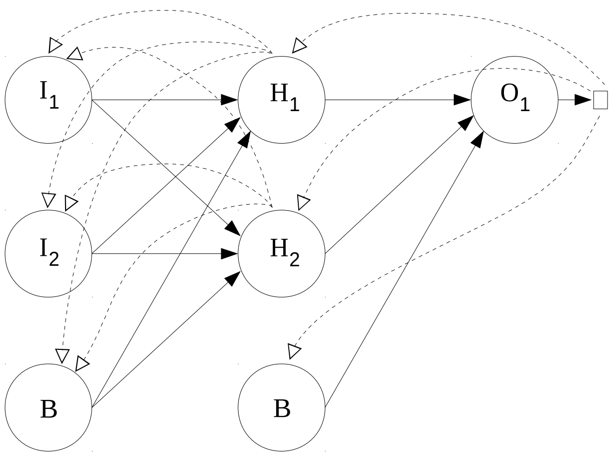
< Artificial Intelligence >--

XOR Hello World using FFANN Backpropagation

Revision #: 3
4. September 2016



Training a Feed Forward Artificial Neural Network (FFANN) using Backpropagation Training to solve a simple XOR example.

XOR example is like a Hello World to a Neural Network.

Full calculations are shown for the first two epochs only to understand the flow and equations. The rest of the epochs will be calculated using our C program ffann_backprop.c.

These calculations are tested against output from Encog framework. The results from these calculations are almost the same with output from Encog framework.

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Document Revision History		
Rev. #	Date	Comment
2	4. Sep. 2016	Completed draft.
3	4. Sep. 2016	Remove duplicated due to copy/paste.

List of equations

Summation for neuron n at training pattern p :

$$S_n^p = \sum_j w_{jn} y_j^p$$

where,

- p - Training pattern index number or training iteration index number
- n - Neuron index number for current layer
- j - Neuron index number from input layer
- w_{jn} - Weight between neuron j and neuron n
- y_j^p - The output of neuron j at training pattern p
- t - epoch number
- α - momentum
- γ - learning rate (gain)

Sigmoid activation and it's derivative:

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = f(x)(1-f(x))$$

Delta for output neuron:

$$\delta_o^p = \frac{\partial E_o^p}{\partial S_o^p} = \frac{\partial E_o^p}{\partial f(S_o^p)} \frac{\partial f(S_o^p)}{\partial S_o^p}$$

where,

- $\frac{\partial E_o^p}{\partial S_o^p}$ - The partial derivative of output error with respect to S_o^p . The gradient from E_o^p to S_o^p
- $\frac{\partial E_o^p}{\partial f(S_o^p)}$ - The gradient from E_o^p to $f(S_o^p)$
- $\frac{\partial f(S_o^p)}{\partial S_o^p}$ - The gradient from $f(S_o^p)$ to S_o^p

Delta for hidden neuron:

$$\delta_h^p = \frac{\partial E_o^p}{\partial S_h^p} = \frac{\partial E_o^p}{\partial f(S_h^p)} \frac{\partial f(S_h^p)}{\partial S_h^p}$$

where,

- $\frac{\partial E_o^p}{\partial S_h^p}$ - The gradient from E_o^p to S_h^p
- $\frac{\partial E_o^p}{\partial f(S_h^p)}$ - The gradient from E_o^p to $f(S_h^p)$
- $\frac{\partial f(S_h^p)}{\partial S_h^p}$ - The gradient from $f(S_h^p)$ to S_h^p

Amount required to change the w_j at epoch t :

$$\Delta_p w_j(t) = -\gamma \frac{\partial E_o^p}{\partial w_j} + \alpha \Delta_p w_j(t-1)$$

where,

- p - training pattern index number or training iteration index number
- w - weight
- j - weight index number
- t - epoch number
- α - momentum
- γ - learning rate (gain)
- $\frac{\partial E_o^p}{\partial w_j}$ - The gradient from E_o^p to w_j

Common error calculation formulas:

$$SSE = \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d_o^p - f(S_o^p)\right)^2$$

$$LSE = \frac{1}{2} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d_o^p - f(S_o^p)\right)^2$$

$$MSE = \frac{1}{P \cdot N_o} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d_o^p - f(S_o^p)\right)^2$$

$$RMSE = \sqrt{\frac{1}{P \cdot N_o} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d_o^p - f(S_o^p)\right)^2}$$

where,

- P - Total number of training patterns
- N_o - Total number of output units (classes)
- d - Desired output (unit)
- $f(S_o^p)$ - Actual neuron output (unit)
- α - momentum
- γ - learning rate (gain)

Brief explanation

Our training data set:

Input1	Input2	Desired Output1
0	0	0
1	0	1
0	1	1
1	1	0

There are 4 training patterns in the data set:

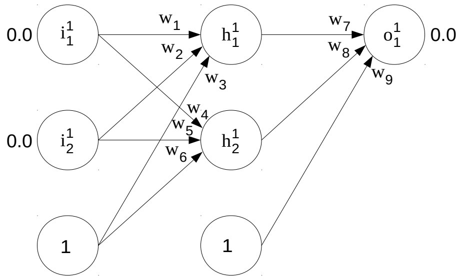
1) When Input1 = 0 and Input2 = 0, the output should be 0.
2) When Input1 = 1 and Input2 = 0, the output should be 1.
3) When Input1 = 0 and Input2 = 1, the output should be 1.
4) When Input1 = 1 and Input2 = 1, the output should be 0.

The next page is the calculations for FFANN Backpropagation. We will be using training rate $\gamma = 0.7$ and momentum $\alpha = 0.3$ when updating the weight values.

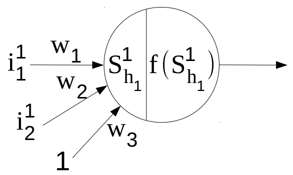
References

1. Kattan, Ali Abdullah, Rosni Geem, Zong Woo, "Computer Networks : Artificial Neural Network Training and Software Implementation Techniques".
2. Alavala, Chennakesava R., "Fuzzy Logic and Neural Networks : Basic Concepts & Application".
3. Jeff Heaton, “Programming Neural Networks with Encog3 in Java”.
4. R. Lippmann, "An introduction to computing with neural nets," in *IEEE ASSP Magazine*, vol. 4, no. 2, pp. 4-22, Apr 1987. doi: 10.1109/MASSP.1987.1165576
5. Matt Mazur. A Step by Step Backpropagation Example, 2015, https://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/.

Epoch 1 : Iteration 1

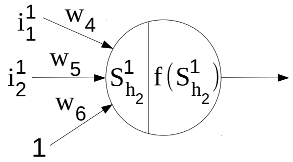


$w_1 = 0.1000$ $\quad w_4 = 0.4000$ $\quad w_7 = 0.7000$
$w_2 = 0.2000$ $\quad w_5 = 0.5000$ $\quad w_8 = 0.8000$
$w_3 = 0.3000$ $\quad w_6 = 0.6000$ $\quad w_9 = 0.9000$



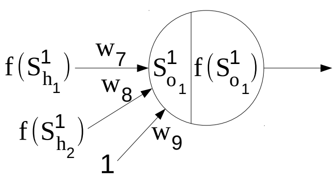
$$
\begin{aligned}
S_{h_1}^1 &= \sum_j w_{jh_1} y_j^1 \\
&= w_1 i_1^1 + w_2 i_2^1 + w_3(1) \\
&= 0.1000(0.0000)+0.2000(0.0000)+0.3000(1) \\
&= 0.3000
\end{aligned}
$$

$$
\begin{aligned}
f(S_{h_1}^1) &= \frac{1}{1+e^{-S_{h_1}^1}} \\
&= \frac{1}{1+e^{-0.3000}} \\
&= 0.5744
\end{aligned}
$$



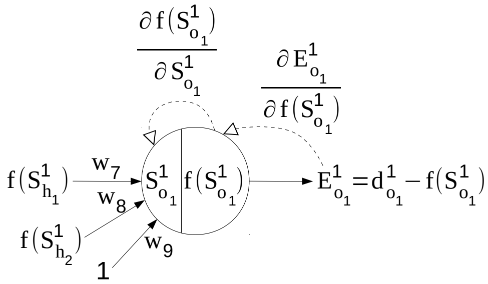
$$
\begin{aligned}
S_{h_2}^1 &= \sum_j w_{jh_2} y_j^1 \\
&= w_4 i_1^1 + w_5 i_2^1 + w_6(1) \\
&= 0.4000(0.0000)+0.5000(0.0000)+0.6000(1) \\
&= 0.6000
\end{aligned}
$$

$$
\begin{aligned}
f(S_{h_2}^1) &= \frac{1}{1+e^{-S_{h_2}^1}} \\
&= \frac{1}{1+e^{-0.6000}} \\
&= 0.6457
\end{aligned}
$$



$$
\begin{aligned}
S_{o_1}^1 &= \sum_j w_{jo_1} y_j^1 \\
&= w_7 f(S_{h_1}^1) + w_8 f(S_{h_2}^1) + w_9(1) \\
&= 0.7000(0.5744)+0.8000(0.6457)+0.9000(1) \\
&= 1.8186
\end{aligned}
$$

$$
\begin{aligned}
f(S_{o_1}^1) &= \frac{1}{1+e^{-S_{o_1}^1}} \\
&= \frac{1}{1+e^{-1.8186}} \\
&= 0.8604
\end{aligned}
$$

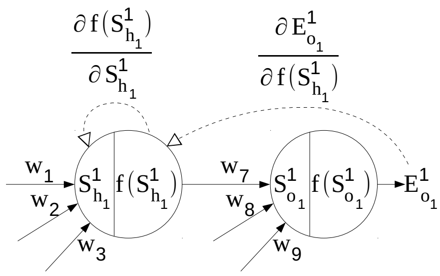


$$
\begin{aligned}
\delta_{o_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \\
&= \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} &= -\left(d_{o_1}^1 - f(S_{o_1}^1)\right) \\
&= -(0.0000-0.8604) \\
&= -(-0.8604) \\
&= 0.8604
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} &= f'(S_{o_1}^1) \\
&= f(S_{o_1}^1)\left(1-f(S_{o_1}^1)\right) \\
&= 0.8604(1-0.8604) \\
&= 0.1201
\end{aligned}
$$

$$
\begin{aligned}
\delta_{o_1}^1 &= 0.8604(0.1201) \\
&= 0.1033
\end{aligned}
$$

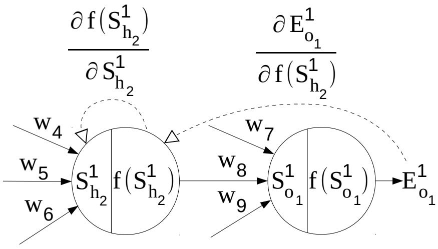


$$
\begin{aligned}
\delta_{h_1}^1 &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \\
&= \frac{\partial E_o^1}{\partial f(S_{h_1}^1)} \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^1}{\partial f(S_{h_1}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^1}{\partial S_o^1} \frac{\partial S_o^1}{\partial f(S_{h_1}^1)} \\
&= \sum_{o=1}^{N_o} \delta_o^1 w_{h_1 o} \\
&= \delta_{o_1}^1 w_7 \\
&= 0.1033(0.7000) \\
&= 0.0723
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} &= f'(S_{h_1}^1) \\
&= f(S_{h_1}^1)\left(1-f(S_{h_1}^1)\right) \\
&= 0.5744(1-0.5744) \\
&= 0.2445
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_1}^1 &= 0.0723(0.2445) \\
&= 0.0177
\end{aligned}
$$



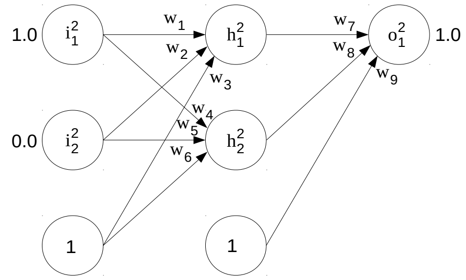
$$
\begin{aligned}
\delta_{h_2}^1 &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \\
&= \frac{\partial E_o^1}{\partial f(S_{h_2}^1)} \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^1}{\partial f(S_{h_2}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^1}{\partial S_o^1} \frac{\partial S_o^1}{\partial f(S_{h_2}^1)} \\
&= \sum_{o=1}^{N_o} \delta_o^1 w_{h_2 o} \\
&= \delta_{o_1}^1 w_8 \\
&= 0.1033(0.8000) \\
&= 0.0826
\end{aligned}
$$

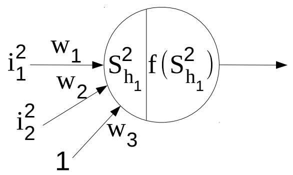
$$
\begin{aligned}
\frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} &= f'(S_{h_2}^1) \\
&= f(S_{h_2}^1)\left(1-f(S_{h_2}^1)\right) \\
&= 0.6457(1-0.6457) \\
&= 0.2288
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_2}^1 &= 0.0826(0.2288) \\
&= 0.0189
\end{aligned}
$$

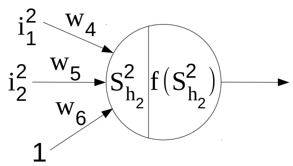
Epoch 1 : Iteration 2



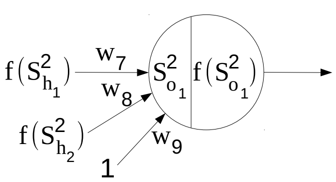
$w_1 = 0.1000$ $\quad w_4 = 0.4000$ $\quad w_7 = 0.7000$
$w_2 = 0.2000$ $\quad w_5 = 0.5000$ $\quad w_8 = 0.8000$
$w_3 = 0.3000$ $\quad w_6 = 0.6000$ $\quad w_9 = 0.9000$



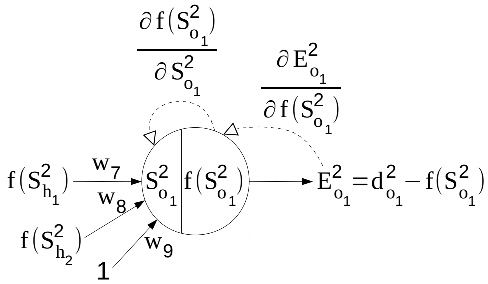
$$
\begin{aligned}
S_{h_1}^2 &= \sum_j w_{jh_1} y_j^2 \\
&= w_1 i_1^2 + w_2 i_2^2 + w_3(1) \\
&= 0.1000(1.0000)+0.2000(0.0000)+0.3000(1) \\
&= 0.4000 \\
f(S_{h_1}^2) &= \frac{1}{1+e^{-S_{h_1}^2}} \\
&= \frac{1}{1+e^{-0.4000}} \\
&= 0.5987
\end{aligned}
$$



$$
\begin{aligned}
S_{h_2}^2 &= \sum_j w_{jh_2} y_j^2 \\
&= w_4 i_1^2 + w_5 i_2^2 + w_6(1) \\
&= 0.4000(1.0000)+0.5000(0.0000)+0.6000(1) \\
&= 1.0000 \\
f(S_{h_2}^2) &= \frac{1}{1+e^{-S_{h_2}^2}} \\
&= \frac{1}{1+e^{-1.0000}} \\
&= 0.7311
\end{aligned}
$$



$$
\begin{aligned}
S_{o_1}^2 &= \sum_j w_{jo_1} y_j^2 \\
&= w_7 f(S_{h_1}^2) + w_8 f(S_{h_2}^2) + w_9(1) \\
&= 0.7000(0.5987)+0.8000(0.7311)+0.9000(1) \\
&= 1.9040 \\
f(S_{o_1}^2) &= \frac{1}{1+e^{-S_{o_1}^2}} \\
&= \frac{1}{1+e^{-1.9040}} \\
&= 0.8703
\end{aligned}
$$

