Penalized Regression

University of St. Gallen
School of Management, Economics, Law,
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Assignment 2

Data Analytics I: Predictive Econometrics
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submitted by

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Contents

Requirements

To solve the following tasks, the required libraries and the data sets are loaded first.

```
library(glmnet)
library(corrplot)
library(ggplot2)
library(dplyr)

load("GHA/student-mat-train.RData")
load("GHA/student-mat-test.RData")
```

Exercise 1

There are 214 observations in the training data set and 143 observations in the test data set.

```
(n_obs_train <- nrow(train))
```

```
## [1] 214
```

```
(n_obs_test <- nrow(test))
```

```
## [1] 143
```

Exercise 2

The average grade is ~11.64, the minimum grade is 4 and the maximum grade is 19. All numbers were calculated using the training data.

```
(avg_grade <- mean(train$G3))
```

```
## [1] 11.64019
```

```
(min_grade <- min(train$G3))
```

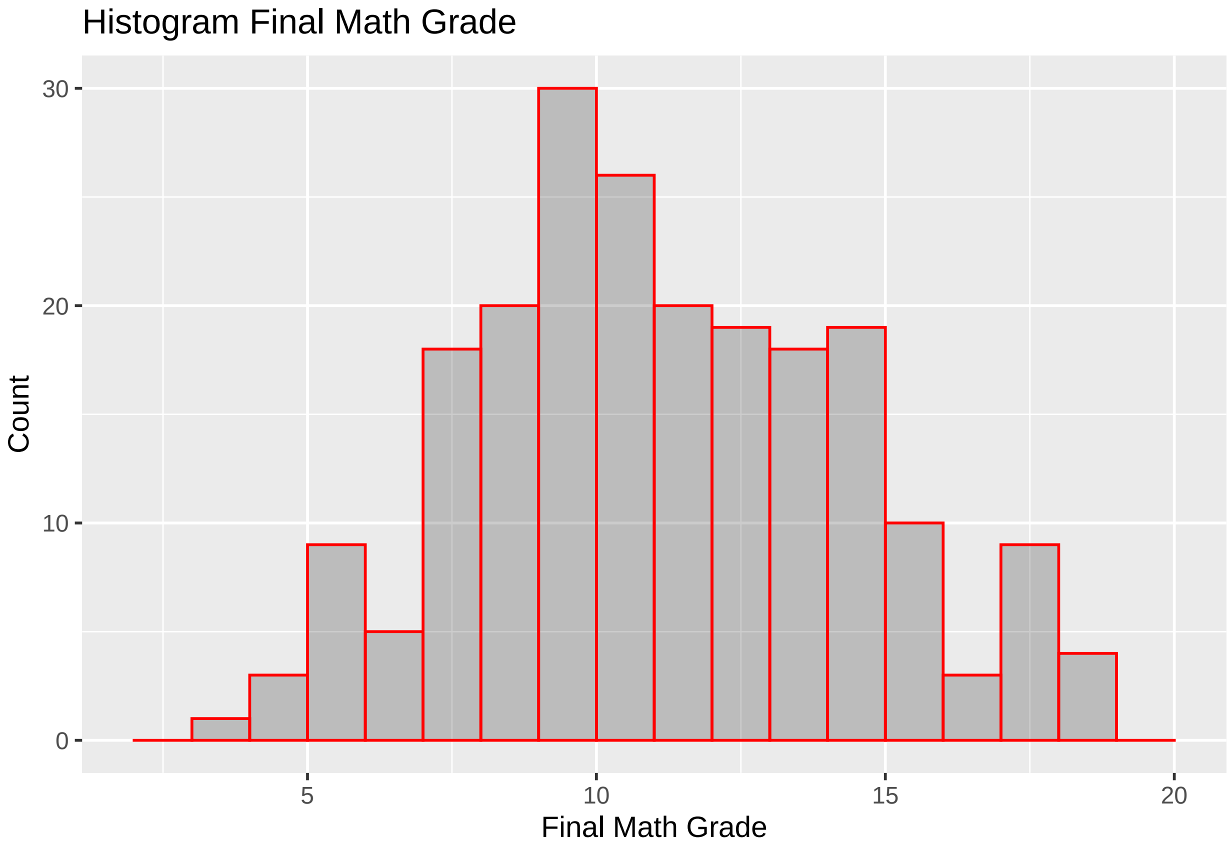
```
## [1] 4
```

```
(max_grade <- max(train$G3))
```

```
## [1] 19
```

