JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Collection of Nuclear astrophysics exercises

FYSS3440

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ESA/Hubble, NASA

Contents

Introduction

0.1 | DISCLAIMER

DISCLAIMER: This is a collection of exercises to help you with the exams of FYSS3440. These notes are not meant to replace the lecture notes or the demo sessions where the full solution to the exercises was discussed and presented. The current collection hasn't been through any evaluation and might contain typos or/and mistakes. Use as a supplement to your notes. Please refer mistakes you might find (typo or solution mistake) at stynikas@jyu.fi.

0.2 | Purpose

This is a collection of the exercises and the solutions for the nuclear Astrophysics course FYSS3440. In this course, we covered the basic concepts of nuclear astrophysics. This is not a stand-alone guide to the Nuclear astrophysics course but rather an effort to summarize the most important concepts through some exercises. We will usually refer to codes that we used. These can be found either in their original source that would be cited or at kone.phys.jyu.fi computer cluster. Each chapter corresponds to an exercise set.

list of symbols

Table 0.1: List of symbols

Symbol	dimension	Unit	Value (constants)
X_i	Mass fraction	-	
Y_i	Abundance	-	
n_i	number density	cm^{-3}	
N_i	number of species	-	
N_A	Avogadro's numbers	mol^{-1}	$6.02214076 * 10^{23}$
ρ	Density	$g \cdot cm^{-1}$	

1 | Introduction to nuclear astrophysics and Astrophysical aspects for nuclear astrophysics

1. Let's consider astrophysical plasma, which contains 820 g 1H, 200 g 4He and 30 g 15O, and assume that the mass density is constant throughout the plasma. You will need to use $NA = 6.0221409 * 10^{23} mol^{-1}$.

- i. Calculate mass fraction X, abundance Y for ^{1}H, ^{4}He and ^{15}O.
- ii. What is the electron abundance Ye of the plasma?
- iii. For a density of $1.8x10^{6} g/cm^{3}$, calculate the electron density and the number of helium nuclei per cm^{3}.

Answer:
We can define the mass fractions X as:

$$X_i = \frac{m_i}{m_{tot}} \tag{1.1}$$

i.e. for Hydrogen this yields:

$$X_{^1H} = \frac{m_{^1H}}{m_{tot}} = \frac{820g}{(820 + 200 + 30)g} = 0.781$$

Similarly:

$$X_{^4He} = 0.190, X_{^{15}O} = 0.029$$

We can verify our results by summing over the mass fractions:

$$\sum_i m_i = 1 \tag{1.2}$$

We can now define the abundance Y_i in terms of the mass fraction X_i as:

$$Y_i = \frac{X_i}{A_i} \tag{1.3}$$

i.e.

$$Y_{^1H} = \frac{X_{^1H}}{X_{^1H}} = \frac{0.781}{1.} = 0.781$$

similarly:

$$Y_{^4He} = 0.0476,$$
$$X_{^{15}O} = 0.029$$

To calculate the Y_e one needs to apply the conservation of charge. The amount of protons and electrons in the plasma is the same. thus, we can simply calculate the abundance of proton atoms:

$$Y_e = Y_p = \sum_i Y_i Z_i, \tag{1.4}$$

Which yields $Y_e = 0.891$.
To calculate the electron density we need to use the Avogadro's number then n_i is defined as:

$$n_i = \rho N_A Y_i, \tag{1.5}$$

yielding:

$$n_e = 9.663 * 10^{23} cm^3,$$
$$n_{^4He} = 5.161 * 10^{28} cm^3.$$

---

2. Let's consider similar plasma as in problem 1. Temperature of the plasma has suddenly become higher, and it is hot enough for ^{15}O nuclei to capture an alpha particle: $^{15}O +^{4} He \Longleftrightarrow^{19} Ne$.

- i. Calculate mass fractions and abundances for different constituents of the plasma after all ^{15}O nuclei have been destroyed by alpha captures.

- ii. Did the electron abundance Ye change? If it changed, explain why, and give the new value.
- iii. Did the electron density change?

Answer:
Since all the ^{15}O particles recombined with α particles, $X_{^{15}O} = 0$, $Y_{^{15}O} = 0$. Furthermore the abundance and mass fraction oh H remains unchanged. To find the amount of α particles that were used we need to calculate the number of each species in the plasma:

$$N_i = \frac{m_i}{A_i m_u} \approx \frac{m_i}{A_i} N_A \tag{1.6}$$

Before the reaction:

$$N_{^{15}O} = 1.2 * 10^{24}, \text{ and } N_{^4He} = 3 * 10^{25}.$$

After the reaction we will have:

$$N_{^4He} = 3 * 10^{25} - 1.2 * 10^{24}, \text{ and } N_{^{19}Ne} = 1.2 * 10^{24}.$$

The mass of these species is now given by:

$$N_i = \frac{m_i}{A_i m_u} \approx \frac{m_i}{A_i} N_A \tag{1.7}$$

Yielding:

$$m_{^4He} = 192g \text{ and } m_{^{19}Ne} = 38g$$

The calculation of X_i and Y_i becomes then trivial for ^4He and ^{19}Ne:

$$X_{^4He} = 0.183 \text{ and } X_{^{19}Ne} = 0.036$$
$$Y_{^4He} = 0.046 \text{ and } Y_{^{19}Ne} = 0.002$$

Due to the conservation of charge and mass the absence of weak reactions the total number of protons-electrons in the plasma remains the same and thus the n_e and Y_e remain both unchanged.

---

3. Let's consider the plasma from problem 1 again. This time the temperature is lower so that 15O will decay completely to ^{15}N via $\beta+$ decay: $^{15}O \Longleftrightarrow ^{15} N + \nu_e + e^+$.

- i. Calculate mass fractions and abundances for different constituents of the plasma after all ^{15}O nuclei have been destroyed to ^{15}O.
- ii. Did the electron abundance Ye change? If it changed, explain why and give the new value.
- iii. Did the electron density change?

Answer:
In contrast to problem 2. there is a weak reaction involved. In this case protons are converted to neutrons, so we expect the Y_e to be smaller accordingly. Since all the ^{15}O is depleted we can assume, ignoring the loss of the positrons and neutrinos, that $m_{^{15}O} = m_{^{15}Ne}$.
Therefore $X_{^{15}Ne} = 0.029$, $Y_{^{15}Ne} = 0.002$. calculating Y_e using Eq. 1.4 we get:

$$Y_e = 0.0889$$

---

4. In stellar plasma, excited states are thermally populated and can undergo beta decay to the ground or excited states of the daughter nucleus. As a result, the beta-decay half-life can be quite different in stellar environment compared to the half-lives measured in laboratories. For a given nuclide in a non-degenerate plasma at thermodynamic equilibrium, the thermal population probability of a state i is:

$$P_i = \frac{N_i}{N_{tot}} = \frac{g_i e^{-E_i/kT}}{\sum_i g_i e^{-E_i/kT}}, \tag{1.8}$$

where $g_i = 2J_i + 1$ is the statistical weight, J_i is the spin and E_i is the excitation energy of the state i. k is the Boltzmann constant and T is the plasma temperature. The denominator(the sum over all states i) is called the partition function G. Calculate the stellar half-life of 26Al when the plasma temperature is $T = 2GK$. ^{26}Al has a 5+ ground state with a half-life of $7.17x10^5y$ while the first excited, 0+ state at $228keV$ decays via superallowed beta decay with a half-life of $6.345s$ to the ground state of $26Mg$ (see figure). Above $T > 0.4GK$ these states are in thermal equilibrium. You will need the Boltzmann's constant expressed in MeV: $k = 0.0862MeV/GK^{-1}$.

Answer:

We can calculate the stellar decay constant as:

$$\lambda^* = P_{g.s.}\lambda_{g.s.} + P_m\lambda_m \tag{1.9}$$

The probability to decay P can be expressed as:

$$\begin{aligned} P_{g.s.} &= \frac{Ng.s.}{N_{tot}} = \frac{g_{g.s.}e^{-E_{g.s.}/kT}}{g_{g.s.}e^{-E_{g.s.}/kT} + g_m e^{-E_m/kT}} \\ P_m &= \frac{Nm}{N_{tot}} = \frac{g_m e^{-E_m/kT}}{g_{g.s.}e^{-E_{g.s.}/kT} + g_m e^{-E_m/kT}} \end{aligned} \tag{1.10}$$

We can calculate the partition functions:

$$\begin{aligned} g_{g.s.} &= 2J_{g.s.} + 1 = 11 \\ , g_m &= 2J_m + 1 = 1. \end{aligned} \tag{1.11}$$

Now $E_{g.s.} = 0MeV$ and $E_m = 0.228MeV$, $T = 2GK$, $k = 0.0862MeV/GK$ Then we get $P_{g.s} = 0.9764$ and $P_m = 0.0236$. Now we can express the half life:

$$T_{1/2} = ln(2)/\lambda^*_\beta \tag{1.12}$$

where expressing all half lifes in seconds we can write:

$$\lambda_\beta* = \frac{0.9764ln(2)}{6345s} + \frac{0.0236ln(2)}{6.345s} = 0.002584s^{-1}, \tag{1.13}$$

yielding, $T_{1/2} = 268s$.

---

5. Express the luminosity L, temperature T and radius R of a main sequence star with i. $M1 = 3M\odot$ and ii. $M2 = 0.3M\odot$ in terms of solar luminosity $L\odot$, surface temperature $T\odot$, radius $R\odot$.

Answer:

We know that:

$$L \propto M^4 \tag{1.14}$$

Therefore we can write:

$$\frac{L_{star}}{L\odot} = \frac{M^4_{star}}{M\odot^4} \xrightarrow{L_{star}} = \frac{M^4_{star}}{M\odot^4} L\odot \tag{1.15}$$

yielding:

$$\begin{aligned} i.L_{star} &= 81L\odot, \\ ii.L_{star} &= 0.0081L\odot \end{aligned}$$

To estimate the surface temperature we have 2 options. first is to use the H-R diagram (Fig. **??**) Alternatively, we can use the approximation that $L \propto T^{5.5}$ (fitting a line on the main sequence). Solving in terms of T we find:

$$T \propto L^{0.18} \tag{1.16}$$

We can write:

$$T_{star} = \frac{L^{0.18}_{star}}{L\odot^{0.18}} T\odot \tag{1.17}$$

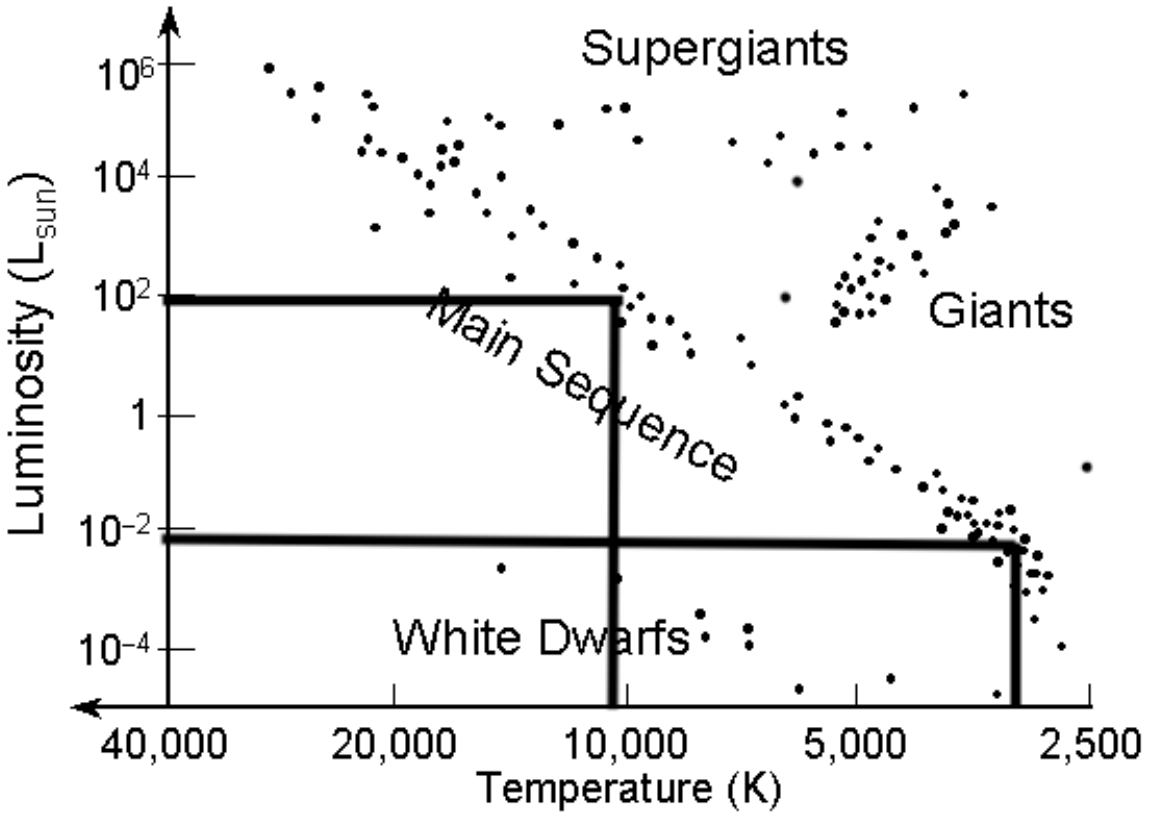

Figure 1.1: H-R diagram

Yielding for:

$$i. L_{star} = 81 L\odot \rightarrow \frac{(81 L\odot)^{0.18}}{L\odot^{0.18}} = 81^{0.18} T\odot = 2.2 T\odot \approx 13000 K,$$
$$ii. L_{star} = 81 L\odot \rightarrow \frac{(81 L\odot)^{0.18}}{L\odot^{0.0018}} = 0.0018^{0.18} T\odot = 0.42 T\odot \approx 2500 K,$$

