Parameter and Signal Estimation

Laboratory 5, DEPI

Objective

Experiment with Maximum Likelihood, Maximum A Posteriori and Minimum Mean Squared Error estimation for a basic signal.

Theoretical aspects

Exercises

1. Generate a 100-samples long sinusoidal signal with frequency $f = 0.02$, and add over it normal noise with distribution $\mathcal{N}(0, \sigma^2 = 2)$. Name the resulting vector `data`. Plot the `data` vector.

2. Estimate the frequency $\hat{f}$ of the signal via Maximum Likelihood estimation, based only on the `data` vector.

   - Write the mathematical expression of the likelihood function $w(\mathbf{r}|f)$
   - Compute numerically the value of likelihood function for f going from 0 to 0.5, in 200 equally-spaced values
   - Maximum Likelihood: choose $\hat{f}_{ML}$ as the value which maximizes the likelihood
   - Display $\hat{f}_{ML}$, and plot the resulting sinusoidal along the original
   - Try changing the length of the data. How is the estimation accuracy affected?
   - Try changing the variance of the noise. How is the estimation accuracy affected?

3. Suppose that for f we know a *prior distribution $w(f)$, displayed on the whiteboard. Modify the previous example to implement Bayesian estimation.

   - Multiply the computed likelihood function from previous exercise with the prior distribution, for each point. The result is the *posterior* distribution.

- Maximum A Posteriori: choose $\hat{f}_{MAP}$ as the value which maximizes the posterior distribution
- Minimum Mean Squared Error: : choose $\hat{f}_{MMSE}$ as the average value of the posterior distribution
- Display $\hat{f}_{MAP}$ and $\hat{f}_{MMSE}$, and plot the resulting sinusoidal signals along the original and the ML one

4. *Signal inpainting (recover missing parts of signal).* Randomly replace 20 samples from `data` with 0, to simulate missing data. Rerun exercise 3 and estimate the original signal. Plot the result(s) against the starting data (with the missing samples) to visualize the result.

Final questions

1. TBD