FN 4329: INVESTMENTS AND PORTFOLIO MANAGEMENT

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Contents

1 Question 1

Decide on the allocation of your budget to each of the above instruments. Rationalize your choice based on your own investment philosophy. You should fill out a risk tolerance questionnaire and supply both the questionnaire and its suggested allocation (in the project's Appendix). In your discussion, please explain the choice of your investment vehicles and their significance in your portfolio. Label this as your investment policy statement (IPS).

Investment Policy Statement We are classified as young investors, with a small initial capital available. A summary of our investment philosophy is as follows:

Objectives

- Long-term growth and capital appreciation
- Risk-profile: Aggressive
- Time horizon: Greater than 10 years
- Short-term liquidity needs: Minimal

Portfolio Selection Guidelines Long-term investment performance is generally determined by the underlying asset's performance. From a historical point of view, stock assets tend to achieve higher rates of return, along with greater volatility. Fixed income assets (i.e bonds) yield lower rates of return and bear less risk.

Based on our risk tolerance profile (see Appendix) and aim for long-term capital growth, the portfolio asset allocation will be:

- 70% Stocks
- 20% Bonds
- 10% Commodities

The selected assets are traded on US exchanges. The individual composition of holdings will be selected from the following asset classes:

Equity Equities are the main investment instrument of our portfolio, which will implement the required capital growth. A total of 6 stocks are selected, with the following characteristics:

- Company Size: 4 Large, 1 Mid, 1 Small
- Stock Type: 4 Growth, 1 Aggressive Growth, 1 High-Dividend

Bonds Fixed income assets are of secondary importance, due to our minimal needs for generation of current income. Instead of two individual corporate bonds, 2 bond mutual funds are selected, which invest at least 85% of their assets in corporate bonds.

Commodities For a portfolio that has a long-term horizon, allocating a small amount to commodities offers some protection against inflation and reduces the overall portfolio risk. Commodities historically move opposite from the market and therefore yield a negative correlation regarding other selected securities. This fact, as explained in 5.1, offers risk diversification benefits. A total of 2 precious metals are selected.

A more elaborate analysis of the final selected assets is performed in Section 2.

Risk-free Rate The 10-Year U.S. Treasury Bond is chosen as the risk-free asset, as it matches the time horizon of our investment.

Benchmark Index The S&P 500 index is used as the benchmark index of the portfolio.

2 Question 2

Outline your investment objectives, constraints and strategies and discuss the relative importance of risk and return in your investment decision making.

Objectives and Constraints The portfolio's main objective is long-term growth and capital appreciation, which is defined as a rise in an investment's market price. Given our young age, the limited initial capital available is the main constrain. Because of our risk-tolerance profile, we do not explicitly specify a loss limit. As a general rule, if a holding asset presents a one-year loss of its value greater that 20%, then it should be considered for exclusion from the portfolio.

Investment Strategy The main investment vehicles are stocks highlighted for their growth potential. We mainly focus on the technology sector or sectors that directly benefit from advancements in technology and innovations. A small amount is also allocated to corporate bond funds of medium credit quality (A and/or BBB) and to precious metals, for diversification purposes.

The final selected assets from each category are presented and analysed below:

2.1 Stocks

A total of six stocks are selected for the portfolio:

Broadcom Inc. (Large/High-Dividend) Broadcom Inc. is a global leader in designing and developing semiconductor and software infrastructure solutions. Broadcom's product portfolio serves data centres, networking & telecom hardware and high-end smartphone components. In January 2020, Broadcom announced a $15 billion deal with Apple to supply upcoming iPhone products with wireless chips. With new 5G technology well under-way towards global scale deployment, Broadcom will gain the opportunity to expand its business and develop 5G compatible components. Even though currently is categorised as a "value" type stock, we believe it has a good growth potential in the future.

ServiceNow Inc. (Large/Growth) ServiceNow Inc. is a provider of cloud-based software solutions to structure, manage and automate business processes for global enterprises. The company delivers services based on a Software as a Service (SaaS) model, and its main focus is IT service management. However, more recently it expanded the available workflow solutions to other departments, such as Customer Support, Human Resources and Security Operations. It targets a wide range of industries including financial services, healthcare, government and education.

Tesla Inc. (Large/Growth) Tesla Inc. is a leading auto manufacturer company for next-generation electric vehicles, while also a designer and operator of sustainable energy systems. Tesla is a global leader in electric mobility and will benefit from the increasing global social responsibility regarding fuel emissions. At the moment Tesla operates 3 Gigafactories worldwide, while also constructing a forth one in Berlin. It is expected to begin operations in July 2021 and will enable Tesla to establish a stronger presence in Europe. Global vehicle deliveries were 367,656 units in 2019 and we expect this figure to grow in the upcoming years, as more advancements in battery technology will allow electric vehicles to increase their autonomy and become a wider commercial product.

Amazon Inc. (Large/Growth) Amazon Inc. is one of the biggest online retailers, recently reaching the $1 trillion market cap for the second time in the last 3 years. Its core business are online product and digital media sales, however we believe a higher potential lies in its Amazon Web Services Department. AWS offers solutions for machine learning, big data, IoT, cloud-computing and storage application. All of these technological fields are expected to greatly advance in the upcoming years and therefore further boosting the company growth.

Iridium Communications Inc. (Small/Growth) Iridium Communications Inc. is a leader in providing global voice and data services with satellite communications. It operates and maintains a vast network with over 70 satellites in orbit and related ground infrastructure, achieving true global coverage. In 2018 a partnership was announced with Amazon Web Services, aiming to develop a satellite network for future IoT applications. The company's commercial end user base includes business and organisations of various sectors, including maritime, aviation, government & military, emergency services, oil & gas, mining and transportation.

Broadridge Financial Solutions Inc. (Mid/Aggressive Growth) Broadridge Financial Solutions Inc. is a provider of communications and technology-driven solutions to investors, banks, brokerage offices, mutual funds and corporate issuers. The company delivers a range of solutions, including communication platforms, securities processing and data analytics that aid its clients in improving their quality of customer services, along the whole life cycle of an investment.

2.2 Bonds

A total of two corporate bond mutual funds are selected for the portfolio:

Payden Corporate Bond Mutual Fund The fund invests in a variety of debt instruments and income generation securities. It invests at least 85% of its assets in corporate bonds of mainly A and BBB credit rating. It may also use derivative products, such as CDS in order to gain exposure to the corporate debt market.

Western Asset Corporate Bond Mutual Fund The fund invests at least 80% of its assets in corporate debt securities, while 10% of the assets might be non-U.S dollar nominated fixed income securities of foreign issuers. The majority of holding assets are of A and BBB credit rating.

2.3 Commodities

A total of two precious metals are selected as commodities:

Gold After reaching historical lows in 2015, gold has since then seen a significant rise in value. The precious metal is traditionally considered a safe investment during rough economic times and a hedge against inflation. Recently investors are turning to gold as safe houses instead of government bonds, as the latter had negative inflation-adjusted returns in 2019. Due to this fact investors have shifted their focus to gold, because even though is does not generate any return, it also does not have any holding costs and thus is more favourable for the time being.

Palladium Palladium is now more expensive than the four major precious metals, as its value has almost doubled over the last year. Palladium's main commercial use is in catalytic converters of car's exhaust systems, that help turn toxic gas emissions into less harmful ones. This impressive rise in price can be solely explained by the fact that demand outstrips supply. Palladium is a secondary product from mining operations of other metals, such as platinum or nickel and thus miners have less control over the extracted quantities in response to prices rising. Palladium production is projected to fall short of demand for the 8^{th} straight year and therefore the metal is expected to maintain its current market value.

2.4 Overview

The portfolio aims in capital appreciation and focuses on assets, such as stocks, that traditionally are more volatile and thus bear more risk. However they may also generate higher returns compared to less risky assets, hence their selection as the main investment instrument of the present portfolio.

3 Question 3

Provide a preliminary discussion of the effects of the short investment horizon in your decision(s) and diversification strategy

The investor's time horizon plays a crucial role in choosing the instruments of a portfolio. As stated previously, our main objective is long-term capital appreciation and the primary selected asset are stocks. Stocks are widely considered as risky assets, but they also tend to generate higher returns over long periods of time compared to other less risky assets, such as bonds.

If an investor wants to make an investment of short-term horizon, then stocks probably should not be the preferred asset, as their prices usually have substantial fluctuations over short periods of time, affected by temporary financial, social as well as geopolitical news and events. Therefore the desired return might not be realised, leaving the investor short on his target. Bonds should be a more favourable choice in this occasion, as they are less risky (in terms of variability) and provide a relatively steady generation of income over the short period.

However, this does not mean that a long-term portfolio should not include bonds. One of their important roles is that they reduce overall volatility, as their returns present very low or negative correlation values with stock returns. Additionally, they offer some protection against any emergency short-term capital or liquidity needs that may arise, in the form of regular interest payments.

In conclusion, our diversification strategy can be summarized as follows:

- Even though we focused on growth style stocks, our choices included companies of different market capitalizations (Large, Mid, Small) and industry sectors (i.e Semiconductors, Communication Services, Online Retail and Cloud Services).
- Two corporate bond mutual funds were included in the portfolio, for the reasons analysed above.
- Two commodities were also included, as a hedge against inflation and for risk reduction, since they exhibit low or negative correlation values with stock assets.

4 Question 4

Compute the descriptive statistics for each instrument. Explain each metric you computed from the perspective of the investor. Provide graphs as well.

