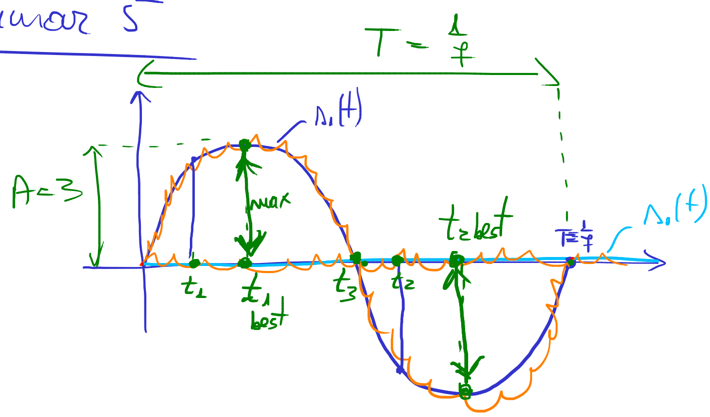
Seminar 5



(1) $H_1: s_1(t) = 3 \cdot \sin(2\pi f_1 t)$

$H_0: s_0(t) = 0$

$r = [1.1 \quad 4.4]$

$t_1 = \frac{0.125}{f_1}$

$t_2 = \frac{0.625}{f_1}$

$\frac{1}{f_1} = T$

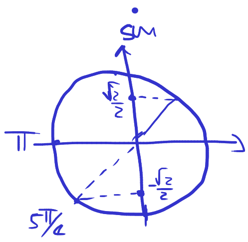
$D = ?$ M.L.

$H_0: \rightarrow [0 \quad 0 \quad 0] = s_0$

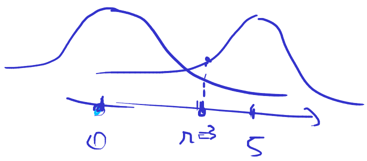
$H_1: \rightarrow \left[\underbrace{\frac{3\sqrt{2}}{2}}_{2.12} \quad \underbrace{-\frac{3\sqrt{2}}{2}}_{-2.12} \quad 0\right] = s_1$

$s_1(t_1) = 3 \cdot \sin\left(2\pi \cancel{f_1} \cdot \frac{0.125}{\cancel{f_1}}\right) = 3 \cdot \sin(\underbrace{0.25\pi}_{\pi/4}) = \frac{3\sqrt{2}}{2}$

$s_1(t_2) = 3 \cdot \sin\left(2\pi \cancel{f_1} \cdot \frac{0.625}{\cancel{f_1}}\right) = 3 \cdot \sin(\underbrace{1.25\pi}_{5\pi/4}) = -\frac{3\sqrt{2}}{2}$



Gaussian noise:



$d(r, s_0) \underset{H_0}{\overset{H_1}{\gtrless}} d(r, s_1)$

$d(a, b) = \sqrt{(a_1-b_1)^2 + \ldots + (a_N-b_N)^2}$

$d(r, s_0) = \sqrt{(1.1-0)^2 + (4.4-0)^2} = 4.53$

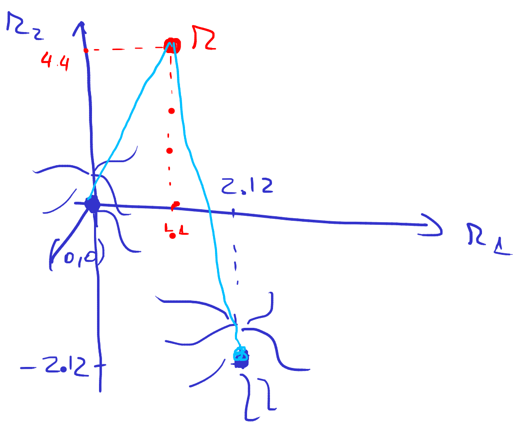
$d(r, s_1) = \sqrt{(1.1-2.12)^2 + (4.4+2.12)^2} = 6.59$