Exercise 3

```
(final_grade_hist <- ggplot(data=train, aes(G3)) +
    geom_histogram(breaks=seq(2,20, by=1),
                   col="red",
                   fill="black",
                   alpha = 0.2)+
    labs(title="Histogram Final Math Grade", x="Final Math Grade", y="Count"))
```



Exercise 4

When doing causal modeling there are independent variables (x_1,..,x_n) which are considered as the cause of the dependent variable (y), therefore one would expect a direct impact of the independent variables on the dependent variable. For predictive modelling the goal is to establish a method that allows to make predictions of the dependent variable (y) based on the known independent variables (x_1,..,x_n).

Exercise 5

```
(OLS1 <- lm(G3 ~ . ,
            data=select(train, G3, Medu, Fedu, studytime, schoolsup, higher)))
```

```
##
```

```
## Call:
## lm(formula = G3 ~ ., data = select(train, G3, Medu, Fedu, studytime,
##     schoolsup, higher))
##
## Coefficients:
## (Intercept)         Medu         Fedu    studytime    schoolsup       higher
##     9.38701      0.36742      0.07675      0.60662     -3.36832      0.77327
```

```
(summary(OLS1))
```

```
##
## Call:
## lm(formula = G3 ~ ., data = select(train, G3, Medu, Fedu, studytime,
##     schoolsup, higher))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.4668 -2.1690 -0.1981  2.0630  7.0630
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.38701    1.05127   8.929 2.29e-16 ***
## Medu         0.36742    0.24753   1.484   0.1392
## Fedu         0.07675    0.24727   0.310   0.7566
## studytime    0.60662    0.24803   2.446   0.0153 *
## schoolsup   -3.36832    0.67412  -4.997 1.24e-06 ***
## higher       0.77327    1.02224   0.756   0.4502
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.041 on 208 degrees of freedom
## Multiple R-squared:  0.1501, Adjusted R-squared:  0.1297
## F-statistic: 7.346 on 5 and 208 DF,  p-value: 2.312e-06
```

```
(OLS2 <- lm(G3 ~ . + .^2,
            data=select(train, G3, Medu, Fedu, studytime, schoolsup, higher)))
```

```
##
## Call:
## lm(formula = G3 ~ . + .^2, data = select(train, G3, Medu, Fedu,
##     studytime, schoolsup, higher))
##
## Coefficients:
##         (Intercept)                 Medu                 Fedu
##           13.172132             0.145263            -1.427466
##           studytime            schoolsup               higher
##          -0.464677            1.920704           -4.432522
##          Medu:Fedu      Medu:studytime      Medu:schoolsup
##          -0.001922            0.105788           -2.611720
##        Medu:higher      Fedu:studytime      Fedu:schoolsup
##           0.322940           -0.499887            1.271388
##        Fedu:higher  studytime:schoolsup    studytime:higher
##           1.939871           -0.210424            2.074920
##   schoolsup:higher
##          -1.165641
```

```
(summary(OLS2))
```

```
##
## Call:
## lm(formula = G3 ~ . + .^2, data = select(train, G3, Medu, Fedu,
##     studytime, schoolsup, higher))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6603 -2.0887 -0.0921  1.8277  7.8154
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)         13.172132   5.437629   2.422  0.01632 *
## Medu                 0.145263   1.064137   0.137  0.89156
## Fedu                -1.427466   2.050517  -0.696  0.48715
## studytime           -0.464677   2.785748  -0.167  0.86769
## schoolsup            1.920704   4.596735   0.418  0.67652
## higher              -4.432522   5.533668  -0.801  0.42409
## Medu:Fedu           -0.001922   0.217956  -0.009  0.99297
## Medu:studytime       0.105788   0.312408   0.339  0.73525
## Medu:schoolsup      -2.611720   0.899135  -2.905  0.00409 **
## Medu:higher          0.322940   1.040274   0.310  0.75656
## Fedu:studytime      -0.499887   0.298323  -1.676  0.09538 .
## Fedu:schoolsup       1.271388   0.844657   1.505  0.13386
## Fedu:higher          1.939871   2.119975   0.915  0.36128
## studytime:schoolsup -0.210424   0.851999  -0.247  0.80518
## studytime:higher     2.074920   2.739937   0.757  0.44978
## schoolsup:higher    -1.165641   4.726445  -0.247  0.80546
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.01 on 198 degrees of freedom
## Multiple R-squared:  0.2076, Adjusted R-squared:  0.1476
## F-statistic: 3.459 on 15 and 198 DF,  p-value: 3.007e-05
```