Descriptive statistics are brief descriptive coefficients that summarize a given data set. In our case, this dataset is daily returns of each instrument, over the last 5 years. These coefficients enable the reader to understand the features of the dataset and derive quantitative insights on its nature.

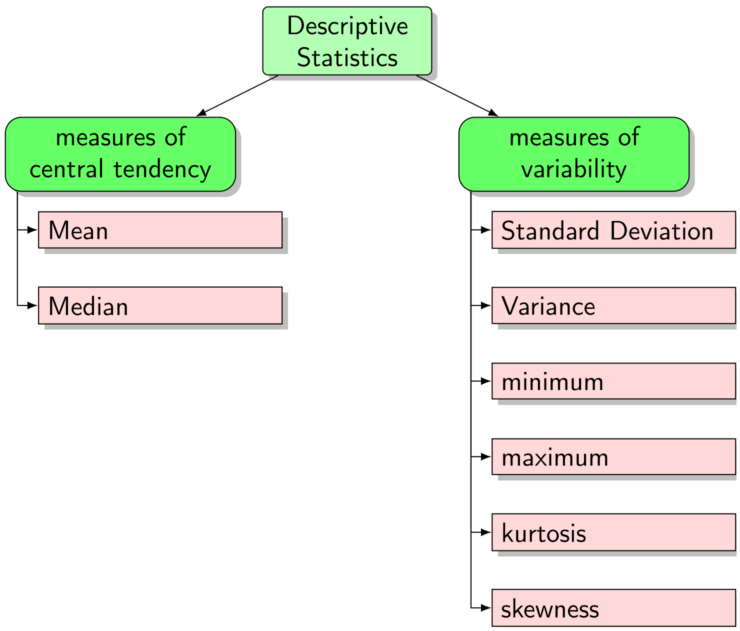


Figure 1: Taxonomy of descriptive statistics

Let $x_i, i \in [n]$ be the sample, i.e. the returns of an instrument.

4.1 Measures of central tendency

Measures of central tendency describe the centre position of a distribution for a dataset.

Mean The mean, denoted as $\bar{x}$ is the sum of the sampled values divided by the number of items in the sample:

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n} \tag{4.1}$$

From the investor's perspective, the mean describes the average performance of the instrument. If $\bar{x} > 0$ then the instrument increases in value on average.

Median The median of data sample can be thought as the middle value of dataset, separating the higher from the lower half. It is a more robust measure than the mean, since it is not affected by outliers. The median can inform the investor on whether the returns are positive or negative on most time instances.

4.2 Measures of Variability

Measures of variability describe how the data is distributed within the set.

Standard Deviation The Standard deviation expresses the variability of a population. It indicates the extent to which the values tend to be close to the mean. It is commonly used to measure confidence in statistics. For this reason, it is a measure of risk in economic terms. It is given by the following formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{4.2}$$

From the perspective of the investor, an instrument with high standard deviation carries more risk and should have higher returns to accommodate for this variability.

Minimum & Maximum Minimum and maximum describe the range of values. In economic terms, the maximum and minimum of returns reflect events that shape the price of an instrument. An example would be the economic crisis of 2008, or more recently the announcement of Tesla's (`TSLA`) new vehicle:

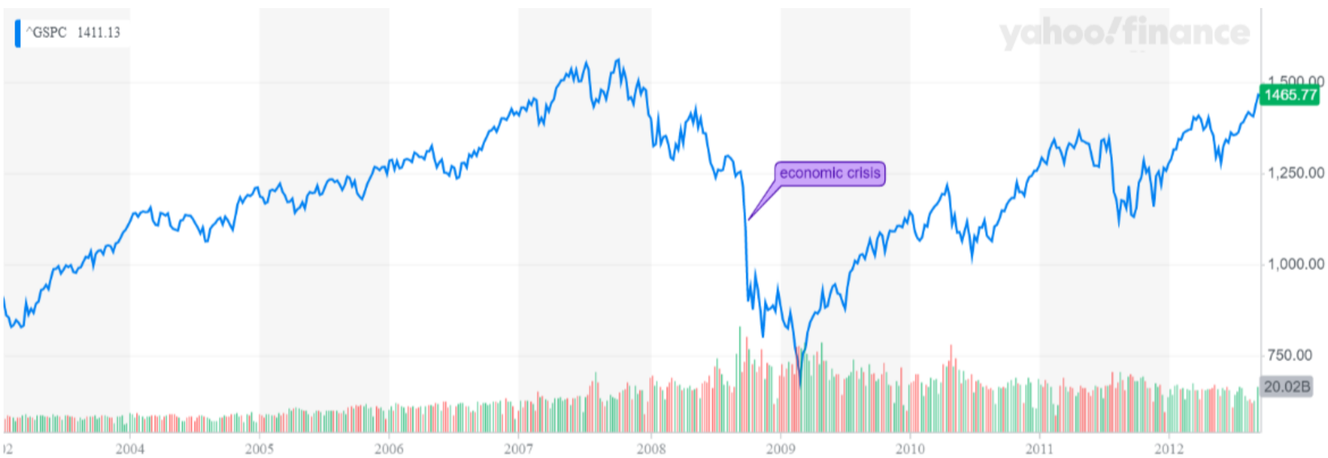


Figure 2: The minimum of the `S&P500` returns would occur on the day of the economic crisis for this period.

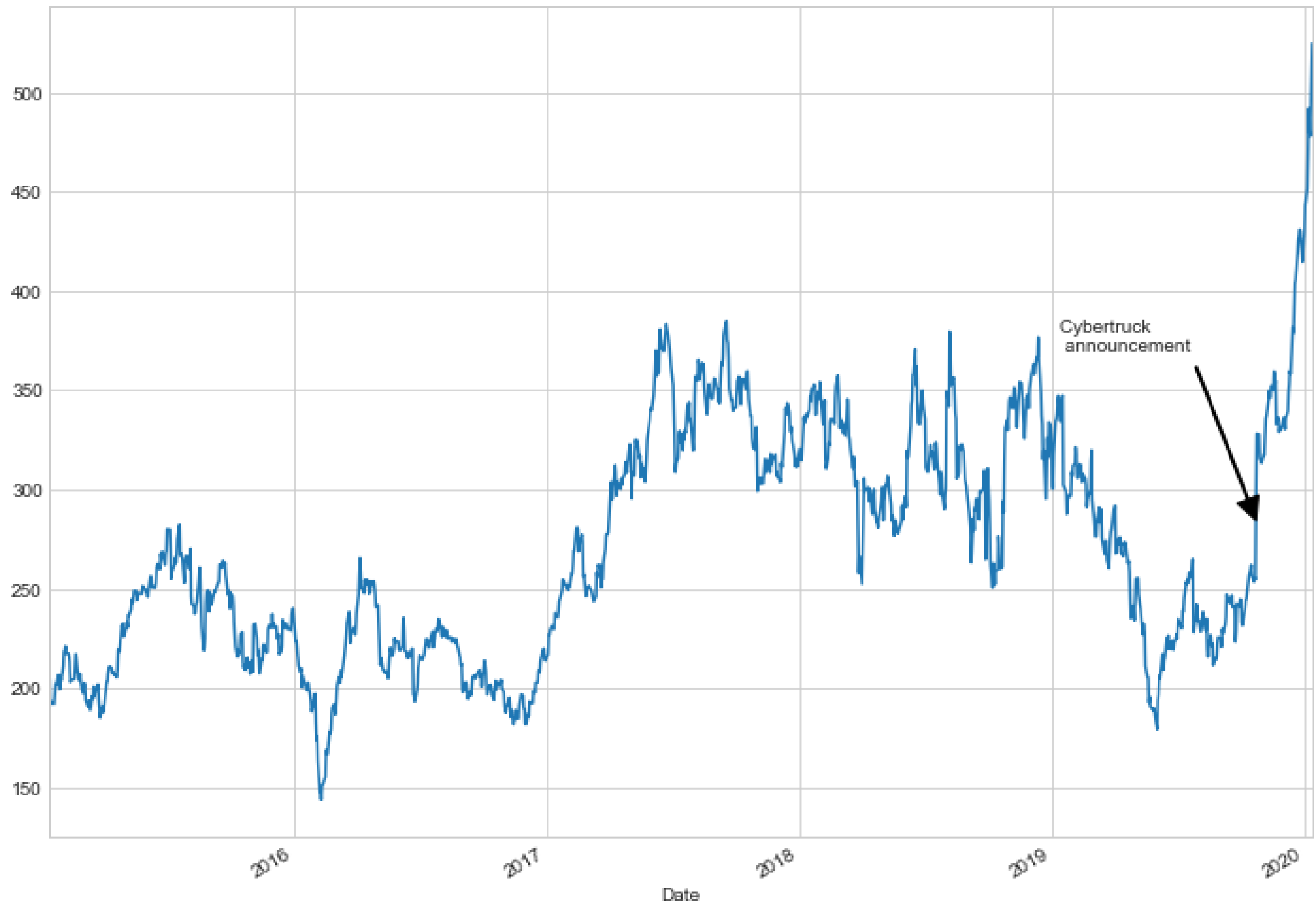


Figure 3: Tesla's announcement of the Cybertruck resulted in a steep price increase.

Kurtosis Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution. Data sets with high kurtosis tend to have heavy tails, or outliers, whilst data sets with low kurtosis lack outliers. The kurtosis is given by the following formula:

$$\mathrm{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X-\bar{x}}{\sigma}\right)^4\right] \tag{4.3}$$

For investors, high kurtosis of the returns distribution implies that the investor might experience occasional extreme returns.

