Seminar 3 - Maximum Likelihood decision

DEDP

1. Consider the problem of deciding between two possible signals, $s_0(t) = cos(2\pi t)$ and $s_1(t) = sin(2\pi t)$. The receiver takes one sample, at time $t_0 = 0.75$, and the obtained value is $r = 3.5$.

    a. Write the expressions of the two conditional distributions of the sample
    b. Considering that the noise is white gaussian noise, what is the decision based on the Maximum Likelihood criterion?
    c. What is the **best** moment t_0 for sampling, in order to best discriminate between the signals? Justify.
    d. What is the **worst** moment t_0 for sampling, in order to discriminate between the signals? Justify.
    e. Repeat a. in the case the signal 0 is affected uniform noise $\mathcal{U}[-4, 4]$? What is the problem here
    f. What is the maximum variance of a uniform noise, in order to be able to take a decision with the ML criterion for $r = 3.5$?

2. A signal can have four possible values: -6, -2, 2, 6. Each value lasts for 1 second. The signal is affected by white noise with normal distribution. The receiver takes 1 sample per second. Using ML criterion, decide what signal has been transmitted, if the received samples are:

$$4, 6.6, -5.2, 1.1, 0.3, -1.5, 7, -7, 4.4$$

3. A signal can have two possible values, $s_0 = -3$ or $s_1 = 3$. The signal is affected by gaussian noise with distribution $\mathcal{N}(0, 1)$. The receiver performs ML decision based on a single sample.

    a. What is the maximum variance σ^2 of the noise, such that the probability of wrongly detecting s_1 if the true signal is s_0 is at most 10^{-3}
    b. If the noise variance is $\sigma^2 = 0.5$, what is the minimum gap between the two signal levels $(s_1 - s_0)$ such that the probability of correct detection if the true signal is s_1 is at least 0.9999?