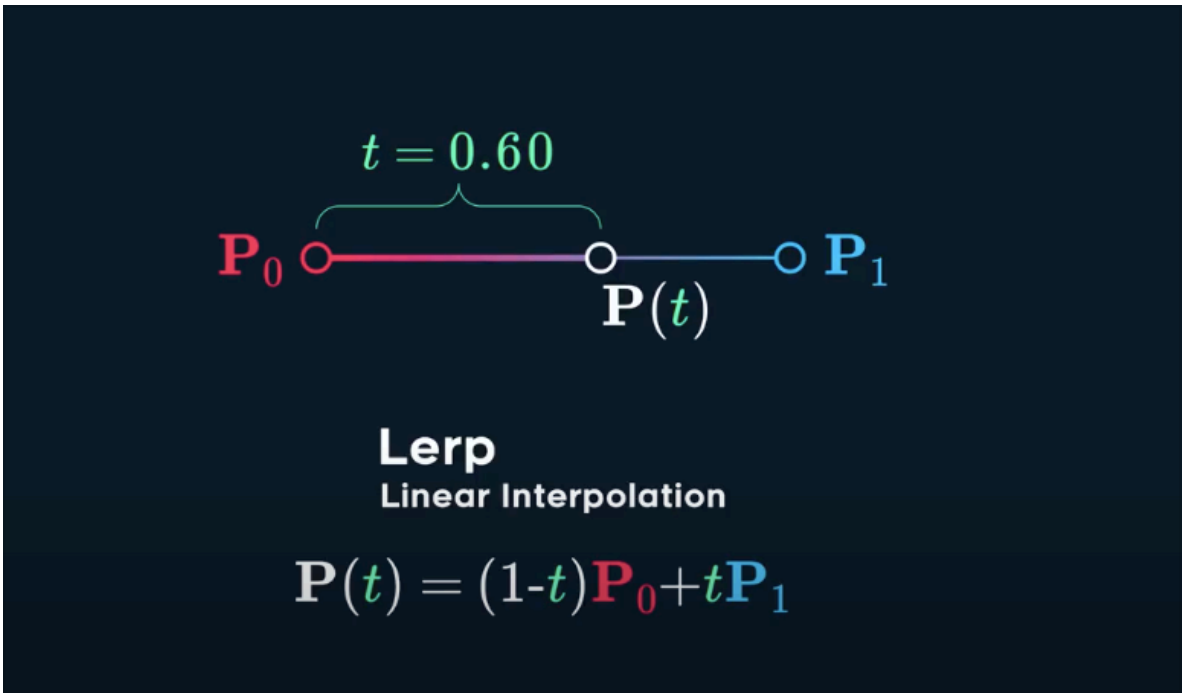
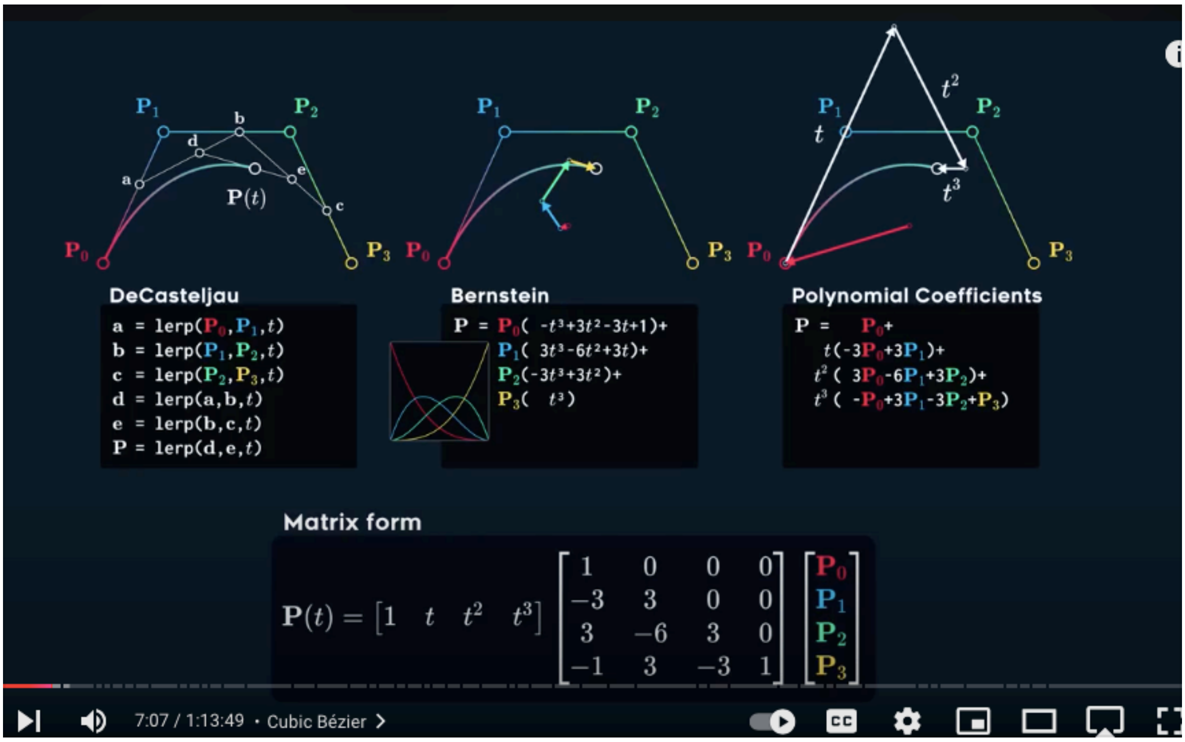
Unit 3 Curve & Surfaces