Skewness Skewness is a measure of asymmetry. To be more precise, this coefficient indicates whether the tail of the distribution is on the left or the right of the mean/median value. Let us consider a simple example:

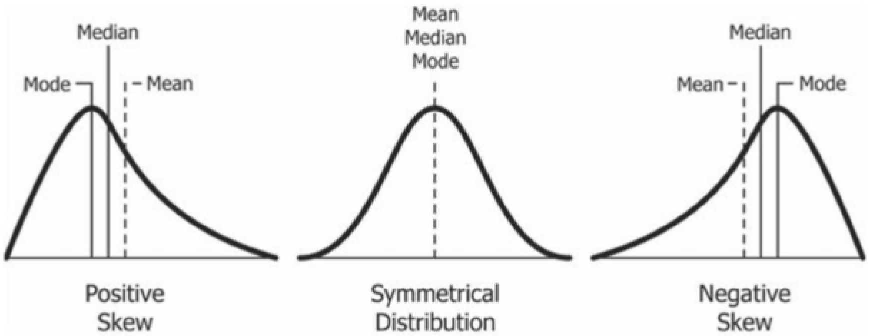


Figure 4: Distribution diagrams for three different scew values (Source: Wikipedia)

The skewness is given by the following formula:

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right] \tag{4.4}$$

Skewness is a very important measure for investors. Standard Deviation is commonly associated with the estimation of an instrument's risk, but it has a major flaw in assuming a normal distribution. Since few return distributions resemble a normal distribution, skewness is a better measure for predicting performance.

Most asset returns are skewed, either left or right. An investor can utilize this information to better predict future returns. A positively-skewed investment return means that there were frequent small losses and a few large gains. The opposite is true for negatively-skewed distributions.

A summary of the descriptive statistics of the selected asset's daily returns is given in the table below. A histogram for these statistics is presented in Figure 5.

	AVGO	NOW	TSLA	AMZN	IRDM	BR	PYACX	SIGAX	gold	palladium	S&P 500
mean (%)	0.1072	0.1459	0.1162	0.1660	0.1150	0.0891	0.0043	0.0034	0.0235	0.0899	0.0425
median (%)	0.1113	0.2066	0.0604	0.1374	0.1185	0.0910	0.0000	0.0000	0.0000	0.1505	0.0504
std (%)	2.0410	2.2267	2.8508	1.8480	2.6336	1.2677	0.2672	0.2282	0.9882	1.7089	0.8450
var (%)	4.1656	4.9583	8.1271	3.4151	6.9358	1.6071	0.0714	0.0521	0.9765	2.9202	0.7140
min (%)	-13.7447	-15.6561	-13.9015	-7.8197	-11.1297	-9.7015	-2.0777	-1.2893	-4.3193	-7.4046	-4.0979
max (%)	14.7054	14.0685	17.6692	14.1311	22.2393	11.1618	0.7712	0.6683	5.0975	7.2825	4.9594
kurtosis	5.3069	6.5043	5.0603	9.8273	6.4736	9.2869	3.4009	1.2845	4.5577	1.4909	3.8721
skewness	0.2343	-0.3319	0.3027	1.0068	0.6237	-0.1879	-0.7082	-0.4345	0.3295	-0.1472	-0.4688

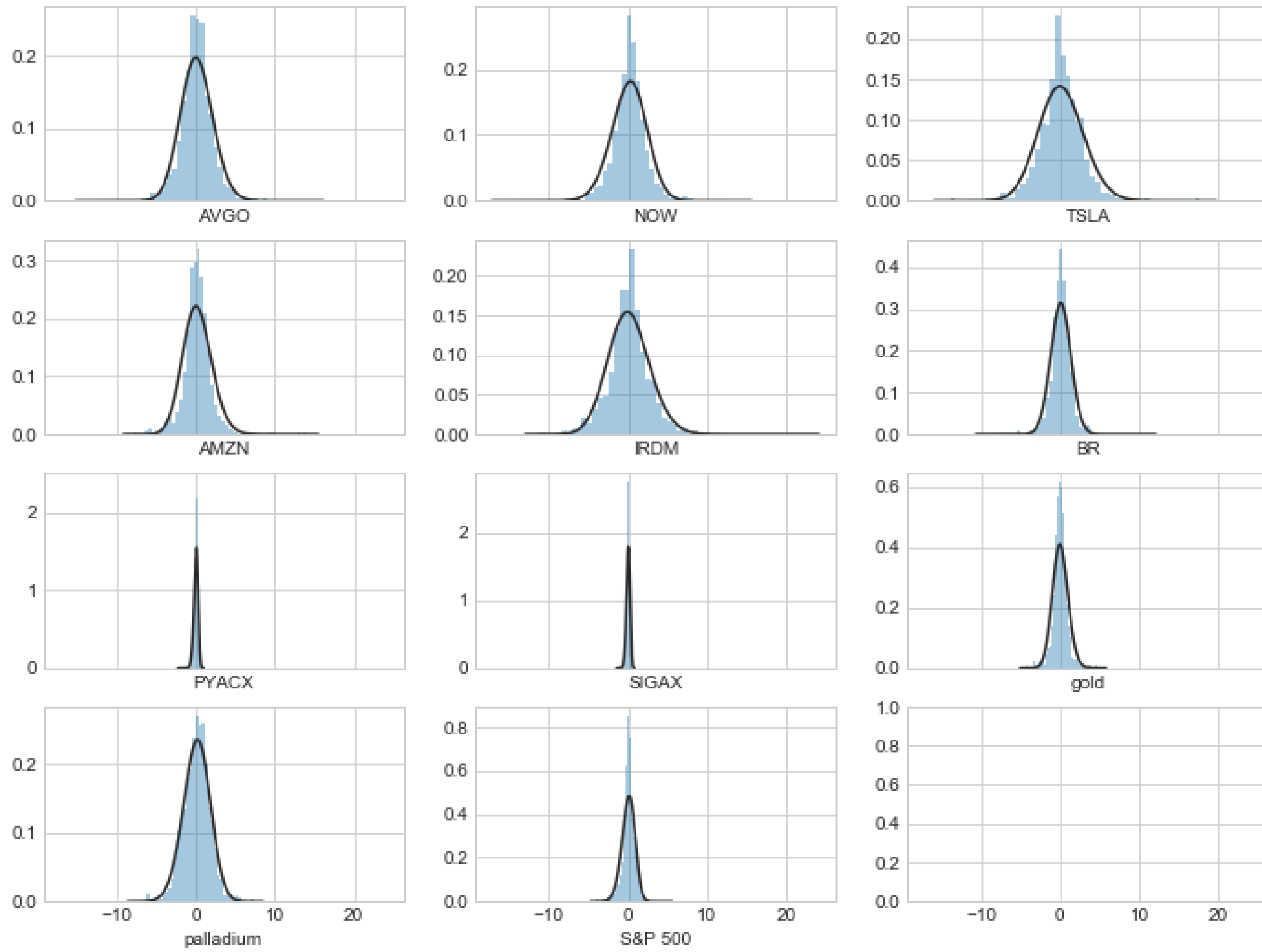


Figure 5: Histogram of the returns. A bell curve approximation is fitted. Note that the x-axis is common among the histograms. This allows us to grasp the "spread" or variability of each asset. It is easy to see that bonds are less risky than commodities and especially stocks.

5 Question 5

Compute additional metrics for the assets such as the correlation and covariance matrices, for the entire and two subperiods (of your own choosing), if needed. Interpret your findings. Also, compute each fund's alpha, beta, R-square. Interpret your findings from the perspective of the investor

5.1 Covariance and Correlation

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X,Y) = \mathbb{E}[(X-\bar{x})(Y-\bar{y})] \tag{5.1}$$

In our case these random variables are the daily returns of the various instruments. The element (i,j) of the covariance matrix is the covariance between instruments i and j, as presented below (in percentage values):

	AVGO	NOW	TSLA	AMZN	IRDM	BR	PYACX	SIGAX	gold	palladium	S&P 500
AVGO	0.04166	0.01663	0.01737	0.01309	0.01260	0.00864	-0.00085	-0.00028	-0.00168	0.00511	0.00966
NOW	0.01663	0.04958	0.01884	0.01871	0.01682	0.01191	-0.00057	0.00004	-0.00176	0.00490	0.01047
TSLA	0.01737	0.01884	0.08127	0.01615	0.01173	0.00837	-0.00072	-0.00017	-0.00200	0.00543	0.00903
AMZN	0.01309	0.01871	0.01615	0.03415	0.01633	0.00977	-0.00040	-0.00009	-0.00113	0.00364	0.00944
IRDM	0.01260	0.01682	0.01173	0.01633	0.06936	0.00968	-0.00114	-0.00047	-0.00110	0.00535	0.01012
BR	0.00864	0.01191	0.00837	0.00977	0.00968	0.01607	-0.00036	-0.00012	-0.00104	0.00291	0.00657
PYACX	-0.00085	-0.00057	-0.00072	-0.00040	-0.00114	-0.00036	0.00071	0.00051	0.00072	-0.00002	-0.00048
SIGAX	-0.00028	0.00004	-0.00017	-0.00009	-0.00047	-0.00012	0.00051	0.00052	0.00052	0.00021	-0.00014
gold	-0.00168	-0.00176	-0.00200	-0.00113	-0.00110	-0.00104	0.00072	0.00052	0.00976	0.00377	-0.00108
palladium	0.00511	0.00490	0.00543	0.00364	0.00535	0.00291	-0.00002	0.00021	0.00377	0.02920	0.00338
S&P 500	0.00966	0.01047	0.00903	0.00944	0.01012	0.00657	-0.00048	-0.00014	-0.00108	0.00338	0.00714

The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y} \tag{5.2}$$

The correlation ranges between -1 and 1 and it measures the linear dependence between two variables.

