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ISMT S-136 Time Series Analysis with Python
Part I of Assignment 4

Find the AR and MA representations of x_t if

(a) x_t is the following AR(1) process: $x_t = 0.7x_{t-1} + w_t$ with $w_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_w^2)$

(b) x_t is the following MA(1) process: $x_t = w_t + 0.3w_{t-1}$ with $w_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_w^2)$

SOLUTION:

a) $x_t = 0{,}7 \cdot x_{t-1} + w_t \quad / - 0{,}7x_{t-1}$

$x_t - 0{,}7 x_{t-1} = w_t \quad /$ Backward shift operator

$x_t - 0{,}7 B x_t = w_t \cdot 1$

$\underbrace{(1 - 0{,}7B)}_{\phi(B)} x_t = \underbrace{w_t \cdot 1}_{\theta(B)}$

$\phi(B) x_t = w_t \cdot 1 \xrightarrow{\text{MA-representation}} x_t = \frac{w_t}{\phi(B)} = \frac{1}{\phi(B)} \cdot w_t$ iff $|0{,}7| \leq 1$

↳ process is causal and the MA-representation exists

b) $x_t = w_t + 0{,}3\, w_{t-1}$

$x_t = w_t + 0{,}3 \cdot B w_t$

$x_t = (1 + 0{,}3B) w_t$

$w_t = \frac{x_t}{1 + 0{,}3B} \xrightarrow{\text{AR-representation}} w_t = \sum_{j=0}^{\infty} (-0{,}3)^j \cdot x_{t-j} = x_t + (-0{,}3) \cdot x_{t-1} + (-0{,}3)^2 \cdot x_{t-2} \ldots$

iff $|0{,}3| \neq 1$

↳ process is invertible and the AR-representation exists