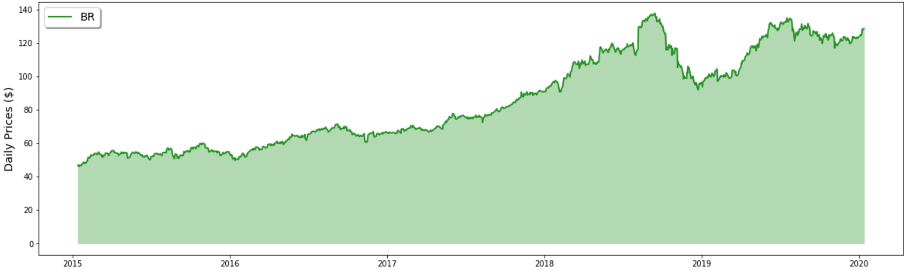
AMZN - Amazon Inc.

Stocks

IRDM - Iridium Communications Inc.

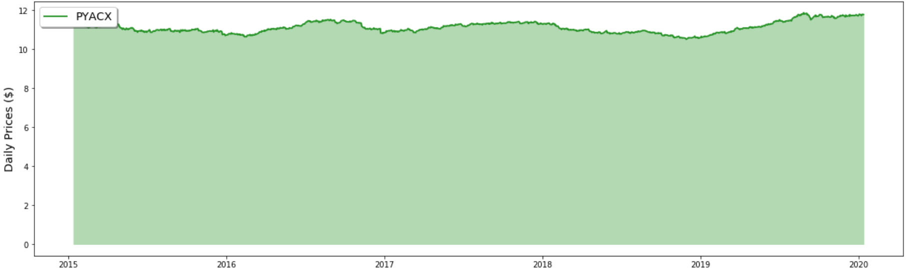
Stocks

BR - Broadridge Financial Solutions Inc.