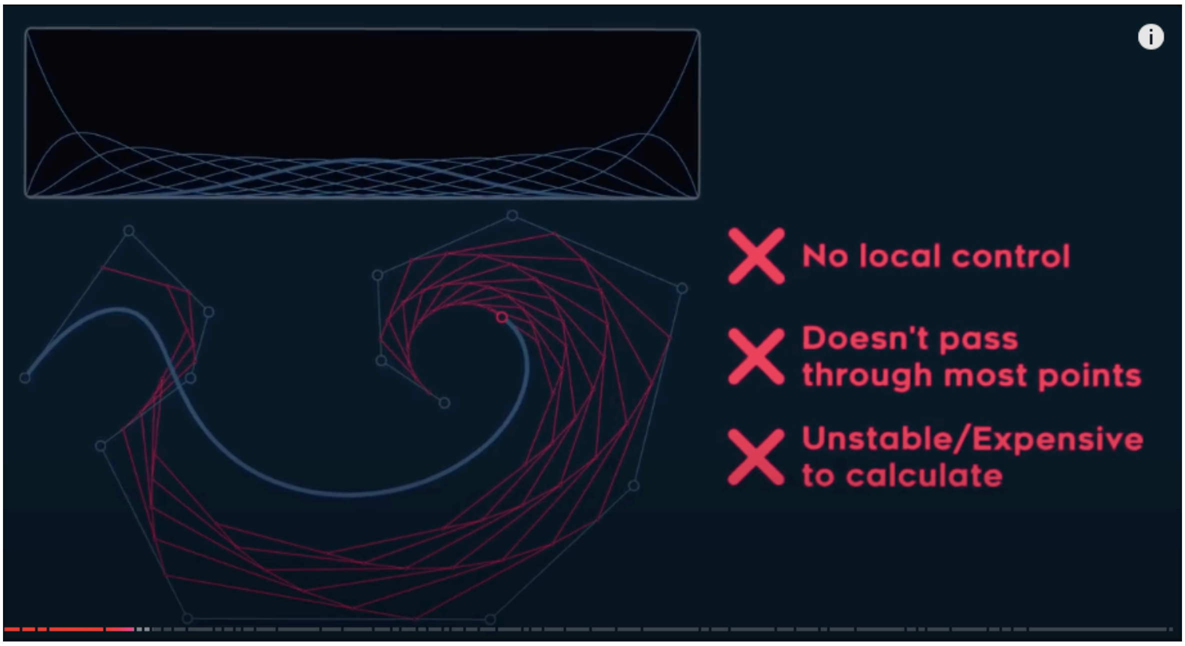
LERP:



