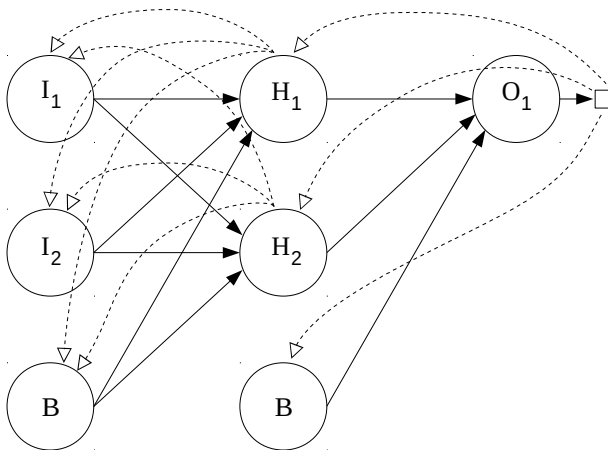


< Artificial Intelligence >-----

XOR Hello World using FFANN Backpropagation

Revision #: 2
4. September 2016



Training a Feed Forward Artificial Neural Network (FFANN) using Backpropagation Training to solve a simple XOR example.

XOR example is like a Hello World to a Neural Network.

Full calculations are shown for the first two epochs only to understand the flow and equations. The rest of the epochs will be calculated using our C program `ffann_backprop.c`.

These calculations are tested against output from Encog framework. The results from these calculations are almost the same with output from Encog framework.

List of equationsSummation for neuron n at training pattern p :

$$S_n^p = \sum_j w_{jn} y_j^p$$

where,

- p - Training pattern index number or training iteration index number
 n - Neuron index number for current layer
 j - Neuron index number from input layer
 w_{jn} - Weight between neuron j and neuron n
 y_j^p - The output of neuron j at training pattern p
 t - epoch number
 α - momentum
 γ - learning rate (gain)

Sigmoid activation and it's derivative:

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = f(x)(1-f(x))$$

Delta for output neuron:

$$\delta_o^p = \frac{\partial E_o^p}{\partial S_o^p}$$

$$= \frac{\partial E_o^p}{\partial f(S_o^p)} \frac{\partial f(S_o^p)}{\partial S_o^p}$$

where,

- $\frac{\partial E_o^p}{\partial S_o^p}$ - The partial derivative of output error with respect to S_o^p . The gradient from E_o^p to S_o^p
 $\frac{\partial E_o^p}{\partial f(S_o^p)}$ - The gradient from E_o^p to $f(S_o^p)$
 $\frac{\partial f(S_o^p)}{\partial S_o^p}$ - The gradient from $f(S_o^p)$ to S_o^p

Delta for hidden neuron:

$$\delta_h^p = \frac{\partial E_o^p}{\partial S_h^p}$$

$$= \frac{\partial E_o^p}{\partial f(S_h^p)} \frac{\partial f(S_h^p)}{\partial S_h^p}$$

where,

- $\frac{\partial E_o^p}{\partial S_h^p}$ - The gradient from E_o^p to S_h^p
 $\frac{\partial E_o^p}{\partial f(S_h^p)}$ - The gradient from E_o^p to $f(S_h^p)$
 $\frac{\partial f(S_h^p)}{\partial S_h^p}$ - The gradient from $f(S_h^p)$ to S_h^p

Amount required to change the w_j at epoch t :

$$\Delta_p w_j(t) = -\gamma \frac{\partial E_o^p}{\partial w_j} + \alpha \Delta_p w_j(t-1)$$

where,

- p - training pattern index number or training iteration index number
 w - weight
 j - weight index number
 t - epoch number
 α - momentum
 γ - learning rate (gain)
 $\frac{\partial E_o^p}{\partial w_j}$ - The gradient from E_o^p to w_j

Common error calculation formulas:

$$SSE = \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2$$

$$LSE = \frac{1}{2} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2$$

$$MSE = \frac{1}{P \cdot N_o} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2$$

$$RMSE = \sqrt{\frac{1}{P \cdot N_o} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2}$$

where,

- P - Total number of training patterns
 N_o - Total number of output units (classes)
 d - Desired output (unit)
 $f(S_o^p)$ - Actual neuron output (unit)
 α - momentum
 γ - learning rate (gain)
 $\frac{\partial E_o^p}{\partial w_j}$ - The gradient from E_o^p to w_j

Brief explanation

Our training data set:

Input1	Input2	Desired Output1
0	0	0
1	0	1
0	1	1
1	1	0

There are 4 training patterns in the data set:

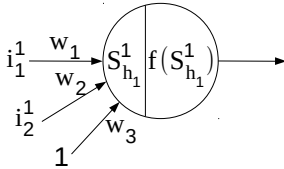
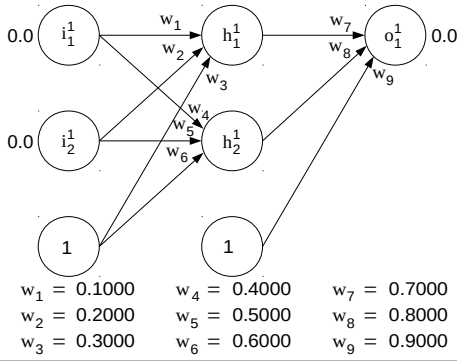
- 1) When Input1 = 0 and Input2 = 0, the output should be 0.
- 2) When Input1 = 1 and Input2 = 0, the output should be 1.
- 3) When Input1 = 0 and Input2 = 1, the output should be 1.
- 4) When Input1 = 1 and Input2 = 1, the output should be 0.

The next page is the calculations for FFANN Backpropagation.

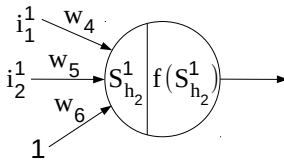
We will be using training rate $\gamma = 0.7$ and momentum $\alpha = 0.3$ when updating the weight values.References

1. Kattan, Ali Abdullah, Rosni Geem, Zong Woo, "Computer Networks : Artificial Neural Network Training and Software Implementation Techniques".
2. Alavala, Chennakesava R., "Fuzzy Logic and Neural Networks : Basic Concepts & Application".
3. Jeff Heaton, "Programming Neural Networks with Encog3 in Java".
4. R. Lippmann, "An introduction to computing with neural nets," in *IEEE ASSP Magazine*, vol. 4, no. 2, pp. 4-22, Apr 1987. doi: 10.1109/MASSP.1987.1165576
5. Matt Mazur. A Step by Step Backpropagation Example, 2015, <https://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/>.

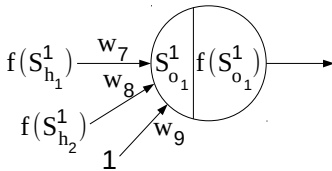
Epoch 1 : Iteration 1



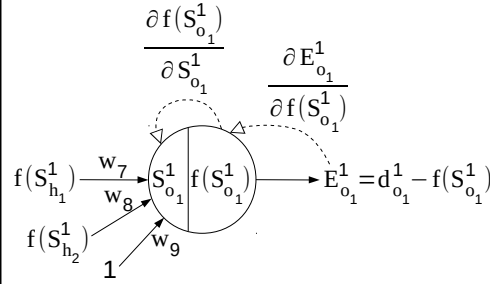
$$\begin{aligned}
 S_{h_1}^1 &= \sum_j w_{jh_1} y_j^1 \\
 &= w_1 i_1^1 + w_2 i_2^1 + w_3 (1) \\
 &= 0.1000(0.0000) + 0.2000(0.0000) + 0.3000(1) \\
 &= 0.3000 \\
 f(S_{h_1}^1) &= \frac{1}{1 + e^{-S_{h_1}^1}} \\
 &= \frac{1}{1 + e^{-0.3000}} \\
 &= 0.5744
 \end{aligned}$$



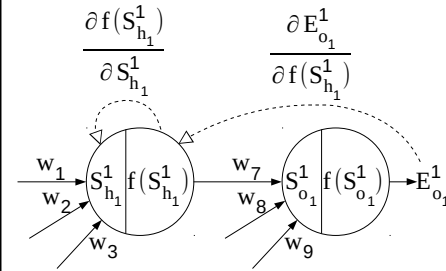
$$\begin{aligned}
 S_{h_2}^1 &= \sum_j w_{jh_2} y_j^1 \\
 &= w_4 i_1^1 + w_5 i_2^1 + w_6 (1) \\
 &= 0.4000(0.0000) + 0.5000(0.0000) + 0.6000(1) \\
 &= 0.6000 \\
 f(S_{h_2}^1) &= \frac{1}{1 + e^{-S_{h_2}^1}} \\
 &= \frac{1}{1 + e^{-0.6000}} \\
 &= 0.6457
 \end{aligned}$$



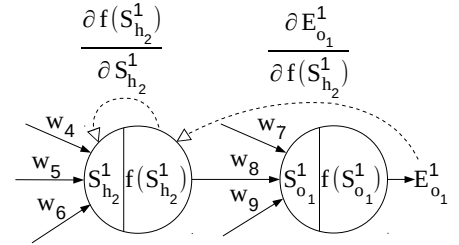
$$\begin{aligned}
 S_{o_1}^1 &= \sum_j w_{jo_1} y_j^1 \\
 &= w_7 f(S_{h_1}^1) + w_8 f(S_{h_2}^1) + w_9 (1) \\
 &= 0.7000(0.5744) + 0.8000(0.6457) + 0.9000(1) \\
 &= 1.8186 \\
 f(S_{o_1}^1) &= \frac{1}{1 + e^{-S_{o_1}^1}} \\
 &= \frac{1}{1 + e^{-1.8186}} \\
 &= 0.8604
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} &= -(d_{o_1}^1 - f(S_{o_1}^1)) \\
 &= -(0.0000 - 0.8604) \\
 &= -(-0.8604) \\
 &= 0.8604 \\
 \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} &= f'(S_{o_1}^1) \\
 &= f(S_{o_1}^1)(1 - f(S_{o_1}^1)) \\
 &= 0.8604(1 - 0.8604) \\
 &= 0.1201 \\
 \delta_{o_1}^1 &= 0.8604(0.1201) \\
 &= 0.1033
 \end{aligned}$$

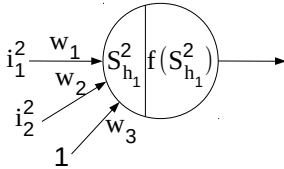
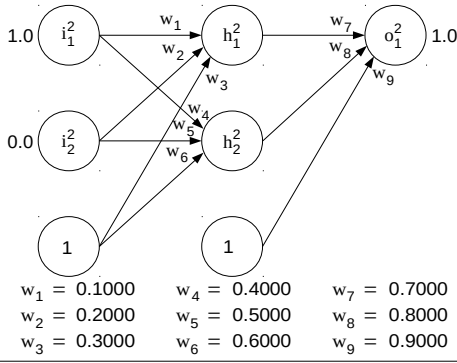


$$\begin{aligned}
 \delta_{h_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{h_1}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{h_1}^1)} \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{h_1}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial f(S_{h_1}^1)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^1 w_{h_1 o} \\
 &= \delta_{o_1}^1 w_7 \\
 &= 0.1033(0.7000) \\
 &= 0.0723 \\
 \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} &= f'(S_{h_1}^1) \\
 &= f(S_{h_1}^1)(1 - f(S_{h_1}^1)) \\
 &= 0.5744(1 - 0.5744) \\
 &= 0.2445 \\
 \delta_{h_1}^1 &= 0.0723(0.2445) \\
 &= 0.0177
 \end{aligned}$$

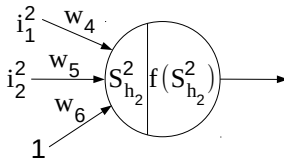


$$\begin{aligned}
 \delta_{h_2}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{h_2}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{h_2}^1)} \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{h_2}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial f(S_{h_2}^1)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^1 w_{h_2 o} \\
 &= \delta_{o_1}^1 w_8 \\
 &= 0.1033(0.8000) \\
 &= 0.0826 \\
 \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} &= f'(S_{h_2}^1) \\
 &= f(S_{h_2}^1)(1 - f(S_{h_2}^1)) \\
 &= 0.6457(1 - 0.6457) \\
 &= 0.2288 \\
 \delta_{h_2}^1 &= 0.0826(0.2288) \\
 &= 0.0189
 \end{aligned}$$

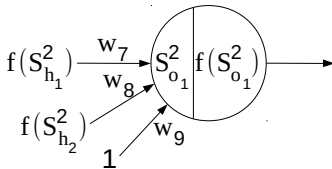
Epoch 1 : Iteration 2



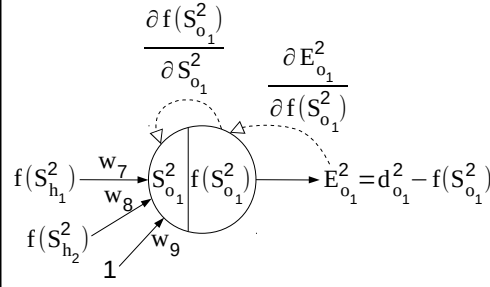
$$\begin{aligned}
 S_{h_1}^2 &= \sum_j w_{jh_1} y_j^2 \\
 &= w_1 i_1^2 + w_2 i_2^2 + w_3 (1) \\
 &= 0.1000(1.0000) + 0.2000(0.0000) + 0.3000(1) \\
 &= 0.4000 \\
 f(S_{h_1}^2) &= \frac{1}{1 + e^{-S_{h_1}^2}} \\
 &= \frac{1}{1 + e^{-0.4000}} \\
 &= 0.5987
 \end{aligned}$$



