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ISMT S-136 Time Series Analysis with Python

Part I of Assignment 1

Suppose we observe $x_t = t$ for all $t = 1, 2, \ldots, n$. Show that for any fixed h, the sample ACF $\hat{\rho}(h) \to 1$ as $n \to +\infty$. Please notice that $\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$.

Hint: For any m:

$$\sum_{t=1}^{m} t = \frac{m(m+1)}{2} \text{ and } \sum_{t=1}^{m} t^2 = \frac{m(m+1)(2m+1)}{6}.$$

SOLUTION:

$$\hat{\rho}(h) = Corr(x_{t+h}, x_t) = \frac{Cov(x_{t+h}, x_t)}{\sqrt{Var(x_{t+h}) \cdot Var(x_t)}} = \frac{\hat{\gamma}(t+h, t)}{\sqrt{\hat{\gamma}(h,h) \cdot \hat{\gamma}(t,t)}}$$

$$\hat{\gamma}(h) = \hat{\gamma}(t+h, t) = Cov(t+h, t) = E[(x_{t+h} - \bar{x})(x_t - \bar{x})]$$

$$\bar{x} = \frac{1}{n} \cdot \sum_{t=1}^{n} t = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}, \quad \lim_{n\to\infty} \frac{n+1}{2} = \infty$$

$$\Rightarrow \text{ as } \bar{x} \to \infty: \hat{\gamma}(h) = E[(x_{t+h} - \bar{x})(x_t - \bar{x})] \to \infty \Rightarrow \lim_{n\to\infty} \hat{\gamma}(h) = \infty$$

$$\Rightarrow \text{ also: } \lim_{n\to\infty} \gamma(h,h), \gamma(t,t) = \infty$$

$$\Rightarrow \underline{\hat{\rho}(h) = \frac{\hat{\gamma}(t+h,t)}{\sqrt{\hat{\gamma}(h,h) \cdot \hat{\gamma}(t,t)}} = \frac{\infty}{\infty} = 1 \quad \text{for } n \to \infty}$$