Unit-1 Set Theory & Logics

Sets and Their Representations

A set is a collection of distinct objects or elements. It is often denoted by curly braces {}.

Example:
- A = {1, 2, 3, 4} - A set of natural numbers from 1 to 4.

Representations:
- **Roster Method:** Listing all elements.
- **Set-Builder Notation:** Describing the elements using a rule.

Empty Set

The empty set, denoted by ∅ or {}, is a set that contains no elements.

Example:
- The set of even prime numbers greater than 2.

Finite and Infinite Sets

- **Finite Set:** A set that has a definite number of elements.
- **Infinite Set:** A set that has an unlimited number of elements.

Example:
- {1, 2, 3} is a finite set.
- The set of all natural numbers is an infinite set.

Equal and Equivalent Sets

- **Equal Sets:** Two sets are equal if they contain the same elements.
- **Equivalent Sets:** Two sets are equivalent if they have the same number of elements.

Example:
- {1, 2, 3} and {3, 2, 1} are equal sets.
- {1, 2} and {a, b} are equivalent sets.

Subsets

A set A is a subset of set B if all elements of A are also elements of B.

Notation: $A \subseteq B$

Example:
- {1, 2} is a subset of {1, 2, 3}.

Power Set

The power set of a set A is the set of all subsets of A.

Notation: P(A)

Example:

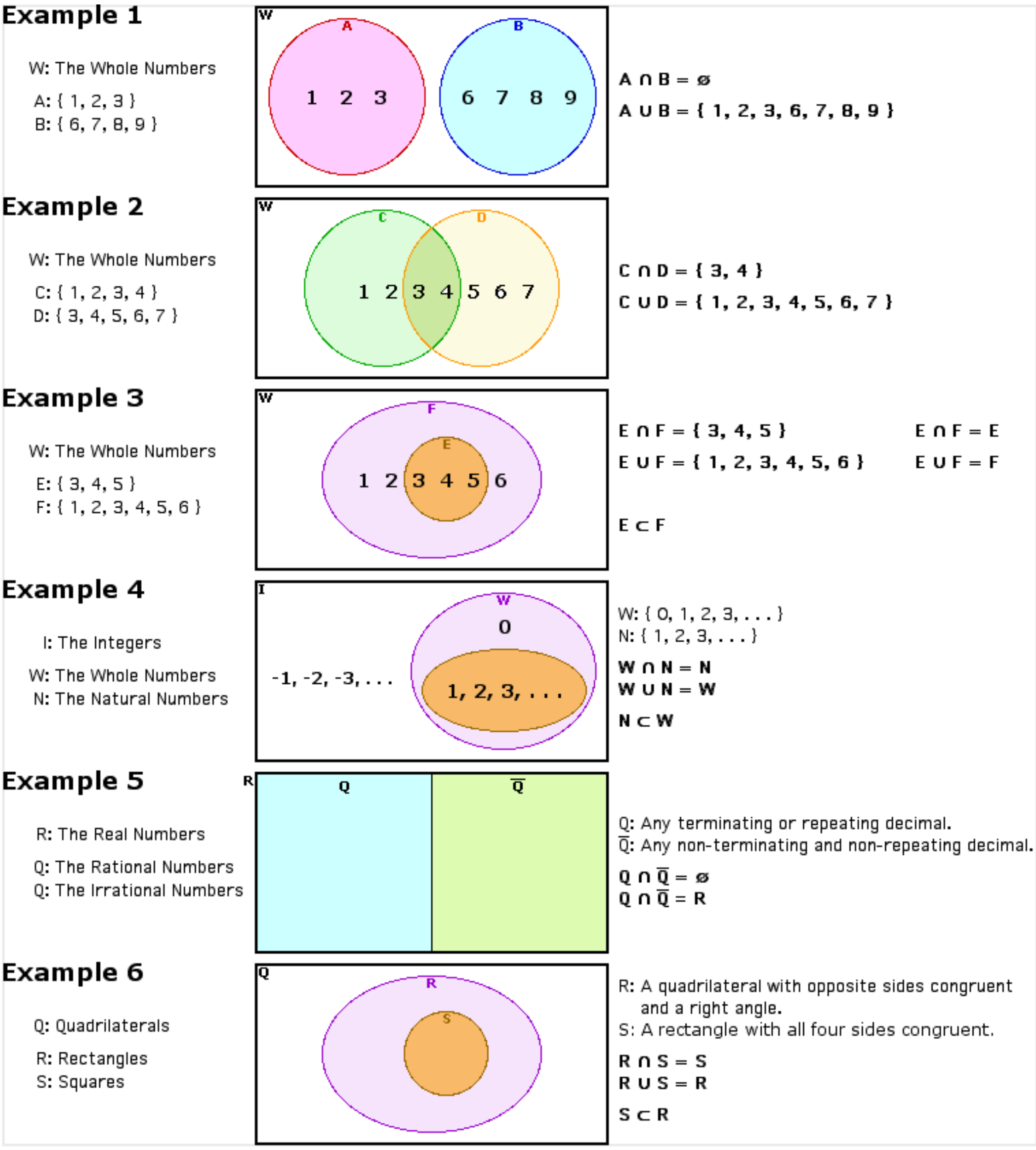
- If A = {1, 2}, then P(A) = {{}, {1}, {2}, {1, 2}}.

