Signal detection with multiple samples

Laboratory 4, DEPI

Objective

Visualize the signal detection rules for detection of signals based on multiple samples

Theoretical aspects

For detecting a constant signal with two possible levels B (hypothesis H_0) and A (hypothesis H_1), affected by Gaussian noise, based on multiple samples, the decision formula is:

$$\sum(r_i - B)^2 \underset{H_0}{\overset{H_1}{\gtrless}} \sum(r_i - A)^2 + 2\sigma^2 \ln K$$

.

For Maximum Likelihood criterion, $K = 1$. In this case, the equation can be interpreted geometrically as **choosing minimum Euclidean distance from point $\mathbf{r} = [r_1, r_2, ...r_N]$ to points $\mathbf{B} = [B, B, ...]$ and $\mathbf{A} = [A, A, ...]$.**

The Euclidian distance between two points $\mathbf{x} = [x_1, x_2, ...x_N]$ and $\mathbf{y} = [y_1, y_2, ...y_N]$ is

$$d(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$$

.

The general formula thus becomes:

$$[d(r, B)]^2 \underset{H_0}{\overset{H_1}{\gtrless}} [d(r, A)]^2 + 2\sigma^2 \ln K$$

.

Exercises

1. Simulate detection of a constant signal with two levels $B = 0$ and $A = 5$, based on two samples, as follows:

   - Generate a vector `data` of 1000 values 0 or 1, with equal probability (hint: use `rand()` and compare to 0.5).
   - Generate a 1000×2 matrix `points` of data samples defined as:
     - row i of `points` is (B, B) if `data(i) is 0`, or
     - row i of `points` is (A, A) if `data(i) is 1`.
   - Add over it a random noise with normal distribution $\mathcal{N}(0, \sigma^2 = 2)$ (use a noise matrix of same size 1000×2). The result should be saved as the matrix `received`.
   - Implement the Maximum Likelihood decision rule for each row i of the received samples. The result (0 or 1) should be saved as vector `decision`.
   - Compare the decision result with the original data vector, and classify the result in a 1000-long vector `result` with the following values:
     - `result(i) = 0` if for bit i there was a correct rejection
     - `result(i) = 1` if for bit i there was a false alarm
     - `result(i) = 2` if for bit i there was a miss
     - `result(i) = 3` if for bit i there was a correct detection
   - Generate a **scatter plot** displaying the received data samples with different coloring of the decisions (*Hint*: check Matlab function `scatter()`).
   - What are the decision regions? What is the decision rule, geometrically?

2. Repeat Exercise 1 for Minimum Probability of Error criterion with probabilities $P(H_0) = 2/3$ and $P(H_1) = 1/3$

   - Change the threshold value for comparing `rand()` to generate 0 with probability 2/3 and 1 with probability 1/3.
   - Modify the detection rule taking into account $K = \frac{P(H_0)}{P(H_1)}$.
   - Visualize the data. How does it look like?

3. Repeat Exercise 1 for four values of K: $K = 1$, $K = 5$, $K = 25$, $K = 125$. Display the 4 scatter plots as subfigures of a figure window, arranged 2×2.

4. Repeat Exercise 2 for 3 samples.

   - The matrix's sizes should be 1000×3.
   - Detection rule should consider all three samples.

Final questions

1. In a practical scenario, what is the disadvantage of using multiple samples for detection, compared to just 1?