***University Physics Volume I***

**Unit 1: Mechanics**

**Chapter 13: Gravitation**

**Conceptual Questions**

1. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in science, and why was this action at a distance ultimately accepted?

Solution

The ultimate truth is experimental verification. Field theory was developed to help explain how force is exerted without objects being in contact for both gravity and electromagnetic forces that act at the speed of light. It has only been since the twentieth century that we have been able to measure that the force is not conveyed immediately.

3. Must engineers take Earth’s rotation into account when constructing very tall buildings at any location other than the equator or very near the poles?

Solution

The centripetal acceleration is not directed along the gravitational force and therefore the correct line of the building (i.e., the plumb bob line) is not directed towards the center of Earth. But engineers use either a plumb bob or a transit, both of which respond to both the direction of gravity and acceleration. No special consideration for their location on Earth need be made.

5. It was shown that the energy required to lift a satellite into a *low* Earth orbit (the change in potential energy) is only a small fraction of the kinetic energy needed to keep it in orbit. Is this true for larger orbits? Is there a trend to the ratio of kinetic energy to change in potential energy as the size of the orbit increases?

Solution

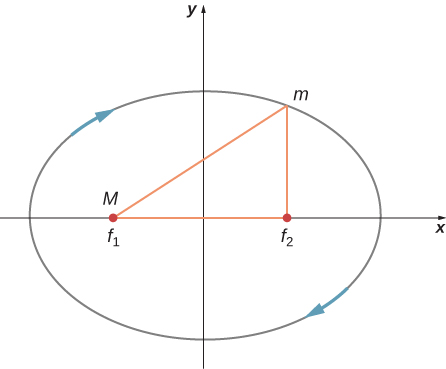
As we move to larger orbits, the change in potential energy increases, whereas the orbital velocity decreases. Hence, the ratio is highest near Earth’s surface (technically infinite if we orbit at Earth’s surface with no elevation change), moving to zero as we reach infinitely far away.

7. Many satellites are placed in geosynchronous orbits. What is special about these orbits? For a global communication network, how many of these satellites would be needed?

Solution

The period of the orbit must be 24 hours. But in addition, the satellite must be located in an equatorial orbit and orbiting in the same direction as Earth’s rotation. All three criteria must be met for the satellite to remain in one position relative to Earth’s surface. At least three satellites are needed, as two on opposite sides of Earth cannot communicate with each other. (This is not technically true, as a wavelength could be chosen that provides sufficient diffraction. But it would be totally impractical.)

9. In the diagram below for a satellite in an elliptical orbit about a much larger mass, indicate where its speed is the greatest and where it is the least. What conservation law dictates this behavior? Indicate the directions of the force, acceleration, and velocity at these points. Draw vectors for these same three quantities at the two points where the *y*-axis intersects (along the semi-minor axis) and from this determine whether the speed is increasing decreasing, or at a max/min.



Solution

The speed is greatest where the satellite is closest to the large mass and least where farther away—at the periapsis and apoapsis, respectively. It is conservation of angular momentum that governs this relationship. But it can also be gleaned from conservation of energy, the kinetic energy must be greatest where the gravitational potential energy is the least (most negative). The force, and hence acceleration, is always directed towards *M* in the diagram, and the velocity is always tangent to the path at all points. The acceleration vector has a tangential component along the direction of the velocity at the upper location on the *y*-axis; hence, the satellite is speeding up. Just the opposite is true at the lower position.

11. The principle of equivalence states that all experiments done in a lab in a uniform gravitational field cannot be distinguished from those done in a lab that is not in a gravitational field but is uniformly accelerating. For the latter case, consider what happens to a laser beam at some height shot perfectly horizontally to the floor, across the accelerating lab. (View this from a nonaccelerating frame outside the lab.) Relative to the height of the laser, where will the laser beam hit the far wall? What does this say about the effect of a gravitational field on light? Does the fact that light has no mass make any difference to the argument?

Solution

The laser beam will hit the far wall at a lower elevation than it left, as the floor is accelerating upward. Relative to the lab, the laser beam “falls.” So we would expect this to happen in a gravitational field. The mass of light, or even an object with mass, is not relevant.

**Problems**

13. Evaluate the magnitude of gravitational force between two 5-kg spherical steel balls separated by a center-to-center distance of 15 cm.

Solution

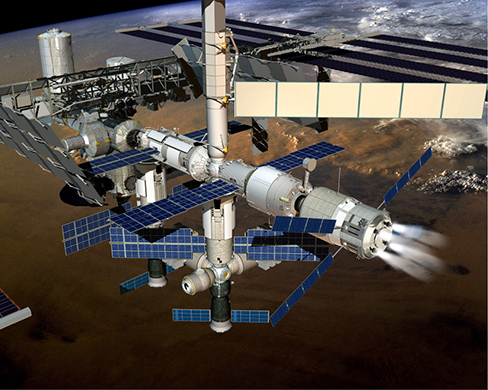


15. Astrology makes much of the position of the planets at the moment of one’s birth. The only known force a planet exerts on Earth is gravitational. (a) Calculate the gravitational force exerted on a 4.20-kg baby by a 100-kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the force on the baby due to Jupiter if it is at its closest distance to Earth, some  away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

Solution

a. ; b. The mass of Jupiter is

17. The International Space Station has a mass of approximately 370,000 kg. (a) What is the force on a 150-kg suited astronaut if she is 20 m from the center of mass of the station? (b) How accurate do you think your answer would be?



