Portfolio Management
FN 4329

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American College of Greece

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Contents

todo

Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

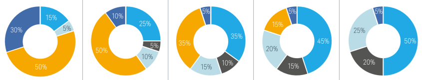
Questionnaire

SELECT AN INVESTMENT STRATEGY

These investment strategies show how investors might allocate their money among investments in various categories. Please note that these examples are not based on market forecasts, but simply reflect an established approach to investing—allocating dollars among different investment categories. Keep in mind that it's important to periodically review your investment strategy to make sure it continues to be consistent with your goals.

If one of the investment strategies below matches your Investor Profile, you can use this information to help you create an asset allocation plan.

Conservative allocation	Moderately conservative	Moderate allocation	Moderately aggressive	Aggressive allocation
Average annual return: 7.6% Best year: 22.8% Worst year: -4.6%	Average annual return: 8.8% Best year: 27.0% Worst year: -12.5%	Average annual return: 9.5% Best year: 30.9% Worst year: -20.9%	Average annual return: 10.0% Best year: 34.4% Worst year: -29.5%	Average annual return:10.3% Best year: 39.9% Worst year: -36.0%
For investors who seek current income and stability and are less concerned about growth.	For investors who seek current income and stability, with modest potential for increase in the value of their investments.	For long-term investors who don't need current income and want some growth potential. Likely to entail some fluctuations in value, but presents less volatility than the overall equity market.	For long-term investors who want good growth potential and don't need current income. Entails a fair amount of volatility, but not as much as a portfolio invested exclusively in equities.	For long-term investors who want high growth potential and don't need current income. May entail substantial year-to-year volatility in value in exchange for potentially high long-term returns.



■ Large-Cap Equity ■ Small-Cap Equity ■ International Equity ■ Fixed Income ■ Cash Investments

Brokerage Products: Not FDIC-Insured ▪ No Bank Guarantee ▪ May Lose Value

Source: Schwab Center for Financial Research with data provided by Morningstar, Inc. The return figures for 1970-2017 are the compounded annual average and the minimum and maximum annual total returns of hypothetical asset allocation plans. The asset allocation plans are weighted averages of the performance of the indices used to represent each asset class in the plans, include reinvestment of dividends and interest, and are rebalanced annually. The indices representing each asset class in the historical asset allocation plans are S&P 500® Index (large-cap stocks); CRSP 6–8 Index for the period 1970–1978 and Russell 2000® Index for the period 1979–2017 (small-cap stocks); MSCI EAFE® Net of Taxes (international stocks); Ibbotson Intermediate-Term Government Bond Index for the period 1970–1975 and Bloomberg Barclays U.S. Aggregate Bond Index for the period 1976–2017 (fixed income); and Ibbotson U.S. 30-day Treasury Bill Index for the period 1970–1977 and Citigroup 3-month U.S. Treasury Bills for the period 1978–2017 (cash investments). Indices are unmanaged, do not incur fees or expenses, and cannot be invested in directly. Past performance is no guarantee of future results.

charles SCHWAB

Own your tomorrow.

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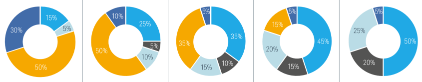

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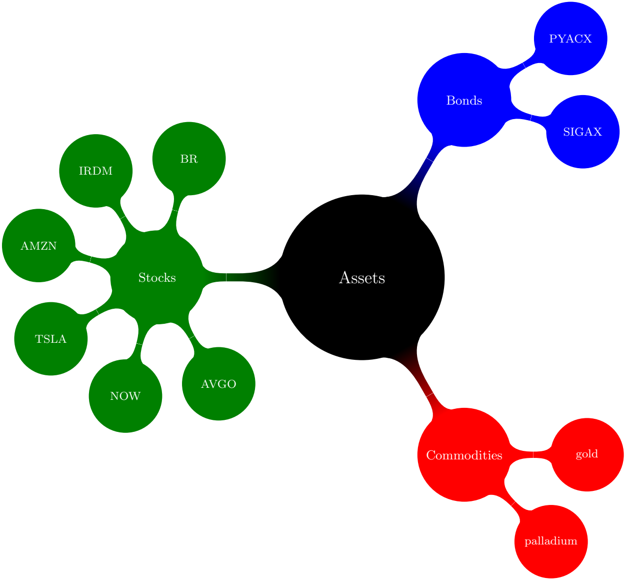
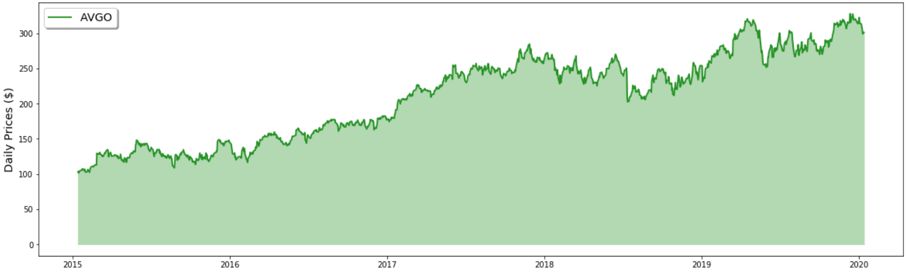
Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

