DEDP Exam 2019-2020

Exercises (17p)

1. Let A be a continuous r.v. with distribution $\mathcal{U}\left[-4, 4\right]$

   a. (1p) Draw the probability density of A (including its height)
   b. (1p) Compute the probability that A is positive
   c. (1p) Compute the variance σ^2 of A
   d. (1p) We define a new random variable B as $B = A + 2$. What is the distribution of B?

2. Consider detection between two possible constant signals, $s_0(t) = -2$ and $s_1(t) = 2$. The signals are affected by AWGN with distribution $\mathcal{N}\left(\mu = 0, \sigma^2 = 3\right)$. The probabilities of the two hypotheses are $P(H_0) = 3/4$, $P(H_1) = 1/4$. The receiver takes one sample, at time $t_0 = 1$, and the obtained value is $r = 0.5$.

   a. (1p) Sketch the two conditional distributions
   b. (1p) What are the decision regions R_0 and R_1 for the Minimum Probability of Error criterion? Justify.
   c. (2p) Compute the probability of miss, in case of the Maximum Likelihood criterion.
   d. (2p) Johnny uses his own special criterion. If we know that for Johnny the probability $P(D_0|H_0) = \frac{1}{2}$, find Johnny's decision regions R_0' and R_1'.

3. Consider receiving a signal which can be either $s_1(t) = \sin(\pi t)$ (hypothesis H_1) or $s_0(t) = -1$ (hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 4)$. The receiver samples the signal at times $t_0 = 0.5$, $t_1 = 1.5$ and $t_2 = 2.5$, and reads the values $r_0 = 0$, $r_1 = -0.6$ and $r_2 = 0.5$.

   a. (2p) What is the decision according to Maximum Likelihood criterion?
   b. (1p) If we want to take decisions using only 2 samples (not 3), which are the best 2 sample times to keep from t_1, t_2, t_3? (pick best 2 out of 3)

4. (4p) Consider the received signal $r(t) = \underbrace{At^2 - 1}_{s_\Theta(t)} + noise$, which is sampled at time moments $t_i = [0, 2, 4]$, and the values are $r_i = [-0.9, -5.1, -16.6]$. The noise has normal distribution $\mathcal{N}(0, \sigma^2 = 3)$. Estimate the unknown parameter A using Maximum Likelihood estimation.

Theory (16p)

1. (2p) What is the difference between a **discrete** random variable and a **continuous** random variable?

2. (2p) Let A be a random variable with any distribution $w_A(r)$. Define a new random variable B as $B = A + 4$. Answer the following questions:

   a. Do A and B have the same average value?
   b. Do A and B have the same variance?
   c. Do A and B have the same average squared value?

Justify.

(*Hint*: take $w_A(r)$ as a uniform or a normal distribution, for example).

3. (4p) Deduce the Minimum Probability of Error criterion: prove that minimizing the total probability of error (false alarms + miss) leads to the formula

$$\frac{w(r|H_1)}{w(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{p(H_0)}{p(H_1)}$$

4. (2p) If the **noise** added to a signal is **reduced to a half** ("înjumătățit"), how does the Signal-to-Noise Ratio (SNR) change (explain in words why):

   a. SNR increases
   b. SNR decreases
   c. SNR remains the same

5. (2p) Consider receiving a signal which can be either $s_1(t) = 5$ (hypothesis H_1) or $s_0(t) = 0$ (hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2)$. The receiver takes some samples and then takes a decision, using the Maximum Likelihood criterion.

   If we double the number of samples, will the decision be improved?

   a. If we double the number of samples, the decision will be **improved**
   b. If we double the number of samples, the decision will be **worse**
   c. If we double the number of samples, the decision will be **the same**

   Justify.

6. (3p) Prove that the quadratic cost function $C(\epsilon)$ leads to the formula of the MMSE estimator:

$$\hat{\Theta}_{MMSE} = \int_{-\infty}^{\infty} \Theta w(\Theta|r) d\Theta$$

   Starting point: it is known that the expected cost is $C = \int_{-\infty}^{\infty} C(\epsilon) w(\Theta|r) d\Theta$

7. (1p) Write the mathematical definitions of the Maximum Likelihood (ML) and Maximum A Posteriori (MAP) estimators

Notes: 33p total, solve 30p for grade 10. 3p are awarded from start. Time available: 2h