$$\frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Seminar 3

(1)

$\Lambda_0(t) = -1$

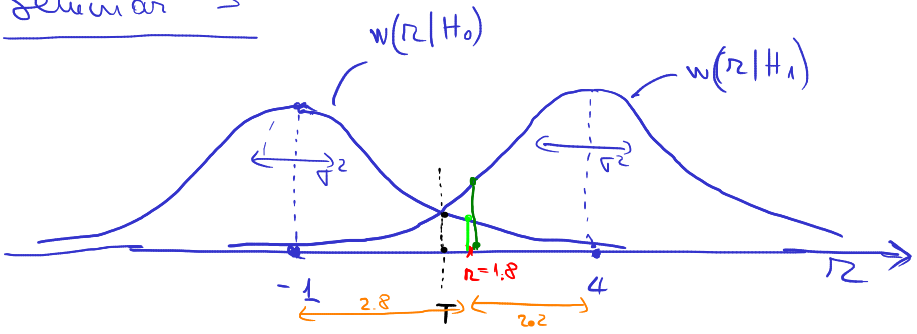
$\Lambda_1(t) = 4$

$N(\mu=0, \sigma^2=4)$

$r = r(t_0) = 1.8$

$t_0 = 0.75$

a)



$$w(r|H_0) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r+1)^2}{8}}$$

$$w(r|H_1) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(r-4)^2}{8}}$$

b). $r = 1.8$ $\quad d(r,-1) = 2.8$, $d(r,4) = 2.2$ $\Rightarrow D_1$

OR: from the graphic $w(r|H_1)\big|_{r=1.8} > w(r|H_0)\big|_{r=1.8} \Rightarrow D_1$ for r

OR

$$w(r|H_1)\Big|_{r=1.8} = \frac{1}{2\sqrt{2\pi}} e^{-\frac{2.2^2}{8}} = 0.108$$

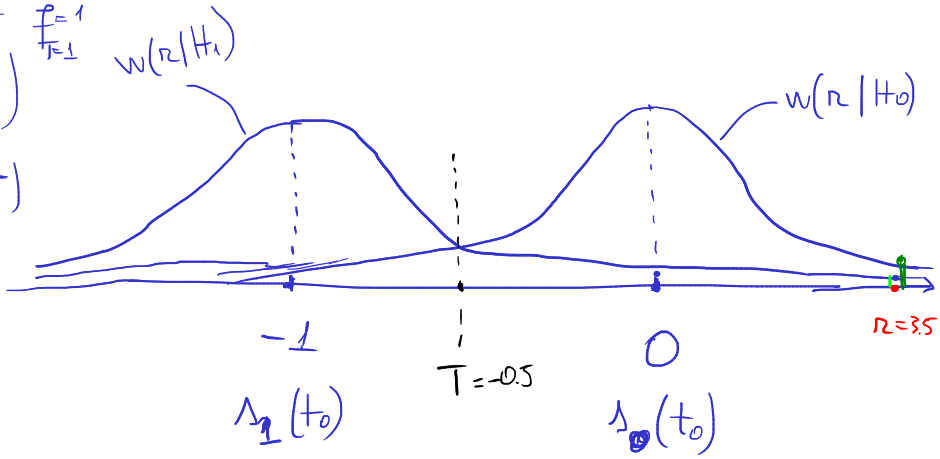
$$w(r|H_0)\Big|_{r=1.8} = \frac{1}{2\sqrt{2\pi}} e^{-\frac{2.8^2}{8}} = 0.074$$

$\Rightarrow$ $\boxed{D_1}$ (largest value)

(2) H_0: $s_0(t) = \cos(2\pi t)$ $\quad 2\pi f t$, $f = \frac{1}{T} = 1$

H_1: $s_1(t) = \sin(2\pi t)$

$N(\mu = 0, \sigma^2 = 4)$



$t_0 = 0.75$

$r = r(t_0) = 3.5$

$\Lambda_1(t_0)$ $\qquad$ $\Lambda_0(t_0)$

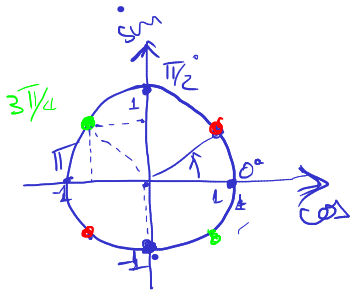
$w(r|H_0) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{r^2}{8}}$

$w(r|H_1) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(r+1)^2}{8}}$

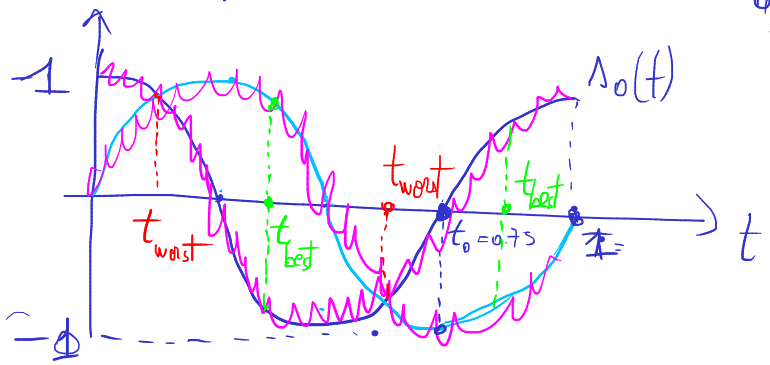
H_0: $\cos(2\pi \cdot 0.75) = \cos(1.5\pi) = 0$

H_1: $\sin(2\pi \cdot 0.75) = \sin(1.5\pi) = -1$

$w(r|H_0)\big|_{3.5} > w(r|H_1)\big|_{3.5} \Rightarrow \Lambda_0$



d).



