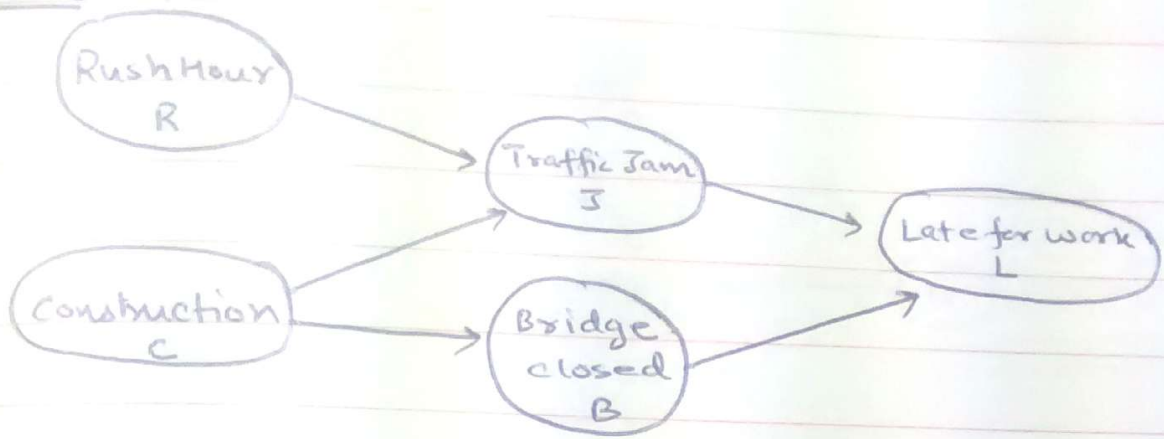


Problem 1

1.(a)



(b) $P(L/J, B)$ $P(B/C)$ $P(J/C, B)$ $P(R)$ $P(C)$

4 2 4 1 1

12 parameters.

Collaboration Questions

Did you receive any help from anyone in solving this assignment?

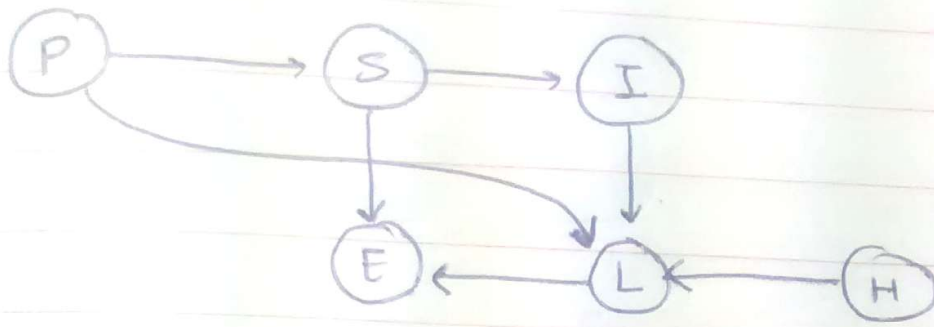
→ No

Did you give any help to anyone in solving this assignment?

→ No.

Time Spent : 7 hours.

2(a)



Assumption: "If Harry's professor is evil, Harry is more likely to spend his time studying in library". I assume this ~~the~~ dependence exist beside the relationship via the act of sneaking out.

(b) $P(E/S, L) \quad P(L/H, I, P) \quad P(I/S) \quad P(S/P) \quad P(P) \quad P(H)$

4 8 2 2 1 1

└──┘

18 parameters

(c) ~~No~~. Yes.

$P(L/H, I, P)$: still 8 parameters

Since $P = \text{"Neutral" and "Good"}$ are combined.

$P(S/P)$: still 2 parameters

But,

$P(P)$: 2 parameters instead of 1.

\therefore 19 parameters in all.

Problem 2

2.1

1. (a) $T \perp C$ True
- (b) $T \perp G$ False
- (c) $O \perp C$ True
- (d) $O \perp H$ True
- (e) $T \perp G | E$ True
- (f) $O \perp C | G$ False
- (g) $E \perp H | C$ True
- (h) $C \perp G | O, H$ False
- (i) $C \perp G | O, E, H$ True
- (j) $T \perp C \perp O \perp G | E, H$ False.

2(a) None.

$E \rightarrow G$, $H \rightarrow G$, $O \rightarrow E \rightarrow G$,
 $T \rightarrow E \rightarrow G$, $C \rightarrow E \rightarrow G$.

None of these paths can be d-separated.

(b) (E, T, O) .

All paths from (E, T, O) to H are blocked
 either by $C (\dots \rightarrow C \rightarrow \dots)$ or $G (\dots \rightarrow G \leftarrow \dots)$.

3. $P(G|E, H) P(E|O, T, C) P(H|C) P(T) P(C) P(O)$

$$3. \quad P(S_1 = A / \vec{O}) = \frac{\alpha_1^A \beta_1^A}{\alpha_{\text{End}}} = 0.582$$

$$P(S_1 = B / \vec{O}) = \frac{\alpha_1^B \beta_1^B}{\alpha_{\text{End}}} = 0.418$$

most likely $S_1 = A$

$$P(S_2 = A / \vec{O}) = \frac{\alpha_2^A \beta_2^A}{\alpha_{\text{End}}} = 0.181$$

$$P(S_2 = B / \vec{O}) = \frac{\alpha_2^B \beta_2^B}{\alpha_{\text{End}}} = 0.819$$

most likely $S_2 = B$

$$P(S_3 = A / \vec{O}) = \frac{\alpha_3^A \beta_3^A}{\alpha_{\text{End}}} = 0.776$$

$$P(S_3 = B / \vec{O}) = \frac{\alpha_3^B \beta_3^B}{\alpha_{\text{End}}} = 0.224$$

most likely $S_3 = A$

4. No. This is found using Viterbi Algorithm.
Because forward-backward algorithm allows us to calculate

$$P(S_1 = s_1 / \vec{O}), P(S_2 = s_2 / \vec{O}), P(S_3 = s_3 / \vec{O})$$

$$\neq P(S_1, S_2, S_3 / \vec{O})$$

Not conditionally independent.

