

PROBLEM SET 4

Fixed Income

You are advised, but not obliged, to work on this problem set in groups of up to three people. Groups can change for different problem sets. You are free to use any software you are familiar with. Each group has to hand in one solution sheet (available for each problem set on Canvas) together with the unique calculation file (Matlab/R/Julia or whatever software you used). Answers in the solution sheet should be typed (or written legibly). The deadline is December 16 midnight. Late submissions will not be accepted by the system. Please round your estimated values when presenting them in the solution sheet to enhance readability.

Assume a face value of 100 for all bonds mentioned in this problem set. Please use the data file 'ToF_2021_PS-4_data_1-2.csv' for Problem 1 and 2. It contains forward prices of the FTSE 100 over different maturities τ (in years), observed on July 31, 2021. The 'maturity = 0' entry represents the current price of the index. Assume a continuous dividend yield of 0.05% p.a.

Problem 1. (20 points) - *From Forwards to Interest*

- a) (9 points) Based on the forward prices, compute (i) the continuously compounded-, (ii) the effective and (iii) the simple spot interest rates. Visualize your results in a graph with rates on the y- and maturity on the x-axis. Provide an interpretation.
- b) (7 points) Transform your results from a) into zero-coupon prices. Show the corresponding plot and leave some short comments.
- c) (4 points) What is the forward rate of a contract entering in 3 years and selling in 20 years?

Problem 2. (40 points) - *Bond price, yield-to-maturity and duration*

Consider a bond of 40 years maturity and annual coupons of 1.

- a) (4 points) Calculate the bond price given the yield curve as derived in Problem 1.
- b) (4 points) What is the yield-to-maturity of this bond?
- c) (6 points) Calculate the Dollar-, adjusted- and Macaulay's duration of the bond.

- d) (6 points) Consider four different scenarios of how the yield curve potentially changes. Let Y denote the effective rate from *a)* and Y' the new one, the scenarios are described as follows:

scenario	new yield curve
1	$Y' = Y + 0.005$
2	$Y' = Y - 0.005$
3	$Y' = Y \times 1.3$
4	$Y' = Y \times 0.3 + \tau/4000$

Visualize the new yield curves and briefly describe the changes.

- e) (10 points) Given the scenarios from *d)*, calculate the new bond prices and yield-to-maturities for each scenario. Report your results.
- f) (10 points) Use the duration from *c)* and the changes in the yield-to-maturity from *e)* to approximate the new prices. Show a plot with scenario prices (precise and approximated) on the y-axis and yield-to-maturity on the x-axis. Interpret the graph, is the duration-approximation a good one?

Problem 3. (40 points) - *Yield Curve, etc.*

The intended data file for this problem is '*ToF_2021_PS-4_data_3.csv*'. It contains interest rates of government (GVT) and corporate (CORP) bonds of different maturities (e.g., *_10Y* for ten years, *_3M* for three months, etc.).

- a) (4 points) Select the interest rates observed on July 15, 2021 of the government bonds and plot the yield curve. Are there any special patterns worth to mention?
- b) (6 points) Transform the yield curve from *a)* into zero coupon bond prices. Plot the term structure of zero-bond prices.
- c) (8 points) Use the results from *b)* to fit a cubic discount function (LN 16.5.2). Report the coefficients and add the fitted line to the plot.
- d) (5 points) Based on the fitted values, what is the price of a zero-bond of seven years maturity?
- e) (5 points) Further, calculate the price of a semi-annual coupon bond with a maturity of three years, paying $\frac{1}{2}$ every 6 months (first payment in 6 months, last payment in three years). Use fitted rates for those coupon payments that occur on dates not covered by the data.

f) (6 points) Calculate and visualize the time-series of level, slope and curvature of the government interest rates. Briefly comment on special patterns you observe.

g) (6 points) Calculate the following time-series:

(i) $\text{CORP_AA_10Y} - \text{GVT_10Y}$

(ii) $\text{CORP_BBB_10Y} - \text{GVT_10Y}$

Starting in July 2018, show the time-series plot and provide an interpretation. What do they describe?