

Seminar 8 - Criteria / Multiple samples

DEDP

1. Consider the detection of a signal with two possible levels, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by noise with triangular distribution $[-5, 5]$. The receiver takes one sample $r = 3.5$.
 - a. Find the decision for the sample $r = 3.5$ considering the following criteria:
 - Maximum Likelihood criterion.
 - Minimum probability of error criterion, if $P(H_0) = \frac{3}{4}$ and $P(H_1) = \frac{1}{4}$.
 - Minimum risk criterion, if $P(H_0) = \frac{3}{4}$ and $P(H_1) = \frac{1}{4}$ and the costs are:
 - $C_{00} = 0$
 - $C_{11} = 0$
 - $C_{01} = 5$
 - $C_{10} = 2$
 - b. What is the probability of false alarm, $P(D_1 \cap H_0)$, for the third criterion above?
2. A signal can have two values, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 5 samples with values $\{1.1, 4.4, 3.7, 4.1, 3.8\}$.
 - a. What is decision according to Maximum Likelihood criterion?
 - b. What is decision according to Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
 - c. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
3. Consider detecting a signal $s_1(t) = 3 \sin(2\pi f_1 t)$ that can be present (hypothesis H_1) or not ($s_0(t) = 0$, hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 2 samples.
 - a. What are the best sample times t_1 and t_2 to maximize detection performance?
 - b. The receiver takes 2 samples with values $\{1.1, 4.4\}$, at sample times $t_1 = \frac{0.125}{f_1}$ and $t_2 = \frac{0.625}{f_1}$. What is decision according to Maximum Likelihood criterion?
 - c. What if we take the decision with Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?

- d. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
 - e. What if the receiver takes an extra third sample at time $t_3 = \frac{0.5}{f_1}$. Will the detection be improved?
4. A signal can have two values, -4 (hypothesis H_0) or 5 (hypothesis H_1). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 3 samples with values $\{1.1, 4.4, 2.2\}$.
- a. What is decision according to Maximum Likelihood criterion? Use both interpretations.