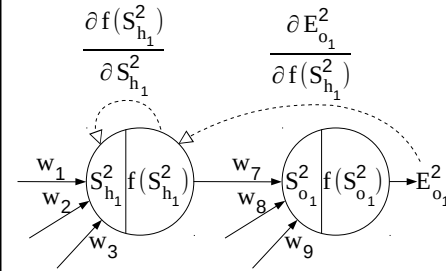
$$\begin{aligned}
 S_{h_2}^2 &= \sum_j w_{jh_2} y_j^2 \\
 &= w_4 i_1^2 + w_5 i_2^2 + w_6 (1) \\
 &= 0.4000(1.0000) + 0.5000(0.0000) + 0.6000(1) \\
 &= 1.0000 \\
 f(S_{h_2}^2) &= \frac{1}{1 + e^{-S_{h_2}^2}} \\
 &= \frac{1}{1 + e^{-1.0000}} \\
 &= 0.7311
 \end{aligned}$$



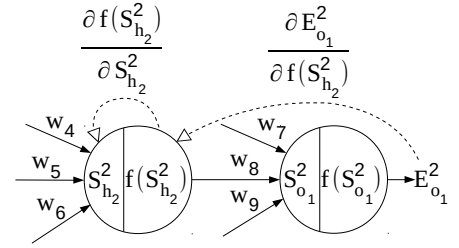
$$\begin{aligned}
 S_{o_1}^2 &= \sum_j w_{jo_1} y_j^2 \\
 &= w_7 f(S_{h_1}^2) + w_8 f(S_{h_2}^2) + w_9 (1) \\
 &= 0.7000(0.5987) + 0.8000(0.7311) + 0.9000(1) \\
 &= 1.9040 \\
 f(S_{o_1}^2) &= \frac{1}{1 + e^{-S_{o_1}^2}} \\
 &= \frac{1}{1 + e^{-1.9040}} \\
 &= 0.8703
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} \frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} &= -(d_{o_1}^2 - f(S_{o_1}^2)) \\
 &= -(1.0000 - 0.8703) \\
 &= -0.1297 \\
 \frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2} &= f'(S_{o_1}^2) \\
 &= f(S_{o_1}^2)(1 - f(S_{o_1}^2)) \\
 &= 0.8703(1 - 0.8703) \\
 &= 0.1129 \\
 \delta_{o_1}^2 &= -0.1297(0.1129) \\
 &= -0.0146
 \end{aligned}$$

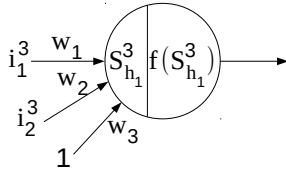
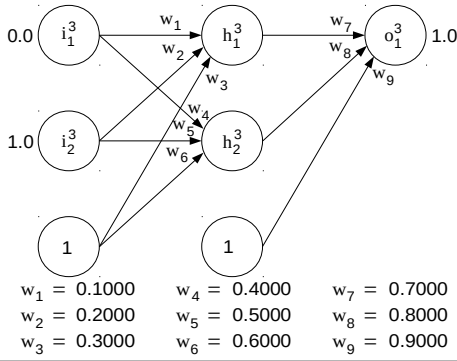


$$\begin{aligned}
 \delta_{h_1}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{h_1}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{h_1}^2)} \frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{h_1}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial f(S_{h_1}^2)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^2 w_{h_1 o} \\
 &= \delta_{o_1}^2 w_7 \\
 &= -0.0146(0.7000) \\
 &= -0.0102 \\
 \frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2} &= f'(S_{h_1}^2) \\
 &= f(S_{h_1}^2)(1 - f(S_{h_1}^2)) \\
 &= 0.5987(1 - 0.5987) \\
 &= 0.2403 \\
 \delta_{h_1}^2 &= -0.0102(0.2403) \\
 &= -0.0025
 \end{aligned}$$

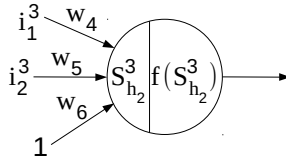


$$\begin{aligned}
 \delta_{h_2}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{h_2}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{h_2}^2)} \frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{h_2}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial f(S_{h_2}^2)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^2 w_{h_2 o} \\
 &= \delta_{o_1}^2 w_8 \\
 &= -0.0146(0.8000) \\
 &= -0.0117 \\
 \frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2} &= f'(S_{h_2}^2) \\
 &= f(S_{h_2}^2)(1 - f(S_{h_2}^2)) \\
 &= 0.7311(1 - 0.7311) \\
 &= 0.1966 \\
 \delta_{h_2}^2 &= -0.0117(0.1966) \\
 &= -0.0023
 \end{aligned}$$

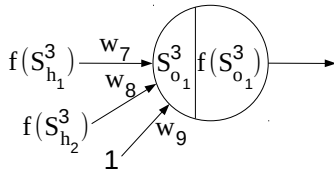
Epoch 1 : Iteration 3



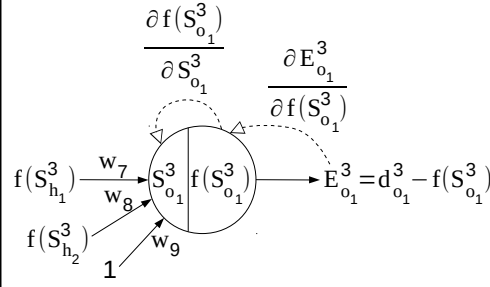
$$\begin{aligned}
 S_{h_1}^3 &= \sum_j w_{jh_1} y_j^3 \\
 &= w_1 i_1^3 + w_2 i_2^3 + w_3 (1) \\
 &= 0.1000(0.0000) + 0.2000(1.0000) + 0.3000(1) \\
 &= 0.5000 \\
 f(S_{h_1}^3) &= \frac{1}{1 + e^{-S_{h_1}^3}} \\
 &= \frac{1}{1 + e^{-0.5000}} \\
 &= 0.6225
 \end{aligned}$$



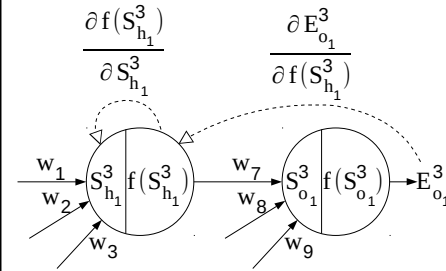
$$\begin{aligned}
 S_{h_2}^3 &= \sum_j w_{jh_2} y_j^3 \\
 &= w_4 i_1^3 + w_5 i_2^3 + w_6 (1) \\
 &= 0.4000(0.0000) + 0.5000(1.0000) + 0.6000(1) \\
 &= 1.1000 \\
 f(S_{h_2}^3) &= \frac{1}{1 + e^{-S_{h_2}^3}} \\
 &= \frac{1}{1 + e^{-1.1000}} \\
 &= 0.7503
 \end{aligned}$$



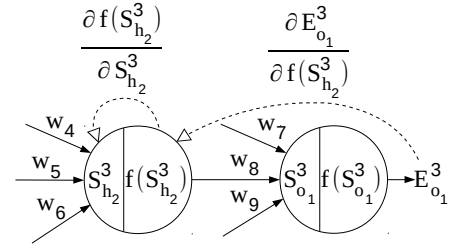
$$\begin{aligned}
 S_{o_1}^3 &= \sum_j w_{jo_1} y_j^3 \\
 &= w_7 f(S_{h_1}^3) + w_8 f(S_{h_2}^3) + w_9 (1) \\
 &= 0.7000(0.6225) + 0.8000(0.7503) + 0.9000(1) \\
 &= 1.9360 \\
 f(S_{o_1}^3) &= \frac{1}{1 + e^{-S_{o_1}^3}} \\
 &= \frac{1}{1 + e^{-1.9360}} \\
 &= 0.8739
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{o_1}^3)} \frac{\partial f(S_{o_1}^3)}{\partial S_{o_1}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{o_1}^3)} &= -(d_{o_1}^3 - f(S_{o_1}^3)) \\
 &= -(1.0000 - 0.8739) \\
 &= -(0.1261) \\
 &= -0.1261 \\
 \frac{\partial f(S_{o_1}^3)}{\partial S_{o_1}^3} &= f'(S_{o_1}^3) \\
 &= f(S_{o_1}^3)(1 - f(S_{o_1}^3)) \\
 &= 0.8739(1 - 0.8739) \\
 &= 0.1102 \\
 \delta_{o_1}^3 &= -0.1261(0.1102) \\
 &= -0.0139
 \end{aligned}$$

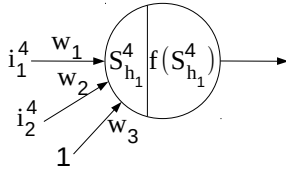
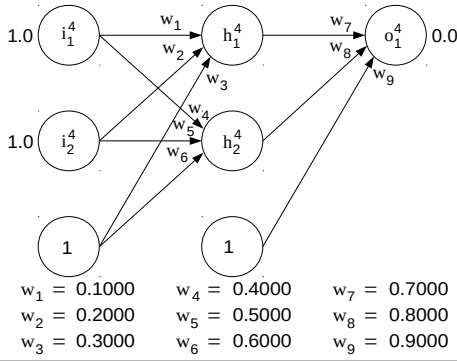


$$\begin{aligned}
 \delta_{h_1}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{h_1}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{h_1}^3)} \frac{\partial f(S_{h_1}^3)}{\partial S_{h_1}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{h_1}^3)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial f(S_{h_1}^3)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^3 w_{h_1 o} \\
 &= \delta_{o_1}^3 w_7 \\
 &= -0.0139(0.7000) \\
 &= -0.0097 \\
 \frac{\partial f(S_{h_1}^3)}{\partial S_{h_1}^3} &= f'(S_{h_1}^3) \\
 &= f(S_{h_1}^3)(1 - f(S_{h_1}^3)) \\
 &= 0.6225(1 - 0.6225) \\
 &= 0.2350 \\
 \delta_{h_1}^3 &= -0.0097(0.2350) \\
 &= -0.0023
 \end{aligned}$$

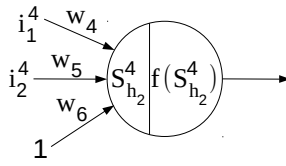


$$\begin{aligned}
 \delta_{h_2}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{h_2}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{h_2}^3)} \frac{\partial f(S_{h_2}^3)}{\partial S_{h_2}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{h_2}^3)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial f(S_{h_2}^3)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^3 w_{h_2 o} \\
 &= \delta_{o_1}^3 w_8 \\
 &= -0.0139(0.8000) \\
 &= -0.0111 \\
 \frac{\partial f(S_{h_2}^3)}{\partial S_{h_2}^3} &= f'(S_{h_2}^3) \\
 &= f(S_{h_2}^3)(1 - f(S_{h_2}^3)) \\
 &= 0.7503(1 - 0.7503) \\
 &= 0.1873 \\
 \delta_{h_2}^3 &= -0.0111(0.1873) \\
 &= -0.0021
 \end{aligned}$$

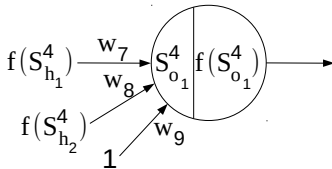
Epoch 1 : Iteration 4



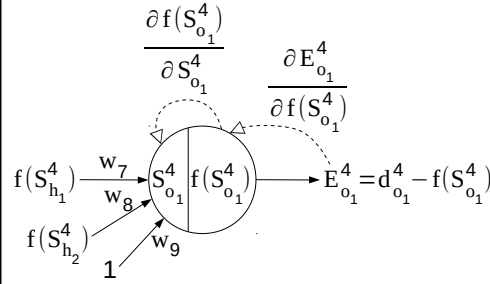
$$\begin{aligned}
 S_{h_1}^4 &= \sum_j w_{jh_1} y_j^4 \\
 &= w_1 i_1^4 + w_2 i_2^4 + w_3 (1) \\
 &= 0.1000(1.0000) + 0.2000(1.0000) + 0.3000(1) \\
 &= 0.6000 \\
 f(S_{h_1}^4) &= \frac{1}{1 + e^{-S_{h_1}^4}} \\
 &= \frac{1}{1 + e^{-0.6000}} \\
 &= 0.6457
 \end{aligned}$$



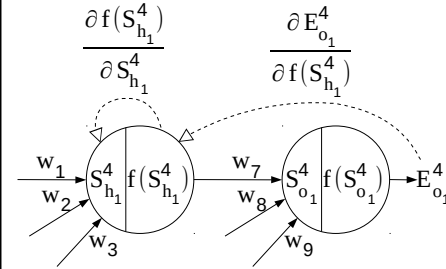
$$\begin{aligned}
 S_{h_2}^4 &= \sum_j w_{jh_2} y_j^4 \\
 &= w_4 i_1^4 + w_5 i_2^4 + w_6 (1) \\
 &= 0.4000(1.0000) + 0.5000(1.0000) + 0.6000(1) \\
 &= 1.5000 \\
 f(S_{h_2}^4) &= \frac{1}{1 + e^{-S_{h_2}^4}} \\
 &= \frac{1}{1 + e^{-1.5000}} \\
 &= 0.8176
 \end{aligned}$$



