# Portfolio Management
## FN 4329

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American College of Greece

Spring Semester 2020

# Contents


1. Portfolio Construction
   - Asset Allocation
   - Security Selection
   - Diversification strategy
2. Theory
   - Descriptive Statistics
   - Financial Metrics
3. Building the Portfolio
   - Naive allocation
   - Portfolio Optimization
   - Optimal Overall Portfolio

# todo

## Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

# Questionnaire



PERSONAL INVESTORS

Vanguard

## Investor questionnaire

**1. I plan to begin taking money from my investments in ...**

- ○ 1 year or less
- ○ 1 – 2 years
- ○ 3 – 5 years
- ○ 6 – 10 years
- ○ 11 – 15 years
- ◉ More than 15 years

**2. As I withdraw money from these investments, I plan to spend it over a period of ...**

- ◉ 2 years or less
- ○ 3 – 5 years
- ○ 6 – 10 years
- ○ 11 – 15 years
- ○ More than 15 years

**3. When making a long-term investment, I plan to keep the money invested for ...**

- ○ 1 – 2 years
- ○ 3 – 4 years
- ○ 5 – 6 years
- ○ 7 – 8 years
- ◉ More than 8 years

**4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would …** ***(If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)***

- ○ Sell all of the remaining investment.
- ○ Sell a portion of the remaining investment.
- ◉ Hold onto the investment and sell nothing.
- ○ Buy more of the investment.

**5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.**

- ○ Strongly disagree
- ◉ Disagree
- ○ Somewhat agree
- ○ Agree
- ○ Strongly agree

**6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.**

- ◉ Strongly disagree
- ○ Disagree
- ○ Somewhat agree
- ○ Agree
- ○ Strongly agree

**7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.**

- ○ Strongly disagree
- ○ Disagree
- ◉ Somewhat agree
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**8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would …** ***(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)***

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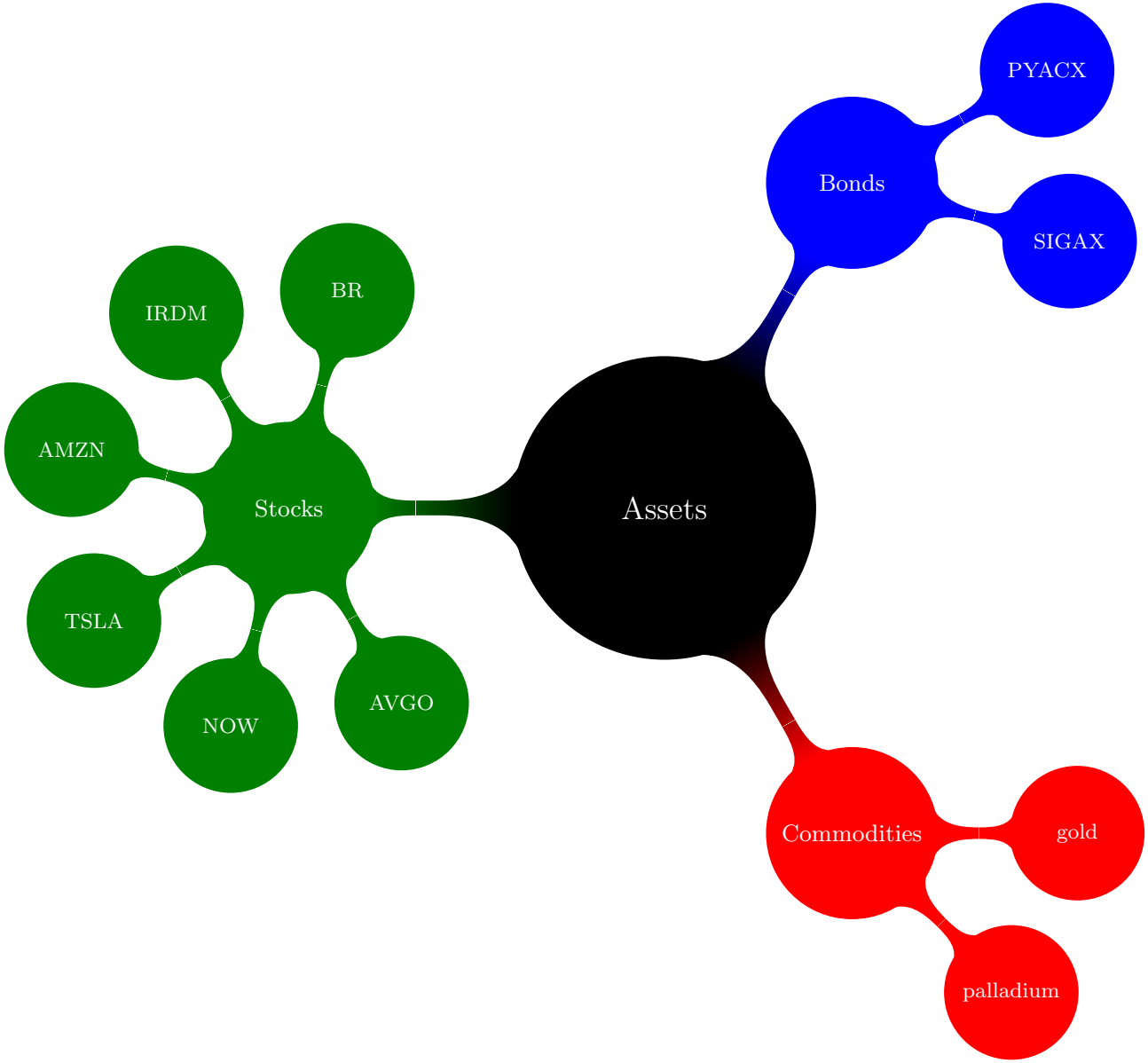
**8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ...** ***(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)***



## Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

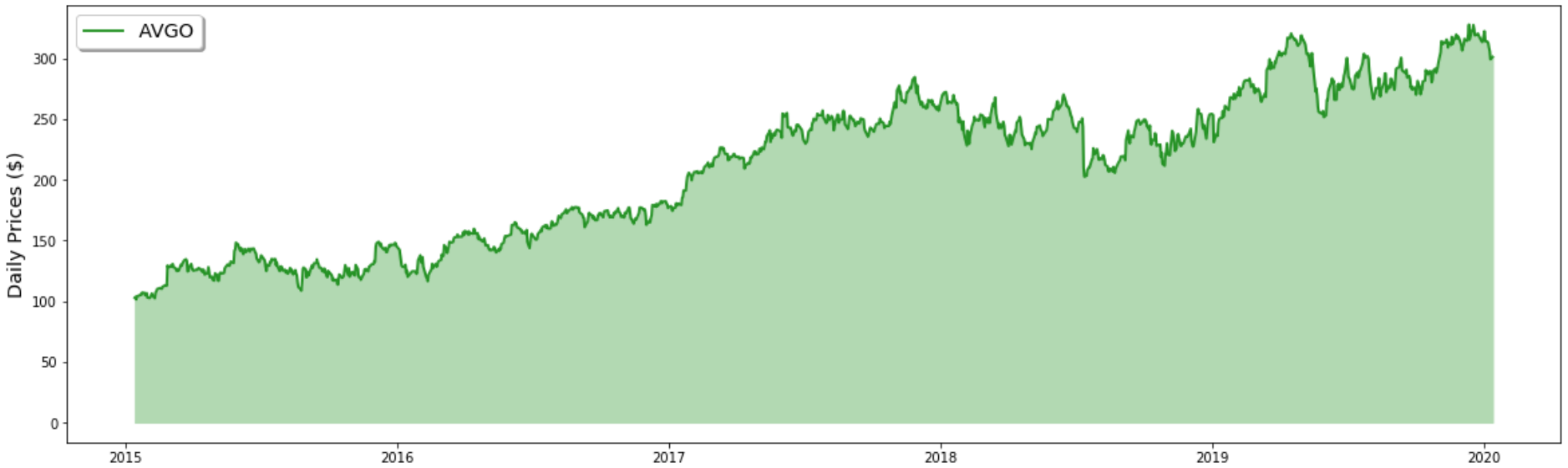
# Security Selection



# Stocks

## AVGO - Broadcom Inc

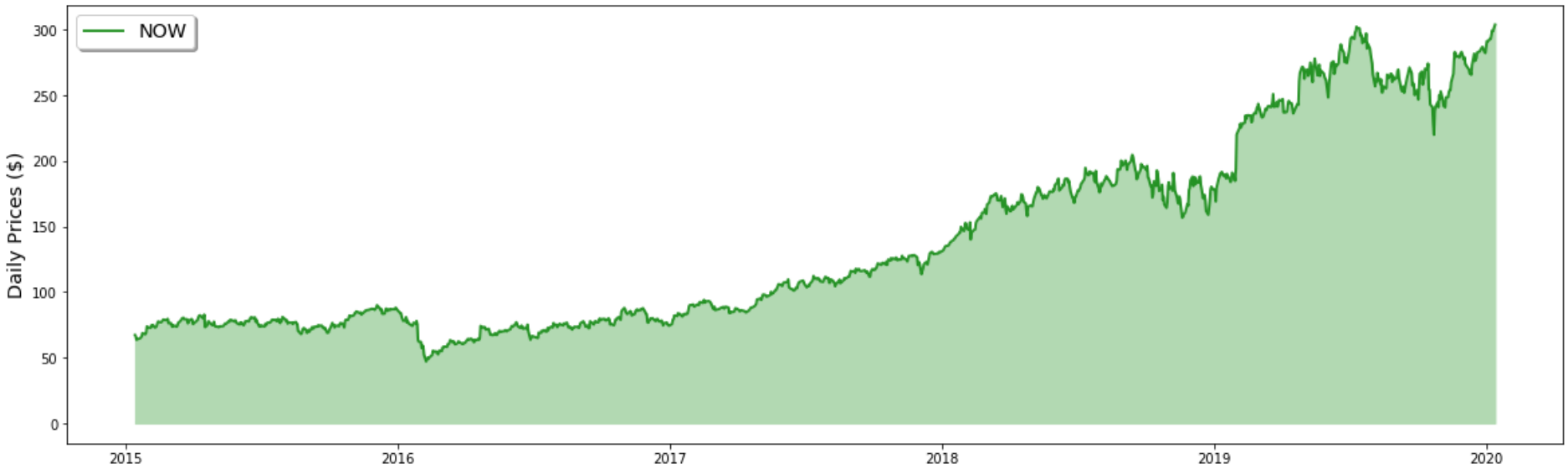
- Large/High-Dividend
- Wireless chips and semiconductor manufacturer
- Announced 15$ billion deal with Apple Inc. in January 2020
- Potential for gains with 5G deployment



