NAME: Niklas Kampe

ISMT S-136 Time Series Analysis with Python

Part I of Assignment 3

Let

$$x_t = \sum_{j=-\infty}^{\infty} \psi_j w_{t-j}, \quad \text{where} \quad w_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_w^2).$$

Assuming $\sum_{j=-\infty}^{\infty} \psi_j^2 < \infty$, prove that

(a) $\mathrm{E}[x_t] = 0$

(b) $\gamma(t+h, t) = \sigma_w^2 \sum_{j=-\infty}^{\infty} \psi_{j+h}\psi_j \quad \text{for} \quad h \geq 0$

SOLUTION:

a) from $w_t \overset{\text{iid}}{\sim} N(0, \sigma_w^2)$: $E[w_t] = 0$

$$E[x_t] = E\Big[\sum_{j=-\infty}^{\infty} \psi_j \cdot w_{t-j}\Big] = \sum_{-\infty}^{\infty} E[\psi_j \cdot w_{t-j}] = \sum_{-\infty}^{\infty} E[\psi_j] \cdot E[w_{t-j}] = \sum_{-\infty}^{\infty} E[\psi_j] \cdot 0 = \underline{\underline{0}}$$

b) $$\gamma(t+h, t) = Cov(x_{t+h}, x_t) = Cov\Big(\sum_{k=-\infty}^{\infty} \psi_k \cdot w_{t+h-k}, \sum_{j=-\infty}^{\infty} \psi_j \cdot w_{t-j}\Big)$$

$$= \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \psi_k \psi_j \, Cov(w_{t+h-k}, w_{t-j}) = \sum_{k=-\infty}^{\infty} \psi_k \psi_{k-h} \underbrace{Cov(w_{t+h-k}, w_{t+h-k})}_{\sigma_w^2}$$

$$= \underline{\underline{\sigma_w^2 \cdot \sum_{j=-\infty}^{\infty} \psi_{j+h} \cdot \psi_j}}$$