Problem Set 0

All parts are due Sunday, September 8 at 6PM.

Name: Your Name

Problem 0-1. $A = \{1, 2, 4, 8, 16\}$ and $B = \{-1, 1, 3, 5\}$

(a) $A \cap B = \{1\}$

(b) $|A \cup B| = |\{1, 2, 4, 8, 5, 3, 16, -1\}| = 8$

(c) $|A - B| = |\{2, 4, 8, 16\}| = 4$

Problem 0-2.

(a) $\mathbb{P}(0) = \frac{2}{5} * \frac{1}{4} = \frac{1}{10}$

$\mathbb{P}(1) = \frac{3}{5}$

$\mathbb{P}(2) = \frac{3}{5} * \frac{2}{4} = \frac{3}{10}$

X	0	1	2
$\mathbb{P}$	$\frac{1}{10}$	$\frac{6}{10}$	$\frac{3}{10}$

$\mathbb{E}[X] = \sum_{i=1}^{3} x_i p_i = 0 * 0.1 + 1 * 0.6 + 2 * 0.3 = 1.2$

(b)

Y	0	1	2
$\mathbb{P}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\mathbb{E}[Y] = 1$

(c) $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y] = 1.2 + 1 = 2.2$

Problem 0-3.

(a) $A \equiv B \mod 2$

$(606 - 360) \mod 2 = 0 \Rightarrow true$

(b) $A \equiv B \mod 3$

$(606 - 360) \mod 3 = 0 \Rightarrow true$

(c) $A \equiv B \mod 4$

$(606 - 360) \mod 4 = 2 \Rightarrow false$

Problem 0-4. Base case: $n = 0$ then $\sum_{i=0}^{0} a^i = \frac{1-a^0}{1-a} = 1$ Assume that is true for $n = k$. We want to show that the statement is true for $n = k + 1$.

$\sum_{i=0}^{k+1} a^i = \frac{1-a^{k+2}}{1-a}$

$\sum_{i=0}^{k+1} a^i = a^{k+1} + \sum_{i=0}^{k} a^i = a^k + \frac{1-a^{k+1}}{1-a} = \frac{a^{k+1}(1-a)}{1-a} + \frac{1-a^{k+1}}{1-a} = \frac{a^{k+1}(1-a)+(1-a^{k+1})}{1-a} = \frac{a^{k+1}-a^{k+2}+1-a^{k+1}}{1-a} = \frac{1-a^{k+2}}{1-a}$, as desired.

Problem 0-5. Base case: A tree contains only one vertex without edges. It means it can be colored to red or blue color. Assume that is true for k vertexes of the tree. We want to show that the statement is true for $k + 1$ vertexes. From the condition, we have that the vertex v connected with only one edge with vertex u. If we delete the edge which connects v and u edges, we will have the T' tree. The T' tree contains k vertexes. By the induction hypothesis, the tree consisting k vertexes doesn't have the adjacent vertices with the same color. Thus, the color of u vertex must be colored to opposite color than vertex v. Therefore, anyone edge in the tree doesn't connect vertexes with the same color.