We can finally find the radius from the Eq. :

$$\frac{L_{star}}{L\odot} = \frac{R^2_{star}}{R^2\odot} \frac{T^4_{star}}{T^4\odot} \tag{1.18}$$

and we can express the radius as:

$$R_{star} = \sqrt{\frac{L_{star}}{L\odot}} \frac{T^2\odot}{T^2_{star}} \tag{1.19}$$

and we get:

$$\text{i.} R_{star} = 1.82 R\odot$$
$$\text{ii.} \ R_{star} = 0.52 R\odot$$

---

6. Fig. 1.2 shows the solar abundances according to Lodders et al. Fig. 1.3 shows the binding energies per nucleon. The abundance pattern of Fig. 1 exhibits some very interesting features. Can you use Fig. 2 to explain them?

- i. The odd A nuclei are less abundant. Why is this happening?
- ii. The sharp peak at the area $50 < A < 60$. Why does the peak exist in this region? Which elements contribute to the creation of the peak structure?

- iii. The sudden drop of abundances after the peak at $50 < A < 60$.

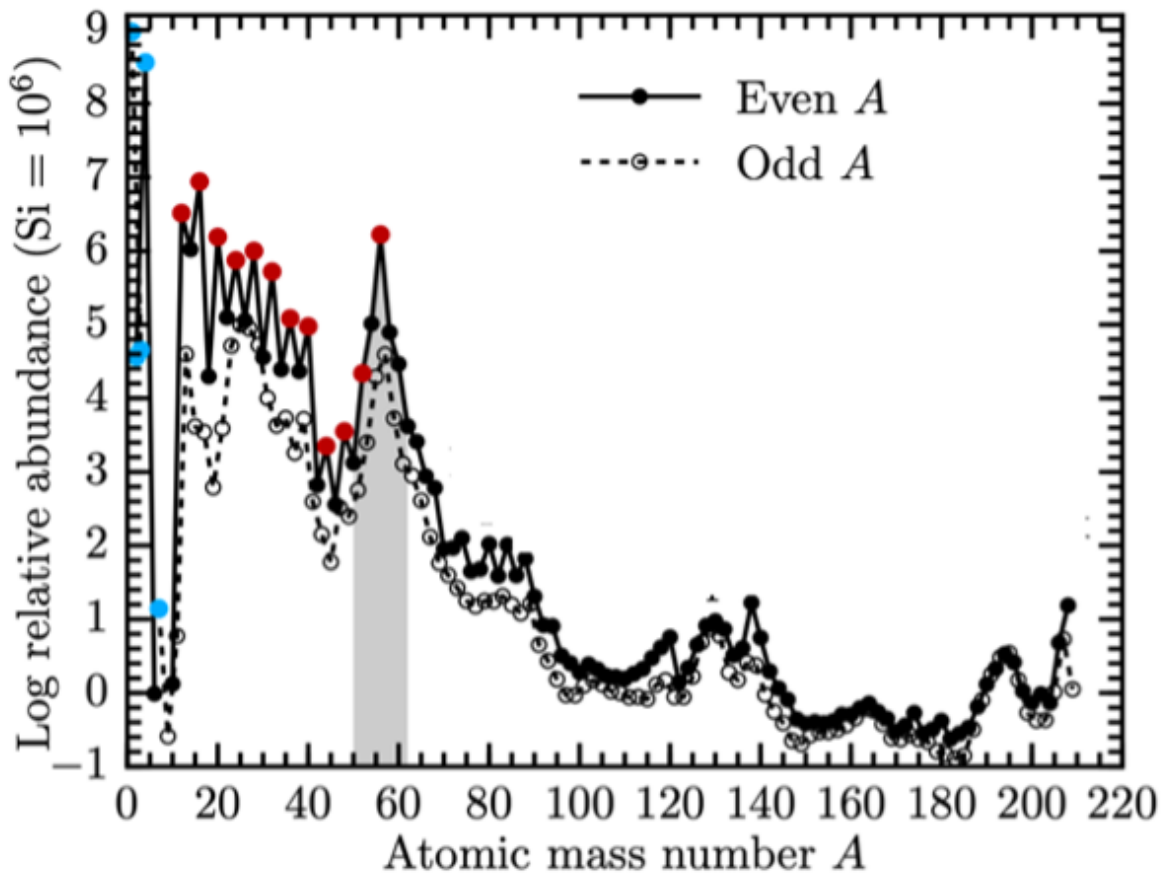

Figure 1.2: Solar abundances. Figure from [5]

Answer:
We observe that Fig. 1.3 B/A (binding energy per nucleon)is increasing as a function of A until we reach the iron (Fe) region. In the same moment, we observe in Fig. 1.2 that abundances peak in the same region. Anything heavier than Fe is much less abundant. When Fe is produced in the star no further energy can be extracted from nuclear fusion. Heavier elements cannot be made in stars through fusion, but through other processes i.e. neutron capture processes in extreme scenarios (explosive nucleosynthesis).

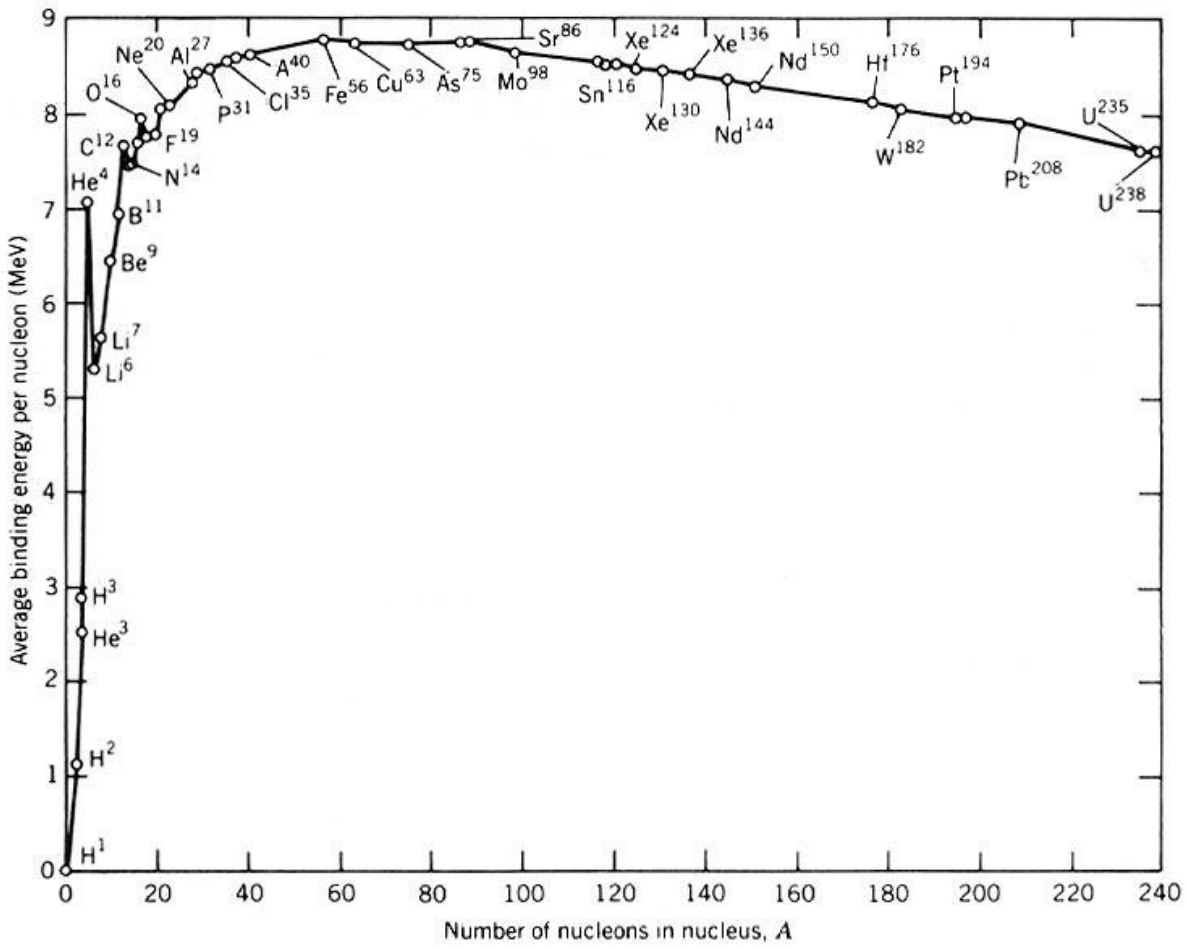

Figure 1.3: Binding energy/A vs A

2 | Reaction rates

1. Attempting to understand the curve of binding energies (B) vs A leads us to the semi-empirical mass formula which employs a few parameters to characterize the variation of B vs A. The semi-empirical mass formula.

- a) The first term is called volume term and reads $B = \alpha_\nu A$. What does the linear dependence of B to A mean? Think about the strong force. What would happen if strong force could act over longer distances? How would you write the volume term?
- b) The second term is the surface term $-a_s A^{2/3}$ has a negative sign. Why do we include this correction factor?
- c) Third term is the Coulomb term we can approximate Coulomb term as $-a_c Z(Z-1)/A^{1/3}$. Why do we include this term? Explain the Z(Z-1) component.
- symmetry term accomplish? What configuration does it favor?
- e) Finally, we introduce a pairing term δ. δ is expressed as $\delta = 0$ for odd A, $\delta = a_p A^{-3/4}$ for even Z even N and finally $\delta = -a_p A^{-3/4}$ for an odd N odd Z nucleus. What does this term account for?
- f) Plot each term progressively (i.e. first term, first term, and second term, etc) per nucleon vs A.

Assume: $\alpha_\nu = 15.5MeV, -a_s = 16.8MeV, -a_c = 0.72MeV, -a_{sym.} = 23MeV, a_p = 34MeV$. Compare the results you get with the binding energy/nucleon vs A. Is the liquid drop model a good assumption?
Answer:

- Due to the short range of strong force each nucleon interacts only with the neighboring nucleons. As the volume increases (with the increasing number of nucleons), more nucleons have interactions with their close neighbors, so the binding energy increases linearly. If the range of strong force was longer, the nucleons would interact with all the other nucleons in the nucleus. Thus, there would be $\approx 1/2 * A(A-1)$ pairs that would interact with each other, and the volume term would be proportional to A^2.
- The surface term is a correction to the volume term as it does not take into account the nucleons which are near the surface, which do not have the same amount of neighbors to interact with, thus the negative sign.
- The Coulomb term is needed to take into account the electrostatic repulsion between protons in the nucleon. Thus it has a negative sign as it decreases the binding energy caused by the strong force. As neutrons have no net charge, and thus are not affected by the Coulomb force, the term only includes protons.
- The asymmetry term favours the nucleons which have an equal or nearly equal number of protons and neutrons as this term has a minus sign as well. As nucleons are fermions, two of them cannot reside in the same state, instead, they need to occupy higher energy states.
- The pairing term takes into account the spin interaction as coupled spins are energetically favoured.