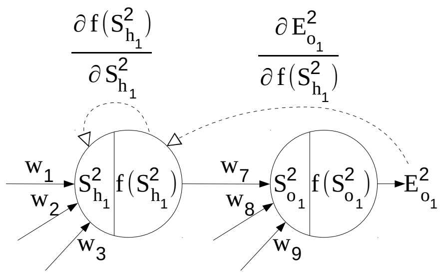


$$
\begin{aligned}
\delta_{o_1}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \\
&= \frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} \frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} &= -\left(d_{o_1}^2 - f(S_{o_1}^2)\right) \\
&= -(1.0000-0.8703) \\
&= -(0.1297) \\
&= -0.1297
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2} &= f'(S_{o_1}^2) \\
&= f(S_{o_1}^2)\left(1-f(S_{o_1}^2)\right) \\
&= 0.8703(1-0.8703) \\
&= 0.1129
\end{aligned}
$$

$$
\begin{aligned}
\delta_{o_1}^2 &= -0.1297(0.1129) \\
&= -0.0146
\end{aligned}
$$

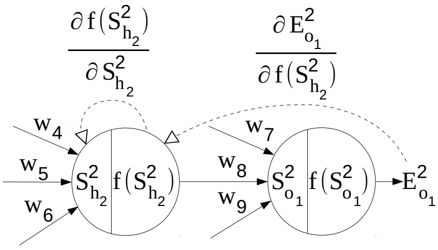


$$
\begin{aligned}
\delta_{h_1}^2 &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \\
&= \frac{\partial E_o^2}{\partial f(S_{h_1}^2)} \frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^2}{\partial f(S_{h_1}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^2}{\partial S_o^2} \frac{\partial S_o^2}{\partial f(S_{h_1}^2)} \\
&= \sum_{o=1}^{N_o} \delta_o^2 w_{h_1 o} \\
&= \delta_{o_1}^2 w_7 \\
&= -0.0146(0.7000) \\
&= -0.0102
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2} &= f'(S_{h_1}^2) \\
&= f(S_{h_1}^2)\left(1-f(S_{h_1}^2)\right) \\
&= 0.5987(1-0.5987) \\
&= 0.2403
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_1}^2 &= -0.0102(0.2403) \\
&= -0.0025
\end{aligned}
$$



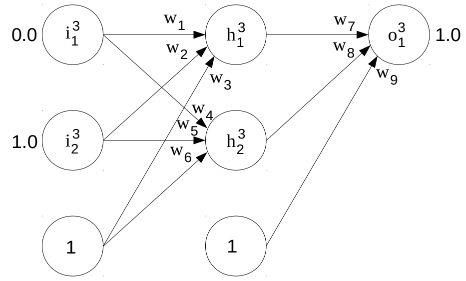
$$
\begin{aligned}
\delta_{h_2}^2 &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \\
&= \frac{\partial E_o^2}{\partial f(S_{h_2}^2)} \frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^2}{\partial f(S_{h_2}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^2}{\partial S_o^2} \frac{\partial S_o^2}{\partial f(S_{h_2}^2)} \\
&= \sum_{o=1}^{N_o} \delta_o^2 w_{h_2 o} \\
&= \delta_{o_1}^2 w_8 \\
&= -0.0146(0.8000) \\
&= -0.0117
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2} &= f'(S_{h_2}^2) \\
&= f(S_{h_2}^2)\left(1-f(S_{h_2}^2)\right) \\
&= 0.7311(1-0.7311) \\
&= 0.1966
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_2}^2 &= -0.0117(0.1966) \\
&= -0.0023
\end{aligned}
$$

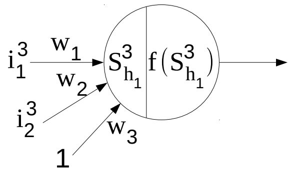
Epoch 1 : Iteration 3



$w_1 = 0.1000 \quad w_4 = 0.4000 \quad w_7 = 0.7000$

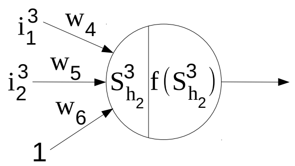
$w_2 = 0.2000 \quad w_5 = 0.5000 \quad w_8 = 0.8000$

$w_3 = 0.3000 \quad w_6 = 0.6000 \quad w_9 = 0.9000$



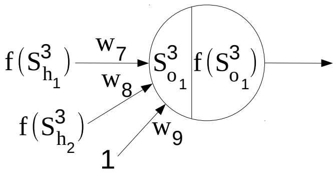
$$
\begin{aligned}
S^3_{h_1} &= \sum_j w_{jh_1} y^3_j \\
&= w_1 i^3_1 + w_2 i^3_2 + w_3(1) \\
&= 0.1000(0.0000) + 0.2000(1.0000) + 0.3000(1) \\
&= 0.5000
\end{aligned}
$$

$$
\begin{aligned}
f(S^3_{h_1}) &= \frac{1}{1+e^{-S^3_{h_1}}} \\
&= \frac{1}{1+e^{-0.5000}} \\
&= 0.6225
\end{aligned}
$$



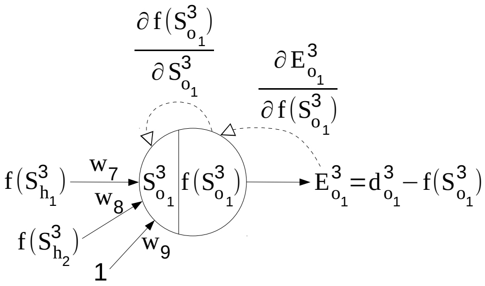
$$
\begin{aligned}
S^3_{h_2} &= \sum_j w_{jh_2} y^3_j \\
&= w_4 i^3_1 + w_5 i^3_2 + w_6(1) \\
&= 0.4000(0.0000) + 0.5000(1.0000) + 0.6000(1) \\
&= 1.1000
\end{aligned}
$$

$$
\begin{aligned}
f(S^3_{h_2}) &= \frac{1}{1+e^{-S^3_{h_2}}} \\
&= \frac{1}{1+e^{-1.1000}} \\
&= 0.7503
\end{aligned}
$$



$$
\begin{aligned}
S^3_{o_1} &= \sum_j w_{jo_1} y^3_j \\
&= w_7 f(S^3_{h_1}) + w_8 f(S^3_{h_2}) + w_9(1) \\
&= 0.7000(0.6225) + 0.8000(0.7503) + 0.9000(1) \\
&= 1.9360
\end{aligned}
$$

$$
\begin{aligned}
f(S^3_{o_1}) &= \frac{1}{1+e^{-S^3_{o_1}}} \\
&= \frac{1}{1+e^{-1.9360}} \\
&= 0.8739
\end{aligned}
$$

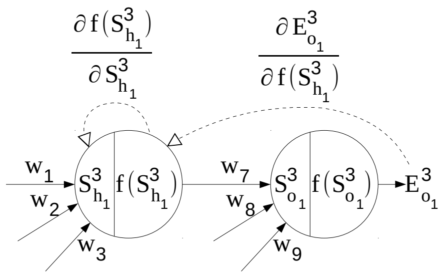


$$
\begin{aligned}
\delta^3_{o_1} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \\
&= \frac{\partial E^3_{o_1}}{\partial f(S^3_{o_1})} \frac{\partial f(S^3_{o_1})}{\partial S^3_{o_1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial f(S^3_{o_1})} &= -\left(d^3_{o_1} - f(S^3_{o_1})\right) \\
&= -(1.0000 - 0.8739) \\
&= -(0.1261) \\
&= -0.1261
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S^3_{o_1})}{\partial S^3_{o_1}} &= f'(S^3_{o_1}) \\
&= f(S^3_{o_1})\left(1 - f(S^3_{o_1})\right) \\
&= 0.8739(1 - 0.8739) \\
&= 0.1102
\end{aligned}
$$

$$
\begin{aligned}
\delta^3_{o_1} &= -0.1261(0.1102) \\
&= -0.0139
\end{aligned}
$$

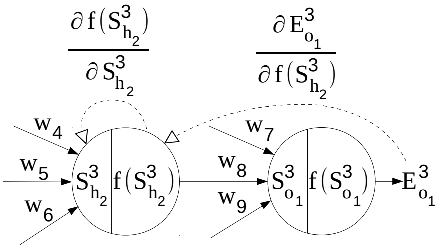


$$
\begin{aligned}
\delta^3_{h_1} &= \frac{\partial E^3_o}{\partial S^3_{h_1}} \\
&= \frac{\partial E^3_o}{\partial f(S^3_{h_1})} \frac{\partial f(S^3_{h_1})}{\partial S^3_{h_1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^3_o}{\partial f(S^3_{h_1})} &= \sum_{o=1}^{N_o} \frac{\partial E^3_o}{\partial S^3_o} \frac{\partial S^3_o}{\partial f(S^3_{h_1})} \\
&= \sum_{o=1}^{N_o} \delta^3_o w_{h_1 o} \\
&= \delta^3_{o_1} w_7 \\
&= -0.0139(0.7000) \\
&= -0.0097
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S^3_{h_1})}{\partial S^3_{h_1}} &= f'(S^3_{h_1}) \\
&= f(S^3_{h_1})\left(1 - f(S^3_{h_1})\right) \\
&= 0.6225(1 - 0.6225) \\
&= 0.2350
\end{aligned}
$$

$$
\begin{aligned}
\delta^3_{h_1} &= -0.0097(0.2350) \\
&= -0.0023
\end{aligned}
$$



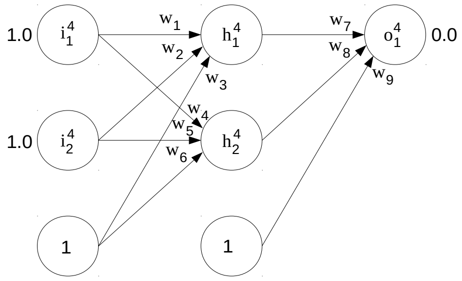
$$
\begin{aligned}
\delta^3_{h_2} &= \frac{\partial E^3_o}{\partial S^3_{h_2}} \\
&= \frac{\partial E^3_o}{\partial f(S^3_{h_2})} \frac{\partial f(S^3_{h_2})}{\partial S^3_{h_2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^3_o}{\partial f(S^3_{h_2})} &= \sum_{o=1}^{N_o} \frac{\partial E^3_o}{\partial S^3_o} \frac{\partial S^3_o}{\partial f(S^3_{h_2})} \\
&= \sum_{o=1}^{N_o} \delta^3_o w_{h_2 o} \\
&= \delta^3_{o_1} w_8 \\
&= -0.0139(0.8000) \\
&= -0.0111
\end{aligned}
$$

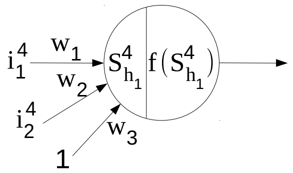
$$
\begin{aligned}
\frac{\partial f(S^3_{h_2})}{\partial S^3_{h_2}} &= f'(S^3_{h_2}) \\
&= f(S^3_{h_2})\left(1 - f(S^3_{h_2})\right) \\
&= 0.7503(1 - 0.7503) \\
&= 0.1873
\end{aligned}
$$

$$
\begin{aligned}
\delta^3_{h_2} &= -0.0111(0.1873) \\
&= -0.0021
\end{aligned}
$$

Epoch 1 : Iteration 4

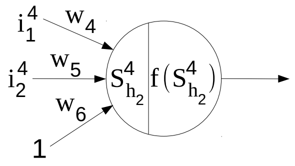


$$
\begin{array}{lll}
w_1 = 0.1000 & w_4 = 0.4000 & w_7 = 0.7000 \\
w_2 = 0.2000 & w_5 = 0.5000 & w_8 = 0.8000 \\
w_3 = 0.3000 & w_6 = 0.6000 & w_9 = 0.9000
\end{array}
$$



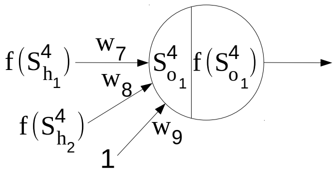
$$
\begin{aligned}
S^4_{h_1} &= \sum_j w_{jh_1} y^4_j \\
&= w_1 i^4_1 + w_2 i^4_2 + w_3(1) \\
&= 0.1000(1.0000)+0.2000(1.0000)+0.3000(1) \\
&= 0.6000
\end{aligned}
$$

$$
\begin{aligned}
f(S^4_{h_1}) &= \frac{1}{1+e^{-S^4_{h_1}}} \\
&= \frac{1}{1+e^{-0.6000}} \\
&= 0.6457
\end{aligned}
$$



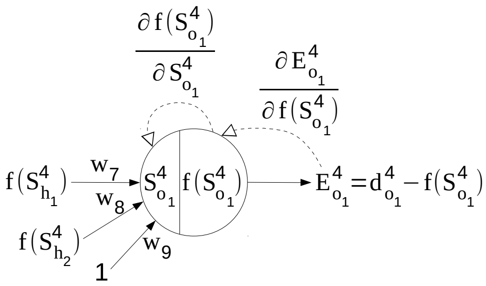
$$
\begin{aligned}
S^4_{h_2} &= \sum_j w_{jh_2} y^4_j \\
&= w_4 i^4_1 + w_5 i^4_2 + w_6(1) \\
&= 0.4000(1.0000)+0.5000(1.0000)+0.6000(1) \\
&= 1.5000
\end{aligned}
$$

$$
\begin{aligned}
f(S^4_{h_2}) &= \frac{1}{1+e^{-S^4_{h_2}}} \\
&= \frac{1}{1+e^{-1.5000}} \\
&= 0.8176
\end{aligned}
$$



$$
\begin{aligned}
S^4_{o_1} &= \sum_j w_{jo_1} y^4_j \\
&= w_7 f(S^4_{h_1}) + w_8 f(S^4_{h_2}) + w_9(1) \\
&= 0.7000(0.6457)+0.8000(0.8176)+0.9000(1) \\
&= 2.0061
\end{aligned}
$$

$$
\begin{aligned}
f(S^4_{o_1}) &= \frac{1}{1+e^{-S^4_{o_1}}} \\
&= \frac{1}{1+e^{-2.0061}} \\
&= 0.8814
\end{aligned}
$$

