Parameter and Signal Estimation

Laboratory 6, DEDP

Objective

Experiment with Maximum Likelihood, Maximum A Posteriori and Minimum Mean Squared Error estimation for a basic signal.

Theoretical aspects

Exercises

1. Generate a 300-samples long sinusoidal signal $s_\Theta = \sin(2\pi f n)$ with frequency $f = 0.02$, and add over it normal noise with distribution $\mathcal{N}(0, \sigma^2 = 2)$. Name the resulting vector `r`. Plot the `r` vector.

2. Estimate the frequency $\hat{f}$ of the signal via Maximum Likelihood estimation, based only on the `r` vector.

   - Write the mathematical expression of the Maximum Likelihood estimation in case of Gaussian noise (`Hint:` based on the Euclidean distance)
   - Generate 1000 candidate frequencies f_k equally spaced from 0 to 0.5
   - Compute the Euclidean distance between `r` and a sine signal with each candidate frequency
   - Maximum Likelihood: choose $\hat{f}_{ML}$ as the candidate frequency which minimizes the Euclidean distance
   - Display $\hat{f}_{ML}$, and plot the resulting sinusoidal along the original
   - Try changing the length of the data. How is the estimation accuracy affected?
   - Try changing the variance of the noise. How is the estimation accuracy affected?

3. TO UPDATE: Suppose that for f we know a *prior distribution* $w(f)$, displayed on the whiteboard. Modify the previous example to implement Bayesian estimation.

- Multiply the computed likelihood function from previous exercise with the prior distribution, for each point. The result is the *posterior* distribution.
- Maximum A Posteriori: choose $\hat{f}_{MAP}$ as the value which maximizes the posterior distribution
- Minimum Mean Squared Error: : choose $\hat{f}_{MMSE}$ as the average value of the posterior distribution
- Display $\hat{f}_{MAP}$ and $\hat{f}_{MMSE}$, and plot the resulting sinusoidal signals along the original and the ML one

4. *Signal inpainting (recover missing parts of signal)*. Randomly replace 20 samples from `data` with 0, to simulate missing data. Rerun exercise 3 and estimate the original signal. Plot the result(s) against the starting data (with the missing samples) to visualize the result.

Final questions

1. TBD