$\Rightarrow d(r, s_0) < d(r, s_1)$

$\Rightarrow \boxed{D_0}$

b). t_1 best $= 0.25 \cdot \frac{1}{f}$

t_2 best $= 0.75 \cdot \frac{1}{f}$



c)

d). $\boxed{d(r, s_0)^2 \gtrless d(r, s_1)^2 + 2\sigma^2 \ln K}$,

$K = \begin{cases} 1, & \text{M.L.} \\ \frac{P(H_0)}{P(H_1)}, & \text{M.P.E.} \\ C_{10}-C_{00} \ldots, & \text{M.R.} \end{cases}$

c). $d(r,s_0)^2 = 4.53^2$

$d(r,s_1)^2 = 6.59^2$

$$\underbrace{4.53^2}_{} \gtrless \underbrace{6.59^2}_{} + \underbrace{2\cdot\sigma^2\cdot\ln K}_{}$$

c). $K = 2$ $\qquad \underbrace{2\cdot 1\cdot \ln(2)}_{>0}$

$$4.53^2 < 6.59^2 + 2\cdot 1\cdot \ln(2) \Rightarrow s_0$$

d). $K = \frac{10-0}{20-5}\cdot\frac{2/3}{1/3} = \frac{20}{15}$

$$4.53^2 < 6.59^2 + 2\cdot 1\cdot \ln\frac{20}{15} \Rightarrow s_0$$

(2) $r = [1.1 \quad 4.4 \quad 3.7 \quad 4.1 \quad 3.8]$

$s_0(t) = 0$ $\qquad H_0 : s_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]$

$s_1(t) = 6$ $\qquad H_1 : s_1 = [6 \quad 6 \quad 6 \quad 6 \quad 6]$

$\boxed{\text{Gaussian noise}}$: $\qquad r = [1.1 \quad 4.4 \quad 3.7 \quad 4.1 \quad 3.8]$

$$\underbrace{d(r,s_0)^2}_{65.51} \underset{H_0}{\overset{H_1}{\gtrless}} \underbrace{d(r,s_1)^2}_{40.31} + 2\underbrace{\sigma}_{1}{}^2\cdot\ln K$$

$d(r,s_0)^2 = (1.1-0)^2 + (4.4-0)^2 + 3.7^2 + 4.1^2 + 3.8^2 = 65.51$

$d(r,s_1)^2 = (1.1-6)^2 + (4.4-6)^2 + (3.7-6)^2 + (4.1-6)^2 + (3.8-6)^2 = 40.31$

a) M.L.: $K = 1 \Rightarrow s_1$

b). M.P.E: $K = \frac{P(H_0)}{P(H_1)} = 2 \Rightarrow$ $\quad 2\sigma^2 \ln K = 2\cdot 1\cdot\ln 2 = 1.38$

$65.51 \overset{H_1}{>} 40.31 + 1.38 \Rightarrow \boxed{s_1}$

c) M.R. : $K = \frac{(C_{10} - C_{00})}{(C_{01} - C_{11})} \cdot \frac{P(H_0)}{P(H_1)} = \frac{10}{15} \cdot 2 = \frac{20}{15}$

$$2 \cdot \sigma^2 \cdot \ln K = \cancel{2 \cdot 1} = 0.57$$

$$65.51 \overset{H_1}{>} 40.31 + 0.57 \Rightarrow \boxed{D_1}$$

d) M.P.E. : $K = \frac{P(H_0)}{P(H_1)} = \frac{P(H_0)}{1 - P(H_0)}$

Decision is D_0 when $2 \cdot \underbrace{\sigma^2}_{1} \cdot \ln(K) > \underbrace{(65.51 - 40.31)}_{25.2}$