# Stocks

## NOW - ServiceNow Inc.

- Large/Growth
- Cloud-based solutions provider to global enterprises
- Software as a Service (SaaS) business model
- IT, Customer Support, HR and Security services



# Stocks

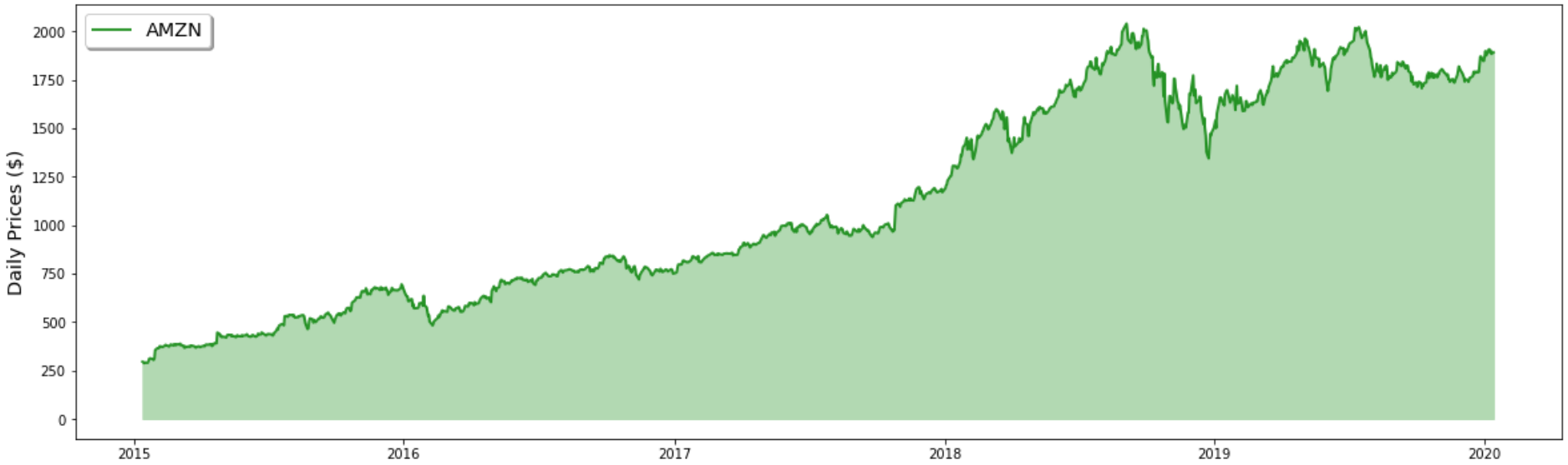
## TSLA - Tesla Inc.

- Large/Growth
- Leader in next-generation electric vehicles
- Advancements in battery technology will allow for increased autonomy
- $4^{th}$ Gigafactory in Berlin to begin operations by July 2021

# Stocks

## AMZN - Amazon Inc.

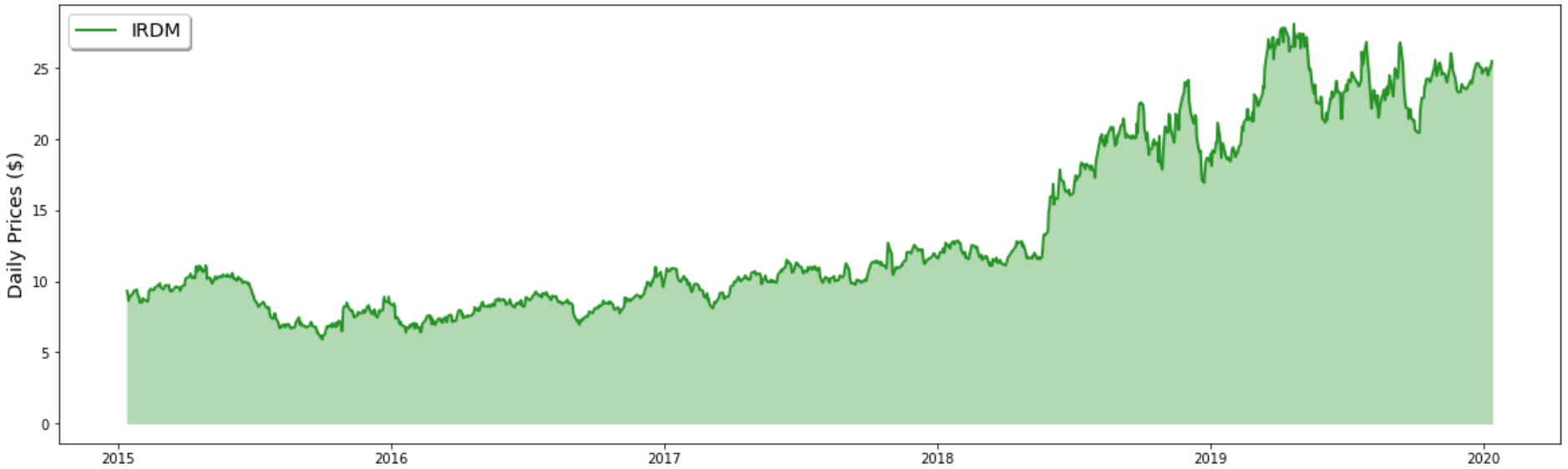
- Large/Growth
- Biggest online retailer
- Online product and digital media sales
- AWS offers solutions for Machine Learning, Big Data, IoT and Cloud-Computing



# Stocks

## IRDM - Iridium Communications Inc.

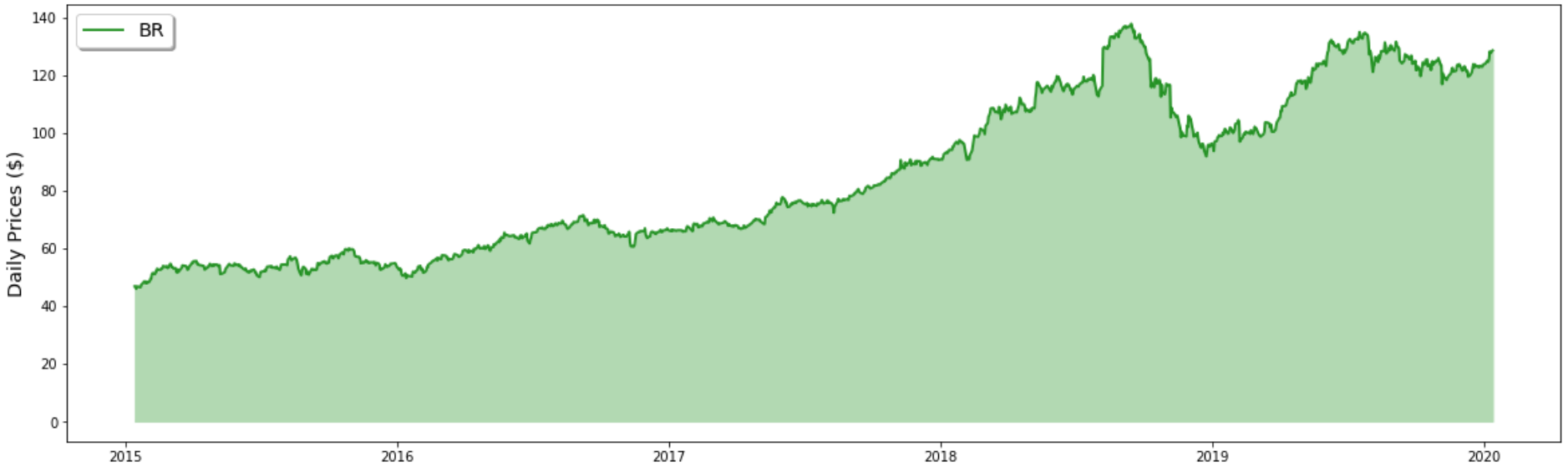
- Small/Growth
- Lead provider of satellite communications, with over 70 satellites in orbit
- Announced partnership with AWS for future applications in 2018
- Wide commercial end base, from maritime and aviation to oil & gas suppliers



# Stocks

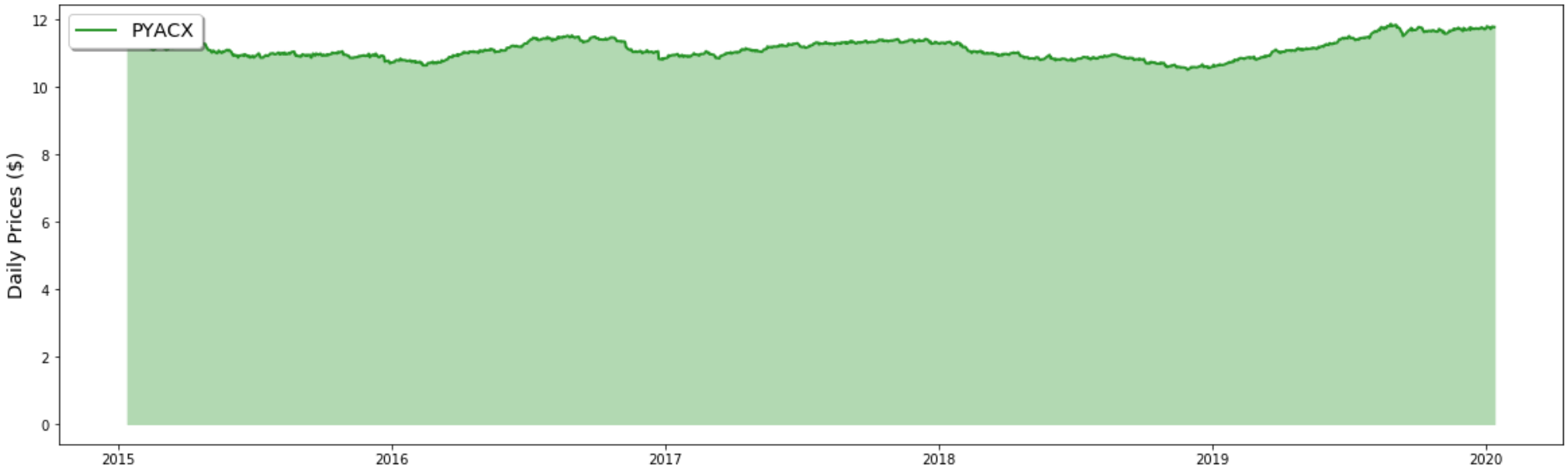
## BR - Broadridge Financial Solutions Inc.