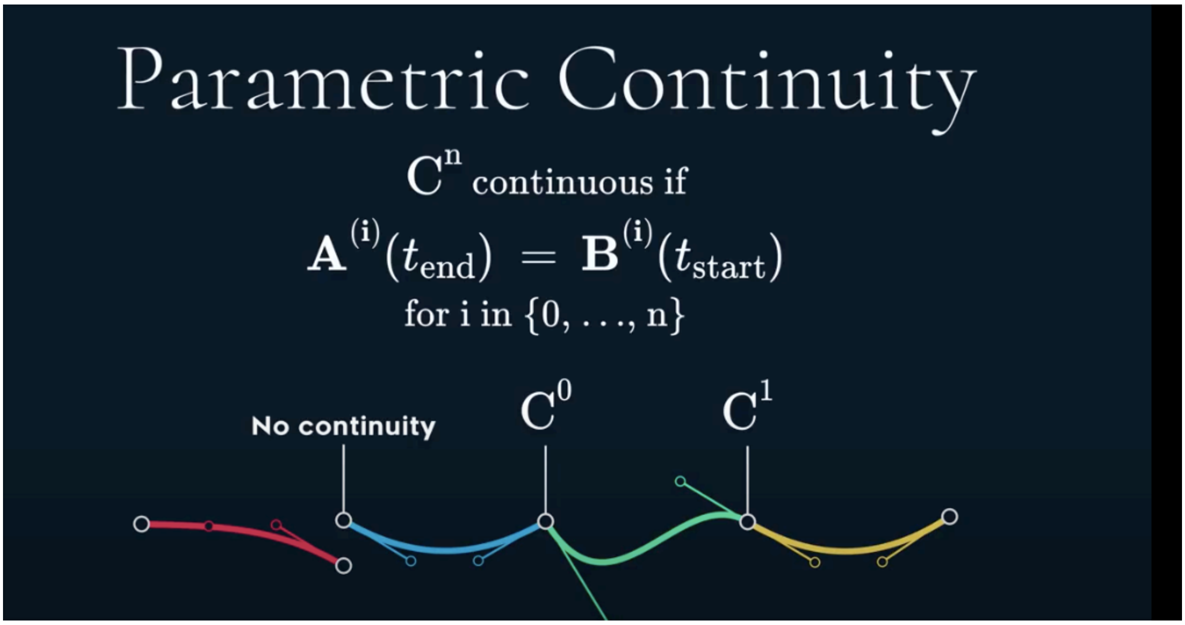
CUBIC BEZIER CURVE

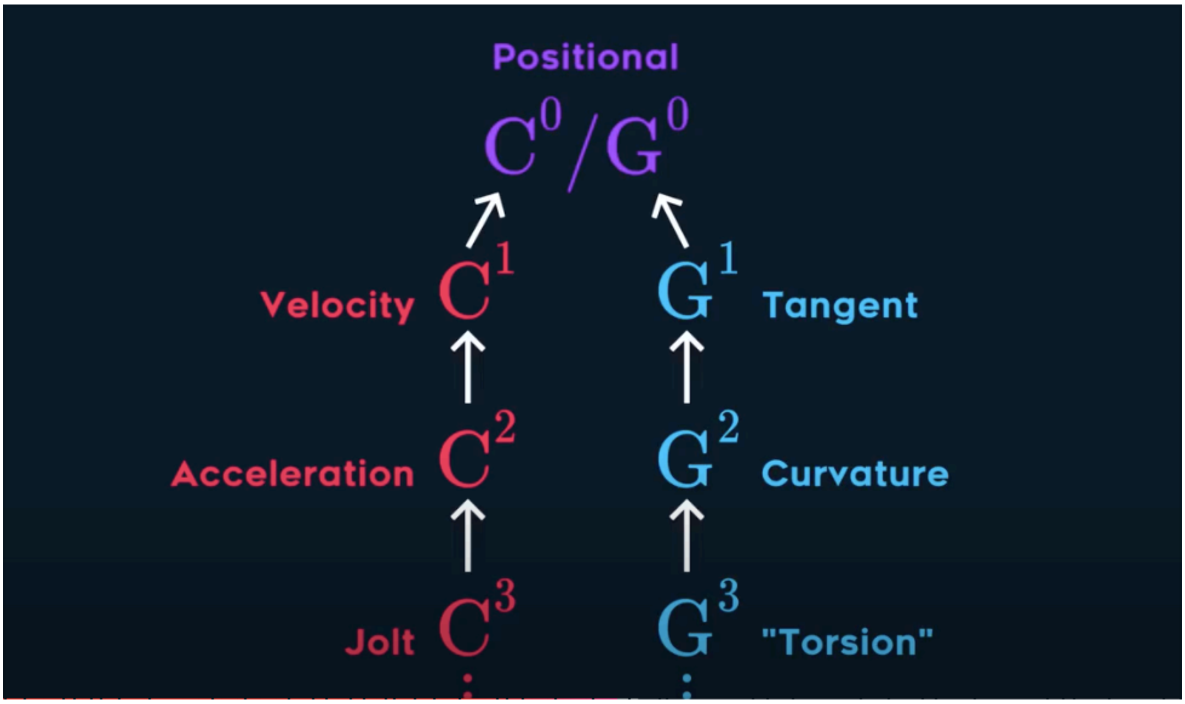


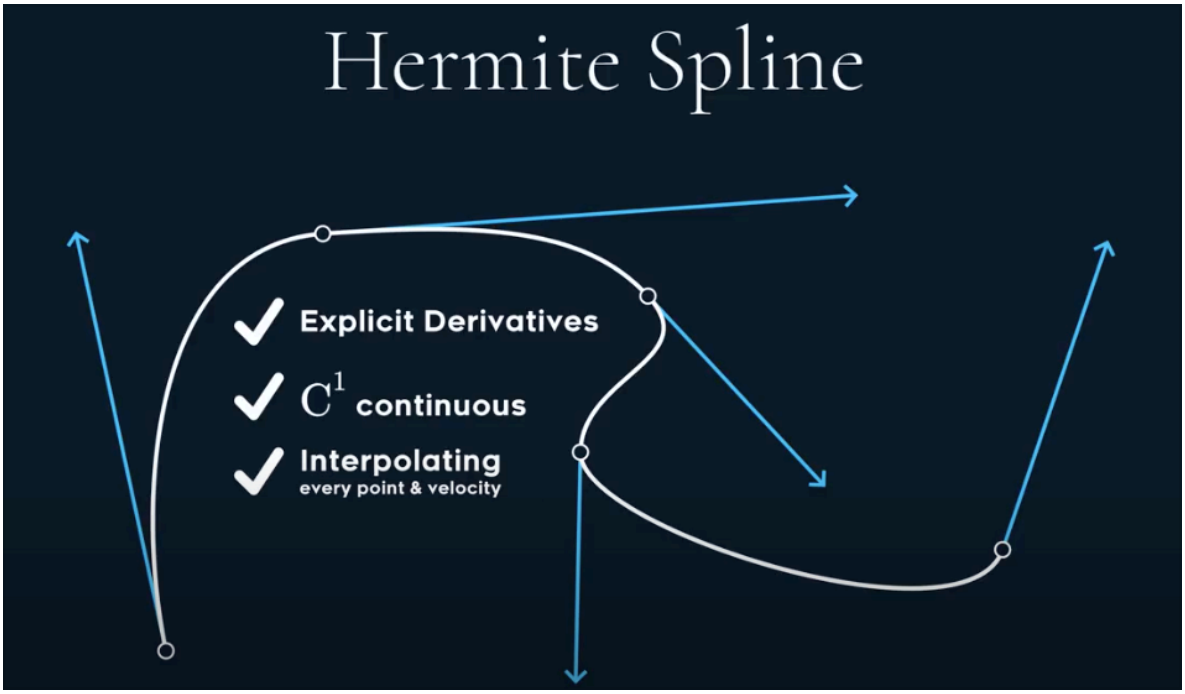
Bézier curve constraints



Bezier Spline Curve


Parametric Continuity
C^n continuous if
$\mathbf{A}^{(\mathbf{i})}(t_{\text{end}}) = \mathbf{B}^{(\mathbf{i})}(t_{\text{start}})$
for i in {0, . . ., n}
No continuity
C^0
C^1