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1,1), & \text{indicating the degree of linear dependence between the variables} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases} \tag{5.3}$$

The correlation matrix is presented below:

	AVGO	NOW	TSLA	AMZN	IRDM	BR	PYACX	SIGAX	gold	palladium	S&P 500
AVGO	1.00000	0.36602	0.29851	0.34694	0.23444	0.33376	-0.15515	-0.05926	-0.08318	0.14655	0.56030
NOW	0.36602	1.00000	0.29685	0.45479	0.28687	0.42206	-0.09508	0.00769	-0.08010	0.12879	0.55637
TSLA	0.29851	0.29685	1.00000	0.30646	0.15619	0.23173	-0.09462	-0.02675	-0.07085	0.11149	0.37479
AMZN	0.34694	0.45479	0.30646	1.00000	0.33547	0.41702	-0.08134	-0.02104	-0.06178	0.11528	0.60447
IRDM	0.23444	0.28687	0.15619	0.33547	1.00000	0.28986	-0.16254	-0.07767	-0.04224	0.11885	0.45495
BR	0.33376	0.42206	0.23173	0.41702	0.28986	1.00000	-0.10708	-0.04245	-0.08267	0.13438	0.61364
PYACX	-0.15515	-0.09508	-0.09462	-0.08134	-0.16254	-0.10708	1.00000	0.84259	0.27093	-0.00480	-0.21479
SIGAX	-0.05926	0.00769	-0.02675	-0.02104	-0.07767	-0.04245	0.84259	1.00000	0.23236	0.05408	-0.07076
gold	-0.08318	-0.08010	-0.07085	-0.06178	-0.04224	-0.08267	0.27093	0.23236	1.00000	0.22312	-0.12976
palladium	0.14655	0.12879	0.11149	0.11528	0.11885	0.13438	-0.00480	0.05408	0.22312	1.00000	0.23437
S&P 500	0.56030	0.55637	0.37479	0.60447	0.45495	0.61364	-0.21479	-0.07076	-0.12976	0.23437	1.00000

Both correlation and covariance matrices are symmetric.

The correlation matrix is also shown in Figure 6. In Figure 7 another visualization of the effect of correlation coefficients is presented.

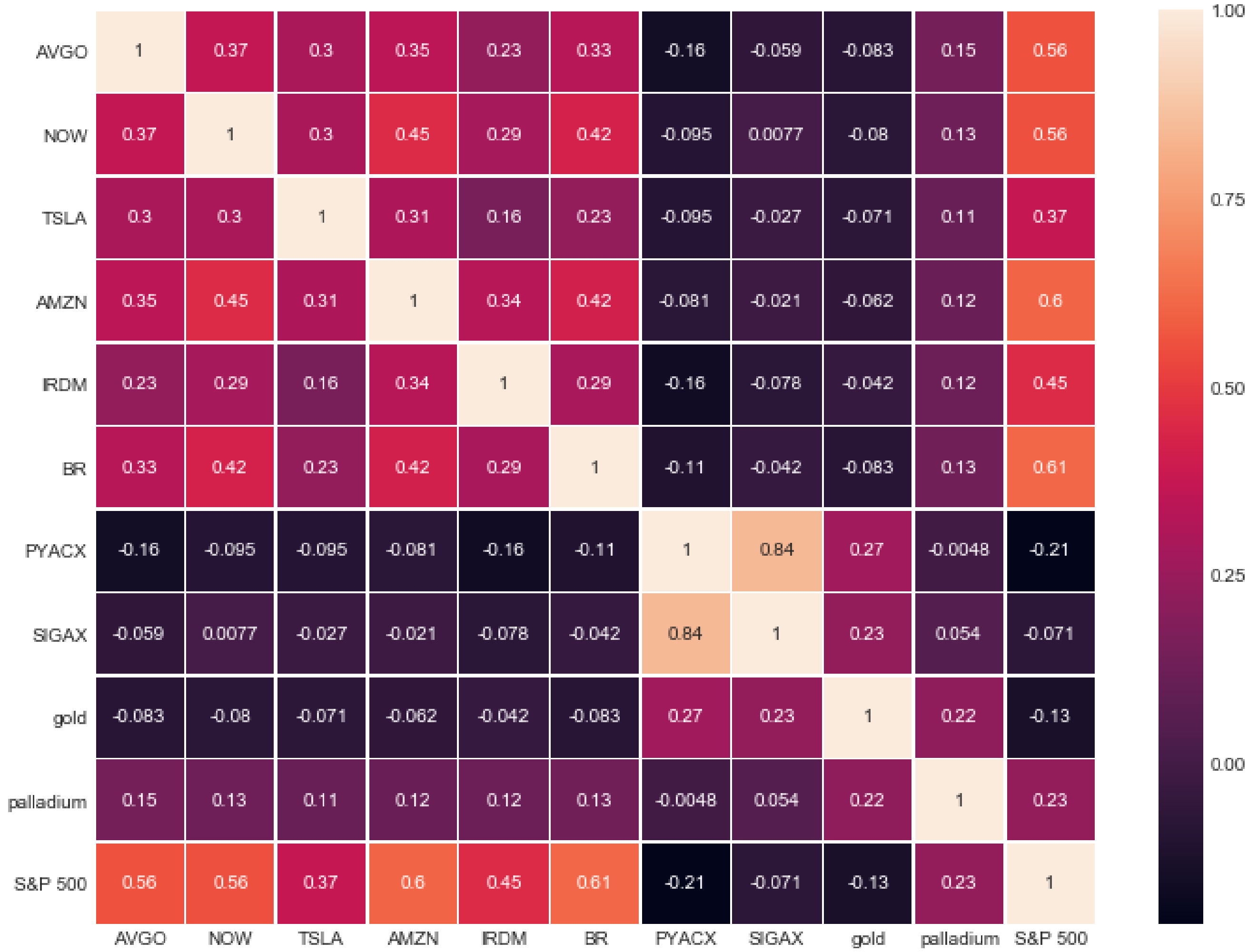


Figure 6: The correlation matrix for our instruments.

The correlation coefficient is a powerful tool for investors. It is a measure of strength of the linear relationship between price movements of two individual securities and hence can be used to gain diversification benefits. Two negatively correlated securities have less risk (in terms of volatility), compared to two positively correlated ones. Therefore investors can reduce the overall risk due to price fluctuations, by including negatively correlated assets in their portfolio.

We can observe from Figure 6 that the inclusion of both bonds and commodities in our portfolio indeed provides risk diversification, as they present either very low or negative correlation values with stock securities. Additionally the choice of stocks of different company sizes and sectors also offers diversification benefits, since the correlation of their prices is relatively weak (below 0.5).