- Mid/Aggressive Growth
- Fintech company that provides solutions for investors, banks, brokerage offices and mutual funds
- Products include communication platforms, securities processing and financial data analytics



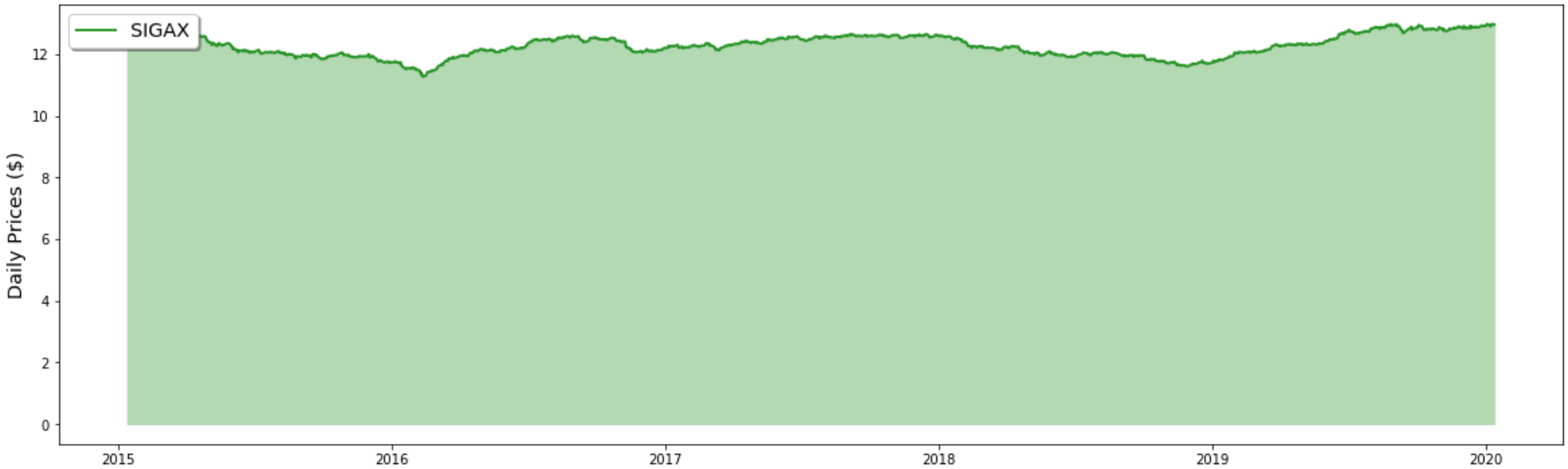
# Bonds

## PYACX - Payden Corporate Bond Mutual Fund



# Bonds

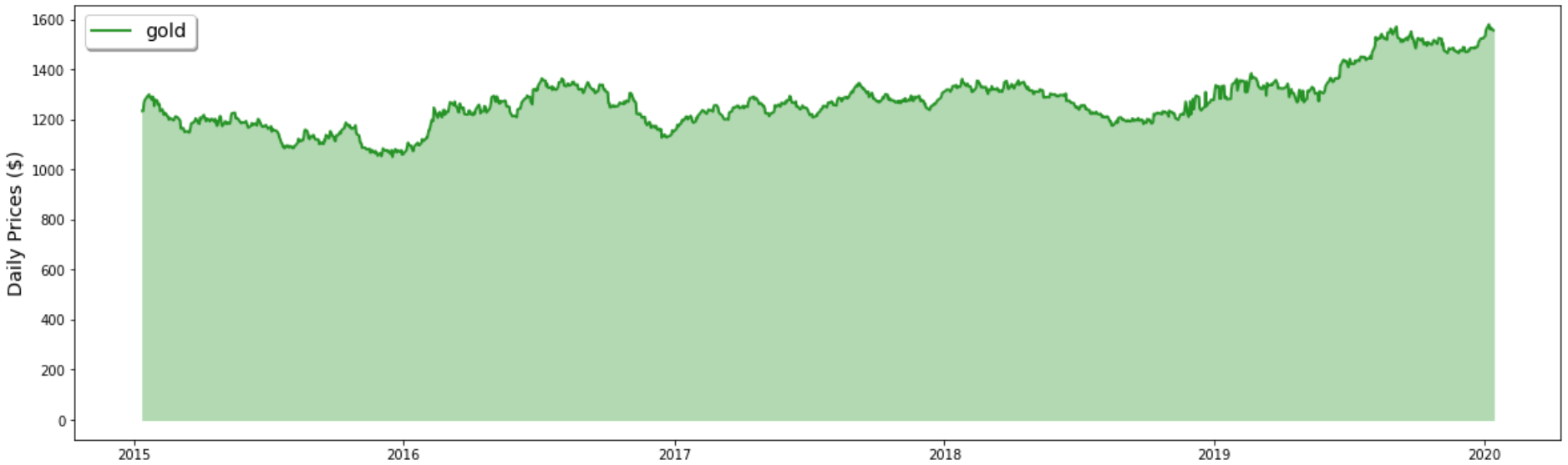
## SIGAX - Western Asset Corporate Bond Mutual Fund



# Commodities

## gold

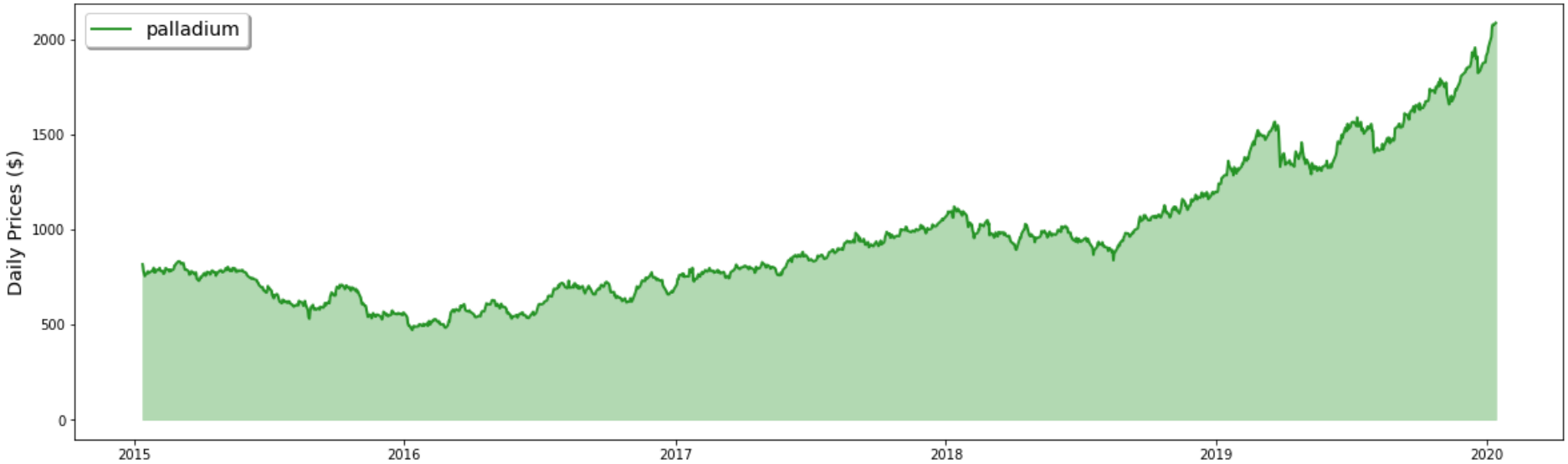
- Traditionally safe investment during rough economic times and hedge against inflation
- Recently preferred over government bonds, as the latter presented negative inflation-adjusted returns in 2019
- Does not generate return and has no holding costs



# Commodities

## palladium

- Metal that is used in catalytic converters, turning toxic gas emissions into less harmful ones
- Secondary product from mining operations of other metals
- As miners have less control over the extracted quantities, demand outstrips supply
- Price has doubled over the last year



