Portfolio Management

FN 4329

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American College of Greece

Spring Semester 2020

Contents

todo

Todo

- fix contents (example: Intro, Theory, Results)
- better resolution on images

Questionnaire

PERSONAL INVESTORS

Vanguard

Investor questionnaire

1. I plan to begin taking money from my investments in ...

- ○ 1 year or less
- ○ 1 – 2 years
- ○ 3 – 5 years
- ○ 6 – 10 years
- ○ 11 – 15 years
- ◉ More than 15 years

2. As I withdraw money from these investments, I plan to spend it over a period of ...

- ◉ 2 years or less
- ○ 3 – 5 years
- ○ 6 – 10 years
- ○ 11 – 15 years
- ○ More than 15 years

3. When making a long-term investment, I plan to keep the money invested for ...

- ○ 1 – 2 years
- ○ 3 – 4 years
- ○ 5 – 6 years
- ○ 7 – 8 years
- ◉ More than 8 years

4. From September 2008 through November 2008, stocks lost more than 31%. If I owned a stock investment that lost about 31% in 3 months, I would ... *(If you owned stocks or stock funds during this period, select the answer that corresponds to your actual behavior.)*

- ○ Sell all of the remaining investment.
- ○ Sell a portion of the remaining investment.
- ◉ Hold onto the investment and sell nothing.
- ○ Buy more of the investment.

5. Generally, I prefer investments with little or no fluctuation in value, and I'm willing to accept the lower return associated with these investments.

- ○ Strongly disagree
- ◉ Disagree
- ○ Somewhat agree
- ○ Agree
- ○ Strongly agree

6. During market declines, I tend to sell portions of my riskier assets and invest the money in safer assets.

- ◉ Strongly disagree
- ○ Disagree
- ○ Somewhat agree
- ○ Agree
- ○ Strongly agree

7. I would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative.

- ○ Strongly disagree
- ○ Disagree
- ◉ Somewhat agree
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8. From September 2008 through October 2008, bonds lost nearly 4%. If I owned a bond investment that lost almost 4% in 2 months, I would ... *(If you owned bonds or bond funds during this period, select the answer that corresponds to your actual behavior.)*

Questionnaire

1/30/2020 Vanguard - Investor Questionnaire

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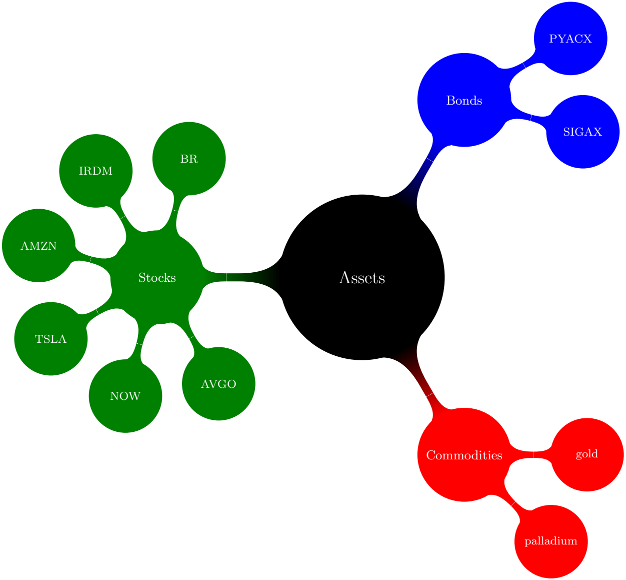
https://personal.vanguard.com/us/FundsInvQuestionnaire 1/2

Asset Allocation

- 70% Stocks
- 20% Bonds
- 10% Commodities

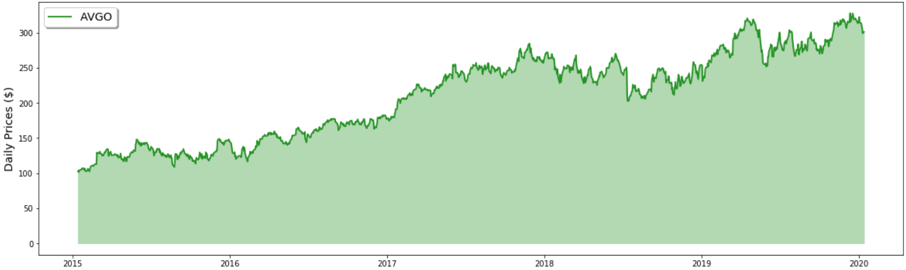
Security Selection



Stocks

AVGO - Broadcom Inc

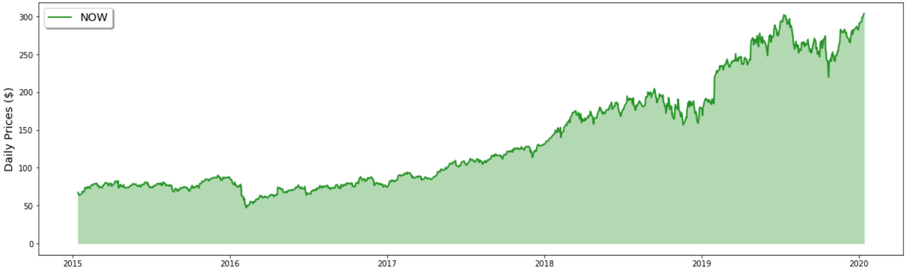
- Large/High-Dividend
- Wireless chips and semiconductor manufacturer
- Announced 15$ billion deal with Apple Inc. in January 2020
- Potential for gains with 5G deployment



Stocks

NOW - ServiceNow Inc.