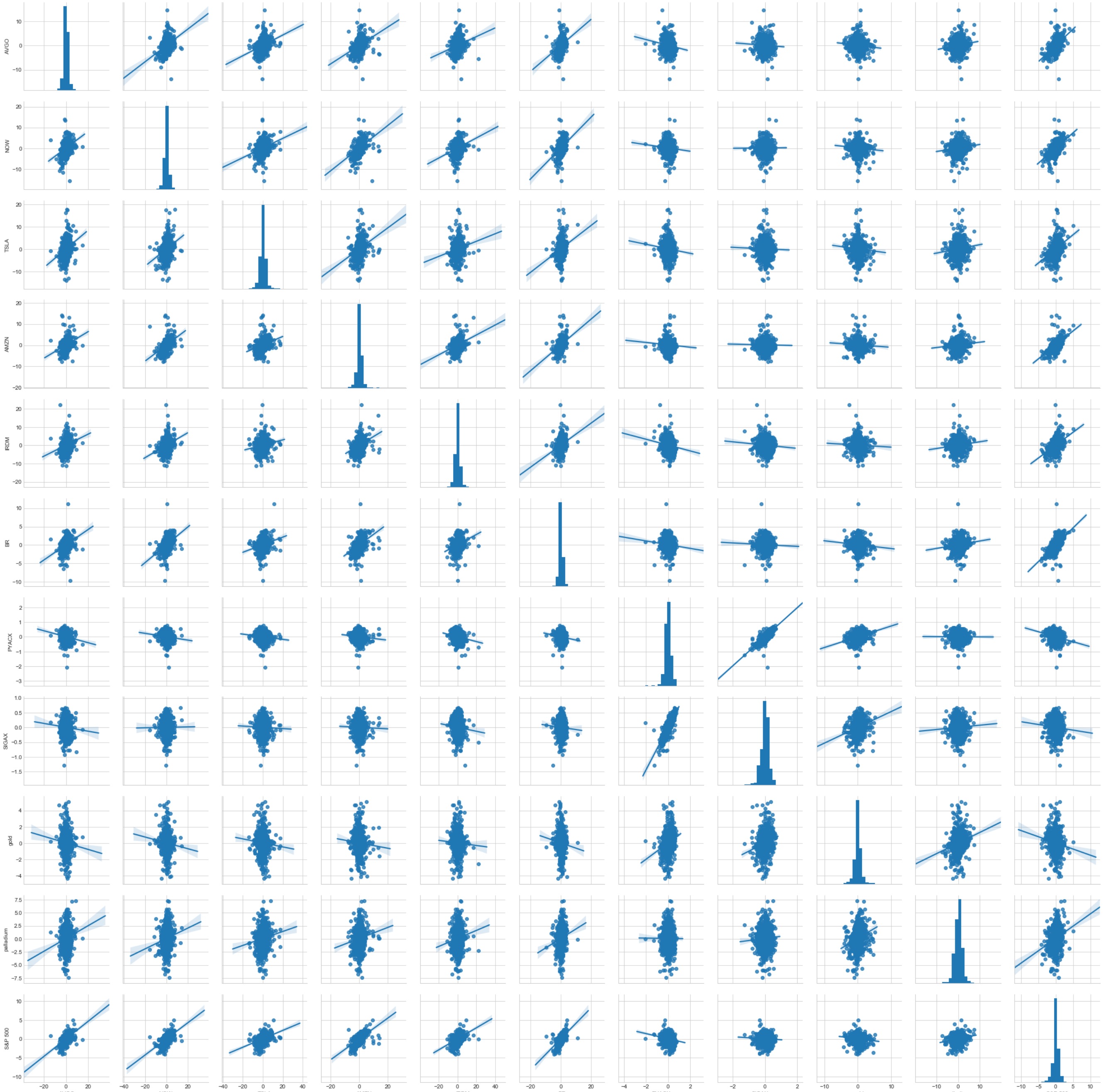


Figure 7: An illustration of the correlation matrix. When the correlation coefficient is negative the slope of the regression line is negative.

5.2 Beta

The beta coefficient measures the systematic risk of an individual stock compared to risk of the entire market. The beta formula is:

$$\beta = \frac{\mathrm{cov}(R_e, R_m)}{\mathrm{var}(R_m)} \tag{5.4}$$

where R_e is the return of the individual stock and R_m is the return of the overall market. In order to calculate β, a regression model has to be fitted on the data points from an individual stock's returns against those of the market. Then β is the slope of the aforementioned line.

Even though the formula is straightforward, the data selection is not. The result depends on the time frame and frequency of historical data selected. Hence, many different β values can be found online. We use daily returns of the last 5 years as our data. To be more specific, let $\mathcal{D} = \{d_0, d_1, \ldots, d_n\}$ be the closing prices of the individual stock. Then, $\mathcal{P} = \{p_1, \ldots, p_n\}$ is the set of the percent changes of said closing prices where $p_i = \frac{d_i - d_{i-1}}{d_{i-1}} \times 100\%$. Note that $\mathcal{P} = R_e$ (see (5.4)). By the same token, R_m is calculated using the historical closing prices of the market index (`S&P 500`), as it is widely considered as an index for representation of the entire market. Using these data points and equation (5.4), β is calculated.

Beta coefficient helps investors to understand how volatile or risky a security is compared to the market. If β has a value of 1.0, then the security is as volatile as the market and it does not add any risk to the portfolio, but also does not increase the potential excess returns. If β has a value of 1.3, then the security is assumed to have 30% more volatility compared to the market. If the market goes up by 10%, then the security will go up by 13%, and vice versa. Therefore an investor can use β to theoretically deduct how much risk a security will add to a diversified portfolio. However in order for β to have a meaningful value, a high R-squared value is required between the security and the benchmark used in calculations.

5.3 Alpha

By finding β we can proceed to calculate α. To be more precise α denotes the excess return. The Capital Asset Pricing Model (CAPM) is given by:

$$\mathbb{E}(R_i) = R_f + \beta_i(\mathbb{E}(R_m) - R_f) \tag{5.5}$$

where $\mathbb{E}(R_i)$ is the expected return of the individual asset and R_f is the risk-free rate. Then, α is calculated by subtracting the expected returns from the actual mean portfolio returns $\bar{R}$:

$$\alpha = \bar{R} - \mathbb{E}(R) \tag{5.6}$$

The α coefficient can be directly calculate from the linear regression model between the security's returns and those of the market. α is used from investors to determine whether or not a security has exceeded expectations in term of returns. Positive values of α indicate that the security is underpriced, whereas negative values indicate an overvalued security. An α value of 0 denotes that a security is fairly priced.

5.4 R-squared

R-squared or coefficient of variation is calculated using the following formula:

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}} \tag{5.7}$$

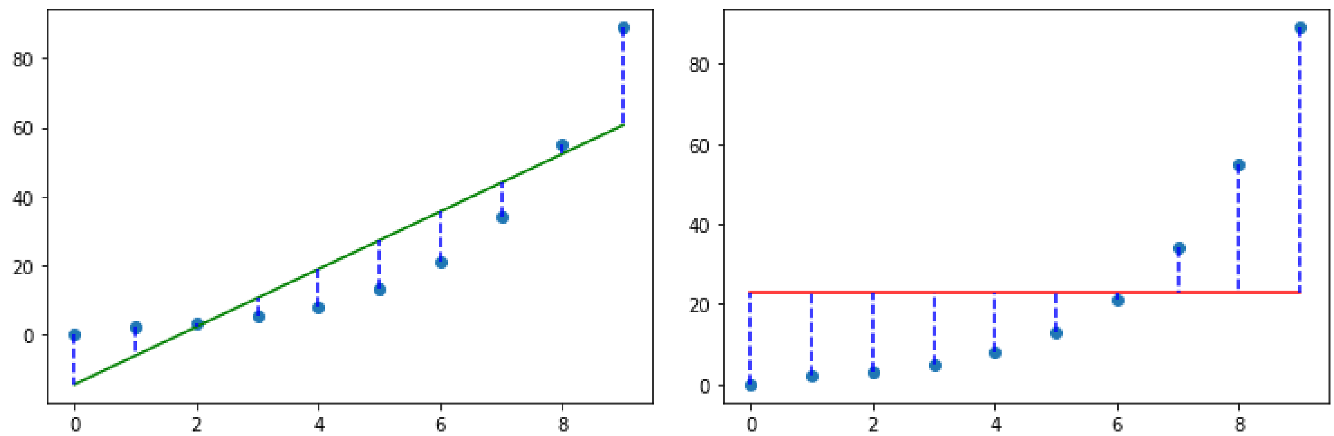
R^2 is a statistical measure that indicates the proportion of the variation of a dependent variable that can be explained using the independent variables of a simple regression model. An illustrative explanation is adduced using the following figures. In Figure 8a a simple regression model is fitted using the data provided. The explained variation is the sum of squared distances from the regression line divided by the number of points. Said distances are denoted with a blue colour. The total variation is calculated by the same token but with a crucial difference: the line is horizontal and denotes the mean of the data, as illustrated in Figure 8b.

In investments, the R^2 measure denotes the percentage of a security's price movements that can be explained by movements of the benchmark index. R^2 essentially measures the correlation between an asset's price changes and that of the benchmark. It is also used as statistical technique to asses the trustworthiness of a *beta* coefficient. High R^2 values (above 0.7) indicate a more useful β figure.

5.5 Sharpe Ratio

The Sharpe ratio is named after Nobel Laureate William Sharpe. It is a measure that relates the return of an investment to its risk; it is the risk-adjusted return.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{5.8}$$



(a) Explained Variation

(b) Total Variation

Figure 8: R-squared calculation

where

- R_p = Return of a portfolio
- R_f = Risk-free rate
- σ_p = standard variation of the portfolio's excess return

Risk-free rate The *trailing* 5 year average (daily interval) of the 10-Year Treasury Constant Maturity Rate is used as the Risk-free rate.

Methodology Since we are working with daily data, the following methodology is used to calculate annualized return and standard deviation of a security or portfolio:

- From daily prices, calculate the daily returns $r_i = \frac{d_i - d_{i-1}}{d_{i-1}}$
- Calculate the mean $\bar{r}_{daily}$ and standard deviation σ_{daily}.
- The annualized return is given from the following equation:

$$\text{Annualized Return (\%)} = [(1 + \bar{r}_{daily})^{252} - 1] * 100 \tag{5.9}$$

- The annualized standard deviation is given from the following equation:

$$\text{Annualized Standard Deviation (\%)} = \sigma_{daily} * \sqrt{252} * 100 \tag{5.10}$$

The value 252 represents the total number of trading days that occur within a 1-year period.

Using the historical data for each asset and the analysis presented above, we calculate α, β and the R^2 using `Python`. The results are presented below:

	AVGO	NOW	TSLA	AMZN	IRDM	BR	PYACX	SIGAX	gold	palladium
alpha	0.14213	0.24718	0.17711	0.32815	0.15867	0.13206	-0.00576	-0.01215	0.05087	0.17817
beta	1.35333	1.46614	1.26445	1.32199	1.41795	0.92064	-0.06792	-0.01911	-0.15174	0.47398
R-squared	0.31393	0.30955	0.14046	0.36539	0.20698	0.37655	0.04613	0.00501	0.01684	0.05493

6 Question 6

Calculate the return/risk of your risky portfolio. Explain each step in your analysis. You must use EXCEL's mmult functions for this part of the analysis.

Up until this point we have selected the assets that we will include in our risky portfolio and we have calculated important descriptive statistics and metrics for each individual security. In this section we will calculate the risk and return of the risky portfolio, using a vector of weights. Each security is assigned a weight, representing a portion of the available capital that is to be invested in it.

