```
In[4]:= poly[n_, x_] := Total[A# x^# & /@ Range[0, n]]
                              суммировать      диапазон

        sol[k_] := Module[{n = k, podintegr, left, eqn, coefeqn, s},
                   программный модуль

          podintegr = poly[n, (m ω^2 x^2 + m v^2)/2] Exp[-(m v^2)/(ℏ ω)];
                                                     показательная функция

          left = Integrate[podintegr, {v, -∞, ∞}, Assumptions → {m > 0, ℏ > 0, ω > 0}];
                 интегрировать                     предположения

          eqn = (left == 1/(2^n n!) Sqrt[(m ω)/(π ℏ)] HermiteH[n, Sqrt[(m ω)/ℏ] x]^2);

          coefeqn = (# == 0 & /@ CoefficientList[First[eqn] - Last[eqn], x]);
                                 список коэффици⋯ первый         последний

          s = Solve[coefeqn, Table[A_i, {i, 0, n}]];
              решить уравне⋯ таблица значений

          s = Simplify[s, {m > 0, ω > 0, ℏ > 0}];
              упростить

          {podintegr, left, eqn, coefeqn, s};
          Print["\n\n\nДля n = ", n];
          печатать

          Print["Интеграл:\n ∫_{-∞}^{∞}", podintegr, "ⅆv ="];
          печатать

           (*Print["= ",Simplify[left,{m>0,ω>0,ℏ>0}]]*)
             печатать     упростить

          Print["= ", left, "\n = ", 1/(2^n n!) Sqrt[(m ω)/(π ℏ)] H_n[Sqrt[(m ω)/ℏ] x]^2];
          печатать

          Print["Уравнение: \n", eqn];
          печатать

          Print["Система уравнений на коэффициенты: \n",
          печатать

            Grid[Transpose@{DeleteCases[coefeqn, True]}, Alignment → Left]];
                 транспозиция удалить случаи по обра⋯ истина  выравнива⋯  слева

          Print["Решение системы: \n", Grid[First@s /. Rule → List, Frame → All]];
                                       та⋯  первый      прав⋯  спи⋯  рамка   всё

          Print["Полином: \n", P_n, "(x) = ", poly[n, x] /. First@s]
          печатать                                          первый
         ]
        (*sol[2]*)
        sol /@ Range[0, 10];
               диапазон
```

Для n = 0

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} A_0 \, dv =$$

$$= \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} A_0$$

$$= \frac{\sqrt{\frac{m \omega}{\hbar}} H_0\left[x \sqrt{\frac{m \omega}{\hbar}}\right]^2}{\sqrt{\pi}}$$

Уравнение:

$$\sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} A_0 == \frac{\sqrt{\frac{m \omega}{\hbar}}}{\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$-\frac{\sqrt{\frac{m \omega}{\hbar}}}{\sqrt{\pi}} + \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} A_0 == 0$$

Решение системы:

A_0	$\frac{m}{\pi \hbar}$

Полином:

$$P_0(x) = \frac{m}{\pi \hbar}$$

Для n = 1

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2} \left(m v^2 + m x^2 \omega^2\right) A_1\right) dv =$$

$$= \frac{1}{4} \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} \left(4 A_0 + \omega \left(2 m x^2 \omega + \hbar\right) A_1\right)$$

$$= \frac{\sqrt{\frac{m \omega}{\hbar}} H_1\left[x \sqrt{\frac{m \omega}{\hbar}}\right]^2}{2 \sqrt{\pi}}$$

Уравнение:

$$\frac{1}{4} \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} \left(4 A_0 + \omega \left(2 m x^2 \omega + \hbar\right) A_1\right) == \frac{2 m x^2 \omega \sqrt{\frac{m \omega}{\hbar}}}{\sqrt{\pi} \hbar}$$

Система уравнений на коэффициенты:

$$\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\,A_0+\frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_1==0$$

$$-\frac{2\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar}+\frac{1}{2}\,m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}\,A_1==0$$

Решение системы:

A_0	$-\frac{m}{\pi\hbar}$
A_1	$\frac{4\,m}{\pi\,\omega\,\hbar^2}$

Полином:

$$P_1(x)\ =\ \frac{4\,m\,x}{\pi\,\omega\,\hbar^2}-\frac{m}{\pi\,\hbar}$$

Для $n = 2$

Интеграл:

$$\int_{-\infty}^{\infty}e^{-\frac{m\,v^2}{\omega\,\hbar}}\left(A_0+\frac{1}{2}\left(m\,v^2+m\,x^2\,\omega^2\right)A_1+\frac{1}{4}\left(m\,v^2+m\,x^2\,\omega^2\right)^2A_2\right)dv\ =$$

$$=\ \frac{1}{16}\sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\left(16\,A_0+\omega\left(4\left(2\,m\,x^2\,\omega+\hbar\right)A_1+\omega\left(4\,m^2\,x^4\,\omega^2+4\,m\,x^2\,\omega\,\hbar+3\,\hbar^2\right)A_2\right)\right)$$

$$=\ \frac{\sqrt{\frac{m\,\omega}{\hbar}}\,H_2\left[x\sqrt{\frac{m\,\omega}{\hbar}}\right]^2}{8\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{16}\sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\left(16\,A_0+\omega\left(4\left(2\,m\,x^2\,\omega+\hbar\right)A_1+\omega\left(4\,m^2\,x^4\,\omega^2+4\,m\,x^2\,\omega\,\hbar+3\,\hbar^2\right)A_2\right)\right)==$$

$$\frac{\left(-2+\frac{4\,m\,x^2\,\omega}{\hbar}\right)^2\sqrt{\frac{m\,\omega}{\hbar}}}{8\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$-\frac{\sqrt{\frac{m\omega}{\hbar}}}{2\sqrt{\pi}}+\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\,A_0+\frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\,\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_2==0$$

$$\frac{2\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar}+\frac{1}{2}\,m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}\,A_1+\frac{1}{4}\,m\sqrt{\pi}\,\omega^3\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_2==0$$

$$-\frac{2\,m^2\,\omega^2\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^2}+\frac{1}{4}\,m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}}\,A_2==0$$

Решение системы:

A_0	$\frac{m}{\pi \hbar}$
A_1	$-\frac{8 m}{\pi \omega \hbar^2}$
A_2	$\frac{8 m}{\pi \omega^2 \hbar^3}$

Полином:

$$P_2(x) = \frac{8 m x^2}{\pi \omega^2 \hbar^3} - \frac{8 m x}{\pi \omega \hbar^2} + \frac{m}{\pi \hbar}$$

Для $n = 3$

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2}\left(m v^2 + m x^2 \omega^2\right) A_1 + \frac{1}{4}\left(m v^2 + m x^2 \omega^2\right)^2 A_2 + \frac{1}{8}\left(m v^2 + m x^2 \omega^2\right)^3 A_3\right) dv =$$

$$= \frac{1}{64}\sqrt{\pi}\sqrt{\frac{\omega \hbar}{m}} \left(64 A_0 + 4 \omega \left(4\left(2 m x^2 \omega + \hbar\right) A_1 + \omega\left(4 m^2 x^4 \omega^2 + 4 m x^2 \omega \hbar + 3 \hbar^2\right) A_2\right) + \omega^3\left(8 m^3 x^6 \omega^3 + 12 m^2 x^4 \omega^2 \hbar + 18 m x^2 \omega \hbar^2 + 15 \hbar^3\right) A_3\right)$$

$$= \frac{\sqrt{\frac{m \omega}{\hbar}}\, H_3\left[x\sqrt{\frac{m \omega}{\hbar}}\right]^2}{48\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{64}\sqrt{\pi}\sqrt{\frac{\omega \hbar}{m}} \left(64 A_0 + 4 \omega \left(4\left(2 m x^2 \omega + \hbar\right) A_1 + \omega\left(4 m^2 x^4 \omega^2 + 4 m x^2 \omega \hbar + 3 \hbar^2\right) A_2\right) + \omega^3\left(8 m^3 x^6 \omega^3 + 12 m^2 x^4 \omega^2 \hbar + 18 m x^2 \omega \hbar^2 + 15 \hbar^3\right) A_3\right) = \frac{\left(-12 x \sqrt{\frac{m \omega}{\hbar}} + 8 x^3 \left(\frac{m \omega}{\hbar}\right)^{3/2}\right)^2 \sqrt{\frac{m \omega}{\hbar}}}{48\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$\sqrt{\pi}\sqrt{\frac{\omega \hbar}{m}} A_0 + \frac{1}{4}\sqrt{\pi}\,\omega \hbar \sqrt{\frac{\omega \hbar}{m}} A_1 + \frac{3}{16}\sqrt{\pi}\,\omega^2 \hbar^2 \sqrt{\frac{\omega \hbar}{m}} A_2 + \frac{15}{64}\sqrt{\pi}\,\omega^3 \hbar^3 \sqrt{\frac{\omega \hbar}{m}} A_3 == 0$$

$$-\frac{3 m \omega \sqrt{\frac{m \omega}{\hbar}}}{\sqrt{\pi}\,\hbar} + \frac{1}{2} m \sqrt{\pi}\,\omega^2 \sqrt{\frac{\omega \hbar}{m}} A_1 + \frac{1}{4} m \sqrt{\pi}\,\omega^3 \hbar \sqrt{\frac{\omega \hbar}{m}} A_2 + \frac{9}{32} m \sqrt{\pi}\,\omega^4 \hbar^2 \sqrt{\frac{\omega \hbar}{m}} A_3 == 0$$

$$\frac{4 m^2 \omega^2 \sqrt{\frac{m \omega}{\hbar}}}{\sqrt{\pi}\,\hbar^2} + \frac{1}{4} m^2 \sqrt{\pi}\,\omega^4 \sqrt{\frac{\omega \hbar}{m}} A_2 + \frac{3}{16} m^2 \sqrt{\pi}\,\omega^5 \hbar \sqrt{\frac{\omega \hbar}{m}} A_3 == 0$$

$$-\frac{4 m^3 \omega^3 \sqrt{\frac{m \omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^3} + \frac{1}{8} m^3 \sqrt{\pi}\,\omega^6 \sqrt{\frac{\omega \hbar}{m}} A_3 == 0$$

Решение системы:

A_0	$-\frac{m}{\pi\hbar}$
A_1	$\frac{12\,m}{\pi\,\omega\,\hbar^2}$
A_2	$-\frac{24\,m}{\pi\,\omega^2\,\hbar^3}$
A_3	$\frac{32\,m}{3\,\pi\,\omega^3\,\hbar^4}$

Полином:

$$P_3(x) = \frac{32\,m\,x^3}{3\,\pi\,\omega^3\,\hbar^4} - \frac{24\,m\,x^2}{\pi\,\omega^2\,\hbar^3} + \frac{12\,m\,x}{\pi\,\omega\,\hbar^2} - \frac{m}{\pi\,\hbar}$$

Для n = 4

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m\,v^2}{\omega\,\hbar}} \left(A_0 + \frac{1}{2}\left(m\,v^2 + m\,x^2\,\omega^2\right) A_1 + \frac{1}{4}\left(m\,v^2 + m\,x^2\,\omega^2\right)^2 A_2 + \frac{1}{8}\left(m\,v^2 + m\,x^2\,\omega^2\right)^3 A_3 + \frac{1}{16}\left(m\,v^2 + m\,x^2\,\omega^2\right)^4 A_4\right) dv =$$

$$= \frac{1}{256}\sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\Big(256\,A_0 + \omega\,\Big(64\left(2\,m\,x^2\,\omega + \hbar\right) A_1 + 4\,\omega\left(4\left(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2\right) A_2 + \omega\left(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3\right) A_3\right) + \omega^3\left(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4\right) A_4\Big)\Big)$$

$$= \frac{\sqrt{\frac{m\,\omega}{\hbar}}\; H_4\left[x\sqrt{\frac{m\,\omega}{\hbar}}\right]^2}{384\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{256}\sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\Big(256\,A_0 + \omega\,\Big(64\left(2\,m\,x^2\,\omega + \hbar\right) A_1 + 4\,\omega\left(4\left(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2\right) A_2 + \omega\left(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3\right) A_3\right) + \omega^3\left(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4\right) A_4\Big)\Big) == \frac{\left(12 + \frac{16\,m^2\,x^4\,\omega^2}{\hbar^2} - \frac{48\,m\,x^2\,\omega}{\hbar}\right)^2 \sqrt{\frac{m\,\omega}{\hbar}}}{384\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$-\frac{3\sqrt{\frac{m\omega}{\hbar}}}{8\sqrt{\pi}}+\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\,A_0+\frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_1+$$
$$\frac{3}{16}\sqrt{\pi}\,\omega^2\,\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\,\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\,\hbar^4\sqrt{\frac{\omega\hbar}{m}}\,A_4==0$$

$$\frac{3\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar}+\frac{1}{2}m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}\,A_1+$$
$$\frac{1}{4}m\sqrt{\pi}\,\omega^3\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{9}{32}m\sqrt{\pi}\,\omega^4\,\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{15}{32}m\sqrt{\pi}\,\omega^5\,\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_4==0$$

$$-\frac{7\,m^2\,\omega^2\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^2}+\frac{1}{4}m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{3}{16}m^2\sqrt{\pi}\,\omega^5\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{9}{32}m^2\sqrt{\pi}\,\omega^6\,\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_4==0$$

$$\frac{4\,m^3\,\omega^3\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^3}+\frac{1}{8}m^3\sqrt{\pi}\,\omega^6\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{1}{8}m^3\sqrt{\pi}\,\omega^7\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_4==0$$

$$-\frac{2\,m^4\,\omega^4\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^4}+\frac{1}{16}m^4\sqrt{\pi}\,\omega^8\sqrt{\frac{\omega\hbar}{m}}\,A_4==0$$