Exercise 6

```
(OLS3 <- lm(G3 ~ . ,
            data=select(train, G3, Medu, Fedu, studytime, schoolsup, higher, Pstatus, famrel, fai
```

```
##
## Call:
## lm(formula = G3 ~ ., data = select(train, G3, Medu, Fedu, studytime,
##     schoolsup, higher, Pstatus, famrel, failures, famsup, internet))
##
## Coefficients:
## (Intercept)         Medu         Fedu    studytime    schoolsup       higher
##    9.649486     0.350090     0.007509     0.597455    -3.151785     0.284839
##     Pstatus       famrel     failures       famsup     internet
##    0.022675     0.272672    -1.016545    -0.891842     0.562597
```

```
(summary(OLS3))
```

```
##
## Call:
## lm(formula = G3 ~ ., data = select(train, G3, Medu, Fedu, studytime,
##     schoolsup, higher, Pstatus, famrel, failures, famsup, internet))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.0295 -2.1703 -0.0742  1.9681  7.1631
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.649486   1.345550   7.171 1.36e-11 ***
## Medu         0.350090   0.248736   1.407   0.1608
## Fedu         0.007509   0.242057   0.031   0.9753
## studytime    0.597455   0.247898   2.410   0.0168 *
## schoolsup   -3.151785   0.657969  -4.790 3.21e-06 ***
## higher       0.284839   1.002100   0.284   0.7765
## Pstatus      0.022675   0.610754   0.037   0.9704
## famrel       0.272672   0.230688   1.182   0.2386
## failures    -1.016545   0.317847  -3.198   0.0016 **
## famsup      -0.891842   0.449153  -1.986   0.0484 *
## internet     0.562597   0.542256   1.038   0.3007
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.943 on 203 degrees of freedom
## Multiple R-squared:  0.2233, Adjusted R-squared:  0.185
## F-statistic: 5.836 on 10 and 203 DF,  p-value: 1.013e-07
```

```r
(OLS4 <- lm(G3 ~ . + .^2,
            data=select(train, G3, Medu, Fedu, studytime, schoolsup, higher, Pstatus, famrel, fai
```

```
## 
## Call:
## lm(formula = G3 ~ . + .^2, data = select(train, G3, Medu, Fedu, 
##     studytime, schoolsup, higher, Pstatus, famrel, failures, 
##     famsup, internet))
## 
## Coefficients:
##         (Intercept)                 Medu                 Fedu  
##             8.00486              2.65008             -2.33596  
##           studytime            schoolsup               higher  
##            -0.74497             -1.03745              0.15984  
##             Pstatus               famrel             failures  
##            -3.81423              1.04447              0.04181  
##              famsup             internet            Medu:Fedu  
##             3.05249             -1.90817             -0.03842  
##      Medu:studytime       Medu:schoolsup          Medu:higher  
##            -0.15811             -2.85143             -0.54863  
##        Medu:Pstatus          Medu:famrel        Medu:failures  
##            -0.91279             -0.36014             -1.01657  
##         Medu:famsup        Medu:internet       Fedu:studytime  
##             0.74973             -0.59083             -0.32178  
##      Fedu:schoolsup          