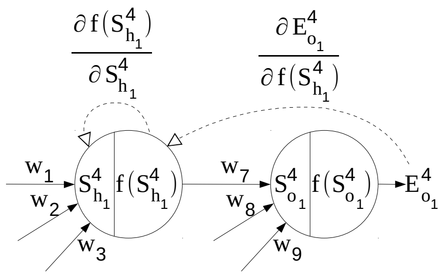


$$
\begin{aligned}
\delta^4_{o_1} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \\
&= \frac{\partial E^4_{o_1}}{\partial f(S^4_{o_1})} \frac{\partial f(S^4_{o_1})}{\partial S^4_{o_1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial f(S^4_{o_1})} &= -\left(d^4_{o_1} - f(S^4_{o_1})\right) \\
&= -(0.0000-0.8814) \\
&= -(-0.8814) \\
&= 0.8814
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S^4_{o_1})}{\partial S^4_{o_1}} &= f'(S^4_{o_1}) \\
&= f(S^4_{o_1})\left(1-f(S^4_{o_1})\right) \\
&= 0.8814(1-0.8814) \\
&= 0.1045
\end{aligned}
$$

$$
\begin{aligned}
\delta^4_{o_1} &= 0.8814(0.1045) \\
&= 0.0921
\end{aligned}
$$

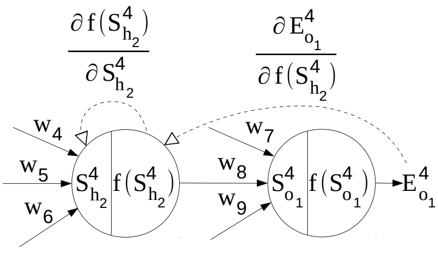


$$
\begin{aligned}
\delta^4_{h_1} &= \frac{\partial E^4_o}{\partial S^4_{h_1}} \\
&= \frac{\partial E^4_o}{\partial f(S^4_{h_1})} \frac{\partial f(S^4_{h_1})}{\partial S^4_{h_1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^4_o}{\partial f(S^4_{h_1})} &= \sum_{o=1}^{N_o} \frac{\partial E^4_o}{\partial S^4_o} \frac{\partial S^4_o}{\partial f(S^4_{h_1})} \\
&= \sum_{o=1}^{N_o} \delta^4_o w_{h_1 o} \\
&= \delta^4_{o_1} w_7 \\
&= 0.0921(0.7000) \\
&= 0.0645
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S^4_{h_1})}{\partial S^4_{h_1}} &= f'(S^4_{h_1}) \\
&= f(S^4_{h_1})\left(1-f(S^4_{h_1})\right) \\
&= 0.6457(1-0.6457) \\
&= 0.2288
\end{aligned}
$$

$$
\begin{aligned}
\delta^4_{h_1} &= 0.0645(0.2288) \\
&= 0.0148
\end{aligned}
$$



$$
\begin{aligned}
\delta^4_{h_2} &= \frac{\partial E^4_o}{\partial S^4_{h_2}} \\
&= \frac{\partial E^4_o}{\partial f(S^4_{h_2})} \frac{\partial f(S^4_{h_2})}{\partial S^4_{h_2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E^4_o}{\partial f(S^4_{h_2})} &= \sum_{o=1}^{N_o} \frac{\partial E^4_o}{\partial S^4_o} \frac{\partial S^4_o}{\partial f(S^4_{h_2})} \\
&= \sum_{o=1}^{N_o} \delta^4_o w_{h_2 o} \\
&= \delta^4_{o_1} w_8 \\
&= 0.0921(0.8000) \\
&= 0.0737
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S^4_{h_2})}{\partial S^4_{h_2}} &= f'(S^4_{h_2}) \\
&= f(S^4_{h_2})\left(1-f(S^4_{h_2})\right) \\
&= 0.8176(1-0.8176) \\
&= 0.1491
\end{aligned}
$$

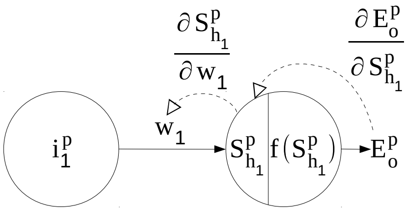
$$
\begin{aligned}
\delta^4_{h_2} &= 0.0737(0.1491) \\
&= 0.0110
\end{aligned}
$$

$$
\begin{aligned}
SSE &= \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \left(d^1_{o_1} - f(S^1_{o_1})\right)^2 + \left(d^2_{o_1} - f(S^2_{o_1})\right)^2 + \\
&\quad \left(d^3_{o_1} - f(S^3_{o_1})\right)^2 + \left(d^4_{o_1} - f(S^4_{o_1})\right)^2 \\
&= (0.0000-0.8604)^2 + (1.0000-0.8703)^2 + \\
&\quad (1.0000-0.8739)^2 + (0.0000-0.8814)^2 \\
&= 1.5499
\end{aligned}
$$

$$
\begin{aligned}
LSE &= \frac{1}{2} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \frac{1}{2} 1.5499 \\
&= 0.7750
\end{aligned}
$$

$$
\begin{aligned}
MSE &= \frac{1}{P \cdot N_o} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \frac{1}{4(1)} 1.5499 \\
&= 0.3875
\end{aligned}
$$

$$
\begin{aligned}
RMSE &= \sqrt{MSE} \\
&= \sqrt{0.3875} \\
&= 0.6225
\end{aligned}
$$



$$\Delta_1 w_1(2) = -\gamma \frac{\partial E_o^1}{\partial w_1} + \alpha \Delta_1 w_1(1)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_1} &= \frac{\partial E_o^1}{\partial S_{h_1}^1}\frac{\partial S_{h_1}^1}{\partial w_1}\\ &= \delta_{h_1}^1 i_1^1\\ &= 0.0177(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_1 w_1(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_1(1)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_2 w_1(2) = -\gamma \frac{\partial E_o^2}{\partial w_1} + \alpha \Delta_2 w_1(1)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_1} &= \frac{\partial E_o^2}{\partial S_{h_1}^2}\frac{\partial S_{h_1}^2}{\partial w_1}\\ &= \delta_{h_1}^2 i_1^2\\ &= -0.0025(1.0000)\\ &= -0.0025\end{aligned}$$

$$\begin{aligned}\Delta_2 w_1(2) &= -\gamma(-0.0025) + \alpha \Delta_2 w_1(1)\\ &= -0.7(-0.0025) + 0.3(0.0000)\\ &= 0.0018\end{aligned}$$

$$\Delta_3 w_1(2) = -\gamma \frac{\partial E_o^3}{\partial w_1} + \alpha \Delta_3 w_1(1)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_1} &= \frac{\partial E_o^3}{\partial S_{h_1}^3}\frac{\partial S_{h_1}^3}{\partial w_1}\\ &= \delta_{h_1}^3 i_1^3\\ &= -0.0023(0.0000)\\ &= 0.0000\end{aligned}$$

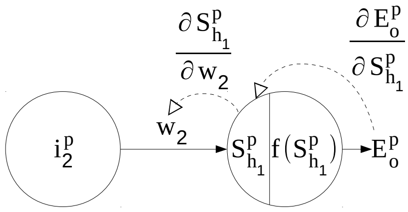
$$\begin{aligned}\Delta_3 w_1(2) &= -\gamma(0.0000) + \alpha \Delta_3 w_1(1)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_4 w_1(2) = -\gamma \frac{\partial E_o^4}{\partial w_1} + \alpha \Delta_4 w_1(1)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_1} &= \frac{\partial E_o^4}{\partial S_{h_1}^4}\frac{\partial S_{h_1}^4}{\partial w_1}\\ &= \delta_{h_1}^4 i_1^4\\ &= 0.0148(1.0000)\\ &= 0.0148\end{aligned}$$

$$\begin{aligned}\Delta_4 w_1(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_1(1)\\ &= -0.7(0.0148) + 0.3(0.0000)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_1 &= \left(\sum_{p=1}^{P} \Delta_p w_1(2)\right) + w_1\\ &= \left(\Delta_1 w_1(2) + \Delta_2 w_1(2) + \Delta_3 w_1(2) + \Delta_4 w_1(2)\right) + w_1\\ &= \left(0.0000 + 0.0018 + 0.0000 + (-0.0104)\right) + 0.1000\\ &= 0.0914\end{aligned}$$



$$\Delta_1 w_2(2) = -\gamma \frac{\partial E_o^1}{\partial w_2} + \alpha \Delta_1 w_2(1)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_2} &= \frac{\partial E_o^1}{\partial S_{h_1}^1}\frac{\partial S_{h_1}^1}{\partial w_2}\\ &= \delta_{h_1}^1 i_2^1\\ &= 0.0177(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_1 w_2(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_2(1)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_2 w_2(2) = -\gamma \frac{\partial E_o^2}{\partial w_2} + \alpha \Delta_2 w_2(1)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_2} &= \frac{\partial E_o^2}{\partial S_{h_1}^2}\frac{\partial S_{h_1}^2}{\partial w_2}\\ &= \delta_{h_1}^2 i_2^2\\ &= -0.0025(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_2 w_2(2) &= -\gamma(0.0000) + \alpha \Delta_2 w_2(1)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_3 w_2(2) = -\gamma \frac{\partial E_o^3}{\partial w_2} + \alpha \Delta_3 w_2(1)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_2} &= \frac{\partial E_o^3}{\partial S_{h_1}^3}\frac{\partial S_{h_1}^3}{\partial w_2}\\ &= \delta_{h_1}^3 i_2^3\\ &= -0.0023(1.0000)\\ &= -0.0023\end{aligned}$$

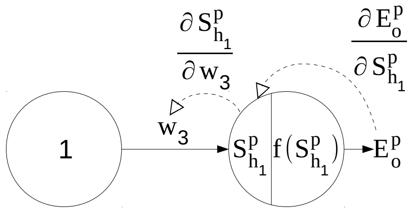
$$\begin{aligned}\Delta_3 w_2(2) &= -\gamma(-0.0023) + \alpha \Delta_3 w_2(1)\\ &= -0.7(-0.0023) + 0.3(0.0000)\\ &= 0.0016\end{aligned}$$

$$\Delta_4 w_2(2) = -\gamma \frac{\partial E_o^4}{\partial w_2} + \alpha \Delta_4 w_2(1)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_2} &= \frac{\partial E_o^4}{\partial S_{h_1}^4}\frac{\partial S_{h_1}^4}{\partial w_2}\\ &= \delta_{h_1}^4 i_2^4\\ &= 0.0148(1.0000)\\ &= 0.0148\end{aligned}$$

$$\begin{aligned}\Delta_4 w_2(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_2(1)\\ &= -0.7(0.0148) + 0.3(0.0000)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_2 &= \left(\sum_{p=1}^{P} \Delta_p w_2(2)\right) + w_2\\ &= \left(\Delta_1 w_2(2) + \Delta_2 w_2(2) + \Delta_3 w_2(2) + \Delta_4 w_2(2)\right) + w_2\\ &= \left(0.0000 + 0.0000 + 0.0016 + (-0.0104)\right) + 0.2000\\ &= 0.1912\end{aligned}$$



$$\Delta_1 w_3(2) = -\gamma \frac{\partial E_o^1}{\partial w_3} + \alpha \Delta_1 w_3(1)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_3} &= \frac{\partial E_o^1}{\partial S_{h_1}^1}\frac{\partial S_{h_1}^1}{\partial w_3}\\ &= \delta_{h_1}^1(1)\\ &= 0.0177(1.0000)\\ &= 0.0177\end{aligned}$$

$$\begin{aligned}\Delta_1 w_3(2) &= -\gamma(0.0177) + \alpha \Delta_1 w_3(1)\\ &= -0.7(0.0177) + 0.3(0.0000)\\ &= -0.0124\end{aligned}$$

$$\Delta_2 w_3(2) = -\gamma \frac{\partial E_o^2}{\partial w_3} + \alpha \Delta_2 w_3(1)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_3} &= \frac{\partial E_o^2}{\partial S_{h_1}^2}\frac{\partial S_{h_1}^2}{\partial w_3}\\ &= \delta_{h_1}^2(1)\\ &= -0.0025(1.0000)\\ &= -0.0025\end{aligned}$$

$$\begin{aligned}\Delta_2 w_3(2) &= -\gamma(-0.0025) + \alpha \Delta_2 w_3(1)\\ &= -0.7(-0.0025) + 0.3(0.0000)\\ &= 0.0018\end{aligned}$$

$$\Delta_3 w_3(2) = -\gamma \frac{\partial E_o^3}{\partial w_3} + \alpha \Delta_3 w_3(1)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_3} &= \frac{\partial E_o^3}{\partial S_{h_1}^3}\frac{\partial S_{h_1}^3}{\partial w_3}\\ &= \delta_{h_1}^3(1)\\ &= -0.0023(1.0000)\\ &= -0.0023\end{aligned}$$

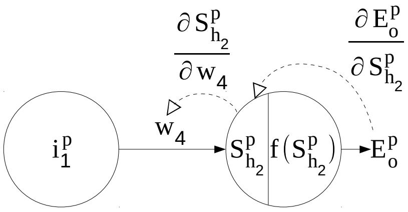
$$\begin{aligned}\Delta_3 w_3(2) &= -\gamma(-0.0023) + \alpha \Delta_3 w_3(1)\\ &= -0.7(-0.0023) + 0.3(0.0000)\\ &= 0.0016\end{aligned}$$

$$\Delta_4 w_3(2) = -\gamma \frac{\partial E_o^4}{\partial w_3} + \alpha \Delta_4 w_3(1)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_3} &= \frac{\partial E_o^4}{\partial S_{h_1}^4}\frac{\partial S_{h_1}^4}{\partial w_3}\\ &= \delta_{h_1}^4(1)\\ &= 0.0148(1.0000)\\ &= 0.0148\end{aligned}$$

$$\begin{aligned}\Delta_4 w_3(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_3(1)\\ &= -0.7(0.0148) + 0.3(0.0000)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_3 &= \left(\sum_{p=1}^{P} \Delta_p w_3(2)\right) + w_3\\ &= \left(\Delta_1 w_3(2) + \Delta_2 w_3(2) + \Delta_3 w_3(2) + \Delta_4 w_3(2)\right) + w_3\\ &= \left(-0.0124 + 0.0018 + 0.0016 + (-0.0104)\right) + 0.3000\\ &= 0.2806\end{aligned}$$