Security Selection



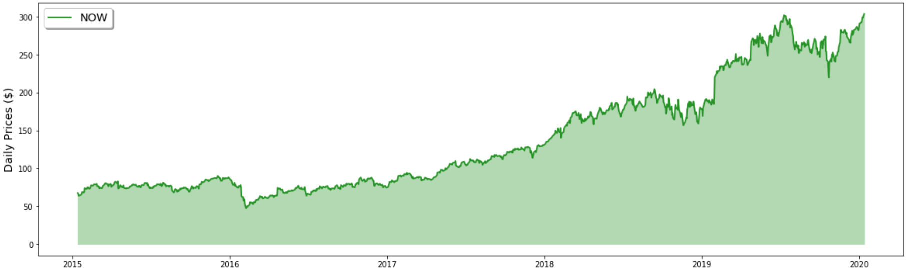
- Large/High-Dividend
- Wireless chips and semiconductor manufacturer
- Announced 15$ billion deal with Apple Inc. in January 2020
- Potential for gains with 5G deployment



Stocks

NOW - ServiceNow Inc.

- Large/Growth
- Cloud-based solutions provider to global enterprises
- Software as a Service (SaaS) business model
- IT, Customer Support, HR and Security services



Stocks

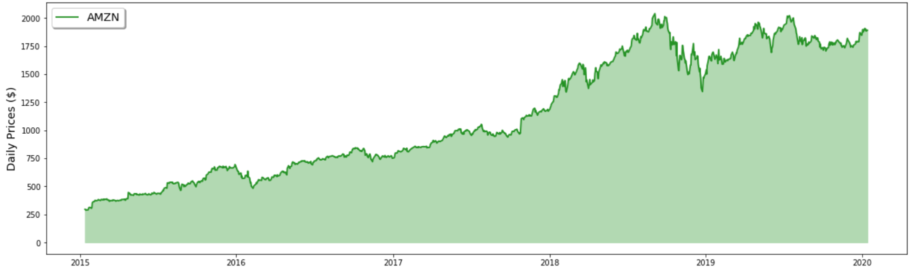
TSLA - Tesla Inc.

- Large/Growth
- Leader in next-generation electric vehicles
- Advancements in battery technology will allow for increased autonomy
- 4^{th} Gigafactory in Berlin to begin operations by July 2021

Stocks

AMZN - Amazon Inc.

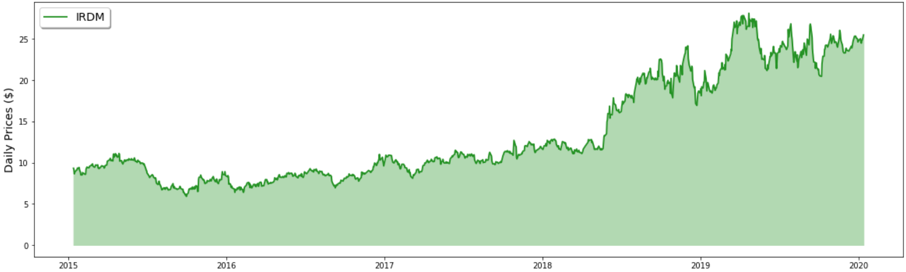
- Large/Growth
- Biggest online retailer
- Online product and digital media sales
- AWS offers solutions for Machine Learning, Big Data, IoT and Cloud-Computing



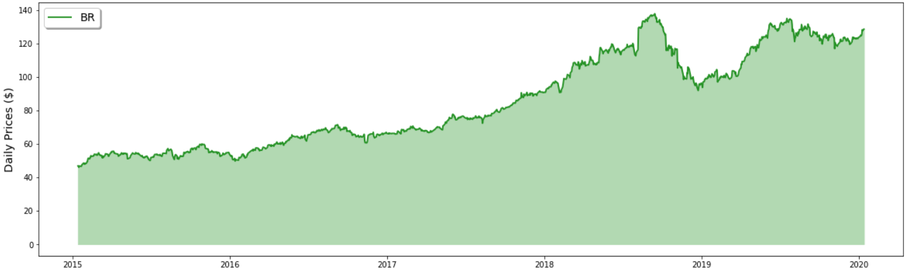
Stocks

IRDM - Iridium Communications Inc.