Fedu:higher         Fedu:Pstatus  
##             1.41207              1.45072              0.10424  
##         Fedu:famrel        Fedu:failures          Fedu:famsup  
##             0.22857              0.34034             -0.49855  
##       Fedu:internet  studytime:schoolsup     studytime:higher  
##             1.11916             -0.43875              1.74282  
##   studytime:Pstatus     studytime:famrel   studytime:failures  
##             1.28299              0.09156             -0.61083  
##    studytime:famsup   studytime:internet     schoolsup:higher  
##             0.79983              0.10897              0.67617  
##   schoolsup:Pstatus     schoolsup:famrel   schoolsup:failures  
##            -0.95843              0.16037              1.93873  
##    schoolsup:famsup   schoolsup:internet       higher:Pstatus  
##             1.18057              0.27075              2.66107  
##       higher:famrel      higher:failures        higher:famsup  
##            -0.14551              1.51356             -4.68077  
##     higher:internet       Pstatus:famrel     Pstatus:failures  
##                  NA              1.04521             -0.19850  
##      Pstatus:famsup     Pstatus:internet      famrel:failures  
##            -0.71384              0.89706             -0.26244  
##       famrel:famsup      famrel:internet      failures:famsup  
##            -0.72776              0.21125              0.67703  
##   failures:internet      famsup:internet  
##            -0.39309             0.95906
```

```
(summary(OLS4))
```

```
##
## Call:
## lm(formula = G3 ~ . + .^2, data = select(train, G3, Medu, Fedu,
##     studytime, schoolsup, higher, Pstatus, famrel, failures,
##     famsup, internet))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.302 -1.638  0.000  1.569  7.129
##
## Coefficients: (1 not defined because of singularities)
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)          8.00486   19.71735   0.406  0.68530
## Medu                 2.65008    2.18285   1.214  0.22653
## Fedu                -2.33596    5.62321  -0.415  0.67840
## studytime           -0.74497    5.53425  -0.135  0.89309
## schoolsup           -1.03745   10.53794  -0.098  0.92170
## higher               0.15984   19.57437   0.008  0.99350
## Pstatus             -3.81423    6.70935  -0.568  0.57050
## famrel               1.04447    4.90832   0.213  0.83176
## failures             0.04181    2.33857   0.018  0.98576
## famsup               3.05249    5.01243   0.609  0.54340
## internet            -1.90817    2.90639  -0.657  0.51242
## Medu:Fedu           -0.03842    0.25236  -0.152  0.87919
## Medu:studytime      -0.15811    0.34987  -0.452  0.65196
## Medu:schoolsup      -2.85143    1.06173  -2.686  0.00801 **
## Medu:higher         -0.54863    1.68739  -0.325  0.74551
## Medu:Pstatus        -0.91279    0.98786  -0.924  0.35689
## Medu:famrel         -0.36014    0.40866  -0.881  0.37951
## Medu:failures       -1.01657    0.63945  -1.590  0.11388
## Medu:famsup          0.74973    