Positional
C^0/G^0
Velocity
C^1
G^1
Tangent
Acceleration
C^2
G^2
Curvature
Jolt
C^3
G^3
"Torsion"



In computer graphics, we often
need to draw different types of objects onto the screen. Objects are not flat all the time and we need to draw curves many times to draw an object.

Types of Curves
A curve is an infinitely large set of points. Each point has two neighbors except endpoints. Curves can be broadly classified into three categories – **explicit, implicit,** and **parametric curves**.

Implicit Curves
Implicit curve representations define the set of points on a curve by employing a procedure that can test to see if a point in on the curve. Usually, an implicit curve is defined by an implicit function of the form –
f(x, y) = 0
It can represent multivalued curves (multiple y values for an x value). A common example is the circle, whose implicit representation is
$\mathbf{x^2 + y^2 - R^2 = 0}$

Explicit Curves
A mathematical function y = f(x) can be plotted as a curve. Such a function is the explicit representation of the curve. The explicit representation is not general, since it cannot represent vertical lines and is also single-valued. For each value of x, only a single value of y

is normally computed by the function.

Parametric Curves

Curves having parametric form are called parametric curves. The explicit and implicit curve representations can be used only when the function is known. In practice the parametric curves are used. A two-dimensional parametric curve has the following form –

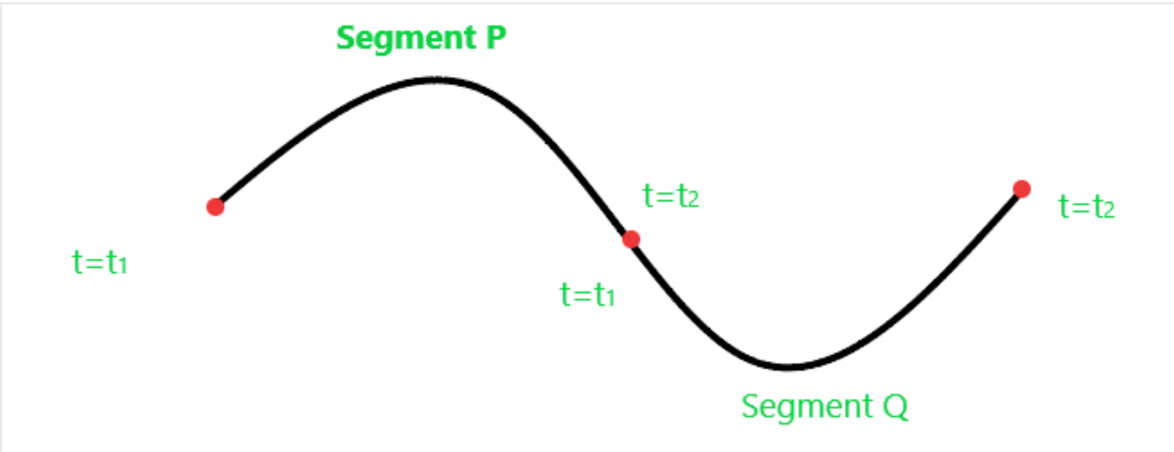
P(t) = f(t), g(t) or P(t) = x(t), y(t)

The functions f and g become the (x, y) coordinates of any point on the curve, and the points are obtained when the parameter t is varied over a certain interval [a, b], normally [0, 1].

