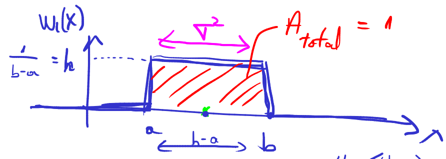
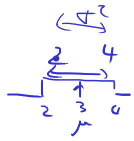
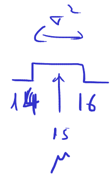
29.10.2020

Seminar 2

1) a) $w_1(x) = U[a,b]$







$$\overline{\mu} = \int_{-\infty}^{\infty} x \cdot w_1(x)\, dx = \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a} \left.\frac{x^2}{2}\right|_a^b = \frac{1}{\cancel{(b-a)} \cdot 2} \cdot \underbrace{(b^2-a^2)}_{\cancel{(b-a)}(b+a)} = \frac{a+b}{2}$$

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2\, w_1(x) = \int_a^b x^2 \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a} \left.\frac{x^3}{3}\right|_a^b = \frac{1}{\cancel{b-a}} \cdot \frac{\overbrace{b^3-a^3}^{\cancel{(b-a)}(a^2+b^2+ab)}}{3} = \frac{a^2+ab+b^2}{3}$$

$$\boxed{\sigma^2 = \overline{x^2} - (\overline{\mu})^2} = \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2 = \frac{{}^{4/}a^2+ab+b^2}{3} - \frac{{}^{3/}a^2+b^2+2ab}{4} = \frac{4a^2+4ab+4b^2-3a^2-3b^2-6ab}{12}$$

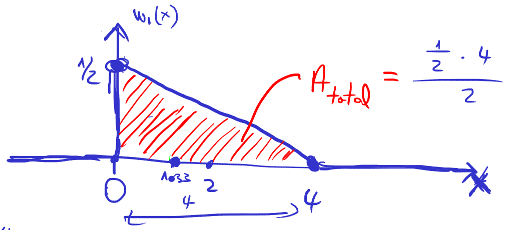
$$= \frac{a^2-2ab+b^2}{12} = \frac{(a-b)^2}{12} = \frac{(b-a)^2}{12}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2\, w_1(x)\, dx = \int_{-\infty}^{\infty} (x^2 - 2\mu x + \mu^2) \cdot w_1(x)\, dx = \underbrace{\int_{-\infty}^{\infty} x^2 w_1(x)}_{\overline{x^2}} \underbrace{- 2 \cdot \mu \cdot \underbrace{\int_{-\infty}^{\infty} x\, w_1(x)}_{\mu} + \mu^2 \cdot \underbrace{\int_{-\infty}^{\infty} 1\, dx}_{}}_{-\mu^2}$$

μ^2

$f(x) = ax+b$

b) $w_1(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}x, & x \in [0,4] \\ 0, & \text{everywhere else} \end{cases}$



$A_{total} = \frac{\frac{1}{2} \cdot 4}{2} = 1$

$$\overline{\mu} = \int_{-\infty}^{\infty} x \cdot w_1(x)\, dx = \int_0^4 x \cdot \left(\frac{1}{2} - \frac{1}{8}x\right) dx = \frac{1}{2}\int_0^4 x\, dx - \frac{1}{8}\int_0^4 x^2\, dx = \frac{1}{2} \cdot \left.\frac{x^2}{2}\right|_0^4 - \frac{1}{8} \left.\frac{x^3}{3}\right|_0^4$$

$$= \frac{1}{4}(4^2 - 0^2) - \frac{1}{8 \cdot 3}(4^3 - 0^3) = 4 - \frac{\cancel{64}^{8}}{\cancel{24}_{3}} = \frac{4}{3} = 1.33$$