Let R_i be the returns data sample for the instrument i and $\mathcal{R} = (R_1, R_2, \cdots, R_n)$ be the collection of those random variables. Let $\vec{w}$ be the vector of weights that represents the allocation of the various instruments in our risky portfolio. Obviously, $\|\vec{w}\|_1 = 1$. Finally, let K be the associated covariance matrix.

Then the return of the risky portfolio is a weighted average of the expected returns:

$$R_{\text{risky portfolio}} = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) \tag{6.1}$$

and the risk of the risky portfolio is measured by its standard deviation:

$$\begin{aligned}
\sigma_{\text{risky portfolio}} &= \sqrt{\vec{w}^\top K \vec{w}} && (6.2)\\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \text{cov}_{1,2} & \dots & \text{cov}_{1,n} \\ \text{cov}_{2,1} & \sigma_2^2 & \dots & \text{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}_{n,1} & \text{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} && (6.3)\\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{cov}_{i,j}} && (6.4)
\end{aligned}$$

As an initial approach, available capital is allocated uniformly between assets belonging in the same class. In other words, let $w\%$ be the percentage allocated to a class of n assets (i.e. stocks, bonds or commodities). Then, each security of this class receives a $\frac{w\%}{n}$ allocation. As stated in the portfolio's IPS (1), we have $w_{stocks} = 70\%$, $w_{bonds} = 20\%$ and $w_{commodities} = 10\%$ Hence, the weights associated with the securities are:

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 11.67 \\ 11.67 \\ 11.67 \\ 11.67 \\ 11.67 \\ 11.67 \\ 10.0 \\ 10.0 \\ 5.0 \\ 5.0 \end{bmatrix} \% \tag{6.5}$$

This naive allocation yields the following results:

	Naive Risky Portfolio
Return (%)	27.444
Risk (%)	15.7565
Sharpe Ratio	1.5977

7 Question 7

Derive and graph the Capital Allocation Line. Graph the Efficient Frontier with your available investment instruments (assets) and superimpose your CAL. Discuss the various options you may have and finalize your optimal point.

By taking various random weight allocations we are able to create portfolios with different risks and returns. If we choose a large enough sample size ($n = 2,000,000$), this will be a fair representation of the all the risky portfolios we can construct (portfolio universe). The results of the simulation are illustrated in Figure 9.

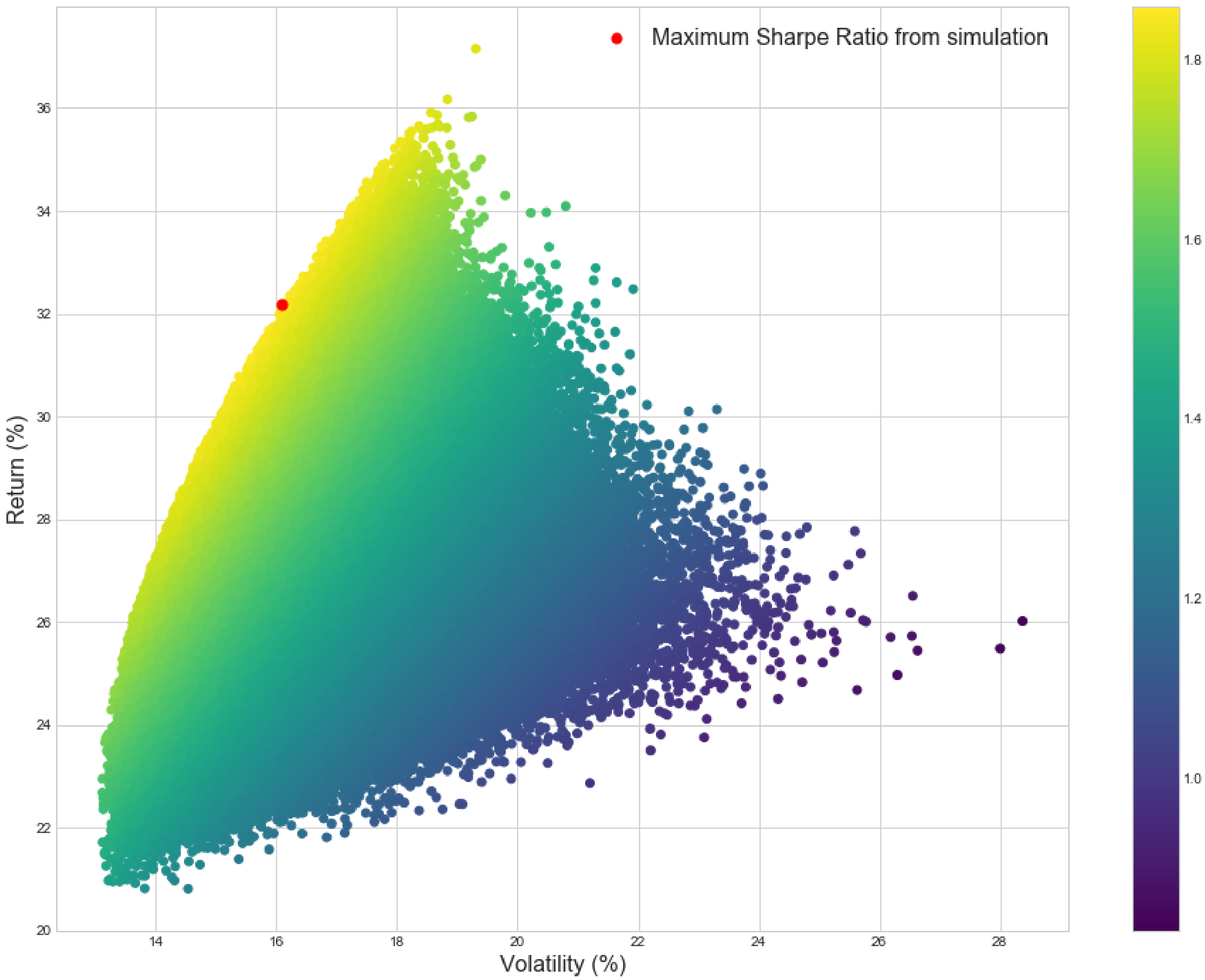


Figure 9: $n = 2,000,000$ random portfolios are created and plotted in a common graph. The portfolio with the highest Sharpe Ratio from the simulation is annotated.

The above graph is conducive with the theoretical one, with respect to its shape. By using a large sample size, the range of annualized returns and volatilities is known. To find the Efficient Frontier an optimization problem must be solved. The Efficient Frontier is essentially the $Minimum - Variance$ Frontier, therefore for any given portfolio return we seek to minimize the portfolio volatility. By solving numerous such optimization problems, points on the Efficient Frontier are obtained and graphed. The results are illustrated in Figure 10.

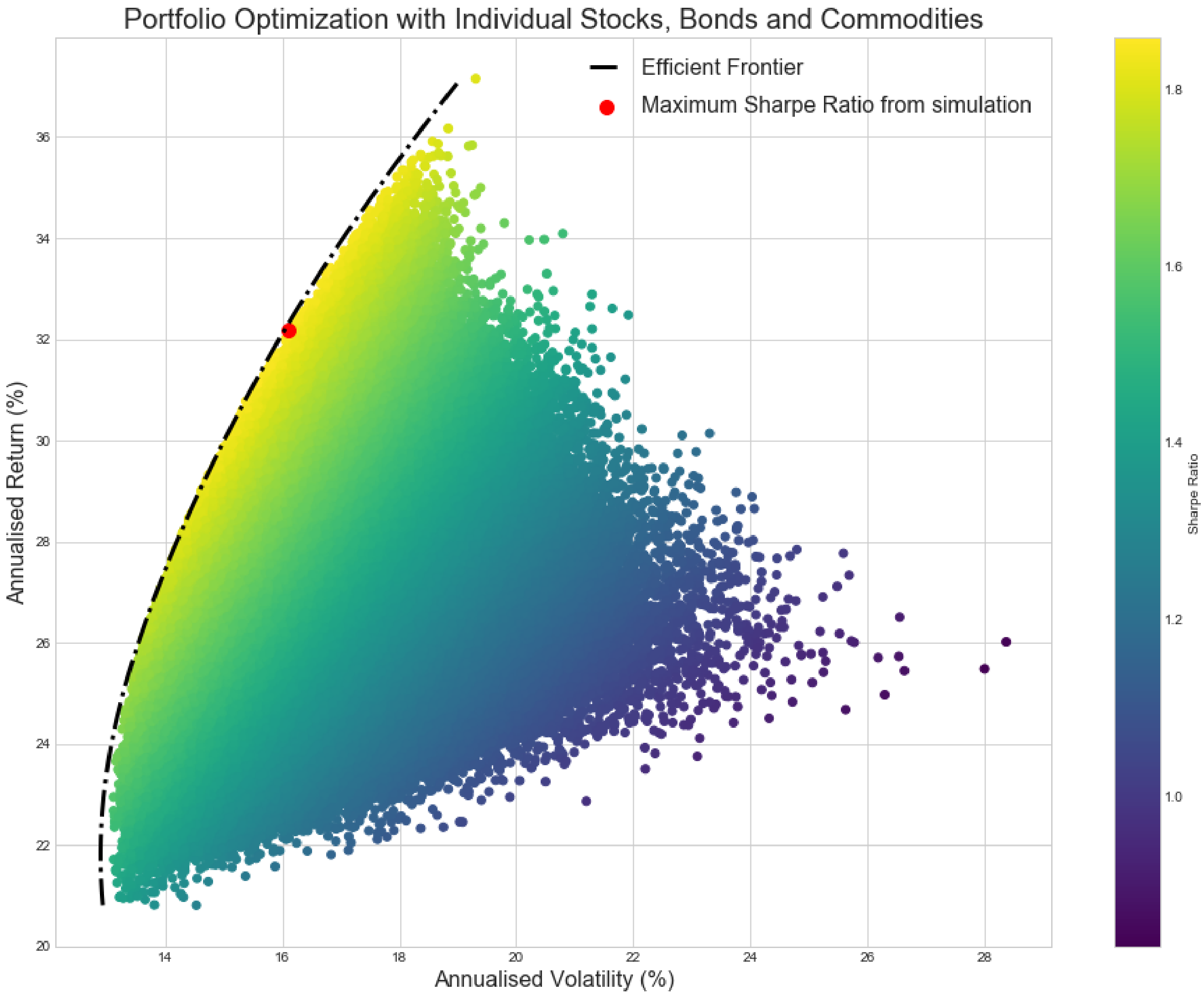


Figure 10: The Efficient Frontier.

