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ISMT S-136 Time Series Analysis with Python
Part I of Final

Let x_t be the following ARMA(p,q) model:

$$x_t = 0.9x_{t-1} - 0.2x_{t-2} + w_t + 0.1w_{t-1}, \text{ where } w_t \sim wn(0, \sigma_w^2).$$

(a) Identify p and q. Please make sure to explicitly show whether AR and MA polynomials have common factors.

(b) Determine whether x_t is causal and/or invertible. You can use Python to check it, but please demonstrate this analytically by examining the AR and MA polynomials.

SOLUTION:

a) $x_t - 0,9x_{t-1} + 0,2x_{t-2} = w_t + 0,1w_{t-1}$

$(1-0,9B+0,2B^2)x_t = (1+0,1B)w_t$

$\underbrace{(1-0,4B)(1-0,5B)}_{=\phi(B)} x_t = \underbrace{(1+0,1B)}_{=\theta(B)} w_t$

$\Rightarrow$ no factor redundancy, therefore ARMA(2,1)

b) AR polynomial: $\phi(z) = 1-0,9z+0,2z^2$

$0 = 1-0,9z+0,2z^2$

$0 = (1-0,4z)(1-0,5z)$

$z_1 = 2,5 \overset{!}{>} 1 \; ; \; z_2 = 2 \overset{!}{>} 1 \Rightarrow |z_1| \overset{!}{>} 1 \; ; \; |z_2| \overset{!}{>} 1$

$\Rightarrow$ x_t is causal ($\phi(z) \neq 0$ for all $|z| \leq 1$)

MA polynomial: $\theta(z) = 1+0,1z$

$0 = 1+0,1z$

$z = -10 \overset{!}{<} 1 \Rightarrow |z| \overset{!}{>} 1$

$\Rightarrow$ x_t is invertible ($\theta(z) \neq$ for all $|z| \leq 1$)