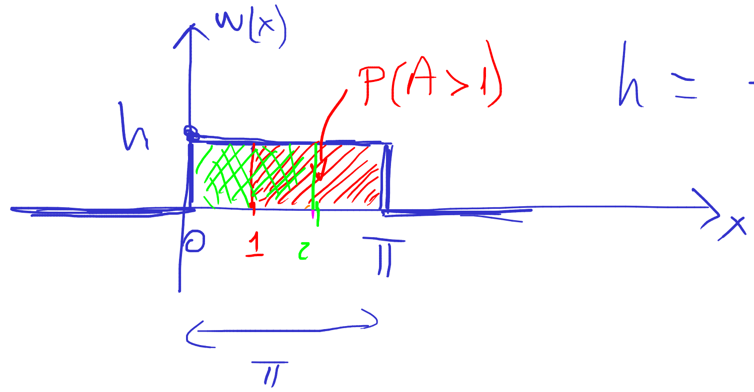
Seminar 1

① a)

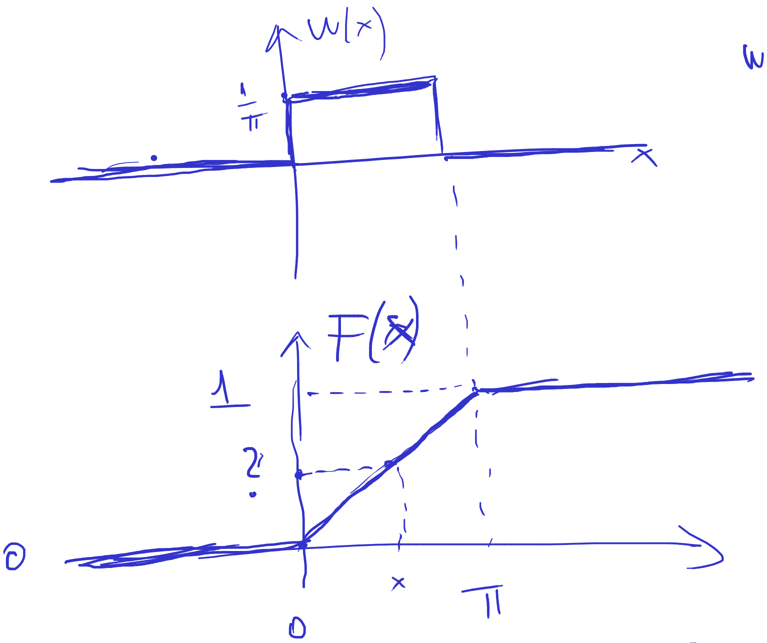


$h = \frac{1}{\pi}$ (because area = 1)

b). $P(A > 1) = h \cdot (\pi - 1) = \frac{\pi - 1}{\pi} = 0.68$

c). $P(A \in (0, 2)) = h \cdot 2 = \frac{2}{\pi} = \ldots$

d). $CDF = F(x) =$



$w(x) = \begin{cases} 0, & x \notin [0, \pi] \\ \frac{1}{\pi}, & x \in [0, \pi) \end{cases}$

$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{\pi} \cdot x, & x \in (0, \pi) \\ 1, & x \ge \pi \end{cases}$

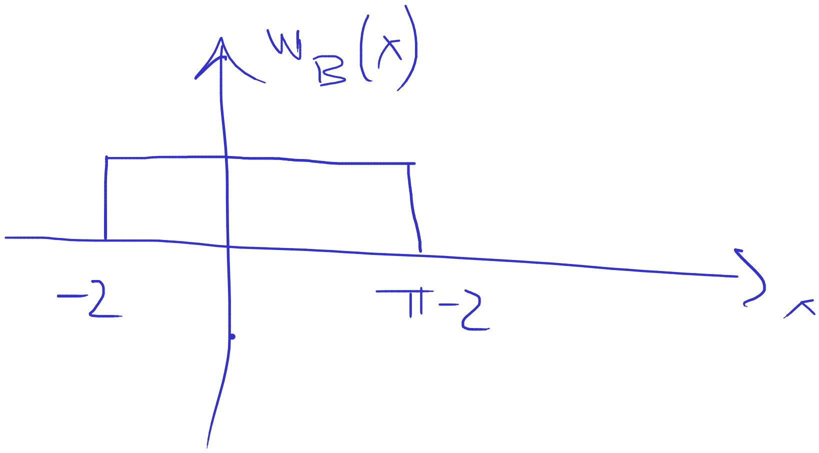
π 1

x ..... ?