The semi-empirical mass formula gives a rather good estimation of stable nuclei binding energies when all the corrections are taken into account.
The comparisons can be seen in Fig. 2.1.

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2. Complete the following reactions: a.

- i. $^{27}Al + p \rightarrow ... + n$
- ii. $^{32Si} + \alpha \rightarrow ... + \gamma$
- iii. $^{197}Au +^{12} C \rightarrow ... + \alpha$
- iv. $^{116}Sn + ... \rightarrow^{116} Sn + p$
- v. $^{56}Ni + n \rightarrow ... + \gamma$

b. Calculate the Q value of these reactions. You can find the masses of these isotopes here AME20

Answer:

- i. $^{27}Al + p \rightarrow^{27} Si + n$
  $Q = (26.981538408 + 1.007825031898 - 26.986704687 - 1.00866491590)uc^2 = -5.59MeV$
- ii. $^{32Si} + \alpha \rightarrow^{36} Si + \gamma$
  $Q = (31.974151538 + 4.00260325413 - 35.967080692)uc^2 = 9.01MeV$
- iii. $^{197}Au +^{12} C \rightarrow^{205} Bi + \alpha$
  $Q = (196.966570103 + 12.0000000 - 204.977385182 - 4.00260325413)uc^2 = -12.5MeV$
- iv. $^{116}Sn + p \rightarrow^{116} Sn + p$
  $Q = (115.901742825 + 1.007825031898 - 115.901742825 - 1.007825031898)uc^2 = 0.0MeV$
- v. $^{56}Ni + n \rightarrow^{57} Ni + \gamma$
  $Q = (55.942127761 + 1.00866491590 - 56.939791394)uc^2 = 10.2MeV$

3.Talys Calculation: Talys is a Hauser Feshbach statistical code, and it can be used to calculate cross sections and reaction rates. Talys can run simple. You need an input file which should include at least these keywords:

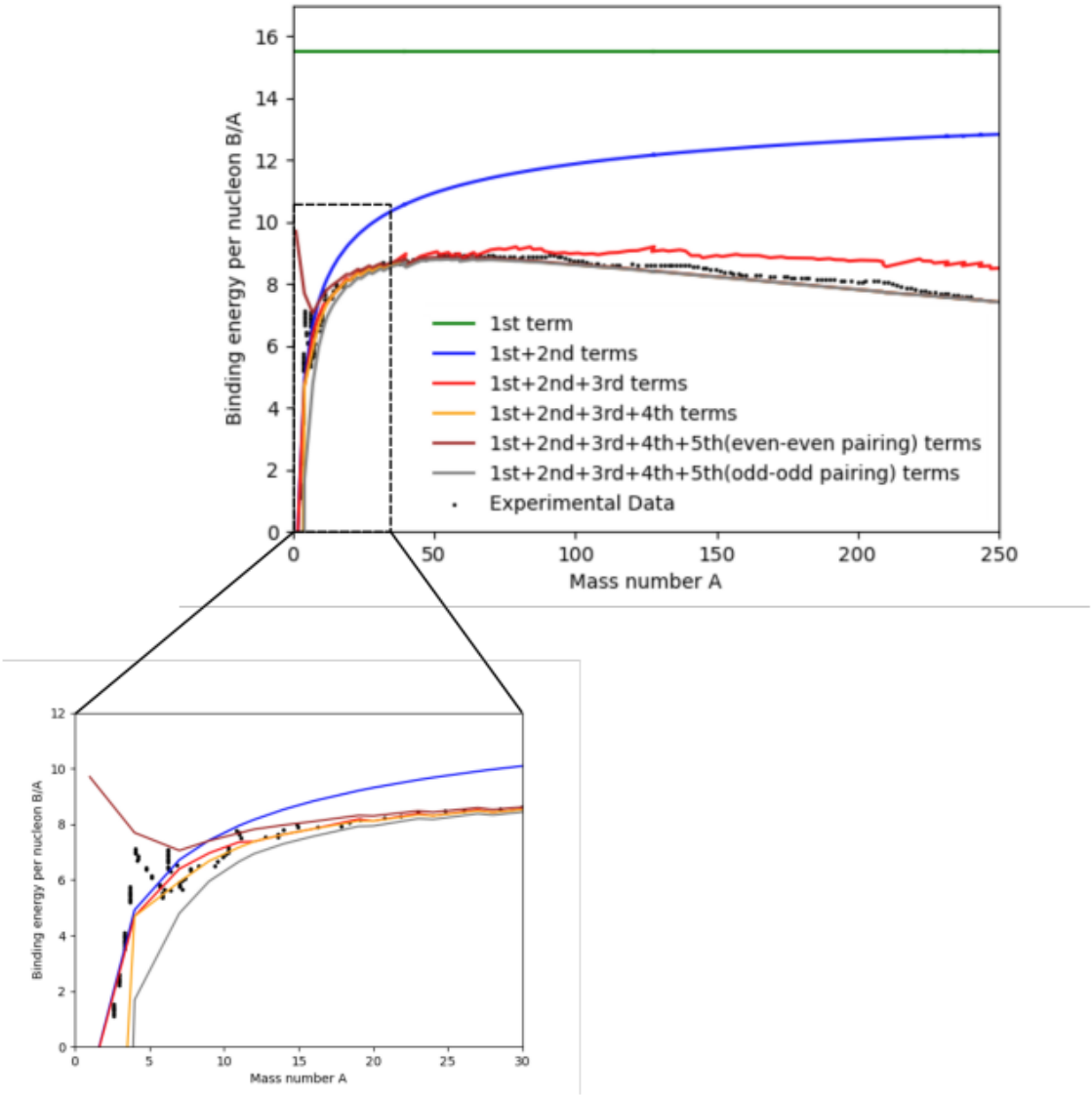

Figure 2.1: Liquid drop mass formula compared with experimental data. Fig. by A. Jaries

- Energy X where X the energy in MeV.
- Projectile (n, p, alpha, d, gamma)
- Element (Z number of the target)
- Mass (mass number of the target)

Talys code can be found here: TALYS [4] and can be installed in Linux or Mac OS machines following the relatively easy installation instructions. You can then run talys by simply typing in the command line:

```
talys<input>output.
```

Alternatively you can connect to the university server via command line at: kone.phys.jyu.fi with your username i.e.

```
ssh stynikas@kone.phys.jyu.fi
```

and your password. You do not need to sign-up you already have an account. After you connect run the following commands:

```
cd /automisc/data/FYSS4456/talys/students
mkdir 'Your_name_last_name'
cp talys /'Your_name_last_name'
cd 'Your_name_last_name'
talys<input>out_'your'name'
```

The results of your calculation are now on the

```
out_'your'name'file.
```

- Perform a simple calculation of the reaction 56Ni(n,g) from previous exercise with Energy 1 MeV. Check that everything works fine. Give the calculated cross section.
- Include the keyword astro y. This will enable calculations of reaction rates for a range of T such that $0.001 < T < 10GK$. Repeat step a).
- Bruslib is an online database of reaction rates http://www.astro.ulb.ac.be/bruslib/. Find the reaction rates of 56Ni. Compare your calculations with the ones from Bruslib.
- The JINA Reaclib is a reaction rate library using a seven-parameter fit to store reaction rates. The reaction then is given by:

$$\lambda = exp[a_0 + \sum_{i=1}^{i=5}(a_i T9^{(2i-5)/3}) + a6lnT9] \tag{2.1}$$

  Download the latest version find the 56Ni(n,g) and use the Eq. 2.1 to retrieve reaction rates in the range of T=0.001 GK and T=10.0 GK. Are the rates the same?
- In Talys you can include direct contributions. Include the keyword racap y and redo the calculation. Is the calculated reaction rate higher in this case?
- Plot all the results together and label each one of the lines.

Answer:
When the direct contribution is enabled we see a slight increase in the calculated reaction rate. Differences between a factor of 3 in most cases can be observed between the calculated reaction rates and online libraries, as the result of using different implementations of statistical quantities such as optical potentials, nuclear level densities and γ - strength functions. The calculated reactions and the comparison to the online libraries can be seen in Fig. 2.2. The routines used to plot the reaction rate can be found in the appendix

4. Calculate the resonant proton capture rate for $^{56}Ni(p,\gamma)^{57}Cu$ leading to a $1/2^-$ state at $E_x = 1106(4)$

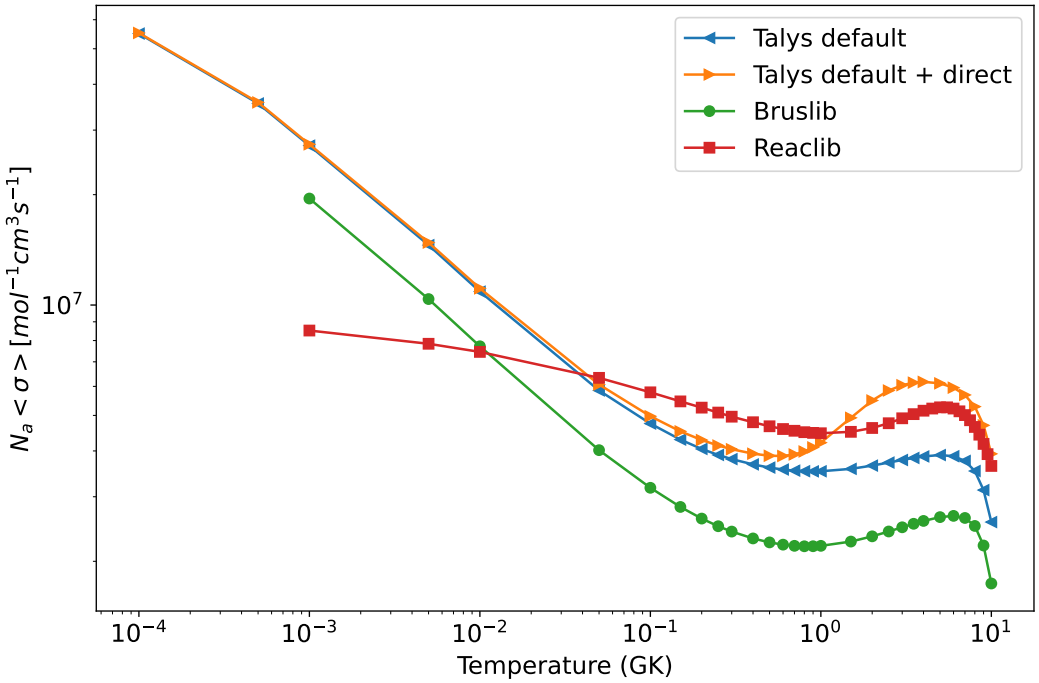

Figure 2.2: Calculated reaction rates using Talys, with and without direct contributions, compared to the online libraries Bruslib and reaclib.

keV 1n ^{57}Cu. Assume temperature $T = 0.7GK$. a. With a JYFLTRAP value $Q_{p,\gamma} = 689.7(5)kev$,$\Gamma_\gamma = 4.23 \times 10^{-3}eV$ and $\Gamma_p = 2.26 * 10^{-7}eV$. b. With the AME2012 values.