- Large/Growth
- Cloud-based solutions provider to global enterprises
- Software as a Service (SaaS) business model
- IT, Customer Support, HR and Security services



Stocks

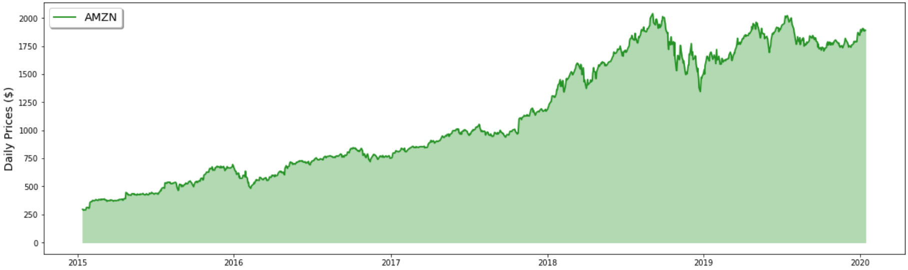
TSLA - Tesla Inc.

- Large/Growth
- Leader in next-generation electric vehicles
- Advancements in battery technology will allow for increased autonomy
- 4^{th} Gigafactory in Berlin to begin operations by July 2021

Stocks

AMZN - Amazon Inc.

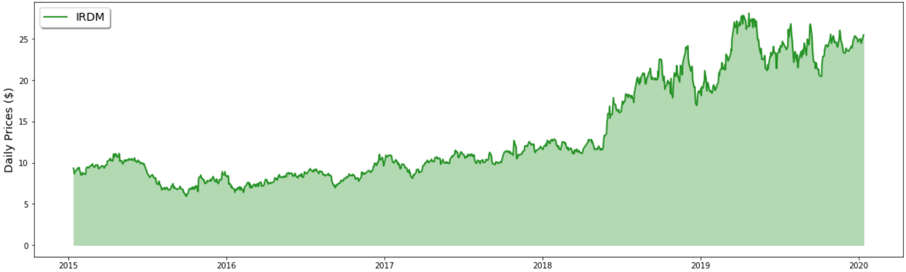
- Large/Growth
- Biggest online retailer
- Online product and digital media sales
- AWS offers solutions for Machine Learning, Big Data, IoT and Cloud-Computing



Stocks

IRDM - Iridium Communications Inc.

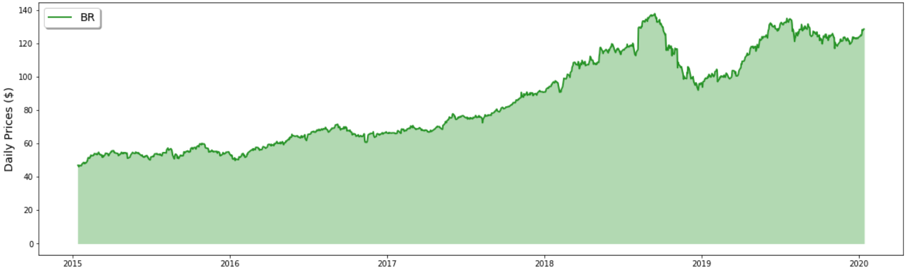
- Small/Growth
- Lead provider of satellite communications, with over 70 satellites in orbit
- Announced partnership with AWS for future applications in 2018
- Wide commercial end base, from maritime and aviation to oil & gas suppliers



Stocks

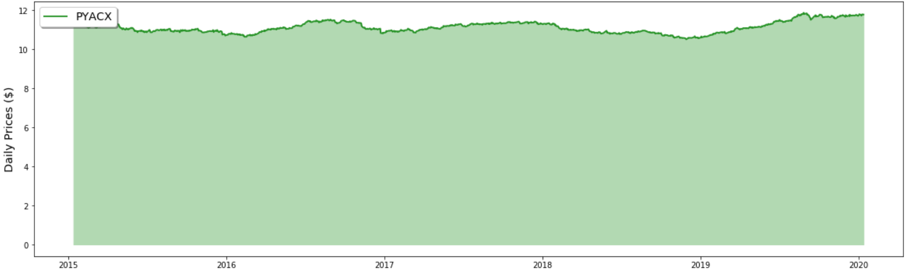
BR - Broadridge Financial Solutions Inc.