$? = \frac{x}{\pi}$

$\frac{1}{\pi} x = \int_{-\infty}^{x} \frac{1}{\pi} dx$

e). $B = A - 2$



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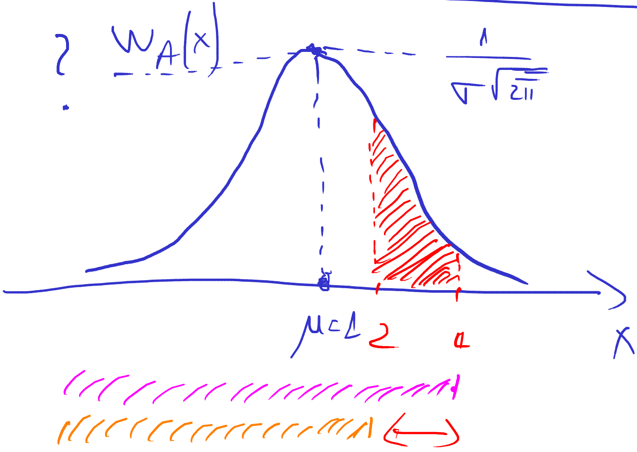
② $N(\mu = 1, \sigma^2 = 2)$?



a) $P(A \in [2, 4]) =$

$$= F(4) - F(2) = 0.223$$

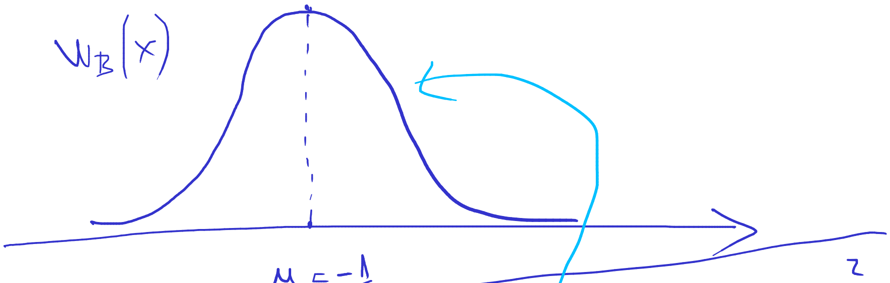
$$F(x) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma \cdot \sqrt{2}}\right)\right)$$

$$\rightarrow F(4) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{4-1}{\sqrt{2}\cdot\sqrt{2}}\right)\right) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{3}{2}\right)\right) = 0.983$$

$$\rightarrow F(2) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{2-1}{\sqrt{2}\cdot\sqrt{2}}\right)\right) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{1}{2}\right)\right) = 0.76$$

b). $B = A - 2$

$w_B(x) = ?$



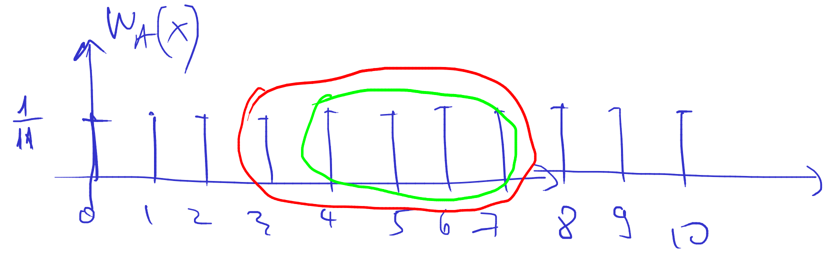
$$\boxed{w(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

c). max is reached for $x = \mu = 1$

$$w_A(1) = w_A(\mu) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-0} = \frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{\sqrt{2}\cdot\sqrt{2\pi}}$$

③ a) # realizations (0, 1, ... 10)

b).



c) $P(\text{impar}) = \frac{5}{11}$

d). $P(A \in [3,7]) = \frac{5}{11}$

$P(A \in (3,7]) = \frac{4}{11}$

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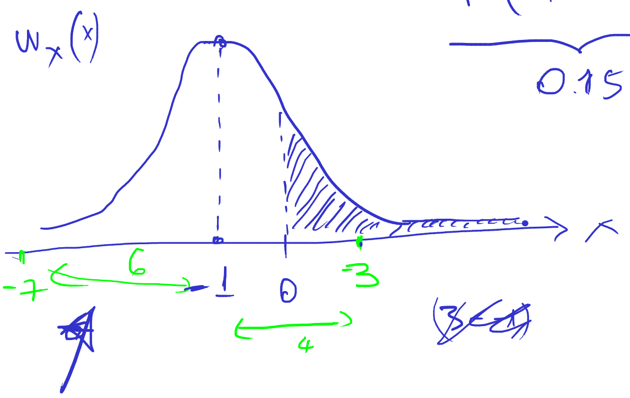
④ X, Y, Z $\quad \mathcal{N}(\mu = -1, \sigma^2 = 1)$ $\quad \boxed{\underline{\text{independent}}}$!