Answer:

Assuming a narrow resonant we can write:

$$N_A < \sigma v >= 1.54 \times 10^{11}(\mu T_9)^{-3/2}(\omega\gamma)e^{\frac{-11.605E_r}{T_9}}\frac{cm^3}{s \times mol}. \tag{2.2}$$

First we need to calculate the reduced mass:

$$\mu = \frac{M_{^{56}}NiM_{^1}H}{M_{^{56}}Ni + M_{^1}H} = 0.98998187u, \tag{2.3}$$

and

$$E_r = E_x - S_p = (1106 - 689.7)keV = 416keV = 0.4163MeV. \tag{2.4}$$

We can then calculate ω_γ:

$$\omega_\gamma = \frac{\Gamma_p\Gamma_\gamma}{\Gamma_p + \Gamma_\gamma} = \frac{2J_\gamma + 1}{(2J_p + 1)(2J_\gamma + 1)} = 2.26 \times 10^{-13}MeV \tag{2.5}$$

Now we can write:

$$N_A < \sigma v >= 1.54 \times 10^{11}(0.98245u0.7GK)^{-3/2}(2.25987 \times 10^{-13}MeV)e^{\frac{-11.605\times0.4162}{0.7GK}} = 6.139 \times 10^{-5}\frac{cm^3}{s \times mol} \tag{2.6}$$

Following the same procedure with AME2012 values we find $Q = 0.415663936MeV$, and we get:

$$N_A < \sigma v >= 6.204 \times 10^{-5}\frac{cm^3}{s \times mol} \tag{2.7}$$

5. Calculate the Gamow peak and Gamow window for the 16O + 16O at T=1GK.

Answer:

We can calculate the Gamow peak as:

$$E_0 = 0.122(z_1^2 Z_2^2 \mu T_9^2)^{1/3} = 0.122(8^2 8^2 \frac{16 \cdot 16}{16+16}(1GK)^5)^{1/6} = 1.33954 = 1.340 MeV \tag{2.8}$$

where μ is the reduced mass. So we can calculate the Gamow window as:

$$E_0 \pm \Delta = 3.410 \pm 1.34 MeV \tag{2.9}$$

---

6. Why do we use reaction rates and not cross-sections? How is the reaction rate defined?
In stellar interiors, nuclides not only exist in their ground states but also in different thermally excited states and a thermodynamic equilibrium holds locally to a very good approximation [3]. Therefore, most of the nuclear astrophysics calculations have made use of nuclear reaction rates evaluated within the statistical model. The assumption of a thermodynamic equilibrium combined with the compound nucleus cross-sections for the various excited states then allows to produce Maxwellian-averaged reaction rates, which is an important input for stellar evolution models.
The effective stellar rate of $\alpha \rightarrow \alpha'$ in the entrance channel at temperature T taking due account of the contributions of the various target excited states is expressed as:

$$N_A < \sigma v > (T) = \left(\frac{8}{\pi m}\right)) \frac{N_A}{(kT)^{3/2} G(T)} \int_0^\infty \sum_\mu \frac{2J_\mu + 1}{2I_0 + 1} \times \sigma^\mu_{\alpha\alpha'}(E) exp\left(- \frac{E + E^\mu_x}{kT} \right) dE. \tag{2.10}$$

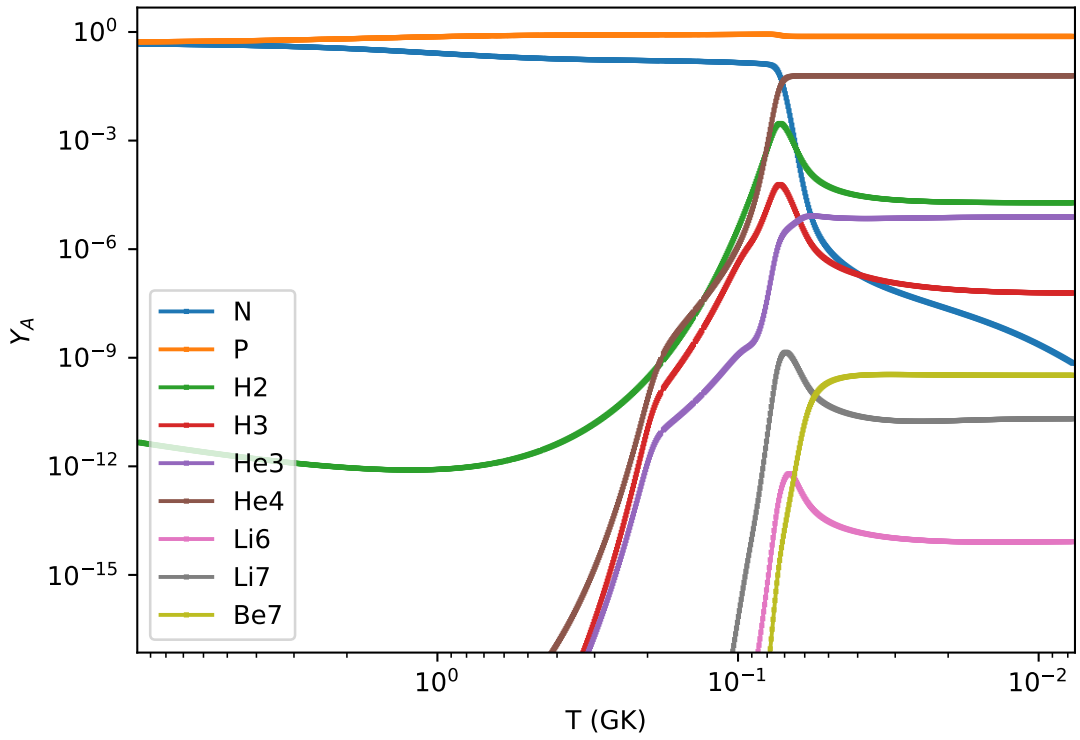

Figure 3.1: Abundances vs T of the code big bang nucleosynthesis code Parthenope. The associated routine for this plot is given in the appendix (A.3)

3 | Big Bang nucleosynthesis - Hydrogen burning

1. Big bang nucleosynthesis is responsible for the creation of H, He, and traces of Li and Be. The code Parthenope can calculate the abundances of elements created in the Big Bang nucleosynthesis. To run the code login in to kone:

```
ssh your_username@kone.phys.jyu.fi
```

You will find the code under the directory:

```
cd /automisc/data/stynikas/parthenope3.0
```

and follow the next steps to create a directory with a copy of the code

```
mkdir name_lastname
cd name_lastname
cp ../* .
./parthenope3.0
```

After executing the code you will get the following options: (i) (e) option I is more user-friendly and comes with instructions. Use this option. Follow the instructions and run the code for the default parameters set. The file nuclides3.0.out contains the corresponding calculated abundances. Plot together N, P, H2, H3, He3, He4, Li6, Li7, Be7 as a function of temperature (T). The results you acquired are the latest estimations for the production of these elements.

Answer:
The results of the Big Bang Nucleosynthesis code (BBN) are presented in Fig. 3.1. The associated routine for this plot is given in the appendix (A.3). We observe as the temperature drops neutrons and protons start to recombine creating mostly ^4He but also traces of heavier nuclei (Li,Be).

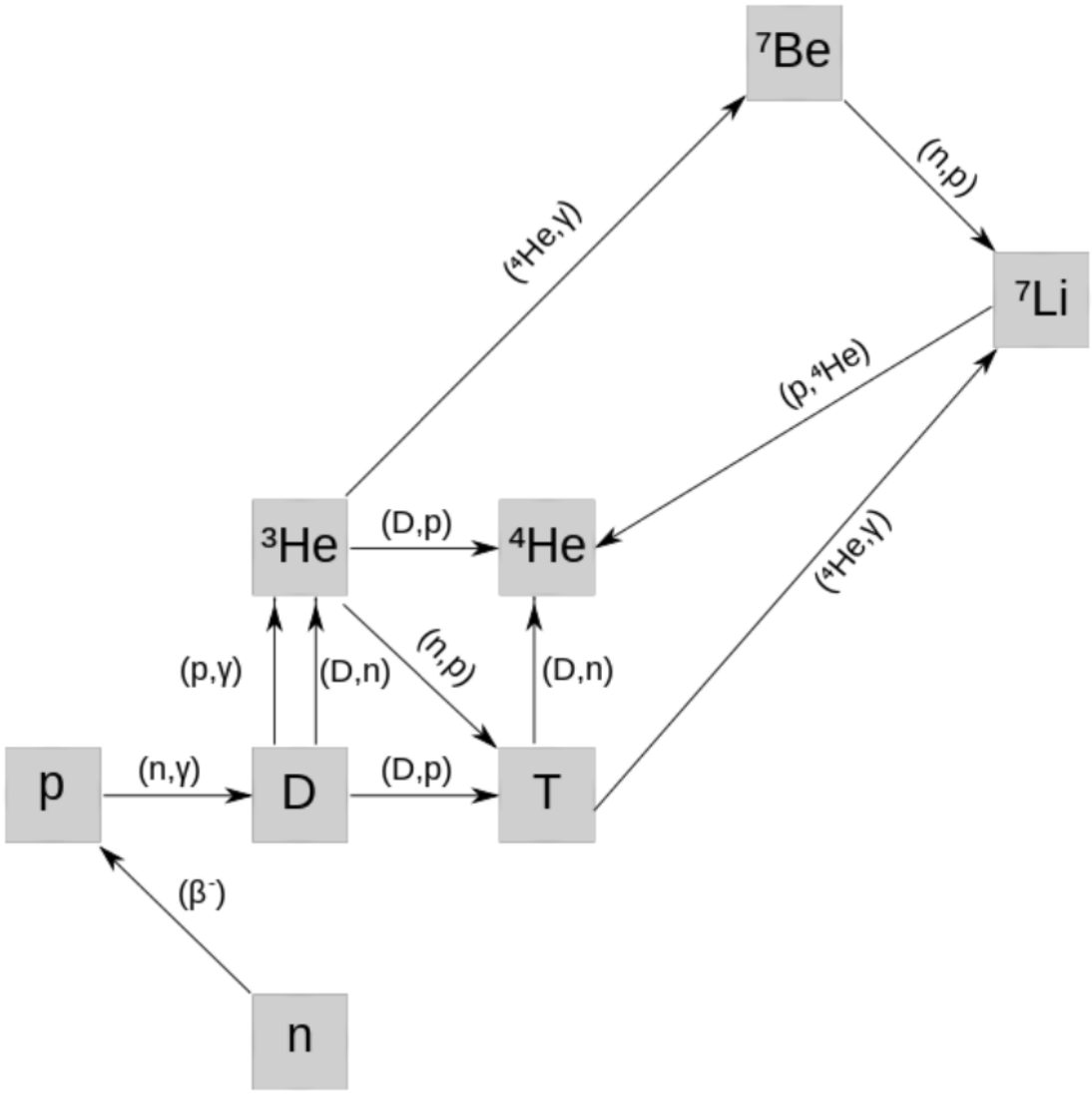

Figure 3.2: Big Bang nucleosynthesismain nuclear reaction chains

2. The code you used is a small nuclear reaction network code. In principle, the abundance of each element can be described at any point by an ordinary differential equation. Write the O.D.E. for the abundance of 4He. Use Fig. 1 to see the main reaction channels. (see also lecture notes about equilibrium to get some inspiration)

Answer:
The exercise indicates to use Fig 3.2 ro write the differential equation. In this figure we only have arrows towards ^4He indicating only creation channels for ^4He. We can write a general O.D.E. as:

$$\frac{dN_i}{dt} = \left[\sum_{j,k} N_j N_k < \sigma v >_{jk\to i} + \sum_l \lambda_{\beta,l\to i} N_l + \sum_m \lambda_{\gamma,m\to i} N_m\right] - \left[\sum_n N_n N_i < \sigma v >_{ni} + \sum_o \lambda_{\beta,i\to o} N_i + \sum_p \lambda_{\gamma,i\to p} N_i\right]. \tag{3.1}$$

Furthermore, if nonidentical particles ($j \neq k$) create two nuclei, then $N_j N_k < \sigma v >_{jk\to i} \to 2N_j N_k < \sigma v >_{jk\to i}$ else if the particles are identical $j = k$) $N_j N_k < \sigma v >_{jk\to i} \to N_j N_k < \sigma v >_{jk\to i} \cdot 1/2$
In this case non only ^7Li creates two nuclei of ^74He via the:

$$^{7}Li + p = 2^{4}He, \tag{3.2}$$

and the other 2 reactions listed in Fig 3.2 are:

$$\begin{aligned} T + D =^{4} He + n, \\ ^{3}He + D =^{4} He + p \end{aligned} \tag{3.3}$$

No γ induced photo-dissociations or decays are mentioned. Therefore the O.D.E. for ^{4}He can be written as:

$$\frac{d^{4}He}{dt} = \left[N_D N_T < \sigma v >_{DT \rightarrow ^{4}He} + N_D N_{^{3}He} < \sigma v >_{D^{3}He \rightarrow ^{4}He} + 2 \cdot N_{^{7}Li} N_p < \sigma v >_{D^{7}Li \rightarrow ^{4}He} \right] \tag{3.4}$$

---

3. About 3 s after the onset of the Big Bang, the neutron-proton ratio became frozen when the temperature was still as high as 10^{10} K ($kT = 0.8$ MeV). About $250s$ later, fusion reactions took place converting neutrons and protons into ^{4}He. Essentially all neutrons were converted to ^{4}He. Calculate the abundances of H and ^{4}He after the primordial nucleosynthesis. The neutron half-life is 10.24 min and the neutron-proton mass difference is $1.29 MeV/c^2$. Compare to Solar System mass fractions: 74 % (1H) and 24.0 % (^{4}He) compare to your results from the code from step 1.