0.61857   1.212  0.22730
## Medu:internet       -0.59083    0.67187  -0.879  0.38052
## Fedu:studytime      -0.32178    0.33980  -0.947  0.34510
## Fedu:schoolsup       1.41207    0.96285   1.467  0.14448
## Fedu:higher          1.45072    5.53069   0.262  0.79343
## Fedu:Pstatus         0.10424    0.80597   0.129  0.89725
## Fedu:famrel          0.22857    0.38702   0.591  0.55563
## Fedu:failures        0.34034    0.67323   0.506  0.61389
## Fedu:famsup         -0.49855    0.63597  -0.784  0.43426
## Fedu:internet        1.11916    0.71180   1.572  0.11787
## studytime:schoolsup -0.43875    0.94950  -0.462  0.64465
## studytime:higher     1.74282    5.29531   0.329  0.74249
## studytime:Pstatus    1.28299    1.13311   1.132  0.25923
## studytime:famrel     0.09156    0.28279   0.324  0.74654
## studytime:failures  -0.61083    0.81245  -0.752  0.45326
## studytime:famsup     0.79983    0.63934   1.251  0.21277
## studytime:internet   0.10897    0.81962   0.133  0.89440
## schoolsup:higher     0.67617    9.41038   0.072  0.94281
## schoolsup:Pstatus   -0.95843    2.60822  -0.367  0.71376
## schoolsup:famrel     0.16037    1.43567   0.112  0.91120
## schoolsup:failures   1.93873    1.64480   1.179  0.24028
## schoolsup:famsup     1.18057    2.35659   0.501  0.61709
## schoolsup:internet   0.27075    2.11487   0.128  0.89830
## higher:Pstatus       2.66107    4.74288   0.561  0.57554
## higher:famrel       -0.14551    4.88194  -0.030  0.97626
## higher:failures      1.51356    1.98140   0.764  0.44607
## higher:famsup       -4.68077    4.62750  -1.012  0.31331
## higher:internet           NA         NA      NA       NA
## Pstatus:famrel       1.04521    0.69232   1.510  0.13310
## Pstatus:failures    -0.19850    1.71396  -0.116  0.90795
## Pstatus:famsup      -0.71384    1.84148  -0.388  0.69880
## Pstatus:internet     0.89706    1.95701   0.458  0.64730
## famrel:failures     -0.26244    0.39874  -0.658  0.51138
## famrel:famsup       -0.72776    0.67389  -1.080  0.28180
## famrel:internet      0.21125    0.72523   0.291  0.77121
## failures:famsup      0.67703    0.98499   0.687  0.49286
## failures:internet   -0.39309    1.02403  -0.384  0.70159
## famsup:internet      0.95906    1.41469   0.678  0.49880
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.919 on 159 degrees of freedom
## Multiple R-squared:  0.4014, Adjusted R-squared:  0.1981
## F-statistic: 1.974 on 54 and 159 DF,  p-value: 0.0006072
```

```
MSE_IS_OLS1 <- mean((train$G3 - OLS1$fitted.values)^2)
MSE_IS_OLS2 <- mean((train$G3 - OLS2$fitted.values)^2)
MSE_IS_OLS3 <- mean((train$G3 - OLS3$fitted.values)^2)
MSE_IS_OLS4 <- mean((train$G3 - OLS4$fitted.values)^2)

fit_OLS1 <- predict(OLS1, newdata = test)
MSE_OOS_OLS1 <- mean((test$G3 - fit_OLS1)^2)

fit_OLS2 <- predict(OLS2, newdata = test)
MSE_OOS_OLS2 <- mean((test$G3 - fit_OLS2)^2)

fit_OLS3 <- predict(OLS3, newdata = test)
MSE_OOS_OLS3 <- mean((test$G3 - fit_OLS3)^2)

fit_OLS4 <- predict(OLS4, newdata = test)
MSE_OOS_OLS4 <- mean((test$G3 - fit_OLS4)^2)
```