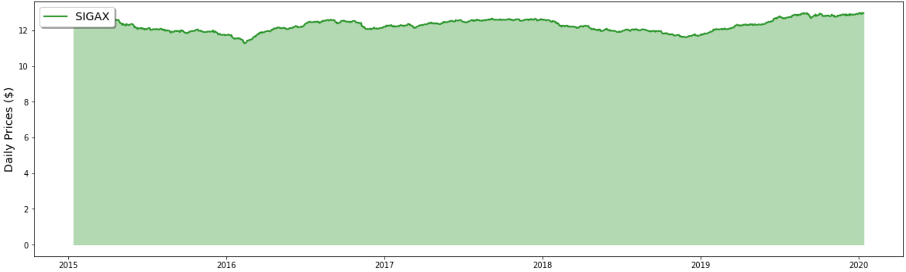
Bonds

PYACX - Payden Corporate Bond Mutual Fund



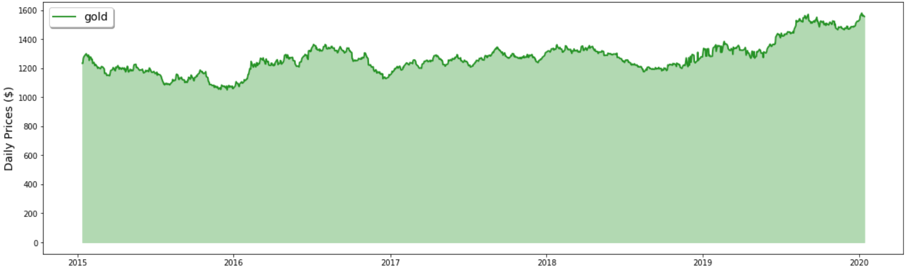
Bonds

SIGAX - Western Asset Corporate Bond Mutual Fund



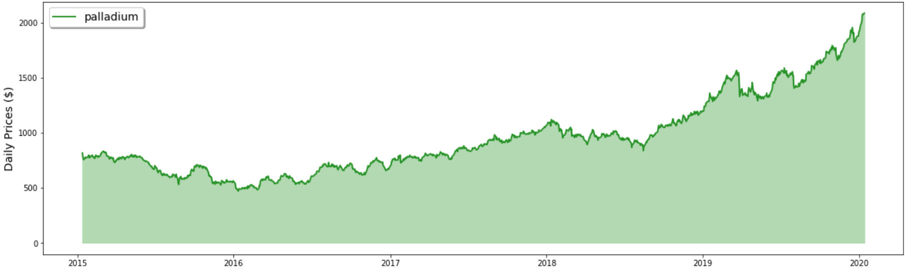
Commodities

gold



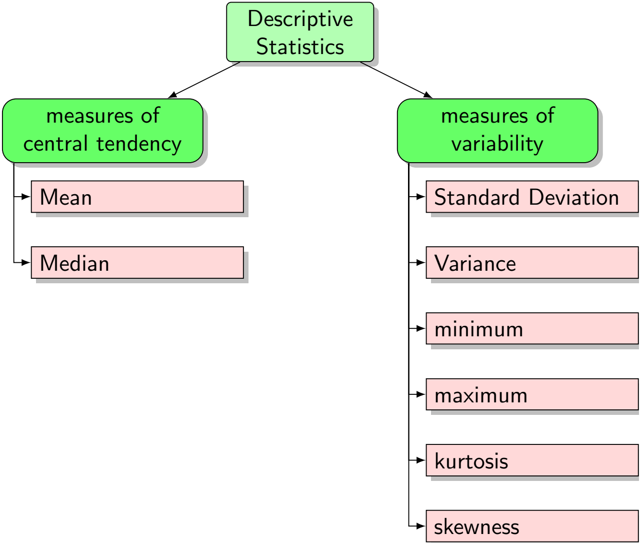
Commodities

palladium



Contents

Descriptive statistics Taxonomy



Mean

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- i.e. the middle value

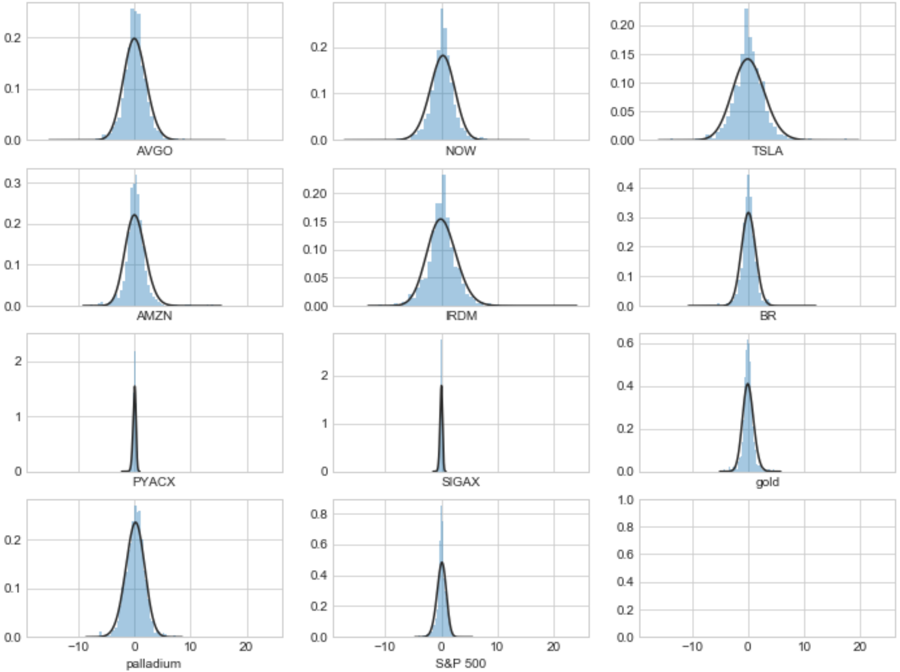
- i.e. the middle value
- Why?