- Small/Growth
- Lead provider of satellite communications, with over 70 satellites in orbit
- Announced partnership with AWS for future applications in 2018
- Wide commercial end base, from maritime and aviation to oil & gas suppliers



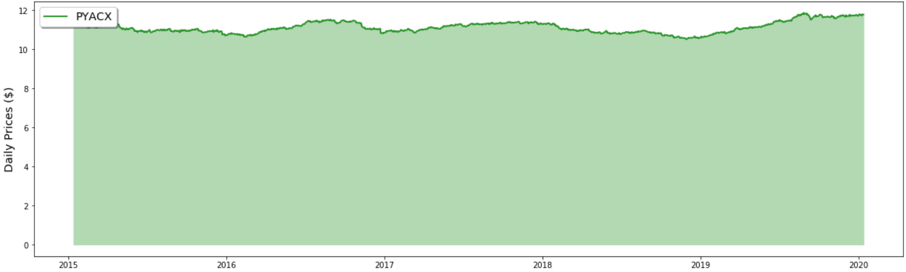
- Mid/Aggressive Growth
- Fintech company that provides solutions for investors, banks, brokerage offices and mutual funds
- Products include communication platforms, securities processing and financial data analytics



Bonds

PYACX - Payden Corporate Bond Mutual Fund

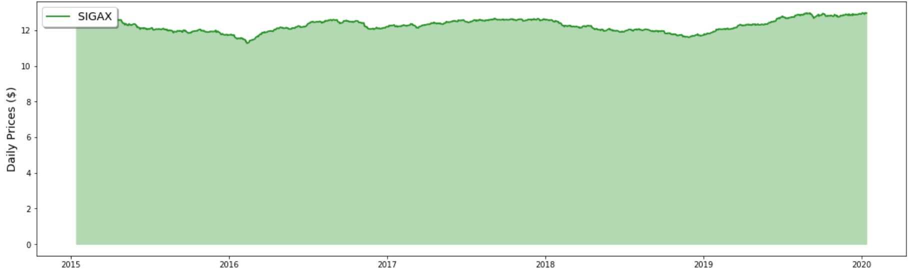
- The fund invests in a variety of debt instruments and income generations securities, with at least 85% of its assets allocated in corporate bonds of mainly A and BBB credit rating
- 5-star ranking from Morningstar



Bonds

SIGAX - Western Asset Corporate Bond Mutual Fund

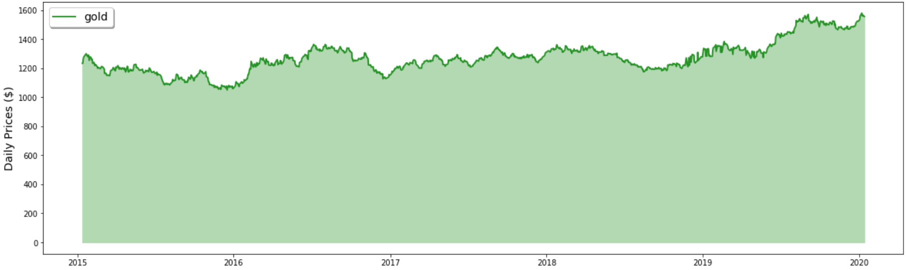
- The fund invests at least 80% of its assets in corporate debt securities, while 10% of the assets might be non-U.S dollar nominated fixed income securities of foreign issuers.



Commodities

gold

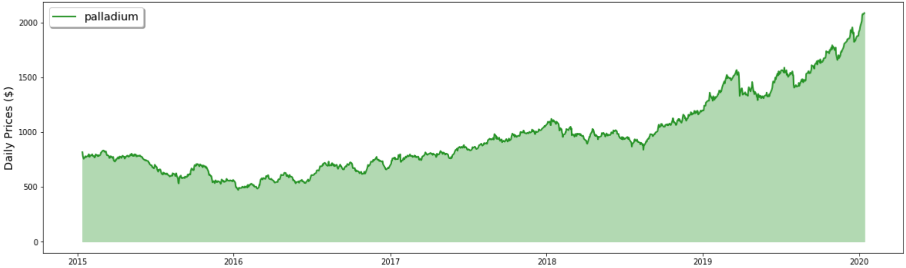
- Traditionally safe investment during rough economic times and hedge against inflation
- Recently preferred over government bonds, as the latter presented negative inflation-adjusted returns in 2019
- Does not generate return and has no holding costs



