

Seminar 7

ML, MAP and MMSE estimation

DEDP

1. We want to estimate a robot's true position Θ along a linear road.

We have one imprecise position sensor (e.g. GPS-like) which provides values affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 2)$.

We take one position reading from the sensor, and we obtain a position value $r_1 = 40$ meters.

From previous measurements, we know that the robot is located somewhere around position 35 meters, the position having a Gaussian distribution $\mathcal{N}(35, \sigma^2 = 2)$ (prior distribution).

- a. Estimate the true position using ML estimation
 - b. Estimate the true position using MAP estimation
 - c. Estimate the true position using MMSE estimators
 - d. What if the prior distribution is more imprecise and has $\sigma^2 = 20$?
 - e. What if the prior distribution is more precise and has $\sigma^2 = 0.2$?
2. Repeat Exercise 1, but instead of taking one position reading from the sensor, assume we take three separate readings, yielding positions $r_1 = 40$, $r_2 = 38.1$ and $r_3 = 39.2$
 - a. Estimate again the true position using ML, MAP, MMSE estimation
 - b. Does the prior distribution have a stronger or a weaker influence now?

Another example, identical to exercise 1, but with different text and values:

1. We want to estimate today's temperature accurately.

We have one thermometer which provides imprecise values, affected by Gaussian noise $\mathcal{N}(0, \sigma^2 = 2)$ (lousy thermometer).

We take one thermometer reading, with the value $r_1 = 0$ degrees.

From historical data, we know that this time of the year the temperature is around -5 degrees, being distributed with Gaussian distribution $\mathcal{N}(-5, \sigma^2 = 2)$.

- a. Estimate the true temperature using ML estimation
- b. Estimate the true temperature using MAP estimation
- c. Estimate the true temperature using MMSE estimators