Решение системы:

A_0	$\frac{m}{\pi\hbar}$
A_1	$-\frac{16\,m}{\pi\,\omega\,\hbar^2}$
A_2	$\frac{48\,m}{\pi\,\omega^2\,\hbar^3}$
A_3	$-\frac{128\,m}{3\,\pi\,\omega^3\,\hbar^4}$
A_4	$\frac{32\,m}{3\,\pi\,\omega^4\,\hbar^5}$

Полином:

$$P_4(x)\;=\;\frac{32\,m\,x^4}{3\,\pi\,\omega^4\,\hbar^5}-\frac{128\,m\,x^3}{3\,\pi\,\omega^3\,\hbar^4}+\frac{48\,m\,x^2}{\pi\,\omega^2\,\hbar^3}-\frac{16\,m\,x}{\pi\,\omega\,\hbar^2}+\frac{m}{\pi\,\hbar}$$

Для $n = 5$

Интеграл:

$$\int_{-\infty}^{\infty}e^{-\frac{m v^2}{\omega\hbar}}\left(A_0+\frac{1}{2}\left(m v^2+m x^2\omega^2\right)A_1+\frac{1}{4}\left(m v^2+m x^2\omega^2\right)^2A_2+\right.$$
$$\left.\frac{1}{8}\left(m v^2+m x^2\omega^2\right)^3A_3+\frac{1}{16}\left(m v^2+m x^2\omega^2\right)^4A_4+\frac{1}{32}\left(m v^2+m x^2\omega^2\right)^5A_5\right)dv\;=$$

$$= \frac{1}{1024}\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\left(1024\,A_0+\omega\left(256\left(2\,m\,x^2\,\omega+\hbar\right)A_1+\omega\left(64\left(4\,m^2\,x^4\,\omega^2+4\,m\,x^2\,\omega\,\hbar+3\,\hbar^2\right)A_2+\omega\left(16\left(8\,m^3\,x^6\,\omega^3+12\,m^2\,x^4\,\omega^2\,\hbar+18\,m\,x^2\,\omega\,\hbar^2+15\,\hbar^3\right)A_3+\omega\left(4\left(16\,m^4\,x^8\,\omega^4+32\,m^3\,x^6\,\omega^3\,\hbar+72\,m^2\,x^4\,\omega^2\,\hbar^2+120\,m\,x^2\,\omega\,\hbar^3+105\,\hbar^4\right)A_4+\omega\left(32\,m^5\,x^{10}\,\omega^5+80\,m^4\,x^8\,\omega^4\,\hbar+240\,m^3\,x^6\,\omega^3\,\hbar^2+600\,m^2\,x^4\,\omega^2\,\hbar^3+1050\,m\,x^2\,\omega\,\hbar^4+945\,\hbar^5\right)A_5\right)\right)\right)\right)\right)$$

$=$

$$\frac{\sqrt{\frac{m\,\omega}{\hbar}}\;H_5\left[x\sqrt{\frac{m\,\omega}{\hbar}}\right]^2}{3840\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{1024}\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\left(1024\,A_0+\omega\left(256\left(2\,m\,x^2\,\omega+\hbar\right)A_1+\omega\left(64\left(4\,m^2\,x^4\,\omega^2+4\,m\,x^2\,\omega\,\hbar+3\,\hbar^2\right)A_2+\omega\left(16\left(8\,m^3\,x^6\,\omega^3+12\,m^2\,x^4\,\omega^2\,\hbar+18\,m\,x^2\,\omega\,\hbar^2+15\,\hbar^3\right)A_3+\omega\left(4\left(16\,m^4\,x^8\,\omega^4+32\,m^3\,x^6\,\omega^3\,\hbar+72\,m^2\,x^4\,\omega^2\,\hbar^2+120\,m\,x^2\,\omega\,\hbar^3+105\,\hbar^4\right)A_4+\omega\left(32\,m^5\,x^{10}\,\omega^5+80\,m^4\,x^8\,\omega^4\,\hbar+240\,m^3\,x^6\,\omega^3\,\hbar^2+600\,m^2\,x^4\,\omega^2\,\hbar^3+1050\,m\,x^2\,\omega\,\hbar^4+945\,\hbar^5\right)A_5\right)\right)\right)\right)\right) ==$$

$$\frac{\left(120\,x\sqrt{\frac{m\,\omega}{\hbar}}-160\,x^3\left(\frac{m\,\omega}{\hbar}\right)^{3/2}+32\,x^5\left(\frac{m\,\omega}{\hbar}\right)^{5/2}\right)^2\sqrt{\frac{m\,\omega}{\hbar}}}{3840\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}A_0+\frac{1}{4}\sqrt{\pi}\,\omega\hbar\sqrt{\frac{\omega\hbar}{m}}A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\hbar^4\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{945\sqrt{\pi}\,\omega^5\hbar^5\sqrt{\frac{\omega\hbar}{m}}A_5}{1024}==0$$

$$-\frac{15\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{4\sqrt{\pi}\,\hbar}+\frac{1}{2}m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}A_1+\frac{1}{4}m\sqrt{\pi}\,\omega^3\hbar\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{9}{32}m\sqrt{\pi}\,\omega^4\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{15}{32}m\sqrt{\pi}\,\omega^5\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{525}{512}m\sqrt{\pi}\,\omega^6\hbar^4\sqrt{\frac{\omega\hbar}{m}}A_5==0$$

$$\frac{10\,m^2\omega^2\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^2}+\frac{1}{4}m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{3}{16}m^2\sqrt{\pi}\,\omega^5\hbar\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{9}{32}m^2\sqrt{\pi}\,\omega^6\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{75}{128}m^2\sqrt{\pi}\,\omega^7\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_5==0$$

$$-\frac{26\,m^3\omega^3\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^3}+\frac{1}{8}m^3\sqrt{\pi}\,\omega^6\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{1}{8}m^3\sqrt{\pi}\,\omega^7\hbar\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{15}{64}m^3\sqrt{\pi}\,\omega^8\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_5==0$$

$$\frac{8\,m^4\omega^4\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^4}+\frac{1}{16}m^4\sqrt{\pi}\,\omega^8\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{5}{64}m^4\sqrt{\pi}\,\omega^9\hbar\sqrt{\frac{\omega\hbar}{m}}A_5==0$$

$$-\frac{4\,m^5\omega^5\sqrt{\frac{m\omega}{\hbar}}}{15\sqrt{\pi}\,\hbar^5}+\frac{1}{32}m^5\sqrt{\pi}\,\omega^{10}\sqrt{\frac{\omega\hbar}{m}}A_5==0$$

Решение системы:

A_0	$-\frac{m}{\pi\hbar}$
A_1	$\frac{20\,m}{\pi\omega\hbar^2}$
A_2	$-\frac{80\,m}{\pi\omega^2\hbar^3}$
A_3	$\frac{320\,m}{3\pi\omega^3\hbar^4}$
A_4	$-\frac{160\,m}{3\pi\omega^4\hbar^5}$
A_5	$\frac{128\,m}{15\pi\omega^5\hbar^6}$

Полином:

$$P_5(x)\;=\;\frac{128\,m\,x^5}{15\pi\omega^5\hbar^6}-\frac{160\,m\,x^4}{3\pi\omega^4\hbar^5}+\frac{320\,m\,x^3}{3\pi\omega^3\hbar^4}-\frac{80\,m\,x^2}{\pi\omega^2\hbar^3}+\frac{20\,m\,x}{\pi\omega\hbar^2}-\frac{m}{\pi\hbar}$$

Для $n = 6$

Интеграл:

$$\int_{-\infty}^{\infty}e^{-\frac{m v^2}{\omega\hbar}}\left(A_0+\frac{1}{2}\left(m v^2+m x^2\omega^2\right)A_1+\frac{1}{4}\left(m v^2+m x^2\omega^2\right)^2A_2+\frac{1}{8}\left(m v^2+m x^2\omega^2\right)^3A_3+\frac{1}{16}\left(m v^2+m x^2\omega^2\right)^4A_4+\frac{1}{32}\left(m v^2+m x^2\omega^2\right)^5A_5+\frac{1}{64}\left(m v^2+m x^2\omega^2\right)^6A_6\right)dv\;=$$

$$= \frac{1}{4096}\sqrt{\pi}\ \sqrt{\frac{\omega\hbar}{m}}\ \Big(4096\,A_0 + \omega\,\Big(1024\,\big(2\,m\,x^2\,\omega + \hbar\big)\,A_1 + \omega\,\Big(256\,\big(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2\big)\,A_2 + \omega\,\Big(64\,\big(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3\big)\,A_3 + \omega\,\Big(16\,\big(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4\big)\,A_4 + \omega\,\Big(4\,\big(32\,m^5\,x^{10}\,\omega^5 + 80\,m^4\,x^8\,\omega^4\,\hbar + 240\,m^3\,x^6\,\omega^3\,\hbar^2 + 600\,m^2\,x^4\,\omega^2\,\hbar^3 + 1050\,m\,x^2\,\omega\,\hbar^4 + 945\,\hbar^5\big)\,A_5 + \omega\,\big(64\,m^6\,x^{12}\,\omega^6 + 192\,m^5\,x^{10}\,\omega^5\,\hbar + 720\,m^4\,x^8\,\omega^4\,\hbar^2 + 2400\,m^3\,x^6\,\omega^3\,\hbar^3 + 6300\,m^2\,x^4\,\omega^2\,\hbar^4 + 11\,340\,m\,x^2\,\omega\,\hbar^5 + 10\,395\,\hbar^6\big)\,A_6\Big)\Big)\Big)\Big)\Big)\Big)$$

$$=$$

$$\frac{\sqrt{\frac{m\,\omega}{\hbar}}\ H_6\left[x\,\sqrt{\frac{m\,\omega}{\hbar}}\right]^2}{46\,080\,\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{4096}\sqrt{\pi}\ \sqrt{\frac{\omega\hbar}{m}}\ \Big(4096\,A_0 + \omega\,\Big(1024\,\big(2\,m\,x^2\,\omega + \hbar\big)\,A_1 + \omega\,\Big(256\,\big(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2\big)\,A_2 + \omega\,\Big(64\,\big(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3\big)\,A_3 + \omega\,\Big(16\,\big(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4\big)\,A_4 + \omega\,\Big(4\,\big(32\,m^5\,x^{10}\,\omega^5 + 80\,m^4\,x^8\,\omega^4\,\hbar + 240\,m^3\,x^6\,\omega^3\,\hbar^2 + 600\,m^2\,x^4\,\omega^2\,\hbar^3 + 1050\,m\,x^2\,\omega\,\hbar^4 + 945\,\hbar^5\big)\,A_5 + \omega\,\big(64\,m^6\,x^{12}\,\omega^6 + 192\,m^5\,x^{10}\,\omega^5\,\hbar + 720\,m^4\,x^8\,\omega^4\,\hbar^2 + 2400\,m^3\,x^6\,\omega^3\,\hbar^3 + 6300\,m^2\,x^4\,\omega^2\,\hbar^4 + 11\,340\,m\,x^2\,\omega\,\hbar^5 + 10\,395\,\hbar^6\big)\,A_6\Big)\Big)\Big)\Big)\Big)\Big) == \frac{\left(-120 + \frac{64\,m^3\,x^6\,\omega^3}{\hbar^3} - \frac{480\,m^2\,x^4\,\omega^2}{\hbar^2} + \frac{720\,m\,x^2\,\omega}{\hbar}\right)^2\sqrt{\frac{m\,\omega}{\hbar}}}{46\,080\,\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$-\frac{5\sqrt{\frac{m\omega}{\hbar}}}{16\sqrt{\pi}}+\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}A_0+\frac{1}{4}\sqrt{\pi}\,\omega\hbar\sqrt{\frac{\omega\hbar}{m}}A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\hbar^4\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{945\sqrt{\pi}\,\omega^5\hbar^5\sqrt{\frac{\omega\hbar}{m}}A_5}{1024}+\frac{10\,395\sqrt{\pi}\,\omega^6\hbar^6\sqrt{\frac{\omega\hbar}{m}}A_6}{4096}==0$$

