Seminar 4
Decision criteria
DEDP

1. A vehicle airbag system detects a crash by evaluating a sensor which provides two values: $s_0(t) = 0$ (no crash) or $s_1(t) = 5$ (crashing) The signal is affected by gaussian noise $\mathcal{N}$ ($\mu = 0, \sigma^2 = 1$). The costs of the scenarios are: $C_{00} = 0$, $C_{01} = 100$, $C_{10} = 10$, $C_{11} = -100$
   a. Find the decision taken based on a sample $r = 3.1$
   b. Find the decision regions R_0 and R_1.

2. An information source provides two messages with probabilities $p(a_0) = \frac{2}{3}$ and $p(a_1) = \frac{1}{3}$. The messages are encoded as constant signals with values -5 (a_0) and 5 (a_1). The signals are affected by noise with uniform distribution $U[-5, 5]$. The receiver takes one sample r.
   a. Find the decision regions according to the Neyman-Pearson criterion, considering $P_{fa} \leq 10^{-2}$
   b. What is the probability of correct detection, in this case?

3. A signal can have two values, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 5 samples with values $\{1.1, 4.4, 3.7, 4.1, 3.8\}$.
   a. What is decision according to Maximum Likelihood criterion?
   b. What is decision according to Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
   c. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
   d. How large should $P(H_0)$ be, in order for the decision to be D_0?

4. Consider detecting a signal $s_1(t) = 3\sin(2\pi f_1 t)$ that can be present (hypothesis H_1) or not ($s_0(t) = 0$, hypothesis H_0). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 2 samples.
   a. What are the best sample times t_1 and t_2 to maximize detection performance?
   b. The receiver takes 2 samples with values $\{1.1, 4.4\}$, at sample times $t_1 = \frac{0.125}{f_1}$ and $t_2 = \frac{0.625}{f_1}$. What is decision according to Maximum Likelihood criterion?

c. What if we take the decision with Minimum Probability of Error criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$?
d. What is the decision according to Minimum Risk Criterion, assuming $P(H_0) = 2/3$ and $P(H_1) = 1/3$, and $C_{00} = 0$, $C_{10} = 10$, $C_{01} = 20$, $C_{11} = 5$?
e. What if the receiver takes an extra third sample at time $t_3 = \frac{0.5}{f_1}$. Will the detection be improved?