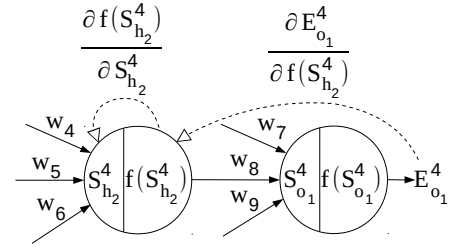
$$\begin{aligned}
 S_{o_1}^4 &= \sum_j w_{jo_1} y_j^4 \\
 &= w_7 f(S_{h_1}^4) + w_8 f(S_{h_2}^4) + w_9 (1) \\
 &= 0.7000(0.6457) + 0.8000(0.8176) + 0.9000(1) \\
 &= 2.0061 \\
 f(S_{o_1}^4) &= \frac{1}{1 + e^{-S_{o_1}^4}} \\
 &= \frac{1}{1 + e^{-2.0061}} \\
 &= 0.8814
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{o_1}^4)} \frac{\partial f(S_{o_1}^4)}{\partial S_{o_1}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{o_1}^4)} &= -(d_{o_1}^4 - f(S_{o_1}^4)) \\
 &= -(0.0000 - 0.8814) \\
 &= -(-0.8814) \\
 &= 0.8814 \\
 \frac{\partial f(S_{o_1}^4)}{\partial S_{o_1}^4} &= f'(S_{o_1}^4) \\
 &= f(S_{o_1}^4)(1 - f(S_{o_1}^4)) \\
 &= 0.8814(1 - 0.8814) \\
 &= 0.1045 \\
 \delta_{o_1}^4 &= 0.8814(0.1045) \\
 &= 0.0921
 \end{aligned}$$

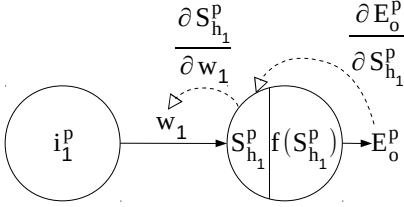


$$\begin{aligned}
 \delta_{h_1}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{h_1}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{h_1}^4)} \frac{\partial f(S_{h_1}^4)}{\partial S_{h_1}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{h_1}^4)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial f(S_{h_1}^4)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^4 w_{h_1 o} \\
 &= \delta_{o_1}^4 w_7 \\
 &= 0.0921(0.7000) \\
 &= 0.0645 \\
 \frac{\partial f(S_{h_1}^4)}{\partial S_{h_1}^4} &= f'(S_{h_1}^4) \\
 &= f(S_{h_1}^4)(1 - f(S_{h_1}^4)) \\
 &= 0.6457(1 - 0.6457) \\
 &= 0.2288 \\
 \delta_{h_1}^4 &= 0.0645(0.2288) \\
 &= 0.0148
 \end{aligned}$$



$$\begin{aligned}
 \delta_{h_2}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{h_2}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{h_2}^4)} \frac{\partial f(S_{h_2}^4)}{\partial S_{h_2}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{h_2}^4)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial f(S_{h_2}^4)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^4 w_{h_2 o} \\
 &= \delta_{o_1}^4 w_8 \\
 &= 0.0921(0.8000) \\
 &= 0.0737 \\
 \frac{\partial f(S_{h_2}^4)}{\partial S_{h_2}^4} &= f'(S_{h_2}^4) \\
 &= f(S_{h_2}^4)(1 - f(S_{h_2}^4)) \\
 &= 0.8176(1 - 0.8176) \\
 &= 0.1491 \\
 \delta_{h_2}^4 &= 0.0737(0.1491) \\
 &= 0.0110
 \end{aligned}$$

$$\begin{aligned}
 SSE &= \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= (d_{o_1}^1 - f(S_{o_1}^1))^2 + (d_{o_1}^2 - f(S_{o_1}^2))^2 + \\
 &\quad (d_{o_1}^3 - f(S_{o_1}^3))^2 + (d_{o_1}^4 - f(S_{o_1}^4))^2 \\
 &= (0.0000 - 0.8604)^2 + (1.0000 - 0.8703)^2 + \\
 &\quad (1.0000 - 0.8739)^2 + (0.0000 - 0.8814)^2 \\
 &= 1.5499 \\
 LSE &= \frac{1}{2} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= \frac{1}{2} \cdot 1.5499 \\
 &= 0.7750 \\
 MSE &= \frac{1}{P \cdot N_o} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= \frac{1}{4(1)} \cdot 1.5499 \\
 &= 0.3875 \\
 RMSE &= \sqrt{MSE} \\
 &= \sqrt{0.3875} \\
 &= 0.6225
 \end{aligned}$$



$$\Delta_1 w_1(2) = -\gamma \frac{\partial E_o^1}{\partial w_1} + \alpha \Delta_1 w_1(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_1} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_1} \\ &= \delta_{h_1}^1 i_1^1 \\ &= 0.0177(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_1(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_1(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_1(2) = -\gamma \frac{\partial E_o^2}{\partial w_1} + \alpha \Delta_2 w_1(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_1} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_1} \\ &= \delta_{h_1}^2 i_1^2 \\ &= -0.0025(1.0000) \\ &= -0.0025 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_1(2) &= -\gamma(-0.0025) + \alpha \Delta_2 w_1(1) \\ &= -0.7(-0.0025) + 0.3(0.0000) \\ &= 0.0018 \end{aligned}$$

$$\Delta_3 w_1(2) = -\gamma \frac{\partial E_o^3}{\partial w_1} + \alpha \Delta_3 w_1(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_1} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_1} \\ &= \delta_{h_1}^3 i_1^3 \\ &= -0.0023(0.0000) \\ &= 0.0000 \end{aligned}$$

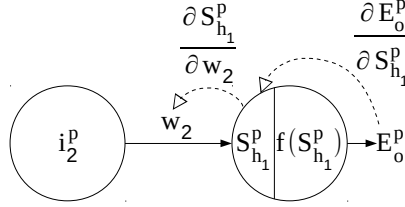
$$\begin{aligned} \Delta_3 w_1(2) &= -\gamma(0.0000) + \alpha \Delta_3 w_1(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_4 w_1(2) = -\gamma \frac{\partial E_o^4}{\partial w_1} + \alpha \Delta_4 w_1(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_1} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_1} \\ &= \delta_{h_1}^4 i_1^4 \\ &= 0.0148(1.0000) \\ &= 0.0148 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_1(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_1(1) \\ &= -0.7(0.0148) + 0.3(0.0000) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_1 &= \left(\sum_{p=1}^P \Delta_p w_1(2) \right) + w_1 \\ &= (\Delta_1 w_1(2) + \Delta_2 w_1(2) + \Delta_3 w_1(2) + \Delta_4 w_1(2)) + w_1 \\ &= (0.0000 + 0.0018 + 0.0000 + (-0.0104)) + 0.1000 \\ &= 0.0914 \end{aligned}$$



$$\Delta_1 w_2(2) = -\gamma \frac{\partial E_o^1}{\partial w_2} + \alpha \Delta_1 w_2(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_2} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_2} \\ &= \delta_{h_1}^1 i_2^1 \\ &= 0.0177(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_2(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_2(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_2(2) = -\gamma \frac{\partial E_o^2}{\partial w_2} + \alpha \Delta_2 w_2(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_2} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_2} \\ &= \delta_{h_1}^2 i_2^2 \\ &= -0.0025(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_2(2) &= -\gamma(0.0000) + \alpha \Delta_2 w_2(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_3 w_2(2) = -\gamma \frac{\partial E_o^3}{\partial w_2} + \alpha \Delta_3 w_2(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_2} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_2} \\ &= \delta_{h_1}^3 i_2^3 \\ &= -0.0023(1.0000) \\ &= -0.0023 \end{aligned}$$

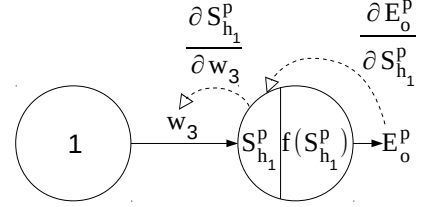
$$\begin{aligned} \Delta_3 w_2(2) &= -\gamma(-0.0023) + \alpha \Delta_3 w_2(1) \\ &= -0.7(-0.0023) + 0.3(0.0000) \\ &= 0.0016 \end{aligned}$$

$$\Delta_4 w_2(2) = -\gamma \frac{\partial E_o^4}{\partial w_2} + \alpha \Delta_4 w_2(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_2} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_2} \\ &= \delta_{h_1}^4 i_2^4 \\ &= 0.0148(1.0000) \\ &= 0.0148 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_2(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_2(1) \\ &= -0.7(0.0148) + 0.3(0.0000) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_2 &= \left(\sum_{p=1}^P \Delta_p w_2(2) \right) + w_2 \\ &= (\Delta_1 w_2(2) + \Delta_2 w_2(2) + \Delta_3 w_2(2) + \Delta_4 w_2(2)) + w_2 \\ &= (0.0000 + 0.0000 + 0.0016 + (-0.0104)) + 0.2000 \\ &= 0.1912 \end{aligned}$$



$$\Delta_1 w_3(2) = -\gamma \frac{\partial E_o^1}{\partial w_3} + \alpha \Delta_1 w_3(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_3} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_3} \\ &= \delta_{h_1}^1(1) \\ &= 0.0177(1.0000) \\ &= 0.0177 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_3(2) &= -\gamma(0.0177) + \alpha \Delta_1 w_3(1) \\ &= -0.7(0.0177) + 0.3(0.0000) \\ &= -0.0124 \end{aligned}$$

$$\Delta_2 w_3(2) = -\gamma \frac{\partial E_o^2}{\partial w_3} + \alpha \Delta_2 w_3(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_3} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_3} \\ &= \delta_{h_1}^2(1) \\ &= -0.0025(1.0000) \\ &= -0.0025 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_3(2) &= -\gamma(-0.0025) + \alpha \Delta_2 w_3(1) \\ &= -0.7(-0.0025) + 0.3(0.0000) \\ &= 0.0018 \end{aligned}$$

$$\Delta_3 w_3(2) = -\gamma \frac{\partial E_o^3}{\partial w_3} + \alpha \Delta_3 w_3(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_3} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_3} \\ &= \delta_{h_1}^3(1) \\ &= -0.0023(1.0000) \\ &= -0.0023 \end{aligned}$$

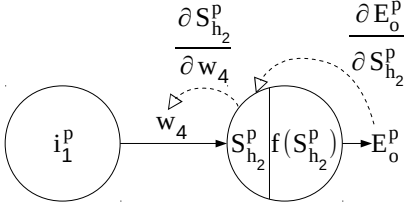
$$\begin{aligned} \Delta_3 w_3(2) &= -\gamma(-0.0023) + \alpha \Delta_3 w_3(1) \\ &= -0.7(-0.0023) + 0.3(0.0000) \\ &= 0.0016 \end{aligned}$$

$$\Delta_4 w_3(2) = -\gamma \frac{\partial E_o^4}{\partial w_3} + \alpha \Delta_4 w_3(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_3} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_3} \\ &= \delta_{h_1}^4(1) \\ &= 0.0148(1.0000) \\ &= 0.0148 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_3(2) &= -\gamma(0.0148) + \alpha \Delta_4 w_3(1) \\ &= -0.7(0.0148) + 0.3(0.0000) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_3 &= \left(\sum_{p=1}^P \Delta_p w_3(2) \right) + w_3 \\ &= (\Delta_1 w_3(2) + \Delta_2 w_3(2) + \Delta_3 w_3(2) + \Delta_4 w_3(2)) + w_3 \\ &= (-0.0124 + 0.0018 + 0.0016 + (-0.0104)) + 0.3000 \\ &= 0.2806 \end{aligned}$$



$$\Delta_1 w_4(2) = -\gamma \frac{\partial E_o^1}{\partial w_4} + \alpha \Delta_1 w_4(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_4} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_4} \\ &= \delta_{h_2}^1 i_1^1 \\ &= 0.0189(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_4(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_4(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_4(2) = -\gamma \frac{\partial E_o^2}{\partial w_4} + \alpha \Delta_2 w_4(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_4} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_4} \\ &= \delta_{h_2}^2 i_1^2 \\ &= -0.0023(1.0000) \\ &= -0.0023 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_4(2) &= -\gamma(-0.0023) + \alpha \Delta_2 w_4(1) \\ &= -0.7(-0.0023) + 0.3(0.0000) \\ &= 0.0016 \end{aligned}$$

$$\Delta_3 w_4(2) = -\gamma \frac{\partial E_o^3}{\partial w_4} + \alpha \Delta_3 w_4(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_4} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_4} \\ &= \delta_{h_2}^3 i_1^3 \\ &= -0.0021(0.0000) \\ &= 0.0000 \end{aligned}$$

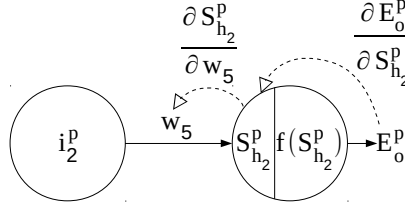
$$\begin{aligned} \Delta_3 w_4(2) &= -\gamma(0.0000) + \alpha \Delta_3 w_4(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_4 w_4(2) = -\gamma \frac{\partial E_o^4}{\partial w_4} + \alpha \Delta_4 w_4(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_4} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_4} \\ &= \delta_{h_2}^4 i_1^4 \\ &= 0.0110(1.0000) \\ &= 0.0110 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_4(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_4(1) \\ &= -0.7(0.0110) + 0.3(0.0000) \\ &= -0.0077 \end{aligned}$$

$$\begin{aligned} w_4 &= \left(\sum_{p=1}^P \Delta_p w_4(2) \right) + w_4 \\ &= (\Delta_1 w_4(2) + \Delta_2 w_4(2) + \Delta_3 w_4(2) + \Delta_4 w_4(2)) + w_4 \\ &= (0.0000 + 0.0016 + 0.0000 + (-0.0077)) + 0.4000 \\ &= 0.3939 \end{aligned}$$



$$\Delta_1 w_5(2) = -\gamma \frac{\partial E_o^1}{\partial w_5} + \alpha \Delta_1 w_5(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_5} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_5} \\ &= \delta_{h_2}^1 i_2^1 \\ &= 0.0189(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_5(2) &= -\gamma(0.0000) + \alpha \Delta_1 w_5(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_5(2) = -\gamma \frac{\partial E_o^2}{\partial w_5} + \alpha \Delta_2 w_5(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_5} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_5} \\ &= \delta_{h_2}^2 i_2^2 \\ &= -0.0023(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_5(2) &= -\gamma(0.0000) + \alpha \Delta_2 w_5(1) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_3 w_5(2) = -\gamma \frac{\partial E_o^3}{\partial w_5} + \alpha \Delta_3 w_5(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_5} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_5} \\ &= \delta_{h_2}^3 i_2^3 \\ &= -0.0021(1.0000) \\ &= -0.0021 \end{aligned}$$