The Capital Allocation Line (CAL) is a linear representation of all possible combinations of risk-free and risky assets. It aids investors in deciding how to split their available capital between risk-free assets and the risky portfolio. For the naive portfolio in question, the CAL equation (ϵ_{naive}) is given below:

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \implies \epsilon_{naive} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f \tag{7.1}$$

As illustrated in Figure 11, the naive risky portfolio of uniform weights does not plot on the EF, therefore is not the optimal point. We know the following from our "Foundations of Investments" course:

- The optimal risky portfolio should plot on the EF.
- The optimal risky portfolio has the highest Sharpe between other risky portfolios on the EF.
- The CAL of the optimal risky portfolio is tangent to the EF.

Based on the above, an optimization problem that maximizes the Sharpe Ratio must be solved, in order to find the optimal vector of weights. This analysis is performed in Section 9.

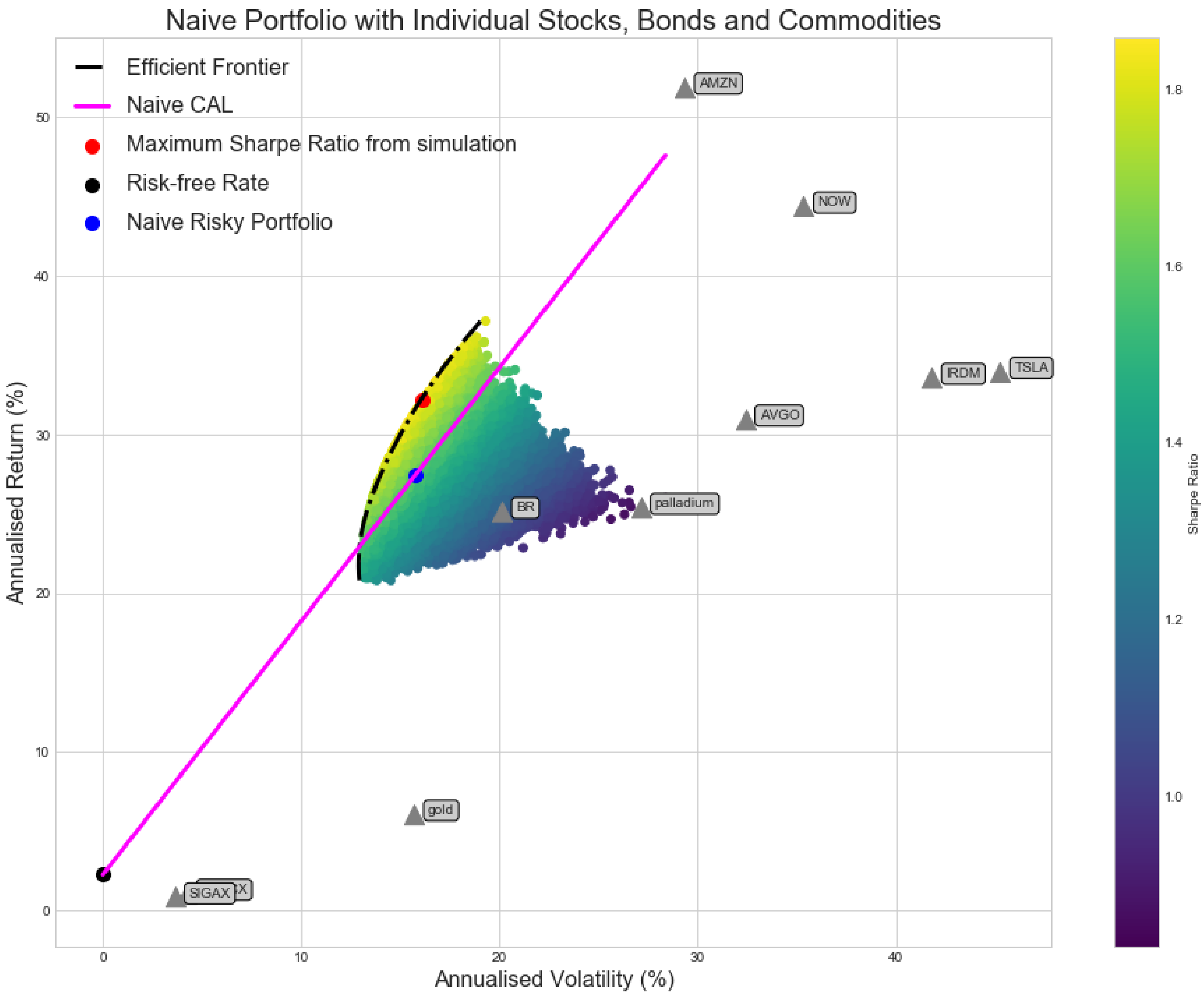


Figure 11: CAL of the naive portfolio and EF. The naive portfolio does not plot on the EF and does not have the maximum Sharpe Ratio, therefore the naive weight allocation to risky assets is not the optimal.

8 Question 8

Keep a track record of the macro- and microeconomic events that influenced your assets/portfolio and offer explanations for some of them, if you deem necessary. Your explanations should also be accompanied with some quantitative verification.

9 Question 9

Measure and evaluate your overall portfolio's performance and compare it with the passive investment strategy. In this step, you should apply EXCEL's Solver to evaluate several possible outcomes (in terms of risk and return) and explain each outcome. In that endeavour, compute the various performance measures we have learned. Decide on the best outcome for you. Discuss.

As we examined in Section 7, the naive approach is not optimal when constructing a portfolio of risky assets. In this part we will examine how optimization analysis helps investors determine the optimal risky portfolio and how the optimal overall allocation between risk-free and risky assets is derived.

According to Markowitz, father of Modern Portfolio Theory, the portfolio construction process can be separated into two different independent tasks:

- Determination of the optimal risky portfolio, using the investor's selected assets. This part is purely technical.
- Determination of the overall portfolio. This part includes the investor's decision for allocation of funds between risk-free assets and the risky portfolio. This choice depends on the investor's personal preference and his/her risk profile.

The above concept is known as the Separation Theorem. We can therefore break our final choice into two separate problems.

9.1 Optimal Risky Portfolio

As illustrated in Figure 10, the Efficient Frontier consists of $minimum - variance$ portfolios. Therefore the optimality criterion lies in the Sharpe Ratio associated with each risky portfolio. The portfolio with the highest Sharpe Ratio offers the best $risk - adjusted\ returns$ among all the available combinations in the opportunity set.

In order to find the optimal portfolio, we ought to solve the following optimization problem:

$$
\begin{aligned}
\max \quad & \frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

where $\vec{w}$ is the weight vector associated with the n assets, $\vec{w}_{\mathcal{S}}$, $\vec{w}_{\mathcal{B}}$ and $\vec{w}_{\mathcal{C}}$ are the weights associated with the stock, bond and commodity assets and sum up to w_s, w_b and w_c, respectively. Obviously, the concatenation of these vectors is the original weight vector, i.e. $\mathcal{S} \sqcup \mathcal{B} \sqcup \mathcal{C} = \{1, \cdots, n\}$. In other words, the objective corresponds to the Sharpe Ratio and the constraints construct the feasibility region of the optimization problem given the demands dictated by the suggested allocation resulting in the weights w_s, w_b and w_c.

By solving the above optimization problem the optimal weights of securities in the risky portfolio are calculated:

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \% \tag{9.1}$$

This optimal allocation yields the following results:

	Optimal Risky Portfolio
Return (%)	32.7164
Risk (%)	16.3185
Sharpe Ratio	1.8658
Beta (β)	0.9067

The analysis showed that `gold` and `SIGAX` have zero weights. We have two choices:

- Exclude these securities from the optimal portfolio, as they have no effect on its risk and return or
- Substitute them with other securities of the same class.