Commodities

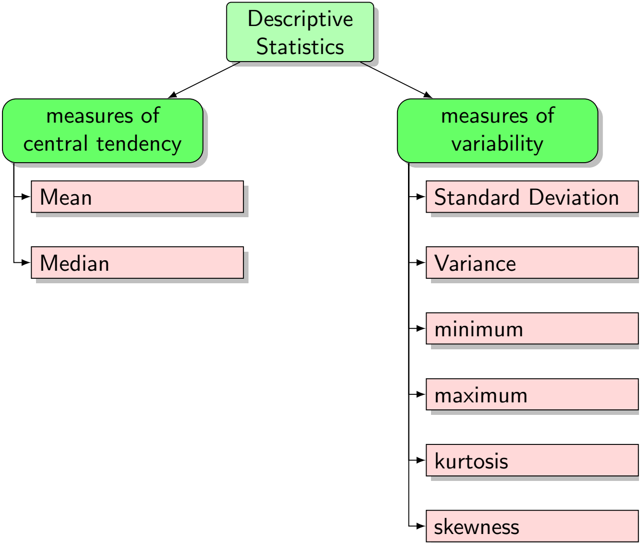
palladium

- Metal that is used in catalytic converters, turning toxic gas emissions into less harmful ones
- Secondary product from mining operations of other metals
- As miners have less control over the extracted quantities, demand outstrips supply
- Price has doubled over the last year



Contents

Descriptive statistics Taxonomy



Measures of central tendency

Mean

Definition

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

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- The arithmetic mean of sampled values
- From the investor's perspective, the mean describes the average performance of the instrument. If $\bar{x} > 0$ then the instrument increases in value on average.

Definition

The middle value of a given dataset, separating the higher and lower half

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Definition

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- Usually preferred over the arithmetic mean
- Why? robust w.r.t. outliers
- Indicates whether returns are positive or negative on most time instances.

Definition

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

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- Measures confidence in statistics $\Longrightarrow$ risk in finance

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- Quantifies how much "spread out" are the data around the mean
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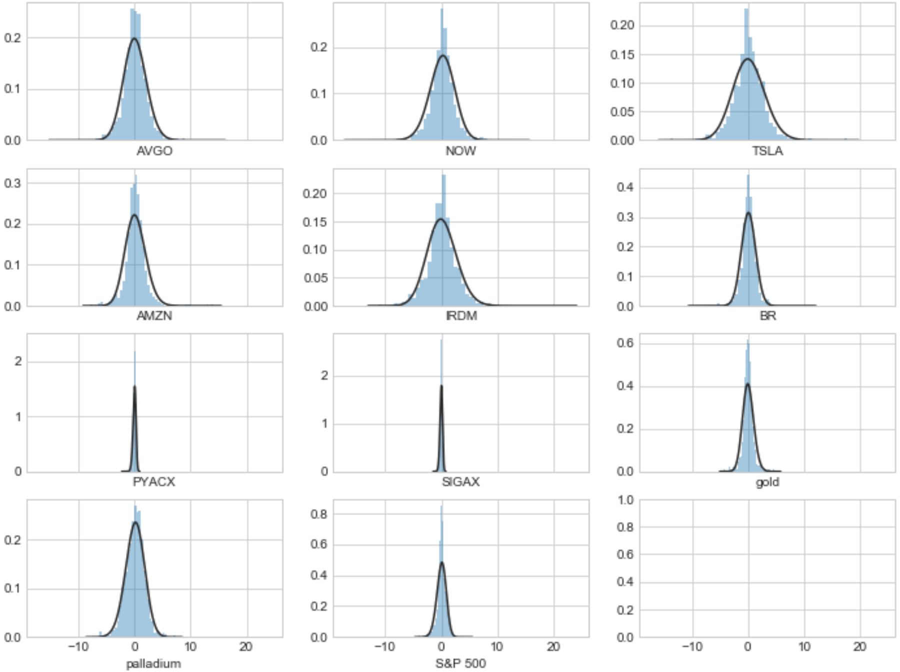
- BUT assumes normal distribution $\rightarrow$ Skewness, Kurtosis


AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500

Measures of Variability
Minimum & Maximum

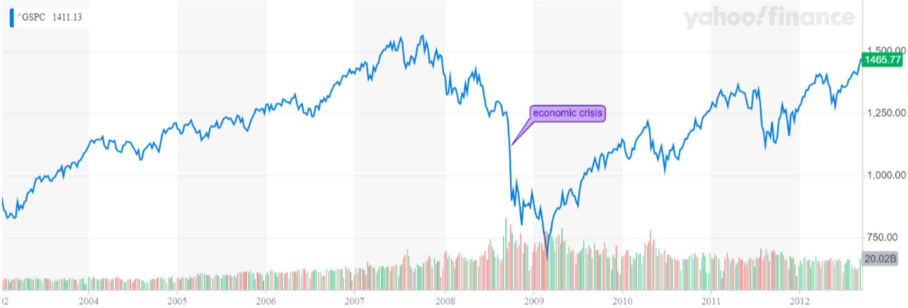
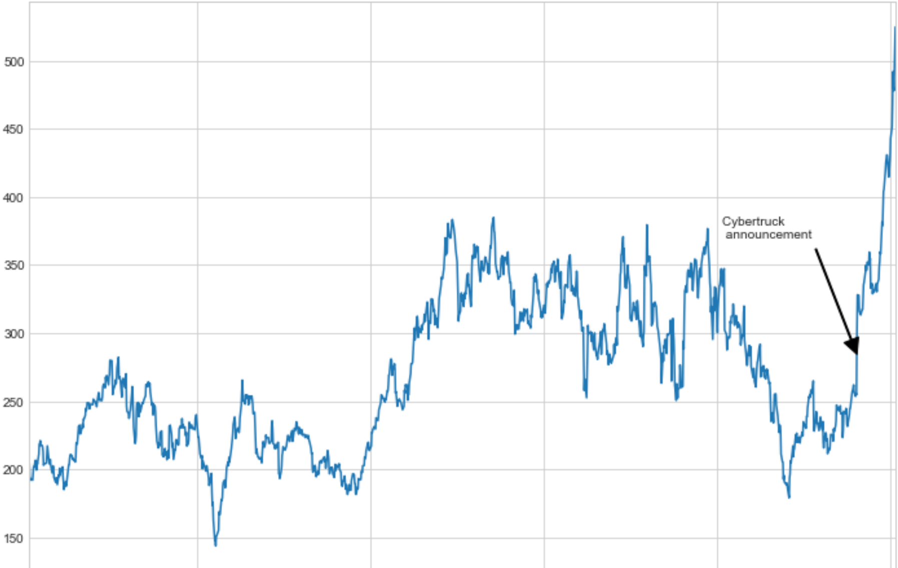


