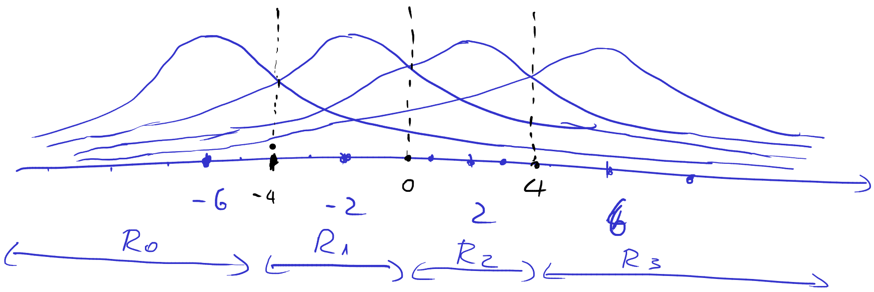
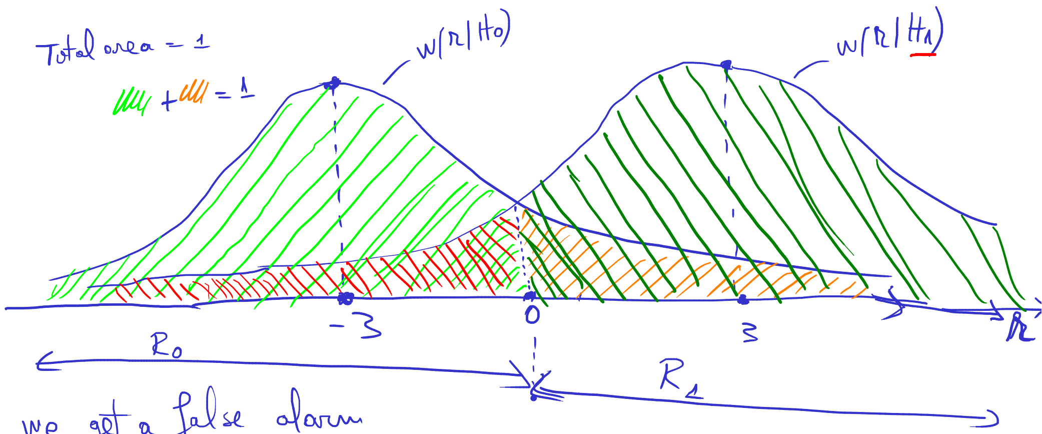
Seminar 4

(1) 4.1 6.6 −5.2 1.1 0.3 −1.5 7 −7 4.4

$\downarrow$

6 6 −6 2 2 −2 6 −6 6





(2) $s_0(t) = -3$

$s_1(t) = 3$

noise = $\mathcal{N}(\mu = 0, \sigma^2 = 1)$

a) (In hypothesis H_0) we get a false alarm ($D_1 \cap H_0$) when $\underline{r > 0}$

b). $P(D_0 | H_0) = P_{cr} = \int_{-\infty}^{0} w(r|H_0)\,dr = F(0) - \underbrace{F(-\infty)}_{0} = 0.999$

$P(D_1 | H_0) = P_{fa} = \int_{0}^{\infty} \underline{w(r|H_0)}\,dr = \underbrace{F(\infty)}_{1} - F(0) = 1 - \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{0+3}{1\cdot\sqrt{2}}\right)\right) = 0.001$

$P(D_0 | H_1) = P_m = \int_{-\infty}^{0} w(r|H_1)\,dr = F(0) - \underbrace{F(-\infty)}_{0} = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{0-3}{1\cdot\sqrt{2}}\right)\right) = 0.001$

$P(D_1 | H_1) = P_{cd} = \int_{0}^{\infty} w(r|H_1)\,dr = F(\infty) - F(0) = 0.999$

$$\boxed{F(x) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)}$$

$P(D_0 \cap H_0) = \underbrace{P(D_0 | H_0)}_{0.999} \cdot P(H_0) = \ldots$

$P(D_1 \cap H_0) = \underbrace{P(D_1 | H_0)}_{0.001} \cdot \underline{P(H_0)} = \ldots$