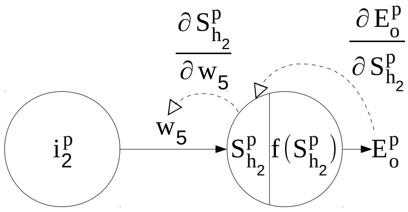
$$\begin{aligned}
\Delta_1 w_4(2) &= -\gamma \frac{\partial E^1_o}{\partial w_4} + \alpha \Delta_1 w_4(1) \\
\frac{\partial E^1_o}{\partial w_4} &= \frac{\partial E^1_o}{\partial S^1_{h_2}} \frac{\partial S^1_{h_2}}{\partial w_4} \\
&= \delta^1_{h_2} i^1_1 \\
&= 0.0189(0.0000) \\
&= 0.0000 \\
\Delta_1 w_4(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_4(1) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_4(2) &= -\gamma \frac{\partial E^2_o}{\partial w_4} + \alpha \Delta_2 w_4(1) \\
\frac{\partial E^2_o}{\partial w_4} &= \frac{\partial E^2_o}{\partial S^2_{h_2}} \frac{\partial S^2_{h_2}}{\partial w_4} \\
&= \delta^2_{h_2} i^2_1 \\
&= -0.0023(1.0000) \\
&= -0.0023 \\
\Delta_2 w_4(2) &= -\gamma(-0.0023) + \alpha \Delta_2 w_4(1) \\
&= -0.7(-0.0023) + 0.3(0.0000) \\
&= 0.0016
\end{aligned}$$

$$\begin{aligned}
\Delta_3 w_4(2) &= -\gamma \frac{\partial E^3_o}{\partial w_4} + \alpha \Delta_3 w_4(1) \\
\frac{\partial E^3_o}{\partial w_4} &= \frac{\partial E^3_o}{\partial S^3_{h_2}} \frac{\partial S^3_{h_2}}{\partial w_4} \\
&= \delta^3_{h_2} i^3_1 \\
&= -0.0021(0.0000) \\
&= 0.0000 \\
\Delta_3 w_4(2) &= -\gamma(0.0000) + \alpha \Delta_3 w_4(1) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_4(2) &= -\gamma \frac{\partial E^4_o}{\partial w_4} + \alpha \Delta_4 w_4(1) \\
\frac{\partial E^4_o}{\partial w_4} &= \frac{\partial E^4_o}{\partial S^4_{h_2}} \frac{\partial S^4_{h_2}}{\partial w_4} \\
&= \delta^4_{h_2} i^4_1 \\
&= 0.0110(1.0000) \\
&= 0.0110 \\
\Delta_4 w_4(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_4(1) \\
&= -0.7(0.0110) + 0.3(0.0000) \\
&= -0.0077
\end{aligned}$$

$$\begin{aligned}
w_4 &= \left(\sum_{p=1}^{P} \Delta_p w_4(2) \right) + w_4 \\
&= \left(\Delta_1 w_4(2) + \Delta_2 w_4(2) + \Delta_3 w_4(2) + \Delta_4 w_4(2) \right) + w_4 \\
&= \left(0.0000 + 0.0016 + 0.0000 + (-0.0077) \right) + 0.4000 \\
&= 0.3939
\end{aligned}$$



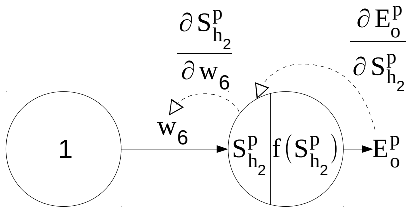
$$\begin{aligned}
\Delta_1 w_5(2) &= -\gamma \frac{\partial E^1_o}{\partial w_5} + \alpha \Delta_1 w_5(1) \\
\frac{\partial E^1_o}{\partial w_5} &= \frac{\partial E^1_o}{\partial S^1_{h_2}} \frac{\partial S^1_{h_2}}{\partial w_5} \\
&= \delta^1_{h_2} i^1_2 \\
&= 0.0189(0.0000) \\
&= 0.0000 \\
\Delta_1 w_5(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_5(1) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_5(2) &= -\gamma \frac{\partial E^2_o}{\partial w_5} + \alpha \Delta_2 w_5(1) \\
\frac{\partial E^2_o}{\partial w_5} &= \frac{\partial E^2_o}{\partial S^2_{h_2}} \frac{\partial S^2_{h_2}}{\partial w_5} \\
&= \delta^2_{h_2} i^2_2 \\
&= -0.0023(0.0000) \\
&= 0.0000 \\
\Delta_2 w_5(2) &= -\gamma(0.0000) + \alpha \Delta_2 w_5(1) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_3 w_5(2) &= -\gamma \frac{\partial E^3_o}{\partial w_5} + \alpha \Delta_3 w_5(1) \\
\frac{\partial E^3_o}{\partial w_5} &= \frac{\partial E^3_o}{\partial S^3_{h_2}} \frac{\partial S^3_{h_2}}{\partial w_5} \\
&= \delta^3_{h_2} i^3_2 \\
&= -0.0021(1.0000) \\
&= -0.0021 \\
\Delta_3 w_5(2) &= -\gamma(-0.0021) + \alpha \Delta_3 w_5(1) \\
&= -0.7(-0.0021) + 0.3(0.0000) \\
&= 0.0015
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_5(2) &= -\gamma \frac{\partial E^4_o}{\partial w_5} + \alpha \Delta_4 w_5(1) \\
\frac{\partial E^4_o}{\partial w_5} &= \frac{\partial E^4_o}{\partial S^4_{h_2}} \frac{\partial S^4_{h_2}}{\partial w_5} \\
&= \delta^4_{h_2} i^4_2 \\
&= 0.0110(1.0000) \\
&= 0.0110 \\
\Delta_4 w_5(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_5(1) \\
&= -0.7(0.0110) + 0.3(0.0000) \\
&= -0.0077
\end{aligned}$$

$$\begin{aligned}
w_5 &= \left(\sum_{p=1}^{P} \Delta_p w_5(2) \right) + w_5 \\
&= \left(\Delta_1 w_5(2) + \Delta_2 w_5(2) + \Delta_3 w_5(2) + \Delta_4 w_5(2) \right) + w_5 \\
&= \left(0.0000 + 0.0000 + 0.0015 + (-0.0077) \right) + 0.5000 \\
&= 0.4938
\end{aligned}$$



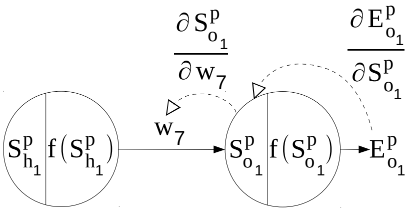
$$\begin{aligned}
\Delta_1 w_6(2) &= -\gamma \frac{\partial E^1_o}{\partial w_6} + \alpha \Delta_1 w_6(1) \\
\frac{\partial E^1_o}{\partial w_6} &= \frac{\partial E^1_o}{\partial S^1_{h_2}} \frac{\partial S^1_{h_2}}{\partial w_6} \\
&= \delta^1_{h_2}(1) \\
&= 0.0189(1.0000) \\
&= 0.0189 \\
\Delta_1 w_6(2) &= -\gamma(0.0189) + \alpha \Delta_1 w_6(1) \\
&= -0.7(0.0189) + 0.3(0.0000) \\
&= -0.0132
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_6(2) &= -\gamma \frac{\partial E^2_o}{\partial w_6} + \alpha \Delta_2 w_6(1) \\
\frac{\partial E^2_o}{\partial w_6} &= \frac{\partial E^2_o}{\partial S^2_{h_2}} \frac{\partial S^2_{h_2}}{\partial w_6} \\
&= \delta^2_{h_2}(1) \\
&= -0.0023(1.0000) \\
&= -0.0023 \\
\Delta_2 w_6(2) &= -\gamma(-0.0023) + \alpha \Delta_2 w_6(1) \\
&= -0.7(-0.0023) + 0.3(0.0000) \\
&= 0.0016
\end{aligned}$$

$$\begin{aligned}
\Delta_3 w_6(2) &= -\gamma \frac{\partial E^3_o}{\partial w_6} + \alpha \Delta_3 w_6(1) \\
\frac{\partial E^3_o}{\partial w_6} &= \frac{\partial E^3_o}{\partial S^3_{h_2}} \frac{\partial S^3_{h_2}}{\partial w_6} \\
&= \delta^3_{h_2}(1) \\
&= -0.0021(1.0000) \\
&= -0.0021 \\
\Delta_3 w_6(2) &= -\gamma(-0.0021) + \alpha \Delta_3 w_6(1) \\
&= -0.7(-0.0021) + 0.3(0.0000) \\
&= 0.0015
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_6(2) &= -\gamma \frac{\partial E^4_o}{\partial w_6} + \alpha \Delta_4 w_6(1) \\
\frac{\partial E^4_o}{\partial w_6} &= \frac{\partial E^4_o}{\partial S^4_{h_2}} \frac{\partial S^4_{h_2}}{\partial w_6} \\
&= \delta^4_{h_2}(1) \\
&= 0.0110(1.0000) \\
&= 0.0110 \\
\Delta_4 w_6(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_6(1) \\
&= -0.7(0.0110) + 0.3(0.0000) \\
&= -0.0077
\end{aligned}$$

$$\begin{aligned}
w_6 &= \left(\sum_{p=1}^{P} \Delta_p w_6(2) \right) + w_6 \\
&= \left(\Delta_1 w_6(2) + \Delta_2 w_6(2) + \Delta_3 w_6(2) + \Delta_4 w_6(2) \right) + w_6 \\
&= \left(-0.0132 + 0.0016 + 0.0015 + (-0.0077) \right) + 0.6000 \\
&= 0.5822
\end{aligned}$$



$$\Delta_1 w_7(2) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_7} + \alpha \Delta_1 w_7(1)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_7} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_7} \\
&= \delta^1_{o_1} f(S^1_{h_1}) \\
&= 0.1033(0.5744) \\
&= 0.0593
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_7(2) &= -\gamma(0.0593) + \alpha \Delta_1 w_7(1) \\
&= -0.7(0.0593) + 0.3(0.0000) \\
&= -0.0415
\end{aligned}$$

$$\Delta_2 w_7(2) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_7} + \alpha \Delta_2 w_7(1)$$

$$\begin{aligned}
\frac{\partial E^2_{o}}{\partial w_7} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_7} \\
&= \delta^2_{o_1} f(S^2_{h_1}) \\
&= -0.0146(0.5987) \\
&= -0.0087
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_7(2) &= -\gamma(-0.0087) + \alpha \Delta_2 w_7(1) \\
&= -0.7(-0.0087) + 0.3(0.0000) \\
&= 0.0061
\end{aligned}$$

$$\Delta_3 w_7(2) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_7} + \alpha \Delta_3 w_7(1)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_7} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_7} \\
&= \delta^3_{o_1} f(S^3_{h_1}) \\
&= -0.0139(0.6225) \\
&= -0.0087
\end{aligned}$$

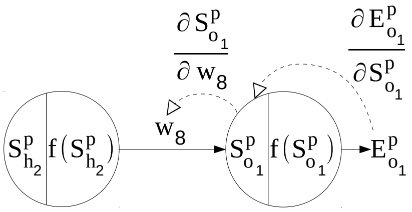
$$\begin{aligned}
\Delta_3 w_7(2) &= -\gamma(-0.0087) + \alpha \Delta_3 w_7(1) \\
&= -0.7(-0.0087) + 0.3(0.0000) \\
&= 0.0061
\end{aligned}$$

$$\Delta_4 w_7(2) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_7} + \alpha \Delta_4 w_7(1)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_7} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_7} \\
&= \delta^4_{o_1} f(S^4_{h_1}) \\
&= 0.0921(0.6457) \\
&= 0.0595
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_7(2) &= -\gamma(0.0595) + \alpha \Delta_4 w_7(1) \\
&= -0.7(0.0595) + 0.3(0.0000) \\
&= -0.0417
\end{aligned}$$

$$\begin{aligned}
w_7 &= \left(\sum_{p=1}^{P} \Delta_p w_7(2) \right) + w_7 \\
&= \left(\Delta_1 w_7(2) + \Delta_2 w_7(2) + \Delta_3 w_7(2) + \Delta_4 w_7(2) \right) + w_7 \\
&= \left(-0.0415 + 0.0061 + 0.0061 + (-0.0417) \right) + 0.7000 \\
&= 0.6290
\end{aligned}$$



$$\Delta_1 w_8(2) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_8} + \alpha \Delta_1 w_8(1)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_8} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_8} \\
&= \delta^1_{o_1} f(S^1_{h_2}) \\
&= 0.1033(0.6457) \\
&= 0.0667
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_8(2) &= -\gamma(0.0667) + \alpha \Delta_1 w_8(1) \\
&= -0.7(0.0667) + 0.3(0.0000) \\
&= -0.0467
\end{aligned}$$

$$\Delta_2 w_8(2) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_8} + \alpha \Delta_2 w_8(1)$$

$$\begin{aligned}
\frac{\partial E^2_{o_1}}{\partial w_8} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_8} \\
&= \delta^2_{o_1} f(S^2_{h_2}) \\
&= -0.0146(0.7311) \\
&= -0.0107
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_8(2) &= -\gamma(-0.0107) + \alpha \Delta_2 w_8(1) \\
&= -0.7(-0.0107) + 0.3(0.0000) \\
&= 0.0075
\end{aligned}$$

$$\Delta_3 w_8(2) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_8} + \alpha \Delta_3 w_8(1)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_8} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_8} \\
&= \delta^3_{o_1} f(S^3_{h_2}) \\
&= -0.0139(0.7503) \\
&= -0.0104
\end{aligned}$$

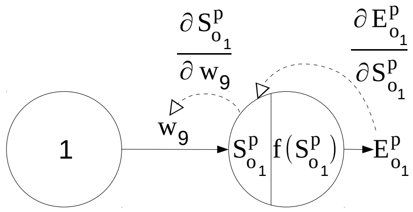
$$\begin{aligned}
\Delta_3 w_8(2) &= -\gamma(-0.0104) + \alpha \Delta_3 w_8(1) \\
&= -0.7(-0.0104) + 0.3(0.0000) \\
&= 0.0073
\end{aligned}$$

$$\Delta_4 w_8(2) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_8} + \alpha \Delta_4 w_8(1)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_8} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_8} \\
&= \delta^4_{o_1} f(S^4_{h_2}) \\
&= 0.0921(0.8176) \\
&= 0.0753
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_8(2) &= -\gamma(0.0753) + \alpha \Delta_4 w_8(1) \\
&= -0.7(0.0753) + 0.3(0.0000) \\
&= -0.0527
\end{aligned}$$

$$\begin{aligned}
w_8 &= \left(\sum_{p=1}^{P} \Delta_p w_8(2) \right) + w_8 \\
&= \left(\Delta_1 w_8(2) + \Delta_2 w_8(2) + \Delta_3 w_8(2) + \Delta_4 w_8(2) \right) + w_8 \\
&= \left(-0.0467 + 0.0075 + 0.0073 + (-0.0527) \right) + 0.8000 \\
&= 0.7154
\end{aligned}$$



$$\Delta_1 w_9(2) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_9} + \alpha \Delta_1 w_9(1)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_9} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_9} \\
&= \delta^1_{o_1}(1) \\
&= 0.1033(1.0000) \\
&= 0.1033
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_9(2) &= -\gamma(0.1033) + \alpha \Delta_1 w_9(1) \\
&= -0.7(0.1033) + 0.3(0.0000) \\
&= -0.0723
\end{aligned}$$

$$\Delta_2 w_9(2) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_9} + \alpha \Delta_2 w_9(1)$$

$$\begin{aligned}
\frac{\partial E^2_{o_1}}{\partial w_9} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_9} \\
&= \delta^2_{o_1}(1) \\
&= -0.0146(1.0000) \\
&= -0.0146
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_9(2) &= -\gamma(-0.0146) + \alpha \Delta_2 w_9(1) \\
&= -0.7(-0.0146) + 0.3(0.0000) \\
&= 0.0102
\end{aligned}$$

$$\Delta_3 w_9(2) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_9} + \alpha \Delta_3 w_9(1)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_9} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_9} \\
&= \delta^3_{o_1}(1) \\
&= -0.0139(1.0000) \\
&= -0.0139
\end{aligned}$$

$$\begin{aligned}
\Delta_3 w_9(2) &= -\gamma(-0.0139) + \alpha \Delta_3 w_9(1) \\
&= -0.7(-0.0139) + 0.3(0.0000) \\
&= 0.0097
\end{aligned}$$