Answer:

To calculate the initial neutron to proton ratio we use:

$$Y_n = \frac{X_n}{A_n} = X_n = \frac{n_n}{n_p + n_n} = \frac{1}{1 + \frac{n_p}{n_n}} \tag{3.5}$$

where we can approximate $n_n/n_p = e^{-Q/kT}$ where Q is $Q = m_n - m_p = 1.29 MeV/c^2$,thus: $Y_n^{t1} = 0.166$ and $Y_p^{t1} = 0.834$.

When fusion reactions begin to operate creating ^{4}He all neutrons will be converted to ^{4}He. Therefore we need to find the abundance of neutrons at time $t = 250s$.

$$Y_n^{t2} = Y_n^{t1} \cdot e^{-t2-t1)\lambda} \tag{3.6}$$

where λ is the decay constant calculated as:

$$\lambda = \frac{ln2}{T_{1/2}}, \tag{3.7}$$

with $T_{1/2} = 10.24$ min for neutrons we finally find:

$$Y_n^{t2} = 0.125 \tag{3.8}$$

Assuming all neutrons are converted to ^{4}He:

$$Y_{^{4}He}^{t2} = \frac{Y_n^{t2}}{2} = 6.125 \tag{3.9}$$

Note this result is abundance of ^{4}He. To convert this to mass fraction we need to multiply by A. We see that results from solar system, code ($Y_{^{4}He}^{t2} = 0.0617$, and our calculation match.

---

4. In the ppI, ppII and ppIII chains as well as in the CNO cycle, the total energy released is the same: Q=26.73 MeV. However, the energy carried away by the escaping neutrinos differs. The mean energy for neutrinos emitted in the: $p + p \rightarrow d + e^{+} + \nu_e$ reaction is 0.26 MeV and in the 8B $\beta+$ decay 7.30 MeV. For 15O and 13N + decays you can approximate that the mean neutrino energy is 0.6E,max. For the EC decay of 7Be, 90% of the decays go to the ground state of 7Li and 10% to the first excited state at 477.6 keV. Calculate the energy carried away by the neutrinos in: a) ppI chain, b) ppII chain, c) ppIII chain and d) CNO cycle.

Answer:

- In the ppI cycle, neutrinos are only created in the deuterium formation. Therefore, the energycarried away by neutrinos according to the problem is 0.26 MeV from the $p + p \rightarrow d + e^+ + \nu_e$ mean energy carried by neutrinos is $E = 0.26$ MeV.
- In the ppII cycle, neutrinos are also created in the EC decay of ^7Be. For decay to the ground state 90% of the total energy of neutrinos is $E_\nu = 0.26MeV + 0.862MeV = 1.12MeV$. In case that the decay goes to the excited state 10%, we have $E_\nu = 0.26MeV + (0.860.48)MeV = 0.64MeV$. So the energy carried away by neutrinos is approximately $E_\nu = 0.9 * 1.12 + 0.1 * 0.64 = 1.07$ MeV.
- In the ppIII cycle, neutrinos are created in the deuterium formation and $^8B\beta^+$ decay. The energy carried away is therefore $E_\nu = 0.26MeV + 7.30MeV = 7.56MeV$.
- In the CNO1 cycle, energy is carried away by the neutrinos created in the ^{15}O and $^{13}N\beta^+$ decay and the energy escaping by neutrinos is calculated as, $E_\nu 0.6(2.22 + 2.75)MeV = 2.97MeV$.

---

5. Calculate the total energy released in the CNO1 cycle by computing the energy release in each of the six steps of the cycle: $^{12}C(p,\gamma)^{13}N(\beta+)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(\beta+)^{15}N(p,\alpha)12C$. Which reaction releases most of the energy? Use the Atomic Mass Evaluation 2020 tables AME20 for Q-value calculations.

Answer:

Table 3 contains the masses of nuclei participating in the CNO cycle.

Nuclei Mass (u)	(AME2020)
p	1.007825031898(14)
4He	4.00260325413(16)
12C	12.0000000(0)
13N	13.005738609(289)
13C	13.00335483534(25)
14N	14.00307400425(24)
15O	15.003065636(526)
15N	15.00010889827(62)

We use this table to calculate the Q values:

$$\begin{aligned} Q_{(^{12}C(p,\gamma)^{13}N)} &= 1.943MeV \\ Q_{(^{13}N(\beta+)^{13}C)} &= 2.220MeV \\ Q_{(13C(p,\gamma)^{14}N)} &= 7.551MeV \\ Q_{(^{14}N(p,\gamma)^{15}O)} &= 7.297 \\ Q_{(^{15}O(\beta+)^{15}N)} &= 2.754 \\ Q_{(^{15}N(p,\alpha)12C)} &= 4.965MeV \end{aligned} \tag{3.10}$$

The total energy released is $Q_{tot} = 26.731MeV$ as expected

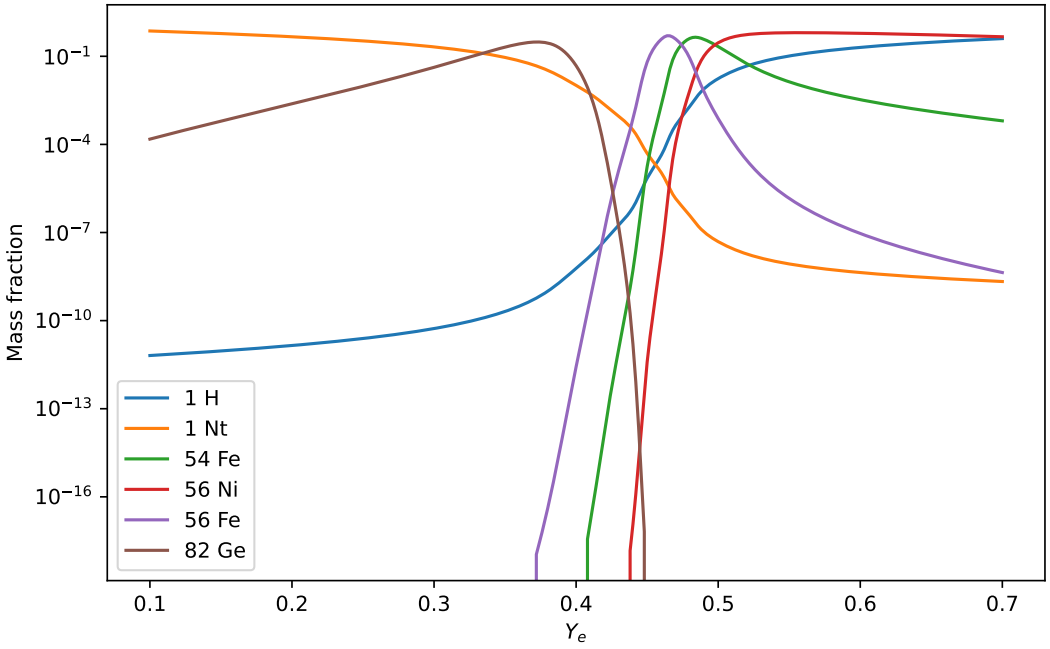

Figure 4.1: Mass fraction vs Y_e results at $T = 5GK$ and $\rho = 10^7$ using a NSE code.

4 | NSE - He burning

Above $T \approx 5 \cdot 10^9$K forward and inverse reactions are balanced and the abundances are in a state of nuclear statistical equilibrium (NSE). With the NSE approximation, we can accurately calculate nuclear abundances in a hot dense environment. a) Use the NSE code to calculate the nuclear abundances as a function of temperature for density $\rho = 10^7 gcm^{-3}$, Ye=0.5. b) Use the NSE code to find the abundance of elements at T = 5 GK, $\rho = 10^7 gcm^{-3}$ as a function of Ye. Use Ye values of 0.3-0.5 (steps of 0.02). c) Use the NSE code to find the abundance of elements at T = 5 GK, Ye=0.5 for $\rho = 10^7$ – $10^8 gcm^{-3}$ (steps of 10^7). d) Explain your results to your best comprehension. When does NSE break and why? To use the NSE code:

```
cd /automisc/data/stynikas/NSE_code/nse/students
mkdir firstname_lastname
cp ../../* .
./eos
give den,temp,ye in cgs
results in nz-plane.dat
```

to run multiple calculations automatically you can try a modified version of the bash scripts provided in the Appendix A.4:
If you did a mistake and want to kill the process, use Ctrl-c. A copy of the code can be found in Prof. Frank Timmes website .

Answer:
Running the NSE code according to the instructions we get the following results summarized in Figs. 4.3,4.2,4.1 The code used to 'clean' the results and produce the plots is described in the Appendix A.5 In Fig. 4.1 we observe the dependence of Mass fraction to the Y_e. We observe that the most abundant element for a certain Y_e can be estimated as the most stable (highest binding energy) isotope where $\frac{Z}{A} = Y_e$. In Fig. 4.2 we see that in high T most of the mass is n,p, and He atoms but as the T drops these recombine creating nuclei in the iron peak region. Finally, in Fig. 4.3 we see that for the conditions explored there is no big dependence on the density.

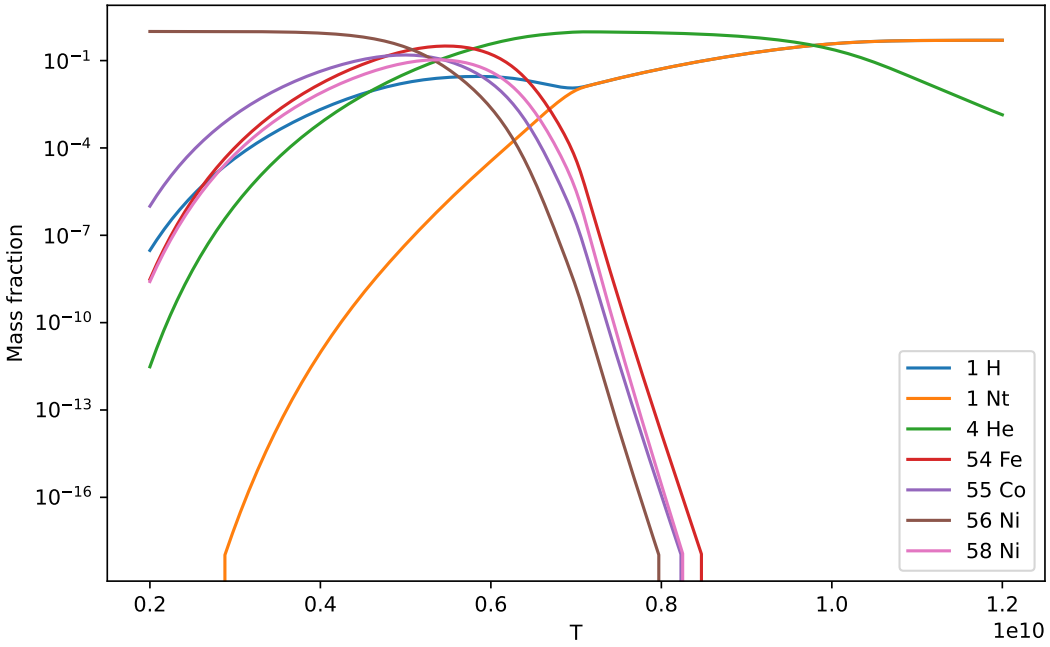

Figure 4.2: Mass fraction vs T results at $Y_e = 0.5GK$ and $\rho = 10^7$using a NSE code.

