Seminar 6

$$d(a,b) = \sqrt{\sum_i (a_i - b_i)^2}$$

1

A:

$$d(x, v_1) = \sqrt{(-2-2)^2 + (5+4)^2} = \sqrt{16+81} = \sqrt{97}$$

$$d(x, v_2) = \sqrt{(-2-1)^2 + (5+5)^2} = \sqrt{109}$$

$$d(x, v_3) = \sqrt{1}$$

$$d(x, v_4) = \sqrt{2}$$

$$d(x, v_5) = \sqrt{116}$$

B:

$$d(x, v_6) = \sqrt{25+16} = \sqrt{41}$$

$$d(x, v_7) = \sqrt{17}$$

$$d(x, v_8) = \sqrt{68}$$

$$d(x, v_9) = \sqrt{26}$$

$$d(x, v_{10}) = \sqrt{4}$$

$\sqrt{2}$	$\sqrt{2}$	$\sqrt{4}$	$\sqrt{17}$	$\sqrt{26}$	$\sqrt{41}$	$\sqrt{68}$	$\sqrt{97}$	$\sqrt{109}$	$\sqrt{116}$
V_3	V_4	V_{10}	V_7	V_9	V_6	V_8	V_1	V_2	V_5
A	A	B	B	B	B	B	A	A	A

$k=1 \Rightarrow \boxed{A}$

$k=3 \Rightarrow \boxed{A}$

$k=5 \Rightarrow \boxed{B}$

$k=7 \Rightarrow \boxed{B}$

$k=9 \Rightarrow \boxed{B}$

2

$$r(t) = \underbrace{a \cdot t^2}_{s_\theta(t)} + \text{noise} \qquad \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

$\mu = 1$

gaussian

$$t = \begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \\ \end{array}$$
$$t = [1, 2, 3, 4, 5]$$

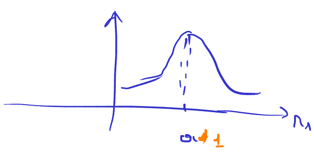
$$r = [1.2,\ 3.7,\ 8.5,\ 18,\ 25.8]$$

$$\hat{a}_{ML} = ?$$

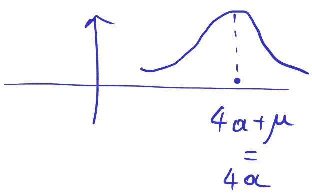
ML estimation: $\hat{a}_{ML} = \underset{a}{\operatorname{argmax}}\ L(a \mid r) = \underset{a}{\operatorname{argmax}} \left(w(r \mid a) \right)$

$$\boxed{w(r \mid a)} = w(r_1 / a) \cdot w(r_2 \mid a) \cdot \ldots \cdot w(r_5 \mid a)$$

$$w(r_1 / a) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r_1 - a)^2}{2\sigma^2}}$$



$$w(r_2 \mid a) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r_2 - 4a)^2}{2\sigma^2}}$$



$$w(r_3 \mid a) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r_3 - 9a)^2}{2\sigma^2}}$$

$$w(r_4 \mid a) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(r_4 - 16a)^2}{2\sigma^2}}$$

$$w(r_5 \mid a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_5 - 25a)^2}{2\sigma^2}}$$

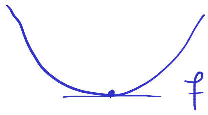
$$\sum (r_i - s_\theta(t_i))^2 = (d(r, s_\theta(t_i)))^2$$

$$\underbrace{w(r \mid a) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^5 \cdot e^{-\frac{\overbrace{\sum (r_i - a \cdot t_i^2)^2}^{D}}{2\sigma^2}}}_{\text{Want maximum}}$$

Want minimum!

Want minimum $\quad D = \sum (r_i - a \cdot t_i^2)^2 = (r_1 - a)^2 + (r_2 - 4a)^2 + (r_3 - 9a)^2 + (r_4 - 16a)^2 + (r_5 - 25a)^2$

$= \text{minimum}$



$f'(x) = 0$

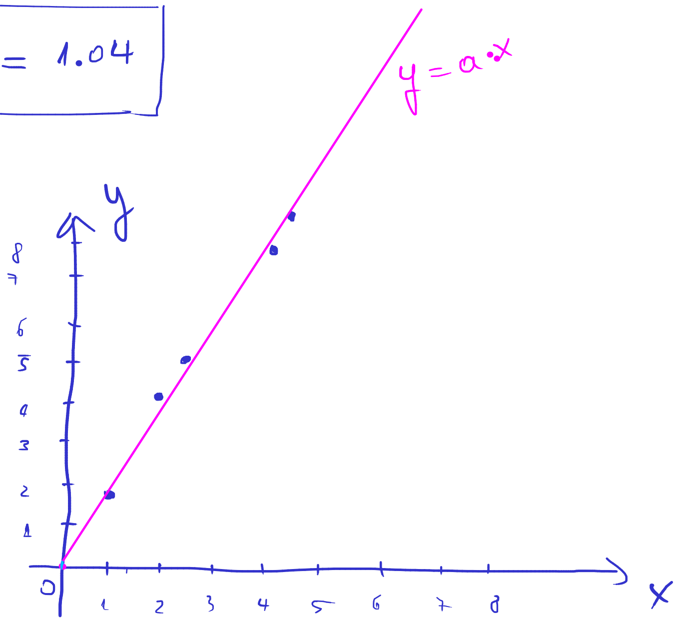
$$\frac{\partial D}{\partial a} = 0 \quad \Rightarrow \quad a = \ldots$$

$$\frac{\partial D}{\partial a} = \cancel{2} \cdot (\overset{1.2}{r_1} - a)(-1) + \cancel{2}(\overset{3.7}{r_2} - 4a) \cdot (-4) + \cancel{2}(\overset{8.5}{r_3} - 9a) \cdot (-9) + \cancel{2}(\overset{18}{r_4} - 16a)(-16) + \cancel{2}(\overset{25.8}{r_5} - 25a)(-25)$$

$= \emptyset$

$$\Leftrightarrow \quad a - 1.2 + 16a - 14.8 + 81a - 76.5 + 256a - 288 + 625a - 645 = 0$$

$$\Leftrightarrow \quad 979 \cdot a = 1025.5 \quad \Rightarrow \quad \boxed{\hat{a}_{ML} = 1.04}$$



③ $\quad y = a \cdot x + \text{noise}$

$t = \cancel{x} = [1 \quad 2 \quad 2.5 \quad 4 \quad 4.3]$

$$\boxed{\begin{array}{l} r = y = [1.8 \quad 4.1 \quad 5.1 \quad 7.9 \quad 8.5] \\ s_\theta = a \cdot x = [a \quad 2a \quad 2.5a \quad 4a \quad 4.3a] \end{array}}$$

Want minimum $\quad D = d(r, s_\theta)^2 = (1.8 - a)^2 + (4.1 - 2a)^2 + (5.1 - 2.5a)^2 + (7.9 - 4a)^2 +$

$+ (8.5 - 4.3a)^2$

$$\frac{\partial D}{\partial a} = 0 \iff \cancel{2}(1.8-a)(-1) + \cancel{2}(4.1-2a)(-2) + \cancel{2}(5.1-2.5a)(-2.5)$$
$$+ \cancel{2}(7.9-4a)(-4) + \cancel{2}(8.5-4.3a)(-4.3) = 0$$

$$a - 1.8 + 4a - 8.2 + 6.25a - 12.75 + 16a - 31.6 + 18.49a - 36.55 = 0$$

$$\iff 45.74a = 90.9 \implies \hat{a}_{ML} = \frac{90.9}{45.74} = 1.98$$