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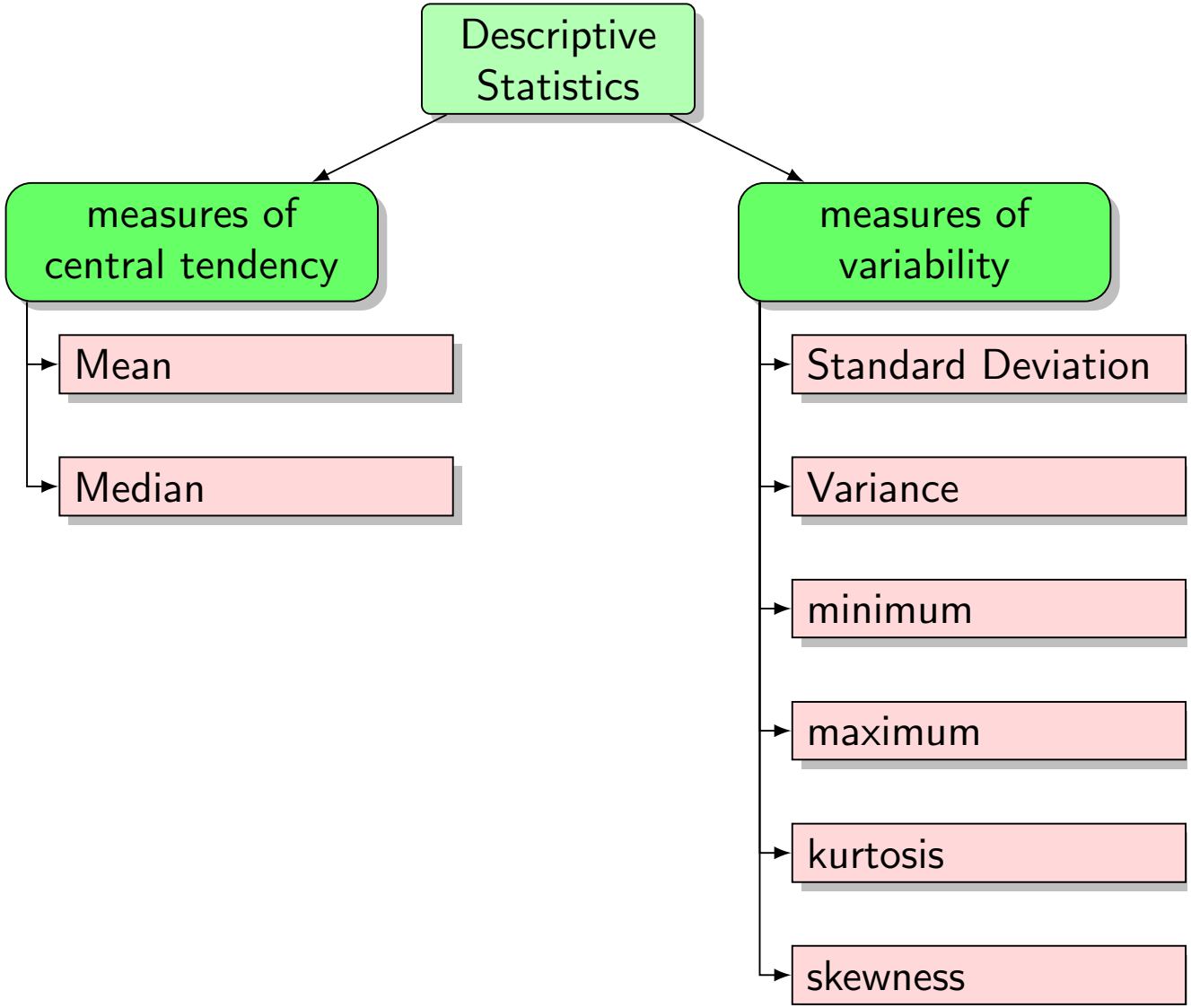
# Descriptive statistics Taxonomy



# Measures of central tendency

## Mean

### Mean

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

# Measures of central tendency

## Median

- i.e. the middle value

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- Why?

# Measures of central tendency

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- Why? robust w.r.t. outliers

# Measures of central tendency

## Median

- i.e. the middle value
- Why? robust w.r.t. outliers
- indicates whether returns are positive or negative on most time instances.

# Measures of Variability

## Standard Deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- how "spread out" are the data around the mean
- measures confidence in statistics $\Longrightarrow$ risk in finance

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# Measures of Variability

## Standard Deviation

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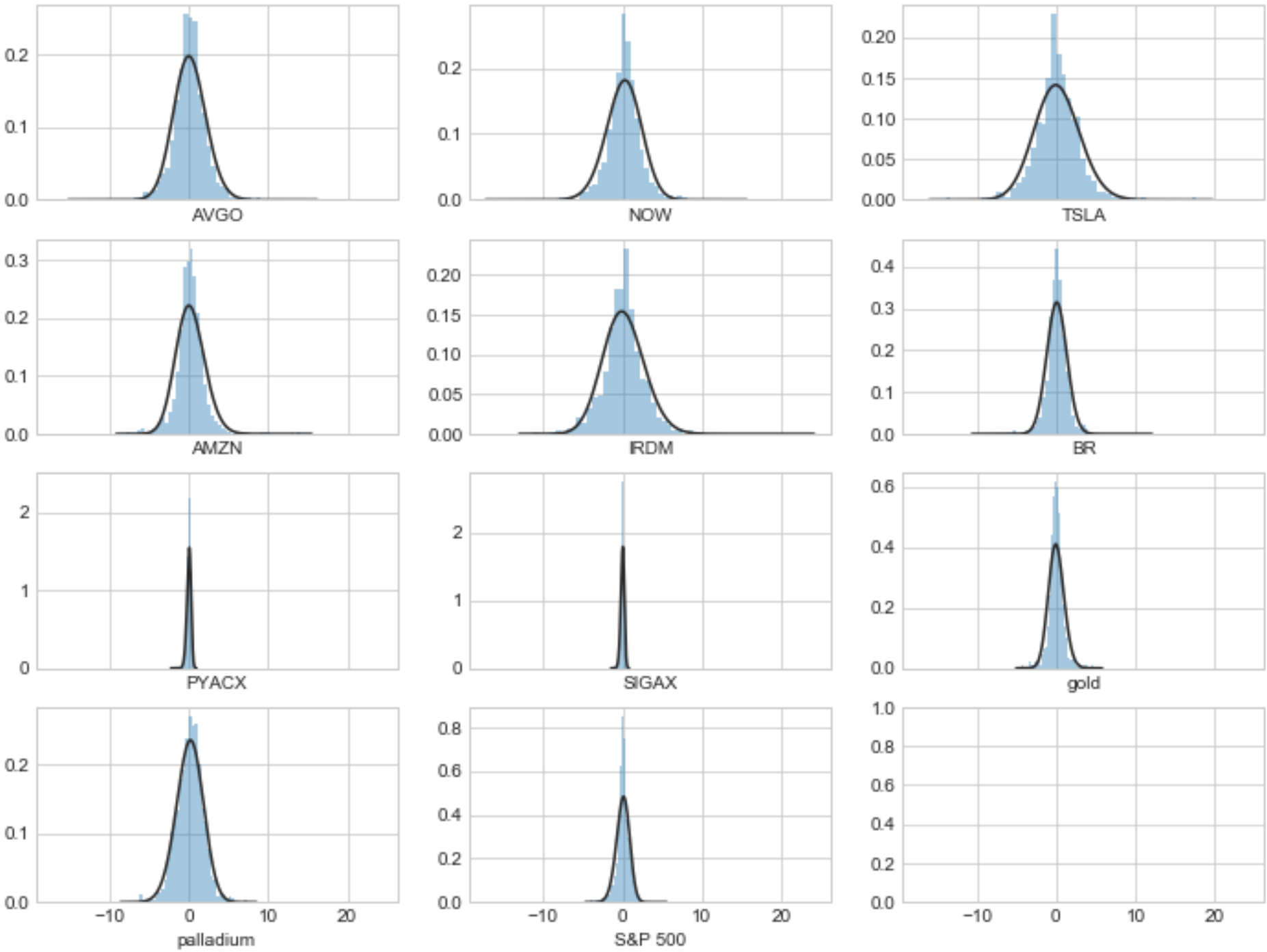
- how "spread out" are the data around the mean
- measures confidence in statistics $\Longrightarrow$ risk in finance
- BUT assumes normal distribution $\rightarrow$ Skewness, Kurtosis


AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500

# Measures of Variability

## Minimum & Maximum

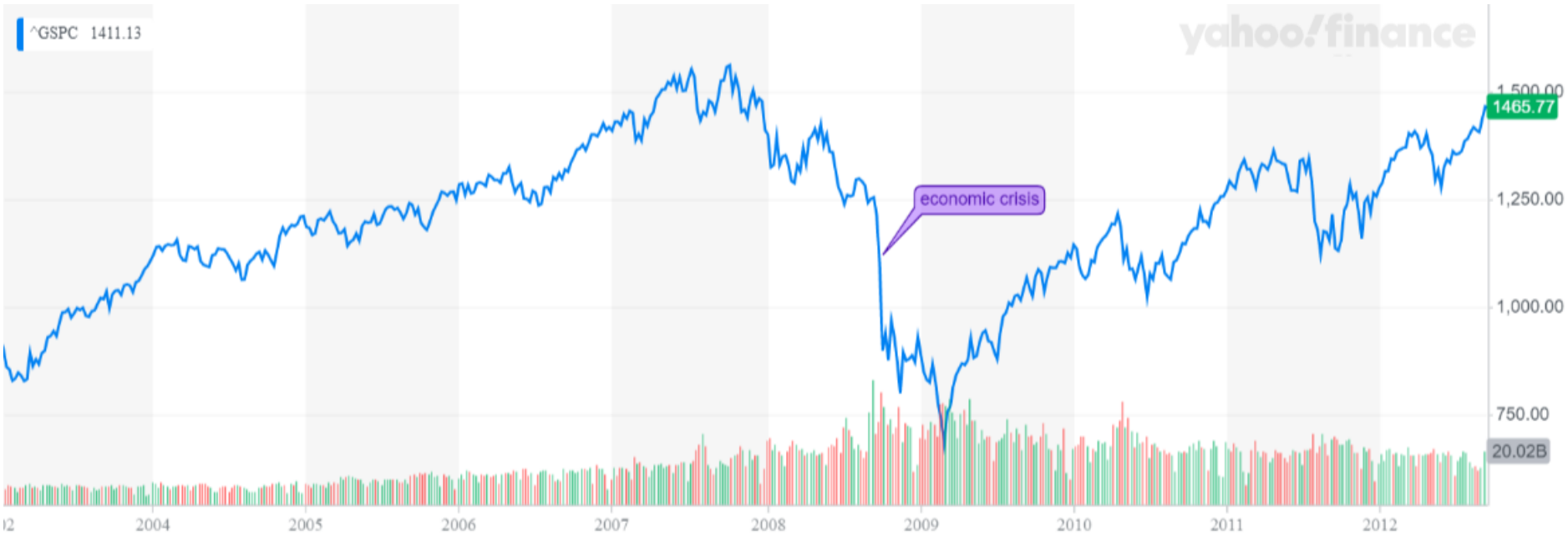
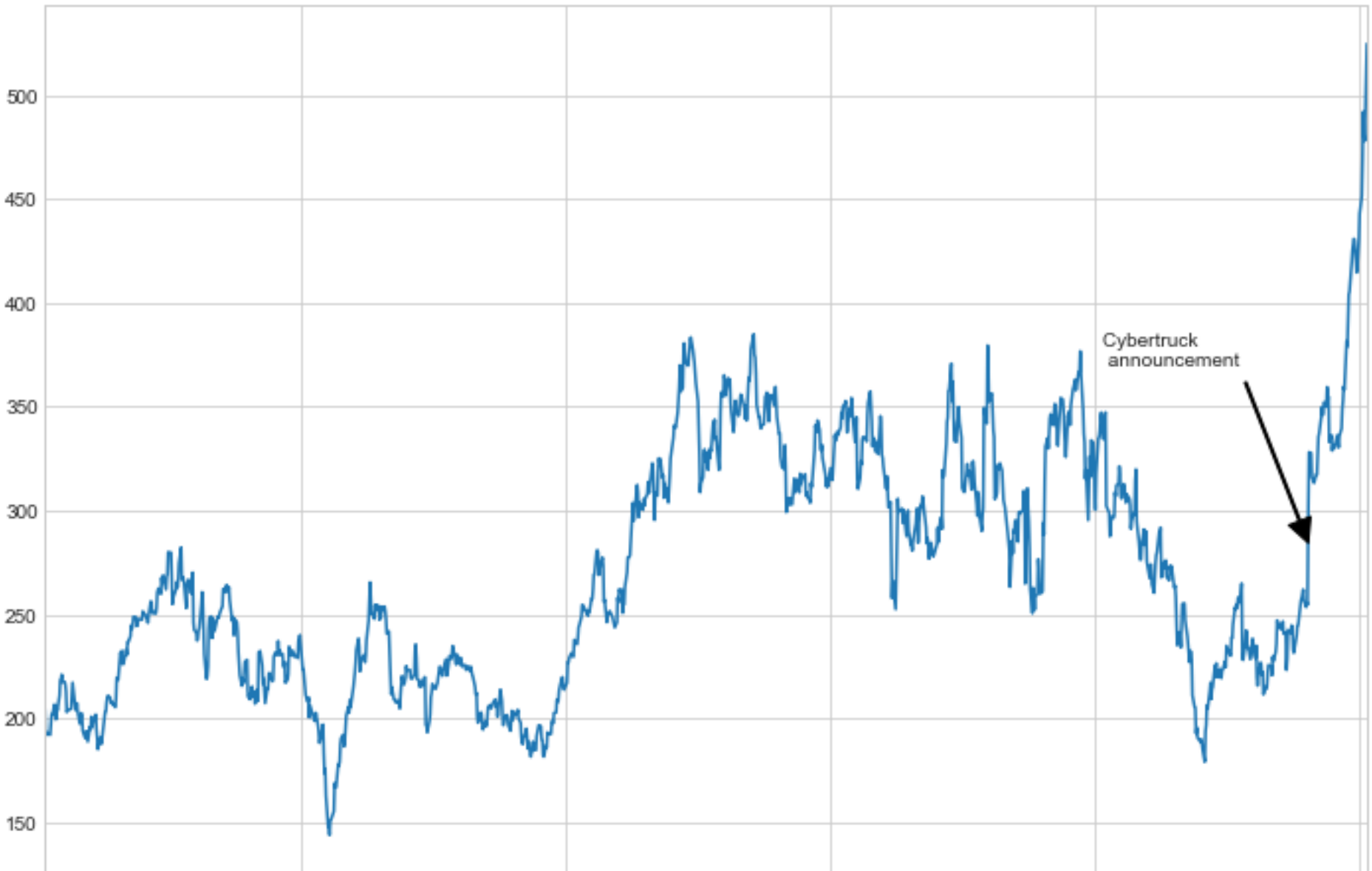



Figure: The minimum of the `S&P500` returns would occur on the day of the economic crisis for this period.

# Measures of Variability

## Minimum & Maximum



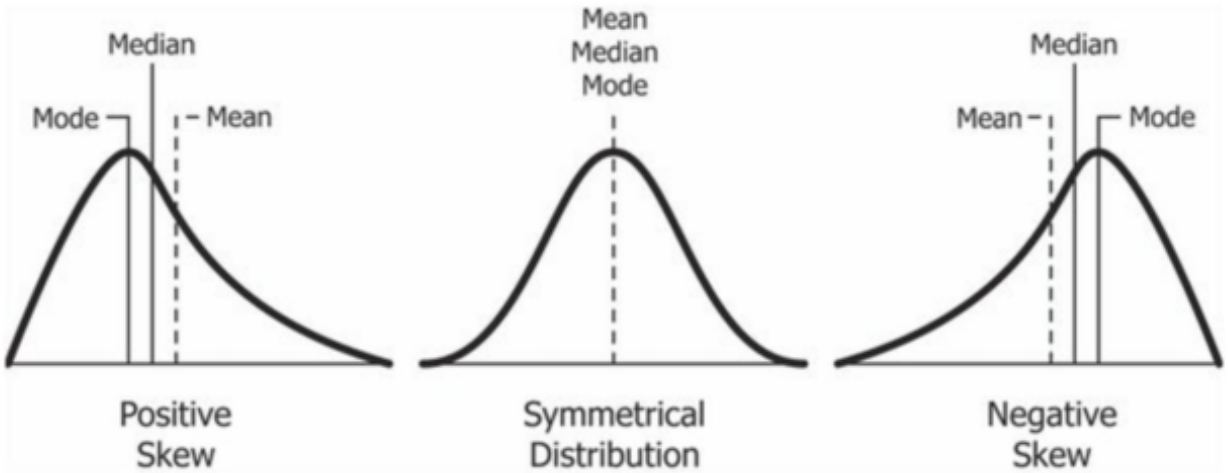
# Measures of Variability

## Skewness

### Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



## Definition

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^4\right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- high kurtosis $\rightarrow$ heavy tails **(outliers)**
- low kurtosis $\rightarrow$ no outliers

# Measures of Variability

## An overview from an investor's perspective

### $\tilde{\mu}_2$ standard deviation $\sigma$

- measure of risk
- assumes normal distribution

### $\tilde{\mu}_3$ skewness

- measure of asymmetry: tail on the left/right
- $\tilde{\mu}_3 > 0$: frequent small losses, few large gains
- $\tilde{\mu}_3 > 0$: frequent small gains, few large losses

### $\tilde{\mu}_4$ kurtosis

- heavy- or light-tailed (relative to a normal distribution)
- high: occasional extreme returns (either positive or negative)

# Descriptive Statistics

## For our assets

| | mean% | median% | std% | var% | min% | max% | kurtosis | skewness |
|---|---|---|---|---|---|---|---|---|
| AVGO | 0.11 | 0.11 | 2.04 | 4.17 | -13.74 | 14.71 | 5.31 | 0.23 |
| NOW | 0.15 | 0.21 | 2.23 | 4.96 | -15.66 | 14.07 | 6.50 | -0.33 |
| TSLA | 0.12 | 0.06 | 2.85 | 8.13 | -13.90 | 17.67 | 5.06 | 0.30 |
| AMZN | 0.17 | 0.14 | 1.85 | 3.42 | -7.82 | 14.13 | 9.83 | 1.01 |
| IRDM | 0.12 | 0.12 | 2.63 | 6.94 | -11.13 | 22.24 | 6.47 | 0.62 |
| BR | 0.09 | 0.09 | 1.27 | 1.61 | -9.70 | 11.16 | 9.29 | -0.19 |
| PYACX | 0.00 | 0.00 | 0.27 | 0.07 | -2.08 | 0.77 | 3.40 | -0.71 |
| SIGAX | 0.00 | 0.00 | 0.23 | 0.05 | -1.29 | 0.67 | 1.28 | -0.43 |
| gold | 0.02 | 0.00 | 0.99 | 0.98 | -4.32 | 5.10 | 4.56 | 0.33 |
| palladium | 0.09 | 0.15 | 1.71 | 2.92 | -7.40 | 7.28 | 1.49 | -0.15 |
| S&P 500 | 0.04 | 0.05 | 0.84 | 0.71 | -4.10 | 4.96 | 3.87 | -0.47 |

# Covariance

## Covariance Definition

Let $X$ and $Y$ be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X,Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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- not so helpful!

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- not so helpful! $\rightarrow$ correlation