- Mid/Aggressive Growth
- Fintech company that provides solutions for investors, banks, brokerage offices and mutual funds
- Products include communication platforms, securities processing and financial data analytics



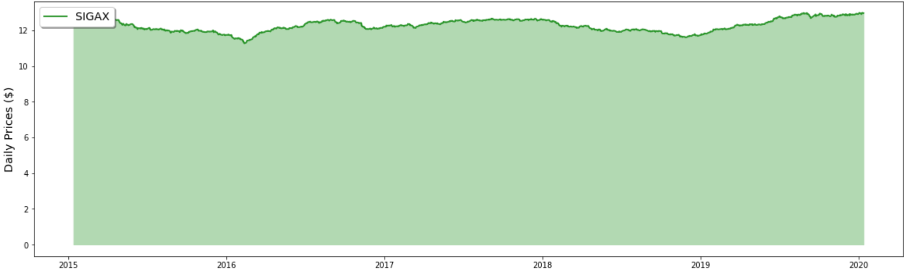
Bonds

PYACX - Payden Corporate Bond Mutual Fund



Bonds

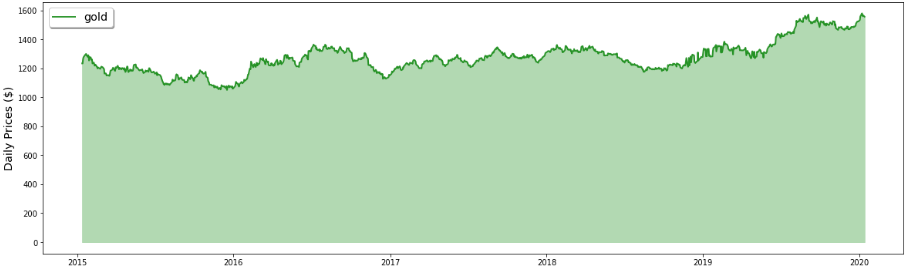
SIGAX - Western Asset Corporate Bond Mutual Fund



Commodities

gold

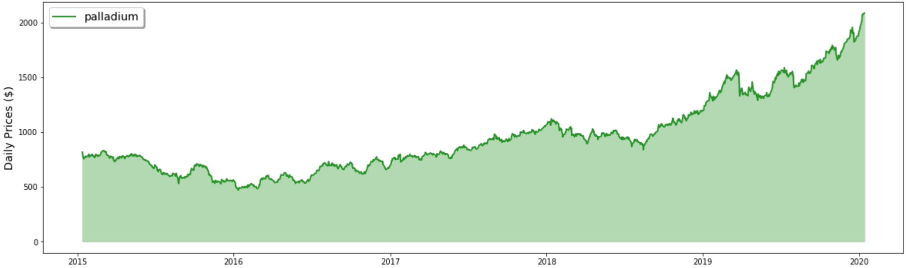
- Traditionally safe investment during rough economic times and hedge against inflation
- Recently preferred over government bonds, as the latter presented negative inflation-adjusted returns in 2019
- Does not generate return and has no holding costs



Commodities

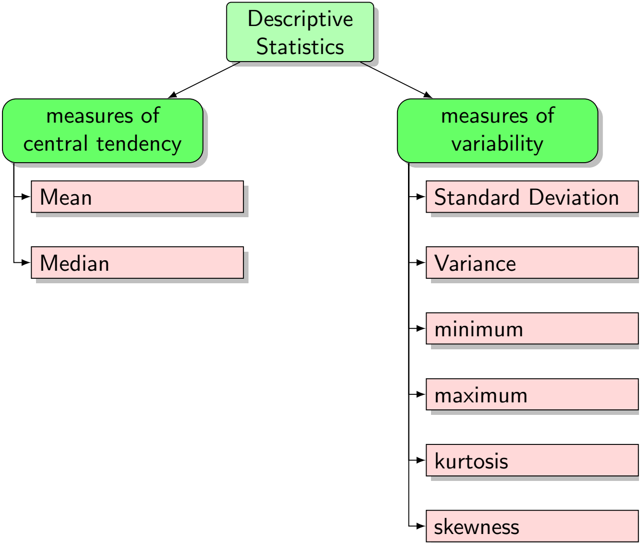
palladium

- Metal that is used in catalytic converters, turning toxic gas emissions into less harmful ones
- Secondary product from mining operations of other metals
- As miners have less control over the extracted quantities, demand outstrips supply
- Price has doubled over the last year



Contents

Descriptive statistics Taxonomy



Definition

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- The arithmetic mean of sampled values
- From the investor's perspective, the mean describes the average performance of the instrument. If $\bar{x} > 0$ then the instrument increases in value on average.

Definition

The middle value of a given dataset, separating the higher and lower half

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- Usually preferred over the arithmetic mean

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- Usually preferred over the arithmetic mean
- Why?

Measures of central tendency
Median

Definition

The middle value of a given dataset, separating the higher and lower half

- Usually preferred over the arithmetic mean
- Why? robust w.r.t. outliers

Definition

The middle value of a given dataset, separating the higher and lower half

- Usually preferred over the arithmetic mean
- Why? robust w.r.t. outliers
- Indicates whether returns are positive or negative on most time instances.

