DEDP Exam 2018-2019

No.1

Exercises (18p)

1. Consider a random variable A with the uniform distribution $\mathcal{U}[-3, 5]$.
   a. (1p) Draw the density function.
   b. (1p) Compute the probability that A is smaller than 3
   c. (2p) Compute the average squared value $\overline{A^2}$;
   d. (2p) Draw the cumulative distribution function (CDF) of A, $F_A(x)$. Justify the shape.

2. Consider detection between two possible signals, $s_0(t) = -4$ and $s_1(t) = 2$. The signals are affected by AWGN with distribution $\mathcal{N}\left(\mu = 0, \sigma^2 = 3\right)$. The probabilities of the two hypotheses are $P(H_0) = 3/4$, $P(H_1) = 1/4$. The receiver takes one sample, at time $t_0 = 3$, and the obtained value is $r = -0.5$.
   a. (1p) What are the decision regions R_0 and R_1 for the Maximum Likelihood criterion? Justify.
   b. (3p) Compute the probability of false alarm, in case of the Maximum Likelihood criterion.
   c. (2p) What is the decision taken with the Minimum Probability of Error criterion?

3. Consider detecting a signal $s(t) = 2\cos(4\pi t)$ that can be present (hypothesis H_1) or absent (signal is equal to zero, hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 5)$. The receiver takes 3 samples at times $t_0 = 0$, $t_1 = 1$ and $t_2 = 2$, with values $r_0 = 0.6$, $r_1 = -0.6$ and $r_2 = -0.3$.
   a. (2p) What is the decision according to Maximum Likelihood criterion?

4. (4p) Consider the received signal $r(t) = \underbrace{At + 3}_{s(t)} + noise$, which is sampled at time moments $t_i = [1, 2, 3]$, and the values are $r_i = [5.9, 7.8, 8]$. The noise has Gaussian distribution $\mathcal{N}(0, \sigma^2 = 4)$. Estimate the unknown parameter A using Maximum Likelihood estimation.

Known:

- $F(x) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$

Theory (14p)

1. (3p) Prove the relation $\sigma^2 = E\{X^2\} - (E\{X\})^2$.
2. (2p) Suppose we have $R_{ff}(2) > 0$ for a stationary random process $f(t)$. What does this tell us about two samples which are 2 seconds apart? Explain why.
3. (2p) Define the Neymar-Pearson decision criterion.
4. (3p) Prove that the output of a **matched filter**, taken at the end of the input signal, is equal to the inner product of the signals. It is known: the convolution of two signals $x[n]$ and $y[n]$ is defined as $\sum_k x[k]y[n-k]$.
5. (2p) Consider signal detection, with the probability of the two hypotheses being $P(H_0) = \frac{1}{2}$ and $P(H_1) = \frac{1}{2}$. How is the **Maximum Likelihood** decision criterion affected when $P(H_0)$ increases and $P(H_1)$ decreases? Explain why.
   a. Decision D_0 becomes more likely, decision D_1 becomes less likely
   b. Decision D_1 becomes more likely, decision D_0 becomes less likely
   c. Decisions are not affected
6. (2p) What is the relation between ML estimator and MAP estimator? Argue that one of them is a particular case of the other.

Notes: 35p total, solve 30p for grade 10. 3p are awarded from start. Time available: 2h