# Correlation

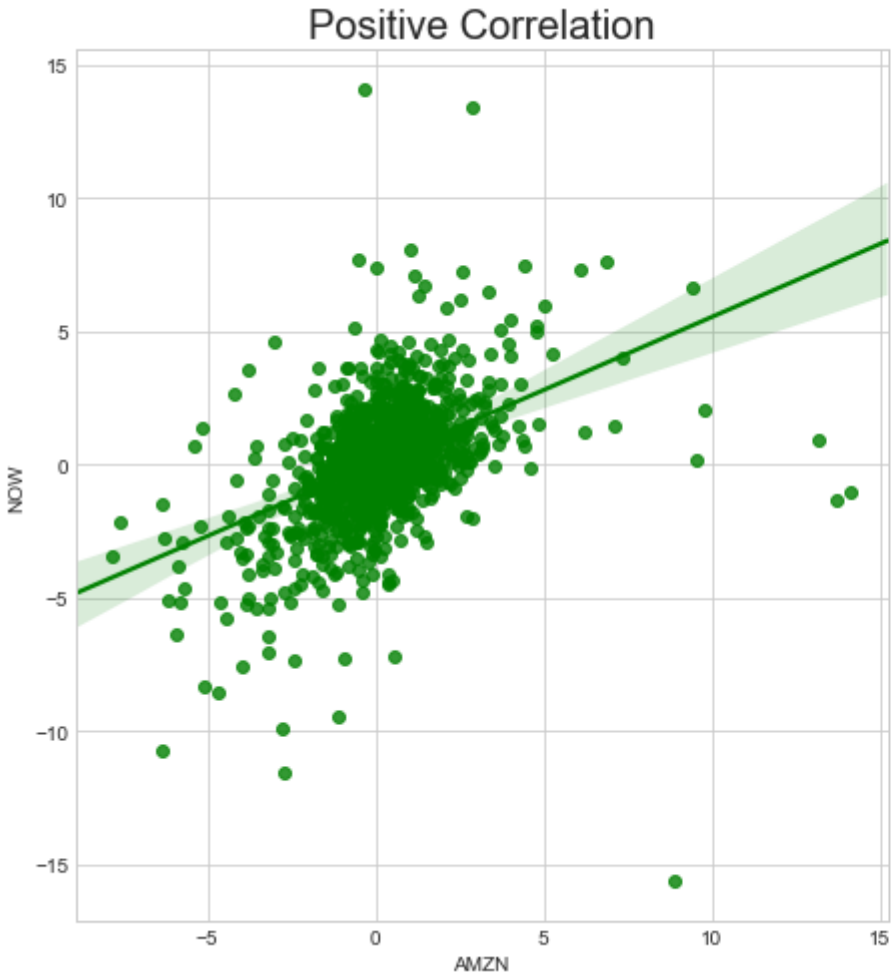
## Definition

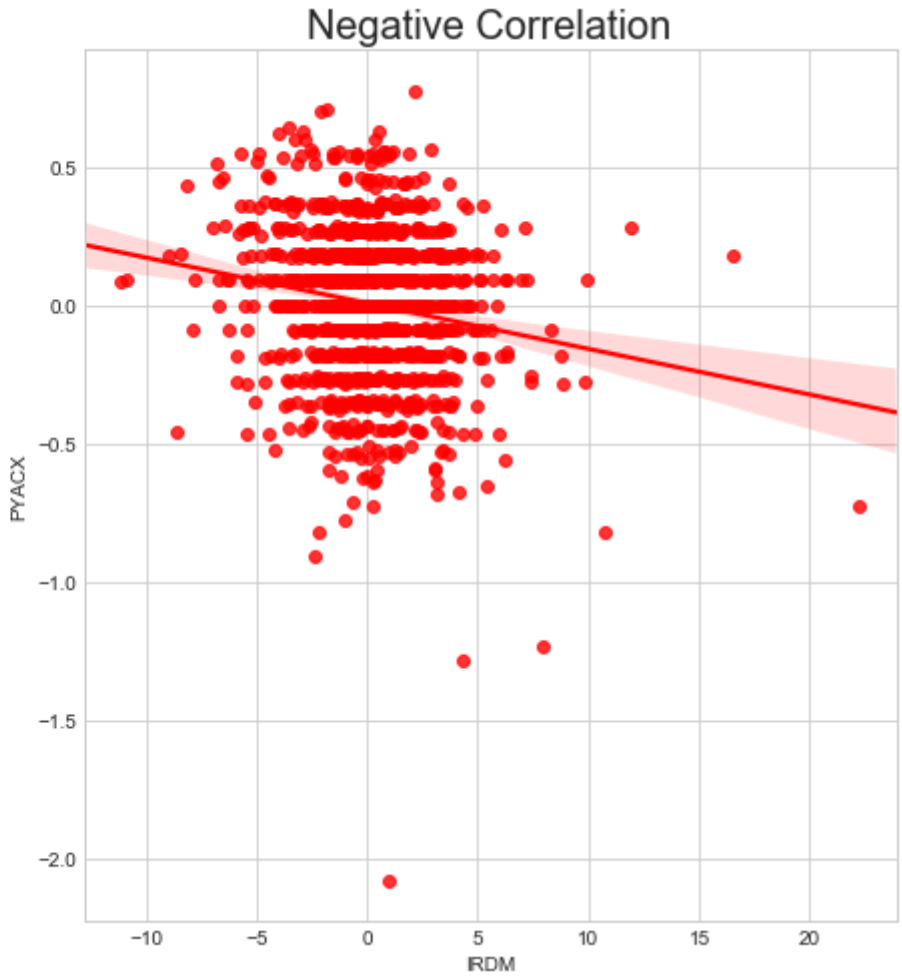
The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

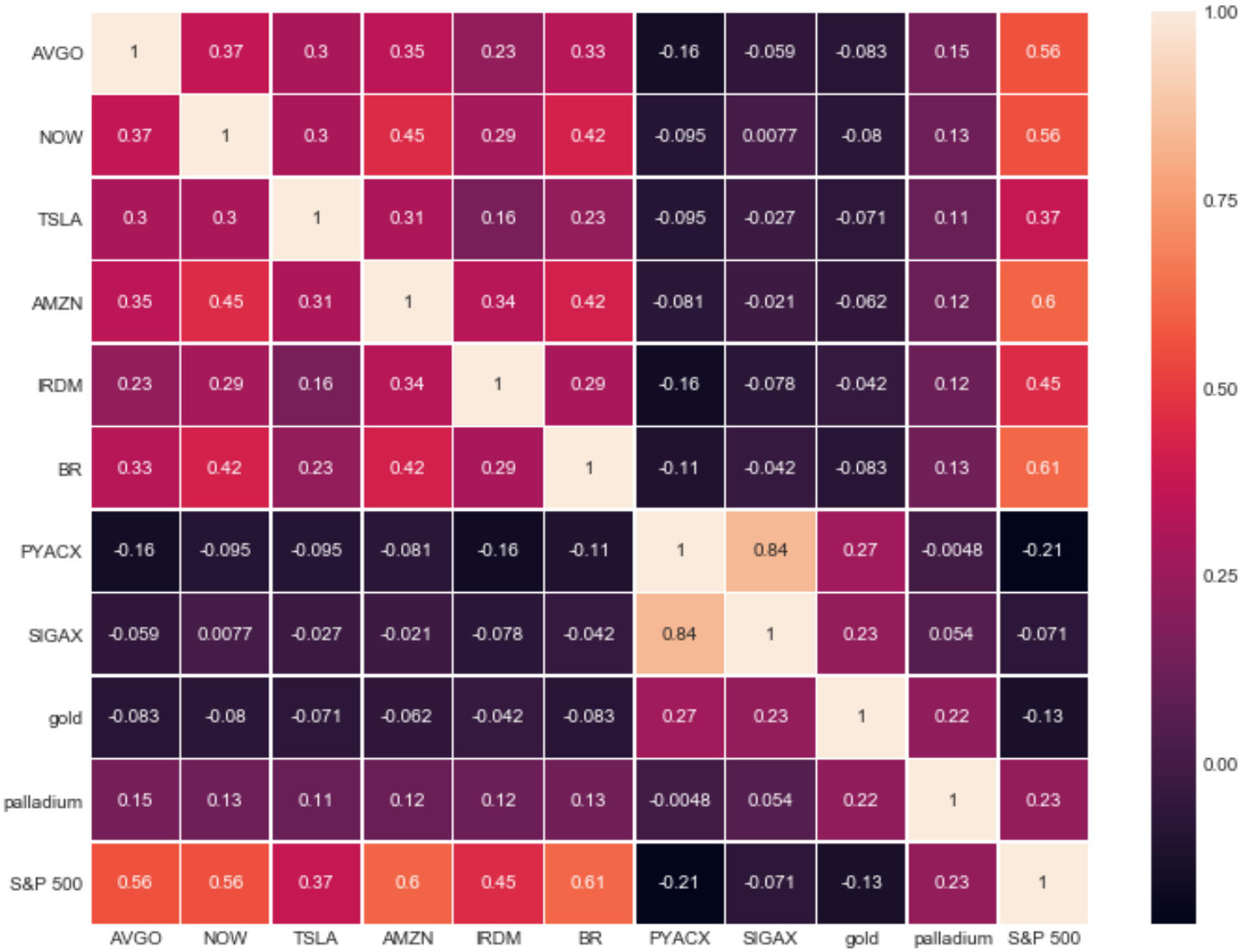
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1,1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

# A closer look at correlation



# Correlation Matrix



# Beta

## Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk, also called unsystematic risk.

$$\beta = \frac{\text{cov}(R_e, R_m)}{\text{var}(R_m)}$$

# alpha

## Definition

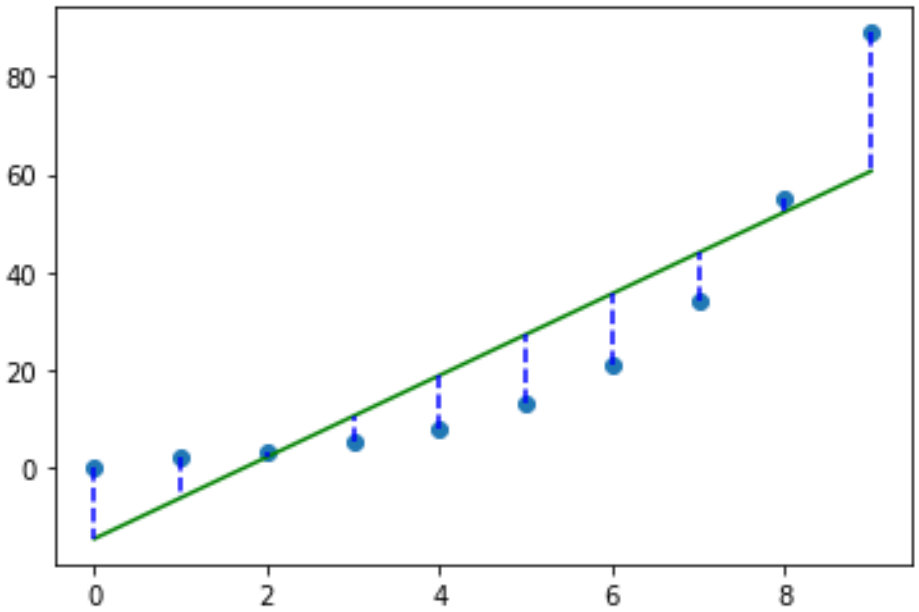
Alpha is the difference between the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

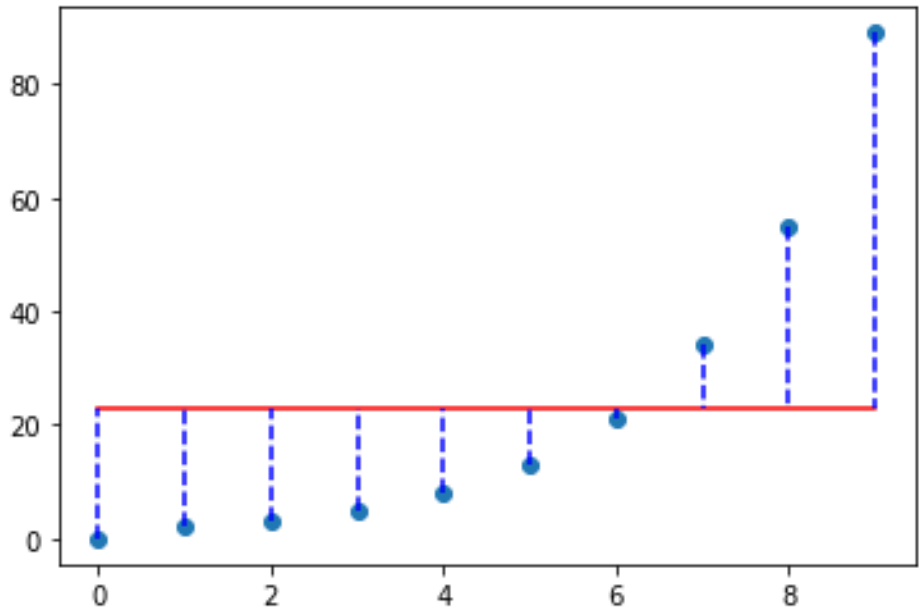
$$\overset{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}$$

# $R$-squared

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$




(a) Explained Variation




(b) Total Variation

# Sharpe Ratio

## Definition

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

measures the risk-adjusted return

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$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

measures the risk-adjusted return
i.e. the average return earned *in excess* of the risk-free rate per unit of volatility or total risk.