Definition

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Quantifies how much "spread out" are the data around the mean
- Measures confidence in statistics $\Longrightarrow$ risk in finance

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- Quantifies how much "spread out" are the data around the mean
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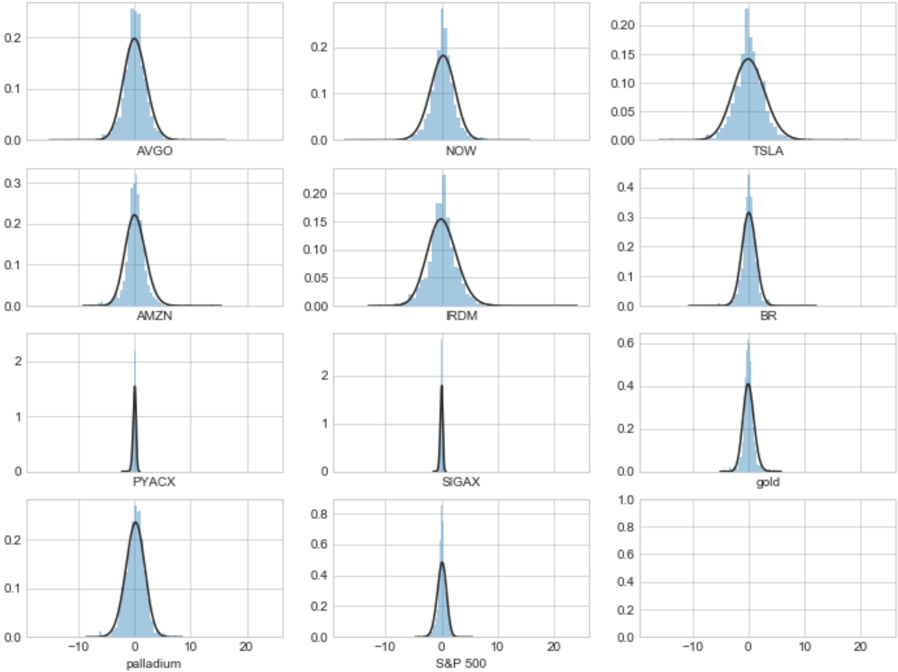
- BUT assumes normal distribution $\rightarrow$ Skewness, Kurtosis


AVGO
NOW
TSLA
AMZN
IRDM
BR
PYACX
SIGAX
gold
palladium
S&P 500

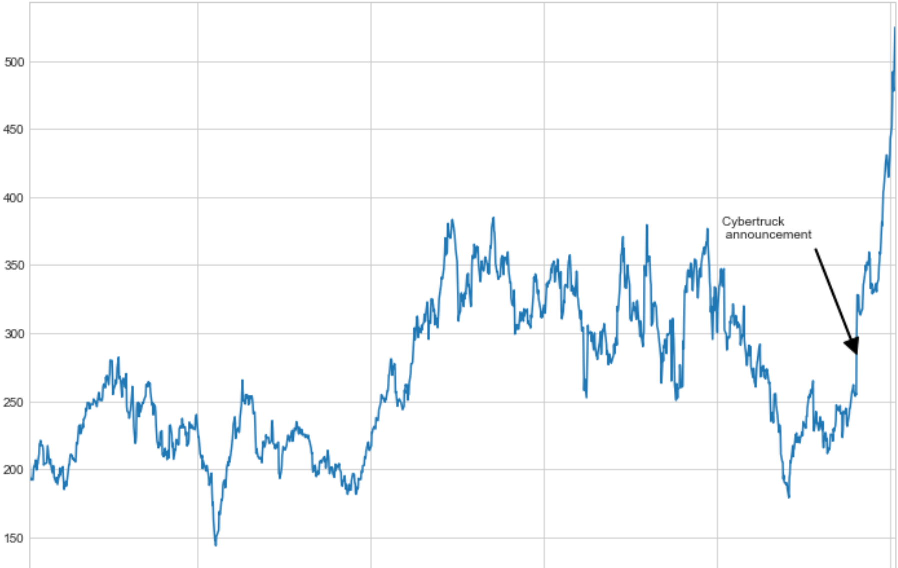
Measures of Variability

Minimum & Maximum

Figure: The minimum of the S&P 500 returns would occur on the day of the economic crisis for this period.

Measures of Variability

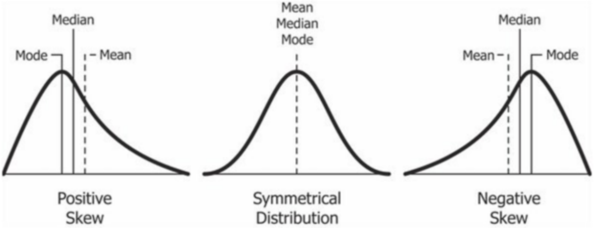
Minimum & Maximum



Definition

$$\tilde{\mu}_3 = \mathbb{E}\left[\left(\frac{X - \bar{x}}{\sigma}\right)^3\right]$$

Skewness is a measure of asymmetry that indicates if the tail of the distribution is on the left or the right.



Definition

$$\text{Kurt}(X) = \tilde{\mu}_4 = \mathbb{E}\left[\left(\frac{X-\bar{x}}{\sigma}\right)^4\right]$$

Kurtosis measures whether the distribution is heavy- or light-tailed relative to a normal distribution

- High kurtosis $\rightarrow$ heavy tails **(outliers)**
- Low kurtosis $\rightarrow$ no outliers

Measures of Variability

An overview from an investor's perspective

$\tilde{\mu}_2$ standard deviation σ

- Measure of risk
- Assumes normal distribution

$\tilde{\mu}_3$ skewness

- Measure of asymmetry: tail on the left/right
- $\tilde{\mu}_3 > 0$: frequent small losses, few large gains
- $\tilde{\mu}_3 < 0$: frequent small gains, few large losses

$\tilde{\mu}_4$ kurtosis

- Heavy- or light-tailed (relative to a normal distribution)
- High: occasional extreme returns (either positive or negative)