- i.e. the middle value
- Why? robust w.r.t. outliers

- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics $\implies$ risk in finance


AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500

Measures of Variability
Minimum & Maximum

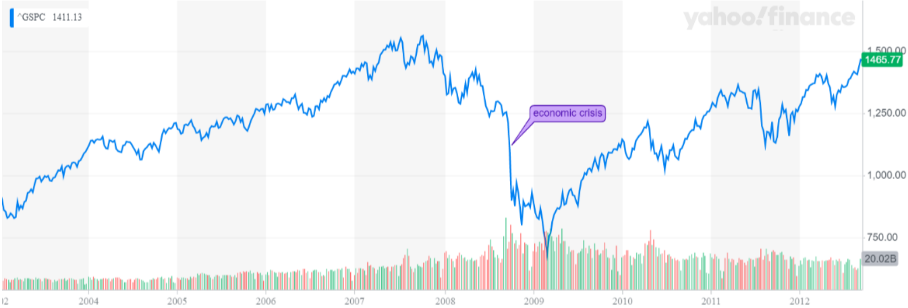
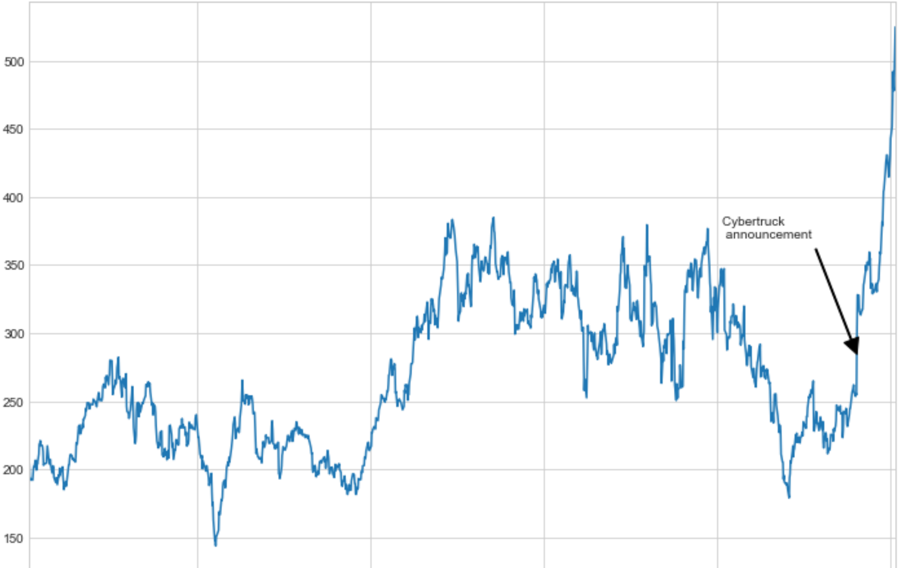


Figure: The minimum of the S&P500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

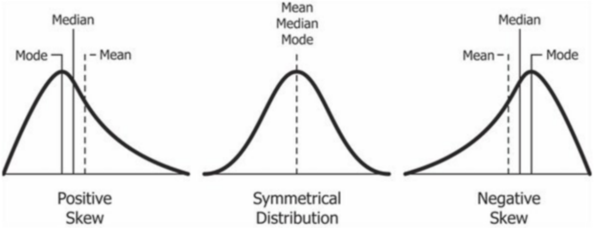
Minimum & Maximum



Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



Definition

$$\mathrm{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^4\right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis $\rightarrow$ heavy tails **(outliers)**
- low kurtosis $\rightarrow$ no outliers

Moments from an investor's perspective

- $\tilde{\mu}_2$ standard deviation σ
- $\tilde{\mu}_3$ skewness
- $\tilde{\mu}_4$ kurtosis

TODO

add what each moments means to understanding security performance

Covariance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

Covariance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

- not so helpful!

