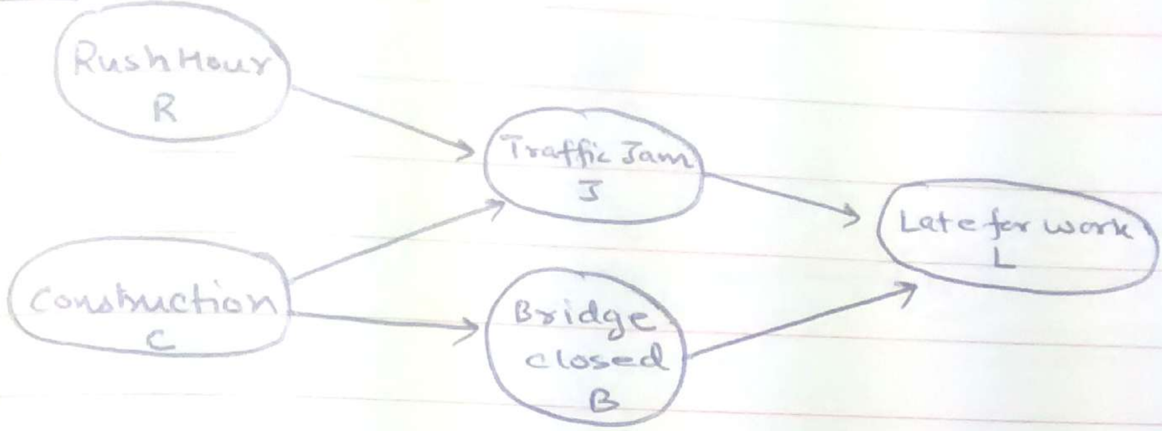
Problem I

1.(a)



(b) $\underbrace{\underset{4}{P(L/J,B)}\ \underset{2}{P(B/C)}\ \underset{4}{P(J/C,R)}\ \underset{1}{P(R)}\ \underset{1}{P(C)}}_{12 \text{ parameters.}}$

Collaboration Questions

Did you receive any help from anyone in solving this assignment?

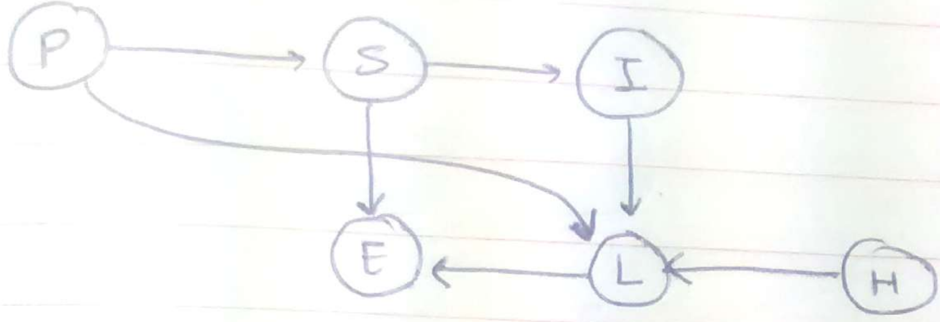
→ No

Did you give any help to anyone in solving this assignment?

→ No.

Time Spent : 7 hours.

2(a)



Assumption : "If harry's professor is evil, Harry is more likely to spend his time studying in library". I assume this ~~t~~ dependence exist beside the relationship via the act of sneaking out.

(b) P(E/S, L) P(L/H, I, P) P(I/S) P(S/P) P(P) P(H)

$$\underbrace{4 \quad 8 \quad 2 \quad 2 \quad 1 \quad 1}_{18 \text{ parameters}}$$

(c) ~~No~~. Yes.

P(L/H, I, P) : still 8 parameters

Since P= "Neutral" and "Good" are combined.

P(S/P) : still 2 parameters

But,

P(P) : 2 parameters instead of 1.

∴ 19 parameters in all.

Scanned by CamScanner

Problem 2

2.1

1. (a) $T \perp C$ True

(b) $T \perp G$ False

(c) $O \perp C$ True

(d) $O \perp H$ True

(e) $T \perp G | E$ True

(f) $O \perp C | G$ False

(g) $E \perp H | C$ True

(h) $C \perp G | O, H$ False

(i) $C \perp G | O, E, H$ True

(j) $T \perp C \perp O \perp G | E, H$ False.

2.(a) None.

$E \to G$, $H \to G$, $O \to E \to G$,

$T \to E \to G$, $C \to E \to G$.

None of these paths can be d-seperated.

(b) (E, T, O).

All paths from (E, T, O) to H are blocked either by C (..← C →..) or G (..→ G ←..).

3. $P(G|E,H)\, P(E|O,T,C)\, P(H|C)\, P(T)\, P(C)\, P(O)$

3. $P(S_1 = A / \vec{O}) = \dfrac{\alpha_1^A \beta_1^A}{\alpha_{End}} = 0.582$

$P(S_1 = B / \vec{O}) = \dfrac{\alpha_1^B \beta_1^B}{\alpha_{End}} = 0.418$

most likely $\underline{S_1 = A}$

$P(S_2 = A / \vec{O}) = \dfrac{\alpha_2^A \beta_2^A}{\alpha_{End}} = 0.181$

$P(S_2 = B / \vec{O}) = \dfrac{\alpha_2^B \beta_2^B}{\alpha_{End}} = 0.819$

most likely $\underline{S_2 = B}$

$P(S_3 = A / \vec{O}) = \dfrac{\alpha_3^A \beta_3^A}{\alpha_{End}} = 0.776$

$P(S_3 = B / \vec{O}) = \dfrac{\alpha_3^B \beta_3^B}{\alpha_{end}} = 0.224$

most likely $\underline{S_3 = A}$

4. __No.__ This is found using Viterbi Algorithm.
Because forward-backward algorithm allows us to calculate

$P(S_1 = s_1 / \vec{O}) \cdot P(S_2 = s_2 / \vec{O}) \cdot P(S_3 = s_3 / \vec{O})$
$\neq P(S_1, S_2, S_3 / \vec{O})$

Not conditionally independent.