# Financial Metrics

## For our assets

| | alpha | beta | R-squared |
|---|---|---|---|
| AVGO | 0.14213 | 1.35333 | 0.31393 |
| NOW | 0.24718 | 1.46614 | 0.30955 |
| TSLA | 0.17711 | 1.26445 | 0.14046 |
| AMZN | 0.32815 | 1.32199 | 0.36539 |
| IRDM | 0.15867 | 1.41795 | 0.20698 |
| BR | 0.13206 | 0.92064 | 0.37655 |
| PYACX | -0.00576 | -0.06792 | 0.04613 |
| SIGAX | -0.01215 | -0.01911 | 0.00501 |
| gold | 0.05087 | -0.15174 | 0.01684 |
| palladium | 0.17817 | 0.47398 | 0.05493 |

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# Risk and return

## Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

## Risk

$$\sigma_p = \sqrt{\vec{w}^\top K \vec{w}}$$

$$= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}}$$

$$= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}$$

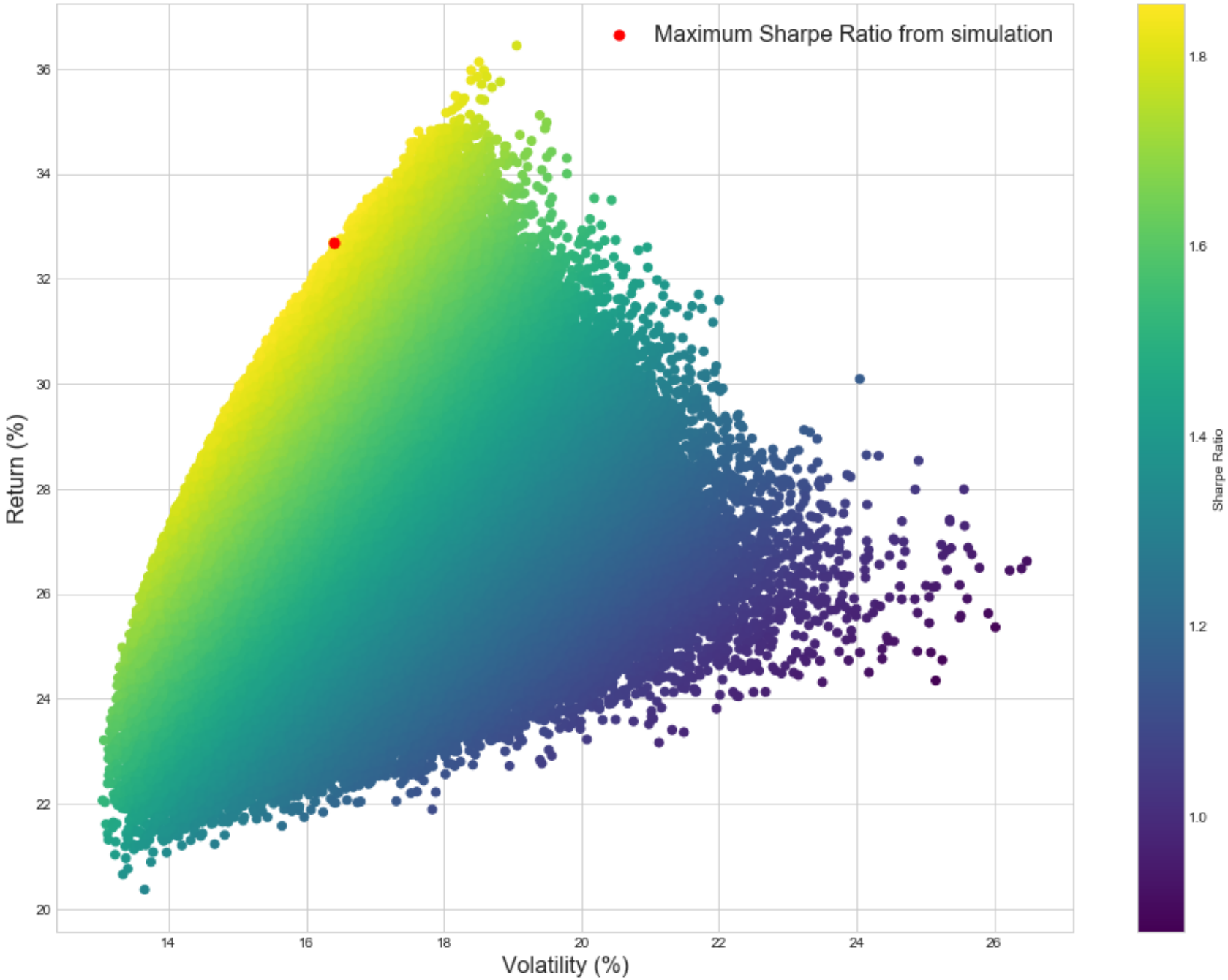
# Naive weight allocation

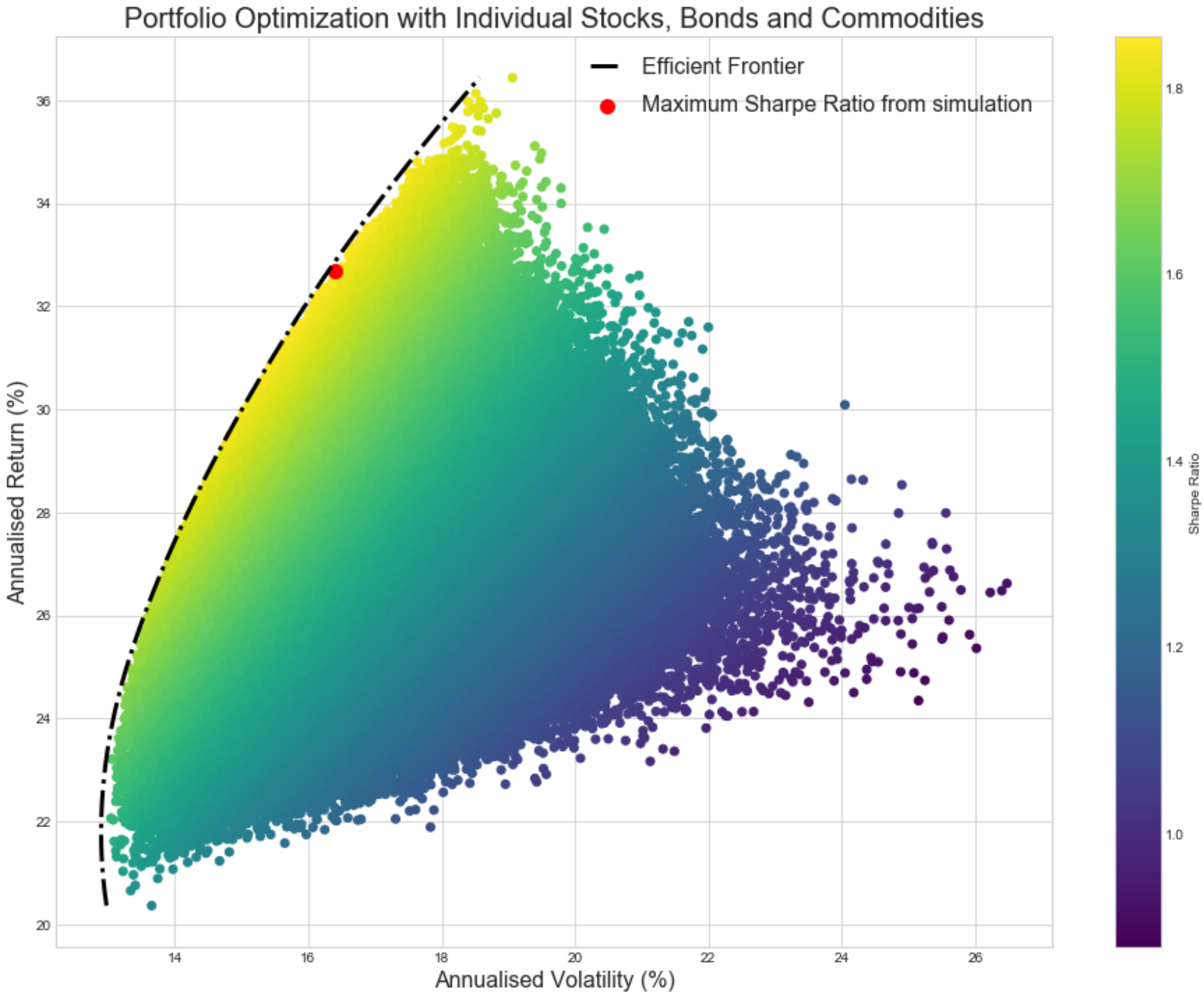
## Remember!