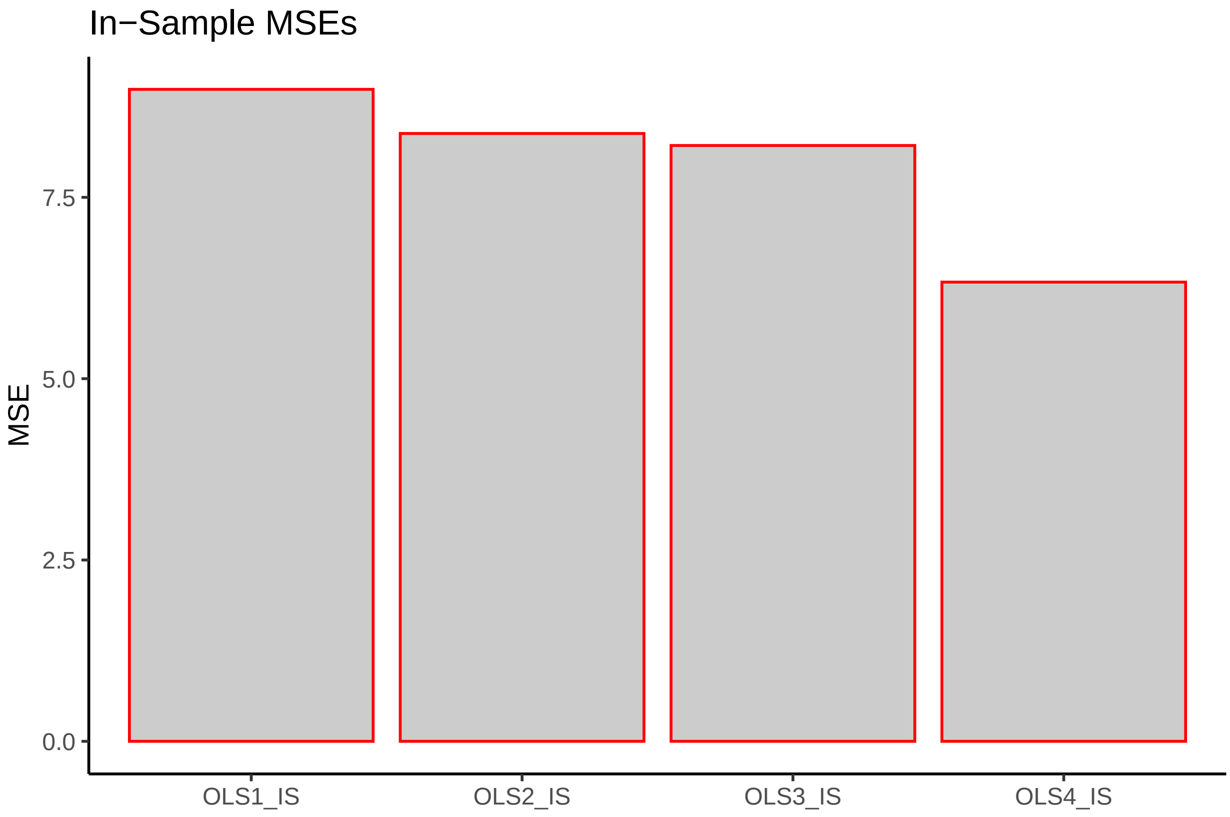
```
(MSE_IS <- data.frame(model = c("OLS1_IS", "OLS2_IS", "OLS3_IS", "OLS4_IS"),
                      MSE = c(MSE_IS_OLS1, MSE_IS_OLS2, MSE_IS_OLS3, MSE_IS_OLS4)))
```

```
##     model      MSE
## 1 OLS1_IS 8.988840
## 2 OLS2_IS 8.380403
## 3 OLS3_IS 8.214626
## 4 OLS4_IS 6.330987
```

