Receiver Operating Characteristic for thresholding-based decision

Laboratory 2, DEPI

Objective

Simulate a binary constant-signal detection system based on thresholding, and characterize the system via the Receiver Operating Characteristic.

Theoretical aspects

Consider a binary message encoded with two constant levels 0 and A, affected by white noise. A sample of the signal is equal to:

$$r = s + n,$$

where s is the true signal (0 or A), and n is the sample of noise.

The receiver decides what is the true signal by comparing the sample r with a threshold T, whose value depends on the specific decision criterion used:

- Maximum Likelihood
- Minimum error probability
- etc.

For one decision, there can be 4 outcomes:

- correct rejection: signal is 0, detection is 0
- false alarm: signal is 0, detection is 1
- miss: signal is 1, detection is 0
- hit (correct detection): signal is 1, detection is 1

The Receiver Operating Characteristic (ROC) curve is the plot of P(hit) against P(false alarm).

Exercises

1. Simulate threshold-based detection with a single sample, as follows:
   - Generate a vector of 1000 values 0 or A, with equal probability (hint: rand $>$ 0.5)
   - Add over it a random noise with normal distribution $\mathcal{N}(0, \sigma^2 = 1)$
   - Compare each element with T to decide which sample is 0 or 1
   - Compare the decision result with the true original vector, and count how many correct detections and how may false alarms have been.
   - Estimate P(hit) and P(false alarm) by dividing the above numbers to the size of the vector
2. Wrap the above code into a function `[phit, pfa] = myThreshDet(T)` that returns the two probabilities for a given `T`. Draw the ROC by running the function for 100 values of T uniformly spaced between 0 and A, and plotting the resulting vector `phit` against `pfa` (
3. Repeat the same simulation for two samples per bit:
   - double the values of the starting vector, making two consecutive 0 or A values, e.g.
     $$[00AA00AAAA00AA...]$$
   - the decision now uses **the average value** of the two consecutive samples of a bit
   - plot the ROC and compare with the first one. Which is better?

Final questions

1. In a practical scenario, what is the disadvantage of using 2 samples for detection, compared to just 1?