Theory of Finance, 2021

MiQEF - University of St.Gallen

PROBLEM SET 2

Risk Measures

You are advised, but not obliged, to work on this problem set in groups of up to three people. Groups can change for different problem sets. You are free to use any software you are familiar with. Each group has to hand in one solution sheet (available for each problem set on Canvas) together with the unique calculation file (Matlab/R/Julia or whatever software you used). Answers in the solution sheet should be typed (or written legibly). The deadline is November 16 midnight. Late submissions will not be accepted by the system. Please round your estimated values when presenting them in the solution sheet to enhance readability.

Please use the data file *ps2_data.csv* for the exercises of this problem set. It contains discrete daily adjusted stock return data for five U.S. large cap firms, spanning the time horizon of 1973 to 2021 as well as the 1-month U.S T-Bill rate (quoted daily, column 'RF'). If not stated otherwise, use 95% confidence levels for value at risk and expected shortfall estimates.

Problem 1. (68 points) - *Estimation, Interpretation*

a) (12 points) Compute the equally weighted portfolio of the 5 stocks (equal weights every day). For each firm and for the portfolio, estimate the following risk measures using the total sample of daily returns: (i) mean absolute deviation, (ii) semi standard deviation, (iii) empirical value at risk, (iv) empirical expected shortfall. Stick to the definitions of the Lecture Notes, Chapter 9.

b) (6 points) Based on your results from *a)*, which investment do you see as the riskiest and which as the safest one? Briefly argue your standpoint.

c) (10 points) Use the equally weighted portfolio and compute the 21-day rolling-window returns.[1] Based on the overlapping return series, compare the monthly empirical VaR with the theoretical (LN Eq.9.8) one. Run the same comparison for the expected shortfall.

d) (10 points) Use the aforementioned monthly return series from *c)* for the tasks *d)* to *h)*. Run a rolling estimation of the semi standard deviation using a 5-year window. Create a plot showing

[1] For each day of the series, compute the return that was realized in the last 21 days. Start at day 22.

the log price development of the equally weighted portfolio and the time-varying semi standard deviation; use dates on the x-axis and ideally two different y-axes such that both time-series are clearly visible.

e) (8 points) Use the plot of *d)* and identify the 5 riskiest time-periods for the portfolio, highlight them in your plot. Does the risk measure relate to historical stock market crashes/crises? If so, to which ones?

f) (6 points) Calculate the mean and the median of the return series. How do you interpret the difference?

g) (8 points) Create a histogram for the returns (with at least 100 bars) and overlap it with a normal density distribution of same mean and same standard deviation as the empirical series. What do you observe in the plot?

h) (8 points) Draw and interpret a Q-Q plot of the monthly returns. Additionally, compute the series' skewness and kurtosis; briefly discuss the estimates.

Problem 2. (32 points) - *Risk Targeting*

Take the daily return series of the equally weighted portfolio from *Problem 1, a)*. The task of *Problem 2* is to balance an investment between the equally weighted portfolio and the risk-free rate to create an overall portfolio which has a monthly theoretical VaR of -5% at a confidence level of 99% (assuming normal distribution). The weight w_e put onto the risky equally weighted portfolio is limited by $[0, 10]$ and total weights should sum up to 1, borrowing at the risk-free rate is allowed. Start with $w_e = 0.5$ for the first month, then vary it to meet the target VaR.

a) (16 points) Re-balance the overall portfolio at each beginning of a month to match the target value-at-risk. Estimate expected mean and standard deviation under the RiskMetrics approach as discussed in the Lecture Notes 9.2.2 using the return sample of the previous month and $\lambda = 0.95$. Compute the weights for the risk-free rate and w_e to achieve the target VaR. Plot the time-series of w_e and the price development of the overall portfolio, report statistics on mean return and volatility.

b) (10 points) Analyzing the risk-targeting strategy, how often was the VaR exceeded? Did the strategy perform well to achieve its risk goals?

c) (6 points) Discuss potential improvements to further enhance the risk-targeting performance.