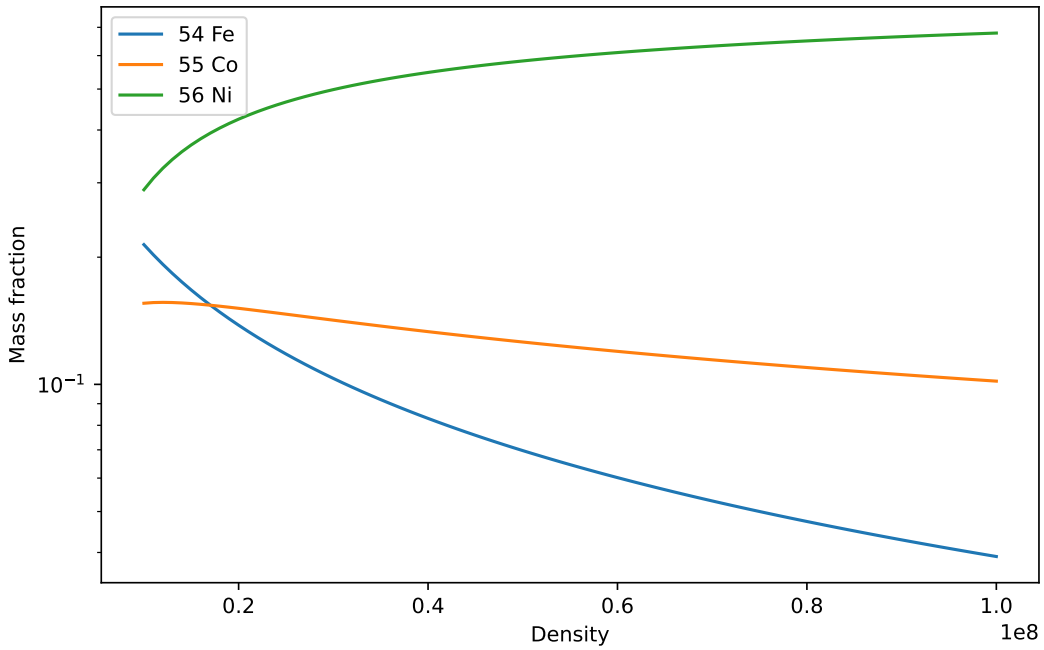

Figure 4.3: Mass fraction vs ρ results at $Y_e = 0.5GK$ and $T = 5GK$ using a NSE code.

2) [!!!! Problematic !!!! requires a lot of work and assumptions to solve analytically] Estimate the abundances $Y(Z,N)$ of ^{56}Ni and ^{56}Fe in the Nuclear Statistical Equilibrium conditions $\rho = 10^7 gcm^{-3}$ g/cm3 and T=5 GK for : a) Ye = 0.5 b) Ye = 0.4 You can assume that the normalized partition functions G are equal to 1. Assume that the total number density of protons and neutrons is $n_{tot} = n_p + n_n = 10^{30} cm^{-3}$ $n_i = \rho N_A$, with Use Atomic Mass Evaluation 2020 mass tables AME20 to calculate the binding energies. c) Compare your results to the NSE calculations you performed before

Answer:
This exercise is difficult to evaluate analytically using the Saha equation. One could find the relative abundance between two nuclei or estimate the abundance using the observations we made in the first exercise since the conditions are similar. Evaluating the Saha equation for each isotope separately would lead to very large numbers that would be challenging to evaluate with simple tools i.e.:

$$Y_{Z,N} = Y_p^Z Y_n^N G(Z,A)(\rho N_A)^{(A-1)} \frac{A^{3/2}}{2^A} (\frac{2\pi\hbar}{m_u kT}) * e^{B(Z,N)/kT} \tag{4.1}$$

As one can obviously observe this equation scales significantly with A.
The best way to approach this problem is by observing that isotopes where $\frac{Z}{A} = Y_e$ are favored. Furthermore, the distribution is usually dominated by a single nucleus. Now one can assume that only isotopes with $\frac{Z}{A} = Y_e$ can be produced and find the relative abundance between them analytically. For $Y_e = 0.5$ the production of double magic ^{40}Ca and ^{56}Ni is favored. Low entropy favors iron group nuclei over light ones therefore the mass fraction of ^{56}Ni should be the dominant heavy element produced. Similar arguments can be made for $Y_e = 0.4$ where ^{82}Ge dominates the abundance and ^{56}Ni, and ^{58}Fe are only traces.

---

3) Calculate the mass defect for the triple-α process in MeV. Compare your results to hydrogen burning. What can you say about the energy produced by He burning compared to H burning? Assuming the star has the same luminosity during both phases of evolution compare the He burning lifetime to H burning.

Answer:
The energy from the triple alpha amounts to:

$$Q = m_{(3\alpha)} - m_{(^{12}C)} = 7.2748 MeV. \tag{4.2}$$

H-burning releases $E = 26.73 MeV$ which ≈ 3.5 times larger. To maintain the same energy output triple alpha needs to happen 3.5 times faster. Furthermore, the number of nucleons involved in the triple-α are 12 while in H burning 4 (3 times higher). This means that to maintain the same energy output, a stars life-time would be 11.5 shorter during triple α compared to H burning.

---

4) Hoyle predicted a resonance in 12C (7.68 MeV). Why can we assume that the formation of ^8Be is in equilibrium with the destruction of it? If we assume: $\mu[12C(Hoylestate)] \leftrightarrow 3\mu\alpha$ express the triple-alpha reaction rate. Present your work and write down any assumptions you made.

Answer:
The astrophysicist Fred Hoyle predicted its existence based on stellar evolution. Without a state of this sort, it's unlikely that carbon would be formed when alpha particles smack into beryllium nuclei in a star. And that would be a serious roadblock to the formation of carbon.
Assuming ssume: $\mu[12C(Hoylestate)] \leftrightarrow 3\mu\alpha$ we can write using Saha equation:

$$Y_{^{12}C}(HS) = 3^{3/2} * Y_\alpha^3 (\frac{\rho}{m_\alpha}^2 (\frac{h}{2\pi m_\alpha kT})^3 e^{Q/kT}. \tag{4.3}$$

The formation of 12C is determined by the decay of the Hoyle state by gamma since we are in an equilibrium and we can write:

$$\frac{dY_{12C}}{dt} = Y_{12C}(HS) \frac{2\pi\Gamma_{rad}}{h}, \tag{4.4}$$

thus:

$$\frac{dY_{12C}}{dt} = \frac{1}{6} 3^{3/2} (\frac{h}{2\pi m_\alpha kT}) \frac{2\pi\Gamma_{rad}}{h} < \alpha\alpha\alpha >, \tag{4.5}$$

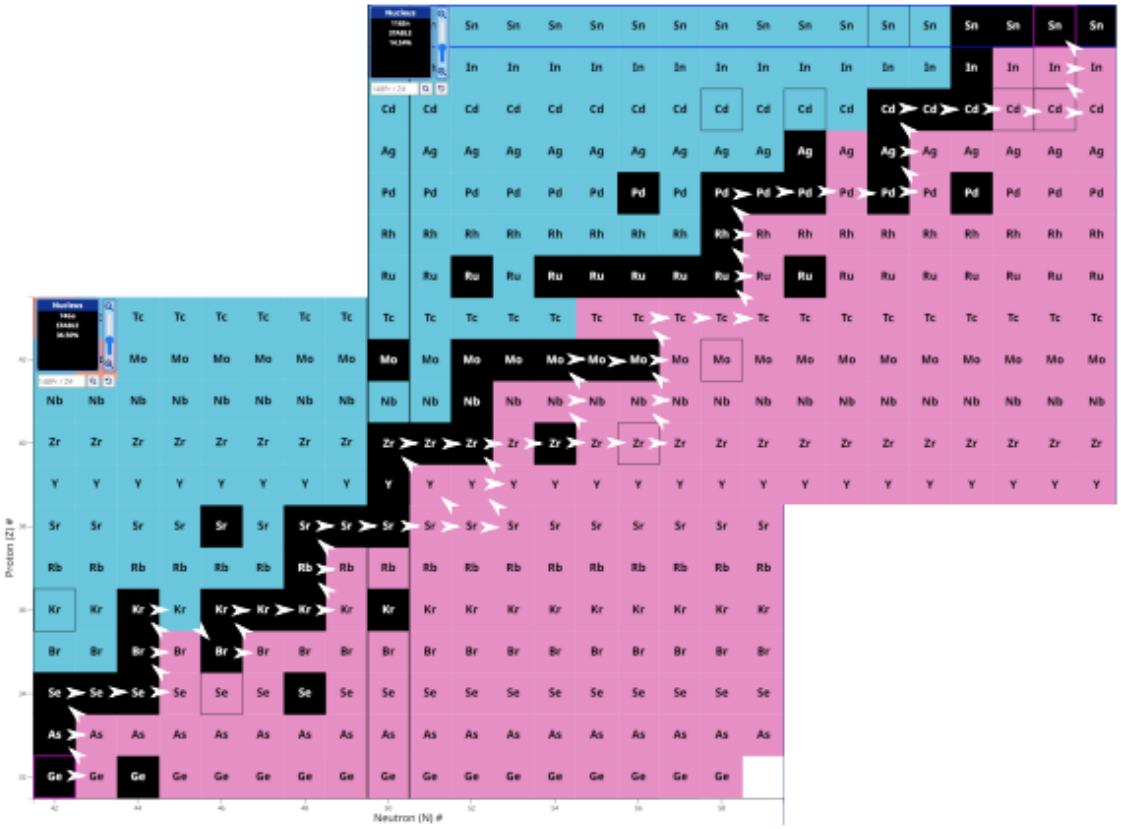

Figure 4.4: Path of the s-process Fi. by A. Jaries.

Which finally if we solve for $< \alpha\alpha\alpha >$ gives:

$$< \alpha\alpha\alpha >= \frac{1}{6} 3^{3/2} \left(\frac{h}{2\pi m_\alpha kT}\right) \frac{2\pi\Gamma_{rad}}{h} e^{Q/kT} \tag{4.6}$$

---

5) a) Trace the s-process path from 74Ge to 116Sn. Identify any stable nuclei in this region that cannot be reached in the s-process, and note which nuclei are shielded from the r-process. b) The s process branches at 147Nd because it has a beta decay half-life of 5.3 days (under stellar conditions). 148Sm and 150Sm are s-only isotopes. The ratio of their neutron capture cross sections corresponding to kT=30 keV is $\langle($ Sm 148 $)\rangle / \langle($ Sm 150 $)\rangle = 0.596$. Use the steady-flow approximation and determine the fraction of the reaction flow that branched into the beta decay at 147Nd. (Hint: use the terrestrial isotopic abundance ratios of 148Sm and 150Sm, see the figure below taken from https://www.nndc.bnl.gov/nudat2/ . The yellow arrows indicate the s process path.)

Answer:
The path of the s-process is on the valley of stability. Assuming a stable nucleus with Z,N s-process will only reach a nucleus with Z,N+1 before it decay back to stability. This is due to the timescale that s-process operates (1000 years) and the low neutron density. Some s-process produced only isotopes are: 76Se, 82Kr, 86Sr, 94Mo, 98Ru, 104Pd, 110Cd and 116Sn. Fig. 4.4 shows a rough sketch of the path of the s-process.

5 | R-process

1) Use https://www.nndc.bnl.gov/nudat3/ and identify stable isotopes which are produced only by the r process (r-only).