- 70% Stocks
- 20% Bonds
- 10% Commodities

## What about $\vec{w}$?

- 70% for 6 Stocks $\rightarrow \frac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow$ 10% each
- 10% for 2 Commodities $\rightarrow$ 5% each

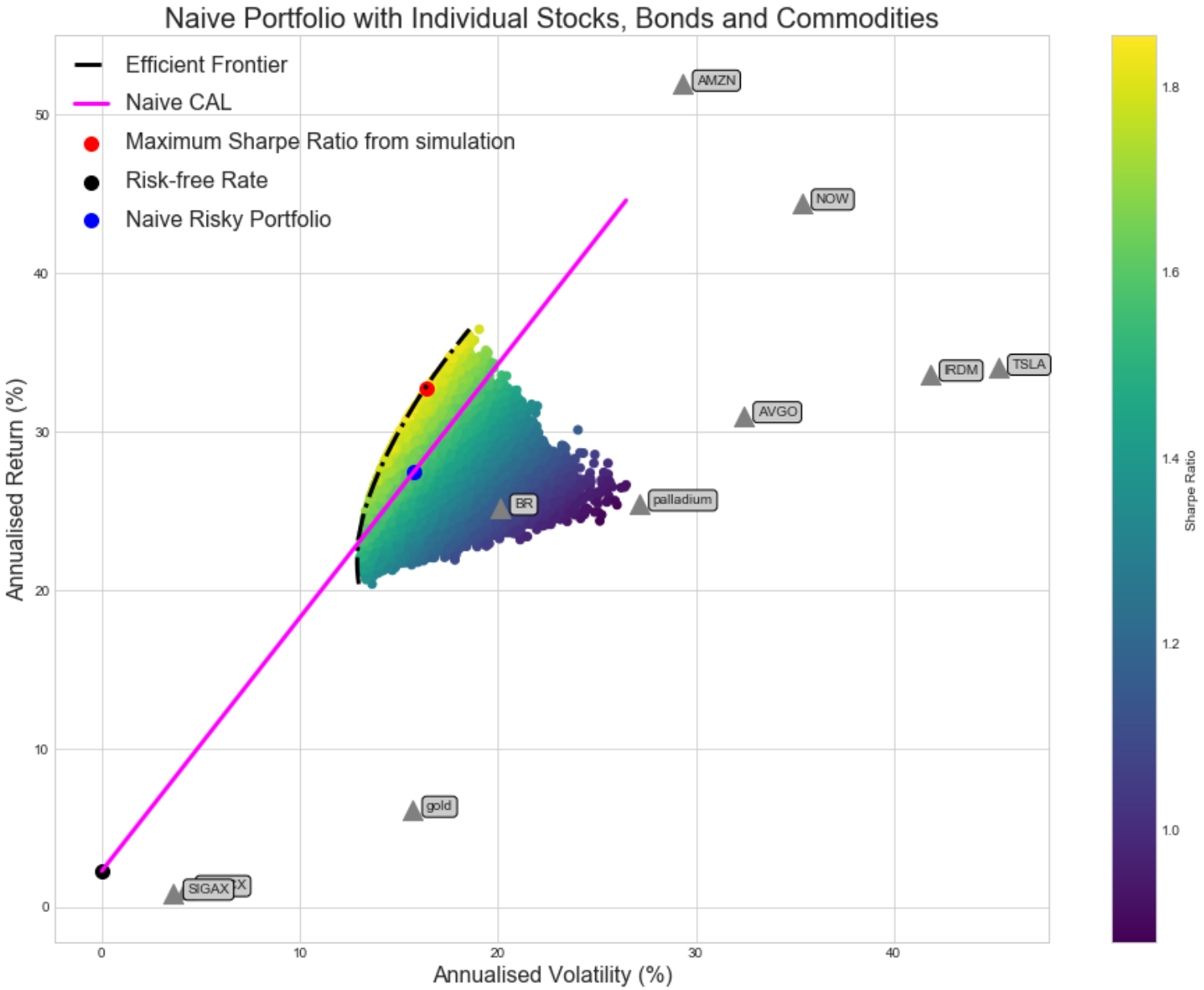

Maximum Sharpe Ratio from simulation
Return (%)
Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Maximum Sharpe Ratio from simulation
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0

# CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \Longrightarrow \epsilon_{naive} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$

# Naive CAL


Naive Portfolio with Individual Stocks, Bonds and Commodities
Efficient Frontier
Naive CAL
Maximum Sharpe Ratio from simulation
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

# But...

We can do better!

# Optimal portfolio

## Optimization problem formulation

$$
\begin{aligned}
\max \quad & \frac{\vec{w}^{\top} \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^{\top} K \vec{w}}} \\
\text{s.t.} \quad & \mathbf{1}^{\top} \vec{w} = 1 \\
& \mathbf{1}^{\top} \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^{\top} \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^{\top} \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

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## Optimization problem formulation

Sharpe Ratio

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$$\text{s.t.} \quad \mathbf{1}^\top \vec{w} = 1$$

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$$w_s + w_b + w_c = 1$$

$$w_i \geq 0 \quad i = 1, \ldots, n$$

# Optimization yields...

## optimal weights

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

# Optimization yields...

## Financial Metrics

| | Optimal Risky Portfolio |
|---|---|
| Return (%) | 32.7164 |
| Risk (%) | 16.3185 |
| Sharpe Ratio | 1.8658 |
| Beta ($\beta$) | 0.9067 |

# Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{CAL} \\ (\sigma_{OPT}, r_{OPT}) \in \epsilon_{CAL} \end{array} \right\}$$

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$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{OPT}}, r_{\text{OPT}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \Longrightarrow \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{OPT}} - r_f}{\sigma_{\text{OPT}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$

# Optimal Overall Portfolio

## Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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## Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

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Maximize w.r.t. $z$:

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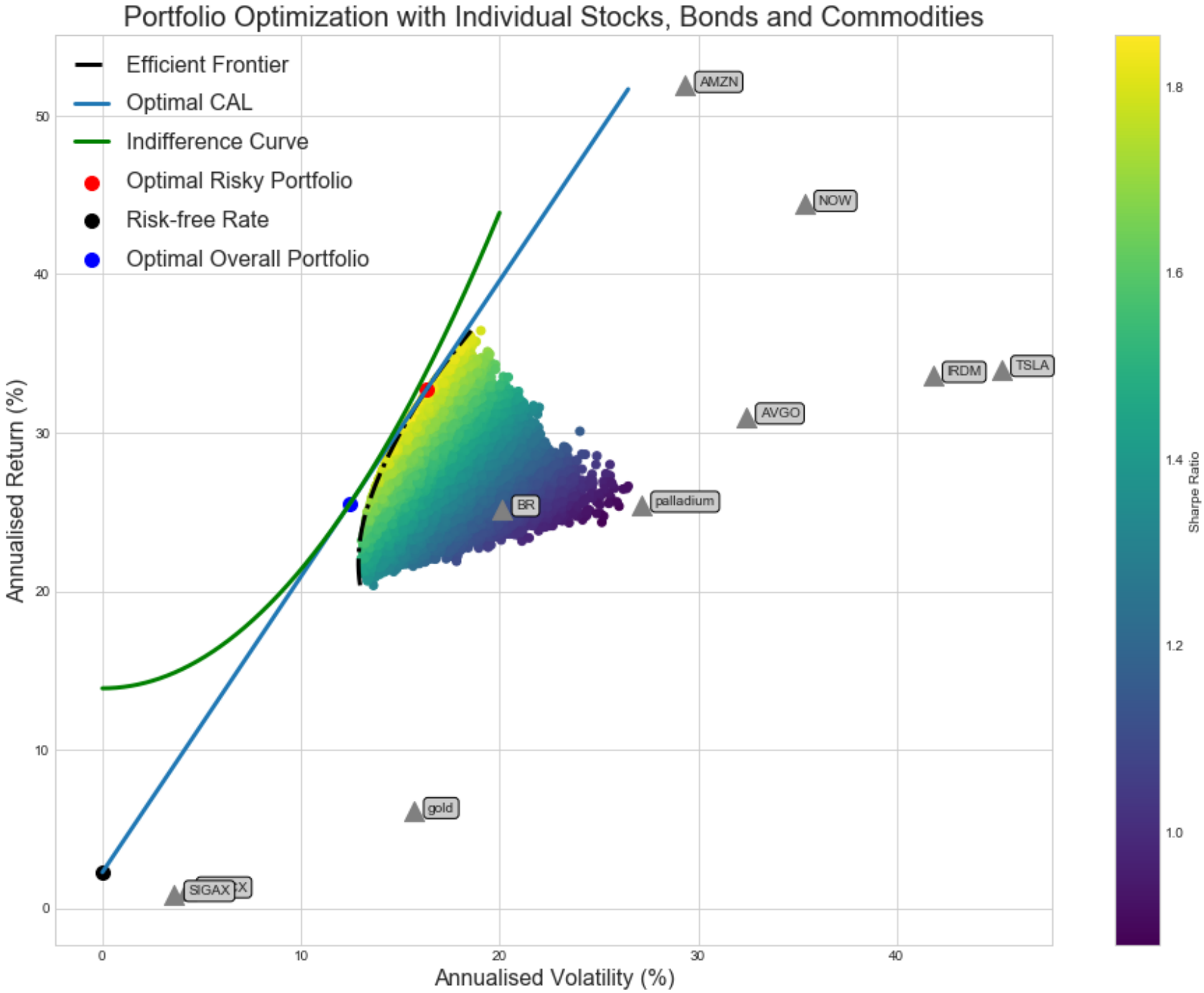
Maximize w.r.t. $z$:

$$\frac{\partial U}{\partial z} = 0$$

$$\implies r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\implies z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$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# Past Performance
(1/13/2015 - 1/13/2020)

| | Optimal Overall Portfolio | S&P 500 |
|---|---|---|
| Return (%) | 25.4769 | 11.3118 |
| Risk (%) | 12.4384 | 13.4138 |
| Sharpe Ratio | 1.8658 | 0.6741 |
| Beta ($\beta$) | 0.6911 | 1.0 |
| Alpha ($\alpha$) (%) | 16.9584 | 0.0 |

# References

# The end!