Problem 2

$P(C=c^1 / O=o^1, G=G^B)$

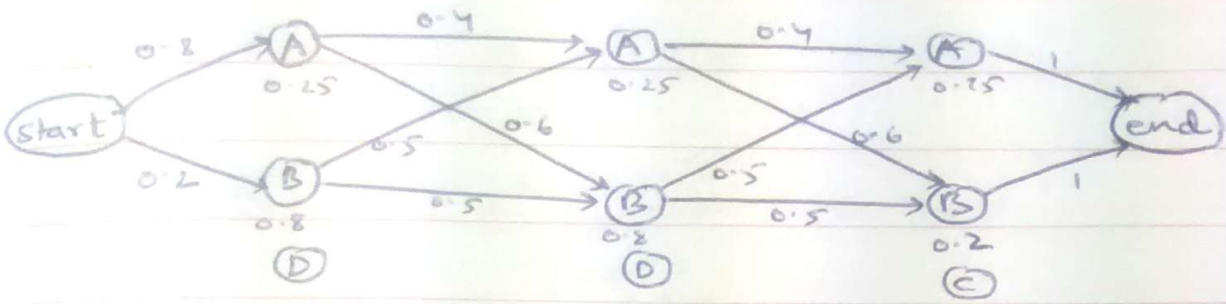
$$= \frac{\sum_{h=0}^{1}\sum_{e=0}^{1}\sum_{t=0}^{1} P(G^B/E^e,H^h)\, P(H^h/c^1)\, P(E^e/T^t, c^1, o^1)\, P(O^1) P(C^1) P(T^t)}{\sum_{h=0}^{1}\sum_{e=0}^{1}\sum_{t=0}^{1}\sum_{c=0}^{1} P(G^B/E^e H^h)\, P(H^h/c^c)\, P(E^e/T^t, c^c, o^1) P(O^1)\, P(C^c)\, P(T^t)}$$

$$= \frac{\left(\begin{array}{l} 0.000324 + 0.000132 + 0.004536 + 0.007128 + \\ 0.007776 + 0.003168 + 0.012096 + 0.019008 \end{array}\right)}{\left(\begin{array}{l} 0.00081 + 0.000324 + 0.00061875 + 0.000132 + \\ 0.000285 + 0.004536 + 0.0037125 + 0.007128 + 0.00162 + \\ 0.007776 + 0.0012375 + 0.003168 + 0.00027 + 0.012096 \\ + 0.000825 + 0.019008 \end{array}\right)}$$

$$= \frac{0.054168}{0.064476} = \boxed{0.840116}$$

4. Gibbs Sampling utilises conditional distribution. generating a sequence of samples constituting a Markov chain. Unlike brute forces which assigns marginal probabilities to variables T, C, in our case, gibbs sampling generates T, C samples with probabilities conditioned on $G=G^B$ and $O=o^1$.

Q.2



1. $\alpha_1^A = 0.8 \times 0.25 \cancel{+ 0.2 \times 0.8} = 0.2$

$\alpha_1^B = 0.2 \times 0.8 = 0.16$

$\alpha_2^A = 0.8 \times 0.25 \times 0.4 \times 0.25 + \alpha_1^B \times 0.5 \times 0.25 = 0.04$

$\alpha_2^B = \alpha_1^A \times 0.6 \times 0.8 + \alpha_1^B \times 0.5 \times 0.8 = 0.16$

$\alpha_3^A = \alpha_2^A \times 0.4 \times 0.75 + \alpha_2^B \times 0.5 \times 0.75 = 0.072$

$\alpha_3^B = \alpha_2^A \times 0.6 \times 0.2 + \alpha_2^B \times 0.5 \times 0.2 = 0.0208$

$\alpha_{End} = \alpha_3^A + \alpha_3^B = 0.0928 = P(O^1=D, O^2=D, O^3=C)$

2. $\beta_3^A = 1$

$\beta_3^B = 1$

$\beta_2^A = \beta_3^A \times 0.75 \times 0.4 + \beta_3^B \times 0.2 \times 0.6 = 0.42$

$\beta_2^B = \beta_3^A \times 0.75 \times 0.5 + \beta_3^B \times 0.2 \times 0.5 = 0.475$

$\beta_1^A = \beta_2^A \times 0.25 \times 0.4 + \beta_2^B \times 0.8 \times 0.6 = 0.27$

$\beta_1^B = \beta_2^A \times 0.25 \times 0.5 + \beta_2^B \times 0.8 \times 0.5 = 0.2425$

$P(O^1=D, O^2=D, O^3=C) = \underline{0.2645} \leftarrow \begin{pmatrix} 0.27 \times 0.8 + \\ 0.2425 \times 0.2 \end{pmatrix}$

Problem 3

1) 1.

$$P(S_t / S_{t-1}) = P(S_t / S_{t-1}, S_{t-2})$$
$$P(O_t | S_t) = P(O_t / S_t, S_{t-1})$$

2. Transition Matrix parameters : $K(K-1)$

Emmission probabilities : $K(P-1)$

Initial probabilities : $(K-1)$

Total parameters = $(K+1)(K-1) + K(P-1)$