Figure: The minimum of the S&P 500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

Minimum & Maximum



Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

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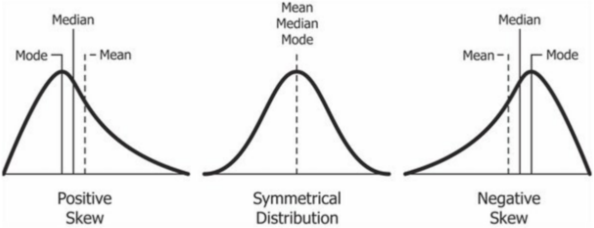

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- High kurtosis $\rightarrow$ heavy tails **(outliers)**
- Low kurtosis $\rightarrow$ no outliers

Measures of Variability

An overview from an investor's perspective

$\tilde{\mu}_2$ standard deviation σ

- Measure of risk
- Assumes normal distribution

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- $\tilde{\mu}_3 > 0$: frequent small losses, few large gains
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$\tilde{\mu}_4$ kurtosis

- Heavy- or light-tailed (relative to a normal distribution)
- High: occasional extreme returns (either positive or negative)

Descriptive Statistics

For our assets

	mean%	median%	std%	var%	min%	max%	kurtosis	skewness
AVGO	0.11	0.11	2.04	4.17	-13.74	14.71	5.31	0.23
NOW	0.15	0.21	2.23	4.96	-15.66	14.07	6.50	-0.33
TSLA	0.12	0.06	2.85	8.13	-13.90	17.67	5.06	0.30
AMZN	0.17	0.14	1.85	3.42	-7.82	14.13	9.83	1.01
IRDM	0.12	0.12	2.63	6.94	-11.13	22.24	6.47	0.62
BR	0.09	0.09	1.27	1.61	-9.70	11.16	9.29	-0.19
PYACX	0.00	0.00	0.27	0.07	-2.08	0.77	3.40	-0.71
SIGAX	0.00	0.00	0.23	0.05	-1.29	0.67	1.28	-0.43
gold	0.02	0.00	0.99	0.98	-4.32	5.10	4.56	0.33
palladium	0.09	0.15	1.71	2.92	-7.40	7.28	1.49	-0.15
S&P 500	0.04	0.05	0.84	0.71	-4.10	4.96	3.87	-0.47

Covariance

Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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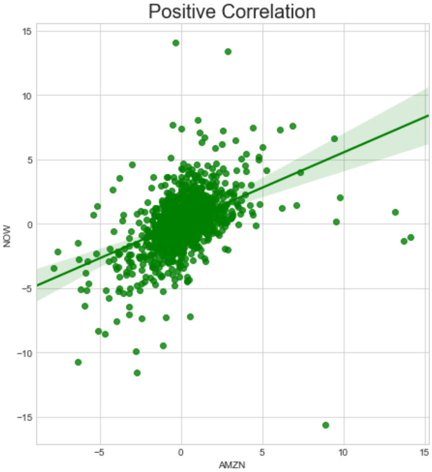
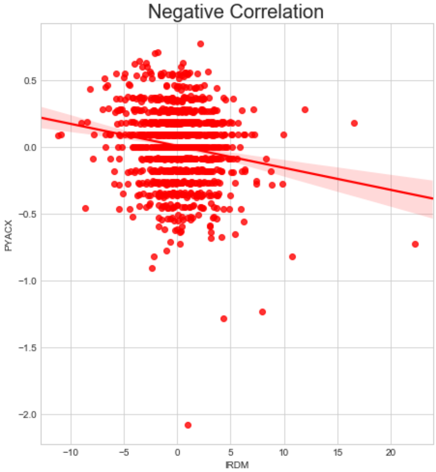
$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1,1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation





Correlation

From an investor's perspective

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- Powerful tool that measures the strength of linear relationship between the price movements of two individual securities
- The total risk of two correlated assets is:

$$\sigma^2 = w_x^2 \cdot \sigma_x^2 + w_y^2 \cdot \sigma_y^2 + 2 \cdot w_x \cdot w_y \cdot \sigma_x \cdot \sigma_y \cdot \rho_{x,y}$$

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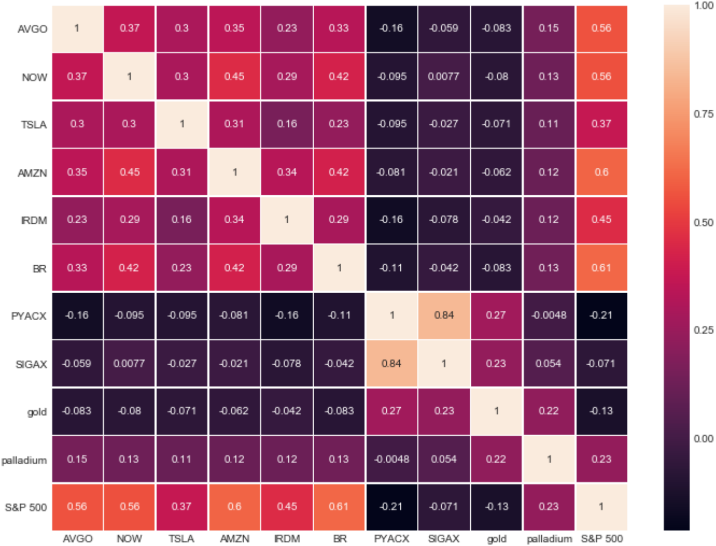
- Negatively correlated assets yield less risk, compared to positively correlated ones
- Investors can diversify risk by including negatively correlated assets in their portfolios

Correlation Matrix



Beta

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The beta coefficient measures the systematic risk of an individual stock compared to the market risk:

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- A high R^2 is required for β to be meaningful

alpha

Definition

Alpha is the difference between the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

$$\overset{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{ R_f + \beta(\mathbb{E}(R_m) - R_f) \right\}$$

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- Investors use α to determine whether or not a security has exceeded expectations in terms of return
- Frequently used to quantify the "added" value of the manager

R-squared

Definition

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$

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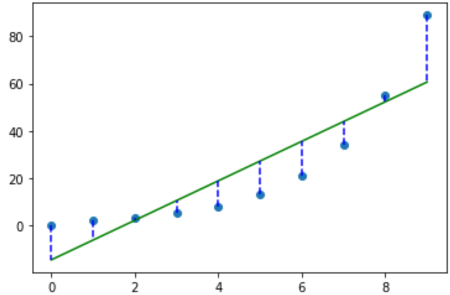
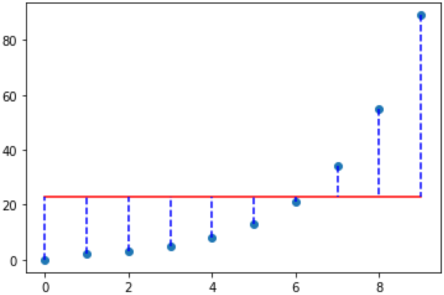


(a) Explained Variation



(b) Total Variation

- $R-squared$ measures the "fitness" of a regression model

Sharpe Ratio

Definition

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- i.e. the average return earned *in excess* of the risk-free rate per unit of volatility or total risk.

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- Measures the risk-adjusted return
- i.e. the average return earned *in excess* of the risk-free rate per unit of volatility or total risk.
- Useful for comparing the performance of investments that have different levels of risk and return

Financial Metrics
For our assets

	alpha	beta	R-squared
AVGO	0.14213	1.35333	0.31393
NOW	0.24718	1.46614	0.30955
TSLA	0.17711	1.26445	0.14046
AMZN	0.32815	1.32199	0.36539
IRDM	0.15867	1.41795	0.20698
BR	0.13206	0.92064	0.37655
PYACX	-0.00576	-0.06792	0.04613
SIGAX	-0.01215	-0.01911	0.00501
gold	0.05087	-0.15174	0.01684
palladium	0.17817	0.47398	0.05493

Contents

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk

$$\begin{aligned}
\sigma_p &= \sqrt{\vec{w}^\top K \vec{w}} \\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}
\end{aligned}$$

Naive weight allocation

Remember!

- 70% Stocks
- 20% Bonds
- 10% Commodities

Naive weight allocation

Remember!

