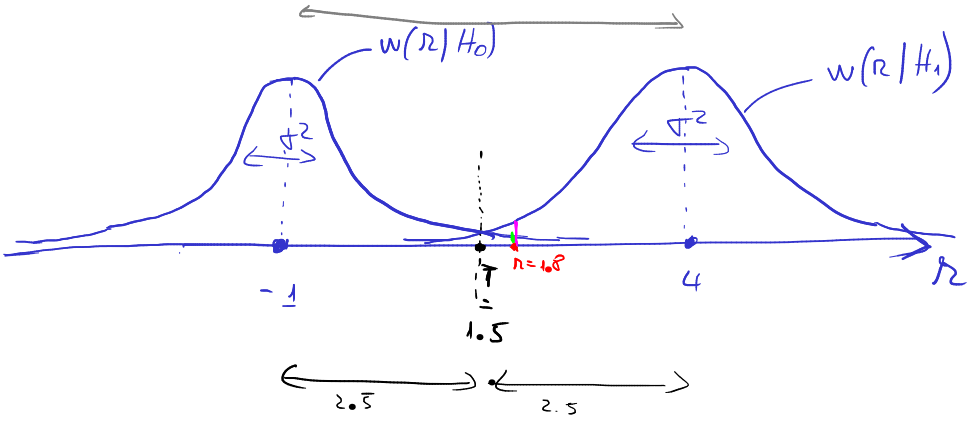
Seminar 3

1

$s_0(t) = -1$

$s_1(t) = 4$

$r = 1.8$



$w(r|H_0) = ?$

$w(r|H_1) = ?$

$r = s(t) + \text{noise}$

$$w(r|H_0) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r+1)^2}{2\sigma^2}} = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r+1)^2}{8}}$$

$$w(r|H_1) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r-4)^2}{8}}$$

b. $1.8 > 1.5 \Rightarrow D_1$

$$w(r|H_1)\Big|_{r=1.8} > w(r|H_0)\Big|_{r=1.8} \Rightarrow D_1$$

$$d(1.8, 4) < d(1.8, -1) \Rightarrow D_1$$

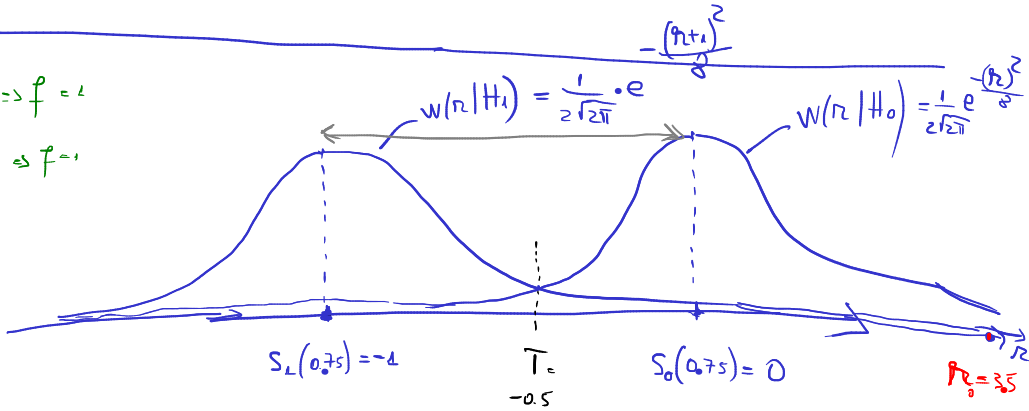
2

$s_0(t) = \cos(2\pi t) \Rightarrow f = 1$

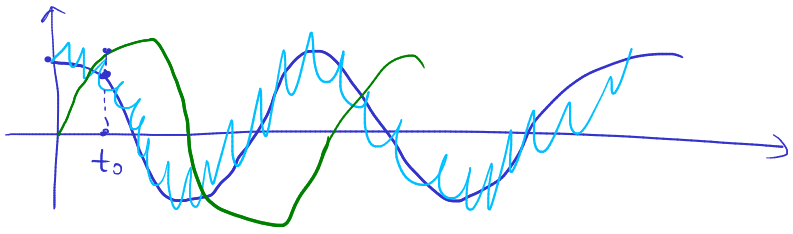
$s_1(t) = \sin(2\pi t) \Rightarrow f = 1$

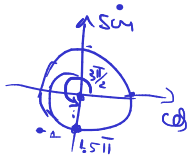
$r(t) = s(t) + \text{noise}$

$r(t_0 = 0.75) = 3.5$



H_0 :