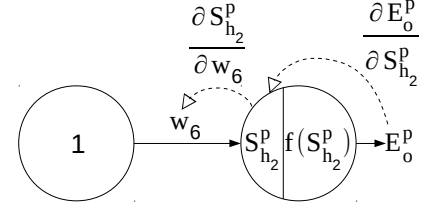
$$\begin{aligned} \Delta_3 w_5(2) &= -\gamma(-0.0021) + \alpha \Delta_3 w_5(1) \\ &= -0.7(-0.0021) + 0.3(0.0000) \\ &= 0.0015 \end{aligned}$$

$$\Delta_4 w_5(2) = -\gamma \frac{\partial E_o^4}{\partial w_5} + \alpha \Delta_4 w_5(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_5} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_5} \\ &= \delta_{h_2}^4 i_2^4 \\ &= 0.0110(1.0000) \\ &= 0.0110 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_5(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_5(1) \\ &= -0.7(0.0110) + 0.3(0.0000) \\ &= -0.0077 \end{aligned}$$

$$\begin{aligned} w_5 &= \left(\sum_{p=1}^P \Delta_p w_5(2) \right) + w_5 \\ &= (\Delta_1 w_5(2) + \Delta_2 w_5(2) + \Delta_3 w_5(2) + \Delta_4 w_5(2)) + w_5 \\ &= (0.0000 + 0.0000 + 0.0015 + (-0.0077)) + 0.5000 \\ &= 0.4938 \end{aligned}$$



$$\Delta_1 w_6(2) = -\gamma \frac{\partial E_o^1}{\partial w_6} + \alpha \Delta_1 w_6(1)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_6} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_6} \\ &= \delta_{h_2}^1(1) \\ &= 0.0189(1.0000) \\ &= 0.0189 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_6(2) &= -\gamma(0.0189) + \alpha \Delta_1 w_6(1) \\ &= -0.7(0.0189) + 0.3(0.0000) \\ &= -0.0132 \end{aligned}$$

$$\Delta_2 w_6(2) = -\gamma \frac{\partial E_o^2}{\partial w_6} + \alpha \Delta_2 w_6(1)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_6} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_6} \\ &= \delta_{h_2}^2(1) \\ &= -0.0023(1.0000) \\ &= -0.0023 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_6(2) &= -\gamma(-0.0023) + \alpha \Delta_2 w_6(1) \\ &= -0.7(-0.0023) + 0.3(0.0000) \\ &= 0.0016 \end{aligned}$$

$$\Delta_3 w_6(2) = -\gamma \frac{\partial E_o^3}{\partial w_6} + \alpha \Delta_3 w_6(1)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_6} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_6} \\ &= \delta_{h_2}^3(1) \\ &= -0.0021(1.0000) \\ &= -0.0021 \end{aligned}$$

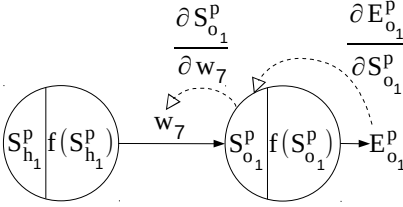
$$\begin{aligned} \Delta_3 w_6(2) &= -\gamma(-0.0021) + \alpha \Delta_3 w_6(1) \\ &= -0.7(-0.0021) + 0.3(0.0000) \\ &= 0.0015 \end{aligned}$$

$$\Delta_4 w_6(2) = -\gamma \frac{\partial E_o^4}{\partial w_6} + \alpha \Delta_4 w_6(1)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_6} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_6} \\ &= \delta_{h_2}^4(1) \\ &= 0.0110(1.0000) \\ &= 0.0110 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_6(2) &= -\gamma(0.0110) + \alpha \Delta_4 w_6(1) \\ &= -0.7(0.0110) + 0.3(0.0000) \\ &= -0.0077 \end{aligned}$$

$$\begin{aligned} w_6 &= \left(\sum_{p=1}^P \Delta_p w_6(2) \right) + w_6 \\ &= (\Delta_1 w_6(2) + \Delta_2 w_6(2) + \Delta_3 w_6(2) + \Delta_4 w_6(2)) + w_6 \\ &= (-0.0132 + 0.0016 + 0.0015 + (-0.0077)) + 0.6000 \\ &= 0.5822 \end{aligned}$$



$$\Delta_1 w_7(2) = -\gamma \frac{\partial E_{o1}^1}{\partial w_7} + \alpha \Delta_1 w_7(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^1}{\partial w_7} &= \frac{\partial E_{o1}^1}{\partial S_{o1}^1} \frac{\partial S_{o1}^1}{\partial w_7} \\ &= \delta_{o1}^1 f(S_{h1}^1) \\ &= 0.1033(0.5744) \\ &= 0.0593 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_7(2) &= -\gamma(0.0593) + \alpha \Delta_1 w_7(1) \\ &= -0.7(0.0593) + 0.3(0.0000) \\ &= -0.0415 \end{aligned}$$

$$\Delta_2 w_7(2) = -\gamma \frac{\partial E_{o1}^2}{\partial w_7} + \alpha \Delta_2 w_7(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^2}{\partial w_7} &= \frac{\partial E_{o1}^2}{\partial S_{o1}^2} \frac{\partial S_{o1}^2}{\partial w_7} \\ &= \delta_{o1}^2 f(S_{h1}^2) \\ &= -0.0146(0.5987) \\ &= -0.0087 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_7(2) &= -\gamma(-0.0087) + \alpha \Delta_2 w_7(1) \\ &= -0.7(-0.0087) + 0.3(0.0000) \\ &= 0.0061 \end{aligned}$$

$$\Delta_3 w_7(2) = -\gamma \frac{\partial E_{o1}^3}{\partial w_7} + \alpha \Delta_3 w_7(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^3}{\partial w_7} &= \frac{\partial E_{o1}^3}{\partial S_{o1}^3} \frac{\partial S_{o1}^3}{\partial w_7} \\ &= \delta_{o1}^3 f(S_{h1}^3) \\ &= -0.0139(0.6225) \\ &= -0.0087 \end{aligned}$$

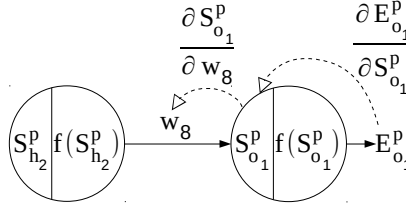
$$\begin{aligned} \Delta_3 w_7(2) &= -\gamma(-0.0087) + \alpha \Delta_3 w_7(1) \\ &= -0.7(-0.0087) + 0.3(0.0000) \\ &= 0.0061 \end{aligned}$$

$$\Delta_4 w_7(2) = -\gamma \frac{\partial E_{o1}^4}{\partial w_7} + \alpha \Delta_4 w_7(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^4}{\partial w_7} &= \frac{\partial E_{o1}^4}{\partial S_{o1}^4} \frac{\partial S_{o1}^4}{\partial w_7} \\ &= \delta_{o1}^4 f(S_{h1}^4) \\ &= 0.0921(0.6457) \\ &= 0.0595 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_7(2) &= -\gamma(0.0595) + \alpha \Delta_4 w_7(1) \\ &= -0.7(0.0595) + 0.3(0.0000) \\ &= -0.0417 \end{aligned}$$

$$\begin{aligned} w_7 &= \left(\sum_{p=1}^P \Delta_p w_7(2) \right) + w_7 \\ &= (\Delta_1 w_7(2) + \Delta_2 w_7(2) + \Delta_3 w_7(2) + \Delta_4 w_7(2)) + w_7 \\ &= (-0.0415 + 0.0061 + 0.0061 + (-0.0417)) + 0.7000 \\ &= 0.6290 \end{aligned}$$



$$\Delta_1 w_8(2) = -\gamma \frac{\partial E_{o1}^1}{\partial w_8} + \alpha \Delta_1 w_8(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^1}{\partial w_8} &= \frac{\partial E_{o1}^1}{\partial S_{o1}^1} \frac{\partial S_{o1}^1}{\partial w_8} \\ &= \delta_{o1}^1 f(S_{h2}^1) \\ &= 0.1033(0.6457) \\ &= 0.0667 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_8(2) &= -\gamma(0.0667) + \alpha \Delta_1 w_8(1) \\ &= -0.7(0.0667) + 0.3(0.0000) \\ &= -0.0467 \end{aligned}$$

$$\Delta_2 w_8(2) = -\gamma \frac{\partial E_{o1}^2}{\partial w_8} + \alpha \Delta_2 w_8(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^2}{\partial w_8} &= \frac{\partial E_{o1}^2}{\partial S_{o1}^2} \frac{\partial S_{o1}^2}{\partial w_8} \\ &= \delta_{o1}^2 f(S_{h2}^2) \\ &= -0.0146(0.7311) \\ &= -0.0107 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_8(2) &= -\gamma(-0.0107) + \alpha \Delta_2 w_8(1) \\ &= -0.7(-0.0107) + 0.3(0.0000) \\ &= 0.0075 \end{aligned}$$

$$\Delta_3 w_8(2) = -\gamma \frac{\partial E_{o1}^3}{\partial w_8} + \alpha \Delta_3 w_8(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^3}{\partial w_8} &= \frac{\partial E_{o1}^3}{\partial S_{o1}^3} \frac{\partial S_{o1}^3}{\partial w_8} \\ &= \delta_{o1}^3 f(S_{h2}^3) \\ &= -0.0139(0.7503) \\ &= -0.0104 \end{aligned}$$

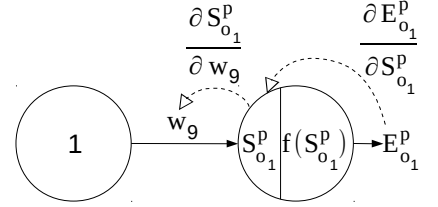
$$\begin{aligned} \Delta_3 w_8(2) &= -\gamma(-0.0104) + \alpha \Delta_3 w_8(1) \\ &= -0.7(-0.0104) + 0.3(0.0000) \\ &= 0.0073 \end{aligned}$$

$$\Delta_4 w_8(2) = -\gamma \frac{\partial E_{o1}^4}{\partial w_8} + \alpha \Delta_4 w_8(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^4}{\partial w_8} &= \frac{\partial E_{o1}^4}{\partial S_{o1}^4} \frac{\partial S_{o1}^4}{\partial w_8} \\ &= \delta_{o1}^4 f(S_{h2}^4) \\ &= 0.0921(0.8176) \\ &= 0.0753 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_8(2) &= -\gamma(0.0753) + \alpha \Delta_4 w_8(1) \\ &= -0.7(0.0753) + 0.3(0.0000) \\ &= -0.0527 \end{aligned}$$

$$\begin{aligned} w_8 &= \left(\sum_{p=1}^P \Delta_p w_8(2) \right) + w_8 \\ &= (\Delta_1 w_8(2) + \Delta_2 w_8(2) + \Delta_3 w_8(2) + \Delta_4 w_8(2)) + w_8 \\ &= (-0.0467 + 0.0075 + 0.0073 + (-0.0527)) + 0.8000 \\ &= 0.7154 \end{aligned}$$



$$\Delta_1 w_9(2) = -\gamma \frac{\partial E_{o1}^1}{\partial w_9} + \alpha \Delta_1 w_9(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^1}{\partial w_9} &= \frac{\partial E_{o1}^1}{\partial S_{o1}^1} \frac{\partial S_{o1}^1}{\partial w_9} \\ &= \delta_{o1}^1 f(1) \\ &= 0.1033(1.0000) \\ &= 0.1033 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_9(2) &= -\gamma(0.1033) + \alpha \Delta_1 w_9(1) \\ &= -0.7(0.1033) + 0.3(0.0000) \\ &= -0.0723 \end{aligned}$$

$$\Delta_2 w_9(2) = -\gamma \frac{\partial E_{o1}^2}{\partial w_9} + \alpha \Delta_2 w_9(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^2}{\partial w_9} &= \frac{\partial E_{o1}^2}{\partial S_{o1}^2} \frac{\partial S_{o1}^2}{\partial w_9} \\ &= \delta_{o1}^2 f(1) \\ &= -0.0146(1.0000) \\ &= -0.0146 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_9(2) &= -\gamma(-0.0146) + \alpha \Delta_2 w_9(1) \\ &= -0.7(-0.0146) + 0.3(0.0000) \\ &= 0.0102 \end{aligned}$$

$$\Delta_3 w_9(2) = -\gamma \frac{\partial E_{o1}^3}{\partial w_9} + \alpha \Delta_3 w_9(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^3}{\partial w_9} &= \frac{\partial E_{o1}^3}{\partial S_{o1}^3} \frac{\partial S_{o1}^3}{\partial w_9} \\ &= \delta_{o1}^3 f(1) \\ &= -0.0139(1.0000) \\ &= -0.0139 \end{aligned}$$

$$\begin{aligned} \Delta_3 w_9(2) &= -\gamma(-0.0139) + \alpha \Delta_3 w_9(1) \\ &= -0.7(-0.0139) + 0.3(0.0000) \\ &= 0.0097 \end{aligned}$$