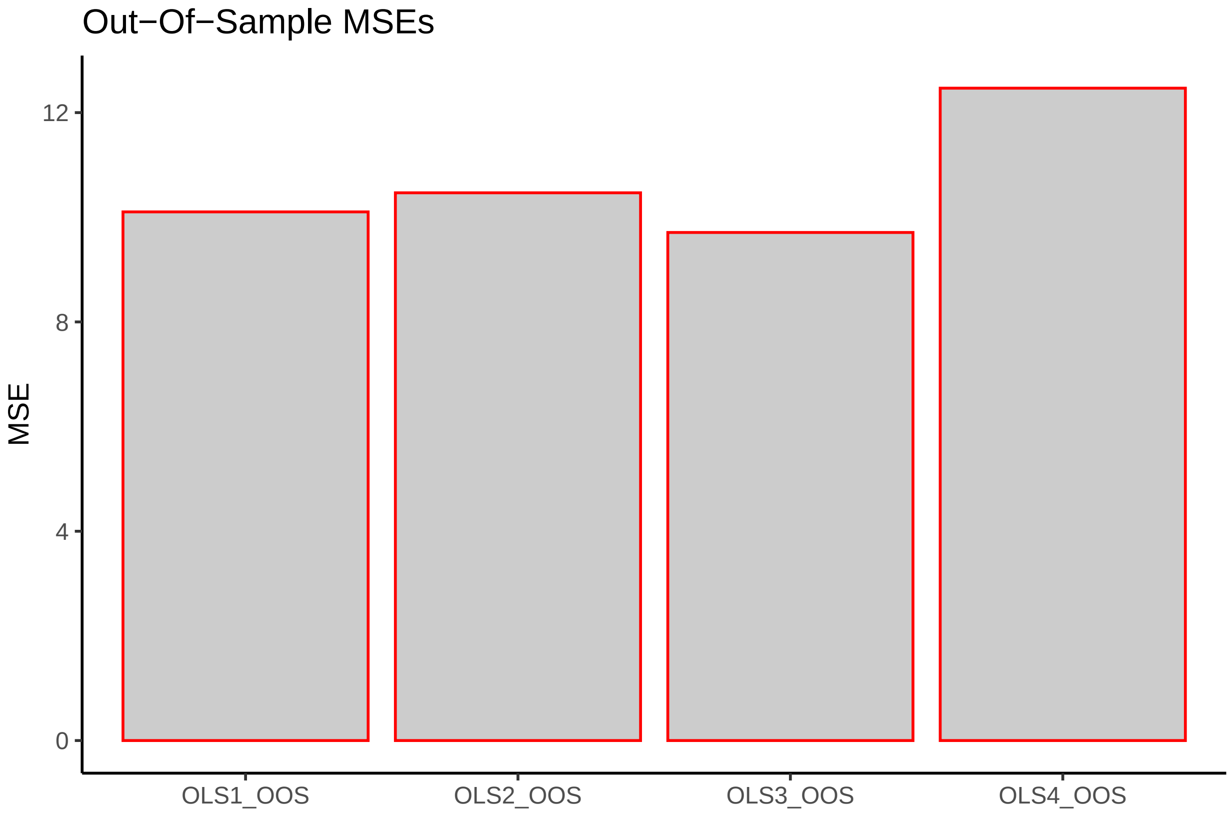
```
(MSE_OOS <- data.frame(model = c("OLS1_OOS", "OLS2_OOS", "OLS3_OOS", "OLS4_OOS"),
                       MSE = c(MSE_OOS_OLS1, MSE_OOS_OLS2, MSE_OOS_OLS3, MSE_OOS_OLS4)))
```

```
##      model       MSE
## 1 OLS1_OOS 10.103001
## 2 OLS2_OOS 10.467642
## 3 OLS3_OOS  9.709007
## 4 OLS4_OOS 12.466627
```

```
(ggplot(MSE_IS, aes(model, MSE)) +
  geom_col(color = "red", fill = 'black', alpha = 0.2) +
  ggtitle("In-Sample MSEs") +
  xlab("") +
  theme_classic())
```



```
(ggplot(MSE_OOS, aes(model, MSE)) +
  geom_col(color = "red", fill = 'black', alpha = 0.2) +
  ggtitle("Out-Of-Sample MSEs") +
  xlab("") +
  theme_classic())
```


Out−Of−Sample MSEs
MSE
OLS1_OOS
OLS2_OOS
OLS3_OOS
OLS4_OOS