Descriptive Statistics
For our assets

	mean%	median%	std%	var%	min%	max%	kurtosis	skewness
AVGO	0.11	0.11	2.04	4.17	-13.74	14.71	5.31	0.23
NOW	0.15	0.21	2.23	4.96	-15.66	14.07	6.50	-0.33
TSLA	0.12	0.06	2.85	8.13	-13.90	17.67	5.06	0.30
AMZN	0.17	0.14	1.85	3.42	-7.82	14.13	9.83	1.01
IRDM	0.12	0.12	2.63	6.94	-11.13	22.24	6.47	0.62
BR	0.09	0.09	1.27	1.61	-9.70	11.16	9.29	-0.19
PYACX	0.00	0.00	0.27	0.07	-2.08	0.77	3.40	-0.71
SIGAX	0.00	0.00	0.23	0.05	-1.29	0.67	1.28	-0.43
gold	0.02	0.00	0.99	0.98	-4.32	5.10	4.56	0.33
palladium	0.09	0.15	1.71	2.92	-7.40	7.28	1.49	-0.15
S&P 500	0.04	0.05	0.84	0.71	-4.10	4.96	3.87	-0.47

Covariance

Definition

Let X and Y be two random variables. Then the covariance is a measure of the joint variability of these two random variables:

$$cov(X, Y) = \mathbb{E}[(X - \bar{x})(Y - \bar{y})]$$

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- Not so helpful!

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- Not so helpful! $\rightarrow$ correlation

Correlation

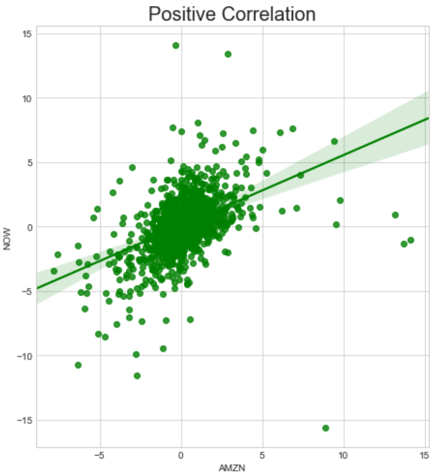
Definition

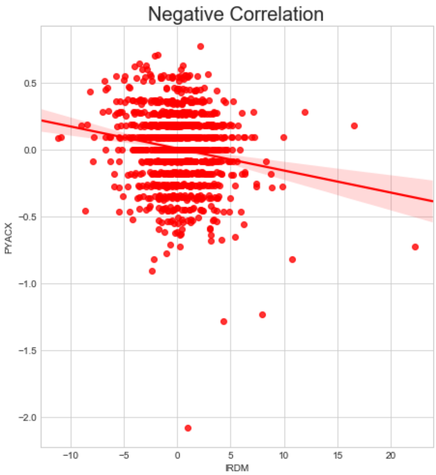
The correlation is the normalization of the covariance.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

$$\rho_{X,Y} \begin{cases} = -1, & \text{perfect decreasing (inverse) linear relationship} \\ \in (-1, 1), & \text{indicating the degree of linear dependence} \\ = 1, & \text{perfect (increasing) linear relationship} \end{cases}$$

A closer look at correlation



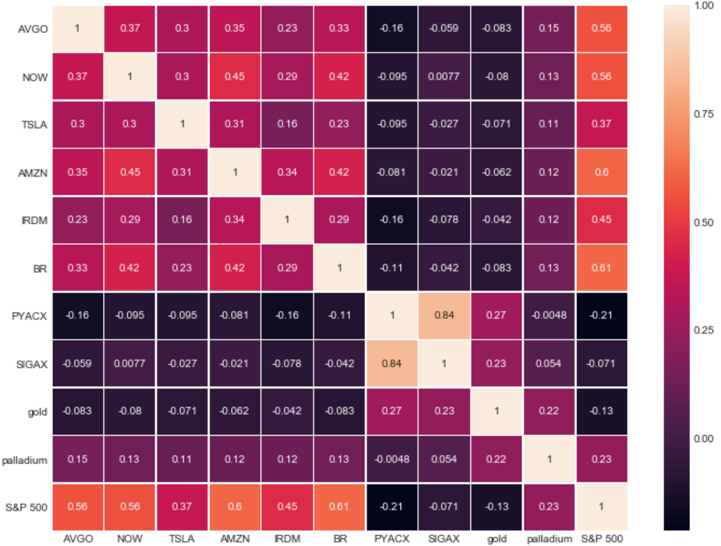


- Powerful tool that measures the strength of linear relationship between the price movements of two individual securities
- The total risk of two correlated assets is:

$$\sigma^2 = w_x^2 \cdot \sigma_x^2 + w_y^2 \cdot \sigma_y^2 + 2 \cdot w_x \cdot w_y \cdot \sigma_x \cdot \sigma_y \cdot \rho_{x,y}$$

- Negatively correlated assets yield less risk, compared to positively correlated ones
- Investors can diversify risk by including negatively correlated assets in their portfolios