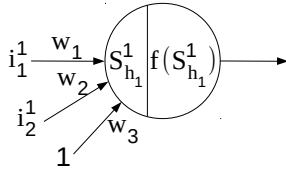
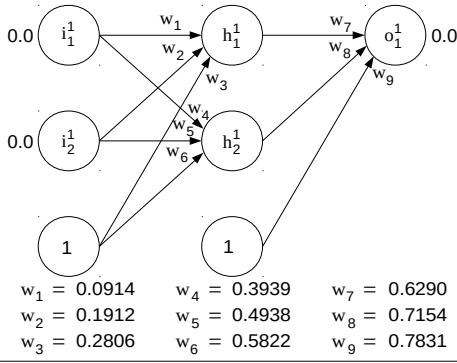
$$\Delta_4 w_9(2) = -\gamma \frac{\partial E_{o1}^4}{\partial w_9} + \alpha \Delta_4 w_9(1)$$

$$\begin{aligned} \frac{\partial E_{o1}^4}{\partial w_9} &= \frac{\partial E_{o1}^4}{\partial S_{o1}^4} \frac{\partial S_{o1}^4}{\partial w_9} \\ &= \delta_{o1}^4 f(1) \\ &= 0.0921(1.0000) \\ &= 0.0921 \end{aligned}$$

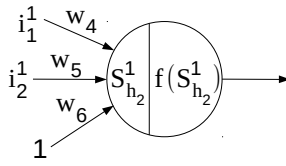
$$\begin{aligned} \Delta_4 w_9(2) &= -\gamma(0.0921) + \alpha \Delta_4 w_9(1) \\ &= -0.7(0.0921) + 0.3(0.0000) \\ &= -0.0645 \end{aligned}$$

$$\begin{aligned} w_9 &= \left(\sum_{p=1}^P \Delta_p w_9(2) \right) + w_9 \\ &= (\Delta_1 w_9(2) + \Delta_2 w_9(2) + \Delta_3 w_9(2) + \Delta_4 w_9(2)) + w_9 \\ &= (-0.0723 + 0.0102 + 0.0097 + (-0.0645)) + 0.9000 \\ &= 0.7831 \end{aligned}$$

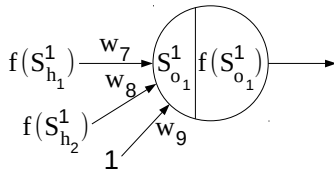
Epoch 2 : Iteration 1



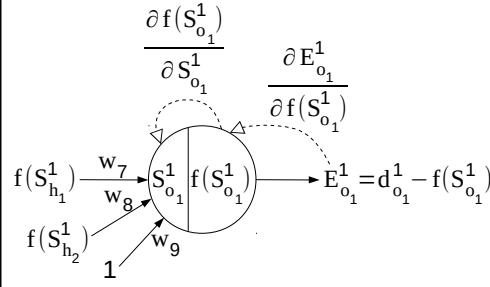
$$\begin{aligned}
 S_{h_1}^1 &= \sum_j w_{jh_1} y_j^1 \\
 &= w_1 i_1^1 + w_2 i_2^1 + w_3 (1) \\
 &= 0.0914(0.0000) + 0.1912(0.0000) + 0.2806(1) \\
 &= 0.2806 \\
 f(S_{h_1}^1) &= \frac{1}{1 + e^{-S_{h_1}^1}} \\
 &= \frac{1}{1 + e^{-0.2806}} \\
 &= 0.5697
 \end{aligned}$$



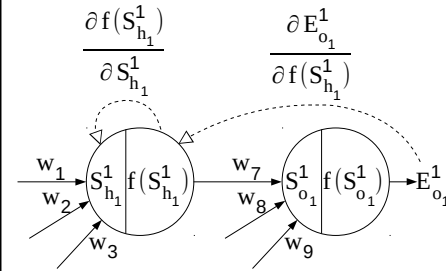
$$\begin{aligned}
 S_{h_2}^1 &= \sum_j w_{jh_2} y_j^1 \\
 &= w_4 i_1^1 + w_5 i_2^1 + w_6 (1) \\
 &= 0.3939(0.0000) + 0.4938(0.0000) + 0.5822(1) \\
 &= 0.5822 \\
 f(S_{h_2}^1) &= \frac{1}{1 + e^{-S_{h_2}^1}} \\
 &= \frac{1}{1 + e^{-0.5822}} \\
 &= 0.6416
 \end{aligned}$$



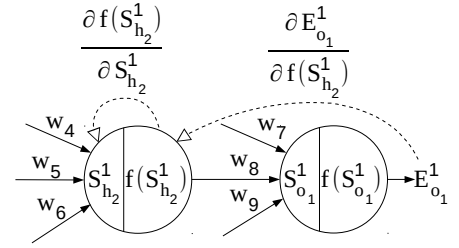
$$\begin{aligned}
 S_{o_1}^1 &= \sum_j w_{jo_1} y_j^1 \\
 &= w_7 f(S_{h_1}^1) + w_8 f(S_{h_2}^1) + w_9 (1) \\
 &= 0.6290(0.5697) + 0.7154(0.6416) + 0.7831(1) \\
 &= 1.6004 \\
 f(S_{o_1}^1) &= \frac{1}{1 + e^{-S_{o_1}^1}} \\
 &= \frac{1}{1 + e^{-1.6004}} \\
 &= 0.8321
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{o_1}^1)} &= -(d_{o_1}^1 - f(S_{o_1}^1)) \\
 &= -(0.0000 - 0.8321) \\
 &= -(-0.8321) \\
 &= 0.8321 \\
 \frac{\partial f(S_{o_1}^1)}{\partial S_{o_1}^1} &= f'(S_{o_1}^1) \\
 &= f(S_{o_1}^1)(1 - f(S_{o_1}^1)) \\
 &= 0.8321(1 - 0.8321) \\
 &= 0.1397 \\
 \delta_{o_1}^1 &= 0.8321(0.1397) \\
 &= 0.1162
 \end{aligned}$$

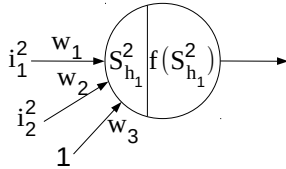
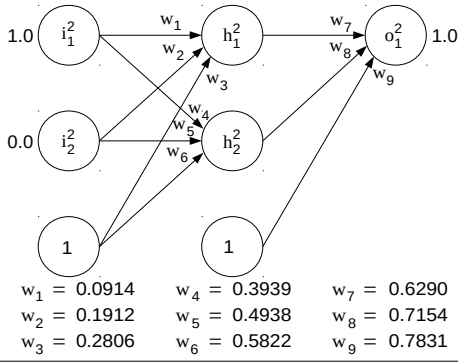


$$\begin{aligned}
 \delta_{h_1}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{h_1}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{h_1}^1)} \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{h_1}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial f(S_{h_1}^1)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^1 w_{h_1 o} \\
 &= \delta_{o_1}^1 w_7 \\
 &= 0.1162(0.6290) \\
 &= 0.0731 \\
 \frac{\partial f(S_{h_1}^1)}{\partial S_{h_1}^1} &= f'(S_{h_1}^1) \\
 &= f(S_{h_1}^1)(1 - f(S_{h_1}^1)) \\
 &= 0.5697(1 - 0.5697) \\
 &= 0.2451 \\
 \delta_{h_1}^1 &= 0.0731(0.2451) \\
 &= 0.0179
 \end{aligned}$$

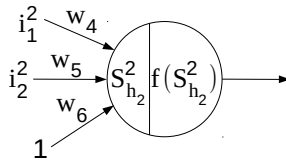


$$\begin{aligned}
 \delta_{h_2}^1 &= \frac{\partial E_{o_1}^1}{\partial S_{h_2}^1} \\
 &= \frac{\partial E_{o_1}^1}{\partial f(S_{h_2}^1)} \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} \\
 \frac{\partial E_{o_1}^1}{\partial f(S_{h_2}^1)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial f(S_{h_2}^1)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^1 w_{h_2 o} \\
 &= \delta_{o_1}^1 w_8 \\
 &= 0.1162(0.7154) \\
 &= 0.0831 \\
 \frac{\partial f(S_{h_2}^1)}{\partial S_{h_2}^1} &= f'(S_{h_2}^1) \\
 &= f(S_{h_2}^1)(1 - f(S_{h_2}^1)) \\
 &= 0.6416(1 - 0.6416) \\
 &= 0.2299 \\
 \delta_{h_2}^1 &= 0.0831(0.2299) \\
 &= 0.0191
 \end{aligned}$$

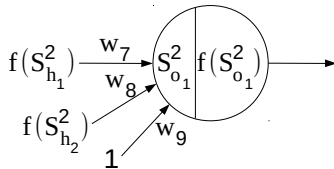
Epoch 2 : Iteration 2



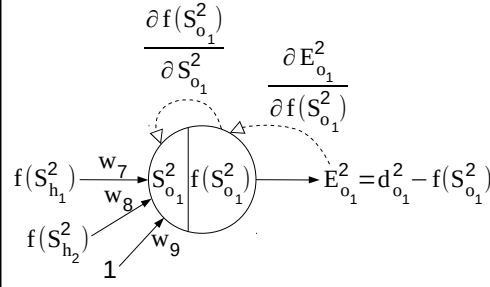
$$\begin{aligned}
 S_{h_1}^2 &= \sum_j w_{jh_1} y_j^2 \\
 &= w_1 i_1^2 + w_2 i_2^2 + w_3 (1) \\
 &= 0.0914(1.0000) + 0.1912(0.0000) + 0.2806(1) \\
 &= 0.3720 \\
 f(S_{h_1}^2) &= \frac{1}{1 + e^{-S_{h_1}^2}} \\
 &= \frac{1}{1 + e^{-0.3720}} \\
 &= 0.5919
 \end{aligned}$$



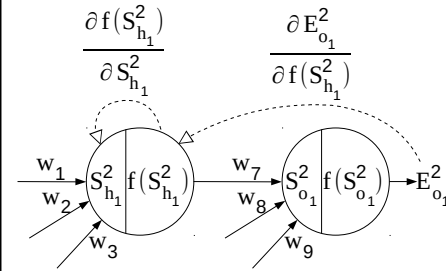
$$\begin{aligned}
 S_{h_2}^2 &= \sum_j w_{jh_2} y_j^2 \\
 &= w_4 i_1^2 + w_5 i_2^2 + w_6 (1) \\
 &= 0.3939(1.0000) + 0.4938(0.0000) + 0.5822(1) \\
 &= 0.9761 \\
 f(S_{h_2}^2) &= \frac{1}{1 + e^{-S_{h_2}^2}} \\
 &= \frac{1}{1 + e^{-0.9761}} \\
 &= 0.7263
 \end{aligned}$$



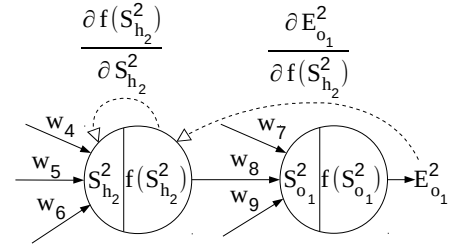
$$\begin{aligned}
 S_{o_1}^2 &= \sum_j w_{jo_1} y_j^2 \\
 &= w_7 f(S_{h_1}^2) + w_8 f(S_{h_2}^2) + w_9 (1) \\
 &= 0.6290(0.5919) + 0.7154(0.7263) + 0.7831(1) \\
 &= 1.6750 \\
 f(S_{o_1}^2) &= \frac{1}{1 + e^{-S_{o_1}^2}} \\
 &= \frac{1}{1 + e^{-1.6750}} \\
 &= 0.8422
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} \frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{o_1}^2)} &= -(d_{o_1}^2 - f(S_{o_1}^2)) \\
 &= -(1.0000 - 0.8422) \\
 &= -(0.1578) \\
 &= -0.1578 \\
 \frac{\partial f(S_{o_1}^2)}{\partial S_{o_1}^2} &= f'(S_{o_1}^2) \\
 &= f(S_{o_1}^2)(1 - f(S_{o_1}^2)) \\
 &= 0.8422(1 - 0.8422) \\
 &= 0.1329 \\
 \delta_{o_1}^2 &= -0.1578(0.1329) \\
 &= -0.0210
 \end{aligned}$$

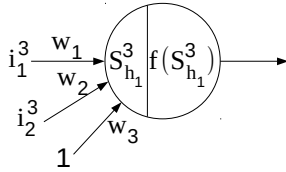
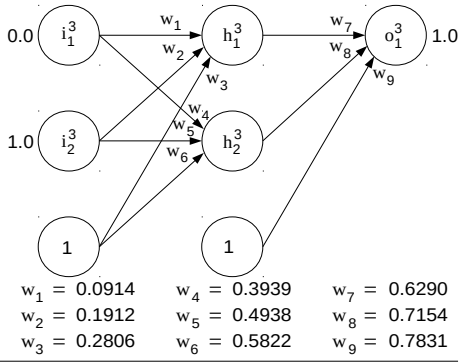


$$\begin{aligned}
 \delta_{h_1}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{h_1}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{h_1}^2)} \frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{h_1}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial f(S_{h_1}^2)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^2 w_{h_1 o} \\
 &= \delta_{o_1}^2 w_7 \\
 &= -0.0210(0.6290) \\
 &= -0.0132 \\
 \frac{\partial f(S_{h_1}^2)}{\partial S_{h_1}^2} &= f'(S_{h_1}^2) \\
 &= f(S_{h_1}^2)(1 - f(S_{h_1}^2)) \\
 &= 0.5919(1 - 0.5919) \\
 &= 0.2416 \\
 \delta_{h_1}^2 &= -0.0132(0.2416) \\
 &= -0.0032
 \end{aligned}$$