$$\frac{15\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{4\sqrt{\pi}\,\hbar}+\frac{1}{2}m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}A_1+\frac{1}{4}m\sqrt{\pi}\,\omega^3\hbar\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{9}{32}m\sqrt{\pi}\,\omega^4\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{15}{32}m\sqrt{\pi}\,\omega^5\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{525}{512}m\sqrt{\pi}\,\omega^6\hbar^4\sqrt{\frac{\omega\hbar}{m}}A_5+\frac{2835\,m\sqrt{\pi}\,\omega^7\hbar^5\sqrt{\frac{\omega\hbar}{m}}A_6}{1024}==0$$

$$-\frac{55\,m^2\omega^2\sqrt{\frac{m\omega}{\hbar}}}{4\sqrt{\pi}\,\hbar^2}+\frac{1}{4}m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}}A_2+\frac{3}{16}m^2\sqrt{\pi}\,\omega^5\hbar\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{9}{32}m^2\sqrt{\pi}\,\omega^6\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{75}{128}m^2\sqrt{\pi}\,\omega^7\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_5+\frac{1575\,m^2\sqrt{\pi}\,\omega^8\hbar^4\sqrt{\frac{\omega\hbar}{m}}A_6}{1024}==0$$

$$\frac{46\,m^3\omega^3\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^3}+\frac{1}{8}m^3\sqrt{\pi}\,\omega^6\sqrt{\frac{\omega\hbar}{m}}A_3+\frac{1}{8}m^3\sqrt{\pi}\,\omega^7\hbar\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{15}{64}m^3\sqrt{\pi}\,\omega^8\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_5+\frac{75}{128}m^3\sqrt{\pi}\,\omega^9\hbar^3\sqrt{\frac{\omega\hbar}{m}}A_6==0$$

$$-\frac{7\,m^4\omega^4\sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^4}+\frac{1}{16}m^4\sqrt{\pi}\,\omega^8\sqrt{\frac{\omega\hbar}{m}}A_4+\frac{5}{64}m^4\sqrt{\pi}\,\omega^9\hbar\sqrt{\frac{\omega\hbar}{m}}A_5+\frac{45}{256}m^4\sqrt{\pi}\,\omega^{10}\hbar^2\sqrt{\frac{\omega\hbar}{m}}A_6==0$$

$$\frac{4\,m^5\omega^5\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^5}+\frac{1}{32}m^5\sqrt{\pi}\,\omega^{10}\sqrt{\frac{\omega\hbar}{m}}A_5+\frac{3}{64}m^5\sqrt{\pi}\,\omega^{11}\hbar\sqrt{\frac{\omega\hbar}{m}}A_6==0$$

$$-\frac{4\,m^6\omega^6\sqrt{\frac{m\omega}{\hbar}}}{45\sqrt{\pi}\,\hbar^6}+\frac{1}{64}m^6\sqrt{\pi}\,\omega^{12}\sqrt{\frac{\omega\hbar}{m}}A_6==0$$

Решение системы:

A_0	$\frac{m}{\pi\hbar}$
A_1	$-\frac{24\,m}{\pi\omega\hbar^2}$
A_2	$\frac{120\,m}{\pi\omega^2\hbar^3}$
A_3	$-\frac{640\,m}{3\pi\omega^3\hbar^4}$
A_4	$\frac{160\,m}{\pi\omega^4\hbar^5}$
A_5	$-\frac{256\,m}{5\pi\omega^5\hbar^6}$
A_6	$\frac{256\,m}{45\pi\omega^6\hbar^7}$

Полином:

$$P_6(x)\;=\;\frac{256\,m\,x^6}{45\pi\omega^6\hbar^7}-\frac{256\,m\,x^5}{5\pi\omega^5\hbar^6}+\frac{160\,m\,x^4}{\pi\omega^4\hbar^5}-\frac{640\,m\,x^3}{3\pi\omega^3\hbar^4}+\frac{120\,m\,x^2}{\pi\omega^2\hbar^3}-\frac{24\,m\,x}{\pi\omega\hbar^2}+\frac{m}{\pi\hbar}$$

Для n = 7

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2}\left(m v^2 + m x^2 \omega^2\right) A_1 + \frac{1}{4}\left(m v^2 + m x^2 \omega^2\right)^2 A_2 + \frac{1}{8}\left(m v^2 + m x^2 \omega^2\right)^3 A_3 + \frac{1}{16}\left(m v^2 + m x^2 \omega^2\right)^4 A_4 + \frac{1}{32}\left(m v^2 + m x^2 \omega^2\right)^5 A_5 + \frac{1}{64}\left(m v^2 + m x^2 \omega^2\right)^6 A_6 + \frac{1}{128}\left(m v^2 + m x^2 \omega^2\right)^7 A_7\right) dv =$$

$$= \frac{1}{16\,384} \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} \Big(16\,384 A_0 + \omega \Big(4096 \left(2 m x^2 \omega + \hbar\right) A_1 + \omega \Big(1024 \left(4 m^2 x^4 \omega^2 + 4 m x^2 \omega \hbar + 3 \hbar^2\right) A_2 + \omega \Big(256 \left(8 m^3 x^6 \omega^3 + 12 m^2 x^4 \omega^2 \hbar + 18 m x^2 \omega \hbar^2 + 15 \hbar^3\right) A_3 + \omega \Big(64 \left(16 m^4 x^8 \omega^4 + 32 m^3 x^6 \omega^3 \hbar + 72 m^2 x^4 \omega^2 \hbar^2 + 120 m x^2 \omega \hbar^3 + 105 \hbar^4\right) A_4 + \omega \Big(16 \left(32 m^5 x^{10} \omega^5 + 80 m^4 x^8 \omega^4 \hbar + 240 m^3 x^6 \omega^3 \hbar^2 + 600 m^2 x^4 \omega^2 \hbar^3 + 1050 m x^2 \omega \hbar^4 + 945 \hbar^5\right) A_5 + \omega \Big(4 \left(64 m^6 x^{12} \omega^6 + 192 m^5 x^{10} \omega^5 \hbar + 720 m^4 x^8 \omega^4 \hbar^2 + 2400 m^3 x^6 \omega^3 \hbar^3 + 6300 m^2 x^4 \omega^2 \hbar^4 + 11\,340 m x^2 \omega \hbar^5 + 10\,395 \hbar^6\right) A_6 + \omega \left(128 m^7 x^{14} \omega^7 + 448 m^6 x^{12} \omega^6 \hbar + 2016 m^5 x^{10} \omega^5 \hbar^2 + 8400 m^4 x^8 \omega^4 \hbar^3 + 29\,400 m^3 x^6 \omega^3 \hbar^4 + 79\,380 m^2 x^4 \omega^2 \hbar^5 + 145\,530 m x^2 \omega \hbar^6 + 135\,135 \hbar^7\right) A_7\Big)\Big)\Big)\Big)\Big)\Big)\Big)$$

$$=$$

$$\frac{\sqrt{\frac{m \omega}{\hbar}}\, H_7\left[x \sqrt{\frac{m \omega}{\hbar}}\right]^2}{645\,120 \sqrt{\pi}}$$

Уравнение:

$$\frac{1}{16\,384} \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} \Big(16\,384 A_0 + \omega \Big(4096 \left(2 m x^2 \omega + \hbar\right) A_1 + \omega \Big(1024 \left(4 m^2 x^4 \omega^2 + 4 m x^2 \omega \hbar + 3 \hbar^2\right) A_2 + \omega \Big(256 \left(8 m^3 x^6 \omega^3 + 12 m^2 x^4 \omega^2 \hbar + 18 m x^2 \omega \hbar^2 + 15 \hbar^3\right) A_3 + \omega \Big(64 \left(16 m^4 x^8 \omega^4 + 32 m^3 x^6 \omega^3 \hbar + 72 m^2 x^4 \omega^2 \hbar^2 + 120 m x^2 \omega \hbar^3 + 105 \hbar^4\right) A_4 + \omega \Big(16 \left(32 m^5 x^{10} \omega^5 + 80 m^4 x^8 \omega^4 \hbar + 240 m^3 x^6 \omega^3 \hbar^2 + 600 m^2 x^4 \omega^2 \hbar^3 + 1050 m x^2 \omega \hbar^4 + 945 \hbar^5\right) A_5 + \omega \Big(4 \left(64 m^6 x^{12} \omega^6 + 192 m^5 x^{10} \omega^5 \hbar + 720 m^4 x^8 \omega^4 \hbar^2 + 2400 m^3 x^6 \omega^3 \hbar^3 + 6300 m^2 x^4 \omega^2 \hbar^4 + 11\,340 m x^2 \omega \hbar^5 + 10\,395 \hbar^6\right) A_6 + \omega \left(128 m^7 x^{14} \omega^7 + 448 m^6 x^{12} \omega^6 \hbar + 2016 m^5 x^{10} \omega^5 \hbar^2 + 8400 m^4 x^8 \omega^4 \hbar^3 + 29\,400 m^3 x^6 \omega^3 \hbar^4 + 79\,380 m^2 x^4 \omega^2 \hbar^5 + 145\,530 m x^2 \omega \hbar^6 + 135\,135 \hbar^7\right) A_7\Big)\Big)\Big)\Big)\Big)\Big)\Big) ==$$

$$\frac{1}{645\,120 \sqrt{\pi}} \left(-1680 x \sqrt{\frac{m \omega}{\hbar}} + 3360 x^3 \left(\frac{m \omega}{\hbar}\right)^{3/2} - 1344 x^5 \left(\frac{m \omega}{\hbar}\right)^{5/2} + 128 x^7 \left(\frac{m \omega}{\hbar}\right)^{7/2}\right)^2 \sqrt{\frac{m \omega}{\hbar}}$$

