4001234110 60 74 رضا سلاجی

Subject ..... Date .....

سوال ۱)

$D(s) = 2s^7 + 9s^6 + 6s^5 + 4s^4 + 8s^3 + 8s^2 + 2s + 6$

s^7	2	6	8	2
s^6	9	4	8	6
s^5	$\frac{-(8-54)}{9} = \frac{46}{9}$	$\frac{56}{9}$	$\frac{6}{9}$	0
s^4	$-\frac{35.55}{\frac{46}{9}}$			
s^3				
s^2				
s^1				
s^0				

→ علامت عوض شده پس ⇒ unstable

$D(s) = 2s^5 + 4s^4 + 2s^3 + 3s^2 + 2s + 5$

s^5	2	2	2
s^4	4	3	5
s^3	$\frac{-(6-8)}{4} = \frac{1}{2}$	$\frac{-(10-8)}{4} = -\frac{1}{2}$	0
s^2	7	5	0
s^1	$\frac{-(\frac{5}{2}+\frac{7}{2})}{7} = -\frac{6}{7}$		
s^0			

→ علامت عوض شده پس ⇒ unstable

4001234-11060 74 رضا انسانی

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$D(s) = s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$-\frac{(22-12)}{1} = \boxed{-10}$				
s^5					
s^4					
s^3					
s^2					
s^1					
s^0					

$\longrightarrow$ علامت عوض شده پس $\Rightarrow$ unstable

سوال 2)

Closed-loop transfer function:

$$T(s) = \frac{k}{s^3 + 18s^2 + 77s + k}$$

s^3	1	77
s^2	18	k
s^1	$\frac{1386-k}{18}$	
s^0	k	

اگر $k < 1386$ همه مثبت می‌شود، پایدار است

اگر $k > 1386$ علامت دو بار عوض می‌شود و دو قطب سمت راست می‌افتد که ناپایداری می‌شود

اگر $k = 1386$ باشد یک row صفری می‌شود پس Critical stability

4001234 11060 74 رضا سلاری سوال 3

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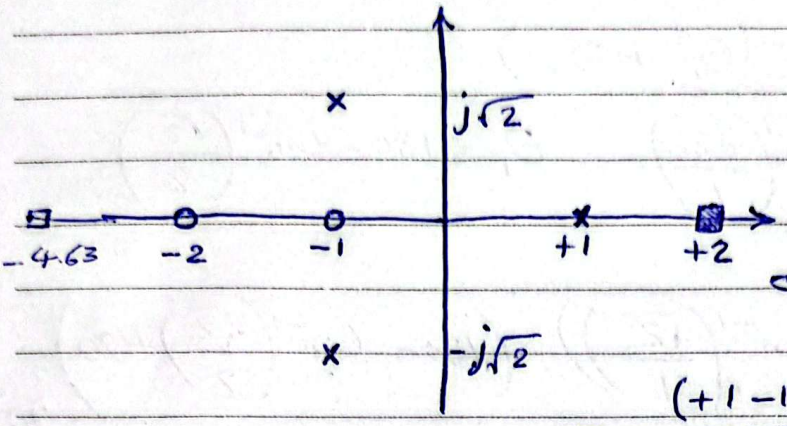
$$GH(S)=\frac{(s+1)(s+2)}{(s^2+2s+3)(s-1)}$$

Zeros: $z_1=-1$, $z_2=-2$

Poles $=+1$, $P_2=-1\pm j\sqrt{2}$



$n-m=1 \rightarrow \angle S_0=\pi$

$$\sigma=\frac{\sum P_i-\sum z_j}{n-m}$$

$$\sigma=\frac{(+1-1+j\sqrt{2}-1-j\sqrt{2})-(-1-2)}{1}=2$$

$$k=-\frac{D(s)}{N(s)}=\frac{-(s^2+2s+3)(s-1)}{(s+1)(s+2)}$$

$$\frac{dk}{ds}=0\rightarrow s^4+6s^3+8s^2+10s+11=0$$

$S_{1,2}=0.016\pm j1.3 \longrightarrow k=0.032\pm j1.18$

$S_3=-1.4\longrightarrow k=-2.1$

$S_4=-4.63\rightarrow k=8.95$

$$1+kGH(s)=0\rightarrow s^3+(k+1)s^2+(3k+1)s+2k-3=0$$

s^3	1	$3k+1$
s^2	$k+1$	$2k-3$
s^1	$\frac{3k^2+2k+4}{k+1}$	
s^0	$2k-3$	

$k+1>0\rightarrow k>-1$

$3k^2+2k+4>0\rightarrow k\in R$

$2k-3>0\rightarrow k>\frac{3}{2}$

اگر $0<k<\frac{3}{2}$ یکی از قطب ها سمت راست می افتد.