Correlation Matrix



Beta

Definition

The beta coefficient measures the systematic risk of an individual stock compared to the market risk:

$$\beta = \frac{\mathrm{cov}(R_e, R_m)}{\mathrm{var}(R_m)}$$

- Investors use β to determine the movement direction and volatility of a security in comparison with the market
- A high R^2 is required for β to be meaningful

alpha

Definition

Alpha is the difference between the realised returns and the expected returns:

$$\alpha = \bar{R} - \mathbb{E}(R)$$

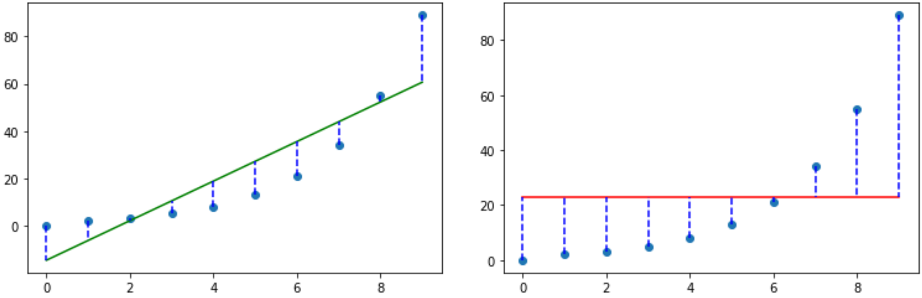
$$\stackrel{\text{CAPM}}{\Longrightarrow} \alpha = \bar{R} - \left\{R_f + \beta(\mathbb{E}(R_m) - R_f)\right\}$$

- Investors use α to determine whether or not a security has exceeded expectations in terms of return
- Frequently used to quantify the "added" value of the manager

R-squared

Definition

$$R^2 = 1 - \frac{\text{Explained Variation}}{\text{Total Variation}}$$



(a) Explained Variation (b) Total Variation

- $R - squared$ measures the "fitness" of a regression model

Sharpe Ratio

Definition

Sharpe Ratio relates the return of an investment to its risk:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

- Measures the risk-adjusted return

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- Measures the risk-adjusted return
- i.e. the average return earned *in excess* of the risk-free rate per unit of volatility or total risk.
- Useful for comparing the performance of investments that have different levels of risk and return

Financial Metrics

For our assets

	alpha	beta	R-squared
AVGO	0.14213	1.35333	0.31393
NOW	0.24718	1.46614	0.30955
TSLA	0.17711	1.26445	0.14046
AMZN	0.32815	1.32199	0.36539
IRDM	0.15867	1.41795	0.20698
BR	0.13206	0.92064	0.37655
PYACX	-0.00576	-0.06792	0.04613
SIGAX	-0.01215	-0.01911	0.00501
gold	0.05087	-0.15174	0.01684
palladium	0.17817	0.47398	0.05493

Contents

Risk and return

Return

$$R_p = \vec{w}^\top \cdot \mathbb{E}(\mathcal{R})$$

Risk

$$\begin{aligned}
\sigma_p &= \sqrt{\vec{w}^\top K \vec{w}} \\
&= \sqrt{\begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \mathrm{cov}_{1,2} & \dots & \mathrm{cov}_{1,n} \\ \mathrm{cov}_{2,1} & \sigma_2^2 & \dots & \mathrm{cov}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}_{n,1} & \mathrm{cov}_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}} \\
&= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{cov}_{i,j}}
\end{aligned}$$

Naive weight allocation

Remember!

- 70% Stocks
- 20% Bonds
- 10% Commodities

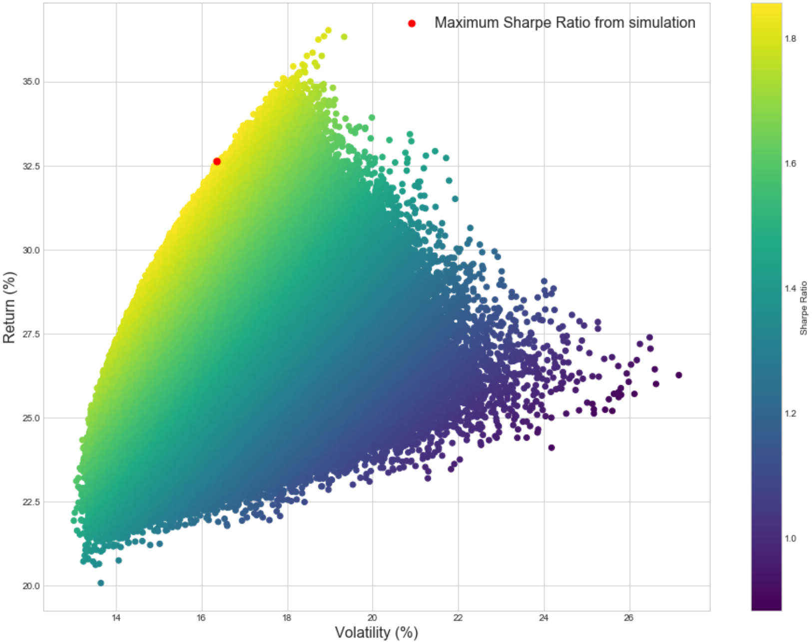
What about $\vec{w}$?