Система уравнений на коэффициенты:

$$\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\,A_0+\frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\hbar^4\sqrt{\frac{\omega\hbar}{m}}\,A_4+\frac{945\sqrt{\pi}\,\omega^5\hbar^5\sqrt{\frac{\omega\hbar}{m}}\,A_5}{1024}+\frac{10\,395\sqrt{\pi}\,\omega^6\hbar^6\sqrt{\frac{\omega\hbar}{m}}\,A_6}{4096}+\frac{135\,135\sqrt{\pi}\,\omega^7\hbar^7\sqrt{\frac{\omega\hbar}{m}}\,A_7}{16\,384}==0$$

$$-\frac{35\,m\,\omega\sqrt{\frac{m\omega}{\hbar}}}{8\sqrt{\pi}\,\hbar}+\frac{1}{2}m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}}\,A_1+\frac{1}{4}m\sqrt{\pi}\,\omega^3\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{9}{32}m\sqrt{\pi}\,\omega^4\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{15}{32}m\sqrt{\pi}\,\omega^5\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_4+\frac{525}{512}m\sqrt{\pi}\,\omega^6\hbar^4\sqrt{\frac{\omega\hbar}{m}}\,A_5+\frac{2835\,m\sqrt{\pi}\,\omega^7\hbar^5\sqrt{\frac{\omega\hbar}{m}}\,A_6}{1024}+\frac{72\,765\,m\sqrt{\pi}\,\omega^8\hbar^6\sqrt{\frac{\omega\hbar}{m}}\,A_7}{8192}==0$$

$$\frac{35\,m^2\omega^2\sqrt{\frac{m\omega}{\hbar}}}{2\sqrt{\pi}\,\hbar^2}+\frac{1}{4}m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}}\,A_2+\frac{3}{16}m^2\sqrt{\pi}\,\omega^5\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{9}{32}m^2\sqrt{\pi}\,\omega^6\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_4+\frac{75}{128}m^2\sqrt{\pi}\,\omega^7\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_5+\frac{1575\,m^2\sqrt{\pi}\,\omega^8\hbar^4\sqrt{\frac{\omega\hbar}{m}}\,A_6}{1024}+\frac{19\,845\,m^2\sqrt{\pi}\,\omega^9\hbar^5\sqrt{\frac{\omega\hbar}{m}}\,A_7}{4096}==0$$

$$-\frac{49\,m^3\omega^3\sqrt{\frac{m\omega}{\hbar}}}{2\sqrt{\pi}\,\hbar^3}+\frac{1}{8}m^3\sqrt{\pi}\,\omega^6\sqrt{\frac{\omega\hbar}{m}}\,A_3+\frac{1}{8}m^3\sqrt{\pi}\,\omega^7\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_4+\frac{15}{64}m^3\sqrt{\pi}\,\omega^8\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_5+\frac{75}{128}m^3\sqrt{\pi}\,\omega^9\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_6+\frac{3675\,m^3\sqrt{\pi}\,\omega^{10}\hbar^4\sqrt{\frac{\omega\hbar}{m}}\,A_7}{2048}==0$$

$$\frac{44\,m^4\omega^4\sqrt{\frac{m\omega}{\hbar}}}{3\sqrt{\pi}\,\hbar^4}+\frac{1}{16}m^4\sqrt{\pi}\,\omega^8\sqrt{\frac{\omega\hbar}{m}}\,A_4+\frac{5}{64}m^4\sqrt{\pi}\,\omega^9\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_5+\frac{45}{256}m^4\sqrt{\pi}\,\omega^{10}\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_6+\frac{525\,m^4\sqrt{\pi}\,\omega^{11}\hbar^3\sqrt{\frac{\omega\hbar}{m}}\,A_7}{1024}==0$$

$$-\frac{62\,m^5\omega^5\sqrt{\frac{m\omega}{\hbar}}}{15\sqrt{\pi}\,\hbar^5}+\frac{1}{32}m^5\sqrt{\pi}\,\omega^{10}\sqrt{\frac{\omega\hbar}{m}}\,A_5+\frac{3}{64}m^5\sqrt{\pi}\,\omega^{11}\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_6+\frac{63}{512}m^5\sqrt{\pi}\,\omega^{12}\hbar^2\sqrt{\frac{\omega\hbar}{m}}\,A_7==0$$

$$\frac{8\,m^6\omega^6\sqrt{\frac{m\omega}{\hbar}}}{15\sqrt{\pi}\,\hbar^6}+\frac{1}{64}m^6\sqrt{\pi}\,\omega^{12}\sqrt{\frac{\omega\hbar}{m}}\,A_6+\frac{7}{256}m^6\sqrt{\pi}\,\omega^{13}\hbar\sqrt{\frac{\omega\hbar}{m}}\,A_7==0$$

$$-\frac{8\,m^7\omega^7\sqrt{\frac{m\omega}{\hbar}}}{315\sqrt{\pi}\,\hbar^7}+\frac{1}{128}m^7\sqrt{\pi}\,\omega^{14}\sqrt{\frac{\omega\hbar}{m}}\,A_7==0$$

Решение системы:

A_0	$-\frac{m}{\pi\hbar}$
A_1	$\frac{28\,m}{\pi\omega\hbar^2}$
A_2	$-\frac{168\,m}{\pi\omega^2\hbar^3}$
A_3	$\frac{1120\,m}{3\pi\omega^3\hbar^4}$
A_4	$-\frac{1120\,m}{3\pi\omega^4\hbar^5}$
A_5	$\frac{896\,m}{5\pi\omega^5\hbar^6}$
A_6	$-\frac{1792\,m}{45\pi\omega^6\hbar^7}$
A_7	$\frac{1024\,m}{315\pi\omega^7\hbar^8}$

Полином:

$$P_7(x) = \frac{1024\, m\, x^7}{315\, \pi\, \omega^7\, \hbar^8} - \frac{1792\, m\, x^6}{45\, \pi\, \omega^6\, \hbar^7} + \frac{896\, m\, x^5}{5\, \pi\, \omega^5\, \hbar^6} - \frac{1120\, m\, x^4}{3\, \pi\, \omega^4\, \hbar^5} + \frac{1120\, m\, x^3}{3\, \pi\, \omega^3\, \hbar^4} - \frac{168\, m\, x^2}{\pi\, \omega^2\, \hbar^3} + \frac{28\, m\, x}{\pi\, \omega\, \hbar^2} - \frac{m}{\pi\, \hbar}$$

Для n = 8

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2}\left(m v^2 + m x^2 \omega^2\right) A_1 + \frac{1}{4}\left(m v^2 + m x^2 \omega^2\right)^2 A_2 + \frac{1}{8}\left(m v^2 + m x^2 \omega^2\right)^3 A_3 + \frac{1}{16}\left(m v^2 + m x^2 \omega^2\right)^4 A_4 + \frac{1}{32}\left(m v^2 + m x^2 \omega^2\right)^5 A_5 + \frac{1}{64}\left(m v^2 + m x^2 \omega^2\right)^6 A_6 + \frac{1}{128}\left(m v^2 + m x^2 \omega^2\right)^7 A_7 + \frac{1}{256}\left(m v^2 + m x^2 \omega^2\right)^8 A_8\right) dv =$$

$$= \frac{1}{65\,536}\sqrt{\pi}\,\sqrt{\frac{\omega\hbar}{m}}\,\Big(65\,536\, A_0 + \omega\,\Big(16\,384\,\left(2\, m\, x^2\, \omega + \hbar\right) A_1 + \omega\,\Big(4096\,\left(4\, m^2\, x^4\, \omega^2 + 4\, m\, x^2\, \omega\, \hbar + 3\, \hbar^2\right) A_2 + \omega\,\Big(1024\,\left(8\, m^3\, x^6\, \omega^3 + 12\, m^2\, x^4\, \omega^2\, \hbar + 18\, m\, x^2\, \omega\, \hbar^2 + 15\, \hbar^3\right) A_3 + \omega\,\Big(256\,\left(16\, m^4\, x^8\, \omega^4 + 32\, m^3\, x^6\, \omega^3\, \hbar + 72\, m^2\, x^4\, \omega^2\, \hbar^2 + 120\, m\, x^2\, \omega\, \hbar^3 + 105\, \hbar^4\right) A_4 + \omega\,\Big(64\,\left(32\, m^5\, x^{10}\, \omega^5 + 80\, m^4\, x^8\, \omega^4\, \hbar + 240\, m^3\, x^6\, \omega^3\, \hbar^2 + 600\, m^2\, x^4\, \omega^2\, \hbar^3 + 1050\, m\, x^2\, \omega\, \hbar^4 + 945\, \hbar^5\right) A_5 + \omega\,\Big(16\,\left(64\, m^6\, x^{12}\, \omega^6 + 192\, m^5\, x^{10}\, \omega^5\, \hbar + 720\, m^4\, x^8\, \omega^4\, \hbar^2 + 2400\, m^3\, x^6\, \omega^3\, \hbar^3 + 6300\, m^2\, x^4\, \omega^2\, \hbar^4 + 11\,340\, m\, x^2\, \omega\, \hbar^5 + 10\,395\, \hbar^6\right) A_6 + \omega\,\Big(4\,\left(128\, m^7\, x^{14}\, \omega^7 + 448\, m^6\, x^{12}\, \omega^6\, \hbar + 2016\, m^5\, x^{10}\, \omega^5\, \hbar^2 + 8400\, m^4\, x^8\, \omega^4\, \hbar^3 + 29\,400\, m^3\, x^6\, \omega^3\, \hbar^4 + 79\,380\, m^2\, x^4\, \omega^2\, \hbar^5 + 145\,530\, m\, x^2\, \omega\, \hbar^6 + 135\,135\, \hbar^7\right) A_7 + \omega\,\left(256\, m^8\, x^{16}\, \omega^8 + 1024\, m^7\, x^{14}\, \omega^7\, \hbar + 5376\, m^6\, x^{12}\, \omega^6\, \hbar^2 + 26\,880\, m^5\, x^{10}\, \omega^5\, \hbar^3 + 117\,600\, m^4\, x^8\, \omega^4\, \hbar^4 + 423\,360\, m^3\, x^6\, \omega^3\, \hbar^5 + 1\,164\,240\, m^2\, x^4\, \omega^2\, \hbar^6 + 2\,162\,160\, m\, x^2\, \omega\, \hbar^7 + 2\,027\,025\, \hbar^8\right) A_8\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)$$

$$= \frac{\sqrt{\frac{m\omega}{\hbar}}\; H_8\left[x\sqrt{\frac{m\omega}{\hbar}}\right]^2}{10\,321\,920\,\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{65\,536}\sqrt{\pi}\,\sqrt{\frac{\omega\,\hbar}{m}}\,\Big(65\,536\,A_0+\omega\,\big(16\,384\,(2\,m\,x^2\,\omega+\hbar)\,A_1+\omega\,\big(4096\,(4\,m^2\,x^4\,\omega^2+4\,m\,x^2\,\omega\,\hbar+3\,\hbar^2)\,A_2+\omega\,\big(1024\,(8\,m^3\,x^6\,\omega^3+12\,m^2\,x^4\,\omega^2\,\hbar+18\,m\,x^2\,\omega\,\hbar^2+15\,\hbar^3)\,A_3+\omega\,\big(256\,(16\,m^4\,x^8\,\omega^4+32\,m^3\,x^6\,\omega^3\,\hbar+72\,m^2\,x^4\,\omega^2\,\hbar^2+120\,m\,x^2\,\omega\,\hbar^3+105\,\hbar^4)\,A_4+\omega\,\big(64\,(32\,m^5\,x^{10}\,\omega^5+80\,m^4\,x^8\,\omega^4\,\hbar+240\,m^3\,x^6\,\omega^3\,\hbar^2+600\,m^2\,x^4\,\omega^2\,\hbar^3+1050\,m\,x^2\,\omega\,\hbar^4+945\,\hbar^5)\,A_5+\omega\,\big(16\,(64\,m^6\,x^{12}\,\omega^6+192\,m^5\,x^{10}\,\omega^5\,\hbar+720\,m^4\,x^8\,\omega^4\,\hbar^2+2400\,m^3\,x^6\,\omega^3\,\hbar^3+6300\,m^2\,x^4\,\omega^2\,\hbar^4+11\,340\,m\,x^2\,\omega\,\hbar^5+10\,395\,\hbar^6)\,A_6+\omega\,\big(4\,(128\,m^7\,x^{14}\,\omega^7+448\,m^6\,x^{12}\,\omega^6\,\hbar+2016\,m^5\,x^{10}\,\omega^5\,\hbar^2+8400\,m^4\,x^8\,\omega^4\,\hbar^3+29\,400\,m^3\,x^6\,\omega^3\,\hbar^4+79\,380\,m^2\,x^4\,\omega^2\,\hbar^5+145\,530\,m\,x^2\,\omega\,\hbar^6+135\,135\,\hbar^7)\,A_7+\omega\,(256\,m^8\,x^{16}\,\omega^8+1024\,m^7\,x^{14}\,\omega^7\,\hbar+5376\,m^6\,x^{12}\,\omega^6\,\hbar^2+26\,880\,m^5\,x^{10}\,\omega^5\,\hbar^3+117\,600\,m^4\,x^8\,\omega^4\,\hbar^4+423\,360\,m^3\,x^6\,\omega^3\,\hbar^5+1\,164\,240\,m^2\,x^4\,\omega^2\,\hbar^6+2\,162\,160\,m\,x^2\,\omega\,\hbar^7+2\,027\,025\,\hbar^8)\,A_8\big)\big)\big)\big)\big)\big)\big)\Big) ==$$

$$\frac{\left(1680+\frac{256\,m^4\,x^8\,\omega^4}{\hbar^4}-\frac{3584\,m^3\,x^6\,\omega^3}{\hbar^3}+\frac{13\,440\,m^2\,x^4\,\omega^2}{\hbar^2}-\frac{13\,440\,m\,x^2\,\omega}{\hbar}\right)^2\sqrt{\frac{m\,\omega}{\hbar}}}{10\,321\,920\,\sqrt{\pi}}$$