Parametric & Geometric Continuity of Curves in Computer Graphics

Prerequisite: **B-spline curve**, **Bezier curve**

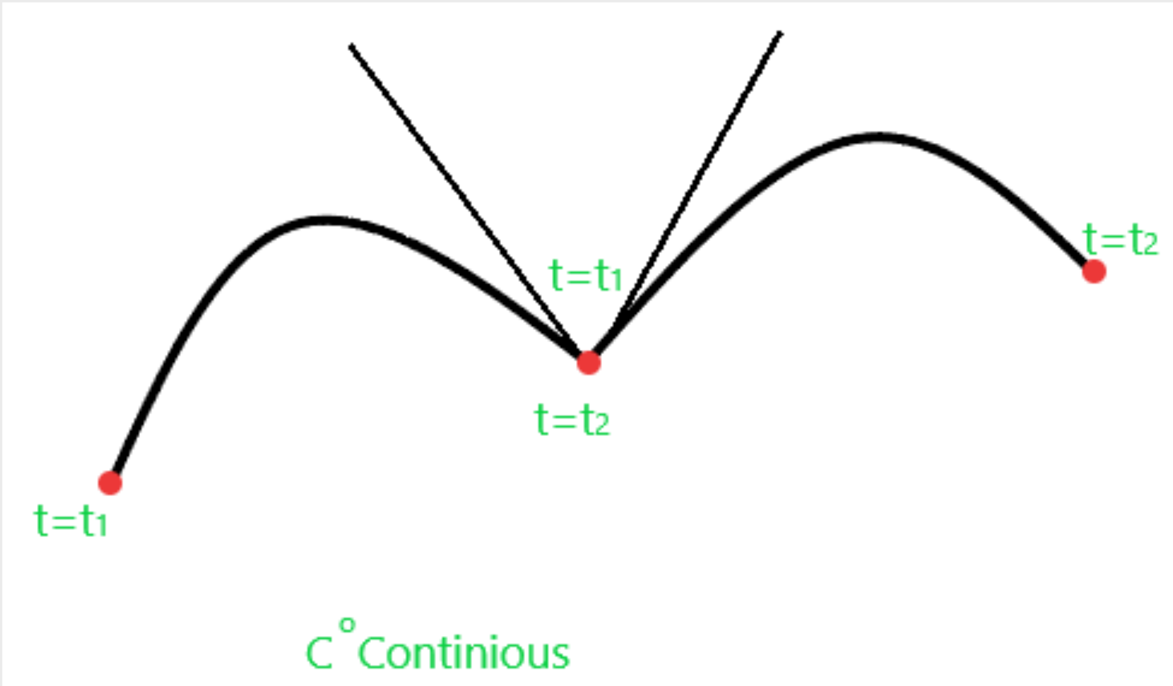
The continuity condition represents that how smoothly a curve transit from one curve segment to another segment. Consider you are given a curve as shown below:



There are three kinds of Parametric continuities that exist:

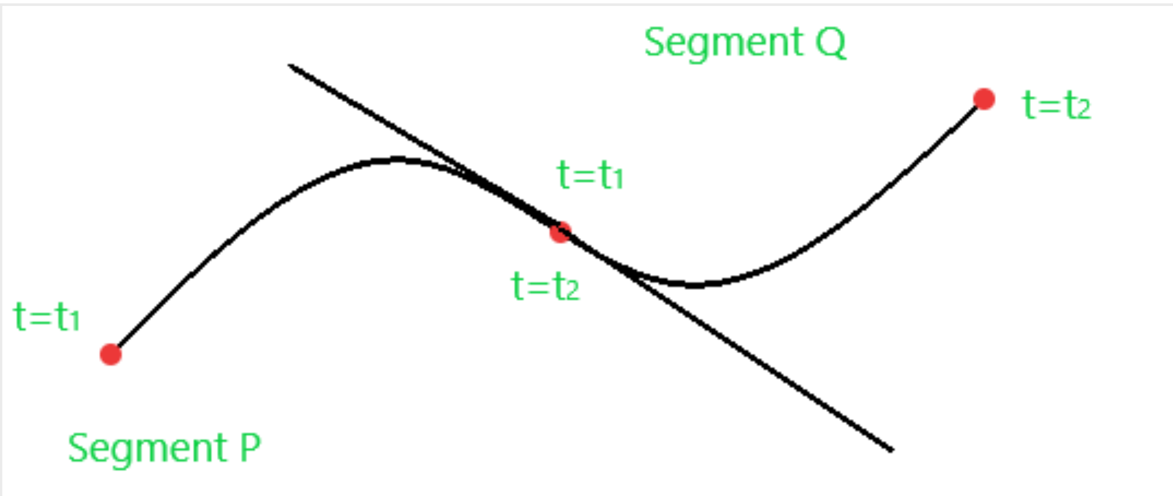
(a) Zero-order parametric continuity(C^0) : A curve is said to be zero-order parametric continuous if both segments of the curve intersect at one endpoint.