- 70% for 6 Stocks $\rightarrow \dfrac{70\%}{6} = 11.67\%$ each
- 20% for 2 Bonds $\rightarrow$ 10% each
- 10% for 2 Commodities $\rightarrow$ 5% each

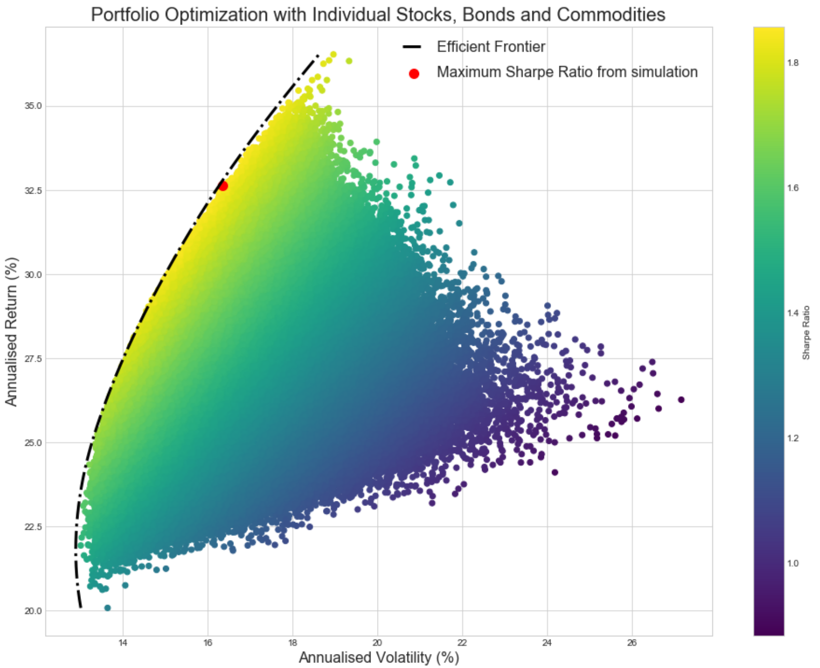
Naive Allocation yields...

Financial Metrics

	Naive Risky Portfolio
Return (%)	27.444
Risk (%)	15.7565
Sharpe Ratio	1.5977
Beta (β)	0.9109



Maximum Sharpe Ratio from simulation
Return (%)
Volatility (%)
Sharpe Ratio
35.0
32.5
30.0
27.5
25.0
22.5
20.0
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Maximum Sharpe Ratio from simulation
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
35.0
32.5
30.0
27.5
25.0
22.5
20.0
14
16
18
20
22
24
26
1.8
1.6
1.4
1.2
1.0

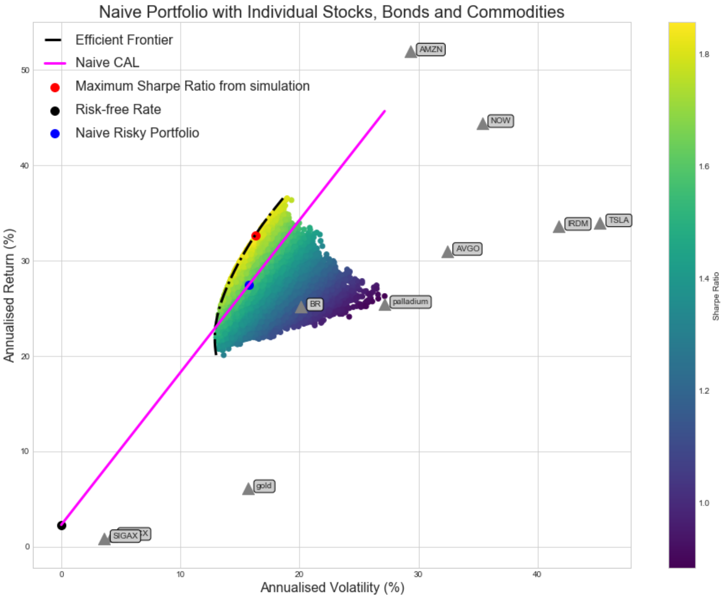
CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\}$$

CAL construction

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{naive}} \\ (\sigma_{\text{naive}}, r_{\text{naive}}) \in \epsilon_{\text{naive}} \end{array} \right\} \implies \epsilon_{naive} : y = \underbrace{\frac{r_{\text{naive}} - r_f}{\sigma_{\text{naive}}}}_{\text{naive Sharpe ratio}} \cdot x + r_f$$

Naive CAL



Naive Portfolio with Individual Stocks, Bonds and Commodities
Efficient Frontier
Naive CAL
Maximum Sharpe Ratio from simulation
Risk-free Rate
Naive Risky Portfolio
AMZN
NOW
IRDM
TSLA
AVGO
BR
palladium
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

We can do better!

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
\text{s.t.} \quad & \mathbf{1}^\top \vec{w} = 1 \\
& \mathbf{1}^\top \vec{w}_{\mathcal{S}} = w_s \\
& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \dots, n
\end{aligned}
$$

Sharpe Ratio

$$
\begin{aligned}
\max \quad & \boxed{\frac{\vec{w}^\top \cdot \mathbb{E}(\mathcal{R}) - r_f}{\sqrt{\vec{w}^\top K \vec{w}}}} \\
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& \mathbf{1}^\top \vec{w}_{\mathcal{B}} = w_b \\
& \mathbf{1}^\top \vec{w}_{\mathcal{C}} = w_c \\
& w_s + w_b + w_c = 1 \\
& w_i \geq 0 \quad i = 1, \ldots, n
\end{aligned}
$$

Optimization yields...

