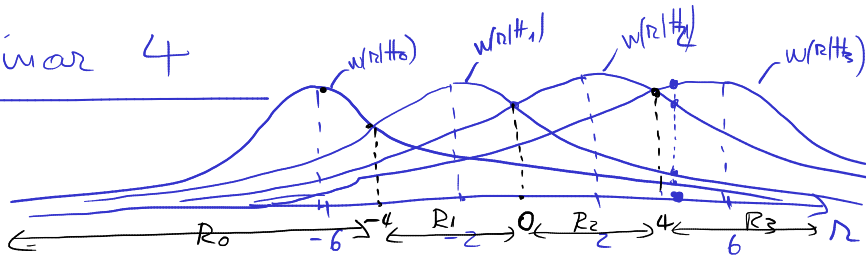
Seminar 4



① $-6 \quad -2 \quad 2 \quad 6$

~~4~~ 4.1 $\quad 6.6 \quad -5.2 \quad 1.1 \quad 0.3 \quad -1.5 \quad 7 \quad -7 \quad 4.4$

ML

$6 \quad 6 \quad -6 \quad +2 \quad 2 \quad -2 \quad 6 \quad -6 \quad 6$

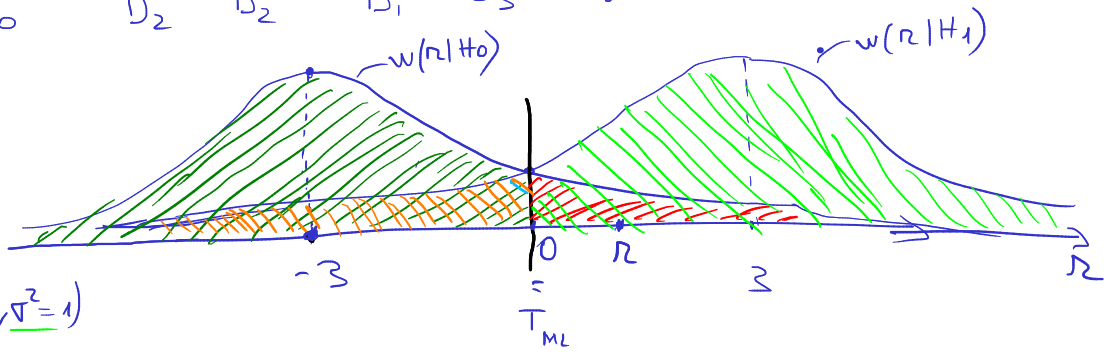
$D_3 \quad D_3 \quad D_0 \quad D_2 \quad D_2 \quad D_1 \quad D_3 \quad D_0 \quad D_3$

② $\Lambda_0 = -3$

$\Lambda_1 = 3$

r

ML noise $= N(\mu = 0, \sigma^2 = 1)$



a) False Alarm : $D_1 \cap H_0$

Answer: $\boxed{r > 0}$ $\Rightarrow D_1 \Rightarrow$ false alarm

b) $P(D_0 | H_0) = \int_{-\infty}^{0} w(r|H_0)\,dr = 1 - 0.001 = 0.999$ (OR: $F(0) - F(-\infty) = \frac{1}{2}\left(1 + \text{erf}\frac{3}{\sqrt{2}}\right) = 0.999$)

$P(D_1 | H_0) = \int_{0}^{\infty} w(r|H_0) = F(\infty) - F(0) = 1 - \frac{1}{2}\left(1 + \text{erf}\left(\frac{0-(-3)}{1\cdot\sqrt{2}}\right)\right) = 0.001$ ($\mu = -3$)

$P(D_0 | H_1) = \int_{-\infty}^{0} w(r|H_1) = F(0) - F(-\infty) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{0-3}{1\cdot\sqrt{2}}\right)\right) = 0.001$

$P(D_1 | H_1) = \int_{0}^{\infty} w(r|H_1) = F(\infty) - F(0) = \ldots = 1 - 0.001 = 0.999$

$F(x) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$

$P(D_0 \cap H_0) =$

$P(D_1 \cap H_0) = P(D_1|H_0) \cdot P(H_0) = \ldots$

$=$

$=$

(3) $C_{00} = 0$

$C_{01} = 100$

$C_{10} = 10$

$C_{11} = -100$

bruit $N(\mu=0, \sigma^2=1)$

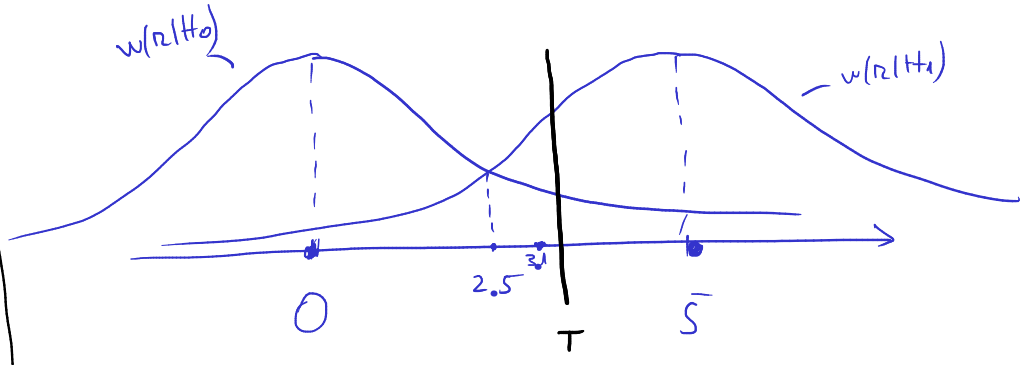
$\begin{cases} P(H_0) = 2/3 \\ P(H_1) = 1/3 \end{cases}$

a) $r = 3.1$, MPE ?