Universal Set

The universal set, denoted by U, is a set that contains all elements under consideration in a particular context.

Venn Diagrams

Venn diagrams are graphical representations of sets using overlapping circles or other sh



apes to show relationships between sets.

Complement of a Set

The complement of a set A, denoted by A', is the set of all elements in the universal set U that are not in A.

Operations on Sets

- **Union:** A ∪ B is the set of elements that are in A or B or both.
- **Intersection:** A ∩ B is the set of elements that are in both A and B.
- **Difference:** A - B is the set of elements that are in A but not in B.

Mathematical Logic: Basic Logical Connections

Propositions : If a statement if either correct or incorrect or true or false is called a proposition or sentence in maths

Otherwise its not ...

Types

1. **Simple Proposition -> where only one subject and one property for true and false**
2. **Compound Proposition -> Where two or more simple propositions combined with several connectives**

Tautology -> At last all true
Contradiction -> At last all false
Contingency -> Not clear / not disclose few are true, few are false ....

Basic Logical Connections

In mathematical logic, we use logical connectives to combine simple statements into compound statements. Here are the three primary logical connectives:

1. **Conjunction (∧):**
   - Represents the logical "and" operation.
   - A conjunction is true only when both of its component statements are true.

    - **Example:** "It is raining and the sun is shining."
2. **Disjunction ($\lor$):**
    - Represents the logical "or" operation.
    - A disjunction is true when at least one of its component statements is true.
    - **Example:** "I will eat an apple or an orange."
3. **Negation ($\neg$):**
    - Represents the logical "not" operation.
    - The negation of a statement is true when the original statement is false, and vice versa.
    - **Example:** "It is not cold outside."

Negation of Compound Statements

To negate a compound statement, apply the negation operator ($\neg$) to the entire statement.

- **Example:**
    - If the statement is "It is raining and the sun is shining," the negation would be "It is not raining or the sun is not shining."
    - If the statement is "I will eat an apple or an orange," the negation would be "I will not eat an apple and I will not eat an orange."

Truth Tables

A truth table is a tabular representation that shows the truth values of a compound statement for all possible combinations of truth values of its component simple statements.

- **Example:**
    - For the compound statement "$p \land q$" (p and q): | p | q | p ∧ q | |---|---|-------| | T | T | T | | T | F | F | | F | T | F | | F | F | F |

Tautologies

A tautology is a compound statement that is always true, regardless of the truth values of its component simple statements.

- **Example:**
    - "$p \lor \neg p$" (p or not p) is a tautology.

Logical Equivalence

Two compound statements are logically equivalent if they have the same truth values for all possible combinations of truth values of their component simple statements.

- **Example:**
  - "$p \rightarrow q$" (if p then q) is logically equivalent to "$\neg p \vee q$" (not p or q).

By understanding these basic logical connectives, negation, truth tables, tautologies, and logical equivalence, you can analyze and evaluate the validity of arguments and logical reasoning.

Implication (→, IF...THEN): The implication connective represents a conditional statement. "p

→ q" is read as "if p, then q." It is true unless "p" is true and "q" is false; otherwise, it is true.

Biconditional Implication (↔, IF AND ONLY IF): The biconditional connective represents a statement of

equivalence. "$p \leftrightarrow q$" is true if and only if "p" and "q" have the same truth value.