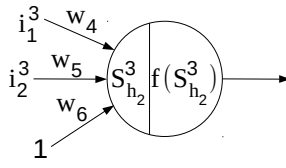


$$\begin{aligned}
 \delta_{h_2}^2 &= \frac{\partial E_{o_1}^2}{\partial S_{h_2}^2} \\
 &= \frac{\partial E_{o_1}^2}{\partial f(S_{h_2}^2)} \frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2} \\
 \frac{\partial E_{o_1}^2}{\partial f(S_{h_2}^2)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial f(S_{h_2}^2)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^2 w_{h_2 o} \\
 &= \delta_{o_1}^2 w_8 \\
 &= -0.0210(0.7154) \\
 &= -0.0150 \\
 \frac{\partial f(S_{h_2}^2)}{\partial S_{h_2}^2} &= f'(S_{h_2}^2) \\
 &= f(S_{h_2}^2)(1 - f(S_{h_2}^2)) \\
 &= 0.7263(1 - 0.7263) \\
 &= 0.1988 \\
 \delta_{h_2}^2 &= -0.0150(0.1988) \\
 &= -0.0030
 \end{aligned}$$

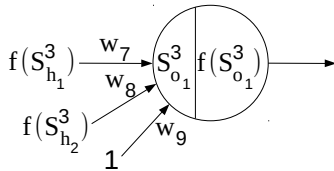
Epoch 2 : Iteration 3



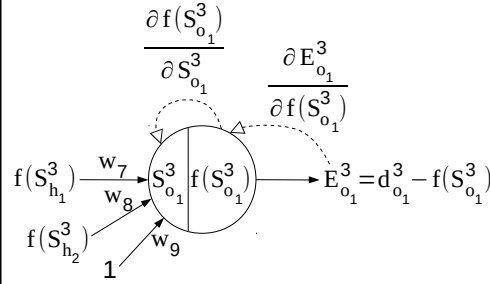
$$\begin{aligned}
 S_{h_1}^3 &= \sum_j w_{jh_1} y_j^3 \\
 &= w_1 i_1^3 + w_2 i_2^3 + w_3 (1) \\
 &= 0.0914(0.0000) + 0.1912(1.0000) + 0.2806(1) \\
 &= 0.4718 \\
 f(S_{h_1}^3) &= \frac{1}{1 + e^{-S_{h_1}^3}} \\
 &= \frac{1}{1 + e^{-0.4718}} \\
 &= 0.6158
 \end{aligned}$$



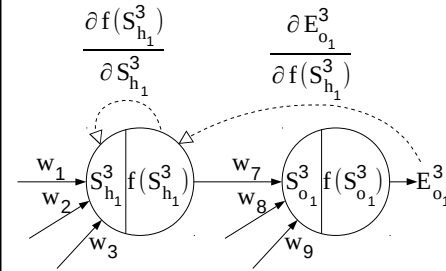
$$\begin{aligned}
 S_{h_2}^3 &= \sum_j w_{jh_2} y_j^3 \\
 &= w_4 i_1^3 + w_5 i_2^3 + w_6 (1) \\
 &= 0.3939(0.0000) + 0.4938(1.0000) + 0.5822(1) \\
 &= 1.0760 \\
 f(S_{h_2}^3) &= \frac{1}{1 + e^{-S_{h_2}^3}} \\
 &= \frac{1}{1 + e^{-1.0760}} \\
 &= 0.7457
 \end{aligned}$$



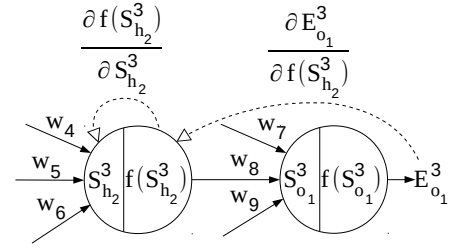
$$\begin{aligned}
 S_{o_1}^3 &= \sum_j w_{jo_1} y_j^3 \\
 &= w_7 f(S_{h_1}^3) + w_8 f(S_{h_2}^3) + w_9 (1) \\
 &= 0.6290(0.6158) + 0.7154(0.7457) + 0.7831(1) \\
 &= 1.7039 \\
 f(S_{o_1}^3) &= \frac{1}{1 + e^{-S_{o_1}^3}} \\
 &= \frac{1}{1 + e^{-1.7039}} \\
 &= 0.8460
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{o_1}^3)} \frac{\partial f(S_{o_1}^3)}{\partial S_{o_1}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{o_1}^3)} &= -(d_{o_1}^3 - f(S_{o_1}^3)) \\
 &= -(1.0000 - 0.8460) \\
 &= -(0.1540) \\
 &= -0.1540 \\
 \frac{\partial f(S_{o_1}^3)}{\partial S_{o_1}^3} &= f'(S_{o_1}^3) \\
 &= f(S_{o_1}^3)(1 - f(S_{o_1}^3)) \\
 &= 0.8460(1 - 0.8460) \\
 &= 0.1303 \\
 \delta_{o_1}^3 &= -0.1540(0.1303) \\
 &= -0.0201
 \end{aligned}$$

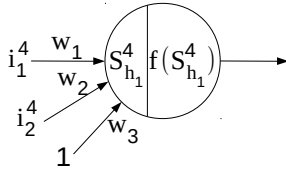
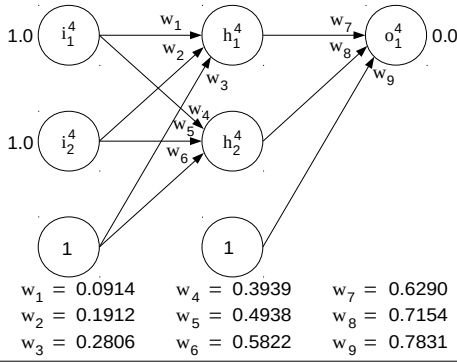


$$\begin{aligned}
 \delta_{h_1}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{h_1}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{h_1}^3)} \frac{\partial f(S_{h_1}^3)}{\partial S_{h_1}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{h_1}^3)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial f(S_{h_1}^3)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^3 w_{h_1 o} \\
 &= \delta_{o_1}^3 w_7 \\
 &= -0.0201(0.6290) \\
 &= -0.0126 \\
 \frac{\partial f(S_{h_1}^3)}{\partial S_{h_1}^3} &= f'(S_{h_1}^3) \\
 &= f(S_{h_1}^3)(1 - f(S_{h_1}^3)) \\
 &= 0.6158(1 - 0.6158) \\
 &= 0.2366 \\
 \delta_{h_1}^3 &= -0.0126(0.2366) \\
 &= -0.0030
 \end{aligned}$$

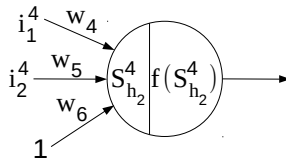


$$\begin{aligned}
 \delta_{h_2}^3 &= \frac{\partial E_{o_1}^3}{\partial S_{h_2}^3} \\
 &= \frac{\partial E_{o_1}^3}{\partial f(S_{h_2}^3)} \frac{\partial f(S_{h_2}^3)}{\partial S_{h_2}^3} \\
 \frac{\partial E_{o_1}^3}{\partial f(S_{h_2}^3)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial f(S_{h_2}^3)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^3 w_{h_2 o} \\
 &= \delta_{o_1}^3 w_8 \\
 &= -0.0201(0.7154) \\
 &= -0.0144 \\
 \frac{\partial f(S_{h_2}^3)}{\partial S_{h_2}^3} &= f'(S_{h_2}^3) \\
 &= f(S_{h_2}^3)(1 - f(S_{h_2}^3)) \\
 &= 0.7457(1 - 0.7457) \\
 &= 0.1896 \\
 \delta_{h_2}^3 &= -0.0144(0.1896) \\
 &= -0.0027
 \end{aligned}$$

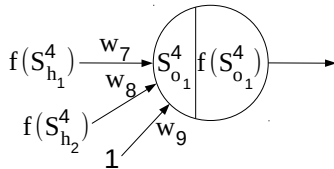
Epoch 2 : Iteration 4



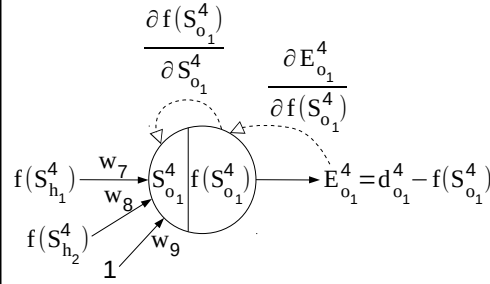
$$\begin{aligned}
 S_{h_1}^4 &= \sum_j w_{jh_1} y_j^4 \\
 &= w_1 i_1^4 + w_2 i_2^4 + w_3 (1) \\
 &= 0.0914(1.0000) + 0.1912(1.0000) + 0.2806(1) \\
 &= 0.5632 \\
 f(S_{h_1}^4) &= \frac{1}{1 + e^{-S_{h_1}^4}} \\
 &= \frac{1}{1 + e^{-0.5632}} \\
 &= 0.6372
 \end{aligned}$$



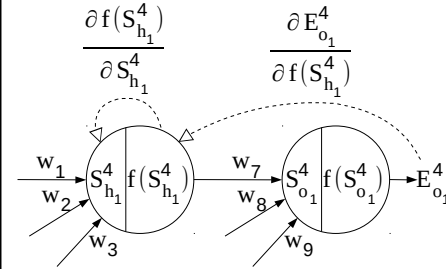
$$\begin{aligned}
 S_{h_2}^4 &= \sum_j w_{jh_2} y_j^4 \\
 &= w_4 i_1^4 + w_5 i_2^4 + w_6 (1) \\
 &= 0.3939(1.0000) + 0.4938(1.0000) + 0.5822(1) \\
 &= 1.4699 \\
 f(S_{h_2}^4) &= \frac{1}{1 + e^{-S_{h_2}^4}} \\
 &= \frac{1}{1 + e^{-1.4699}} \\
 &= 0.8130
 \end{aligned}$$



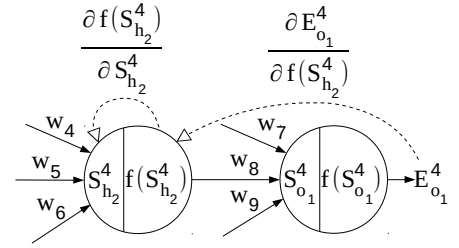
$$\begin{aligned}
 S_{o_1}^4 &= \sum_j w_{jo_1} y_j^4 \\
 &= w_7 f(S_{h_1}^4) + w_8 f(S_{h_2}^4) + w_9 (1) \\
 &= 0.6290(0.6372) + 0.7154(0.8130) + 0.7831(1) \\
 &= 1.7655 \\
 f(S_{o_1}^4) &= \frac{1}{1 + e^{-S_{o_1}^4}} \\
 &= \frac{1}{1 + e^{-1.7655}} \\
 &= 0.8539
 \end{aligned}$$



$$\begin{aligned}
 \delta_{o_1}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{o_1}^4)} \frac{\partial f(S_{o_1}^4)}{\partial S_{o_1}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{o_1}^4)} &= -(d_{o_1}^4 - f(S_{o_1}^4)) \\
 &= -(0.0000 - 0.8539) \\
 &= -(-0.8539) \\
 &= 0.8539 \\
 \frac{\partial f(S_{o_1}^4)}{\partial S_{o_1}^4} &= f'(S_{o_1}^4) \\
 &= f(S_{o_1}^4)(1 - f(S_{o_1}^4)) \\
 &= 0.8539(1 - 0.8539) \\
 &= 0.1248 \\
 \delta_{o_1}^4 &= 0.8539(0.1248) \\
 &= 0.1066
 \end{aligned}$$