Solution

a. ; b. Not very, as the ISS is not even symmetrical, much less spherically symmetrical.

19. (a) What was the acceleration of Earth caused by asteroid Toutatis (see previous problem) at its closest approach? (b) What was the acceleration of Toutatis at this point?

Solution

a. ; b. 

21. (a) What is the acceleration due to gravity on the surface of the Moon? (b) On the surface of Mars? The mass of Mars is  and its radius is .

Solution

a. ; b. 

23. The mass of a particle is 15 kg. (a) What is its weight on Earth? (b) What is its weight on the Moon? (c) What is its mass on the Moon? (d) What is its weight in outer space far from any celestial body? (e) What is its mass at this point?

Solution

a. 147 N; b. 25.5 N; c. 15 kg; d. 0; e. 15 kg

25. The mean diameter of the planet Saturn is  and its mean mass density is  Find the acceleration due to gravity at Saturn’s surface.

Solution



27. The acceleration due to gravity on the surface of a planet is three times as large as it is on the surface of Earth. The mass density of the planet is known to be twice that of Earth. What is the radius of this planet in terms of Earth’s radius?

Solution



29. Find the escape speed of a projectile from the surface of Mars.

Solution

5000 m/s

31. What is the escape speed of a satellite located at the Moon’s orbit about Earth? Assume the Moon is not nearby.

Solution

1440 m/s

33. An average-sized asteroid located  from Earth with mass  is detected headed directly toward Earth with speed of 2.0 km/s. What will its speed be just before it hits our atmosphere? (You may ignore the size of the asteroid.)

Solution

11 km/s

35. (a) What is the change in energy of a 1000-kg payload taken from rest at the surface of Earth and placed at rest on the surface of the Moon? (b) What would be the answer if the payload were taken from the Moon’s surface to Earth? Is this a reasonable calculation of the energy needed to move a payload back and forth?

Solution

a. ; b. ; No. It assumes the kinetic energy is recoverable. This would not even be reasonable if we had an elevator between Earth and the Moon.

37. Two planets in circular orbits around a star have speeds of *v* and 2*v*. (a) What is the ratio of the orbital radii of the planets? (b) What is the ratio of their periods?

Solution

a. 0.25; b. 0.125

39. (a) What is the orbital radius of an Earth satellite having a period of 1.00 h? (b) What is unreasonable about this result?

Solution

a. ; b. This less than the radius of Earth.

41. Find the mass of Jupiter based on the fact that Io, its innermost moon, has an average orbital radius of 421,700 km and a period of 1.77 days.

Solution



43. (a) In order to keep a small satellite from drifting into a nearby asteroid, it is placed in orbit with a period of 3.02 hours and radius of 2.0 km. What is the mass of the asteroid? (b) Does this mass seem reasonable for the size of the orbit?

Solution

a. ; b. The satellite must be outside the radius of the asteroid, so it can’t be larger than this. If it were this size, then its density would be about . This is just above that of water, so this seems quite reasonable.

45. The Sun orbits the Milky Way galaxy once each  with a roughly circular orbit averaging a radius of light-years. (A light-year is the distance traveled by light in 1 year.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun? (b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

Solution

a. ; Yes, the centripetal acceleration is so small it supports the contention that a nearly inertial frame of reference can be located at the Sun. b. 

47. Calculate the mass of the Sun based on data for average Earth’s orbit and compare the value obtained with the Sun’s commonly listed value of 

Solution

 The values are the same within 0.05%.

49. The “mean” orbital radius listed for astronomical objects orbiting the Sun is typically not an integrated average but is calculated such that it gives the correct period when applied to the equation for circular orbits. Given that, what is the mean orbital radius in terms of aphelion and perihelion?

Solution

Compare  and  to see that they differ only in that the circular radius, *r*, is replaced by the semi-major axis, *a*. Therefore, the mean radius is one-half the sum of the aphelion and perihelion, the same as the semi-major axis.

51. The perihelion of the comet Lagerkvist is 2.61 AU and it has a period of 7.36 years. Show that the aphelion for this comet is 4.95 AU.

Solution

The semi-major axis, 3.78 AU is found from the equation for the period. This is one-half the sum of the aphelion and perihelion, giving an aphelion distance of 4.95 AU.

53. Eros has an elliptical orbit about the Sun, with a perihelion distance of 1.13 AU and aphelion distance of 1.78 AU. What is the period of its orbit?

Solution

1.75 years

55. If the Sun were to collapse into a black hole, the point of no return for an investigator would be approximately 3 km from the center singularity. Would the investigator be able to survive visiting even 300 km from the center? Answer this by finding the difference in the gravitational attraction the black holes exerts on a 1.0-kg mass at the head and at the feet of the investigator.