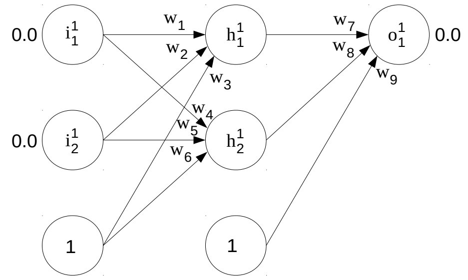
$$\Delta_4 w_9(2) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_9} + \alpha \Delta_4 w_9(1)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_9} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_9} \\
&= \delta^4_{o_1}(1) \\
&= 0.0921(1.0000) \\
&= 0.0921
\end{aligned}$$

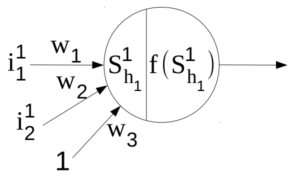
$$\begin{aligned}
\Delta_4 w_9(2) &= -\gamma(0.0921) + \alpha \Delta_4 w_9(1) \\
&= -0.7(0.0921) + 0.3(0.0000) \\
&= -0.0645
\end{aligned}$$

$$\begin{aligned}
w_9 &= \left(\sum_{p=1}^{P} \Delta_p w_9(2) \right) + w_9 \\
&= \left(\Delta_1 w_9(2) + \Delta_2 w_9(2) + \Delta_3 w_9(2) + \Delta_4 w_9(2) \right) + w_9 \\
&= \left(-0.0723 + 0.0102 + 0.0097 + (-0.0645) \right) + 0.9000 \\
&= 0.7831
\end{aligned}$$

Epoch 2 : Iteration 1

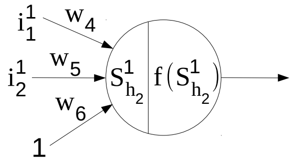


$w_1 = 0.0914$	$w_4 = 0.3939$	$w_7 = 0.6290$
$w_2 = 0.1912$	$w_5 = 0.4938$	$w_8 = 0.7154$
$w_3 = 0.2806$	$w_6 = 0.5822$	$w_9 = 0.7831$



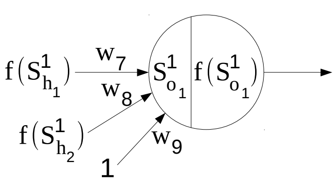
$$
\begin{aligned}
S_{h_1}^1 &= \sum_j w_{jh_1} y_j^1 \\
&= w_1 i_1^1 + w_2 i_2^1 + w_3(1) \\
&= 0.0914(0.0000) + 0.1912(0.0000) + 0.2806(1) \\
&= 0.2806
\end{aligned}
$$

$$
\begin{aligned}
f(S_{h_1}^1) &= \frac{1}{1+e^{-S_{h_1}^1}} \\
&= \frac{1}{1+e^{-0.2806}} \\
&= 0.5697
\end{aligned}
$$



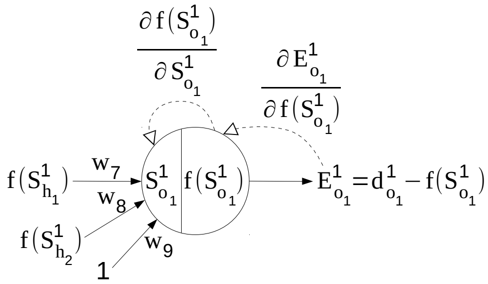
$$
\begin{aligned}
S_{h_2}^1 &= \sum_j w_{jh_2} y_j^1 \\
&= w_4 i_1^1 + w_5 i_2^1 + w_6(1) \\
&= 0.3939(0.0000) + 0.4938(0.0000) + 0.5822(1) \\
&= 0.5822
\end{aligned}
$$

$$
\begin{aligned}
f(S_{h_2}^1) &= \frac{1}{1+e^{-S_{h_2}^1}} \\
&= \frac{1}{1+e^{-0.5822}} \\
&= 0.6416
\end{aligned}
$$



$$
\begin{aligned}
S_{o_1}^1 &= \sum_j w_{jo_1} y_j^1 \\
&= w_7 f(S_{h_1}^1) + w_8 f(S_{h_2}^1) + w_9(1) \\
&= 0.6290(0.5697) + 0.7154(0.6416) + 0.7831(1) \\
&= 1.6004
\end{aligned}
$$

$$
\begin{aligned}
f(S_{o_1}^1) &= \frac{1}{1+e^{-S_{o_1}^1}} \\
&= \frac{1}{1+e^{-1.6004}} \\
&= 0.8321
\end{aligned}
$$

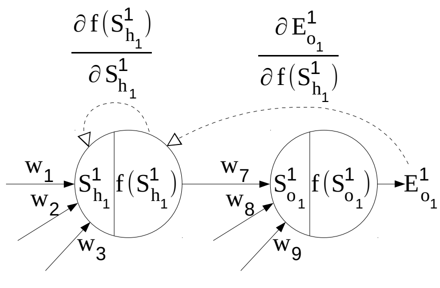


$$
\begin{aligned}
\delta_{o_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \\
&= \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} &= -\left(d_{o_1}^1 - f(S_{o_1}^1)\right) \\
&= -(0.0000 - 0.8321) \\
&= -(-0.8321) \\
&= 0.8321
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} &= f'(S_{o_1}^1) \\
&= f(S_{o_1}^1)\left(1 - f(S_{o_1}^1)\right) \\
&= 0.8321(1 - 0.8321) \\
&= 0.1397
\end{aligned}
$$

$$
\begin{aligned}
\delta_{o_1}^1 &= 0.8321(0.1397) \\
&= 0.1162
\end{aligned}
$$

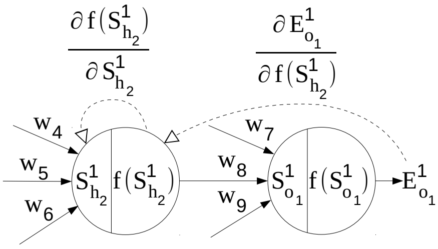


$$
\begin{aligned}
\delta_{h_1}^1 &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \\
&= \frac{\partial E_o^1}{\partial f(S_{h_1}^1)} \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^1}{\partial f(S_{h_1}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^1}{\partial S_o^1} \frac{\partial S_o^1}{\partial f(S_{h_1}^1)} \\
&= \sum_{o=1}^{N_o} \delta_o^1 w_{h_1 o} \\
&= \delta_{o_1}^1 w_7 \\
&= 0.1162(0.6290) \\
&= 0.0731
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} &= f'(S_{h_1}^1) \\
&= f(S_{h_1}^1)\left(1 - f(S_{h_1}^1)\right) \\
&= 0.5697(1 - 0.5697) \\
&= 0.2451
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_1}^1 &= 0.0731(0.2451) \\
&= 0.0179
\end{aligned}
$$



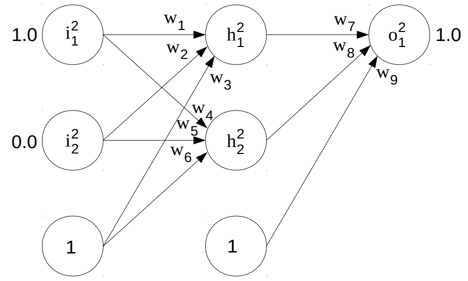
$$
\begin{aligned}
\delta_{h_2}^1 &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \\
&= \frac{\partial E_o^1}{\partial f(S_{h_2}^1)} \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_o^1}{\partial f(S_{h_2}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_o^1}{\partial S_o^1} \frac{\partial S_o^1}{\partial f(S_{h_2}^1)} \\
&= \sum_{o=1}^{N_o} \delta_o^1 w_{h_2 o} \\
&= \delta_{o_1}^1 w_8 \\
&= 0.1162(0.7154) \\
&= 0.0831
\end{aligned}
$$

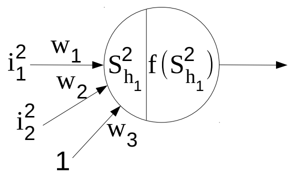
$$
\begin{aligned}
\frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} &= f'(S_{h_2}^1) \\
&= f(S_{h_2}^1)\left(1 - f(S_{h_2}^1)\right) \\
&= 0.6416(1 - 0.6416) \\
&= 0.2299
\end{aligned}
$$

$$
\begin{aligned}
\delta_{h_2}^1 &= 0.0831(0.2299) \\
&= 0.0191
\end{aligned}
$$

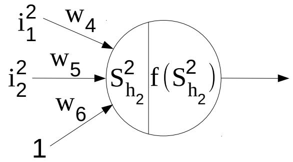
Epoch 2 : Iteration 2



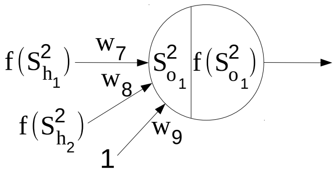
$w_1 = 0.0914$ $w_4 = 0.3939$ $w_7 = 0.6290$
$w_2 = 0.1912$ $w_5 = 0.4938$ $w_8 = 0.7154$
$w_3 = 0.2806$ $w_6 = 0.5822$ $w_9 = 0.7831$



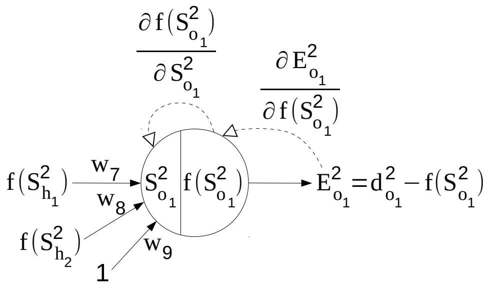
$$S^2_{h_1} = \sum_j w_{jh_1} y^2_j$$
$$= w_1 i^2_1 + w_2 i^2_2 + w_3(1)$$
$$= 0.0914(1.0000) + 0.1912(0.0000) + 0.2806(1)$$
$$= 0.3720$$
$$f(S^2_{h_1}) = \frac{1}{1+e^{-S^2_{h_1}}}$$
$$= \frac{1}{1+e^{-0.3720}}$$
$$= 0.5919$$



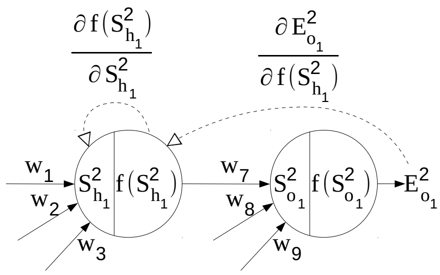
$$S^2_{h_2} = \sum_j w_{jh_2} y^2_j$$
$$= w_4 i^2_1 + w_5 i^2_2 + w_6(1)$$
$$= 0.3939(1.0000) + 0.4938(0.0000) + 0.5822(1)$$
$$= 0.9761$$
$$f(S^2_{h_2}) = \frac{1}{1+e^{-S^2_{h_2}}}$$
$$= \frac{1}{1+e^{-0.9761}}$$
$$= 0.7263$$



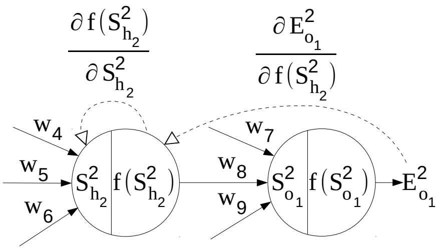
$$S^2_{o_1} = \sum_j w_{jo_1} y^2_j$$
$$= w_7 f(S^2_{h_1}) + w_8 f(S^2_{h_2}) + w_9(1)$$
$$= 0.6290(0.5919) + 0.7154(0.7263) + 0.7831(1)$$
$$= 1.6750$$
$$f(S^2_{o_1}) = \frac{1}{1+e^{-S^2_{o_1}}}$$
$$= \frac{1}{1+e^{-1.6750}}$$
$$= 0.8422$$



$$\delta^2_{o_1} = \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}}$$
$$= \frac{\partial E^2_{o_1}}{\partial f(S^2_{o_1})} \frac{\partial f(S^2_{o_1})}{\partial S^2_{o_1}}$$
$$\frac{\partial E^2_{o_1}}{\partial f(S^2_{o_1})} = -\left(d^2_{o_1} - f(S^2_{o_1})\right)$$
$$= -(1.0000 - 0.8422)$$
$$= -(0.1578)$$
$$= -0.1578$$
$$\frac{\partial f(S^2_{o_1})}{\partial S^2_{o_1}} = f'(S^2_{o_1})$$
$$= f(S^2_{o_1})\left(1 - f(S^2_{o_1})\right)$$
$$= 0.8422(1 - 0.8422)$$
$$= 0.1329$$
$$\delta^2_{o_1} = -0.1578(0.1329)$$
$$= -0.0210$$

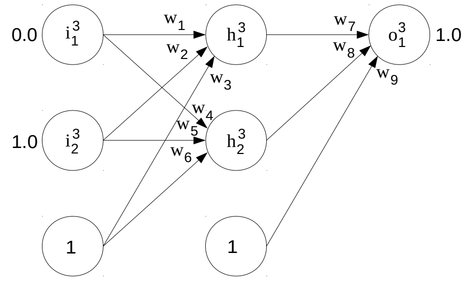


$$\delta^2_{h_1} = \frac{\partial E^2_o}{\partial S^2_{h_1}}$$
$$= \frac{\partial E^2_o}{\partial f(S^2_{h_1})} \frac{\partial f(S^2_{h_1})}{\partial S^2_{h_1}}$$
$$\frac{\partial E^2_o}{\partial f(S^2_{h_1})} = \sum_{o=1}^{N_o} \frac{\partial E^2_o}{\partial S^2_o} \frac{\partial S^2_o}{\partial f(S^2_{h_1})}$$
$$= \sum_{o=1}^{N_o} \delta^2_o w_{h_1 o}$$
$$= \delta^2_{o_1} w_7$$
$$= -0.0210(0.6290)$$
$$= -0.0132$$
$$\frac{\partial f(S^2_{h_1})}{\partial S^2_{h_1}} = f'(S^2_{h_1})$$
$$= f(S^2_{h_1})\left(1 - f(S^2_{h_1})\right)$$
$$= 0.5919(1 - 0.5919)$$
$$= 0.2416$$
$$\delta^2_{h_1} = -0.0132(0.2416)$$
$$= -0.0032$$

