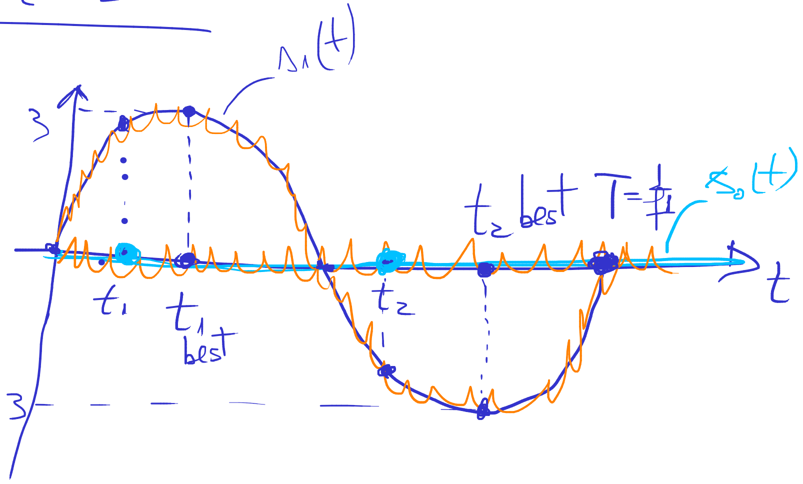
Seminar 5

(1) $s_0(t) = 0$

$s_1(t) = 3\sin(2\pi f_1 t)$



b). $r = [1.1 \quad 4.4]$

ML = ?

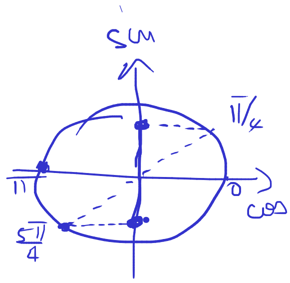
$t_1 = \dfrac{0.125}{f_1} = 0.125 \cdot T$

$t_2 = \dfrac{0.625}{f_1}$

$r = [1.1 \quad 4.4]$

$s_0 = [0 \quad 0]$

$s_1 = \left[\underbrace{\frac{3\sqrt{2}}{2}}_{2.12} \quad \underbrace{\frac{-3\sqrt{2}}{2}}_{-2.12}\right]$



$\boxed{\text{Gaussian noise}}$

$s_1(t_1) = 3\sin\left(2\pi \cancel{f_1} \cdot \frac{0.125}{\cancel{f_1}}\right) = 3 \cdot \sin\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$

$s_1(t_2) = 3\sin\left(2\pi \cancel{f_1} \cdot \frac{0.625}{\cancel{f_1}}\right) = 3 \cdot \sin(1.25\pi) = 3 \cdot \sin\left(\frac{5}{4}\pi\right) = -\frac{3\sqrt{2}}{2}$

Decision: $\underbrace{d(r, s_0)^2}_{20.57} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r, s_1)^2}_{43.5} + \underbrace{2\sigma^2 \ln(K)}_{=0}$

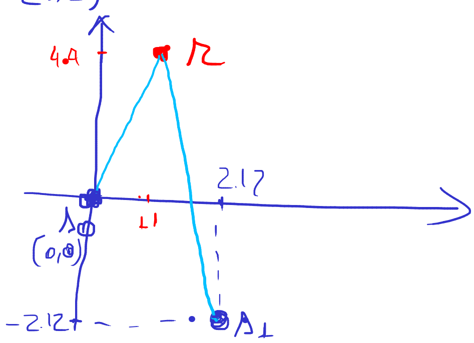
M.L.: K = 1

$d(r, s_0)^2 = (1.1-0)^2 + (4.4-0)^2 = 20.57$

$d(r, s_1)^2 = (1.1-2.12)^2 + (4.4+2.12)^2 = 43.5$

$\Rightarrow d(r,s_0)^2 < d(r,s_1)^2$

$\boxed{s_0}$



c). M.P.E. : $K = \dfrac{P(H_0)}{P(H_1)} = \dfrac{2/3}{1/3} = 2$

Decision : $\underbrace{d(r, s_0)^2}_{20.57} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r, s_1)^2}_{43.5} + \underbrace{2 \cdot \sigma^2 \cdot \ln(K)}_{\underbrace{2 \cdot 1 \cdot \ln(2)}_{1.38}}$

$\Downarrow$

D_0

d). M.R. : $K = \dfrac{(C_{10} - C_{00}) \cdot P(H_0)}{(C_{01} - C_{11}) \cdot P(H_1)} = \dfrac{10 \cdot 2/3}{15 \cdot 1/3} =$

$= \dfrac{20}{15} = 1.33$

$\ln(1.33)$

$\boxed{D_0}$

a) $t_{1\,best} = 0.25\,T = \dfrac{0.25}{f_1}$

$t_{2\,best} = 0.75 \cdot T = \dfrac{0.75}{f}$

e). $t_3 = \dfrac{0.5}{f_1} = 0.5 \cdot T$ No! $s_1(t_3) = 0$

$s_0(t_3) = 0$

2

$s_0(t) = 0$

$s_1(t) = 6$

$\boxed{\text{Gaussian}}$ ✓

a), b), c)