$\Leftrightarrow$

$$\ln K > \frac{25.2}{2} = 12.6 \Leftrightarrow \Big| e^{\cdot}$$

$$\Leftrightarrow \underbrace{e^{\ln K}}_{K} > \underbrace{e^{12.6}} \Leftrightarrow K > 296558 \Leftrightarrow$$

$$\frac{P(H_0)}{1 - P(H_0)} > 296558 \Leftrightarrow$$

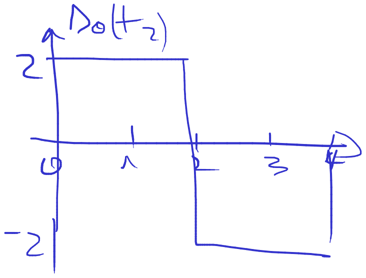
$$P(H_0) > 296558 - 296558 \cdot P(H_0)$$

$\Leftrightarrow$

$$296559 \cdot P(H_0) > 296558 \Leftrightarrow$$

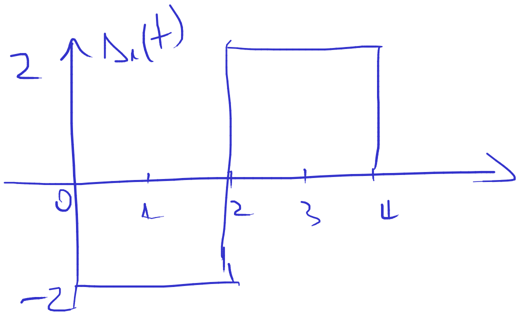
$$\Leftrightarrow \quad P(H_0) > \frac{296558}{296559} = 0.999996$$

3 $\quad s_0(t) \longrightarrow$



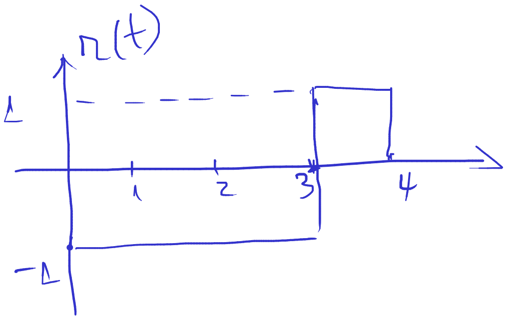
H_0

$s_1(t)$



H_1

$r(t)$



a) $t_1 = 0.5 \qquad s_0 = [2 \quad 2 \quad -2]$

$t_2 = 1.5 \qquad s_1 = [-2 \quad -2 \quad 2]$

$t_3 = 3.5 \qquad r = [-1 \quad -1 \quad 1]$

$$d(r, s_0)^2 = 9 + 9 + 9 = 27$$

$$d(r, s_1)^2 = 1 + 1 + 1 = 3 \implies d(r, s_0) > d(r, s_1)$$

$$\boxed{s_1}$$

b). $$d(r, s_0)^2 = \int \left(r(t) - s_0(t)\right)^2 dt$$

$$d(r, s_1)^2 = \int \left(r(t) - s_1(t)\right)^2 dt$$

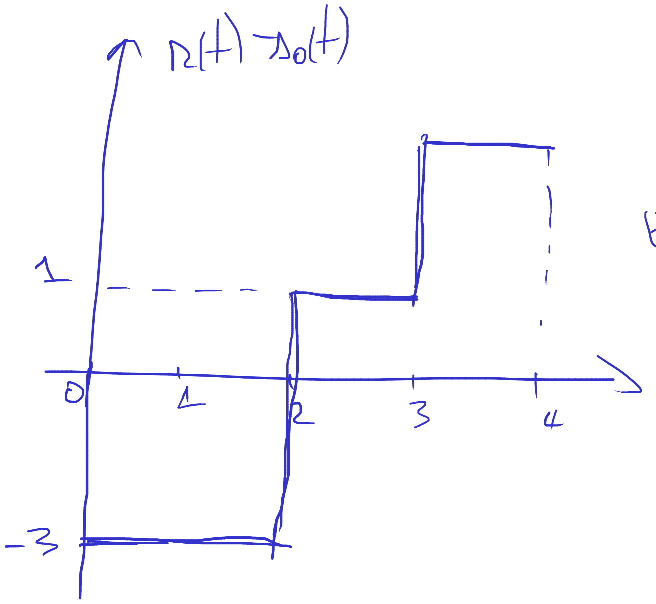
In General:

$a(t)$

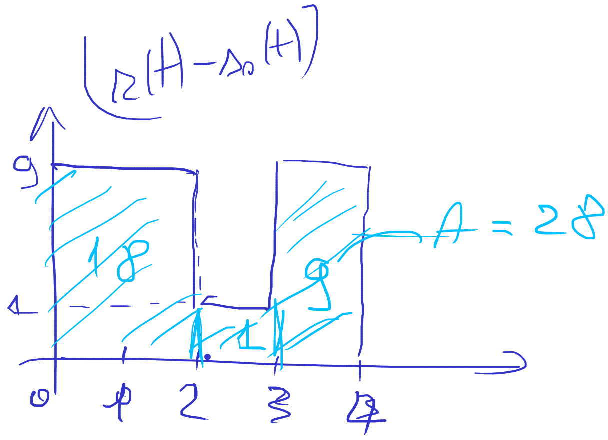
$b(t)$

$$d(a,b) = \sqrt{\int (a(t)-b(t))^2 \, dt}$$

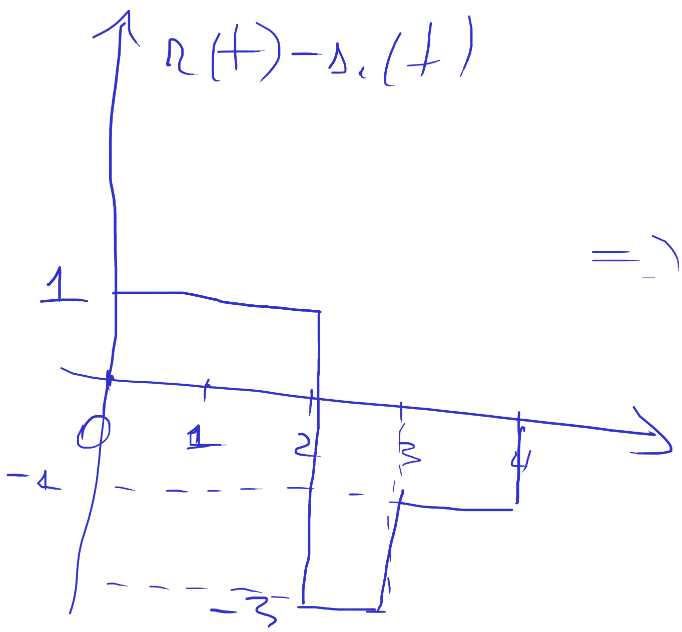
$$d(r,s_0)^2 = \int (\underbrace{r(t)-s_0(t)}_{e_1(t)})^2 \, dt = 28$$



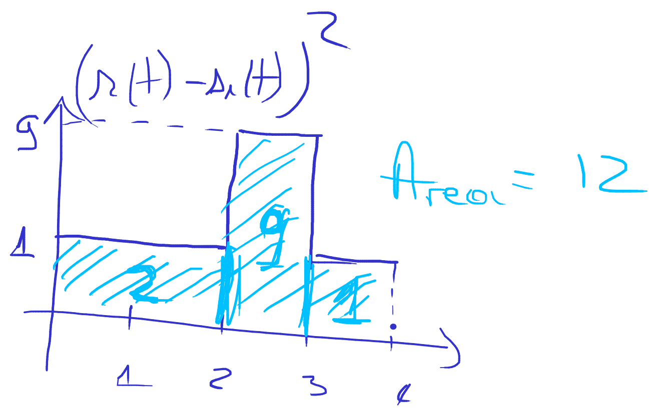
$\Rightarrow$



$$d(r,s_1)^2 = \int (r(t)-s_1(t))^2 \, dt = 12$$



$\Rightarrow$



$$d(r,s_1)^2 < d(r,s_0)^2$$

$$\Rightarrow \boxed{D_1}$$