اگر $k=\frac{3}{2}$ یکی از قطب ها روی $s=0$

$k>\frac{3}{2}\rightarrow$ هیچ قطبی روی راست نمی افتد

9001234110674 رضا سالمی

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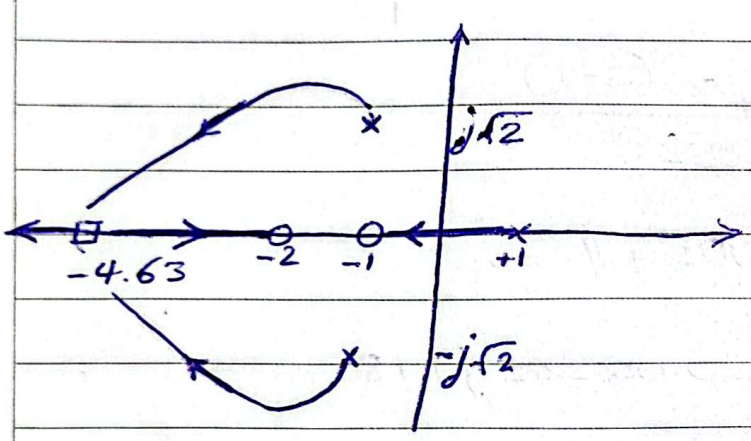
$$\theta_r = \pi + \sum_i \phi_i - \sum_{j \neq r} \theta_j$$

$$\theta_2 = \pi + (\phi_1 + \phi_2) - (\theta_1 + \theta_3)$$

$\phi_1 = 90$, $\phi_2 = = \tan^{-1}\left(\frac{\sqrt{2}}{1.}\right)$ $\theta_1 = 180 - \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$,

$\theta_3 = 90$

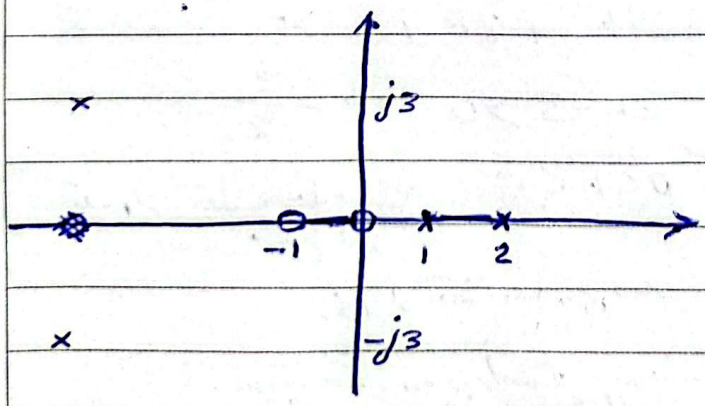
$$\Longrightarrow \theta_2 = 180 + \left(90 + \tan^{-1}\left(\frac{\sqrt{2}}{1}\right)\right) - \left(180 - \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) + 90\right) \approx 90^\circ$$



$$GH(S) = \frac{S(S+1)}{(S-1)(S-2)(S^2+10S+34)}$$

قطب‌ها $S = 1$, $S = 2$, $S = -5 \pm j3$.

صفرها $S = 0$, $S = -1$



رضا الـــــهی 4001234011060 74

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مجانب‌ها

$n - m = 4 - 2 = 2$

مرکز مجانب

$$\frac{(1+2-5+\sqrt{3}-5-\sqrt{3})-(0-1)}{2}$$

$\sigma = -3$

$$k = -\frac{(s-1)(s-2)(s^2+10s+34)}{s(s+1)}$$

$s = -0.4 \quad s = 1.4$

سوال ۴

$$G(s) = \frac{k}{s(s+3)(s+6)} \qquad \zeta = 0.5$$

closed-loop transfer func

$$\frac{G(s)}{1+G(s)}$$

معادله مشخصه

$1+G(s)=0$

معادله مشخصه مرتبه دوم استاندارد

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$e_{ss} = \frac{1}{K_V} \qquad K_V = \lim_{s\to 0} sG(s)$$

$$\frac{G(s)}{1+G(s)} \Rightarrow \frac{k}{s(s+3)(s+6)+k}$$

$s(s+3)(s+6)+k=0$

$s^3 + 9s^2 + 18s + k = 0$

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad 9 = 2\zeta\omega_n \qquad 18 = \omega_n^2$

$\zeta = 0.5 \Rightarrow \omega_n = 9 \qquad \omega_n = \sqrt{18} = 3\sqrt{2}$ ✗

4001234110607 4 رضا

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$S^3 + 9S^2 + 18S + K = 0$

S^3	1	18	$162 - K > 0 \quad K > 0$
S^2	9	K	$0 < K < 162$
S^1	$\frac{162-K}{9}$	0	
S^0	K		

$$S = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$S = -0.5\omega_n \pm j\omega_n\sqrt{1-0.5^2} = -0.5\omega_n \pm j\omega_n\frac{\sqrt{3}}{2}$$

$$S_1 + S_2 + S_3 = -9$$

$$S_1 = -0.5\omega_n + j\omega_n\frac{\sqrt{3}}{2} \qquad S_2 = -0.5\omega_n - j\omega_n\frac{\sqrt{3}}{2}$$

$$S_1 + S_2 = -\omega_n \qquad -\omega_n + S_3 = -9 \qquad S_3 = \omega_n - 9$$

$$S_1 S_2 S_3 = -K \;/\; S_1 S_2 = \omega_n^2 \;/\; \omega_n^2(\omega_n - 9) = -K$$

$$\omega_n = 3\sqrt{2} \qquad (3\sqrt{2})^2(3\sqrt{2} - 9) = -K \qquad 18(3\sqrt{2} - 9) = -K$$

$$54\sqrt{2} - 162 = -K \qquad K = 162 - 54\sqrt{2} \approx \boxed{85.7}$$

$$K_V = \lim_{S\to 0} S\frac{K}{S(S+3)(S+6)} = \frac{K}{18} \qquad e_{SS} = \frac{1}{K_V}$$

$$e_{SS} = \frac{1}{\frac{K}{18}} = \frac{18}{K} \qquad e_{SS} = \frac{18}{85.7} \approx \boxed{0.21}$$