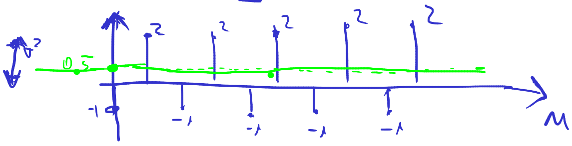
$$\overline{x^2} = \int_0^4 x^2 \cdot \left(\frac{1}{2} - \frac{1}{8}x\right) dx = \frac{1}{2}\int_0^4 x^2 - \frac{1}{8}\int_0^4 x^3\, dx = \frac{1}{2} \cdot \left.\frac{x^3}{3}\right|_0^4 - \frac{1}{8} \cdot \left.\frac{x^4}{4}\right|_0^4 dx = \frac{4^3}{6} - \frac{\cancel{4^4}^{3}\, 8}{\cancel{8 \cdot 4}} =$$

$$= \frac{64}{6} - 8 = \frac{64-48}{6} = \frac{16}{6}$$

$$\sigma^2 = \overline{x^2} - (\mu)^2 = \frac{16}{6} - \left(\frac{4}{3}\right)^2 = \frac{8}{3} - \frac{16}{9} = \frac{24-16}{9} = \frac{8}{9}$$

(2) $f^{(k)} = [-1, 2, -1, 2, -1, 2, -1, 2, -1, 2]$

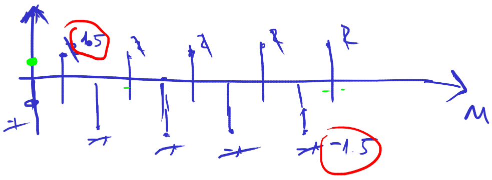
$$\overline{f^{(k)}} = \frac{-1+2-1+2-1+2-1+2-1+2}{10} = \frac{5}{10} = \boxed{\frac{1}{2}}$$



$$\overline{f^{(k)2}} = \frac{(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2}{10} = \frac{25}{10}$$

$$\sigma^2_{f^{(k)}} = \overline{f^{(k)2}} - \left(\overline{f^{(k)}}\right)^2 = \frac{25}{10} - \left(\frac{1}{2}\right)^2 = 2.5 - 0.25 = 2,25$$

$$= \frac{\left(-1-\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2 + \left(-1-\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2 + \dots}{10} = \frac{2.25 + 2.25 + \dots}{10} = 2.25$$



$$R_{xx}[\tau] = \overline{f[t] \cdot f[t+\tau]}$$

$$R_{xx}[0] = \overline{f[t] \cdot f[t]} = \frac{(-1)\cdot(-1) + 2\cdot 2 + (-1)\cdot(-1) + 2\cdot 2 + \dots}{10} = 2.5$$

$$\begin{array}{cccccccccc}
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
\hline
1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4
\end{array} \times \longrightarrow 2.5$$

$$R_{xx}[1] = \overline{f[t] \cdot f[t+1]} = \frac{(-1)\cdot 2 + 2\cdot(-1) + (-1)\cdot 2 + 2\cdot(-1) + (-1)\cdot 2 + 2\cdot(-1) + (-1)\cdot 2 + 2\cdot(-1) + (-1)\cdot 2}{9} = -2$$

periodic

$$\begin{array}{c|ccccccccc|c}
 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & \\
\hline
 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 &
\end{array} \times \longrightarrow -2$$

$$R_{xx}[2] = \overline{f[t] \cdot f[t+2]} = \begin{array}{cc|cccccccc|cc}
 & & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & & \\
\hline
 & & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & &
\end{array} \times \longrightarrow 2.5$$

$$R_{xx}[3] = \dots$$

$$R_{xx}[9] = \begin{array}{cc|cccccccccc}
 & & & & & & & & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & & & & & & & & \\
\hline
 & & & & & & & & -2 & & & & & & & & &
\end{array} \times \longrightarrow -2$$

$$R_{xx}[-1] = \overline{f[t] \cdot f[t-1]}$$

$$\boxed{R_{xx}[-\tau] = R_{xx}[\tau]}$$

$$\begin{array}{cccccccccccc}
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & \times & \\
 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & \\
\hline
 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & & = -2
\end{array}$$

$$R_{fg}[\tau] =$$

f: $\boxed{1 \ -2 \ 1 \ 2 \ 3 \ 0 \ -5 \ -2}$

$\boxed{-2 \ 1 \ 2 \ 3}$

g

$$x = \dots \boxed{\ \ | \ x_{n-2} \ | \ x_{n-1} \ | \ x_n \ | \ x_{n+1} \ |\ } \dots$$

$$h_2 \quad h_1 \quad h_0 \quad h_{-1}$$