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ISMT S-136 Time Series Analysis with Python

Part I of Midterm

Suppose x_t is a stationary process with mean μ_x and autocovariance function $\gamma_x(h)$, $h = 0, \pm 1, \pm 2, \ldots$

Let s_t be a seasonal (deterministic) component with period 12, that is, $s_{t+12} = s_t$ for all t. Assume s_t is not a constant, i.e. $s_1, s_2, \ldots, s_{12}$ are not all equal.

(a) Let $y_t = s_t + x_t$ for all t. Determine whether y_t is stationary. If y_t is stationary, express its mean μ_y and autocovariance function $\gamma_y(h)$ in terms of μ_x, $\gamma_x(h)$, and s_t.

(b) Let now $z_t = (1 - B^{12})y_t$ for all t, where B is the backshift operator. Determine whether z_t is stationary. If z_t is stationary, express its mean μ_z and autocovariance function $\gamma_z(h)$ in terms of μ_x, $\gamma_x(h)$, and s_t.

SOLUTION:

a) $y_t = s_t + x_t = s_{t+12} + x_t$

$E[y_t] = E[s_{t+12} + x_t] = s_{t+12} + E[x_t] = s_t + \mu_x = \mu_y$

$\Rightarrow$ independent of t

$Var(y_t) = Var(s_{t+12} + x_t) = Var(s_{t+12}) + Var(x_t) \Rightarrow$ independent of t

$\Rightarrow y_t = \underline{\underline{\text{weakly stationary}}}$

from before: $\underline{\underline{\mu_y = s_t + \mu_x}}$

$\gamma_y(h) = Cov(s_t + x_t, s_{t-h} + x_{t-h}) = Cov(s_t + x_t, s_{t-12-h} + x_{t-h})$

$= \underbrace{Cov(s_t, s_{t-12-h})}_{0} + \underbrace{Cov(s_t, x_{t-h})}_{0} + \underbrace{Cov(x_t, s_{t-12-h})}_{0} + Cov(x_t, x_{t-h})$

$= \underline{\underline{\gamma_x(h)}}$

b) $z_t = (1-B^{12})y_t = (1-B^{12})(s_t + x_t) = (1-B^{12})(s_{t+12} + x_t)$

$\Rightarrow$ as y_t is stationary: differenced y_t $(= z_t)$ also $\underline{\text{stationary}}$

$\mu_z = E[(1-B^{12})(s_t + x_t)] = (1-B^{12}) \cdot E[s_t + x_t] = (1-B^{12}) \cdot \mu_y$

$= \underline{\underline{(1-B^{12})(s_t + \mu_x)}}$

$\gamma_z(h) = Cov((1-B^{12})(s_{t+12} + x_t), (1-B^{12})(s_{t+12-h} + x_{t-h}))$

$= (1-B^{12})^2 \cdot Cov(s_{t+12} + x_t, s_{t+12-h} + x_{t-h})$

from before $= \underline{\underline{(1-B^{12})^2 \cdot \gamma_x(h)}}$