Seminar 6
k-NN + ML estimation
DEDP

1. Consider the k-NN algorithm with the following training set, composed of 5 vectors of class A and another 5 vectors from class B:

   - Class A:

   $$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \ \mathbf{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \ \mathbf{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \ \mathbf{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

   - Class B:

   $$\mathbf{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \ \mathbf{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \ \mathbf{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \ \mathbf{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \ \mathbf{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

   Compute the class of the vector $\mathbf{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ using the k-NN algorithm, with $k = 1$, $k = 3$, $k = 5$, $k = 7$ and $k = 9$

2. A received signal $r(t) = a \cdot t^2 + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Estimate the parameter a.

   a. use Maximum Likelihood (ML) estimation

3. Fit a linear function $y = ax$ (i.e. estimate a) through the following data points $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$, assuming the noise is $\mathcal{N}(0, \sigma^2 = 1)$

   a. use Maximum Likelihood (ML) estimation