The optimal Sharpe Ratio is also the slope of the Capital Allocation Line. The CAL of the optimal risky portfolio is tangent to the Efficient Frontier, as shown in Figure 12. The equation of the optimal CAL (ϵ_{opt}) is given below:

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{opt}} \\ (\sigma_{\text{opt}}, r_{\text{opt}}) \in \epsilon_{\text{opt}} \end{array} \right\} \Longrightarrow \epsilon_{opt} : y = \underbrace{\frac{r_{\text{opt}} - r_f}{\sigma_{\text{opt}}}}_{\text{max Sharpe ratio}} \cdot x + r_f \tag{9.2}$$

9.2 Optimal Overall Portfolio

The final step in the portfolio construction is to decide on the allocation between risk-free assets and the risky portfolio. This decision is based on the investor's attitude towards risk. An individual's risk profile can quantified by the coefficient of risk aversion, denoted A. Low values of A indicate risk-tolerance, while high values indicate risk-aversion. Various studies have been conducted to determine the range of A and is usually placed between 1 and 6. As young and risk-tolerant investors, we assume that our coefficient of risk-aversion is $A = 1.5$.

Let z be the proportion of funds allocated to the optimal risky portfolio. Then $(1-z)$ is the proportion allocated to risk-free assets. Investors calculate the optimal z by maximizing their utility. Expressed as a function of z, utility is given by the equation below:

enter utility equation here

Therefore the optimal z that maximizes utility is:

enter z* equation here

After calculations, we derive that $z^* = 0.76$ and $U_{max} = 13.87$. By finding different levels of risk an return that yield the same value of maximum utility, we can graph the indifference curve for $U_{max} = 13.87$ and $A = 1.5$. As illustrated in Figure 12, the optimal overall portfolio occurs at the point where the indifference curve is tangent to the CAL of the optimal risky portfolio.

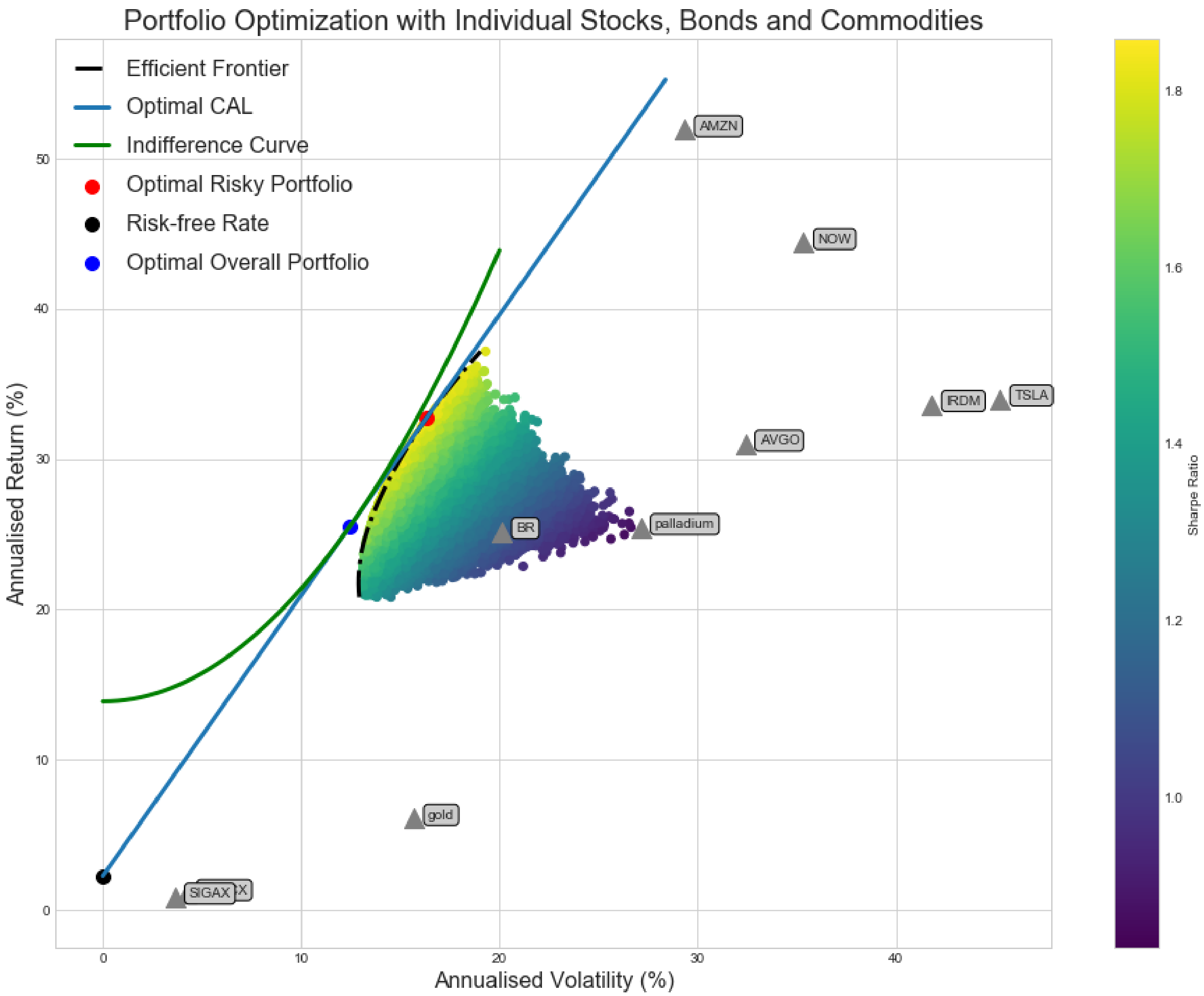


Figure 12: After Sharpe Ratio maximization, the optimal CAL is tangent to the EF. The optimal overall portfolio for the investor occurs at the tangency point between the indifference curve and the CAL.

10 Question 10

Finally, perform a critical evaluation of the project. In other words, what did this project accomplish for you regarding the study and (this simple) application of investment theories and strategies? What would be the implications of constructing, managing and evaluating such a portfolio for your portfolio?

A Questionnaire

1/30/2020 Vanguard - Investor Questionnaire

PERSONAL INVESTORS

Vanguard

Investor questionnaire

1. I plan to begin taking money from my investments in ...

- [ ] 1 year or less
- [ ] 1 – 2 years
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [x] More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- [x] 2 years or less
- [ ] 3 – 5 years
- [ ] 6 – 10 years
- [ ] 11 – 15 years
- [ ] More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- [ ] 1 – 2 years
- [ ] 3 – 4 years
- [ ] 5 – 6 years
- [ ] 7 – 8 years
- [x] More than 8 years

4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would … ***(If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)***

- [ ] Sell all of the remaining investment.
- [ ] Sell a portion of the remaining investment.
- [x] Hold onto the investment and sell nothing.
- [ ] Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- [ ] Strongly disagree
- [x] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

- [x] Strongly disagree
- [ ] Disagree
- [ ] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

- [ ] Strongly disagree
- [ ] Disagree
- [x] Somewhat agree
- [ ] Agree
- [ ] Strongly agree

8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would … ***(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)***

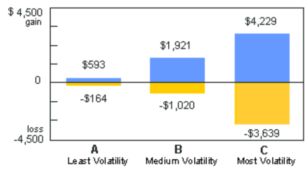
https://personal.vanguard.com/us/FundsInvQuestionnaire 1/2

1/30/2020 Vanguard - Investor Questionnaire

- ○ Sell all of the remaining investment.
- ○ Sell a portion of the remaining investment.
- ◉ Hold onto the investment and sell nothing.
- ○ Buy more of the investment.

9. The chart below shows the greatest 1-year loss and the highest 1-year gain on 3 different hypothetical investments of $10,000.* Given the potential gain or loss in any 1 year, I would invest my money in …

- ○ **A** (loss of $164, gain of $593)
- ◉ **B** (loss of $1,020, gain of $1,921)
- ○ **C** (loss of $3,639, gain of $4,229)



*The maximum gain or loss on an investment is impossible to predict. The ranges shown in the chart are hypothetical and are designed solely to gauge an investor's risk tolerance.

10. My current and future income sources (for example, salary, Social Security, pension) are …

- ○ Very unstable
- ○ Unstable
- ○ Somewhat stable
- ◉ Stable
- ○ Very stable

11. When it comes to investing in stock or bond mutual funds or ETFs—or individual stocks or bonds—I would describe myself as ...

- ◉ Very inexperienced
- ○ Somewhat inexperienced
- ○ Somewhat experienced
- ○ Experienced
- ○ Very experienced

My current asset allocation

Enter the current asset allocation in whole numbers. Your percentages must total 100%. If you don't enter any percentages, the questionnaire will assume that 100% of your assets are in short-term reserves.

Short-term reserves	100.0	%
Bonds	0.0	%
Stocks	0.0	%

Cancel Back Get Results

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B Questionnaire Results

1/30/2020 Vanguard - Investor questionnaire results

PERSONAL INVESTORS

Vanguard

Current Allocation **Suggested Allocation ("30% Bonds & 70% Stocks")**

Be sure to jot these percentages down so you have them handy when you're selecting specific funds and completing your investments.

Have questions about your allocation?

Compare your percentages with other allocation mixes

Review certain factors before reallocating your assets

Already a Vanguard client?

Get a more detailed portfolio analysis (logon required)

This analysis evaluates your current asset allocation—along with your costs, taxes, and risks—using the same methodologies that underlie our personal advice services.

Ready to choose specific funds?

Keep it simple with an all-in-one fund. These funds are designed to help you simplify the way you manage your portfolio and reduce your investment risk.

Get a complete portfolio in a single fund

Or see how you can build your own portfolio with a combination of funds.

Explore different types of Vanguard mutual funds
Get help choosing your Vanguard ETFs®

Want to see all of the funds we offer?

Browse a list of all Vanguard mutual funds & ETFs

Done Back

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