Seminar 3
Maximum Likelihood criterion
DEDP

1. Consider the problem of deciding between two possible signals, $s_0(t) = cos(2\pi t)$ and $s_1(t) = sin(2\pi t)$. The signals are affected by AWGN with distribution $\mathcal{N}$ ($\mu = 0, \sigma^2 = 4$). The receiver takes one sample, at time $t_0 = 0.75$, and the obtained value is $r = 3.5$.
    a. Write the expressions of the two conditional distributions of the sample, and sketch them
    b. What is the decision taken with the Maximum Likelihood criterion?
    c. What is the **best** moment t_0 for sampling, in order to best discriminate between the signals? Justify.
    d. What is the **worst** moment t_0 for sampling, in order to discriminate between the signals? Justify.
    e. Repeat a) and b) in case the noise has uniform distribution $\mathcal{U}[-4, 4]$. Is there a problem with the decision?
    f. What is the maximum variance of a uniform noise, with zero-mean, in order to still be able to take a decision with the ML criterion for $r = 3.5$?

2. A signal can have four possible values: -6, -2, 2, 6. Each value lasts for 1 second. The signal is affected by white noise with normal distribution. The receiver takes 1 sample per second. Using ML criterion, decide what signal has been transmitted, if the received samples are:
$$4, 6.6, -5.2, 1.1, 0.3, -1.5, 7, -7, 4.4$$

3. A signal can have two possible values, $s_0 = -3$ or $s_1 = 3$. The signal is affected by gaussian noise with distribution $\mathcal{N}(0, 1)$. The receiver performs ML decision based on a single sample.
    a. What is the maximum variance σ^2 of the noise, such that the probability of wrongly detecting s_1 if the true signal is s_0 is at most 10^{-3}
    b. If the noise variance is $\sigma^2 = 0.5$, what is the minimum gap between the two signal levels $(s_1 - s_0)$ such that the probability of correct detection if the true signal is s_1 is at least 0.9999?