Seminar 6

1 $x = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

$v_1, \dots v_{10}$

A
- $d(x, v_1) = \sqrt{(-2-2)^2 + (5-(-4))^2} = \sqrt{16+81} = \sqrt{97}$
- $d(x, v_2) = \sqrt{(-2-1)^2 + (5+5)^2} = \sqrt{109}$
- $d(x, v_3) = \sqrt{0+1} = \sqrt{1}$ ✓
- $d(x, v_4) = \sqrt{2}$ ✓
- $d(x, v_5) = \sqrt{116}$

B
- $d(x, v_6) = \sqrt{41}$ ✓
- $d(x, v_7) = \sqrt{17}$ ✓
- $d(x, v_8) = \sqrt{68}$ ✓
- $d(x, v_9) = \sqrt{26}$ ✓
- $d(x, v_{10}) = \sqrt{4}$ ✓

$K = 1$: v_3 ⇒ A

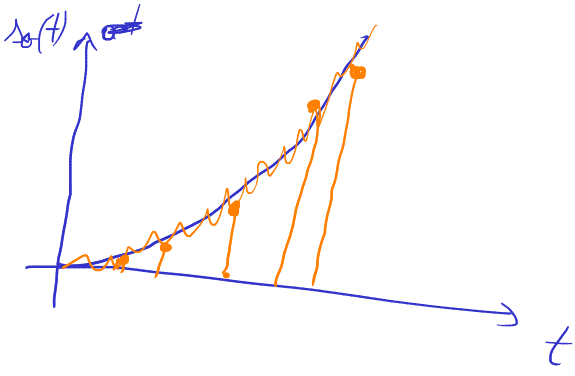
$K = 3$: v_3 v_4 v_{10} ⇒ A
A A B

$K = 5$: v_3 v_4 v_{10} v_7 v_9 ⇒ B
A A B B B

$K = 7$: v_3 v_4 v_{10} v_7 v_9 v_6 v_8 ⇒ B
A A B B B B B

$K = 9$: v_3 v_4 v_{10} v_7 v_9 v_6 v_8 v_1 v_2 ⇒ B
A A B B B B B A A

2 $r(t) = \underbrace{a \cdot t^2}_{s_\theta(t)} + \text{noise}$



$t = [1, 2, 3, 4, 5]$

$r = [1.2 \quad 3.7 \quad 8.5 \quad 18 \quad 25.8]$

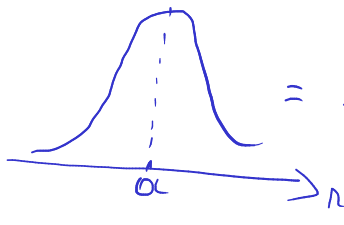
noise is $N(\mu = 0, \sigma^2 = 1)$

$\hat{a}_{ML} = ?$

$t = [1 \quad 2 \quad 3 \quad 4 \quad 5]$

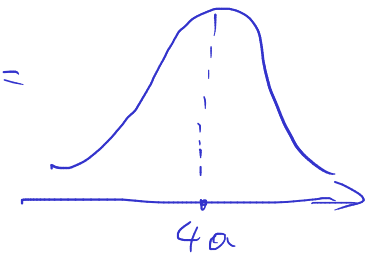
$r = [\underset{r_1}{1.2} \quad \underset{r_2}{3.7} \quad \underset{r_3}{8.5} \quad \underset{r_4}{18} \quad \underset{r_5}{25.8}]$

$s_\theta = [a \quad 4a \quad 9a \quad 16a \quad 25a]$

$w(r_1 | a) =$ 

$= \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r_1 - a)^2}{2\sigma^2}}$

$$w(r_2|a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_2-4a)^2}{2\sigma^2}}$$



$$w(r_3|a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_3-9a)^2}{2\sigma^2}}$$

$$w(r_4|a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_4-16a)^2}{2\sigma^2}}$$

$$w(r_5|a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_5-25a)^2}{2\sigma^2}}$$

Want minimum

$$D \ldots + (25.8-25a)^2 + \ldots$$

$$\ldots + (18-16a)^2 + \ldots$$

$$(1.2-a)^2 + (3.7-4a)^2 + (8.5-9a)^2 + \ldots$$

$$w(\underline{r}|a) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^5 \cdot e^{-\frac{(1.2-a)^2+(3.7-4a)^2+(8.5-9a)^2+\ldots}{2\sigma^2}}$$

Want maximum!

$$\frac{dD}{da} = 0$$

$$2(1.2-a)\cdot(-1) + 2(3.7-4a)(-4) + 2(8.5-9a)(-9) + 2(18-16a)(-16) + 2(25.8-25a)(-25) = 0$$

$$a - 1.2 + 16a - 14.8 + 81a - 76.5 + 256a - 288 + 625a - 645 = 0$$

$$979a - 1025.5 = 0 \Rightarrow \hat{a}_{ML} = \frac{1025.5}{979} = 1.04$$

(3)

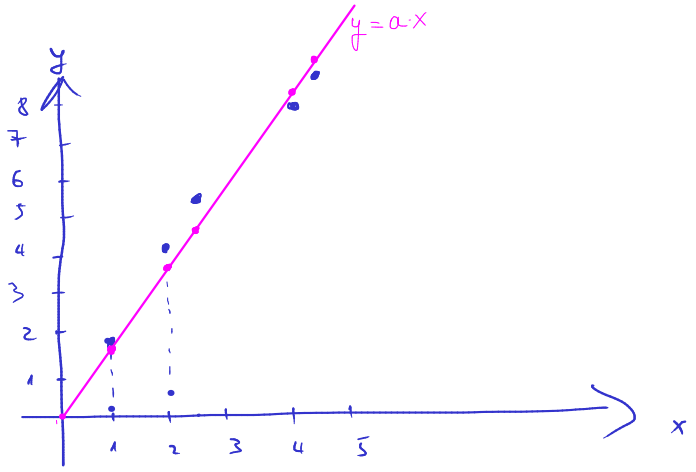
noise is $\mathcal{N}(\mu = 0, \sigma^2 = 1)$

$R = \underbrace{a \cdot x}_{\Delta_\theta(t)} + \text{noise}$

$t = X = [1 \quad 2 \quad 2.5 \quad 4 \quad 4.3]$

$y = R = [1.8 \quad 4.1 \quad 5.1 \quad 7.9 \quad 8.5]$

$\Delta_\theta = [a \quad 2a \quad 2.5a \quad 4a \quad 4.3a]$



$$D = d(R, \Delta_\theta)^2 = (1.8-a)^2 + (4.1-2a)^2 + (5.1-2.5a)^2 + (7.9-4a)^2 + (8.5-4.3a)^2$$

$$\frac{dD}{da} = 2(1.8-a)(-1) + 2(4.1-2a)(-2) + 2(5.1-2.5a)(-2.5) + 2(7.9-4a)(-4) + 2(8.5-4.3a)(-4.3) = 0$$

$$a - 1.8 + 4a - 8.2 + 6.25a - 12.75 + 16a - 31.6 + 18.49a - 36.55 = 0$$

$$\Leftrightarrow \quad 45.75\,a = 90.9 \quad \Rightarrow \quad \hat{a}_{ML} = 1.98$$