$$P(X \geq 0 \text{ AND } Y \geq 0 \text{ AND } Z \geq 0) =$$

$$= \underbrace{P(X \geq 0)}_{0.15} \cdot \underbrace{P(Y \geq 0)}_{0.15} \cdot \underbrace{P(Z \geq 0)}_{0.15} = 0.15^3$$



$$P(X \geq 0) = \int_0^\infty w(x)\, dx = \underbrace{F(\infty)}_{1} - F(0)$$

$$= 1 - \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{0+1}{\sqrt{2}}\right)\right) = 0.15$$

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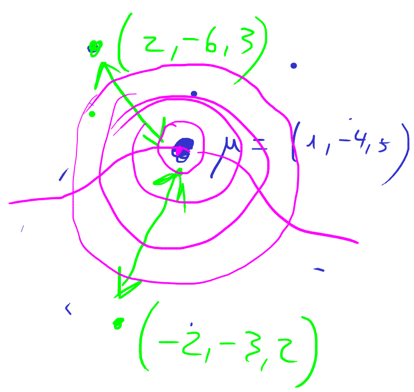
⑤* $\sim$ = "with distribution"

a) $(2,-6,3) = K$

$\mu = (1,-4,5)$

$(-2,-3,2) = L$



$$d(K, \mu) = \sqrt{(2-1)^2 + (-6+4)^2 + (3-5)^2} = \sqrt{1+4+4}$$

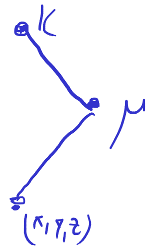
$$d(L, \mu) = \sqrt{(-2-1)^2 + (-3+4)^2 + (2-5)^2} = \sqrt{9+1+9}$$

b).

b). $x, y, z = ?$

$$d(K, \mu) = d((x,y,z), \mu)$$

$$d(K,\mu) = \sqrt{9} = 3$$



$$d((x,y,z), \mu) = 3 = \sqrt{\underbrace{(x-1)^2 + (y+4)^2 + (z-5)^2}_{9}}$$

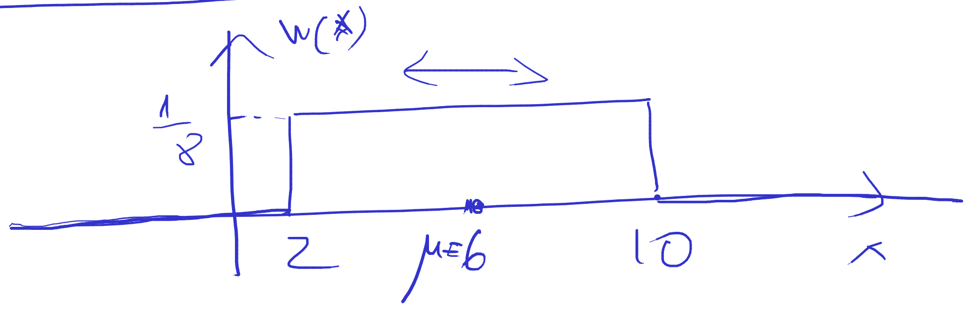
Pick: $x = 1$

$y = 4$

$z = 8$

6

$\mu, \overline{A^2}, \sigma^2$



$$\mu = \int_{-\infty}^{\infty} x \cdot w(x)\, dx = \int_{2}^{10} x \cdot \frac{1}{8}\, dx =$$

$$= \frac{1}{8} \cdot \frac{x^2}{2}\Bigg|_2^{10} = \frac{1}{8}\left(\frac{10^2}{2} - \frac{2^2}{2}\right) =$$

$$= \frac{100-4}{8 \cdot 2} = \frac{96}{16} = 6$$

$$\overline{A^2} = \int_{-\infty}^{\infty} x^2 \cdot w(x)\, dx = \int_{2}^{10} x^2 \cdot \frac{1}{8}\, dx =$$

$$= \frac{1}{8} \cdot \frac{x^3}{3}\Big|_2^{10} = \frac{1}{8}\left(\frac{10^3}{3} - \frac{2^3}{3}\right) = \frac{992}{24} = 41.33$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot w(x)\, dx = \int_{2}^{10} (x-6)^2 \cdot \frac{1}{8}\, dx$$

$$= \frac{1}{8}\left(\int_2^{10} (x^2 - 12x + 36)\, dx\right) =$$

$$= \frac{1}{8}\int_2^{10} x^2 - \frac{12}{8}\int_2^{10} x + \frac{36}{8}\int_2^{10} 1\, dx$$

$$= \frac{1}{8} \cdot \frac{x^3}{3}\Big|_2^{10} - \frac{12}{8}\,\frac{x^2}{2}\Big|_2^{10} + \frac{36}{8}\, x\Big|_2^{10} \quad 5.33$$

$$= \frac{1}{8} \cdot \frac{992}{3} - \frac{12}{8} \cdot \frac{96}{2} + \frac{36}{8} \cdot 8 = \dots\dots$$

$$\boxed{\sigma^2 = \overline{A^2} - \mu^2} = 41.33 - 6^2 = 5.33$$

-FIN-