Answer:

One can do a very simple assumption to estimate if a nucleus is r-process made only. For a certain element, if the last, neutron-rich side isotope with neutron number N=N1 is separated from the next stable isotope with N=N2 and N1¡N2 by at least one relatively short lived isotope (remember that this s-process operates over time scales of thousands of years) then this isotope can be an r-only produced isotope.
You should find ≈ 20 r-only nuclei. Some of these are i.e. ^{71}Zn,^{76}Ge,^{100}Mo,^{104}Ru,^{110}Pd,^{116}Cd, ^{122}Sn,^{124}Sn,^{123}Sb,^{128}Te,^{130}Te ^{134}Xe,^{136}Xe,^{142}Ce,^{150}Nd ^{154}Sm,^{160}Gd,^{170}Er,^{176}Yb,^{178}Pt ^{204}Hg,^{232}Th, ^{235}U,^{235}U,^{244}Pu

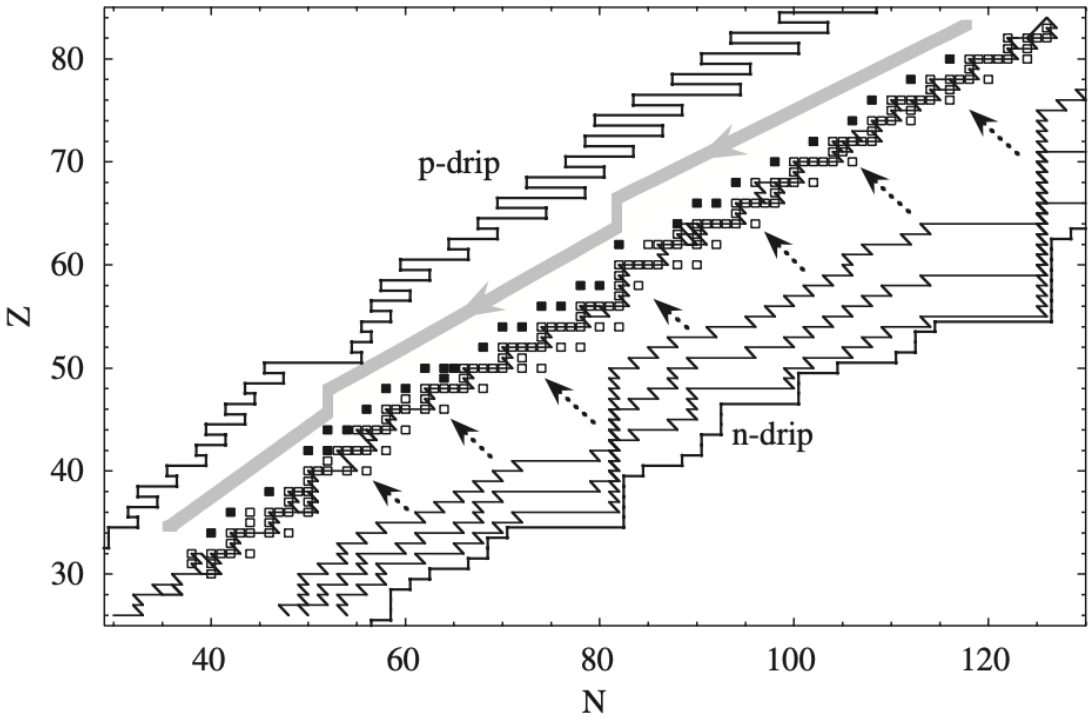

Figure 5.1: r-only and r-dominated nuclei according to [1]

---

2) Follow the instructions of the nuclear network Torch at /automisc/data/stynikas/torch and run the example case for the network. Present the final abundances on an abundance vs A plot. Explain the results of the calculations. What kind of reactions (i.e. neutron captures, alpha captures etc) do you believe are the most important for this calculation?

Answer:

This scenario is a hard nse silicon burning [2] at $T = 9.6$ GK and $\rho = 6.0^9 g/cm^3$. You choose 2 = hydrostatic - constant temperature and density, which means you never fall out of NSE. Therefore all reactions are in equilibrium so they are irrelevant.
The routine that provides this plot can be found in appendix.

---

3) Estimate the r-process contribution to the solar system abundance of the s,r-isotope 125Te. Use values of $N_{Solar}(^{124}Te) = 0.2319$, $N_{Solar}(^{125}Te) = 0.3437$ for the number abundances per 106 Si atoms, respectively. The Maxwellian-averaged neutron-capture cross sections at $kT = 30$ keV for ^{124}Te and ^{125}Te

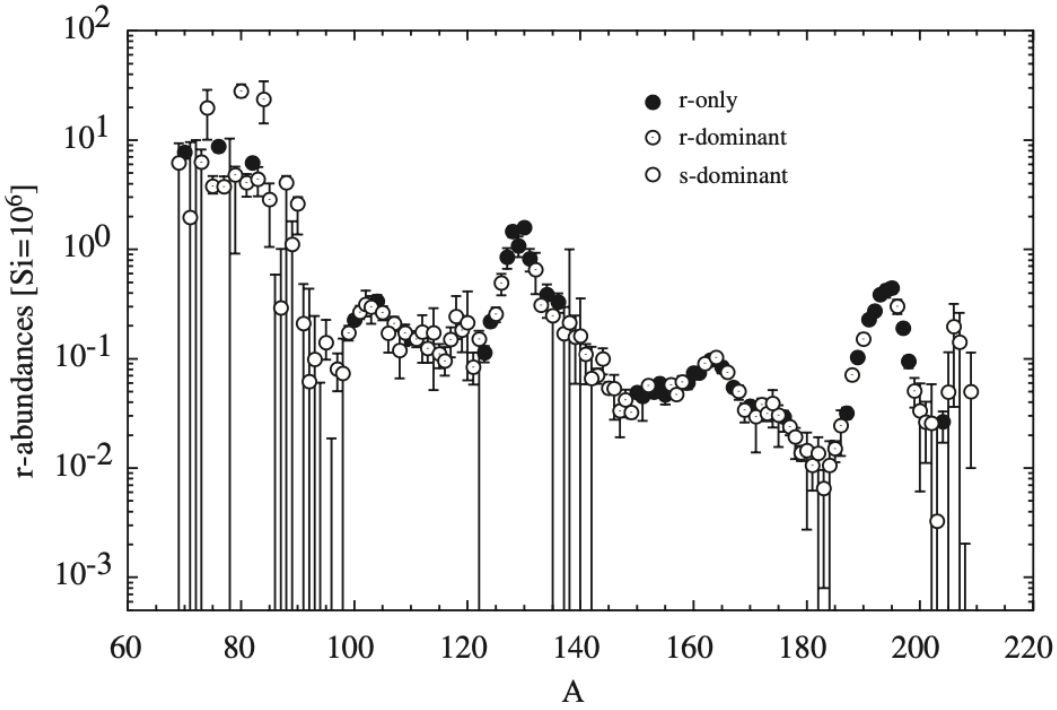

Figure 5.2: r-only and r-dominant and s-dominant nuclei according to [1]

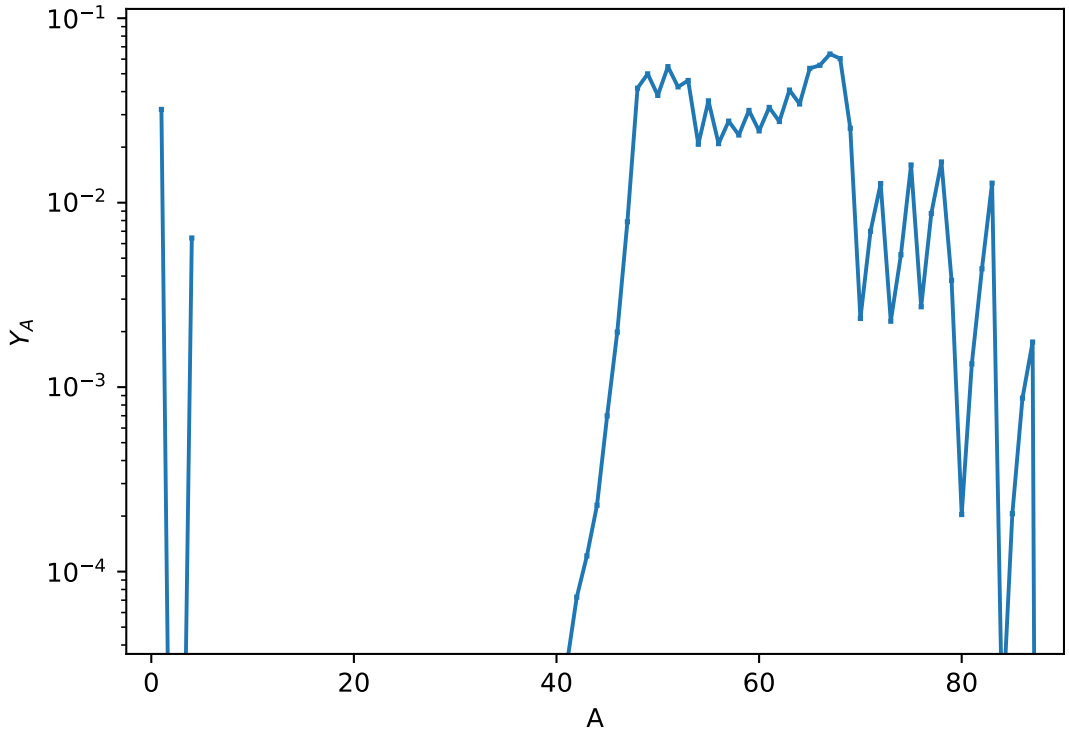

Figure 5.3: Hard nse silicon burning at constant temperature and density

are $< \sigma >^{124} Te = 155 \pm 2$ mb and $< \sigma >^{124} Te = 431 \pm 4$ mb, respectively.

Answer:
$N_{Solar}(^{124}Te) = 0.2319$ (s-only isotope because stable ^{124}Sn blocks the r-process)
$N_{Solar}(^{125}Te) = N_r(^{125}Te) + N_s(^{125}Te) = 0.3437$ (mixture of s- and r- processes) But we can calculate $N_s(^{125}Te)$ as:

$$N_s(^{125}Te) = N_{Solar}(^{124}Te)\frac{< \sigma >_{^{124}Te}}{< \sigma >_{^{125}Te}} = 0.2319\frac{155mb}{431mb} = 0.0834 \tag{5.1}$$

solving now for $N_r(^{125}Te)$:

$$N_r(^{125}Te) = 0.3437 - 0.0834 = 0.26 \tag{5.2}$$

So we find that the main contribution of $N_{Solar}(^{125}Te)$ comes from the r-process.

---

4) Show analytically that in the rapid neutron capture process (r process), for a given neutron density and temperature, the most abundant nucleus within an isotopic chain in (n,γ)-(γ,n) equilibrium (as in the waiting point approximation) is located at a fixed neutron separation energy. Hint: Use the waiting point approximation and the equation for the abundance ratio of two neighbouring nuclei from the lecture notes. Assume that partition functions are g=1 for all heavy nuclei. You only need to calculate the RELATIVE abundance distribution. b) Calculate the neutron separation energy for the most abundant isotope in typical r-process conditions (T = 1.5 GK, neutron density 10^{24} neutrons/cm3).

Answer:
We start with the waiting point approximation:

$$\frac{N(Z,A+1)}{N(Z,A)} = N_n\left(\frac{h^2}{2\pi m_{A_n} kT}\right)^{3/2}\frac{G_{Z,A+1}}{G_{Z,A}}e^{\frac{Q(n,\gamma)}{kT}}\frac{2J_{Z,A+1}}{(2J_{Z,A}+1)(2J_n+1)}. \tag{5.3}$$

If we assume that the partition function is 1, then we can estimate the quantity from Eq. 5.3:

$$\frac{G_{Z,A+1}}{G_{Z,A}}\frac{2J_{Z,A+1}}{(2J_{Z,A}+1)(2J_n+1)} = 1/2 \tag{5.4}$$

The maximum will then be given by the derivative such as:

$$\frac{N(Z,A+1)}{dA} = \frac{N(Z,A+1) - N(Z,A)}{(A+1) - A} = 0 \tag{5.5}$$

$$\frac{N(Z,A+1)}{N(Z,A)} = \frac{N_n}{2}\left(\frac{h^2}{2\pi m_{A_n} kT}\right)^{3/2}e^{\frac{Q(n,\gamma)}{kT}} \tag{5.6}$$

One can estimate the reduced mass by assuming that $m_A >> m_n$ $m_{An} = \frac{m_A m_n}{m_A + m_n} \approx m_n$ and solving for $Q_{(n,\gamma)}$:

$$Q_{(n,\gamma)} = kT.ln\left(\frac{N_n}{2}\left(\frac{h^2}{2\pi m_n kT}\right)^{3/2}\right) \approx 3.1MeV \tag{5.7}$$

but we know that $Q_{(n,\gamma)}(A) = S_n(Z,A+1)$

---

5) The operation of the r-process for a neutron rich fast expanding low entropy scenario can be seen here. a) Is this a hot or cold r-process scenario? b) At which T does the r-process begin to operate? c) At which T neutrons are exhausted and material decays back to stability?

