Theory of Finance, 2021

MiQEF - University of St.Gallen

PROBLEM SET 1

Return Calculations, Portfolio Choice and Mean-Variance Frontier

You are advised, but not obliged, to work on this problem set in groups of up to three people. Groups can change for different problem sets. You are free to use any software you are familiar with. Each group has to hand in one solution sheet (available for each problem set on Canvas) together with the unique calculation file (Matlab/R/Julia or whatever software you used). Answers in the solution sheet should be typed (or written legibly). The deadline is October 19 midnight. Late submissions will not be accepted by the system. Please round your estimated values when presenting them in the solution sheet to enhance readability.

Please use the data file *ps1_data.csv* for the exercises of this problem set. It contains monthly adjusted stock price data for ten German large cap firms, spanning the time horizon of 1996 to 2021.

Problem 1. (26 points) - *Return Comparison*

a) (6 points) Compute monthly discrete and log returns for each of the ten stocks, report mean returns and standard deviations. Additionally, calculate annualized mean returns for the discrete and the log return series.

b) (8 points) Using the monthly return series, what are the maximum differences between discrete and log returns at each stock? Report your results. Further, create two plots with log returns on the x-axis and discrete returns on the y-axis; the first plot should show the stock where the maximum difference was the largest. Respectively, the second plot should relate to the stock where the maximum difference between discrete and log returns are the smallest. Provide a short interpretation of the plots and the observed effect.

c) (6 points) Briefly explain when to use which type of return calculation (discrete or log).

d) (6 points) Compute the discrete return series of the equally weighted portfolio. I you've invested 100€ (theoretically) into the portfolio at July 1996, what would be the worth of your investment at the end of July 2021?

Problem 2. (30 points) - *Diversification Effect*

a) (6 points) Use the discrete return series from Problem 1. Suppose you can pick two stocks (equally weight), which two-stock combination will have the greatest diversification benefit and which one the worst? Explain how you come up with your decision.

b) (12 points) Sort the ten stocks based on their return standard deviation from high to low. Now, based on this sort we want to analyze how standard deviations change if we extend the numbers of stocks in our portfolio. Therefore, fill the table below. Start with the first stock and report its standard deviation (No. of stocks = 1). Next, take the first two stocks and compute the mean of the two standard deviations and the standard deviation of the equally weighted portfolio (No. of stocks = 2). The third column should cover the mixture of the first three stocks and so on.

No. of stocks:	1	2	3	4	5	6	7	8	9	10
Mean St.Dev.:										
Portfolio St.Dev.:										

c) (12 points) Plot the table of *b)* with 'No. of stocks' on the x-axis and volatility (standard deviations) on the y-axis. Provide an interpretation of the plot and explain why there are differences between the two series.

Problem 3. (44 points) - *Mean-Variance Frontier and Portfolio Choice*

For this problem assume a risk-free rate of return of 0.08% per month and work with the discrete stock return-series from before.

a) (8 points) Create a plot with standard deviation on the x-axis and mean return on the y-axis. Indicate each of the ten stocks within this plot. Assume that the past performance is identical to the expected future performance and you can borrow and lend at the risk-free rate without restrictions. If you can only choose one stock, which one would it be? Briefly argue your standpoint.

b) (10 points) Compute the mean-variance efficient frontier (no short-selling restrictions) from the risky stocks and draw it into the plot of *a)*.

c) (10 points) Continuing *b)*, what is the mean return and volatility of the minimum variance portfolio of the ten stocks? Report it's weights and highlight it within the plot.

d) (10 points) The next portfolio we are interested in is the tangency portfolio from the ten stocks. Compute the weights, mean return and volatility. Also, add it to the plot of *a)* and draw the capital market line.

e) (6 points) Consider an investor who can directly invest into the risk-free rate and an ETF perfectly tracking the tangency portfolio. Her risk preferences are described by Eq.2.5 of the Lecture Notes with a risk aversion parameter $k=2$. How does she allocate her portfolio given that she optimizes her utility?