Optimal weights

$$\vec{w} = \begin{bmatrix} w_{\mathrm{AVGO}} \\ w_{\mathrm{NOW}} \\ w_{\mathrm{TSLA}} \\ w_{\mathrm{AMZN}} \\ w_{\mathrm{IRDM}} \\ w_{\mathrm{BR}} \\ w_{\mathrm{PYACX}} \\ w_{\mathrm{SIGAX}} \\ w_{\mathrm{gold}} \\ w_{\mathrm{palladium}} \end{bmatrix} = \begin{bmatrix} 4.0068 \\ 8.6743 \\ 1.0266 \\ 38.2827 \\ 1.32 \\ 16.6897 \\ 20.0 \\ 0.0 \\ 0.0 \\ 10.0 \end{bmatrix} \%$$

- Assets with zero weight allocation can either be excluded or substituted by other assets of the same class

Optimization yields...

Financial Metrics

	Optimal Risky Portfolio	Naive Risky Portfolio
Return (%)	32.7164	27.444
Risk (%)	16.3185	15.7565
Sharpe Ratio	1.8658	1.5977
Beta (β)	0.9067	0.9109

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{opt}}, r_{\text{opt}}) \in \epsilon_{\text{CAL}} \end{array} \right\}$$

Capital Allocation Line

$$\left\{ \begin{array}{c} (0, r_f) \in \epsilon_{\text{CAL}} \\ (\sigma_{\text{opt}}, r_{\text{opt}}) \in \epsilon_{\text{CAL}} \end{array} \right\} \Longrightarrow \epsilon_{CAL} : y = \underbrace{\frac{r_{\text{opt}} - r_f}{\sigma_{\text{opt}}}}_{\text{max Sharpe ratio}} \cdot x + r_f$$

Optimal Overall Portfolio

Which portfolio is the overall optimal?

- i.e. which portfolio along the CAL is the best

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$$\text{Utility} = U = r_f + z \cdot (r_{\text{opt}} - r_f) - 0.05 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z^2$$

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Maximize w.r.t. z:

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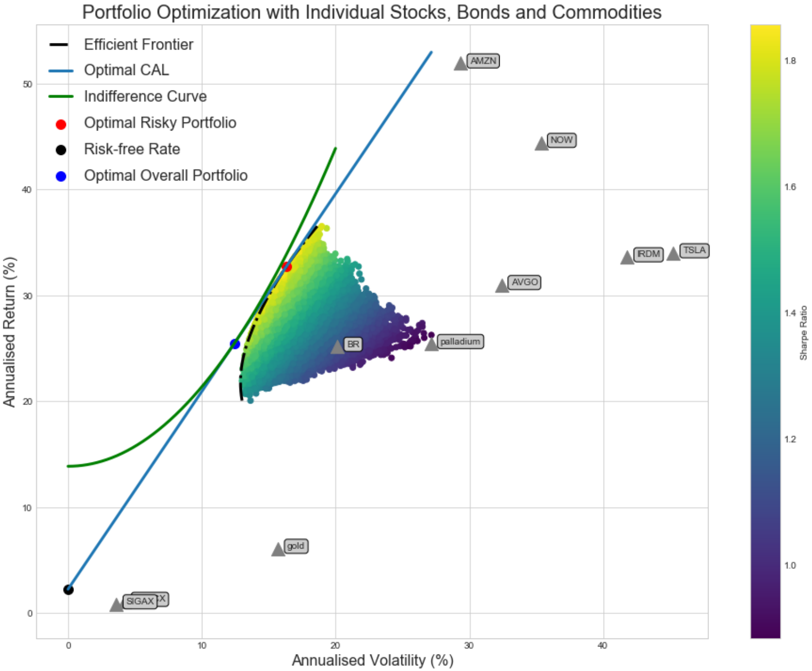
Maximize w.r.t. z:

$$\frac{\partial U}{\partial z} = 0$$

$$\Longrightarrow \; r_{\text{opt}} - r_f - 0.1 \cdot A \cdot \sigma_{\text{opt}}^2 \cdot z = 0$$

$$\Longrightarrow \; z^* = \frac{r_{\text{opt}} - r_f}{0.1 \cdot A \cdot \sigma_{\text{opt}}^2}$$


Portfolio Optimization with Individual Stocks, Bonds and Commodities
Efficient Frontier
Optimal CAL
Indifference Curve
Optimal Risky Portfolio
Risk-free Rate
Optimal Overall Portfolio
AMZN
NOW
TSLA
IRDM
AVGO
palladium
BR
gold
SIGAX
Annualised Return (%)
Annualised Volatility (%)
Sharpe Ratio
0
10
20
30
40
50
1.0
1.2
1.4
1.6
1.8

Past Performance
(1/13/2015 - 1/13/2020)

	Optimal Overall Portfolio	S&P 500
Return (%)	25.4769	11.3118
Risk (%)	12.4384	13.4138
Sharpe Ratio	1.8658	0.6741
Beta (β)	0.6911	1.0
Alpha (α) (%)	16.9584	0.0

Holding Period Returns
(1/14/2020 - Now)

	Optimal Overall Portfolio	S&P 500
HPR (%)	-6.9747	-17.4263

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The end!