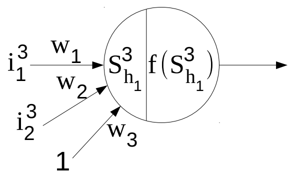


$$\delta^2_{h_2} = \frac{\partial E^2_o}{\partial S^2_{h_2}}$$
$$= \frac{\partial E^2_o}{\partial f(S^2_{h_2})} \frac{\partial f(S^2_{h_2})}{\partial S^2_{h_2}}$$
$$\frac{\partial E^2_o}{\partial f(S^2_{h_2})} = \sum_{o=1}^{N_o} \frac{\partial E^2_o}{\partial S^2_o} \frac{\partial S^2_o}{\partial f(S^2_{h_2})}$$
$$= \sum_{o=1}^{N_o} \delta^2_o w_{h_2 o}$$
$$= \delta^2_{o_1} w_8$$
$$= -0.0210(0.7154)$$
$$= -0.0150$$
$$\frac{\partial f(S^2_{h_2})}{\partial S^2_{h_2}} = f'(S^2_{h_2})$$
$$= f(S^2_{h_2})\left(1 - f(S^2_{h_2})\right)$$
$$= 0.7263(1 - 0.7263)$$
$$= 0.1988$$
$$\delta^2_{h_2} = -0.0150(0.1988)$$
$$= -0.0030$$

Epoch 2 : Iteration 3

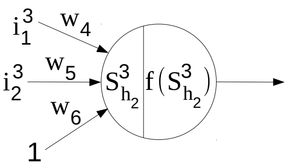


$w_1 = 0.0914$ $w_4 = 0.3939$ $w_7 = 0.6290$
$w_2 = 0.1912$ $w_5 = 0.4938$ $w_8 = 0.7154$
$w_3 = 0.2806$ $w_6 = 0.5822$ $w_9 = 0.7831$



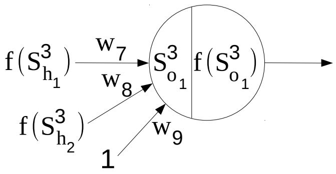
$$\begin{aligned} S^3_{h_1} &= \sum_j w_{jh_1} y^3_j \\ &= w_1 i^3_1 + w_2 i^3_2 + w_3(1) \\ &= 0.0914(0.0000) + 0.1912(1.0000) + 0.2806(1) \\ &= 0.4718 \end{aligned}$$

$$\begin{aligned} f(S^3_{h_1}) &= \frac{1}{1+e^{-S^3_{h_1}}} \\ &= \frac{1}{1+e^{-0.4718}} \\ &= 0.6158 \end{aligned}$$



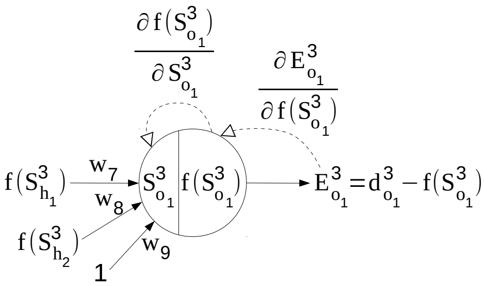
$$\begin{aligned} S^3_{h_2} &= \sum_j w_{jh_2} y^3_j \\ &= w_4 i^3_1 + w_5 i^3_2 + w_6(1) \\ &= 0.3939(0.0000) + 0.4938(1.0000) + 0.5822(1) \\ &= 1.0760 \end{aligned}$$

$$\begin{aligned} f(S^3_{h_2}) &= \frac{1}{1+e^{-S^3_{h_2}}} \\ &= \frac{1}{1+e^{-1.0760}} \\ &= 0.7457 \end{aligned}$$



$$\begin{aligned} S^3_{o_1} &= \sum_j w_{jo_1} y^3_j \\ &= w_7 f(S^3_{h_1}) + w_8 f(S^3_{h_2}) + w_9(1) \\ &= 0.6290(0.6158) + 0.7154(0.7457) + 0.7831(1) \\ &= 1.7039 \end{aligned}$$

$$\begin{aligned} f(S^3_{o_1}) &= \frac{1}{1+e^{-S^3_{o_1}}} \\ &= \frac{1}{1+e^{-1.7039}} \\ &= 0.8460 \end{aligned}$$

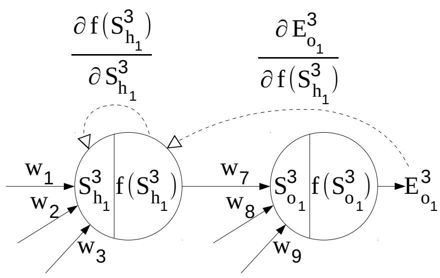


$$\begin{aligned} \delta^3_{o_1} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \\ &= \frac{\partial E^3_{o_1}}{\partial f(S^3_{o_1})} \frac{\partial f(S^3_{o_1})}{\partial S^3_{o_1}} \end{aligned}$$

$$\begin{aligned} \frac{\partial E^3_{o_1}}{\partial f(S^3_{o_1})} &= -\left(d^3_{o_1} - f(S^3_{o_1})\right) \\ &= -(1.0000 - 0.8460) \\ &= -(0.1540) \\ &= -0.1540 \end{aligned}$$

$$\begin{aligned} \frac{\partial f(S^3_{o_1})}{\partial S^3_{o_1}} &= f'(S^3_{o_1}) \\ &= f(S^3_{o_1})\left(1 - f(S^3_{o_1})\right) \\ &= 0.8460(1 - 0.8460) \\ &= 0.1303 \end{aligned}$$

$$\begin{aligned} \delta^3_{o_1} &= -0.1540(0.1303) \\ &= -0.0201 \end{aligned}$$

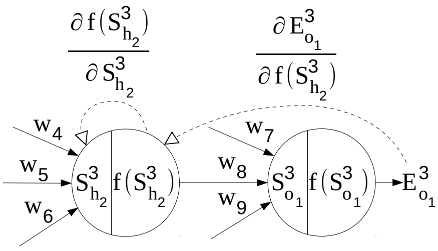


$$\begin{aligned} \delta^3_{h_1} &= \frac{\partial E^3_o}{\partial S^3_{h_1}} \\ &= \frac{\partial E^3_o}{\partial f(S^3_{h_1})} \frac{\partial f(S^3_{h_1})}{\partial S^3_{h_1}} \end{aligned}$$

$$\begin{aligned} \frac{\partial E^3_o}{\partial f(S^3_{h_1})} &= \sum_{o=1}^{N_o} \frac{\partial E^3_o}{\partial S^3_o} \frac{\partial S^3_o}{\partial f(S^3_{h_1})} \\ &= \sum_{o=1}^{N_o} \delta^3_o w_{h_1 o} \\ &= \delta^3_{o_1} w_7 \\ &= -0.0201(0.6290) \\ &= -0.0126 \end{aligned}$$

$$\begin{aligned} \frac{\partial f(S^3_{h_1})}{\partial S^3_{h_1}} &= f'(S^3_{h_1}) \\ &= f(S^3_{h_1})\left(1 - f(S^3_{h_1})\right) \\ &= 0.6158(1 - 0.6158) \\ &= 0.2366 \end{aligned}$$

$$\begin{aligned} \delta^3_{h_1} &= -0.0126(0.2366) \\ &= -0.0030 \end{aligned}$$



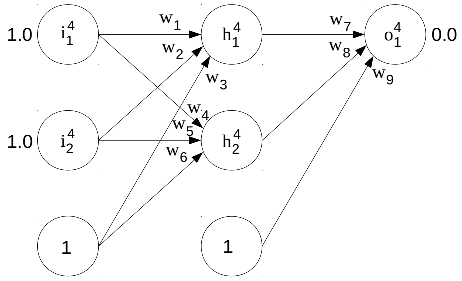
$$\begin{aligned} \delta^3_{h_2} &= \frac{\partial E^3_o}{\partial S^3_{h_2}} \\ &= \frac{\partial E^3_o}{\partial f(S^3_{h_2})} \frac{\partial f(S^3_{h_2})}{\partial S^3_{h_2}} \end{aligned}$$

$$\begin{aligned} \frac{\partial E^3_o}{\partial f(S^3_{h_2})} &= \sum_{o=1}^{N_o} \frac{\partial E^3_o}{\partial S^3_o} \frac{\partial S^3_o}{\partial f(S^3_{h_2})} \\ &= \sum_{o=1}^{N_o} \delta^3_o w_{h_2 o} \\ &= \delta^3_{o_1} w_8 \\ &= -0.0201(0.7154) \\ &= -0.0144 \end{aligned}$$

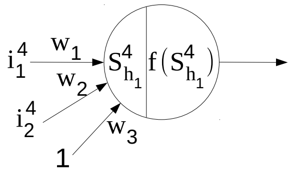
$$\begin{aligned} \frac{\partial f(S^3_{h_2})}{\partial S^3_{h_2}} &= f'(S^3_{h_2}) \\ &= f(S^3_{h_2})\left(1 - f(S^3_{h_2})\right) \\ &= 0.7457(1 - 0.7457) \\ &= 0.1896 \end{aligned}$$

$$\begin{aligned} \delta^3_{h_2} &= -0.0144(0.1896) \\ &= -0.0027 \end{aligned}$$

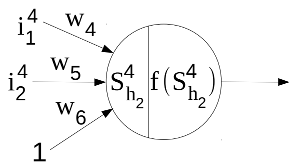
Epoch 2 : Iteration 4



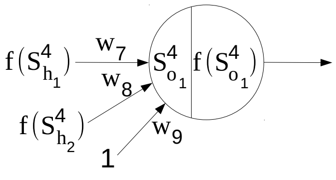
$w_1 = 0.0914$ $w_4 = 0.3939$ $w_7 = 0.6290$
$w_2 = 0.1912$ $w_5 = 0.4938$ $w_8 = 0.7154$
$w_3 = 0.2806$ $w_6 = 0.5822$ $w_9 = 0.7831$



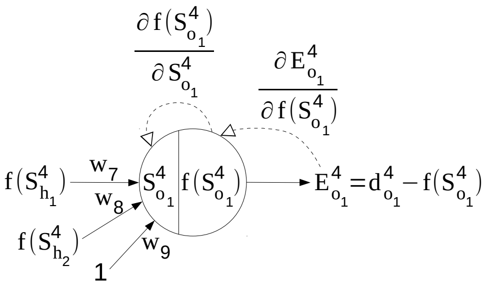
$$
\begin{aligned}
S^4_{h_1} &= \sum_j w_{jh_1} y^4_j \\
&= w_1 i^4_1 + w_2 i^4_2 + w_3(1) \\
&= 0.0914(1.0000)+0.1912(1.0000)+0.2806(1) \\
&= 0.5632 \\
f(S^4_{h_1}) &= \frac{1}{1+e^{-S^4_{h_1}}} \\
&= \frac{1}{1+e^{-0.5632}} \\
&= 0.6372
\end{aligned}
$$



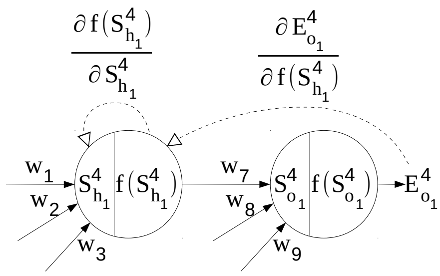
$$
\begin{aligned}
S^4_{h_2} &= \sum_j w_{jh_2} y^4_j \\
&= w_4 i^4_1 + w_5 i^4_2 + w_6(1) \\
&= 0.3939(1.0000)+0.4938(1.0000)+0.5822(1) \\
&= 1.4699 \\
f(S^4_{h_2}) &= \frac{1}{1+e^{-S^4_{h_2}}} \\
&= \frac{1}{1+e^{-1.4699}} \\
&= 0.8130
\end{aligned}
$$



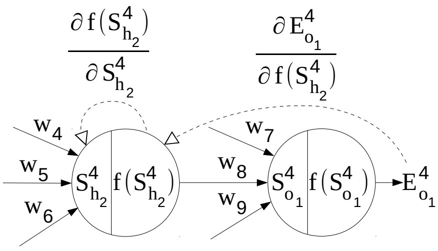
$$
\begin{aligned}
S^4_{o_1} &= \sum_j w_{jo_1} y^4_j \\
&= w_7 f(S^4_{h_1}) + w_8 f(S^4_{h_2}) + w_9(1) \\
&= 0.6290(0.6372)+0.7154(0.8130)+0.7831(1) \\
&= 1.7655 \\
f(S^4_{o_1}) &= \frac{1}{1+e^{-S^4_{o_1}}} \\
&= \frac{1}{1+e^{-1.7655}} \\
&= 0.8539
\end{aligned}
$$



$$
\begin{aligned}
\delta^4_{o_1} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \\
&= \frac{\partial E^4_{o_1}}{\partial f(S^4_{o_1})} \frac{\partial f(S^4_{o_1})}{\partial S^4_{o_1}} \\
\frac{\partial E^4_{o_1}}{\partial f(S^4_{o_1})} &= -\left(d^4_{o_1} - f(S^4_{o_1})\right) \\
&= -(0.0000-0.8539) \\
&= -(-0.8539) \\
&= 0.8539 \\
\frac{\partial f(S^4_{o_1})}{\partial S^4_{o_1}} &= f'(S^4_{o_1}) \\
&= f(S^4_{o_1})\left(1-f(S^4_{o_1})\right) \\
&= 0.8539(1-0.8539) \\
&= 0.1248 \\
\delta^4_{o_1} &= 0.8539(0.1248) \\
&= 0.1066
\end{aligned}
$$