Need $P(H_0)$ and $P(H_1)$ specified in the ~~pb~~ exercise

$P(D_0 \cap H_1) = \underbrace{P(D_0 | H_1)}_{0.001} \cdot P(H_1) = \ldots$

$P(D_1 \cap H_1) = \underbrace{P(D_1 | H_1)}_{0.999} \cdot P(H_1) = \ldots$

3

$s_0 = 0$

$s_1 = 5$

$$\begin{cases} P(H_0) = 2/3 \\ P(H_1) = 1/3 \end{cases}$$

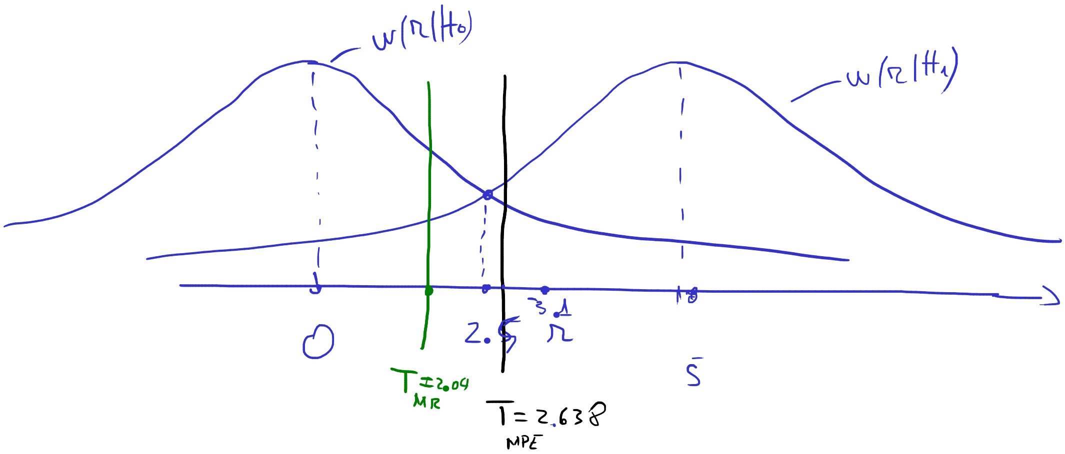
$C_{00} = 0$

$C_{01} = 100$

$C_{10} = 10$

$C_{11} = -100$

$\boxed{r = 3.1}$



M.L. $\Rightarrow$ D_1

a) M.P.E:

a1) $$d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + 2\cdot\sigma^2 \ln \frac{P(H_0)}{P(H_1)}$$

$$(r - s_0)^2 \qquad\qquad (r - s_1)^2$$

$$(3.1 - 0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} (3.1 - 5)^2 + 2\cdot 1\cdot \ln 2$$

$$\underbrace{3.1^2}_{9.61} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{1.9^2}_{3.61} + 1.38 \Rightarrow \boxed{D_1}$$

a2). $$T = \frac{s_0(t) + s_1(t)}{2} + \frac{\sigma^2}{s_1(t_0) - s_0(t_0)} \cdot \ln \frac{P(H_0)}{P(H_1)}$$

$$= \frac{0 + 5}{2} + \frac{1}{5 - 0} \cdot \ln 2 =$$

$$= 2.5 + 0.138$$

$$= 2.638$$

$$\underset{3.1}{r} > \underset{2.638}{T} \Rightarrow \boxed{D_1}$$

b).
$$d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + 2\cdot\sigma^2 \ln \frac{P(H_0)\cdot(C_{10}-C_{00})}{P(H_1)\cdot(C_{01}-C_{11})}$$

$$(r-s_0)^2 \qquad (r-s_1)^2$$

$$9.61 \qquad 3.61 + 2\cdot 1\cdot \ln\left(2\cdot\frac{10-0}{100+100}\right)$$

$$9.61 \gtrless 3.61 + \underbrace{\underbrace{2\cdot\ln\left(\frac{20}{200}\right)}_{-2.3}}_{-4.6}$$

$$9.61 \underset{H_0}{\overset{H_1}{\gtrless}} -0.99 \Rightarrow \boxed{D_1}$$

OR:

b). $$T_{MR} = 2.5 + \frac{1}{5}\cdot \ln\left(\frac{20}{200}\right) = 2.5 + \frac{1}{5}\cdot(-2.3)$$

$$= 2.04$$

$$r > T_{MR} \Rightarrow \boxed{D_1}$$

c). $R_0 = (-\infty, T)$

$R_1 = (T, \infty)$

M.P.E.: $T_{MPE} = 2.638$

$R_0 = (-\infty, 2.638)$

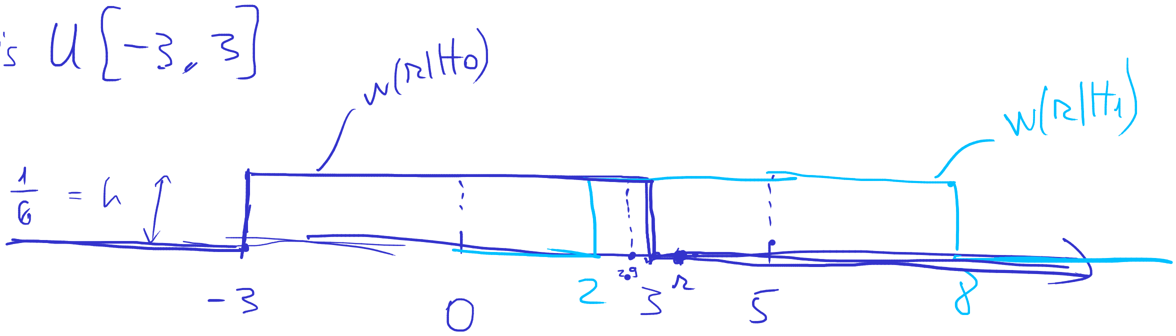
$R_1 = (2.638, \infty)$

M.R.: $T_{MR} = 2.04$

$R_0 = (-\infty, 2.04)$

$R_1 = (2.04, \infty)$

d). Noise is $U[-3, 3]$



a) $r = 3.1$

M.P.E.

$D = ?$

$$\boxed{\frac{w(r|H_1)}{w(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)}{P(H_1)}}$$

$$\frac{1/6}{0} \underset{H_0}{\overset{H_1}{\gtrless}} 2 \Rightarrow D_1$$

If $r = 2.9$?

$$\frac{1/6}{1/6} \underset{H_0}{\overset{H_1}{\gtrless}} 2 \Rightarrow D_0$$

b). $r = 3.1$

M.R.

$$\frac{w(r|H_1)}{w(r|H_0)} \gtrless \frac{P(H_0)\,(C_{10} - C_{00})}{P(H_1)\,(C_{01} - C_{11})}$$

$$\frac{1/6}{0} \overset{H_1}{\gtrless} \ldots$$