Unit - 2

Binary Operation

Binary Operation is an operation defined for any set S such that it takes **two elements from S as input and produces a single element in S as output.** As the name suggests, binary operations **require at least two inputs** as it is defined from the cartesian product of set to set itself.
In this article, we will explore binary operations its definition, properties, types of binary operations, and many more. We will also discuss the applications of binary operations and solve some examples on it. Let's start our learning on the topic "Binary Operation".

What are Binary Operations?

Binary operations are the operations that are performed on two inputs. Some fundamental binary operations are **addition, subtraction, multiplication, and division**. The inputs are known as the **operands.** Binary operations also have several **properties like closure property, associative property, commutative property, identity element, and inverse element.**

Binary Operation Definition

Binary operation is defined as the operation on set S which maps the cartesian product of S to the element that belongs to S. Binary operation * on S with elements a and b can be represented as:
: S × S → S such that for all a, b; a*b ∈ S
×
×
×
is a binary operation
.
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Properties of Binary Operations

Binary operations are the operations performed on two elements of a set and the result also belongs to the same set. Some of the properties of the binary operations are:

- Closure Property in Binary Operations
- Associativity of Binary Operations
- Commutativity of Binary Operations
- Identity Element of Binary Operations
- Inverse Element of Binary Operations

Closure Property in Binary Operations

-> means every operation performed on any two elements of set should have the result in the same set

The closure property in binary operation * on set X with element x and y is defined as:

$x \in X, y \in X \Rightarrow x * y \in X$

If x and y belong to a set X then the result of the binary operation between them will also belong to the set X

Associativity of Binary Operations

Associativity of binary operation * on set X with element x, y and z is defined as:

$(x * y) * z = x * (y * z)$

Commutativity of Binary Operations

Commutativity of binary operation * on set X with element x and y is defined as:

$x * y = y * x$

Identity Element of Binary Operations

***-> here the value of e change** according to the type of operation performed*

-> $a + e = a$ I.e $e = 0$;

-> a × e = a   i.e   e = 1;

Identity element of binary operation * on set X with element x and e is defined as:

x × e = e × x = x

Then, e is called the identity element.

Inverse Element of Binary Operations

->   a^-1 = -a

Inverse element of binary operation * on set X with element x, y and e is defined as:

x * y = y * x = e

Then x is inverse of y and y is inverse of x.

Read More,

- Commutative Property in Maths
- What is Associative Property

Types of Binary Operations

Binary operations are operations which require two inputs. Some of the common types of binary operations are as follows:

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division

Let's discuss these common types in detail as follows:

Binary Addition

Binary addition is an operation on a set A with elements x and y defined as:

+ : A × A → A such that (x, y) → x + y

Binary Subtraction

Consider a set A with elements x and y. Binary subtraction is a closed binary operation on A such that:

- : A × A → A such that (x, y) → x - y

Binary Multiplication

Binary multiplication is a binary operation defined on a set A, where each element x and y in A is paired with the operation symbol ×, resulting in x **×** y, which belongs to the set A. Mathematically this can be written as:

× : A × A → A such that (x, y) → x × y

Binary Division

Binary division on a set A with elements x and y is a binary operation denoted as / and defined as:

/ : A × A → A such that (x, y) → x / y

Binary Operation Table

A binary operation table, also known as a Cayley table or operation table, is a systematic way to display the results of applying a binary operation to elements of a set.

In this table, each row represents one of the elements of the set, and each column represents another element. The cell at the intersection of a row and a column contains the result of applying the binary operation to the corresponding pair of elements.

For example, let's consider a set A={0, 1, 2, 3} with addition modulo(⊕) as operation.

⊕	1	2	3	4
1	2	3	4	1
2	3	4	1	2
3	4	1	2	3
4	1	2	3	4

Applications of Binary Operations

Some of the common applications of binary operations are:

- Binary operations are fundamental in abstract algebra, where they are used to define algebraic structures such as groups, rings, and fields.
- In combinatorics, binary operations are used to study various counting problems, permutations, and combinations.
- Binary operations are extensively used in computer science for bitwise operations, such as AND, OR, XOR, and complement operations, which are fundamental in digital logic and computer arithmetic.
- In electrical engineering, binary operations are essential for digital signal processing, coding theory, and error detection/ correction techniques.

Read More,

- *Set Theory*
- *Algebraic Structure*

Binary Operation Examples

Example: Consider a binary operation * on set X = {1, 2, 3, 4, 5} defined by x×y = min (x, y). With the help of below table find:

(i) Compute (4 * 5) * 1

(ii) Is * commutative?

(iii) Compute (2 * 5) * (1 * 3)

Table:

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Solution:

(i) (4 * 5) * 1

From table

*(4 * 5) = 4*

*(4 * 5) * 1 = 4 * 1*

*(4 * 5) * 1 = 1*

(ii) Is * commutative

*For commutative we have to prove x×y = y*x*

let x = 5 and y = 2

*x*y = 5 * 2 = 2*

*y*x = 2 * 5 = 2*

*Therefore, * is commutative.*

(iii) (2 * 5) * (1 * 3)

From table

*(2 * 5) = 2*

*(1 * 3) = 1*

*(2 * 5) * (1 * 3) = 2 * 1*

(2 * 5) * (1 * 3) = 1

Practice Problems on Binary Operations

Problem : Consider a binary operation * on set X = {a, b, c} defined by below. Find:

(i) Compute (a * b) * c

(ii) Is * commutative?

(iii) Find the identity element of the binary operation.

Table:

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Conclusion

Binary operations are essential math ideas used in many areas like algebra, computer science, engineering, and cryptography. Understanding how these operations work and their properties is essential for solving complex problems and building efficient algorithms. This article gave a basic introduction to this important concept including topics such as properties and types of binary operations.

Binary Operations - FAQs

What is Binary Operation in Maths?

Binary operation in Maths is the operation which is performed on two elements of a set and the result after performing the binary operation also belongs to the same set.

What is Binary Operation on a Set?

*Binary operation on a set is defined as taking two elements from the cartesian product of set and after performing binary operation output also belongs to same set. Binary operation * on S with element a and b can be represented as:*

: S × S → S such that for all a, b; a*b ∈ S

What are the Properties of Binary Operation?

Properties of binary operation are:

- *Closure Property in Binary Operations*
- *Associativity of Binary Operations*
- *Commutativity of Binary Operations*
- *Identity Element of Binary Operations*
- *Inverse Element of Binary Operations*

How to Find Identity Element in Binary Operation?

To find the identity element in binary operation we use: ***x × e = e × x = x where****, e is identity element.*

What is Commutative Property of Binary Operation?

Commutative property of binary operation is defined as:

x × y = y * x

What are these Matrices ?

Matrix is a rectangular array of numbers, symbols, points, or characters each belonging to a specific row and column. A matrix is identified by its order which is given in the form of rows × and columns. The numbers, symbols, points, or characters present inside a matrix are called the elements of a matrix. The location of each element is given by the row and column it belongs to.