Problem 2

$$P(C = c' / O = o', G = G^B)$$

$$= \sum_{h=0}^1 \sum_{e=0}^1 \sum_{t=0}^1 P(G^B / E^e, H^h) P(H^h / C') P(E^e / T^t, C', O') P(O') P(C') P(T^t)$$

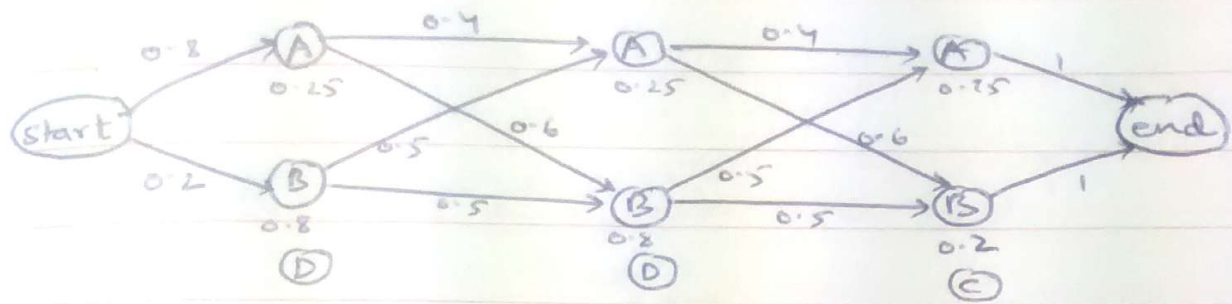
$$\sum_{h=0}^1 \sum_{e=0}^1 \sum_{t=0}^1 \sum_{c=0}^1 P(G^B / E^e, H^h) P(H^h / C') P(E^e / T^t, C', O') P(O') P(C') P(T^t)$$

$$= \frac{(0.000324 + 0.000132 + 0.004536 + 0.007128 + 0.007776 + 0.003168 + 0.012096 + 0.019008)}{1}$$

$$\frac{(0.00081 + 0.000324 + 0.00061875 + 0.000132 + 0.000285 + 0.004536 + 0.0037125 + 0.007128 + 0.00162 + 0.007776 + 0.0012375 + 0.003168 + 0.00027 + 0.012096 + 0.000825 + 0.019008)}{1}$$

$$= \frac{0.054168}{0.064476} \approx \boxed{0.840116}$$

4. ~~Bayes~~ Gibbs Sampling utilises conditional distribution generating a sequence of samples constituting a Markov chain. Unlike brute forces which assigns marginal probabilities to variables T, C, θ in our case, gibbs sampling generates T, C samples with probabilities conditioned on $G = G^B$ and $O = o'$.



$$1. \quad \alpha_1^A = 0.8 \times 0.25 + \cancel{0.2 \times 0.8} = 0.2$$

$$\alpha_1^B = 0.2 \times 0.8 = 0.16$$

$$\alpha_2^A = 0.8 \times 0.25 \times 0.4 \times 0.25 + \alpha_1^B \times 0.5 \times 0.25 = 0.04$$

$$\alpha_2^B = \alpha_1^A \times 0.6 \times 0.8 + \alpha_1^B \times 0.5 \times 0.8 = 0.16$$

$$\alpha_3^A = \alpha_2^A \times 0.4 \times 0.75 + \alpha_2^B \times 0.5 \times 0.75 = 0.072$$

$$\alpha_3^B = \alpha_2^A \times 0.6 \times 0.2 + \alpha_2^B \times 0.5 \times 0.2 = 0.0208$$

$$\alpha_{\text{end}} = \alpha_3^A + \alpha_3^B = 0.0928 = P(O^1=D, O^2=D, O^3=C)$$

$$2. \quad \beta_3^A = 1$$

$$\beta_3^B = 1$$

$$\beta_2^A = \beta_3^A \times 0.75 \times 0.4 + \beta_3^B \times 0.2 \times 0.6 = 0.42$$

$$\beta_2^B = \beta_3^A \times 0.75 \times 0.5 + \beta_3^B \times 0.2 \times 0.5 = 0.475$$

$$\beta_1^A = \beta_2^A \times 0.25 \times 0.4 + \beta_2^B \times 0.8 \times 0.6 = 0.27$$

$$\beta_1^B = \beta_2^A \times 0.25 \times 0.5 + \beta_2^B \times 0.8 \times 0.5 = 0.2425$$

$$P(O^1=D, O^2=D, O^3=C) = \underline{0.2645} \leftarrow (0.27 \times 0.8 + 0.2425 \times 0.2)$$

Problem 3

$$\begin{aligned} P(S_t / S_{t-1}) &= P(S_t / S_{t-1}, S_{t-2}) \\ P(O_t / S_t) &= P(O_t / S_t, S_{t-1}) \end{aligned}$$

2. Transition Matrix parameters: $k(k-1)$

Emission probabilities: $k(p-1)$

Initial probabilities: $(k-1)$

$$\text{Total parameters} = (k+1)(k-1) + k(p-1)$$