Seminar 12

DEDP

1. Consider the ML estimation of a signal with the form

$$s_\Theta(t) = a \cdot t^2 + b \cdot t$$

with two unknowns $\Theta = [a, b]$, from 4 samples $\mathbf{r} = [0, 2.5, -3, 2]$ taken at time moments $\mathbf{t} = [0, 1, 2, 3]$. The noise is AWGN.

Perform 1 iteration of the Gradient Descent algorithm to estimate a and b, starting from the initial values $a_0 = 1$, $b_0 = 0.1$, with $\mu = 0.1$.

2. Consider the k-NN algorithm with the following training set, composed of 5 vectors of class A and another 5 vectors from class B:

   - Class A:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \mathbf{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

   - Class B:

$$\mathbf{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \mathbf{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \mathbf{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \quad \mathbf{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \quad \mathbf{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

   Compute the class of the vector $\mathbf{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ using the k-NN algorithm, with $k = 1$, $k = 3$, $k = 5$, $k = 7$ and $k = 9$

3. Consider the following data

$$\mathbf{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

   Use the k-Means algorithm to find the two centroids $\mathbf{c}_1$ and $\mathbf{c}_2$, starting from two random values $\mathbf{c}_1 = 0.95$ and $\mathbf{c}_2 = 0.96$. Perform 5 iterations of the algorithm.