*Theory of Finance, 2021*

*MiQEF - University of St.Gallen*

# PROBLEM SET 3

*Asset Pricing Models & Portfolio Choice*

*You are advised, but not obliged, to work on this problem set in groups of up to three people. Groups can change for different problem sets. You are free to use any software you are familiar with. Each group has to hand in one solution sheet (available for each problem set on Canvas) together with the unique calculation file (Matlab/R/Julia or whatever software you used). Answers in the solution sheet should be typed (or written legibly). The deadline is November 30 midnight. Late submissions will not be accepted by the system. Please round your estimated values when presenting them in the solution sheet to enhance readability.*

## Problem 1. (72 points) - *Analyzing Beta Sorted Portfolios*

*Problem 1* builds on the data sets *ToF_2021_data_PS-3_beta-pf.csv* and *ToF_2021_data_PS-3_FF.csv*. The first file contains discrete portfolio return data based on U.S. cross-sectional sorts from low (beta1) to high CAPM beta (beta10). The second file covers the five U.S. Fama-French factors. Note: MKT_RF is the market return in excess of the risk-free rate RF. All data is derived from Kenneth R. French's freely accesible data library.[1]

*Problem 1, part I:*

a) (4 points) Report a table of annualized statistics on mean return, volatility and Sharpe ratios of each of the beta-sorted portfolios.

b) (8 points) Run the CAPM regression for each portfolio and extend the table in *a)* by estimated alpha and beta coefficients together with p-values. Additionally, include the coefficient-of-determination (goodness of fit $R^2$).

c) (8 points) Plot the portfolios' mean returns (y-axis) against estimated betas and indicate the market portfolio as well as the security market line. Provide a short interpretation of the plot.

d) (8 points) Next, we are interested into the plot of the portfolio alphas (y-axis) versus betas (x-axis). What do you conclude?

[1] `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.htmls`

e) (8 points) The third plot should show the regression $R^2$ (y-axis) against portfolio betas (x-axis). What pattern do you observe and what economic intuition do you believe behind it?

*Problem 1, part II:*

f) (6 points) For this part we want to create a market-neutral portfolio that is short in the `beta10` portfolio and long in `beta1`. Consider that the weight of the short position is fixed at -1 and you can leverage the long one to make the portfolio market neutral (portfolio beta=0). Calculate and report the required weight for the long position taking the beta estimates for the full data set from before. Further, create a log-price plot (log-price on y-, time on x-axis) of this long-short portfolio and add the series of the market portfolio for comparison.

g) (6 points) Compare the performance between the long-short and the market portfolio based on mean returns and Sharpe ratios. Report the correlation between the two series and answer whether the long-short portfolio is market neutral or not.

h) (14 points) Run the CAPM, Fama-French 3-factor and Fama-French 5-factor regression on the sequence of the long-short portfolio. Provide an interpretation of the models and the effects from extending the number of factors. Also, comment on the multicolinearity across the five factors - is it a problem at our application or not?

i) (10 points) From the found effect we are interested in whether it allows for a profitable trading strategy (market-neutral, leverage only `beta1`), considering the fact that at re-balancing dates we only have historical information available. To create the strategy, start in July 1968 and re-balance the portfolio every month. When estimating betas, note that correlations appear to move more slowly than volatilities.[2] Therefore, use a 5-year backward looking window for the correlations between the portfolios to the market, and a one-year window for volatilities. Analyze the performance of the long-short strategy by running a regression on the Fama-French 5-factor model.

**Problem 2.** (28 points) - *Factor Rotation*

For this problem we use the data set *ToF_2021_data_PS-3_aqr.csv* which covers alternative U.S. factors provided by AQR Capital Management, LLC.[3] The factors are betting-against-beta, value (AQR's definition), small-cap momentum and ESG. The data series represent discrete returns of portfolios whose weights sum up to 1 (long/short plus the risk-free rate). Task of this exercise is to reallocate an

[2] See e.g., Frazzini, A. and Pedersen, L.H. (2014): "Betting against Beta", *Journal of Financial Economics*, 111, p.1-25

[3] Check out `www.aqr.com/Insights/Datasets` for details and more data sets.

investment between different factor portfolios regularly to maximize the expected utility of a CRRA investor. Therefore, assume that the expected utility is described by:

$$\mathrm{E}\, r_p + (1-\gamma)Var(r_p)/2$$

(Lecture Notes 14.2). Hint: discrete returns $\neq$ log returns. Be aware that the AQR factors are given for a different time horizon than the other two data-sets.

a) (8 points) Introduce the equality constraint such that portfolio weights sum up to one and formulate the Lagrangian function. What are the new first order conditions? Write down the formalism that solves the constrained optimization. Hint: Use formula 14.9 in the Lecture Notes.

b) (10 points) Set the risk aversion to $\gamma = 50$. Re-balance your portfolio on a monthly basis and use two-years of historical returns to compute current expected returns and variance. Based on this setting, use your solution from *a)* to caculate the time-varying weights of your optimal portfolio. Visualize your results in a stacked bar plot.

c) (10 points) Report realized mean return, volatility and Sharpe ratio. Run the Fama-French five factor regression (data from Problem 1), report and analyze the regression output. How did your factor-rotation strategy perform?