

Chapter III. Elements of Signal Detection Theory

The model for signal detection

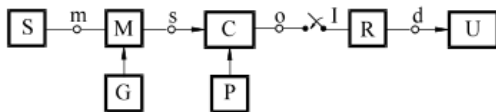


Figure 1: Signal detection model

► Contents:

- Information source: generates messages a_n with probabilities $p(a_n)$
- Modulator: transmits a signal $s_n(t)$ for message a_n
- Channel: adds random noise
- Sampler: takes samples from the signal $s_n(t)$
- Receiver: **decides** what message a_n has been transmitted

Example

- ▶ A simple case (binary):
 - ▶ two messages a_0 and a_1
 - ▶ signals are constants (i.e. 0 for a_0 , 5 for a_1)
 - ▶ take just 1 sample
 - ▶ decide: compare with a threshold
- ▶ General case: many messages, various signals, more samples (or continuous)

Detection for the binary case

- ▶ Receiver guesses between two hypotheses:
 - ▶ H_0 : a_0 has been transmitted
 - ▶ H_1 : a_1 has been transmitted
- ▶ The sample $r = s + n$
 - ▶ if more samples, then they are vectors $\vec{r} = \vec{s} + \vec{n}$
- ▶ Decision based on regions:
 - ▶ if r in region R_0 , then decide D_0 : was a_0
 - ▶ if r in region R_1 , then decide D_1 : was a_1
 - ▶ for single sample, regions are intervals: below/above the threshold
 - ▶ for 2 samples: regions are areas in a 2D plane, etc.
- ▶ Possible errors:
 - ▶ **false alarm**: was a_0 , but decided D_1
 - ▶ probability is $P(D_1 \cap a_0)$
 - ▶ **miss**: was a_1 , but decided D_0
 - ▶ probability is $P(D_0 \cap a_1)$

Minimum risk (cost) criterion

- ▶ How to choose the threshold? We need criteria
 - ▶ In general: how to delimit regions R_i ?
- ▶ Minimum risk (cost) criterion: assign costs to decisions, minimize average cost
 - ▶ C_{ij} = cost of decision D_i when symbol was a_j
 - ▶ C_{00} = cost for good a_0 detection
 - ▶ C_{10} = cost for false alarm
 - ▶ C_{01} = cost for miss
 - ▶ C_{11} = cost for good a_1 detection
- ▶ The risk = the average cost

$$R = C_{00}P(D_0 \cap a_0) + C_{10}P(D_1 \cap a_0) + C_{01}P(D_0 \cap a_1) + C_{11}P(D_1 \cap a_1)$$

- ▶ Minimum risk criterion: **minimize the risk R**

Computations

- ▶ Proof on table:
 - ▶ Use Bayes rule
 - ▶ Notations: $w(r|a_j)$ (*likelihood*)
 - ▶ Probabilities: $\int_{R_i} w(r|a_j) dV$
- ▶ Conclusion, **decision rule is**

$$\frac{w(r|a_1)}{w(r|a_0)} \geq \frac{(C_{10} - C_{00})p(a_0)}{(C_{01} - C_{11})p(a_1)}$$

$$\Lambda(r) \geq K$$

- ▶ Interpretation: effect of costs, probabilities (move threshold)
- ▶ Can also apply logarithm (useful for normal distribution)

$$\ln \Lambda(r) \geq \ln K$$

- ▶ Example at blackboard: random noise with $N(0, \sigma^2)$, one sample

Ideal observer criterion

- ▶ Minimize the probability of decision error P_e
 - ▶ definition of P_e
- ▶ Particular case of minimum risk, with
 - ▶ $C_{00} = C_{11} = 0$ (good decisions bear no cost)
 - ▶ $C_{10} = C_{01}$ (pay the same in case of bad decisions)

$$\frac{w(r|a_1)}{w(r|a_0)} \gtrless \frac{p(a_0)}{p(a_1)}$$

Maximum likelihood criterion

- ▶ Particular case of above, with equal probability of messages

$$\frac{w(r|a_1)}{w(r|a_0)} \geq 1$$

$$\ln \frac{w(r|a_1)}{w(r|a_0)} \geq 0$$

- ▶ Example at blackboard: random noise with $N(0, \sigma^2)$, one sample
- ▶ Example at blackboard: random noise with $N(0, \sigma^2)$, **two** samples