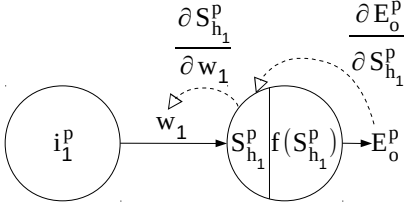


$$\begin{aligned}
 \delta_{h_1}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{h_1}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{h_1}^4)} \frac{\partial f(S_{h_1}^4)}{\partial S_{h_1}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{h_1}^4)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial f(S_{h_1}^4)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^4 w_{h_1 o} \\
 &= \delta_{o_1}^4 w_7 \\
 &= 0.1066(0.6290) \\
 &= 0.0671 \\
 \frac{\partial f(S_{h_1}^4)}{\partial S_{h_1}^4} &= f'(S_{h_1}^4) \\
 &= f(S_{h_1}^4)(1 - f(S_{h_1}^4)) \\
 &= 0.6372(1 - 0.6372) \\
 &= 0.2312 \\
 \delta_{h_1}^4 &= 0.0671(0.2312) \\
 &= 0.0155
 \end{aligned}$$



$$\begin{aligned}
 \delta_{h_2}^4 &= \frac{\partial E_{o_1}^4}{\partial S_{h_2}^4} \\
 &= \frac{\partial E_{o_1}^4}{\partial f(S_{h_2}^4)} \frac{\partial f(S_{h_2}^4)}{\partial S_{h_2}^4} \\
 \frac{\partial E_{o_1}^4}{\partial f(S_{h_2}^4)} &= \sum_{o=1}^{N_o} \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial f(S_{h_2}^4)} \\
 &= \sum_{o=1}^{N_o} \delta_{o_1}^4 w_{h_2 o} \\
 &= \delta_{o_1}^4 w_8 \\
 &= 0.1066(0.7154) \\
 &= 0.0763 \\
 \frac{\partial f(S_{h_2}^4)}{\partial S_{h_2}^4} &= f'(S_{h_2}^4) \\
 &= f(S_{h_2}^4)(1 - f(S_{h_2}^4)) \\
 &= 0.8130(1 - 0.8130) \\
 &= 0.1520 \\
 \delta_{h_2}^4 &= 0.0763(0.1520) \\
 &= 0.0116
 \end{aligned}$$

$$\begin{aligned}
 SSE &= \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= (d_{o_1}^1 - f(S_{o_1}^1))^2 + (d_{o_1}^2 - f(S_{o_1}^2))^2 + \\
 &\quad (d_{o_1}^3 - f(S_{o_1}^3))^2 + (d_{o_1}^4 - f(S_{o_1}^4))^2 \\
 &= (0.0000 - 0.8321)^2 + (1.0000 - 0.8422)^2 + \\
 &\quad (1.0000 - 0.8460)^2 + (0.0000 - 0.8539)^2 \\
 &= 1.4702 \\
 LSE &= \frac{1}{2} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= \frac{1}{2} \cdot 1.4702 \\
 &= 0.7351 \\
 MSE &= \frac{1}{P \cdot N_o} \sum_{p=1}^P \sum_{o=1}^{N_o} (d_o^p - f(S_o^p))^2 \\
 &= \frac{1}{4(1)} \cdot 1.4702 \\
 &= 0.3676 \\
 RMSE &= \sqrt{MSE} \\
 &= \sqrt{0.3676} \\
 &= 0.6063
 \end{aligned}$$



$$\Delta_1 w_1(3) = -\gamma \frac{\partial E_o^1}{\partial w_1} + \alpha \Delta_1 w_1(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_1} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_1} \\ &= \delta_{h_1}^1 i_1^1 \\ &= 0.0179(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_1(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_1(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_1(3) = -\gamma \frac{\partial E_o^2}{\partial w_1} + \alpha \Delta_2 w_1(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_1} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_1} \\ &= \delta_{h_1}^2 i_1^2 \\ &= -0.0032(1.0000) \\ &= -0.0032 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_1(3) &= -\gamma(-0.0032) + \alpha \Delta_2 w_1(2) \\ &= -0.7(-0.0032) + 0.3(0.0018) \\ &= 0.0028 \end{aligned}$$

$$\Delta_3 w_1(3) = -\gamma \frac{\partial E_o^3}{\partial w_1} + \alpha \Delta_3 w_1(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_1} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_1} \\ &= \delta_{h_1}^3 i_1^3 \\ &= -0.0030(0.0000) \\ &= 0.0000 \end{aligned}$$

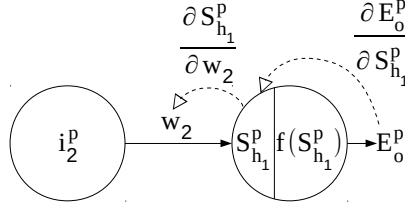
$$\begin{aligned} \Delta_3 w_1(3) &= -\gamma(0.0000) + \alpha \Delta_3 w_1(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_4 w_1(3) = -\gamma \frac{\partial E_o^4}{\partial w_1} + \alpha \Delta_4 w_1(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_1} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_1} \\ &= \delta_{h_1}^4 i_1^4 \\ &= 0.0155(1.0000) \\ &= 0.0155 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_1(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_1(2) \\ &= -0.7(0.0155) + 0.3(-0.0104) \\ &= -0.0140 \end{aligned}$$

$$\begin{aligned} w_1 &= \left(\sum_{p=1}^P \Delta_p w_1(3) \right) + w_1 \\ &= (\Delta_1 w_1(3) + \Delta_2 w_1(3) + \Delta_3 w_1(3) + \Delta_4 w_1(3)) + w_1 \\ &= (0.0000 + 0.0028 + 0.0000 + (-0.0140)) + 0.0914 \\ &= 0.0802 \end{aligned}$$



$$\Delta_1 w_2(3) = -\gamma \frac{\partial E_o^1}{\partial w_2} + \alpha \Delta_1 w_2(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_2} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_2} \\ &= \delta_{h_1}^1 i_2^1 \\ &= 0.0179(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_2(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_2(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_2(3) = -\gamma \frac{\partial E_o^2}{\partial w_2} + \alpha \Delta_2 w_2(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_2} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_2} \\ &= \delta_{h_1}^2 i_2^2 \\ &= -0.0032(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_2(3) &= -\gamma(0.0000) + \alpha \Delta_2 w_2(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_3 w_2(3) = -\gamma \frac{\partial E_o^3}{\partial w_2} + \alpha \Delta_3 w_2(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_2} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_2} \\ &= \delta_{h_1}^3 i_2^3 \\ &= -0.0030(1.0000) \\ &= -0.0030 \end{aligned}$$

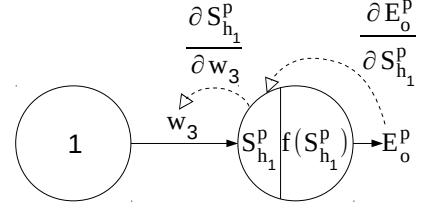
$$\begin{aligned} \Delta_3 w_2(3) &= -\gamma(-0.0030) + \alpha \Delta_3 w_2(2) \\ &= -0.7(-0.0030) + 0.3(0.0016) \\ &= 0.0026 \end{aligned}$$

$$\Delta_4 w_2(3) = -\gamma \frac{\partial E_o^4}{\partial w_2} + \alpha \Delta_4 w_2(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_2} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_2} \\ &= \delta_{h_1}^4 i_2^4 \\ &= 0.0155(1.0000) \\ &= 0.0155 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_2(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_2(2) \\ &= -0.7(0.0155) + 0.3(-0.0104) \\ &= -0.0140 \end{aligned}$$

$$\begin{aligned} w_2 &= \left(\sum_{p=1}^P \Delta_p w_2(3) \right) + w_2 \\ &= (\Delta_1 w_2(3) + \Delta_2 w_2(3) + \Delta_3 w_2(3) + \Delta_4 w_2(3)) + w_2 \\ &= (0.0000 + 0.0000 + 0.0026 + (-0.0140)) + 0.1912 \\ &= 0.1798 \end{aligned}$$



$$\Delta_1 w_3(3) = -\gamma \frac{\partial E_o^1}{\partial w_3} + \alpha \Delta_1 w_3(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_3} &= \frac{\partial E_o^1}{\partial S_{h_1}^1} \frac{\partial S_{h_1}^1}{\partial w_3} \\ &= \delta_{h_1}^1(1) \\ &= 0.0179(1.0000) \\ &= 0.0179 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_3(3) &= -\gamma(0.0179) + \alpha \Delta_1 w_3(2) \\ &= -0.7(0.0179) + 0.3(-0.0124) \\ &= -0.0163 \end{aligned}$$

$$\Delta_2 w_3(3) = -\gamma \frac{\partial E_o^2}{\partial w_3} + \alpha \Delta_2 w_3(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_3} &= \frac{\partial E_o^2}{\partial S_{h_1}^2} \frac{\partial S_{h_1}^2}{\partial w_3} \\ &= \delta_{h_1}^2(1) \\ &= -0.0032(1.0000) \\ &= -0.0032 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_3(3) &= -\gamma(-0.0032) + \alpha \Delta_2 w_3(2) \\ &= -0.7(-0.0032) + 0.3(0.0018) \\ &= 0.0028 \end{aligned}$$

$$\Delta_3 w_3(3) = -\gamma \frac{\partial E_o^3}{\partial w_3} + \alpha \Delta_3 w_3(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_3} &= \frac{\partial E_o^3}{\partial S_{h_1}^3} \frac{\partial S_{h_1}^3}{\partial w_3} \\ &= \delta_{h_1}^3(1) \\ &= -0.0030(1.0000) \\ &= -0.0030 \end{aligned}$$

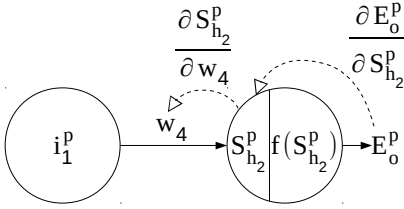
$$\begin{aligned} \Delta_3 w_3(3) &= -\gamma(-0.0030) + \alpha \Delta_3 w_3(2) \\ &= -0.7(-0.0030) + 0.3(0.0016) \\ &= 0.0026 \end{aligned}$$

$$\Delta_4 w_3(3) = -\gamma \frac{\partial E_o^4}{\partial w_3} + \alpha \Delta_4 w_3(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_3} &= \frac{\partial E_o^4}{\partial S_{h_1}^4} \frac{\partial S_{h_1}^4}{\partial w_3} \\ &= \delta_{h_1}^4(1) \\ &= 0.0155(1.0000) \\ &= 0.0155 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_3(3) &= -\gamma(0.0155) + \alpha \Delta_4 w_3(2) \\ &= -0.7(0.0155) + 0.3(-0.0104) \\ &= -0.0140 \end{aligned}$$

$$\begin{aligned} w_3 &= \left(\sum_{p=1}^P \Delta_p w_3(3) \right) + w_3 \\ &= (\Delta_1 w_3(3) + \Delta_2 w_3(3) + \Delta_3 w_3(3) + \Delta_4 w_3(3)) + w_3 \\ &= (-0.0163 + 0.0028 + 0.0026 + (-0.0140)) + 0.2806 \\ &= 0.2557 \end{aligned}$$



$$\Delta_1 w_4(3) = -\gamma \frac{\partial E_o^1}{\partial w_4} + \alpha \Delta_1 w_4(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_4} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_4} \\ &= \delta_{h_2}^1 i_1^1 \\ &= 0.0191(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_4(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_4(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_4(3) = -\gamma \frac{\partial E_o^2}{\partial w_4} + \alpha \Delta_2 w_4(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_4} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_4} \\ &= \delta_{h_2}^2 i_1^2 \\ &= -0.0030(1.0000) \\ &= -0.0030 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_4(3) &= -\gamma(-0.0030) + \alpha \Delta_2 w_4(2) \\ &= -0.7(-0.0030) + 0.3(0.0016) \\ &= 0.0026 \end{aligned}$$

$$\Delta_3 w_4(3) = -\gamma \frac{\partial E_o^3}{\partial w_4} + \alpha \Delta_3 w_4(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_4} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_4} \\ &= \delta_{h_2}^3 i_1^3 \\ &= -0.0027(0.0000) \\ &= 0.0000 \end{aligned}$$

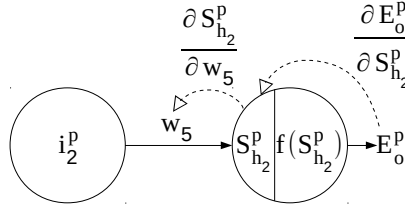
$$\begin{aligned} \Delta_3 w_4(3) &= -\gamma(0.0000) + \alpha \Delta_3 w_4(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_4 w_4(3) = -\gamma \frac{\partial E_o^4}{\partial w_4} + \alpha \Delta_4 w_4(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_4} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_4} \\ &= \delta_{h_2}^4 i_1^4 \\ &= 0.0116(1.0000) \\ &= 0.0116 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_4(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_4(2) \\ &= -0.7(0.0116) + 0.3(-0.0077) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_4 &= \left(\sum_{p=1}^P \Delta_p w_4(3) \right) + w_4 \\ &= (\Delta_1 w_4(3) + \Delta_2 w_4(3) + \Delta_3 w_4(3) + \Delta_4 w_4(3)) + w_4 \\ &= (0.0000 + 0.0026 + 0.0000 + (-0.0104)) + 0.3939 \\ &= 0.3861 \end{aligned}$$



$$\Delta_1 w_5(3) = -\gamma \frac{\partial E_o^1}{\partial w_5} + \alpha \Delta_1 w_5(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_5} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_5} \\ &= \delta_{h_2}^1 i_2^1 \\ &= 0.0191(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_5(3) &= -\gamma(0.0000) + \alpha \Delta_1 w_5(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_2 w_5(3) = -\gamma \frac{\partial E_o^2}{\partial w_5} + \alpha \Delta_2 w_5(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_5} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_5} \\ &= \delta_{h_2}^2 i_2^2 \\ &= -0.0030(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_5(3) &= -\gamma(0.0000) + \alpha \Delta_2 w_5(2) \\ &= -0.7(0.0000) + 0.3(0.0000) \\ &= 0.0000 \end{aligned}$$

$$\Delta_3 w_5(3) = -\gamma \frac{\partial E_o^3}{\partial w_5} + \alpha \Delta_3 w_5(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_5} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_5} \\ &= \delta_{h_2}^3 i_2^3 \\ &= -0.0027(1.0000) \\ &= -0.0027 \end{aligned}$$