b) $r = 3.1$, MR ?

c) R_0, R_1

d) $U[-3,3]$



a) M.L. : $\Rightarrow D_1$

M.P.E. :

$$(r - s_0(t_0))^2 \underset{H_0}{\overset{H_1}{\gtrless}} (r - s_1(t_0))^2 + 2\sigma^2 \ln \frac{P(H_0)}{P(H_1)}$$

$$\underbrace{(3.1 - 0)^2}_{9.61} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{(3.1-5)^2}_{3.61} + \underbrace{2 \cdot 1 \cdot \ln(2)}_{1.38}$$

$$\underbrace{\qquad\qquad}_{5.19}$$

$\Downarrow$

D_1

$$r \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{\frac{s_0(t_0) + s_1(t_0)}{2}}_{\text{middle } T_{ML}} + \frac{\sigma^2}{s_1(t_0) - s_0(t_0)} \cdot \ln\left(\frac{P(H_0)}{P(H_1)}\right)$$

T

$$3.1 \gtrless \underbrace{\frac{0+5}{2}}_{2.5} + \underbrace{\frac{1}{5-0} \ln(2)}_{0.13}$$

$$3.1 \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{2.63}_{T_{MPE}} \Rightarrow \boxed{D_1}$$

b). MR:

$$\Lambda \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{\frac{\Lambda_0(t_0) + \Lambda_1(t_0)}{2} + \frac{\sigma^2}{\Lambda_1(t_0) - \Lambda_0(t_0)} \cdot \ln\left(\frac{(C_{10} - C_{00}) \cdot P(H_0)}{(C_{01} - C_{11}) \cdot P(H_1)}\right)}_{T_{MR}}$$

3.1 $\quad \underbrace{\qquad}_{2.5} + \frac{1}{5-0} \cdot \ln\left(\frac{10-0}{100-(-100)} \cdot 2\right)$

$$2.5 + \underbrace{\frac{1}{5} \cdot \ln\left(\frac{1}{10}\right)}_{-0.46}$$

$$\Lambda \underset{H_0}{\overset{H_1}{\gtrless}} 2.04 \quad \Rightarrow \boxed{D_1}$$
3.1 $\qquad T_{MR}$

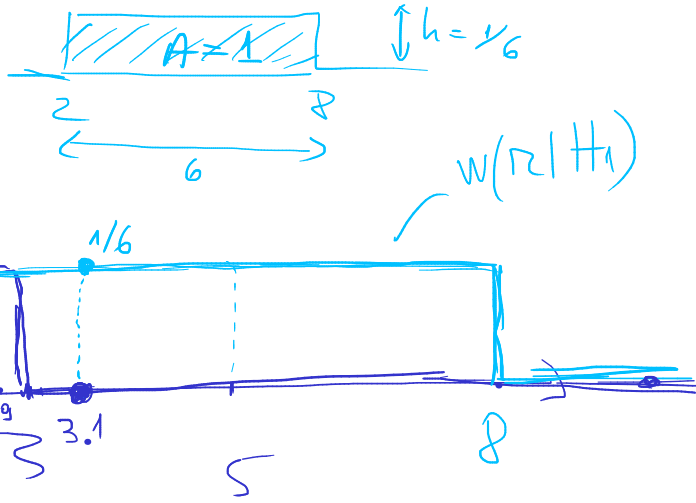
c). M.P.E.: $R_0 = (-\infty, 2.63)$

$R_1 = (2.63, \infty)$

M.R.: $R_0 = (-\infty, 2.04)$

$R_1 = (2.04, \infty)$

d). noise is $\mathcal{U}[-3, 3]$



$r = 3.1$

M.P.E.

$$\Rightarrow \quad \frac{w(r|H_1)}{w(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)}{P(H_1)}$$

$r = 3.1 \Rightarrow \frac{1.6}{0} \underset{H_0}{\overset{H_1}{\gtrless}} 2 \Rightarrow D_1$

$r = 2.9 \Rightarrow \frac{1.6}{1.6} \underset{H_0}{\overset{H_1}{\gtrless}} 2 \Rightarrow D_0$

M.R.

$$\frac{w(r|H_1)}{w(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{(C_{10} - C_{00}) \cdot P(H_0)}{(C_{01} - C_{11}) \cdot P(H_0)}$$

$r = 3.1 \quad \frac{1.6}{0} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{10-0}{100-(-100)} \cdot 2 \Rightarrow D_1$

$r = 2.9 \quad \frac{1.6}{1.6} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{10-0}{100-(-100)} \cdot 2 \Rightarrow D_1$

$1 \qquad 1/10$

R_0, R_1 :

$T_{MPE} =$

$(-3, 2) : D_0$
$(2, 3) : D_0$
$(3, 8) : D_1$

$R_0 = (-3, 3)$
$R_1 = (3, 8)$

$T_{MR} =$

$(-3, 2) : D_0$
$(2, 3) : D_1$
$(3, 8) : D_1$

$R_0 = (-3, 2)$
$R_1 = (2, 8)$