Система уравнений на коэффициенты:

$$-\frac{35\sqrt{\frac{m\,\omega}{\hbar}}}{128\sqrt{\pi}}+\sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\,A_0+\frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_4+\frac{945\sqrt{\pi}\,\omega^5\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_5}{1024}+\frac{10\,395\sqrt{\pi}\,\omega^6\,\hbar^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_6}{4096}+\frac{135\,135\sqrt{\pi}\,\omega^7\,\hbar^7\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{16\,384}+\frac{2\,027\,025\sqrt{\pi}\,\omega^8\,\hbar^8\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{65\,536}==0$$

$$\frac{35\,m\,\omega\sqrt{\frac{m\,\omega}{\hbar}}}{8\sqrt{\pi}\,\hbar}+\frac{1}{2}\,m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_1+\frac{1}{4}\,m\sqrt{\pi}\,\omega^3\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_2+\frac{9}{32}\,m\sqrt{\pi}\,\omega^4\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_3+\frac{15}{32}\,m\sqrt{\pi}\,\omega^5\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_4+\frac{525}{512}\,m\sqrt{\pi}\,\omega^6\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_5+\frac{2835\,m\sqrt{\pi}\,\omega^7\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_6}{1024}+\frac{72\,765\,m\sqrt{\pi}\,\omega^8\,\hbar^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{8192}+\frac{135\,135\,m\sqrt{\pi}\,\omega^9\,\hbar^7\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{4096}==0$$

$$-\frac{175\,m^2\,\omega^2\sqrt{\frac{m\,\omega}{\hbar}}}{8\sqrt{\pi}\,\hbar^2}+\frac{1}{4}\,m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_2+\frac{3}{16}\,m^2\sqrt{\pi}\,\omega^5\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_3+\frac{9}{32}\,m^2\sqrt{\pi}\,\omega^6\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_4+\frac{75}{128}\,m^2\sqrt{\pi}\,\omega^7\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_5+\frac{1575\,m^2\sqrt{\pi}\,\omega^8\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_6}{1024}+\frac{19\,845\,m^2\sqrt{\pi}\,\omega^9\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{4096}+\frac{72\,765\,m^2\sqrt{\pi}\,\omega^{10}\,\hbar^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{4096}==0$$

$$\frac{217\,m^3\,\omega^3\sqrt{\frac{m\,\omega}{\hbar}}}{6\sqrt{\pi}\,\hbar^3}+\frac{1}{8}\,m^3\sqrt{\pi}\,\omega^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_3+\frac{1}{8}\,m^3\sqrt{\pi}\,\omega^7\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_4+\frac{15}{64}\,m^3\sqrt{\pi}\,\omega^8\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_5+\frac{75}{128}\,m^3\sqrt{\pi}\,\omega^9\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_6+\frac{3675\,m^3\sqrt{\pi}\,\omega^{10}\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{2048}+\frac{6615\,m^3\sqrt{\pi}\,\omega^{11}\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{1024}==0$$

$$-\frac{323\,m^4\,\omega^4\sqrt{\frac{m\,\omega}{\hbar}}}{12\sqrt{\pi}\,\hbar^4}+\frac{1}{16}\,m^4\sqrt{\pi}\,\omega^8\sqrt{\frac{\omega\,\hbar}{m}}\,A_4+\frac{5}{64}\,m^4\sqrt{\pi}\,\omega^9\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_5+\frac{45}{256}\,m^4\sqrt{\pi}\,\omega^{10}\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_6+\frac{525\,m^4\sqrt{\pi}\,\omega^{11}\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{1024}+\frac{3675\,m^4\sqrt{\pi}\,\omega^{12}\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{2048}==0$$

$$\frac{10\,m^5\,\omega^5\sqrt{\frac{m\,\omega}{\hbar}}}{\sqrt{\pi}\,\hbar^5}+\frac{1}{32}\,m^5\sqrt{\pi}\,\omega^{10}\sqrt{\frac{\omega\,\hbar}{m}}\,A_5+\frac{3}{64}\,m^5\sqrt{\pi}\,\omega^{11}\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_6+\frac{63}{512}\,m^5\sqrt{\pi}\,\omega^{12}\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_7+\frac{105}{256}\,m^5\sqrt{\pi}\,\omega^{13}\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_8==0$$

$$-\frac{86\,m^6\,\omega^6\sqrt{\frac{m\,\omega}{\hbar}}}{45\sqrt{\pi}\,\hbar^6}+\frac{1}{64}\,m^6\sqrt{\pi}\,\omega^{12}\sqrt{\frac{\omega\,\hbar}{m}}\,A_6+\frac{7}{256}\,m^6\sqrt{\pi}\,\omega^{13}\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_7+\frac{21}{256}\,m^6\sqrt{\pi}\,\omega^{14}\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_8==0$$

$$\frac{8\,m^7\,\omega^7\sqrt{\frac{m\,\omega}{\hbar}}}{45\sqrt{\pi}\,\hbar^7}+\frac{1}{128}\,m^7\sqrt{\pi}\,\omega^{14}\sqrt{\frac{\omega\,\hbar}{m}}\,A_7+\frac{1}{64}\,m^7\sqrt{\pi}\,\omega^{15}\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_8==0$$

$$-\frac{2\,m^8\,\omega^8\sqrt{\frac{m\,\omega}{\hbar}}}{315\sqrt{\pi}\,\hbar^8}+\frac{1}{256}\,m^8\sqrt{\pi}\,\omega^{16}\sqrt{\frac{\omega\,\hbar}{m}}\,A_8==0$$

Решение системы:

A_0	$\frac{m}{\pi \hbar}$
A_1	$-\frac{32 m}{\pi \omega \hbar^2}$
A_2	$\frac{224 m}{\pi \omega^2 \hbar^3}$
A_3	$-\frac{1792 m}{3 \pi \omega^3 \hbar^4}$
A_4	$\frac{2240 m}{3 \pi \omega^4 \hbar^5}$
A_5	$-\frac{7168 m}{15 \pi \omega^5 \hbar^6}$
A_6	$\frac{7168 m}{45 \pi \omega^6 \hbar^7}$
A_7	$-\frac{8192 m}{315 \pi \omega^7 \hbar^8}$
A_8	$\frac{512 m}{315 \pi \omega^8 \hbar^9}$

Полином:

$P_8(x) =$

$$\frac{512 m x^8}{315 \pi \omega^8 \hbar^9} - \frac{8192 m x^7}{315 \pi \omega^7 \hbar^8} + \frac{7168 m x^6}{45 \pi \omega^6 \hbar^7} - \frac{7168 m x^5}{15 \pi \omega^5 \hbar^6} + \frac{2240 m x^4}{3 \pi \omega^4 \hbar^5} - \frac{1792 m x^3}{3 \pi \omega^3 \hbar^4} + \frac{224 m x^2}{\pi \omega^2 \hbar^3} - \frac{32 m x}{\pi \omega \hbar^2} + \frac{m}{\pi \hbar}$$

Для $n = 9$

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2}\left(m v^2 + m x^2 \omega^2\right) A_1 + \frac{1}{4}\left(m v^2 + m x^2 \omega^2\right)^2 A_2 + \frac{1}{8}\left(m v^2 + m x^2 \omega^2\right)^3 A_3 + \frac{1}{16}\left(m v^2 + m x^2 \omega^2\right)^4 A_4 + \frac{1}{32}\left(m v^2 + m x^2 \omega^2\right)^5 A_5 + \frac{1}{64}\left(m v^2 + m x^2 \omega^2\right)^6 A_6 + \frac{1}{128}\left(m v^2 + m x^2 \omega^2\right)^7 A_7 + \frac{1}{256}\left(m v^2 + m x^2 \omega^2\right)^8 A_8 + \frac{1}{512}\left(m v^2 + m x^2 \omega^2\right)^9 A_9\right) dv =$$

$$= \frac{1}{262\,144\,\sqrt{\pi}\,\sqrt{\frac{\omega\,\hbar}{m}}} \Big(262\,144\,A_0 + \omega\,\Big(65\,536\,\big(2\,m\,x^2\,\omega + \hbar\big)\,A_1 + \omega\,\Big(16\,384\,\big(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2\big)\,A_2 + \omega\,\Big(4096\,\big(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3\big)\,A_3 + \omega\,\Big(1024\,\big(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4\big)\,A_4 + \omega\,\Big(256\,\big(32\,m^5\,x^{10}\,\omega^5 + 80\,m^4\,x^8\,\omega^4\,\hbar + 240\,m^3\,x^6\,\omega^3\,\hbar^2 + 600\,m^2\,x^4\,\omega^2\,\hbar^3 + 1050\,m\,x^2\,\omega\,\hbar^4 + 945\,\hbar^5\big)\,A_5 + \omega\,\Big(64\,\big(64\,m^6\,x^{12}\,\omega^6 + 192\,m^5\,x^{10}\,\omega^5\,\hbar + 720\,m^4\,x^8\,\omega^4\,\hbar^2 + 2400\,m^3\,x^6\,\omega^3\,\hbar^3 + 6300\,m^2\,x^4\,\omega^2\,\hbar^4 + 11\,340\,m\,x^2\,\omega\,\hbar^5 + 10\,395\,\hbar^6\big)\,A_6 + \omega\,\Big(16\,\big(128\,m^7\,x^{14}\,\omega^7 + 448\,m^6\,x^{12}\,\omega^6\,\hbar + 2016\,m^5\,x^{10}\,\omega^5\,\hbar^2 + 8400\,m^4\,x^8\,\omega^4\,\hbar^3 + 29\,400\,m^3\,x^6\,\omega^3\,\hbar^4 + 79\,380\,m^2\,x^4\,\omega^2\,\hbar^5 + 145\,530\,m\,x^2\,\omega\,\hbar^6 + 135\,135\,\hbar^7\big)\,A_7 + \omega\,\Big(4\,\big(256\,m^8\,x^{16}\,\omega^8 + 1024\,m^7\,x^{14}\,\omega^7\,\hbar + 5376\,m^6\,x^{12}\,\omega^6\,\hbar^2 + 26\,880\,m^5\,x^{10}\,\omega^5\,\hbar^3 + 117\,600\,m^4\,x^8\,\omega^4\,\hbar^4 + 423\,360\,m^3\,x^6\,\omega^3\,\hbar^5 + 1\,164\,240\,m^2\,x^4\,\omega^2\,\hbar^6 + 2\,162\,160\,m\,x^2\,\omega\,\hbar^7 + 2\,027\,025\,\hbar^8\big)\,A_8 + \omega\,\big(512\,m^9\,x^{18}\,\omega^9 + 2304\,m^8\,x^{16}\,\omega^8\,\hbar + 13\,824\,m^7\,x^{14}\,\omega^7\,\hbar^2 + 80\,640\,m^6\,x^{12}\,\omega^6\,\hbar^3 + 423\,360\,m^5\,x^{10}\,\omega^5\,\hbar^4 + 1\,905\,120\,m^4\,x^8\,\omega^4\,\hbar^5 + 6\,985\,440\,m^3\,x^6\,\omega^3\,\hbar^6 + 19\,459\,440\,m^2\,x^4\,\omega^2\,\hbar^7 + 36\,486\,450\,m\,x^2\,\omega\,\hbar^8 + 34\,459\,425\,\hbar^9\big)\,A_9\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)$$

$$= \frac{\sqrt{\frac{m\,\omega}{\hbar}}\;H_9\left[x\,\sqrt{\frac{m\,\omega}{\hbar}}\right]^2}{185\,794\,560\,\sqrt{\pi}}$$