Solution

19,800 N; this is clearly not survivable

57. What is the Schwarzschild radius for the black hole at the center of our galaxy if it has the mass of 4 million solar masses?

Solution



**Additional Problems**

59. A neutron star is a cold, collapsed star with nuclear density. A particular neutron star has a mass twice that of our Sun with a radius of 12.0 km. (a) What would be the weight of a 100-kg astronaut on standing on its surface? (b) What does this tell us about landing on a neutron star?

Solution

a. ; b. Don’t do it!

61. How far from the center of the Sun would the net gravitational force of Earth and the Sun on a spaceship be zero?

Solution



63. Suppose you can communicate with the inhabitants of a planet in another solar system. They tell you that on their planet, whose diameter and mass are  and  respectively, the record for the high jump is 2.0 m. Given that this record is close to 2.4 m on Earth, what would you conclude about your extraterrestrial friends’ jumping ability?

Solution

The value of *g* for this planet is 3.8 m/s2, which is about one-fourth that of Earth. So they are weak high jumpers.

65. A body of mass 100 kg is weighed at the North Pole and at the equator with a spring scale. What is the scale reading at these two points? Assume that  at the pole.

Solution

At the North Pole, 983 N; at the equator, 980 N

67. Consider the previous problem and include the fact that Earth has an orbital speed about the Sun of 29.8 km/s. (a) What speed relative to Earth would be needed and in what direction should you leave Earth? (b) What will be the shape of the trajectory?

Solution

a. The escape velocity is still 43.6 km/s. By launching from Earth in the direction of Earth’s tangential velocity, you need  relative to Earth. b. The total energy is zero and the trajectory is a parabola.

69. An asteroid has speed 15.5 km/s when it is located 2.00 AU from the sun. At its closest approach, it is 0.400 AU from the Sun. What is its speed at that point?

Solution

61.5 km/s

71. A satellite of mass 1000 kg is in circular orbit about Earth. The radius of the orbit of the satellite is equal to two times the radius of Earth. (a) How far away is the satellite? (b) Find the kinetic, potential, and total energies of the satellite.

Solution

a.  b. **

73. (a) (a) Given the asteroid Vesta which has a diameter of 520 km and mass of , what would be the orbital period for a space probe in a circular orbit of 10.0 km from its surface? (b) Why is this calculation marginally useful at best?

Solution

a.  or about 1.8 hours. This was using the 520 km average diameter. b. Vesta is clearly not very spherical, so you would need to be above the largest dimension, nearly 580 km. More importantly, the nonspherical nature would disturb the orbit very quickly, so this calculation would not be very accurate even for one orbit.

75. (a) Using the information in the previous problem, what velocity do you need to escape the Milky Way galaxy from our present position? (b) Would you need to accelerate a spaceship to this speed relative to Earth?

Solution

a. 323 km/s; b. No, you need only the difference between the solar system’s orbital speed and escape speed, so about 

77. Show that for eccentricity equal to one in  for conic sections, the path is a parabola. Do this by substituting Cartesian coordinates, *x* and *y*, for the polar coordinates, *r* and  and showing that it has the general form for a parabola, 

Solution

Setting  in  we have ; hence,  Expand and collect to show 

79. Given the perihelion distance, *p*, and aphelion distance, *q*, for an elliptical orbit, show that the velocity at perihelion, , is given by  (*Hint:* Use conservation of angular momentum to relate  and , and then substitute into the conservation of energy equation.)

Solution

Substitute directly into the energy equation using  from conservation of angular momentum, and solve for 

**Challenge Problems**

81. A tunnel is dug through the center of a perfectly spherical and airless planet of radius *R*. Using the expression for *g* derived in gravitation Near Earth’s Surface for a uniform density, show that a particle of mass *m* dropped in the tunnel will execute simple harmonic motion. Deduce the period of oscillation of *m* and show that it has the same period as an orbit at the surface.

Solution

From Gravitation Near Earth’s Surface, we have  and from , we get  where the first term is  Then and if we substitute we get the same expression as for the period of orbit *R*.

83. Show that the areal velocity for a circular orbit of radius *r* about a mass *M* is . Does your expression give the correct value for Earth’s areal velocity about the Sun?

Solution

Using the mass of the Sun and Earth’s orbital radius, the equation gives  The value of  gives the same value.

85. Show that for small changes in height *h*, such that   reduces to the expression 

Solution

We start with   where  If , then , and upon substitution, we have

 where we recognize the expression with the parenthesis as  as the definition of *g*.

87. (a) Show that tidal force on a small object of mass *m*, defined as the *difference* in the gravitational force that would be exerted on *m* at a distance at the near and the far side of the object, due to the gravitation at a distance *R* from *M*, is given by  where  is the distance between the near and far side and  (b) Assume you are falling feet first into the black hole at the center of our galaxy. It has mass of 4 million solar masses. What would be the difference between the force at your head and your feet at the Schwarzschild radius (event horizon)? Assume your feet and head each have mass 5.0 kg and are 2.0 m apart. Would you survive passing through the event horizon?

Solution

a. Find the difference in force, ;

b. For the case given, using the Schwarzschild radius from a previous problem, we have a tidal force of . This won’t even be noticed!

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