$s_0(t) = s_1(t)$

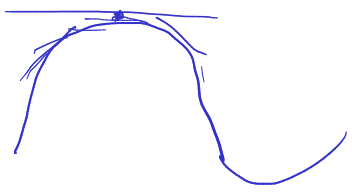
$\cos(2\pi t) = \sin(2\pi t)$

$\pi/4$ $\qquad$ $\pi/2$

$2\pi t = \pi/4 \Leftrightarrow \boxed{t_{worst} = \frac{1}{8}}$

$f(t)$

c). $\max \left| \cos(2\pi t) - \sin(2\pi t) \right|$



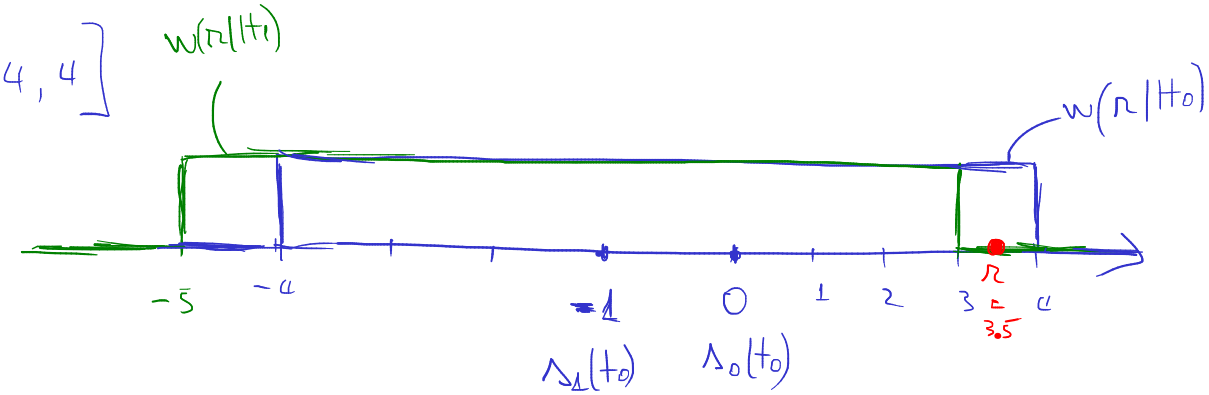
$f'(t) = 0$

$f''(t) < 0$

$t_{best} = \left| \cos(3\pi/4) - \sin(3\pi/4) \right| = \max$

$2\pi t = 3\pi/4 \Leftrightarrow t = \frac{3}{8}$, OR $\frac{7}{8}$

e). $U[-4,4]$



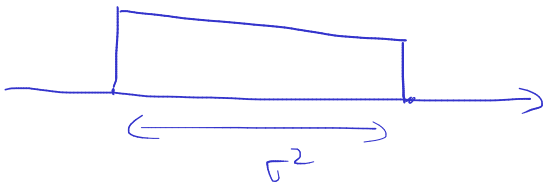
$$w(r|H_0) = \begin{cases} \frac{1}{8}, & r \in [-4,4] \\ 0, & r \notin [-4,4] \end{cases}$$

$$w(r|H_1) = \begin{cases} \frac{1}{8}, & r \in [-5,3] \\ 0, & \text{elsewhere} \end{cases}$$

$r = 3.5$ $\quad w(r|H_0)\Big|_{r=3.5} = \frac{1}{8} \Rightarrow \boxed{D_0}$

$w(r|H_1)\Big|_{r=3.5} = 0$

~~g~~). $U[-4,4]$



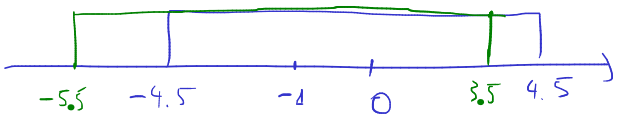
$U[-4.1, 4.1]$

$\vdots$

$U[-4.5, 4.5] \Rightarrow$

$(=)$

Limit case:



$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 w(x)\,dx = \int_{-4.5}^{4.5} (x-0)^2 \cdot \frac{1}{9}\,dx = \frac{1}{9} \cdot \frac{x^3}{3}\Big|_{-4.5}^{4.5}$$

$$= \frac{1}{9} \cdot \frac{2 \cdot 4.5^3}{3} = 6.75$$