Уравнение:

$$\frac{1}{262144}\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\left(262144 A_0+\omega\left(65536\left(2 m x^2 \omega+\hbar\right) A_1+\omega\left(16384\left(4 m^2 x^4 \omega^2+4 m x^2 \omega \hbar+3 \hbar^2\right) A_2+\omega\left(4096\left(8 m^3 x^6 \omega^3+12 m^2 x^4 \omega^2 \hbar+18 m x^2 \omega \hbar^2+15 \hbar^3\right) A_3+\omega\left(1024\left(16 m^4 x^8 \omega^4+32 m^3 x^6 \omega^3 \hbar+72 m^2 x^4 \omega^2 \hbar^2+120 m x^2 \omega \hbar^3+105 \hbar^4\right) A_4+\omega\left(256\left(32 m^5 x^{10} \omega^5+80 m^4 x^8 \omega^4 \hbar+240 m^3 x^6 \omega^3 \hbar^2+600 m^2 x^4 \omega^2 \hbar^3+1050 m x^2 \omega \hbar^4+945 \hbar^5\right) A_5+\omega\left(64\left(64 m^6 x^{12} \omega^6+192 m^5 x^{10} \omega^5 \hbar+720 m^4 x^8 \omega^4 \hbar^2+2400 m^3 x^6 \omega^3 \hbar^3+6300 m^2 x^4 \omega^2 \hbar^4+11340 m x^2 \omega \hbar^5+10395 \hbar^6\right) A_6+\omega\left(16\left(128 m^7 x^{14} \omega^7+448 m^6 x^{12} \omega^6 \hbar+2016 m^5 x^{10} \omega^5 \hbar^2+8400 m^4 x^8 \omega^4 \hbar^3+29400 m^3 x^6 \omega^3 \hbar^4+79380 m^2 x^4 \omega^2 \hbar^5+145530 m x^2 \omega \hbar^6+135135 \hbar^7\right) A_7+\omega\left(4\left(256 m^8 x^{16} \omega^8+1024 m^7 x^{14} \omega^7 \hbar+5376 m^6 x^{12} \omega^6 \hbar^2+26880 m^5 x^{10} \omega^5 \hbar^3+117600 m^4 x^8 \omega^4 \hbar^4+423360 m^3 x^6 \omega^3 \hbar^5+1164240 m^2 x^4 \omega^2 \hbar^6+2162160 m x^2 \omega \hbar^7+2027025 \hbar^8\right) A_8+\omega\left(512 m^9 x^{18} \omega^9+2304 m^8 x^{16} \omega^8 \hbar+13824 m^7 x^{14} \omega^7 \hbar^2+80640 m^6 x^{12} \omega^6 \hbar^3+423360 m^5 x^{10} \omega^5 \hbar^4+1905120 m^4 x^8 \omega^4 \hbar^5+6985440 m^3 x^6 \omega^3 \hbar^6+19459440 m^2 x^4 \omega^2 \hbar^7+36486450 m x^2 \omega \hbar^8+34459425 \hbar^9\right) A_9\right)\right)\right)\right)\right)\right)\right)\right)\right) ==$$

$$\left(\left(30240 x \sqrt{\frac{m\omega}{\hbar}}-80640 x^3\left(\frac{m\omega}{\hbar}\right)^{3/2}+48384 x^5\left(\frac{m\omega}{\hbar}\right)^{5/2}-9216 x^7\left(\frac{m\omega}{\hbar}\right)^{7/2}+512 x^9\left(\frac{m\omega}{\hbar}\right)^{9/2}\right)^2\sqrt{\frac{m\omega}{\hbar}}\right)\Big/\left(185794560\sqrt{\pi}\right)$$

Система уравнений на коэффициенты:

$$\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}} A_0+\frac{1}{4}\sqrt{\pi}\,\omega\hbar\sqrt{\frac{\omega\hbar}{m}} A_1+\frac{3}{16}\sqrt{\pi}\,\omega^2\hbar^2\sqrt{\frac{\omega\hbar}{m}} A_2+\frac{15}{64}\sqrt{\pi}\,\omega^3\hbar^3\sqrt{\frac{\omega\hbar}{m}} A_3+\frac{105}{256}\sqrt{\pi}\,\omega^4\hbar^4\sqrt{\frac{\omega\hbar}{m}} A_4+\frac{945\sqrt{\pi}\,\omega^5\hbar^5\sqrt{\frac{\omega\hbar}{m}} A_5}{1024}+\frac{10395\sqrt{\pi}\,\omega^6\hbar^6\sqrt{\frac{\omega\hbar}{m}} A_6}{4096}+\frac{135135\sqrt{\pi}\,\omega^7\hbar^7\sqrt{\frac{\omega\hbar}{m}} A_7}{16384}+\frac{2027025\sqrt{\pi}\,\omega^8\hbar^8\sqrt{\frac{\omega\hbar}{m}} A_8}{65536}+\frac{34459425\sqrt{\pi}\,\omega^9\hbar^9\sqrt{\frac{\omega\hbar}{m}} A_9}{262144} == 0$$

$$-\frac{315 m\omega\sqrt{\frac{m\omega}{\hbar}}}{64\sqrt{\pi}\,\hbar}+\frac{1}{2} m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\hbar}{m}} A_1+\frac{1}{4} m\sqrt{\pi}\,\omega^3\hbar\sqrt{\frac{\omega\hbar}{m}} A_2+\frac{9}{32} m\sqrt{\pi}\,\omega^4\hbar^2\sqrt{\frac{\omega\hbar}{m}} A_3+\frac{15}{32} m\sqrt{\pi}\,\omega^5\hbar^3\sqrt{\frac{\omega\hbar}{m}} A_4+\frac{525}{512} m\sqrt{\pi}\,\omega^6\hbar^4\sqrt{\frac{\omega\hbar}{m}} A_5+\frac{2835 m\sqrt{\pi}\,\omega^7\hbar^5\sqrt{\frac{\omega\hbar}{m}} A_6}{1024}+\frac{72765 m\sqrt{\pi}\,\omega^8\hbar^6\sqrt{\frac{\omega\hbar}{m}} A_7}{8192}+\frac{135135 m\sqrt{\pi}\,\omega^9\hbar^7\sqrt{\frac{\omega\hbar}{m}} A_8}{4096}+\frac{18243225 m\sqrt{\pi}\,\omega^{10}\hbar^8\sqrt{\frac{\omega\hbar}{m}} A_9}{131072} == 0$$

$$\frac{105 m^2\omega^2\sqrt{\frac{m\omega}{\hbar}}}{4\sqrt{\pi}\,\hbar^2}+\frac{1}{4} m^2\sqrt{\pi}\,\omega^4\sqrt{\frac{\omega\hbar}{m}} A_2+\frac{3}{16} m^2\sqrt{\pi}\,\omega^5\hbar\sqrt{\frac{\omega\hbar}{m}} A_3+\frac{9}{32} m^2\sqrt{\pi}\,\omega^6\hbar^2\sqrt{\frac{\omega\hbar}{m}} A_4+\frac{75}{128} m^2\sqrt{\pi}\,\omega^7\hbar^3\sqrt{\frac{\omega\hbar}{m}} A_5+\frac{1575 m^2\sqrt{\pi}\,\omega^8\hbar^4\sqrt{\frac{\omega\hbar}{m}} A_6}{1024}+$$

$$\frac{19845\, m^2 \sqrt{\pi}\ \omega^9 \hbar^5 \sqrt{\frac{\omega\hbar}{m}}\ A_7}{4096} + \frac{72765\, m^2 \sqrt{\pi}\ \omega^{10} \hbar^6 \sqrt{\frac{\omega\hbar}{m}}\ A_8}{4096} + \frac{1216215\, m^2 \sqrt{\pi}\ \omega^{11} \hbar^7 \sqrt{\frac{\omega\hbar}{m}}\ A_9}{16384} == 0$$

$$-\frac{203\, m^3 \omega^3 \sqrt{\frac{m\omega}{\hbar}}}{4\sqrt{\pi}\ \hbar^3} + \frac{1}{8} m^3 \sqrt{\pi}\ \omega^6 \sqrt{\frac{\omega\hbar}{m}}\ A_3 + \frac{1}{8} m^3 \sqrt{\pi}\ \omega^7 \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_4 + \frac{15}{64} m^3 \sqrt{\pi}\ \omega^8 \hbar^2 \sqrt{\frac{\omega\hbar}{m}}\ A_5 +$$

$$\frac{75}{128} m^3 \sqrt{\pi}\ \omega^9 \hbar^3 \sqrt{\frac{\omega\hbar}{m}}\ A_6 + \frac{3675\, m^3 \sqrt{\pi}\ \omega^{10} \hbar^4 \sqrt{\frac{\omega\hbar}{m}}\ A_7}{2048} + \frac{6615\, m^3 \sqrt{\pi}\ \omega^{11} \hbar^5 \sqrt{\frac{\omega\hbar}{m}}\ A_8}{1024} + \frac{218295\, m^3 \sqrt{\pi}\ \omega^{12} \hbar^6 \sqrt{\frac{\omega\hbar}{m}}\ A_9}{8192} == 0$$

$$\frac{45\, m^4 \omega^4 \sqrt{\frac{m\omega}{\hbar}}}{\sqrt{\pi}\ \hbar^4} + \frac{1}{16} m^4 \sqrt{\pi}\ \omega^8 \sqrt{\frac{\omega\hbar}{m}}\ A_4 + \frac{5}{64} m^4 \sqrt{\pi}\ \omega^9 \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_5 + \frac{45}{256} m^4 \sqrt{\pi}\ \omega^{10} \hbar^2 \sqrt{\frac{\omega\hbar}{m}}\ A_6 +$$

$$\frac{525\, m^4 \sqrt{\pi}\ \omega^{11} \hbar^3 \sqrt{\frac{\omega\hbar}{m}}\ A_7}{1024} + \frac{3675\, m^4 \sqrt{\pi}\ \omega^{12} \hbar^4 \sqrt{\frac{\omega\hbar}{m}}\ A_8}{2048} + \frac{59535\, m^4 \sqrt{\pi}\ \omega^{13} \hbar^5 \sqrt{\frac{\omega\hbar}{m}}\ A_9}{8192} == 0$$

$$-\frac{623\, m^5 \omega^5 \sqrt{\frac{m\omega}{\hbar}}}{30\sqrt{\pi}\ \hbar^5} + \frac{1}{32} m^5 \sqrt{\pi}\ \omega^{10} \sqrt{\frac{\omega\hbar}{m}}\ A_5 + \frac{3}{64} m^5 \sqrt{\pi}\ \omega^{11} \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_6 +$$

$$\frac{63}{512} m^5 \sqrt{\pi}\ \omega^{12} \hbar^2 \sqrt{\frac{\omega\hbar}{m}}\ A_7 + \frac{105}{256} m^5 \sqrt{\pi}\ \omega^{13} \hbar^3 \sqrt{\frac{\omega\hbar}{m}}\ A_8 + \frac{6615\, m^5 \sqrt{\pi}\ \omega^{14} \hbar^4 \sqrt{\frac{\omega\hbar}{m}}\ A_9}{4096} == 0$$

$$\frac{236\, m^6 \omega^6 \sqrt{\frac{m\omega}{\hbar}}}{45\sqrt{\pi}\ \hbar^6} + \frac{1}{64} m^6 \sqrt{\pi}\ \omega^{12} \sqrt{\frac{\omega\hbar}{m}}\ A_6 +$$

$$\frac{7}{256} m^6 \sqrt{\pi}\ \omega^{13} \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_7 + \frac{21}{256} m^6 \sqrt{\pi}\ \omega^{14} \hbar^2 \sqrt{\frac{\omega\hbar}{m}}\ A_8 + \frac{315\, m^6 \sqrt{\pi}\ \omega^{15} \hbar^3 \sqrt{\frac{\omega\hbar}{m}}\ A_9}{1024} == 0$$

$$-\frac{76\, m^7 \omega^7 \sqrt{\frac{m\omega}{\hbar}}}{105\sqrt{\pi}\ \hbar^7} + \frac{1}{128} m^7 \sqrt{\pi}\ \omega^{14} \sqrt{\frac{\omega\hbar}{m}}\ A_7 + \frac{1}{64} m^7 \sqrt{\pi}\ \omega^{15} \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_8 + \frac{27}{512} m^7 \sqrt{\pi}\ \omega^{16} \hbar^2 \sqrt{\frac{\omega\hbar}{m}}\ A_9 == 0$$

$$\frac{16\, m^8 \omega^8 \sqrt{\frac{m\omega}{\hbar}}}{315\sqrt{\pi}\ \hbar^8} + \frac{1}{256} m^8 \sqrt{\pi}\ \omega^{16} \sqrt{\frac{\omega\hbar}{m}}\ A_8 + \frac{9\, m^8 \sqrt{\pi}\ \omega^{17} \hbar \sqrt{\frac{\omega\hbar}{m}}\ A_9}{1024} == 0$$

$$-\frac{4\, m^9 \omega^9 \sqrt{\frac{m\omega}{\hbar}}}{2835\sqrt{\pi}\ \hbar^9} + \frac{1}{512} m^9 \sqrt{\pi}\ \omega^{18} \sqrt{\frac{\omega\hbar}{m}}\ A_9 == 0$$

Решение системы:

A_0	$-\frac{m}{\pi\hbar}$
A_1	$\frac{36\, m}{\pi\omega\hbar^2}$
A_2	$-\frac{288\, m}{\pi\omega^2\hbar^3}$
A_3	$\frac{896\, m}{\pi\omega^3\hbar^4}$
A_4	$-\frac{1344\, m}{\pi\omega^4\hbar^5}$
A_5	$\frac{5376\, m}{5\pi\omega^5\hbar^6}$
A_6	$-\frac{7168\, m}{15\pi\omega^6\hbar^7}$
A_7	$\frac{4096\, m}{35\pi\omega^7\hbar^8}$
A_8	$-\frac{512\, m}{35\pi\omega^8\hbar^9}$
A_9	$\frac{2048\, m}{2835\pi\omega^9\hbar^{10}}$

Полином:

$$P_9(x) = \frac{2048\, m\, x^9}{2835\pi\omega^9\hbar^{10}} - \frac{512\, m\, x^8}{35\pi\omega^8\hbar^9} + \frac{4096\, m\, x^7}{35\pi\omega^7\hbar^8} - \frac{7168\, m\, x^6}{15\pi\omega^6\hbar^7} + \frac{5376\, m\, x^5}{5\pi\omega^5\hbar^6} - \frac{1344\, m\, x^4}{\pi\omega^4\hbar^5} + \frac{896\, m\, x^3}{\pi\omega^3\hbar^4} - \frac{288\, m\, x^2}{\pi\omega^2\hbar^3} + \frac{36\, m\, x}{\pi\omega\hbar^2} - \frac{m}{\pi\hbar}$$

Для n = 10

Интеграл:

$$\int_{-\infty}^{\infty} e^{-\frac{m v^2}{\omega \hbar}} \left(A_0 + \frac{1}{2}\left(m v^2 + m x^2 \omega^2\right) A_1 + \frac{1}{4}\left(m v^2 + m x^2 \omega^2\right)^2 A_2 + \frac{1}{8}\left(m v^2 + m x^2 \omega^2\right)^3 A_3 + \frac{1}{16}\left(m v^2 + m x^2 \omega^2\right)^4 A_4 + \frac{1}{32}\left(m v^2 + m x^2 \omega^2\right)^5 A_5 + \frac{1}{64}\left(m v^2 + m x^2 \omega^2\right)^6 A_6 + \frac{1}{128}\left(m v^2 + m x^2 \omega^2\right)^7 A_7 + \frac{1}{256}\left(m v^2 + m x^2 \omega^2\right)^8 A_8 + \frac{1}{512}\left(m v^2 + m x^2 \omega^2\right)^9 A_9 + \frac{\left(m v^2 + m x^2 \omega^2\right)^{10} A_{10}}{1024}\right) dv =$$

$$= \frac{1}{1\,048\,576} \sqrt{\pi} \sqrt{\frac{\omega \hbar}{m}} \Big(1\,048\,576 A_0 + \omega \big(262\,144 (2 m x^2 \omega + \hbar) A_1 + \omega \big(65\,536 (4 m^2 x^4 \omega^2 + 4 m x^2 \omega \hbar + 3 \hbar^2) A_2 + \omega \big(16\,384 (8 m^3 x^6 \omega^3 + 12 m^2 x^4 \omega^2 \hbar + 18 m x^2 \omega \hbar^2 + 15 \hbar^3) A_3 + \omega \big(4096 (16 m^4 x^8 \omega^4 + 32 m^3 x^6 \omega^3 \hbar + 72 m^2 x^4 \omega^2 \hbar^2 + 120 m x^2 \omega \hbar^3 + 105 \hbar^4) A_4 + \omega \big(1024 (32 m^5 x^{10} \omega^5 + 80 m^4 x^8 \omega^4 \hbar + 240 m^3 x^6 \omega^3 \hbar^2 + 600 m^2 x^4 \omega^2 \hbar^3 + 1050 m x^2 \omega \hbar^4 + 945 \hbar^5) A_5 + \omega \big(256 (64 m^6 x^{12} \omega^6 + 192 m^5 x^{10} \omega^5 \hbar + 720 m^4 x^8 \omega^4 \hbar^2 + 2400 m^3 x^6 \omega^3 \hbar^3 + 6300 m^2 x^4 \omega^2 \hbar^4 + 11\,340 m x^2 \omega \hbar^5 + 10\,395 \hbar^6) A_6 + \omega \big(64 (128 m^7 x^{14} \omega^7 + 448 m^6 x^{12} \omega^6 \hbar + 2016 m^5 x^{10} \omega^5 \hbar^2 + 8400 m^4 x^8 \omega^4 \hbar^3 + 29\,400 m^3 x^6 \omega^3 \hbar^4 + 79\,380 m^2 x^4 \omega^2 \hbar^5 + 145\,530 m x^2 \omega \hbar^6 + 135\,135 \hbar^7) A_7 + \omega \big(16 (256 m^8 x^{16} \omega^8 + 1024 m^7 x^{14} \omega^7 \hbar + 5376 m^6 x^{12} \omega^6 \hbar^2 + 26\,880 m^5 x^{10} \omega^5 \hbar^3 + 117\,600 m^4 x^8 \omega^4 \hbar^4 + 423\,360 m^3 x^6 \omega^3 \hbar^5 + 1\,164\,240 m^2 x^4 \omega^2 \hbar^6 + 2\,162\,160 m x^2 \omega \hbar^7 + 2\,027\,025 \hbar^8) A_8 + \omega \big(4 (512 m^9 x^{18} \omega^9 + 2304 m^8 x^{16} \omega^8 \hbar + 13\,824 m^7 x^{14} \omega^7 \hbar^2 + 80\,640 m^6 x^{12} \omega^6 \hbar^3 + 423\,360 m^5 x^{10} \omega^5 \hbar^4 + 1\,905\,120 m^4 x^8 \omega^4 \hbar^5 + 6\,985\,440 m^3 x^6 \omega^3 \hbar^6 + 19\,459\,440 m^2 x^4 \omega^2 \hbar^7 + 36\,486\,450 m x^2 \omega \hbar^8 + 34\,459\,425 \hbar^9) A_9 + \omega (1024 m^{10} x^{20} \omega^{10} + 5120 m^9 x^{18} \omega^9 \hbar + 34\,560 m^8 x^{16} \omega^8 \hbar^2 + 230\,400 m^7 x^{14} \omega^7 \hbar^3 + 1\,411\,200 m^6 x^{12} \omega^6 \hbar^4 + 7\,620\,480 m^5 x^{10} \omega^5 \hbar^5 + 34\,927\,200 m^4 x^8 \omega^4 \hbar^6 + 129\,729\,600 m^3 x^6 \omega^3 \hbar^7 + 364\,864\,500 m^2 x^4 \omega^2 \hbar^8 + 689\,188\,500 m x^2 \omega \hbar^9 + 654\,729\,075 \hbar^{10}) A_{10}\big)\big)\big)\big)\big)\big)\big)\big)\big)\Big)$$

$$= \frac{\sqrt{\frac{m \omega}{\hbar}}\, H_{10}\left[x \sqrt{\frac{m \omega}{\hbar}}\right]^2}{3\,715\,891\,200 \sqrt{\pi}}$$

Уравнение:

$$
\begin{aligned}
&\frac{1}{1048576}\sqrt{\pi}\sqrt{\frac{\omega\hbar}{m}}\ (1048576\,A_0 + \omega\,(262144\,(2\,m\,x^2\,\omega + \hbar)\,A_1 + \omega\,(65536\,(4\,m^2\,x^4\,\omega^2 + 4\,m\,x^2\,\omega\,\hbar + 3\,\hbar^2)\,A_2 + \\
&\omega\,(16384\,(8\,m^3\,x^6\,\omega^3 + 12\,m^2\,x^4\,\omega^2\,\hbar + 18\,m\,x^2\,\omega\,\hbar^2 + 15\,\hbar^3)\,A_3 + \\
&\omega\,(4096\,(16\,m^4\,x^8\,\omega^4 + 32\,m^3\,x^6\,\omega^3\,\hbar + 72\,m^2\,x^4\,\omega^2\,\hbar^2 + 120\,m\,x^2\,\omega\,\hbar^3 + 105\,\hbar^4)\,A_4 + \\
&\omega\,(1024\,(32\,m^5\,x^{10}\,\omega^5 + 80\,m^4\,x^8\,\omega^4\,\hbar + 240\,m^3\,x^6\,\omega^3\,\hbar^2 + 600\,m^2\,x^4\,\omega^2\,\hbar^3 + 1050\,m\,x^2\,\omega \\
&\hbar^4 + 945\,\hbar^5)\,A_5 + \omega\,(256\,(64\,m^6\,x^{12}\,\omega^6 + 192\,m^5\,x^{10}\,\omega^5\,\hbar + 720\,m^4\,x^8\,\omega^4\,\hbar^2 + \\
&2400\,m^3\,x^6\,\omega^3\,\hbar^3 + 6300\,m^2\,x^4\,\omega^2\,\hbar^4 + 11340\,m\,x^2\,\omega\,\hbar^5 + 10395\,\hbar^6)\,A_6 + \\
&\omega\,(64\,(128\,m^7\,x^{14}\,\omega^7 + 448\,m^6\,x^{12}\,\omega^6\,\hbar + 2016\,m^5\,x^{10}\,\omega^5\,\hbar^2 + 8400\,m^4\,x^8\,\omega^4\,\hbar^3 + \\
&29400\,m^3\,x^6\,\omega^3\,\hbar^4 + 79380\,m^2\,x^4\,\omega^2\,\hbar^5 + 145530\,m\,x^2\,\omega\,\hbar^6 + 135135\,\hbar^7) \\
&A_7 + \omega\,(16\,(256\,m^8\,x^{16}\,\omega^8 + 1024\,m^7\,x^{14}\,\omega^7\,\hbar + 5376\,m^6\,x^{12}\,\omega^6\,\hbar^2 + \\
&26880\,m^5\,x^{10}\,\omega^5\,\hbar^3 + 117600\,m^4\,x^8\,\omega^4\,\hbar^4 + 423360\,m^3\,x^6\,\omega^3\,\hbar^5 + \\
&1164240\,m^2\,x^4\,\omega^2\,\hbar^6 + 2162160\,m\,x^2\,\omega\,\hbar^7 + 2027025\,\hbar^8)\,A_8 + \\
&\omega\,(4\,(512\,m^9\,x^{18}\,\omega^9 + 2304\,m^8\,x^{16}\,\omega^8\,\hbar + 13824\,m^7\,x^{14}\,\omega^7\,\hbar^2 + \\
&80640\,m^6\,x^{12}\,\omega^6\,\hbar^3 + 423360\,m^5\,x^{10}\,\omega^5\,\hbar^4 + 1905120\,m^4 \\
&x^8\,\omega^4\,\hbar^5 + 6985440\,m^3\,x^6\,\omega^3\,\hbar^6 + 19459440\,m^2\,x^4\,\omega^2\,\hbar^7 + \\
&36486450\,m\,x^2\,\omega\,\hbar^8 + 34459425\,\hbar^9)\,A_9 + \omega\,(1024\,m^{10}\,x^{20}\,\omega^{10} + \\
&5120\,m^9\,x^{18}\,\omega^9\,\hbar + 34560\,m^8\,x^{16}\,\omega^8\,\hbar^2 + 230400\,m^7\,x^{14}\,\omega^7\,\hbar^3 + \\
&1411200\,m^6\,x^{12}\,\omega^6\,\hbar^4 + 7620480\,m^5\,x^{10}\,\omega^5\,\hbar^5 + 34927200\,m^4 \\
&x^8\,\omega^4\,\hbar^6 + 129729600\,m^3\,x^6\,\omega^3\,\hbar^7 + 364864500\,m^2\,x^4\,\omega^2\,\hbar^8 + \\
&689188500\,m\,x^2\,\omega\,\hbar^9 + 654729075\,\hbar^{10})\,A_{10}))))))))))\ == \\
&\Bigg(\bigg(-30240 + \frac{1024\,m^5\,x^{10}\,\omega^5}{\hbar^5} - \frac{23040\,m^4\,x^8\,\omega^4}{\hbar^4} + \frac{161280\,m^3\,x^6\,\omega^3}{\hbar^3} - \frac{403200\,m^2\,x^4\,\omega^2}{\hbar^2} + \\
&\frac{302400\,m\,x^2\,\omega}{\hbar}\bigg)^2 \\
&\sqrt{\frac{m\,\omega}{\hbar}}\Bigg) \Big/ \left(3715891200\,\sqrt{\pi}\right)
\end{aligned}
$$