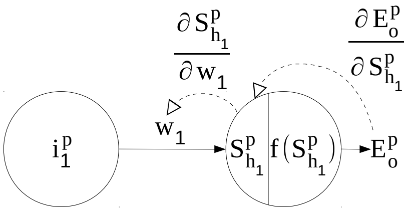


$$
\begin{aligned}
\delta^4_{h_1} &= \frac{\partial E^4_o}{\partial S^4_{h_1}} \\
&= \frac{\partial E^4_o}{\partial f(S^4_{h_1})} \frac{\partial f(S^4_{h_1})}{\partial S^4_{h_1}} \\
\frac{\partial E^4_o}{\partial f(S^4_{h_1})} &= \sum_{o=1}^{N_o} \frac{\partial E^4_o}{\partial S^4_o} \frac{\partial S^4_o}{\partial f(S^4_{h_1})} \\
&= \sum_{o=1}^{N_o} \delta^4_o w_{h_1 o} \\
&= \delta^4_{o_1} w_7 \\
&= 0.1066(0.6290) \\
&= 0.0671 \\
\frac{\partial f(S^4_{h_1})}{\partial S^4_{h_1}} &= f'(S^4_{h_1}) \\
&= f(S^4_{h_1})\left(1-f(S^4_{h_1})\right) \\
&= 0.6372(1-0.6372) \\
&= 0.2312 \\
\delta^4_{h_1} &= 0.0671(0.2312) \\
&= 0.0155
\end{aligned}
$$



$$
\begin{aligned}
\delta^4_{h_2} &= \frac{\partial E^4_o}{\partial S^4_{h_2}} \\
&= \frac{\partial E^4_o}{\partial f(S^4_{h_2})} \frac{\partial f(S^4_{h_2})}{\partial S^4_{h_2}} \\
\frac{\partial E^4_o}{\partial f(S^4_{h_2})} &= \sum_{o=1}^{N_o} \frac{\partial E^4_o}{\partial S^4_o} \frac{\partial S^4_o}{\partial f(S^4_{h_2})} \\
&= \sum_{o=1}^{N_o} \delta^4_o w_{h_2 o} \\
&= \delta^4_{o_1} w_8 \\
&= 0.1066(0.7154) \\
&= 0.0763 \\
\frac{\partial f(S^4_{h_2})}{\partial S^4_{h_2}} &= f'(S^4_{h_2}) \\
&= f(S^4_{h_2})\left(1-f(S^4_{h_2})\right) \\
&= 0.8130(1-0.8130) \\
&= 0.1520 \\
\delta^4_{h_2} &= 0.0763(0.1520) \\
&= 0.0116
\end{aligned}
$$

$$
\begin{aligned}
SSE &= \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \left(d^1_{o_1} - f(S^1_{o_1})\right)^2 + \left(d^2_{o_1} - f(S^2_{o_1})\right)^2 + \\
&\quad \left(d^3_{o_1} - f(S^3_{o_1})\right)^2 + \left(d^4_{o_1} - f(S^4_{o_1})\right)^2 \\
&= (0.0000-0.8321)^2 + (1.0000-0.8422)^2 + \\
&\quad (1.0000-0.8460)^2 + (0.0000-0.8539)^2 \\
&= 1.4702 \\
LSE &= \frac{1}{2} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \frac{1}{2} 1.4702 \\
&= 0.7351 \\
MSE &= \frac{1}{P \cdot N_o} \sum_{p=1}^{P} \sum_{o=1}^{N_o} \left(d^p_o - f(S^p_o)\right)^2 \\
&= \frac{1}{4(1)} 1.4702 \\
&= 0.3676 \\
RMSE &= \sqrt{MSE} \\
&= \sqrt{0.3676} \\
&= 0.6063
\end{aligned}
$$



$$\Delta_1 w_1(3) = -\gamma \frac{\partial E^1_o}{\partial w_1} + \alpha \Delta_1 w_1(2)$$

$$\begin{aligned}
\frac{\partial E^1_o}{\partial w_1} &= \frac{\partial E^1_o}{\partial S^1_{h_1}} \frac{\partial S^1_{h_1}}{\partial w_1} \\
&= \delta^1_{h_1} i^1_1 \\
&= 0.0179(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_1(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_1(2) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\Delta_2 w_1(3) = -\gamma \frac{\partial E^2_o}{\partial w_1} + \alpha \Delta_2 w_1(2)$$

$$\begin{aligned}
\frac{\partial E^2_o}{\partial w_1} &= \frac{\partial E^2_o}{\partial S^2_{h_1}} \frac{\partial S^2_{h_1}}{\partial w_1} \\
&= \delta^2_{h_1} i^2_1 \\
&= -0.0032(1.0000) \\
&= -0.0032
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_1(3) &= -\gamma(-0.0032) + \alpha \Delta_2 w_1(2) \\
&= -0.7(-0.0032) + 0.3(0.0018) \\
&= 0.0028
\end{aligned}$$

$$\Delta_3 w_1(3) = -\gamma \frac{\partial E^3_o}{\partial w_1} + \alpha \Delta_3 w_1(2)$$

$$\begin{aligned}
\frac{\partial E^3_o}{\partial w_1} &= \frac{\partial E^3_o}{\partial S^3_{h_1}} \frac{\partial S^3_{h_1}}{\partial w_1} \\
&= \delta^3_{h_1} i^3_1 \\
&= -0.0030(0.0000) \\
&= 0.0000
\end{aligned}$$

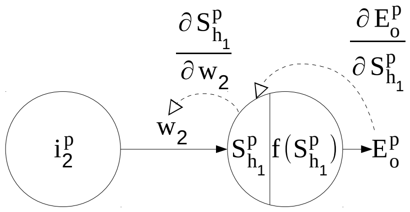
$$\begin{aligned}
\Delta_3 w_1(3) &= -\gamma(0.0000) + \alpha \Delta_3 w_1(2) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\Delta_4 w_1(3) = -\gamma \frac{\partial E^4_o}{\partial w_1} + \alpha \Delta_4 w_1(2)$$

$$\begin{aligned}
\frac{\partial E^4_o}{\partial w_1} &= \frac{\partial E^4_o}{\partial S^4_{h_1}} \frac{\partial S^4_{h_1}}{\partial w_1} \\
&= \delta^4_{h_1} i^4_1 \\
&= 0.0155(1.0000) \\
&= 0.0155
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_1(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_1(2) \\
&= -0.7(0.0155) + 0.3(-0.0104) \\
&= -0.0140
\end{aligned}$$

$$\begin{aligned}
w_1 &= \left(\sum_{p=1}^{P} \Delta_p w_1(3) \right) + w_1 \\
&= \left(\Delta_1 w_1(3) + \Delta_2 w_1(3) + \Delta_3 w_1(3) + \Delta_4 w_1(3) \right) + w_1 \\
&= \left(0.0000 + 0.0028 + 0.0000 + (-0.0140) \right) + 0.0914 \\
&= 0.0802
\end{aligned}$$



$$\Delta_1 w_2(3) = -\gamma \frac{\partial E^1_o}{\partial w_2} + \alpha \Delta_1 w_2(2)$$

$$\begin{aligned}
\frac{\partial E^1_o}{\partial w_2} &= \frac{\partial E^1_o}{\partial S^1_{h_1}} \frac{\partial S^1_{h_1}}{\partial w_2} \\
&= \delta^1_{h_1} i^1_2 \\
&= 0.0179(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_2(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_2(2) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\Delta_2 w_2(3) = -\gamma \frac{\partial E^2_o}{\partial w_2} + \alpha \Delta_2 w_2(2)$$

$$\begin{aligned}
\frac{\partial E^2_o}{\partial w_2} &= \frac{\partial E^2_o}{\partial S^2_{h_1}} \frac{\partial S^2_{h_1}}{\partial w_2} \\
&= \delta^2_{h_1} i^2_2 \\
&= -0.0032(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_2(3) &= -\gamma(0.0000) + \alpha \Delta_2 w_2(2) \\
&= -0.7(0.0000) + 0.3(0.0000) \\
&= 0.0000
\end{aligned}$$

$$\Delta_3 w_2(3) = -\gamma \frac{\partial E^3_o}{\partial w_2} + \alpha \Delta_3 w_2(2)$$

$$\begin{aligned}
\frac{\partial E^3_o}{\partial w_2} &= \frac{\partial E^3_o}{\partial S^3_{h_1}} \frac{\partial S^3_{h_1}}{\partial w_2} \\
&= \delta^3_{h_1} i^3_2 \\
&= -0.0030(1.0000) \\
&= -0.0030
\end{aligned}$$

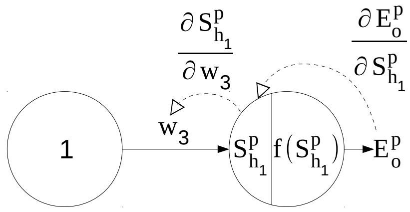
$$\begin{aligned}
\Delta_3 w_2(3) &= -\gamma(-0.0030) + \alpha \Delta_3 w_2(2) \\
&= -0.7(-0.0030) + 0.3(0.0016) \\
&= 0.0026
\end{aligned}$$

$$\Delta_4 w_2(3) = -\gamma \frac{\partial E^4_o}{\partial w_2} + \alpha \Delta_4 w_2(2)$$

$$\begin{aligned}
\frac{\partial E^4_o}{\partial w_2} &= \frac{\partial E^4_o}{\partial S^4_{h_1}} \frac{\partial S^4_{h_1}}{\partial w_2} \\
&= \delta^4_{h_1} i^4_2 \\
&= 0.0155(1.0000) \\
&= 0.0155
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_2(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_2(2) \\
&= -0.7(0.0155) + 0.3(-0.0104) \\
&= -0.0140
\end{aligned}$$

$$\begin{aligned}
w_2 &= \left(\sum_{p=1}^{P} \Delta_p w_2(3) \right) + w_2 \\
&= \left(\Delta_1 w_2(3) + \Delta_2 w_2(3) + \Delta_3 w_2(3) + \Delta_4 w_2(3) \right) + w_2 \\
&= \left(0.0000 + 0.0000 + 0.0026 + (-0.0140) \right) + 0.1912 \\
&= 0.1798
\end{aligned}$$



$$\Delta_1 w_3(3) = -\gamma \frac{\partial E^1_o}{\partial w_3} + \alpha \Delta_1 w_3(2)$$

$$\begin{aligned}
\frac{\partial E^1_o}{\partial w_3} &= \frac{\partial E^1_o}{\partial S^1_{h_1}} \frac{\partial S^1_{h_1}}{\partial w_3} \\
&= \delta^1_{h_1}(1) \\
&= 0.0179(1.0000) \\
&= 0.0179
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_3(3) &= -\gamma(0.0179) + \alpha \Delta_1 w_3(2) \\
&= -0.7(0.0179) + 0.3(-0.0124) \\
&= -0.0163
\end{aligned}$$

$$\Delta_2 w_3(3) = -\gamma \frac{\partial E^2_o}{\partial w_3} + \alpha \Delta_2 w_3(2)$$

$$\begin{aligned}
\frac{\partial E^2_o}{\partial w_3} &= \frac{\partial E^2_o}{\partial S^2_{h_1}} \frac{\partial S^2_{h_1}}{\partial w_3} \\
&= \delta^2_{h_1}(1) \\
&= -0.0032(1.0000) \\
&= -0.0032
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_3(3) &= -\gamma(-0.0032) + \alpha \Delta_2 w_3(2) \\
&= -0.7(-0.0032) + 0.3(0.0018) \\
&= 0.0028
\end{aligned}$$

$$\Delta_3 w_3(3) = -\gamma \frac{\partial E^3_o}{\partial w_3} + \alpha \Delta_3 w_3(2)$$

$$\begin{aligned}
\frac{\partial E^3_o}{\partial w_3} &= \frac{\partial E^3_o}{\partial S^3_{h_1}} \frac{\partial S^3_{h_1}}{\partial w_3} \\
&= \delta^3_{h_1}(1) \\
&= -0.0030(1.0000) \\
&= -0.0030
\end{aligned}$$

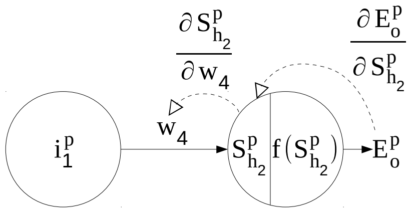
$$\begin{aligned}
\Delta_3 w_3(3) &= -\gamma(-0.0030) + \alpha \Delta_3 w_3(2) \\
&= -0.7(-0.0030) + 0.3(0.0016) \\
&= 0.0026
\end{aligned}$$

$$\Delta_4 w_3(3) = -\gamma \frac{\partial E^4_o}{\partial w_3} + \alpha \Delta_4 w_3(2)$$

$$\begin{aligned}
\frac{\partial E^4_o}{\partial w_3} &= \frac{\partial E^4_o}{\partial S^4_{h_1}} \frac{\partial S^4_{h_1}}{\partial w_3} \\
&= \delta^4_{h_1}(1) \\
&= 0.0155(1.0000) \\
&= 0.0155
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_3(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_3(2) \\
&= -0.7(0.0155) + 0.3(-0.0104) \\
&= -0.0140
\end{aligned}$$

$$\begin{aligned}
w_3 &= \left(\sum_{p=1}^{P} \Delta_p w_3(3) \right) + w_3 \\
&= \left(\Delta_1 w_3(3) + \Delta_2 w_3(3) + \Delta_3 w_3(3) + \Delta_4 w_3(3) \right) + w_3 \\
&= \left(-0.0163 + 0.0028 + 0.0026 + (-0.0140) \right) + 0.2806 \\
&= 0.2557
\end{aligned}$$



$$\Delta_1 w_4(3) = -\gamma \frac{\partial E_o^1}{\partial w_4} + \alpha \Delta_1 w_4(2)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_4} &= \frac{\partial E_o^1}{\partial S_{h_2}^1}\frac{\partial S_{h_2}^1}{\partial w_4}\\ &= \delta_{h_2}^1 i_1^1\\ &= 0.0191(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_1 w_4(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_4(2)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_2 w_4(3) = -\gamma \frac{\partial E_o^2}{\partial w_4} + \alpha \Delta_2 w_4(2)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_4} &= \frac{\partial E_o^2}{\partial S_{h_2}^2}\frac{\partial S_{h_2}^2}{\partial w_4}\\ &= \delta_{h_2}^2 i_1^2\\ &= -0.0030(1.0000)\\ &= -0.0030\end{aligned}$$

$$\begin{aligned}\Delta_2 w_4(3) &= -\gamma(-0.0030) + \alpha \Delta_2 w_4(2)\\ &= -0.7(-0.0030) + 0.3(0.0016)\\ &= 0.0026\end{aligned}$$

$$\Delta_3 w_4(3) = -\gamma \frac{\partial E_o^3}{\partial w_4} + \alpha \Delta_3 w_4(2)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_4} &= \frac{\partial E_o^3}{\partial S_{h_2}^3}\frac{\partial S_{h_2}^3}{\partial w_4}\\ &= \delta_{h_2}^3 i_1^3\\ &= -0.0027(0.0000)\\ &= 0.0000\end{aligned}$$

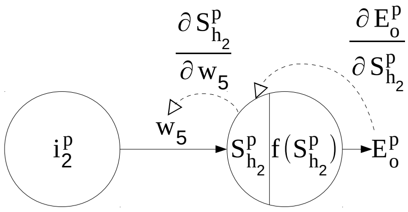
$$\begin{aligned}\Delta_3 w_4(3) &= -\gamma(0.0000) + \alpha \Delta_3 w_4(2)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_4 w_4(3) = -\gamma \frac{\partial E_o^4}{\partial w_4} + \alpha \Delta_4 w_4(2)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_4} &= \frac{\partial E_o^4}{\partial S_{h_2}^4}\frac{\partial S_{h_2}^4}{\partial w_4}\\ &= \delta_{h_2}^4 i_1^4\\ &= 0.0116(1.0000)\\ &= 0.0116\end{aligned}$$

$$\begin{aligned}\Delta_4 w_4(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_4(2)\\ &= -0.7(0.0116) + 0.3(-0.0077)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_4 &= \left(\sum_{p=1}^{P} \Delta_p w_4(3)\right) + w_4\\ &= \left(\Delta_1 w_4(3) + \Delta_2 w_4(3) + \Delta_3 w_4(3) + \Delta_4 w_4(3)\right) + w_4\\ &= \left(0.0000 + 0.0026 + 0.0000 + (-0.0104)\right) + 0.3939\\ &= 0.3861\end{aligned}$$



$$\Delta_1 w_5(3) = -\gamma \frac{\partial E_o^1}{\partial w_5} + \alpha \Delta_1 w_5(2)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_5} &= \frac{\partial E_o^1}{\partial S_{h_2}^1}\frac{\partial S_{h_2}^1}{\partial w_5}\\ &= \delta_{h_2}^1 i_2^1\\ &= 0.0191(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_1 w_5(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_5(2)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_2 w_5(3) = -\gamma \frac{\partial E_o^2}{\partial w_5} + \alpha \Delta_2 w_5(2)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_5} &= \frac{\partial E_o^2}{\partial S_{h_2}^2}\frac{\partial S_{h_2}^2}{\partial w_5}\\ &= \delta_{h_2}^2 i_2^2\\ &= -0.0030(0.0000)\\ &= 0.0000\end{aligned}$$

$$\begin{aligned}\Delta_2 w_5(3) &= -\gamma(0.0000) + \alpha \Delta_2 w_5(2)\\ &= -0.7(0.0000) + 0.3(0.0000)\\ &= 0.0000\end{aligned}$$

$$\Delta_3 w_5(3) = -\gamma \frac{\partial E_o^3}{\partial w_5} + \alpha \Delta_3 w_5(2)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_5} &= \frac{\partial E_o^3}{\partial S_{h_2}^3}\frac{\partial S_{h_2}^3}{\partial w_5}\\ &= \delta_{h_2}^3 i_2^3\\ &= -0.0027(1.0000)\\ &= -0.0027\end{aligned}$$