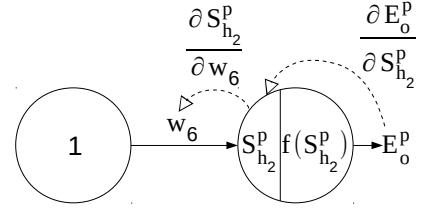
$$\begin{aligned} \Delta_3 w_5(3) &= -\gamma(-0.0027) + \alpha \Delta_3 w_5(2) \\ &= -0.7(-0.0027) + 0.3(0.0015) \\ &= 0.0023 \end{aligned}$$

$$\Delta_4 w_5(3) = -\gamma \frac{\partial E_o^4}{\partial w_5} + \alpha \Delta_4 w_5(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_5} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_5} \\ &= \delta_{h_2}^4 i_2^4 \\ &= 0.0116(1.0000) \\ &= 0.0116 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_5(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_5(2) \\ &= -0.7(0.0116) + 0.3(-0.0077) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_5 &= \left(\sum_{p=1}^P \Delta_p w_5(3) \right) + w_5 \\ &= (\Delta_1 w_5(3) + \Delta_2 w_5(3) + \Delta_3 w_5(3) + \Delta_4 w_5(3)) + w_5 \\ &= (0.0000 + 0.0000 + 0.0023 + (-0.0104)) + 0.4938 \\ &= 0.4857 \end{aligned}$$



$$\Delta_1 w_6(3) = -\gamma \frac{\partial E_o^1}{\partial w_6} + \alpha \Delta_1 w_6(2)$$

$$\begin{aligned} \frac{\partial E_o^1}{\partial w_6} &= \frac{\partial E_o^1}{\partial S_{h_2}^1} \frac{\partial S_{h_2}^1}{\partial w_6} \\ &= \delta_{h_2}^1(1) \\ &= 0.0191(1.0000) \\ &= 0.0191 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_6(3) &= -\gamma(0.0191) + \alpha \Delta_1 w_6(2) \\ &= -0.7(0.0191) + 0.3(-0.0132) \\ &= -0.0173 \end{aligned}$$

$$\Delta_2 w_6(3) = -\gamma \frac{\partial E_o^2}{\partial w_6} + \alpha \Delta_2 w_6(2)$$

$$\begin{aligned} \frac{\partial E_o^2}{\partial w_6} &= \frac{\partial E_o^2}{\partial S_{h_2}^2} \frac{\partial S_{h_2}^2}{\partial w_6} \\ &= \delta_{h_2}^2(1) \\ &= -0.0030(1.0000) \\ &= -0.0030 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_6(3) &= -\gamma(-0.0030) + \alpha \Delta_2 w_6(2) \\ &= -0.7(-0.0030) + 0.3(0.0016) \\ &= 0.0026 \end{aligned}$$

$$\Delta_3 w_6(3) = -\gamma \frac{\partial E_o^3}{\partial w_6} + \alpha \Delta_3 w_6(2)$$

$$\begin{aligned} \frac{\partial E_o^3}{\partial w_6} &= \frac{\partial E_o^3}{\partial S_{h_2}^3} \frac{\partial S_{h_2}^3}{\partial w_6} \\ &= \delta_{h_2}^3(1) \\ &= -0.0027(1.0000) \\ &= -0.0027 \end{aligned}$$

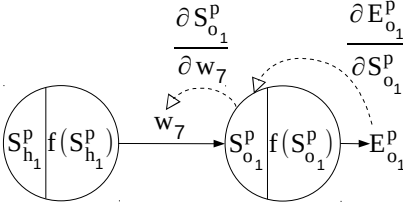
$$\begin{aligned} \Delta_3 w_6(3) &= -\gamma(-0.0027) + \alpha \Delta_3 w_6(2) \\ &= -0.7(-0.0027) + 0.3(0.0015) \\ &= 0.0023 \end{aligned}$$

$$\Delta_4 w_6(3) = -\gamma \frac{\partial E_o^4}{\partial w_6} + \alpha \Delta_4 w_6(2)$$

$$\begin{aligned} \frac{\partial E_o^4}{\partial w_6} &= \frac{\partial E_o^4}{\partial S_{h_2}^4} \frac{\partial S_{h_2}^4}{\partial w_6} \\ &= \delta_{h_2}^4(1) \\ &= 0.0116(1.0000) \\ &= 0.0116 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_6(3) &= -\gamma(0.0116) + \alpha \Delta_4 w_6(2) \\ &= -0.7(0.0116) + 0.3(-0.0077) \\ &= -0.0104 \end{aligned}$$

$$\begin{aligned} w_6 &= \left(\sum_{p=1}^P \Delta_p w_6(3) \right) + w_6 \\ &= (\Delta_1 w_6(3) + \Delta_2 w_6(3) + \Delta_3 w_6(3) + \Delta_4 w_6(3)) + w_6 \\ &= (-0.0173 + 0.0026 + 0.0023 + (-0.0104)) + 0.5822 \\ &= 0.5594 \end{aligned}$$



$$\Delta_1 w_7(3) = -\gamma \frac{\partial E_{o_1}^1}{\partial w_7} + \alpha \Delta_1 w_7(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^1}{\partial w_7} &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial w_7} \\ &= \delta_{o_1}^1 f(S_{h_1}^1) \\ &= 0.1162(0.5697) \\ &= 0.0662 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_7(3) &= -\gamma(0.0662) + \alpha \Delta_1 w_7(2) \\ &= -0.7(0.0662) + 0.3(-0.0415) \\ &= -0.0588 \end{aligned}$$

$$\Delta_2 w_7(3) = -\gamma \frac{\partial E_{o_1}^2}{\partial w_7} + \alpha \Delta_2 w_7(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^2}{\partial w_7} &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial w_7} \\ &= \delta_{o_1}^2 f(S_{h_1}^2) \\ &= -0.0210(0.5919) \\ &= -0.0124 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_7(3) &= -\gamma(-0.0124) + \alpha \Delta_2 w_7(2) \\ &= -0.7(-0.0124) + 0.3(0.0061) \\ &= 0.0105 \end{aligned}$$

$$\Delta_3 w_7(3) = -\gamma \frac{\partial E_{o_1}^3}{\partial w_7} + \alpha \Delta_3 w_7(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^3}{\partial w_7} &= \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial w_7} \\ &= \delta_{o_1}^3 f(S_{h_1}^3) \\ &= -0.0201(0.6158) \\ &= -0.0124 \end{aligned}$$

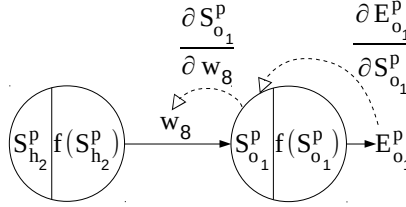
$$\begin{aligned} \Delta_3 w_7(3) &= -\gamma(-0.0124) + \alpha \Delta_3 w_7(2) \\ &= -0.7(-0.0124) + 0.3(0.0061) \\ &= 0.0105 \end{aligned}$$

$$\Delta_4 w_7(3) = -\gamma \frac{\partial E_{o_1}^4}{\partial w_7} + \alpha \Delta_4 w_7(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^4}{\partial w_7} &= \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial w_7} \\ &= \delta_{o_1}^4 f(S_{h_1}^4) \\ &= 0.1066(0.6372) \\ &= 0.0679 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_7(3) &= -\gamma(0.0679) + \alpha \Delta_4 w_7(2) \\ &= -0.7(0.0679) + 0.3(-0.0417) \\ &= -0.0600 \end{aligned}$$

$$\begin{aligned} w_7 &= \left(\sum_{p=1}^P \Delta_p w_7(3) \right) + w_7 \\ &= (\Delta_1 w_7(3) + \Delta_2 w_7(3) + \Delta_3 w_7(3) + \Delta_4 w_7(3)) + w_7 \\ &= (-0.0588 + 0.0105 + 0.0105 + (-0.0600)) + 0.6290 \\ &= 0.5312 \end{aligned}$$



$$\Delta_1 w_8(3) = -\gamma \frac{\partial E_{o_1}^1}{\partial w_8} + \alpha \Delta_1 w_8(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^1}{\partial w_8} &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial w_8} \\ &= \delta_{o_1}^1 f(S_{h_2}^1) \\ &= 0.1162(0.6416) \\ &= 0.0746 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_8(3) &= -\gamma(0.0746) + \alpha \Delta_1 w_8(2) \\ &= -0.7(0.0746) + 0.3(-0.0467) \\ &= -0.0662 \end{aligned}$$

$$\Delta_2 w_8(3) = -\gamma \frac{\partial E_{o_1}^2}{\partial w_8} + \alpha \Delta_2 w_8(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^2}{\partial w_8} &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial w_8} \\ &= \delta_{o_1}^2 f(S_{h_2}^2) \\ &= -0.0210(0.7263) \\ &= -0.0153 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_8(3) &= -\gamma(-0.0153) + \alpha \Delta_2 w_8(2) \\ &= -0.7(-0.0153) + 0.3(0.0075) \\ &= 0.0130 \end{aligned}$$

$$\Delta_3 w_8(3) = -\gamma \frac{\partial E_{o_1}^3}{\partial w_8} + \alpha \Delta_3 w_8(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^3}{\partial w_8} &= \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial w_8} \\ &= \delta_{o_1}^3 f(S_{h_2}^3) \\ &= -0.0201(0.7457) \\ &= -0.0150 \end{aligned}$$

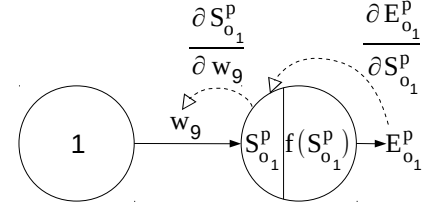
$$\begin{aligned} \Delta_3 w_8(3) &= -\gamma(-0.0150) + \alpha \Delta_3 w_8(2) \\ &= -0.7(-0.0150) + 0.3(0.0073) \\ &= 0.0127 \end{aligned}$$

$$\Delta_4 w_8(3) = -\gamma \frac{\partial E_{o_1}^4}{\partial w_8} + \alpha \Delta_4 w_8(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^4}{\partial w_8} &= \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial w_8} \\ &= \delta_{o_1}^4 f(S_{h_2}^4) \\ &= 0.1066(0.8130) \\ &= 0.0867 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_8(3) &= -\gamma(0.0867) + \alpha \Delta_4 w_8(2) \\ &= -0.7(0.0867) + 0.3(-0.0527) \\ &= -0.0765 \end{aligned}$$

$$\begin{aligned} w_8 &= \left(\sum_{p=1}^P \Delta_p w_8(3) \right) + w_8 \\ &= (\Delta_1 w_8(3) + \Delta_2 w_8(3) + \Delta_3 w_8(3) + \Delta_4 w_8(3)) + w_8 \\ &= (-0.0662 + 0.0130 + 0.0127 + (-0.0765)) + 0.7154 \\ &= 0.5984 \end{aligned}$$



$$\Delta_1 w_9(3) = -\gamma \frac{\partial E_{o_1}^1}{\partial w_9} + \alpha \Delta_1 w_9(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^1}{\partial w_9} &= \frac{\partial E_{o_1}^1}{\partial S_{o_1}^1} \frac{\partial S_{o_1}^1}{\partial w_9} \\ &= \delta_{o_1}^1(1) \\ &= 0.1162(1.0000) \\ &= 0.1162 \end{aligned}$$

$$\begin{aligned} \Delta_1 w_9(3) &= -\gamma(0.1162) + \alpha \Delta_1 w_9(2) \\ &= -0.7(0.1162) + 0.3(-0.0723) \\ &= -0.1030 \end{aligned}$$

$$\Delta_2 w_9(3) = -\gamma \frac{\partial E_{o_1}^2}{\partial w_9} + \alpha \Delta_2 w_9(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^2}{\partial w_9} &= \frac{\partial E_{o_1}^2}{\partial S_{o_1}^2} \frac{\partial S_{o_1}^2}{\partial w_9} \\ &= \delta_{o_1}^2(1) \\ &= -0.0210(1.0000) \\ &= -0.0210 \end{aligned}$$

$$\begin{aligned} \Delta_2 w_9(3) &= -\gamma(-0.0210) + \alpha \Delta_2 w_9(2) \\ &= -0.7(-0.0210) + 0.3(0.0102) \\ &= 0.0178 \end{aligned}$$

$$\Delta_3 w_9(3) = -\gamma \frac{\partial E_{o_1}^3}{\partial w_9} + \alpha \Delta_3 w_9(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^3}{\partial w_9} &= \frac{\partial E_{o_1}^3}{\partial S_{o_1}^3} \frac{\partial S_{o_1}^3}{\partial w_9} \\ &= \delta_{o_1}^3(1) \\ &= -0.0201(1.0000) \\ &= -0.0201 \end{aligned}$$

$$\begin{aligned} \Delta_3 w_9(3) &= -\gamma(-0.0201) + \alpha \Delta_3 w_9(2) \\ &= -0.7(-0.0201) + 0.3(0.0097) \\ &= 0.0170 \end{aligned}$$

$$\Delta_4 w_9(3) = -\gamma \frac{\partial E_{o_1}^4}{\partial w_9} + \alpha \Delta_4 w_9(2)$$

$$\begin{aligned} \frac{\partial E_{o_1}^4}{\partial w_9} &= \frac{\partial E_{o_1}^4}{\partial S_{o_1}^4} \frac{\partial S_{o_1}^4}{\partial w_9} \\ &= \delta_{o_1}^4(1) \\ &= 0.1066(1.0000) \\ &= 0.1066 \end{aligned}$$

$$\begin{aligned} \Delta_4 w_9(3) &= -\gamma(0.1066) + \alpha \Delta_4 w_9(2) \\ &= -0.7(0.1066) + 0.3(-0.0645) \\ &= -0.0940 \end{aligned}$$

$$\begin{aligned} w_9 &= \left(\sum_{p=1}^P \Delta_p w_9(3) \right) + w_9 \\ &= (\Delta_1 w_9(3) + \Delta_2 w_9(3) + \Delta_3 w_9(3) + \Delta_4 w_9(3)) + w_9 \\ &= (-0.1030 + 0.0178 + 0.0170 + (-0.0940)) + 0.7831 \\ &= 0.6209 \end{aligned}$$