Covariance

Covariance Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X,Y) = \mathbb{E}[(X-\bar{x})(Y-\bar{y})]$$

- not so helpful! $\rightarrow$ correlation

Correlation

Definition

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

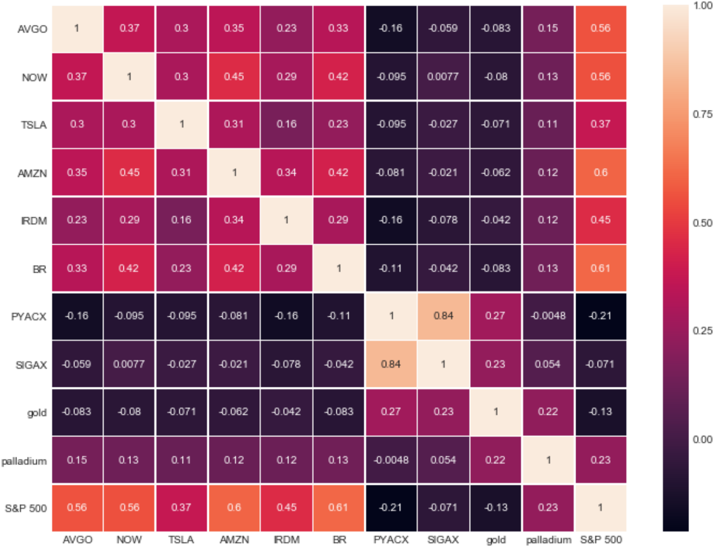
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1,1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation

todo

add regression plots to show the difference

Correlation Matrix



Beta

Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\mathrm{cov}(R_e, R_m)}{\mathrm{var}(R_m)}$$

Definition

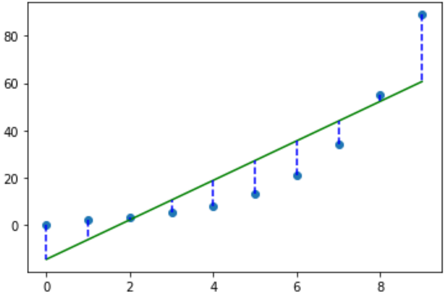
Alpha is the difference between the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

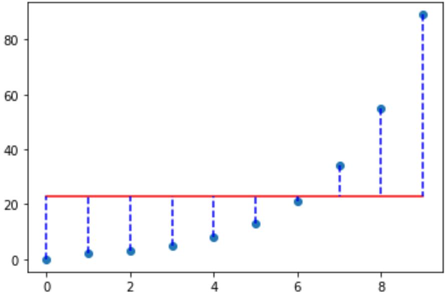
$$\overset{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{R_f + \beta(\mathbb{E}(R_m) - R_f)\right\}$$

R-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation



(b) Total Variation

Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where

- R_p = return of mutual fund
- R_f = risk-free rate
- σ_p = standard variation of the portfolio's excess return

Contents

Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk

$$\begin{aligned}
\sigma_p &= \sqrt{\vec{w}^\top K \vec{w}} \\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}
\end{aligned}$$