- 70% Stocks
- 20% Bonds
- 10% Commodities

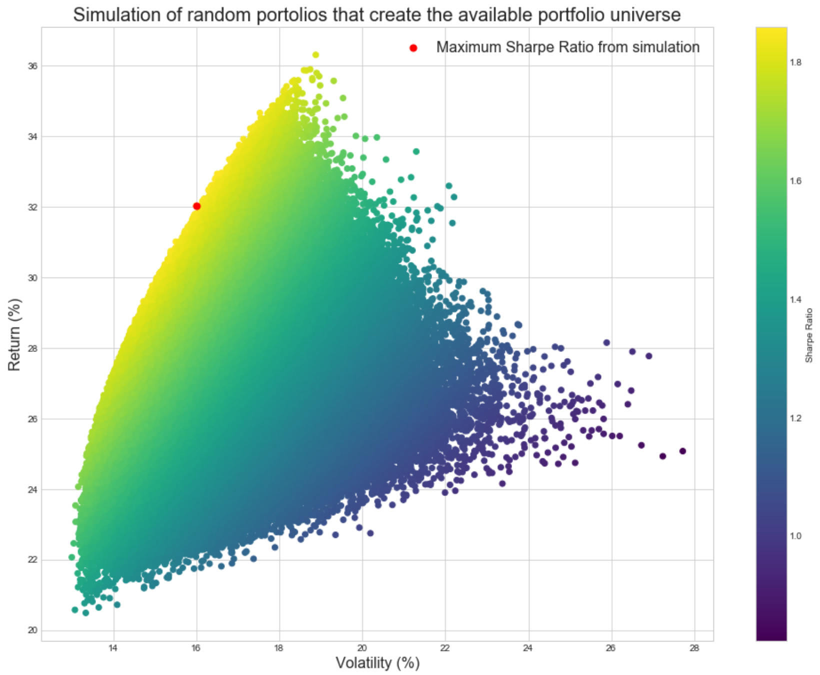
What about $\vec{w}$?

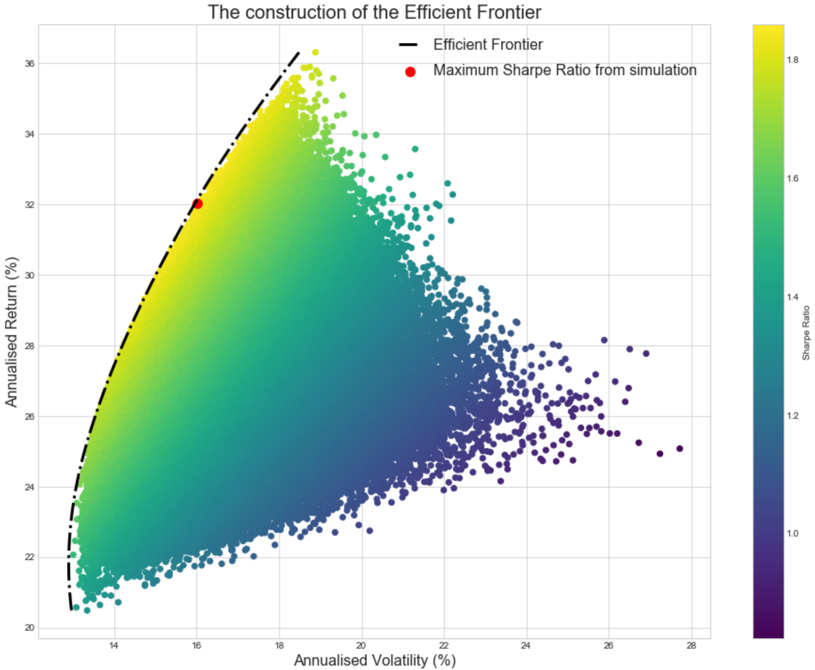
- 70% for 6 Stocks $\rightarrow \dfrac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow$ 10% each
- 10% for 2 Commodities $\rightarrow$ 5% each

Naive Allocation yields...

Financial Metrics

	Naive Risky Portfolio
Return (%)	27.444
Risk (%)	15.7565
Sharpe Ratio	1.5977
Beta (β)	0.9109


Simulation of random portolios that create the available portfolio universe
Maximum Sharpe Ratio from simulation
Return (%)
Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
20
22
24
26
28
1.8
1.6
1.4
1.2
1.0