$r = [1.1 \quad 4.4 \quad 3.7 \quad 4.1 \quad 3.8]$

$s_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]$

$s_1 = [6 \quad 6 \quad 6 \quad 6 \quad 6]$

$$\underbrace{d(r, s_0)^2}_{65.51} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r, s_1)^2}_{40.31} + \underbrace{2 \cdot \sigma^2 \cdot \ln(K)}_{\sigma = 1}$$

$$d(r, s_0)^2 = 1.1^2 + 4.4^2 + 3.7^2 + 4.1^2 + 3.8^2 = 65.51$$

$$d(r, s_1)^2 = (1.1-6)^2 + (4.4-6)^2 + (3.7-6)^2 + (4.1-6)^2 + (3.8-6)^2 =$$

$$= 4.9^2 + 1.6^2 + 2.3^2 + 1.9^2 + 2.2^2 = 40.31$$

a). M.L. : $K = 1 \Rightarrow 2\sigma^2 \ln(K) = 0 \Rightarrow \boxed{D_1}$

b). M.P.E. : $K = \frac{P(H_0)}{P(H_1)} = 2 \Rightarrow 2\sigma^2 \ln 2 = 1.38 \Rightarrow \boxed{D_1}$

c). M.R : $K = \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \cdot \frac{P(H_0)}{P(H_1)} = \frac{10}{15} \cdot 2 = \frac{20}{15} = 1.3 \Rightarrow \boxed{D_1}$

$\Rightarrow 2\sigma^2 \ln K = 2 \cdot \ln(1.3) = \text{small}$

d). D_0 when $\boxed{2 \cdot \underset{1}{\sigma^2} \cdot \ln(K) > 25.2} \Leftrightarrow \ln(K) > 12.6 \,\big|\, e$

$\Leftrightarrow e^{\ln K} > e^{12.6} \Leftrightarrow K > e^{12.6} = 296558$

$\Leftrightarrow \underbrace{K}_{\frac{P(H_0)}{P(H_1) = 1 - P(H_0)}} > 296558 \quad \Leftrightarrow \quad \frac{P(H_0)}{1 - P(H_0)} > 296558$

$\Leftrightarrow \quad P(H_0) > 296558 - 296558 \cdot P(H_0)$

$\Leftrightarrow \quad \boxed{P(H_0) > \frac{296558}{296559}} = 0.999996$

③ $$d(r, s_0)^2 \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)^2 + \cancel{\underbrace{2\sigma^2 \ln(K)}_{=0}}$$

M.L.

a). $s_0 = [2 \quad 2 \quad -2]$

$s_1 = [-2 \quad -2 \quad 2]$

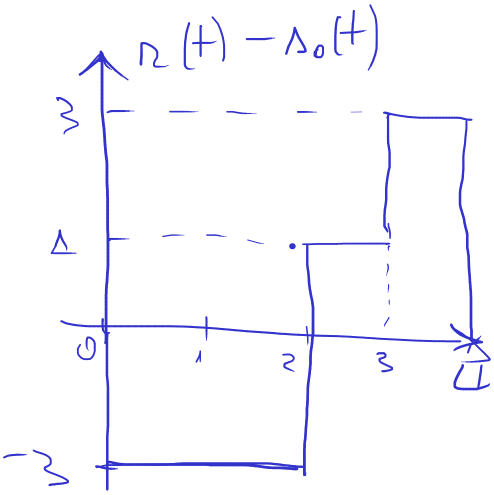
$r = [-1 \quad -1 \quad 1]$

$\sum (r_i - s_0(t_i))^2$

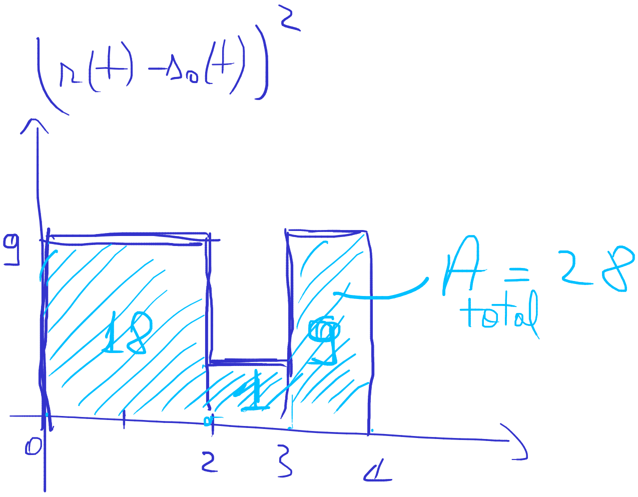
$$\left.\begin{aligned} d(r, s_0)^2 &= (-1-2)^2 + (-1-2)^2 + (1+2)^2 = 27 \\ d(r, s_1)^2 &= (-1+2)^2 + (-1+2)^2 + (1-2)^2 = 3 \end{aligned}\right\} \Rightarrow \boxed{D_1}$$

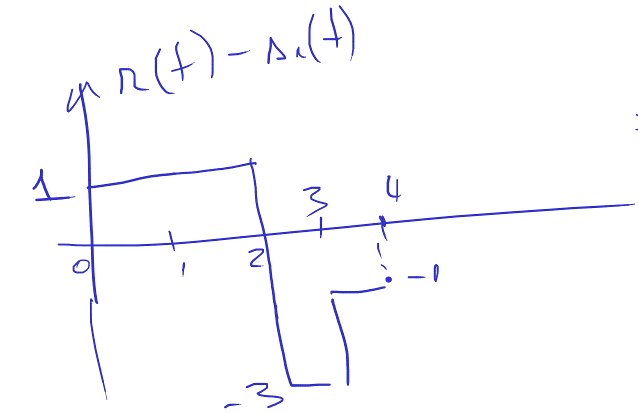
b). $$\left.\begin{aligned} d(r, s_0)^2 &= \int (r(t) - s_0(t))^2 \, dt = 28 \\ d(r, s_1)^2 &= \int (r(t) - s_1(t))^2 \, dt = 12 \end{aligned}\right\} \Rightarrow$$

$\boxed{D_1}$



$\Rightarrow$





$\Rightarrow$