Seminar 6 - ML / MPE criteria

DEDP

1. A vehicle airbag system detects a crash by evaluating a sensor which provides two values: $s_0(t) = 0$ (no crash) or $s_1(t) = 5$ (crashing) The signal is affected by gaussian noise $\mathcal{N}$ $(\mu = 0, \sigma^2 = 1)$. The costs of the scenarios are: $C_{00} = 0$, $C_{01} = 100$, $C_{10} = 10$, $C_{11} = -100$

   a. Find the decision taken based on a sample $r = 3.1$
   b. Find the decision regions R_0 and R_1.

2. An information source provides two messages with probabilities $p(a_0) = \frac{2}{3}$ and $p(a_1) = \frac{1}{3}$. The messages are encoded as constant signals with values -5 (a_0) and 5 (a_1). The signals are affected by noise with uniform distribution $U[-5, 5]$. The receiver takes one sample r.

   a. Find the decision regions according to the Neymar-Pearson criterion, considering $P_{fa} \leq 10^{-2}$
   b. What is the probability of correct detection, in this case?

3. Consider the detection of a signal with two possible levels, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by noise with triangular distribution $[-5, 5]$. The receiver takes one sample $r = 3.5$.

   a. Find the decision for the sample $r = 3.5$ considering the following criteria:
      - Maximum Likelihood criterion.
      - Minimum probability of error criterion, if $P(H_0) = \frac{3}{4}$ and $P(H_1) = \frac{1}{4}$.
      - Minimum risk criterion, if $P(H_0) = \frac{3}{4}$ and $P(H_1) = \frac{1}{4}$ and the costs are:
        - $C_{00} = 0$
        - $C_{11} = 0$
        - $C_{01} = 5$
        - $C_{10} = 2$
   b. What is the probability of false alarm, $P(D_1 \cap H_0)$, for the third criterion above?

4. An information source provides two messages with probabilities $p(a_0) = \frac{2}{3}$ and $p(a_1) = \frac{1}{3}$. The messages are encoded as constant signals with values -5 (a_0) and 5 (a_1). The signals are affected by noise with triangular distribution $[-10, 10]$. The

receiver takes one sample r. Decision is done by comparing r with a threshold value T.

a. Find the threshold value T according to the Neymar-Pearson criterion, considering $P_{fa} \leq 10^{-2}$
b. What is the probability of correct detection, $P(D_1 \cap H_1)$?