The construction of the Efficient Frontier
Efficient Frontier
Maximum Sharpe Ratio from simulation
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
36
34
32
30
28
26
24
22
20
14
16
18
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1.6
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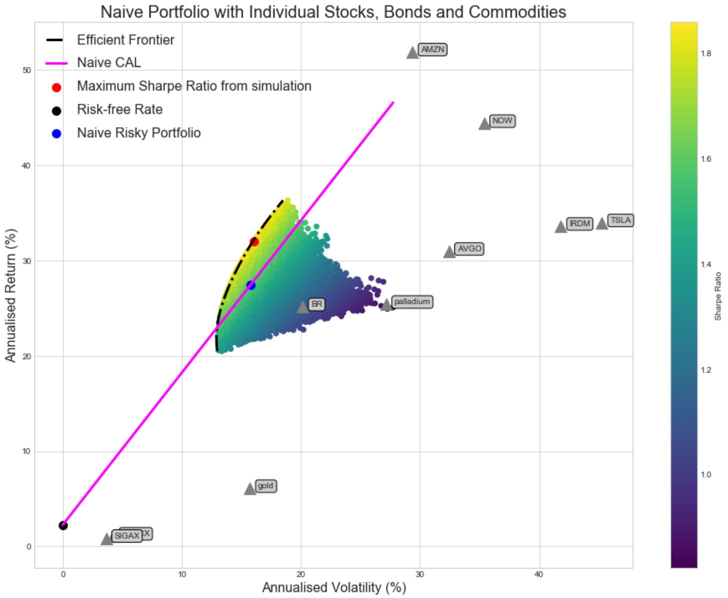
CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\}$$

CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \implies \epsilon_{naive} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe Ratio}} \cdot x + r_f$$

Naive CAL



Naive Portfolio with Individual Stocks, Bonds and Commodities
Efficient Frontier
Naive CAL
Maximum Sharpe Ratio from simulation
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
TSLA
IRDM
AVGO
palladium
BR
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
0
10
20
30
40
50
Sharpe Ratio
1.0
1.2
1.4
1.6
1.8

But...

We can do better!

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Sharpe Ratio

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

$$\vec{w} = \begin{bmatrix} w_{\text{AVGO}} \\ w_{\text{NOW}} \\ w_{\text{TSLA}} \\ w_{\text{AMZN}} \\ w_{\text{IRDM}} \\ w_{\text{BR}} \\ w_{\text{PYACX}} \\ w_{\text{SIGAX}} \\ w_{\text{gold}} \\ w_{\text{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

- Assets with zero weight allocation can either be excluded or substituted by other assets of the same class

Optimization yields...

Financial Metrics

	Optimal Risky Portfolio	Naive Risky Portfolio
Return (%)	32.7164	27.444
Risk (%)	16.3185	15.7565
Sharpe Ratio	1.8658	1.5977
Beta (β)	0.9067	0.9109

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{opt}}, r_{\text{opt}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{opt}}, r_{\text{opt}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \Longrightarrow \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{opt}} - r_f}{\sigma_{\text{opt}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma^2_{\text{opt}} \cdot z^2$$

Maximize w.r.t. z:

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best **for us**?

Utility maximization

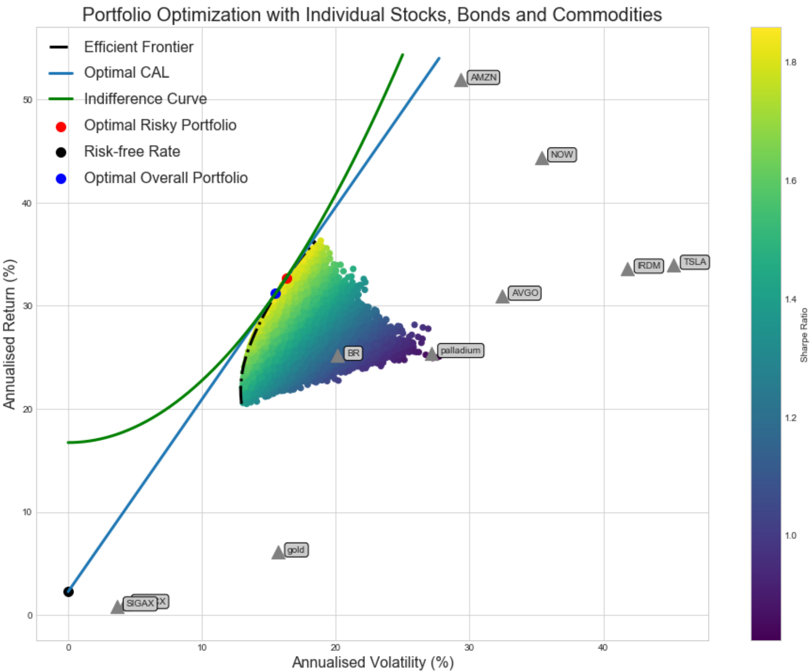
$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

Maximize w.r.t. z:

$$\frac{\partial U}{\partial z} = 0$$

$$\implies r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\implies z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$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Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

Past Performance
(1/13/2015 - 1/13/2020)

	Optimal Overall Portfolio	S&P 500
Return (%)	25.4769	11.3118
Risk (%)	12.4384	13.4138
Sharpe Ratio	1.8658	0.6741
Beta (β)	0.6911	1.0
Alpha (α) (%)	16.9584	0.0

Holding Period Returns
(1/14/2020 - Now)

	Optimal Overall Portfolio	S&P 500
HPR (%)	-6.9747	-17.4263

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The end!