# Signal detection with multiple samples

Laboratory 4, DEPI

### **Objective**

Visualize the signal detection rules for detection of signals based on multiple samples

#### Theoretical aspects

For detecting a constant signal with two possible levels B (hypothesis  $H_0$ ) and A (hypothesis  $H_1$ ), affected by Gaussian noise, based on multiple samples, the decision formula is:

$$\sum (r_i - B)^2 \underset{H_0}{\gtrless} \sum (r_i - A)^2 + 2\sigma^2 \ln K$$

.

For Maximum Likelihood criterion, K=1. In this case, the equation can be interpreted geometrically as **choosing minimum Euclidean distance from point**  $\mathbf{r}=[r_1,r_2,...r_N]$  **to points**  $\mathbf{B}=[B,B,...]$  **and**  $\mathbf{A}=[A,A,...]$ .

The Euclidian distance between two points  $\mathbf{x} = [x_1, x_2, ...x_N]$  and  $\mathbf{y} = [y_1, y_2, ...y_N]$  is

$$d(x,y) = \sqrt{\sum_{i} (x_i - y_i)^2}$$

.

The general formula thus becomes:

$$[d(r,B)]^2 \underset{H_0}{\overset{H_1}{\geq}} [d(r,A)]^2 + 2\sigma^2 \ln K$$

.

#### **Exercises**

- 1. Simulate detection of a constant signal with two levels B=0 and A=5, based on two samples, as follows:
  - Generate a vector data of 1000 values 0 or 1, with equal probability (hint: use rand() and compare to 0.5).
  - Generate a  $1000 \times 2$  matrix points of data samples defined as:
    - row i of points is (B, B) if data(i) is 0, or
    - row i of points is (A, A) if data(i) is 1.
  - Add over it a random noise with normal distribution  $\mathcal{N}(0, \sigma^2 = 2)$  (use a noise matrix of same size  $1000 \times 2$ ). The result should be saved as the matrix received.
  - Implement the Maximum Likelihood decision rule for each row *i* of the received samples. The result (0 or 1) should be saved as vector decision.
  - Compare the decision result with the original data vector, and classify the result in a 1000-long vector result with the following values:
    - result(i) = 0 if for bit i there was a correct rejection
    - result(i) = 1 if for bit *i* there was a false alarm
    - result(i) = 2 if for bit i there was a miss
    - result(i) = 3 if for bit *i* there was a correct detection
  - Generate a **scatter plot** displaying the received data samples with different coloring of the decisions (*Hint*: check Matlab function **scatter()**).
  - What are the decision regions? What is the decision rule, geometrically?
- 2. Repeat Exercise 1 for Minimum Probability of Error criterion with probabilities  $P(H_0) = 2/3$  and  $P(H_1) = 1/3$ 
  - Change the threshold value for comparing rand() to generate 0 with probability 2/3 and 1 with probability 1/3.
  - Modify the detection rule taking into account  $K = \frac{P(H_0)}{P(H_1)}$ .
  - Visualize the data. How does it look like?
- 3. Repeat Exercise 1 for four values of K: K = 1, K = 5, K = 25, K = 125. Display the 4 scatter plots as subfigures of a figure window, arranged  $2 \times 2$ .
- 4. Repeat Exercise 2 for 3 samples.
  - The matrix's sizes should be  $1000 \times 3$ .
  - Detection rule should consider all three samples.

## Final questions

1. In a practical scenario, what is the disadvantage of using multiple samples for detection, compared to just 1?