

# Seminar 6

## k-NN and k-Means + ML estimation

### DEDP

1. Consider the k-NN algorithm with the following training set, composed of 5 vectors of class A and another 5 vectors from class B:

- Class A:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \mathbf{v}_5 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

- Class B:

$$\mathbf{v}_6 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \mathbf{v}_7 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \mathbf{v}_8 = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \quad \mathbf{v}_9 = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \quad \mathbf{v}_{10} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Compute the class of the vector  $\mathbf{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$  using the k-NN algorithm, with  $k = 1$ ,  $k = 3$ ,  $k = 5$ ,  $k = 7$  and  $k = 9$

2. Consider the following data

$$\mathbf{v} = \{v_i\} = [1.1, 0.9, 5.5, 0.6, 5, 6, 1.3, 4.8, 6, 0.8]$$

Use the k-Means algorithm to find the two centroids  $\mathbf{c}_1$  and  $\mathbf{c}_2$ , starting from two random values  $\mathbf{c}_1 = 0.95$  and  $\mathbf{c}_2 = 0.96$ . Perform 5 iterations of the algorithm.

3. A received signal  $r(t) = a \cdot t^2 + \text{noise}$  is sampled at time moments  $t_i = [1, 2, 3, 4, 5]$ , and the values are  $r_i = [1.2, 3.7, 8.5, 18, 25.8]$ . The noise distribution is  $\mathcal{N}(0, \sigma^2 = 1)$ . Estimate the parameter  $a$ .

- a. use Maximum Likelihood (ML) estimation

4. Fit a linear function  $y = ax$  (i.e. estimate  $a$ ) through the following data points  $(x_i, y_i) = (1, 1.8), (2, 4.1), (2.5, 5.1), (4, 7.9), (4.3, 8.5)$ , assuming the noise is  $\mathcal{N}(0, \sigma^2 = 1)$

- a. use Maximum Likelihood (ML) estimation