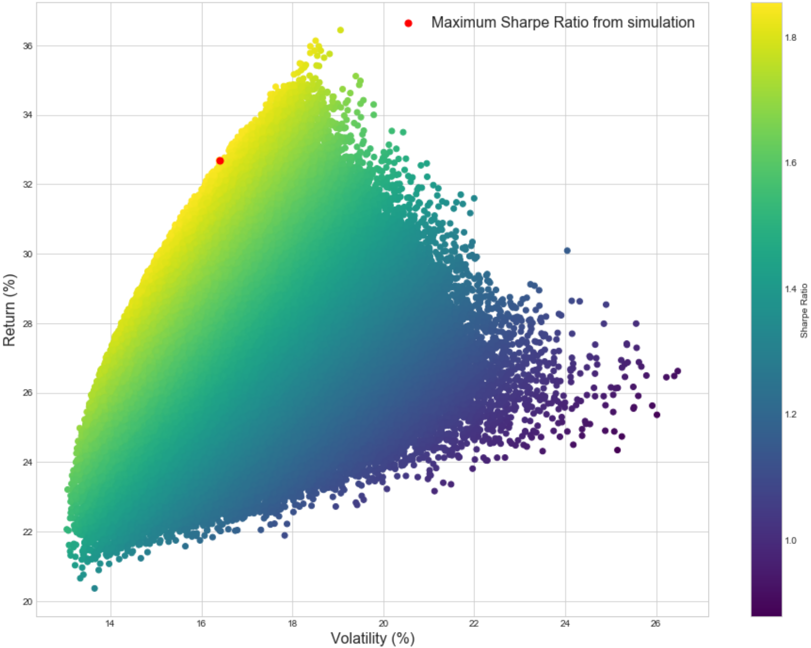
Naive weight allocation

Remember!

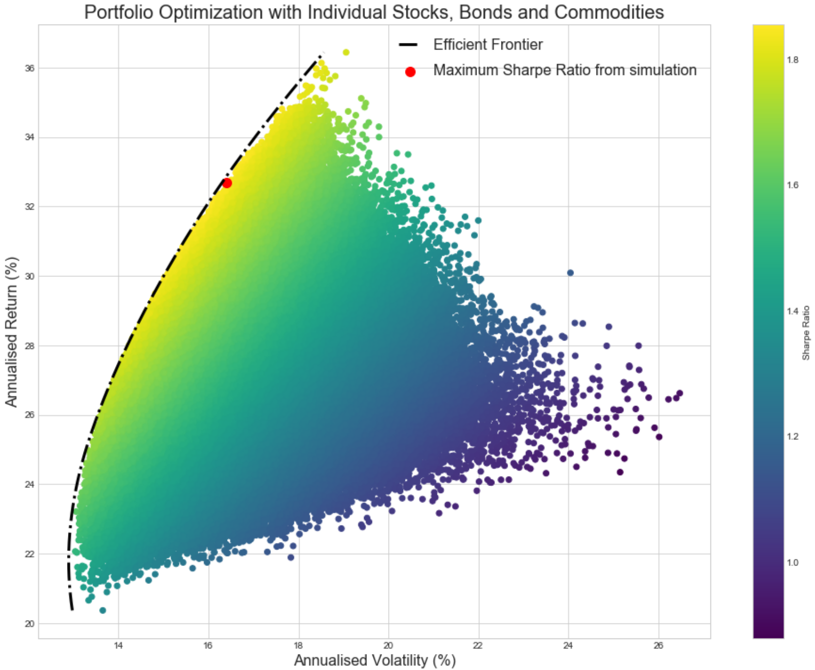
- 70% Stocks
- 20% Bonds
- 10% Commodities

What about $\vec{w}$?

- 70% for 6 Stocks $\rightarrow \dfrac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow$ 10% each
- 10% for 2 Commodities $\rightarrow$ 5% each



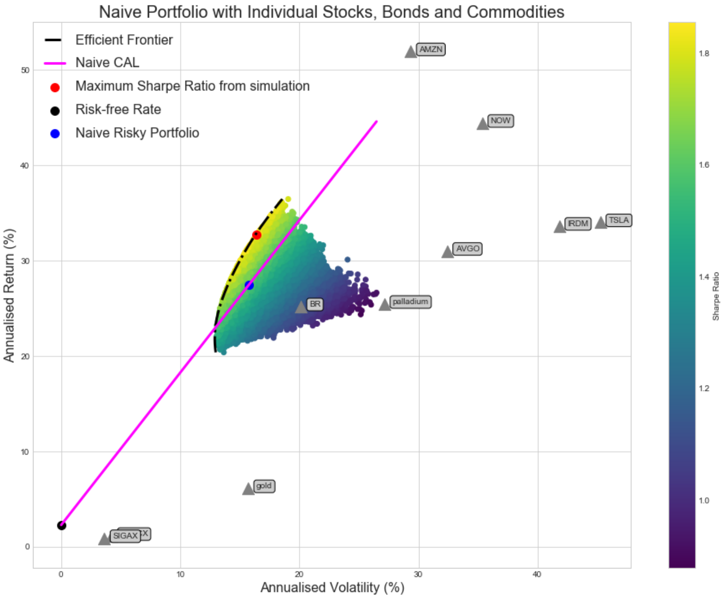
Maximum Sharpe Ratio from simulation
Return (%)
Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Maximum Sharpe Ratio from simulation
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0

CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \implies \epsilon_{naive} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$

Naive CAL



Naive Portfolio with Individual Stocks, Bonds and Commodities
Efficient Frontier
Naive CAL
Maximum Sharpe Ratio from simulation
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
0
10
20
30
40
50
Sharpe Ratio
1.0
1.2
1.4
1.6
1.8

But...

We can do better!

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Sharpe Ratio

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Optimization yields...

optimal weights

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

	Optimal Risky Portfolio
Return (%)	32.7164
Risk (%)	16.3185
Sharpe Ratio	1.8658
Beta (β)	0.9067

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \implies \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{OPT}} - r_f}{\sigma_{\text{OPT}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t. z:

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

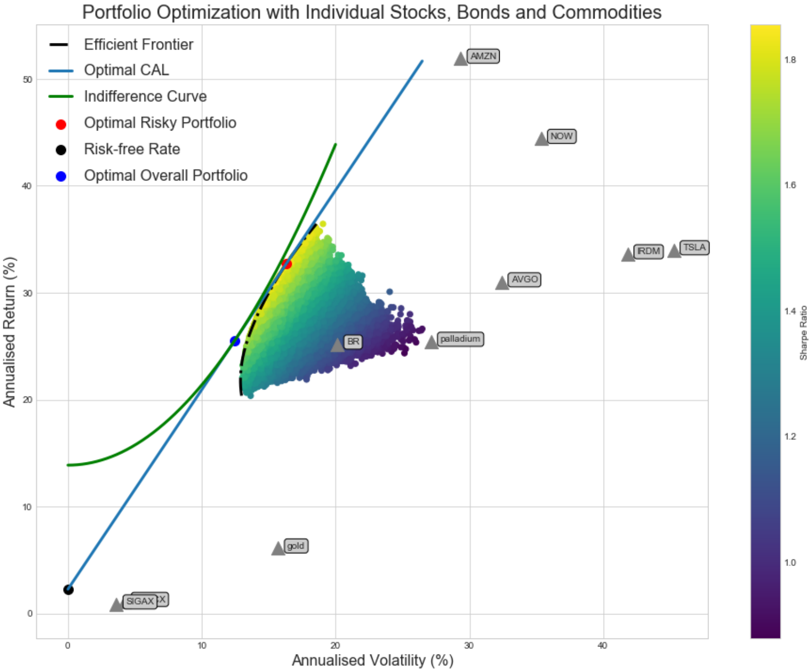
$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t. z:

$$\frac{\partial U}{\partial z} = 0$$

$$\implies r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\implies z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal CAL
Indifference Curve
Optimal Risky Portfolio
Risk-free Rate
Optimal Overall Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

Past Performance (1/13/2015 - 1/13/2020)

	Optimal Overall Portfolio	S&P 500
Return (%)	25.4769	11.3118
Risk (%)	12.4384	13.4138
Sharpe Ratio	1.8658	0.6741
Beta (β)	0.6911	1.0
Alpha (α) (%)	16.9584	0.0

References

Nikiforos Laopodis.
Understanding investments: Theories and strategies.
Routledge, 2012.

Yahoo finance.

Morning star.

Us news money.

Fidelity.

Charles schwab.

Vanguard advisors.

Morgan stanley investment management.

The end!