Система уравнений на коэффициенты:

$$
\begin{aligned}
&-\frac{63\sqrt{\frac{m\,\omega}{\hbar}}}{256\sqrt{\pi}} + \sqrt{\pi}\sqrt{\frac{\omega\,\hbar}{m}}\,A_0 + \frac{1}{4}\sqrt{\pi}\,\omega\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_1 + \frac{3}{16}\sqrt{\pi}\,\omega^2\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_2 + \frac{15}{64}\sqrt{\pi}\,\omega^3\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_3 + \\
&\frac{105}{256}\sqrt{\pi}\,\omega^4\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_4 + \frac{945\sqrt{\pi}\,\omega^5\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_5}{1024} + \frac{10395\sqrt{\pi}\,\omega^6\,\hbar^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_6}{4096} + \frac{135135\sqrt{\pi}\,\omega^7\,\hbar^7\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{16384} + \\
&\frac{2027025\sqrt{\pi}\,\omega^8\,\hbar^8\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{65536} + \frac{34459425\sqrt{\pi}\,\omega^9\,\hbar^9\sqrt{\frac{\omega\,\hbar}{m}}\,A_9}{262144} + \frac{654729075\sqrt{\pi}\,\omega^{10}\,\hbar^{10}\sqrt{\frac{\omega\,\hbar}{m}}\,A_{10}}{1048576} == 0 \\
&\frac{315\,m\,\omega\sqrt{\frac{m\,\omega}{\hbar}}}{64\sqrt{\pi}\,\hbar} + \frac{1}{2}\,m\sqrt{\pi}\,\omega^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_1 + \frac{1}{4}\,m\sqrt{\pi}\,\omega^3\,\hbar\sqrt{\frac{\omega\,\hbar}{m}}\,A_2 + \frac{9}{32}\,m\sqrt{\pi}\,\omega^4\,\hbar^2\sqrt{\frac{\omega\,\hbar}{m}}\,A_3 + \\
&\frac{15}{32}\,m\sqrt{\pi}\,\omega^5\,\hbar^3\sqrt{\frac{\omega\,\hbar}{m}}\,A_4 + \frac{525}{512}\,m\sqrt{\pi}\,\omega^6\,\hbar^4\sqrt{\frac{\omega\,\hbar}{m}}\,A_5 + \frac{2835\,m\sqrt{\pi}\,\omega^7\,\hbar^5\sqrt{\frac{\omega\,\hbar}{m}}\,A_6}{1024} + \frac{72765\,m\sqrt{\pi}\,\omega^8\,\hbar^6\sqrt{\frac{\omega\,\hbar}{m}}\,A_7}{8192} + \\
&\frac{135135\,m\sqrt{\pi}\,\omega^9\,\hbar^7\sqrt{\frac{\omega\,\hbar}{m}}\,A_8}{4096} + \frac{18243225\,m\sqrt{\pi}\,\omega^{10}\,\hbar^8\sqrt{\frac{\omega\,\hbar}{m}}\,A_9}{131072} + \frac{172297125\,m\sqrt{\pi}\,\omega^{11}\,\hbar^9\sqrt{\frac{\omega\,\hbar}{m}}\,A_{10}}{262144} == 0
\end{aligned}
$$

$$-\frac{1995\, m^2\, \omega^2 \sqrt{\frac{m\,\omega}{\hbar}}}{64 \sqrt{\pi}\, \hbar^2} + \frac{1}{4}\, m^2 \sqrt{\pi}\, \omega^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_2 + \frac{3}{16}\, m^2 \sqrt{\pi}\, \omega^5\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_3 + \frac{9}{32}\, m^2 \sqrt{\pi}\, \omega^6\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_4 + \frac{75}{128}\, m^2 \sqrt{\pi}\, \omega^7\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_5 + \frac{1575\, m^2 \sqrt{\pi}\, \omega^8\, \hbar^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_6}{1024} + \frac{19\,845\, m^2 \sqrt{\pi}\, \omega^9\, \hbar^5 \sqrt{\frac{\omega\,\hbar}{m}}\, A_7}{4096} + \frac{72\,765\, m^2 \sqrt{\pi}\, \omega^{10}\, \hbar^6 \sqrt{\frac{\omega\,\hbar}{m}}\, A_8}{4096} + \frac{1\,216\,215\, m^2 \sqrt{\pi}\, \omega^{11}\, \hbar^7 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{16\,384} + \frac{91\,216\,125\, m^2 \sqrt{\pi}\, \omega^{12}\, \hbar^8 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{262\,144} == 0$$

$$\frac{273\, m^3\, \omega^3 \sqrt{\frac{m\,\omega}{\hbar}}}{4 \sqrt{\pi}\, \hbar^3} + \frac{1}{8}\, m^3 \sqrt{\pi}\, \omega^6 \sqrt{\frac{\omega\,\hbar}{m}}\, A_3 + \frac{1}{8}\, m^3 \sqrt{\pi}\, \omega^7\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_4 + \frac{15}{64}\, m^3 \sqrt{\pi}\, \omega^8\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_5 + \frac{75}{128}\, m^3 \sqrt{\pi}\, \omega^9\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_6 + \frac{3675\, m^3 \sqrt{\pi}\, \omega^{10}\, \hbar^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_7}{2048} + \frac{6615\, m^3 \sqrt{\pi}\, \omega^{11}\, \hbar^5 \sqrt{\frac{\omega\,\hbar}{m}}\, A_8}{1024} + \frac{218\,295\, m^3 \sqrt{\pi}\, \omega^{12}\, \hbar^6 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{8192} + \frac{2\,027\,025\, m^3 \sqrt{\pi}\, \omega^{13}\, \hbar^7 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{16\,384} == 0$$

$$-\frac{563\, m^4\, \omega^4 \sqrt{\frac{m\,\omega}{\hbar}}}{8 \sqrt{\pi}\, \hbar^4} + \frac{1}{16}\, m^4 \sqrt{\pi}\, \omega^8 \sqrt{\frac{\omega\,\hbar}{m}}\, A_4 + \frac{5}{64}\, m^4 \sqrt{\pi}\, \omega^9\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_5 + \frac{45}{256}\, m^4 \sqrt{\pi}\, \omega^{10}\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_6 + \frac{525\, m^4 \sqrt{\pi}\, \omega^{11}\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_7}{1024} + \frac{3675\, m^4 \sqrt{\pi}\, \omega^{12}\, \hbar^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_8}{2048} + \frac{59\,535\, m^4 \sqrt{\pi}\, \omega^{13}\, \hbar^5 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{8192} + \frac{1\,091\,475\, m^4 \sqrt{\pi}\, \omega^{14}\, \hbar^6 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{32\,768} == 0$$

$$\frac{1163\, m^5\, \omega^5 \sqrt{\frac{m\,\omega}{\hbar}}}{30 \sqrt{\pi}\, \hbar^5} + \frac{1}{32}\, m^5 \sqrt{\pi}\, \omega^{10} \sqrt{\frac{\omega\,\hbar}{m}}\, A_5 + \frac{3}{64}\, m^5 \sqrt{\pi}\, \omega^{11}\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_6 + \frac{63}{512}\, m^5 \sqrt{\pi}\, \omega^{12}\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_7 + \frac{105}{256}\, m^5 \sqrt{\pi}\, \omega^{13}\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_8 + \frac{6615\, m^5 \sqrt{\pi}\, \omega^{14}\, \hbar^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{4096} + \frac{59\,535\, m^5 \sqrt{\pi}\, \omega^{15}\, \hbar^5 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{8192} == 0$$

$$-\frac{73\, m^6\, \omega^6 \sqrt{\frac{m\,\omega}{\hbar}}}{6 \sqrt{\pi}\, \hbar^6} + \frac{1}{64}\, m^6 \sqrt{\pi}\, \omega^{12} \sqrt{\frac{\omega\,\hbar}{m}}\, A_6 + \frac{7}{256}\, m^6 \sqrt{\pi}\, \omega^{13}\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_7 + \frac{21}{256}\, m^6 \sqrt{\pi}\, \omega^{14}\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_8 + \frac{315\, m^6 \sqrt{\pi}\, \omega^{15}\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{1024} + \frac{11\,025\, m^6 \sqrt{\pi}\, \omega^{16}\, \hbar^4 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{8192} == 0$$

$$\frac{20\, m^7\, \omega^7 \sqrt{\frac{m\,\omega}{\hbar}}}{9 \sqrt{\pi}\, \hbar^7} + \frac{1}{128}\, m^7 \sqrt{\pi}\, \omega^{14} \sqrt{\frac{\omega\,\hbar}{m}}\, A_7 + \frac{1}{64}\, m^7 \sqrt{\pi}\, \omega^{15}\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_8 + \frac{27}{512}\, m^7 \sqrt{\pi}\, \omega^{16}\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_9 + \frac{225\, m^7 \sqrt{\pi}\, \omega^{17}\, \hbar^3 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{1024} == 0$$

$$-\frac{73\, m^8\, \omega^8 \sqrt{\frac{m\,\omega}{\hbar}}}{315 \sqrt{\pi}\, \hbar^8} + \frac{1}{256}\, m^8 \sqrt{\pi}\, \omega^{16} \sqrt{\frac{\omega\,\hbar}{m}}\, A_8 + \frac{9\, m^8 \sqrt{\pi}\, \omega^{17}\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_9}{1024} + \frac{135\, m^8 \sqrt{\pi}\, \omega^{18}\, \hbar^2 \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{4096} == 0$$

$$\frac{4\, m^9\, \omega^9 \sqrt{\frac{m\,\omega}{\hbar}}}{315 \sqrt{\pi}\, \hbar^9} + \frac{1}{512}\, m^9 \sqrt{\pi}\, \omega^{18} \sqrt{\frac{\omega\,\hbar}{m}}\, A_9 + \frac{5\, m^9 \sqrt{\pi}\, \omega^{19}\, \hbar \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{1024} == 0$$

$$-\frac{4\, m^{10}\, \omega^{10} \sqrt{\frac{m\,\omega}{\hbar}}}{14\,175 \sqrt{\pi}\, \hbar^{10}} + \frac{m^{10} \sqrt{\pi}\, \omega^{20} \sqrt{\frac{\omega\,\hbar}{m}}\, A_{10}}{1024} == 0$$

Решение системы:

A_0	$\frac{m}{\pi \hbar}$
A_1	$-\frac{40\,m}{\pi \omega \hbar^2}$
A_2	$\frac{360\,m}{\pi \omega^2 \hbar^3}$
A_3	$-\frac{1280\,m}{\pi \omega^3 \hbar^4}$
A_4	$\frac{2240\,m}{\pi \omega^4 \hbar^5}$
A_5	$-\frac{10\,752\,m}{5 \pi \omega^5 \hbar^6}$
A_6	$\frac{3584\,m}{3 \pi \omega^6 \hbar^7}$
A_7	$-\frac{8192\,m}{21 \pi \omega^7 \hbar^8}$
A_8	$\frac{512\,m}{7 \pi \omega^8 \hbar^9}$
A_9	$-\frac{4096\,m}{567 \pi \omega^9 \hbar^{10}}$
A_{10}	$\frac{4096\,m}{14\,175 \pi \omega^{10} \hbar^{11}}$

Полином:

$$P_{10}(x) = \frac{4096\,m\,x^{10}}{14\,175\,\pi\,\omega^{10}\,\hbar^{11}} - \frac{4096\,m\,x^9}{567\,\pi\,\omega^9\,\hbar^{10}} + \frac{512\,m\,x^8}{7\,\pi\,\omega^8\,\hbar^9} - \frac{8192\,m\,x^7}{21\,\pi\,\omega^7\,\hbar^8} + \frac{3584\,m\,x^6}{3\,\pi\,\omega^6\,\hbar^7} - \frac{10\,752\,m\,x^5}{5\,\pi\,\omega^5\,\hbar^6} + \frac{2240\,m\,x^4}{\pi\,\omega^4\,\hbar^5} - \frac{1280\,m\,x^3}{\pi\,\omega^3\,\hbar^4} + \frac{360\,m\,x^2}{\pi\,\omega^2\,\hbar^3} - \frac{40\,m\,x}{\pi\,\omega\,\hbar^2} + \frac{m}{\pi\,\hbar}$$