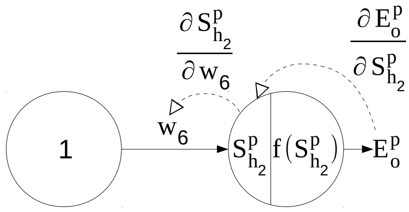
$$\begin{aligned}\Delta_3 w_5(3) &= -\gamma(-0.0027) + \alpha \Delta_3 w_5(2)\\ &= -0.7(-0.0027) + 0.3(0.0015)\\ &= 0.0023\end{aligned}$$

$$\Delta_4 w_5(3) = -\gamma \frac{\partial E_o^4}{\partial w_5} + \alpha \Delta_4 w_5(2)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_5} &= \frac{\partial E_o^4}{\partial S_{h_2}^4}\frac{\partial S_{h_2}^4}{\partial w_5}\\ &= \delta_{h_2}^4 i_2^4\\ &= 0.0116(1.0000)\\ &= 0.0116\end{aligned}$$

$$\begin{aligned}\Delta_4 w_5(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_5(2)\\ &= -0.7(0.0116) + 0.3(-0.0077)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_5 &= \left(\sum_{p=1}^{P} \Delta_p w_5(3)\right) + w_5\\ &= \left(\Delta_1 w_5(3) + \Delta_2 w_5(3) + \Delta_3 w_5(3) + \Delta_4 w_5(3)\right) + w_5\\ &= \left(0.0000 + 0.0000 + 0.0023 + (-0.0104)\right) + 0.4938\\ &= 0.4857\end{aligned}$$



$$\Delta_1 w_6(3) = -\gamma \frac{\partial E_o^1}{\partial w_6} + \alpha \Delta_1 w_6(2)$$

$$\begin{aligned}\frac{\partial E_o^1}{\partial w_6} &= \frac{\partial E_o^1}{\partial S_{h_2}^1}\frac{\partial S_{h_2}^1}{\partial w_6}\\ &= \delta_{h_2}^1(1)\\ &= 0.0191(1.0000)\\ &= 0.0191\end{aligned}$$

$$\begin{aligned}\Delta_1 w_6(3) &= -\gamma(0.0191) + \alpha \Delta_1 w_6(2)\\ &= -0.7(0.0191) + 0.3(-0.0132)\\ &= -0.0173\end{aligned}$$

$$\Delta_2 w_6(3) = -\gamma \frac{\partial E_o^2}{\partial w_6} + \alpha \Delta_2 w_6(2)$$

$$\begin{aligned}\frac{\partial E_o^2}{\partial w_6} &= \frac{\partial E_o^2}{\partial S_{h_2}^2}\frac{\partial S_{h_2}^2}{\partial w_6}\\ &= \delta_{h_2}^2(1)\\ &= -0.0030(1.0000)\\ &= -0.0030\end{aligned}$$

$$\begin{aligned}\Delta_2 w_6(3) &= -\gamma(-0.0030) + \alpha \Delta_2 w_6(2)\\ &= -0.7(-0.0030) + 0.3(0.0016)\\ &= 0.0026\end{aligned}$$

$$\Delta_3 w_6(3) = -\gamma \frac{\partial E_o^3}{\partial w_6} + \alpha \Delta_3 w_6(2)$$

$$\begin{aligned}\frac{\partial E_o^3}{\partial w_6} &= \frac{\partial E_o^3}{\partial S_{h_2}^3}\frac{\partial S_{h_2}^3}{\partial w_6}\\ &= \delta_{h_2}^3(1)\\ &= -0.0027(1.0000)\\ &= -0.0027\end{aligned}$$

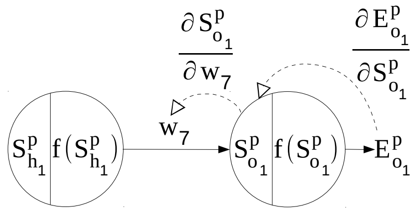
$$\begin{aligned}\Delta_3 w_6(3) &= -\gamma(-0.0027) + \alpha \Delta_3 w_6(2)\\ &= -0.7(-0.0027) + 0.3(0.0015)\\ &= 0.0023\end{aligned}$$

$$\Delta_4 w_6(3) = -\gamma \frac{\partial E_o^4}{\partial w_6} + \alpha \Delta_4 w_6(2)$$

$$\begin{aligned}\frac{\partial E_o^4}{\partial w_6} &= \frac{\partial E_o^4}{\partial S_{h_2}^4}\frac{\partial S_{h_2}^4}{\partial w_6}\\ &= \delta_{h_2}^4(1)\\ &= 0.0116(1.0000)\\ &= 0.0116\end{aligned}$$

$$\begin{aligned}\Delta_4 w_6(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_6(2)\\ &= -0.7(0.0116) + 0.3(-0.0077)\\ &= -0.0104\end{aligned}$$

$$\begin{aligned}w_6 &= \left(\sum_{p=1}^{P} \Delta_p w_6(3)\right) + w_6\\ &= \left(\Delta_1 w_6(3) + \Delta_2 w_6(3) + \Delta_3 w_6(3) + \Delta_4 w_6(3)\right) + w_6\\ &= \left(-0.0173 + 0.0026 + 0.0023 + (-0.0104)\right) + 0.5822\\ &= 0.5594\end{aligned}$$



$$\Delta_1 w_7(3) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_7} + \alpha \Delta_1 w_7(2)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_7} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_7} \\
&= \delta^1_{o_1} f(S^1_{h_1}) \\
&= 0.1162(0.5697) \\
&= 0.0662
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_7(3) &= -\gamma(0.0662) + \alpha \Delta_1 w_7(2) \\
&= -0.7(0.0662) + 0.3(-0.0415) \\
&= -0.0588
\end{aligned}$$

$$\Delta_2 w_7(3) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_7} + \alpha \Delta_2 w_7(2)$$

$$\begin{aligned}
\frac{\partial E^2_{o}}{\partial w_7} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_7} \\
&= \delta^2_{o_1} f(S^2_{h_1}) \\
&= -0.0210(0.5919) \\
&= -0.0124
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_7(3) &= -\gamma(-0.0124) + \alpha \Delta_2 w_7(2) \\
&= -0.7(-0.0124) + 0.3(0.0061) \\
&= 0.0105
\end{aligned}$$

$$\Delta_3 w_7(3) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_7} + \alpha \Delta_3 w_7(2)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_7} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_7} \\
&= \delta^3_{o_1} f(S^3_{h_1}) \\
&= -0.0201(0.6158) \\
&= -0.0124
\end{aligned}$$

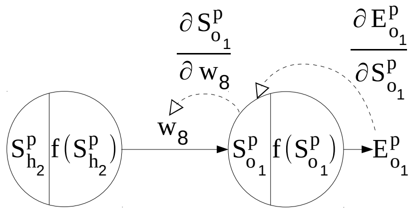
$$\begin{aligned}
\Delta_3 w_7(3) &= -\gamma(-0.0124) + \alpha \Delta_3 w_7(2) \\
&= -0.7(-0.0124) + 0.3(0.0061) \\
&= 0.0105
\end{aligned}$$

$$\Delta_4 w_7(3) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_7} + \alpha \Delta_4 w_7(2)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_7} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_7} \\
&= \delta^4_{o_1} f(S^4_{h_1}) \\
&= 0.1066(0.6372) \\
&= 0.0679
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_7(3) &= -\gamma(0.0679) + \alpha \Delta_4 w_7(2) \\
&= -0.7(0.0679) + 0.3(-0.0417) \\
&= -0.0600
\end{aligned}$$

$$\begin{aligned}
w_7 &= \left(\sum_{p=1}^{P} \Delta_p w_7(3)\right) + w_7 \\
&= \left(\Delta_1 w_7(3) + \Delta_2 w_7(3) + \Delta_3 w_7(3) + \Delta_4 w_7(3)\right) + w_7 \\
&= \left(-0.0588 + 0.0105 + 0.0105 + (-0.0600)\right) + 0.6290 \\
&= 0.5312
\end{aligned}$$



$$\Delta_1 w_8(3) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_8} + \alpha \Delta_1 w_8(2)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_8} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_8} \\
&= \delta^1_{o_1} f(S^1_{h_2}) \\
&= 0.1162(0.6416) \\
&= 0.0746
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_8(3) &= -\gamma(0.0746) + \alpha \Delta_1 w_8(2) \\
&= -0.7(0.0746) + 0.3(-0.0467) \\
&= -0.0662
\end{aligned}$$

$$\Delta_2 w_8(3) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_8} + \alpha \Delta_2 w_8(2)$$

$$\begin{aligned}
\frac{\partial E^2_{o_1}}{\partial w_8} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_8} \\
&= \delta^2_{o_1} f(S^2_{h_2}) \\
&= -0.0210(0.7263) \\
&= -0.0153
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_8(3) &= -\gamma(-0.0153) + \alpha \Delta_2 w_8(2) \\
&= -0.7(-0.0153) + 0.3(0.0075) \\
&= 0.0130
\end{aligned}$$

$$\Delta_3 w_8(3) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_8} + \alpha \Delta_3 w_8(2)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_8} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_8} \\
&= \delta^3_{o_1} f(S^3_{h_2}) \\
&= -0.0201(0.7457) \\
&= -0.0150
\end{aligned}$$

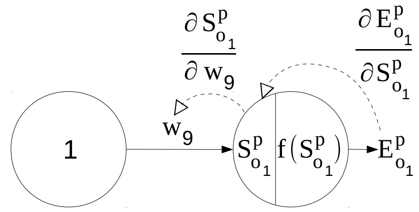
$$\begin{aligned}
\Delta_3 w_8(3) &= -\gamma(-0.0150) + \alpha \Delta_3 w_8(2) \\
&= -0.7(-0.0150) + 0.3(0.0073) \\
&= 0.0127
\end{aligned}$$

$$\Delta_4 w_8(3) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_8} + \alpha \Delta_4 w_8(2)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_8} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_8} \\
&= \delta^4_{o_1} f(S^4_{h_2}) \\
&= 0.1066(0.8130) \\
&= 0.0867
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_8(3) &= -\gamma(0.0867) + \alpha \Delta_4 w_8(2) \\
&= -0.7(0.0867) + 0.3(-0.0527) \\
&= -0.0765
\end{aligned}$$

$$\begin{aligned}
w_8 &= \left(\sum_{p=1}^{P} \Delta_p w_8(3)\right) + w_8 \\
&= \left(\Delta_1 w_8(3) + \Delta_2 w_8(3) + \Delta_3 w_8(3) + \Delta_4 w_8(3)\right) + w_8 \\
&= \left(-0.0662 + 0.0130 + 0.0127 + (-0.0765)\right) + 0.7154 \\
&= 0.5984
\end{aligned}$$



$$\Delta_1 w_9(3) = -\gamma \frac{\partial E^1_{o_1}}{\partial w_9} + \alpha \Delta_1 w_9(2)$$

$$\begin{aligned}
\frac{\partial E^1_{o_1}}{\partial w_9} &= \frac{\partial E^1_{o_1}}{\partial S^1_{o_1}} \frac{\partial S^1_{o_1}}{\partial w_9} \\
&= \delta^1_{o_1}(1) \\
&= 0.1162(1.0000) \\
&= 0.1162
\end{aligned}$$

$$\begin{aligned}
\Delta_1 w_9(3) &= -\gamma(0.1162) + \alpha \Delta_1 w_9(2) \\
&= -0.7(0.1162) + 0.3(-0.0723) \\
&= -0.1030
\end{aligned}$$

$$\Delta_2 w_9(3) = -\gamma \frac{\partial E^2_{o_1}}{\partial w_9} + \alpha \Delta_2 w_9(2)$$

$$\begin{aligned}
\frac{\partial E^2_{o_1}}{\partial w_9} &= \frac{\partial E^2_{o_1}}{\partial S^2_{o_1}} \frac{\partial S^2_{o_1}}{\partial w_9} \\
&= \delta^2_{o_1}(1) \\
&= -0.0210(1.0000) \\
&= -0.0210
\end{aligned}$$

$$\begin{aligned}
\Delta_2 w_9(3) &= -\gamma(-0.0210) + \alpha \Delta_2 w_9(2) \\
&= -0.7(-0.0210) + 0.3(0.0102) \\
&= 0.0178
\end{aligned}$$

$$\Delta_3 w_9(3) = -\gamma \frac{\partial E^3_{o_1}}{\partial w_9} + \alpha \Delta_3 w_9(2)$$

$$\begin{aligned}
\frac{\partial E^3_{o_1}}{\partial w_9} &= \frac{\partial E^3_{o_1}}{\partial S^3_{o_1}} \frac{\partial S^3_{o_1}}{\partial w_9} \\
&= \delta^3_{o_1}(1) \\
&= -0.0201(1.0000) \\
&= -0.0201
\end{aligned}$$

$$\begin{aligned}
\Delta_3 w_9(3) &= -\gamma(-0.0201) + \alpha \Delta_3 w_9(2) \\
&= -0.7(-0.0201) + 0.3(0.0097) \\
&= 0.0170
\end{aligned}$$

$$\Delta_4 w_9(3) = -\gamma \frac{\partial E^4_{o_1}}{\partial w_9} + \alpha \Delta_4 w_9(2)$$

$$\begin{aligned}
\frac{\partial E^4_{o_1}}{\partial w_9} &= \frac{\partial E^4_{o_1}}{\partial S^4_{o_1}} \frac{\partial S^4_{o_1}}{\partial w_9} \\
&= \delta^4_{o_1}(1) \\
&= 0.1066(1.0000) \\
&= 0.1066
\end{aligned}$$

$$\begin{aligned}
\Delta_4 w_9(3) &= -\gamma(0.1066) + \alpha \Delta_4 w_9(2) \\
&= -0.7(0.1066) + 0.3(-0.0645) \\
&= -0.0940
\end{aligned}$$

$$\begin{aligned}
w_9 &= \left(\sum_{p=1}^{P} \Delta_p w_9(3)\right) + w_9 \\
&= \left(\Delta_1 w_9(3) + \Delta_2 w_9(3) + \Delta_3 w_9(3) + \Delta_4 w_9(3)\right) + w_9 \\
&= \left(-0.1030 + 0.0178 + 0.0170 + (-0.0940)\right) + 0.7831 \\
&= 0.6209
\end{aligned}$$