$$P(t_2) = Q(t1)$$



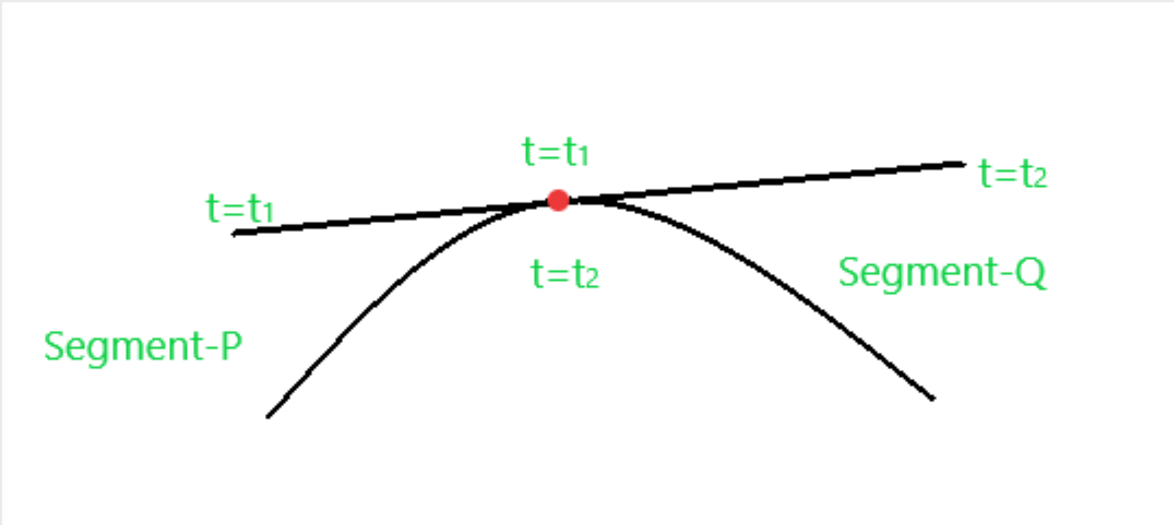
(b) First-order parametric continuity(C^1) : A curve is said to be first-order parametric continuous if it is C^O Continuous and the first-order derivative of segment P at $t=t_2$ is equal to the first-order derivative of segment Q at $t=t_1$. Such kinds of curves have the same tangent line at the intersection point.

$$P'(t_2) = Q'(t1)$$



(c) Second-order parametric continuity(C^2) : A curve is said to be second-order parametric continuous if it is C^O and C^1 Continuous and the second-order derivative of the segment P at $t=t_1$ is equal to the second-order derivative of segment Q at $t=t_2$.

$$P''(t_2) = Q''(t1)$$



Geometric Continuity : It is an alternate method for joining two curve segments, where it requires the parametric derivation of both segments which are proportional to each other rather than equal to each other.

(a) Zero-order geometric continuity(G^0) : It is similar to the zero-order parametric curve continuity condition.

$$P(t_2) = Q(t1)$$

(b) First-order geometric continuity(G^1) : The junction point between two points is said to be G^1 continuous if the coordinate of both curve segments is G^0 continuous and following the below condition:

$$P'(t_2) = k * Q'(t1) \text{ for all x, y, z and } k > 0.$$

(c) Second-order geometric continuity(G^2) : The junction point between two points is said to be G^2 continuous if the coordinate of both curve segments is G^1 continuous and following the below condition:

$$P''(t_2) = k * Q''(t1) \text{ for all x, y, z and } k > 0.$$

Spline Curve - https://people.computing.clemson.edu/~dhouse/courses/405/notes/splines.pdf