DEDP Exam 2017-2018

No.1

Exercises

1. (3p) Compute the average value, the average squared value, and the variance for a stationary random process with a **uniform distribution** $w_1(x) = \mathcal{U}[0, 10]$.

2. (7p) Consider the detection of a constant signal A with two possible values, $A = -2$ (hypothesis H_0) or $A = 2$ (hypothesis H_1). The signal is affected by noise with uniform distribution $[-5, 5]$. The probabilities of the two hypotheses are $P(H_0) = 2/3$, $P(H_1) = 1/3$. The receiver takes a single sample r.

   a. (1p) Draw the two likelihood functions $w(r|H_0)$ and $w(r|H_1)$, and compute their height.
   b. (2p) Find the decision threshold T considering the **Minimum Probability of Error** criterion.
   c. (2p) Compute the probability of **false alarm** and of **correct rejection**, considering the **Minimum Probability of Error** criterion.
   d. (2p) Find a new threshold T' which makes the probability of **false alarm** equal to 1%

3. (3p) Consider detecting a signal $s(t) = 2\cos(4\pi t)$ that can be present (hypothesis H_1) or absent (hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 2 samples at times $t_1 = 0$ and $t_2 = 1/8$, with values $r_1 = 0.6$ and $r_2 = -0.6$.

   a. (2p) What is the decision according to Maximum Likelihood criterion?
   b. (1p) If we could take a third sample, what would be the best sample time t_3? Explain why.

4. (5p) Consider the received signal $r(t) = \underbrace{t - 2A}_{s(t)} + noise$, where A is unknown. The noise has Gaussian distribution $\mathcal{N}(0, \sigma^2 = 4)$. The receiver takes three samples at times $t_1 = 2, t_2 = 4, t_3 = 5$, with values $r_1 = -0.8$, $r_2 = 2.1$, $r_3 = 2.9$. Estimate the parameter A using Maximum Likelihood estimation.

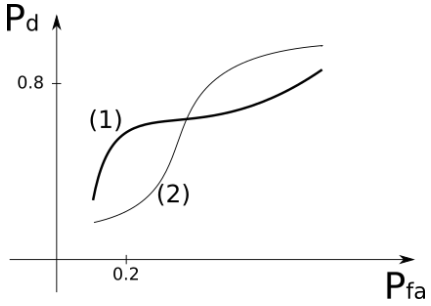
Theory

1. (4p) Prove the relation $\sigma^2 = E\{X^2\} - (E\{X\})^2$.

2. (5p) State the five properties of the autocorrelation function of a stationary random process.

3. (2p)Consider the following graph with two different **Receiver Operating Characteristics** (ROC):

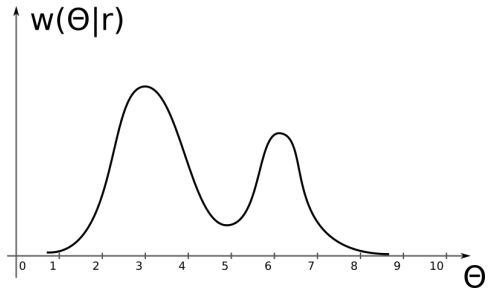
   a. What is a ROC hraph?
   b. Which is the better one for $P_{fa} = 0.2$? Explain (in words).
   c. Which is the better one for $P_d = 0.8$? Explain (in words).



4. (3p) Show that the Maximum Likelihood criterion is a particular case of the Minimum Probability of Error criterion, which in turn is a particular case of the Minimum Risk criterion.

5. (3p) The **a posteriori** distribution of an unknown parameter Θ is shown below.

   a. What is the value of the MAP estimator $\hat{\Theta}_{MAP}$? Explain how you chose it.
   b. Is the value of the MMSE estimator $\hat{\Theta}_{MMSE}$ larger or smaller that that $\hat{\Theta}_{MAP}$? Explain why.



6. (2p) Consider an unknown parameter Θ with value around 10. You can choose between the four estimators below. Which one would you choose and why?

   1. an unbiased estimator with small variance $\sigma^2_{\hat{\Theta}} = 0.1$
   2. a biased estimator with small variance $\sigma^2_{\hat{\Theta}} = 0.1$
   3. an unbiased estimator with large variance $\sigma^2_{\hat{\Theta}} = 10$
   4. a biased estimator with large variance $\sigma^2_{\hat{\Theta}} = 10$

Notes: 40p total, solve 30p for grade 10. 3p are awarded from start. Time available: 2h