Answer:
This particular scenario is a very neutron-rich, fast expanding trajectory representing the dynamical ejecta from the merger. For this trajectory, we can distinguish four different phases.

- Phase 1: Nuclear Statistical Equilibrium. Freeze-out happens at $T \approx 5$ GK
- Phase 2: Cold R-Process. Due to the fast expansion, the temperature and density drop quickly. Material accumulates close to the closed shells. This last between $T \approx 5$ GK and $T \approx 0.5$ GK

- Phase 3: Hot R-Process. Due to nuclear heating, the temperature increases once the full r-process reaction flow is established, despite the ongoing expansion of the ejecta.Notice the reheating and the increase in temperature at $T \approx 0.5$ GK
- Phase 4: The nuclear composition starts to decay towards stability when the neutron supply is depleted. This starts at $T \approx 0.4$ GK. Notice the rapid fall of abundance of fission group nuclei.

for more info take a look at this article

6 | References

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[2] W Raphael Hix and F-K Thielemann. Silicon burning i: Neutronization and the physics of quasi-equilibrium. *arXiv preprint astro-ph/9511088*, 1995.

[3] JA Holmes, SE Woosley, William A Fowler, and BA Zimmerman. Tables of thermonuclear-reaction-rate data for neutron-induced reactions on heavy nuclei. *Atomic Data and Nuclear Data Tables*, 18(4):305–412, 1976.

[4] Arjan J Koning, Stephane Hilaire, and Marieke C Duijvestijn. Talys-1.0. In *International Conference on Nuclear Data for Science and Technology*, pages 211–214. EDP Sciences, 2007.

[5] Jonas Lippuner. *r-Process nucleosynthesis in neutron star mergers with the new nuclear reaction network SkyNet*. PhD thesis, California Institute of Technology, 2018.

A | Appendix

A.1 | Saha equation

```
import numpy as np
###############
# constants   #
###############
k = 1.38*10**(-23)
k_b = 8.617*10**(-5) #in ev
h = 6.626*10**(-34)
me = 9.1*10**(-31)
ion2 = 13.6
pi = 3.14159265359
e = 2.71828

##################
# Saha Equation  #
##################
def saha(temp,n):
    f = (2.*pi*me*k*temp/(h**2))**(3./2.)*e**(-ion2/(k_b*temp))*1./(n)
    return f

################################
# Define conditions of T and n #
################################

#############################################################################
# To create the plots use a for loop where T or n is a variable for example:#
# for i in range (1000,10000,10):
#    T=float(i)
#    n = 10**32
#    ratio=saha(T,n)
#    percent=ratio/(ratio+1)*100
#    T_list.append(T)
#    ratio_list.append(n)
#    percent_list.append(percent)
#    n_list.append(n_list)
# will run conditions between 1000,10000 K and append the results on the
# corresponding list so you can plot them
#############################################################################



#########################################################
############     Main code  ############################
#########################################################


T = 15*10**6 #Tempe
n = 10**32 #number density
ratio=saha(T,n)
percent=ratio/(ratio+1)*100
print('H*/H  is  {}  or  {}%  '.format(ratio,percent))
```

A.2 | Reaction rates comparisons (Talys, bruslib,reaclib(

```
import numpy as np
import matplotlib.pyplot as plt
```

```
#load data from txt files
Tt1,RRt1=np.loadtxt('ni56_nd.txt',unpack=True,skiprows=2,usecols=(0,1))
Tt2,RRt2=np.loadtxt('ni56_wd.txt',unpack=True,skiprows=2,usecols=(0,1))
Tb,RRb=np.loadtxt('ni56_bruslib.txt',unpack=True,skiprows=1,usecols=(0,1))
Tr,RRr=np.loadtxt('ni56_reaclib.txt',unpack=True,skiprows=1,usecols=(0,1))
#define a canvas
font = {'family' : 'normal','size'   : 14.5}
plt.rc('font', **font)
fig, ax = plt.subplots()
fig.set_size_inches(10, 6.5)
#plot the results
ax.semilogy(Tt1,RRt1,marker='<',label='Talys default')
ax.semilogy(Tt2,RRt2,marker='>',label='Talys default + direct')
ax.semilogy(Tb,RRb,marker='o',label='Bruslib')
ax.loglog(Tr,RRr,marker='s',label='Reaclib')
ax.set_xlabel('Temperature (GK)')
ax.set_ylabel(r'$N_a <\sigma>[mol^{-1}cm^3s^{-1}]$')
plt.legend()
plt.show()
```

A.3 | Plot Parthenope results

```
#load the libraries needed
import matplotlib.pyplot as plt
import numpy as np


#Data here are using D instead of the most common E for the scientific notation.
#One way to deal with this is building a rather simple function as bellow
#replacing D with E

def replace_d_exp(s):
    return s.replace(b'D', b'E')



#Define a list of elements that we will plot
elems=['N','P','H2','H3','He3','He4','Li6','Li7','Be7']
#create a canvas
fig,ax=plt.subplots()
fig.set_size_inches(6.5, 4.5)
#read the file
data=np.loadtxt('nuclides3.0.out',converters={n: replace_d_exp for n in
                range(20)},unpack=True,skiprows=1,usecols=(0,5,6,7,8,9,10,11,12,13))
#sum the abundances in the previously created 1d array according to their A value.
for i in range (1,10):
    print (i)
    print(data[i][:])
    ax.loglog(data[0][:],data[i][:],markersize=1,marker='s',label='{}'.format(elems[i-1]))
#plot the results
ax.set_xlabel('T (GK)')
ax.set_ylabel(r'$Y_A$')
ax.set_xlim(10,0.005)
plt.legend()
plt.show()
```

A.4 | Bash script to run multiple NSE calculations automatically examples

```
#!/bin/bash
for i in {30..50..2};
 do
 yes "1e6,5e9,0.${i}" | ./eos
 mv massplot.dat results/massplot_ye0${i}
mv nz-plane.dat results/nz-plane_ye0${i}
 done
```

```
#!/bin/bash
for i in $(seq 400000 5000 4400000);
do
    yes "${i},5000000000,0.38" | ./eos
    mv massfraction results/massfraction_5gk038s_${i}
    mv nz-plane.dat results/nz-plane_5gk038s_${i}
done
```

A.5 | Select and plot NSE results simple python routine

```
import matplotlib.pyplot as plt
import numpy as np
import elements as el #you do not need this it just plots the names of the emlements
#instead of their Z number. Fill free to remove it

#Create empty 3d arrays to fill with the results from the n_z-plane files from
#the code 1 for each quantity we varried

nz_plane_arrays_T = np.zeros((120,80,1001))
nz_plane_arrays_ye = np.zeros((120,80,301))
nz_plane_arrays_den = np.zeros((120,80,91))
index=0


#loop over all the Y_e values we calculated and fill the arrays
for i in range (100, 701, 2):
    n,z,Xi = np.loadtxt('nz-plane_ye_0.{}'.format(i),usecols=(0,1,3),skiprows=1,unpack=True)
    for j in range (0,len(n)):
        nz_plane_arrays_ye[int(n[j])][int(z[j])][index]=10**Xi[j]
    index+=1
index=0
for i in range (200, 1201, 1):
    T=i/100.
    print (T)
    n,z,Xi = np.loadtxt('nz-plane_T{:.2f}'.format(T),usecols=(0,1,3),skiprows=1,unpack=True)
    for j in range (0,len(n)):
        nz_plane_arrays_T[int(n[j])][int(z[j])][index]=10**Xi[j]
    index+=1
index=0
for i in range (10000000, 100000001, 1000000):
    n,z,Xi = np.loadtxt('nz-plane_den_{}'.format(i),usecols=(0,1,3),skiprows=1,unpack=True)
    for j in range (0,len(n)):
        nz_plane_arrays_den[int(n[j])][int(z[j])][index]=10**Xi[j]
    index+=1
fig, ax = plt.subplots()
fig.set_size_inches(8.09, 5)
Ye=np.arange(0.1,0.701,0.002)


```

```
40
41 #First plot Mass_fraction vs Ye alternavely plot Abundances vs Ye, see
42 #commented part bellow with #
43 for n in range (0,120):
44     for z in range (0,80):
45         # Count of all values greater than 0.1 in 1D Numpy array
46         #This is used to gate the plotting to oonly the most abundant isotopes.
47         #if there is at least one instance of this element vs T that has
48         #mass fraction >0.3 then this will be plotted
49         count = np.count_nonzero(nz_plane_arrays_ye[n][z][:] > 0.3)
50         if count>1:
51             ax.semilogy(Ye,nz_plane_arrays_ye[n][z][:],label='{} {}'.format(n+z,el.elems(int(z))))
52 ax.legend()
53 ax.set_xlabel(r'$Y_e$')
54 ax.set_ylabel('Mass fraction')
55
56 #plot abundance instead of Mass fraction#
57 '''for n in range (0,120):
58     for z in range (0,80):
59         # Count of all values greater than 0.1 in 1D Numpy array
60         count = np.count_nonzero(nz_plane_arrays_ye[n][z][:] > 0.3)
61         if count>1:
62             ax.semilogy(Ye,nz_plane_arrays_ye[n][z][:]/float(n+z),linestyle=':'
63             ,label='{} {}'.format(n+z,el.elems(int(z))))
64 ax.legend()
65 ax.set_xlabel(r'$Y_e$')
66 #ax.set_ylabel(r'$Y_A$')'''
67 plt.show()
68
69
70 #Second plot Mass_fraction  T
71 fig, ax = plt.subplots()
72 fig.set_size_inches(8.09, 5)
73 Ye=np.arange(2*10**9,12.01*10**9,1*10**7)
74 for n in range (0,120):
75     for z in range (0,80):
76         # Count of all values greater than 0.1 in 1D Numpy array
77         count = np.count_nonzero(nz_plane_arrays_T[n][z][:] > 0.1)
78         if count>1:
79             ax.semilogy(Ye,nz_plane_arrays_T[n][z][:],label='{} {}'.format(n+z,el.elems(int(z))))
80 ax.legend()
81 ax.set_xlabel('T')
82 ax.set_ylabel('Mass fraction')
83 plt.show()
84
85 #Second plot Mass_fraction vs rho
86 fig, ax = plt.subplots()
87 fig.set_size_inches(8.09, 5)
88 Ye=np.arange(10000000, 100000001, 1000000)
89 for n in range (0,120):
90     for z in range (0,80):
91         # Count of all values greater than 0.1 in 1D Numpy array
92         count = np.count_nonzero(nz_plane_arrays_den[n][z][:] > 0.1)
93         if count>1:
94             ax.semilogy(Ye,nz_plane_arrays_den[n][z][:],label='{} {}'.format(n+z,el.elems(int(z))))
95 ax.legend()
96 ax.set_xlabel('Density')
97 ax.set_ylabel('Mass fraction')
```

```
plt.show()
```

A.6 | Plot torch results

```
#load the libraries needed
import matplotlib.pyplot as plt
import numpy as np
#create an empty 1d array
Y_A=np.empty(200)
Y_A[:]=np.NaN
#create a canvas
fig,ax=plt.subplots()
fig.set_size_inches(6.5, 4.5)
#read the file
n,A,Y=np.loadtxt('foo_final.dat',unpack=True,skiprows=1,usecols=(0,1,2))
#sum the abundances in the previously created 1d array according to their A value.
for i in range (0,len(n)):
    if np.isnan(Y_A[int(A[i])]):
        Y_A[int(A[i])]=(Y[i])
    else:
        Y_A[int(A[i])]+=(Y[i])
A=np.arange(0,200)
#plot the results
ax.semilogy(A,Y_A,markersize=1,marker='s')
ax.set_xlabel('A')
ax.set_ylabel(r'$Y_A$')
plt.show()
```