Binary Phase-Shift Keying (BPSK) modulation

Laboratory 3, DEPI

Objective

Simulate a BPSK modulation system and its decoding performance in white gaussian noise.

Theoretical aspects

Binary Phase-Shift Keying (PSK) modulation is a binary encoding procedure defined as follows:

- for a logical bit 0, send the signal $s_0(t) = A\sin(2\pi f t)$
- for a logical bit 1, send the signal $s_1(t) = A\sin(2\pi f t + \pi) = -A\sin(2\pi f t)$

The difference is in the sign (or phase π) of the signal.

The duration of a bit signal is a multiple of the sine period, $T_{bit} = \frac{1}{f} \cdot k, \quad k \in \mathbb{N}$

The receiver takes a decision with Maximum Likelihood criterion, choosing the **minimum distance** between the received signal $r(t)$ and the two candidate signals $s_0(t) and s_1(t)$.

$$Decision \quad D_i = \arg\min_i \mathbf{d}\left(r(t), s_i(t)\right) = \arg\min_i \|r(t) - s_i(t)\| = \arg\min_i \int \left(r(t) - s_i(t)\right)^2$$

For discrete signals, the integral is replaced with a **sum**.

Note that this distance is simply the **Euclidean (geometric) distance** between signals.

Alternatively, the decision can be found by by multiplying with $\sin(2\pi f t)$ and integrating the result:

$$\pm \int_0^{T_{bit}} A\sin(2\pi f t)\sin(2\pi f t)dt = \pm A \int_0^{T_{bit}} \frac{1 - \cos(2\pi(2f)t)}{2}dt = \pm\frac{AT_{bit}}{2}$$

The sign of the result matches the sign of the original signal:

- If the result is positive, the original bit is 0 (signal amplitude was A)
- If the result is negative, the original bit is 1 (signal amplitude was $-A$)

When the received signal has noise, the values might vary. There can be 4 outcomes:

- correct rejection: original bit is 0, detection is 0
- false alarm: original bit is 0, detection is 1
- miss: original bit is 1, detection is 0
- hit (correct detection): original bit is 1, detection is 1

In general, the result can be compared to a threshold T which might not necessary be 0, but closer to one value or the other. If the distribution of the noise is known, a precise value for T can be found with a decision criterion.

The performance of the detection scheme can be summarized in a Receiver Operating Characteristic (ROC) plot. The Receiver Operating Characteristic (ROC) curve is the plot of P(hit) against P(false alarm).

Exercises

1. Simulate the BPSK sender
   - Generate a vector **data** of 1000 values 0 or 1, with equal probability (hint: use **rand()** and compare to 0.5).
   - Generate a vector **signal** of 100000 values as follows: for each bit in **data**, put a 100-long sine $\pm A \sin(2\pi f n)$ in **signal**. Use $A = 1$, $f = 1/100$. 0 corresponds to $+A$, 1 to $-A$.
   - Plot the resulting signal.
2. Simulate a noisy channel
   - Generate a vector of white gaussian noise with distribution $\mathcal{N}(0, \sigma^2)$, the same length as **signal**, and $\sigma^2 = A/10$.
   - Add the noise to the signal, store result as **r**.
3. Simulate the BPSK receiver
   - For each 100-long piece from **r**, compute the Euclidean distance from that piece to $A \sin(2\pi f n)$ and to $-A \sin(2\pi f n)$. Choose the minimum distance and place the decoded 0 or 1 in a binary vector **decoded**.
   - Compare **decoded** with the original **data** and compute the probability of *correct rejection* and the probability of *false alarm*.

TO CHANGE:

4. Draw the ROC.
   - Wrap all the above into a function [phit pfa] = BPSKsim(T) and call it for different values of T, going from $-50A$ to $50A$.

- Store the results for each case, and at the end plot the graph P(hit) as a function of P(fa).

Final questions

1. When should a value $T \neq 0$ be used?