

# Parameter and Signal Estimation

## Laboratory 6, DEDP

### Objective

Experiment with Maximum Likelihood, Maximum A Posteriori and Minimum Mean Squared Error estimation for a basic signal.

### Theoretical aspects

### Exercises

1. Generate a 100-samples long sinusoidal signal with frequency  $f = 0.02$ , and add over it normal noise with distribution  $\mathcal{N}(0, \sigma^2 = 2)$ . Name the resulting vector **data**. Plot the **data** vector.
2. Estimate the frequency  $\hat{f}$  of the signal via Maximum Likelihood estimation, based only on the **data** vector.
  - Write the mathematical expression of the likelihood function  $w(\mathbf{r}|f)$
  - Compute numerically the value of likelihood function for  $f$  going from 0 to 0.5, in 200 equally-spaced values
  - Maximum Likelihood: choose  $\hat{f}_{ML}$  as the value which maximizes the likelihood
  - Display  $\hat{f}_{ML}$ , and plot the resulting sinusoidal along the original
  - Try changing the length of the data. How is the estimation accuracy affected?
  - Try changing the variance of the noise. How is the estimation accuracy affected?
3. Suppose that for  $f$  we know a *prior distribution*  $w(f)$ , displayed on the whiteboard. Modify the previous example to implement Bayesian estimation.
  - Multiply the computed likelihood function from previous exercise with the prior distribution, for each point. The result is the *posterior* distribution.

- Maximum A Posteriori: choose  $\hat{f}_{MAP}$  as the value which maximizes the posterior distribution
  - Minimum Mean Squared Error: : choose  $\hat{f}_{MMSE}$  as the average value of the posterior distribution
  - Display  $\hat{f}_{MAP}$  and  $\hat{f}_{MMSE}$ , and plot the resulting sinusoidal signals along the original and the ML one
4. *Signal inpainting (recover missing parts of signal)*. Randomly replace 20 samples from `data` with 0, to simulate missing data. Rerun exercise 3 and estimate the original signal. Plot the result(s) against the starting data (with the missing samples) to visualize the result.

## Final questions

1. TBD