H_0 : $s_0(t_0 = 0.75) = \cos(2\pi \cdot 0.75) = \cos(1.5\pi) = 0$

H_1 : $s_1(t_0 = 0.75) = \sin(2\pi \cdot 0.75) = \sin(1.5\pi) = -1$

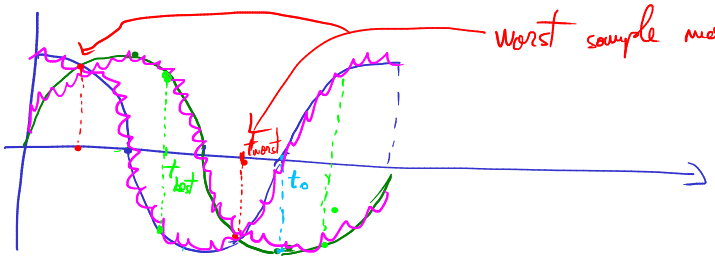
b). For $r = 3.5$

$0.0431 > 0.0158$

$$w(r|H_0)\Big|_{r=3.5} = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(3.5)^2}{8}} = 0.0431 \Rightarrow \boxed{\Delta_0}$$

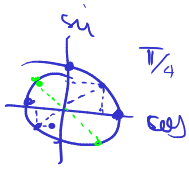
$$w(r|H_1)\Big|_{r=3.5} = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(3.5+1)^2}{8}} = 0.0158$$

c).



d). Worst sample moment = when $s_0(t) = s_1(t)$

$\sin\left(\frac{2\pi t}{\pi/4}\right) = \cos\left(\frac{2\pi t}{\pi/4}\right)$

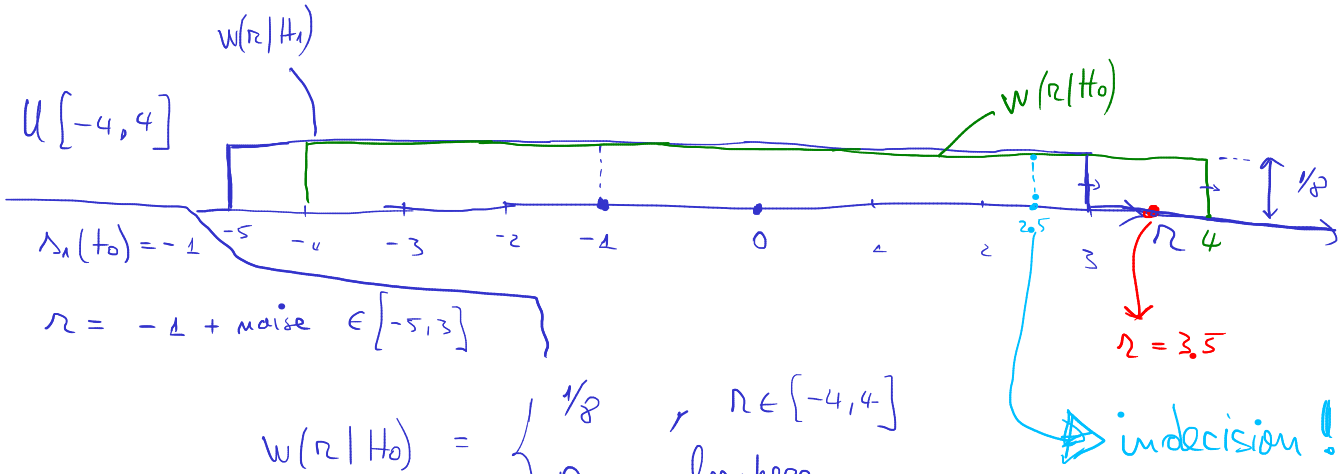


t_{best} = when $|s_1(t) - s_0(t)|$ is maximal

$\sin(3\pi/4) - \cos 3\pi/4$, $2\pi t = \frac{3\pi}{4}$, $t = \frac{3}{8}$

$2\pi t = \pi/4 \Rightarrow \boxed{t_{worst} = \frac{1}{8} \text{ or } 5/8}$

e). $U[-4,4]$



H_1: $s_1(t_0) = -1$

$r = -1 + \text{noise} \in [-5,3]$

$$w(r|H_0) = \begin{cases} 1/8, & r \in [-4,4] \\ 0, & \text{elsewhere} \end{cases}$$

$$w(r|H_1) = \begin{cases} 1/8, & r \in [-5,3] \\ 0, & \text{elsewhere} \end{cases}$$

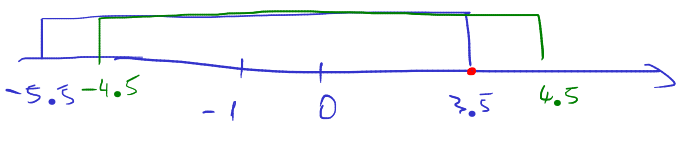
For $r = 3.5$, $w(r|H_0) > w(r|H_1) \Rightarrow \boxed{\Delta_0}$

f). If noise would be $U[-4.1, 4.1]$, $\Delta = \Delta_0$ ✓

$-4.2, 4.2$

$-4.3, 4.3$

$U[-4.5, 4.5]$ = max variance for taking a decision for $r = 3.5$



$\Downarrow$

$$\sigma^2 = \int_{-4.5}^{4.5} (x-0)^2 \frac{1}{8} dx = \dots$$