DEDP Sample Exam

This is a **sample exam sheet**. The exercises / questions are for illustrative purposes only. The exercises shown here are merely the ones from the seminars. In the real exam, they will be changed.

Exercises

1. (1p) Compute the probability that three r.v. X, Y and Z i.i.d. $\mathcal{N}(-1, 1)$ are all positive simultaneously (assume *erf()* is known).
2. (2p) Compute the temporal variance of the following realization of a finite-length random process:
$$v = [-1, 2, -1, 2, -1, 2, -1, 2, -1, 2]$$
3. Consider the detection of a signal with two possible levels, 0 (hypothesis H_0) or 6 (hypothesis H_1). The signal is affected by noise with triangular distribution $[-5, 5]$. The receiver takes one sample $r = 3.5$.
   a. (1p) Draw the graphic of the two functions
   b. (3p) Find the decision for the sample $r = 3.5$ considering the Minimum Probability of Error criterion, if $P(H_0) = \frac{3}{4}$ and $P(H_1) = \frac{1}{4}$.
   c. (3p) What is the probability of false alarm, $P(D_1 \cap H_0)$, for the Maximum Likelihood criterion?
4. (3p) A signal can have two values, -4 (hypothesis H_0) or 5 (hypothesis H_1). The signal is affected by AWGN $\mathcal{N}(0, \sigma^2 = 1)$. The receiver takes 3 samples with values $\{1.1, 4.4, 2.2\}$. What is decision according to Maximum Likelihood criterion?
5. (5p) A received signal $r(t) = a \cdot t^2 + noise$ is sampled at time moments $t_i = [1, 2, 3, 4, 5]$, and the values are $r_i = [1.2, 3.7, 8.5, 18, 25.8]$. The noise distribution is $\mathcal{N}(0, \sigma^2 = 1)$. Estimate the parameter a using Maximum Likelihood (ML) estimation.

Known formulas:

- $F(x) = \frac{1}{2}\left(1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$

Theory

1. (1p) Let X be a random variable obtained by rolling a die. Plot the cumulative distribution function of X.
2. (2p) State the Wiener-Khinchin theorem.
3. (2p) Fill in the blanks: "The minimum probability of error criterion is identical to maximum likelihood criterion when _______________". Justify.
4. (2p) Color ("hașurați") the conditional probability of **correct rejection** (correct decision of non-detection) in case of hypothesis H_0, for the **Maximum Likelihood** criterion, for the two likelihood functions depicted below. Explain (in words) what you colored.

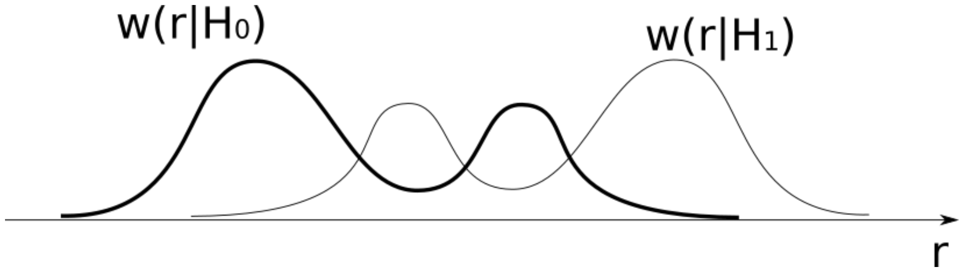

Figure 1:

5. (3p) Consider detection of a constant signal (values 0 or A) based on a single sample r, affected by **Gaussian noise**. The likelihood ratio is compared to some value K, $\frac{w(r|H_1)}{w(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} K$. Find the decision regions R_0 și R_1 (based on value K).
6. (1p) If the **noise** added to a signal is **doubled**, how does the Signal-to-Noise Ratio (SNR) change (explain in words why):
   a. SNR increases
   b. SNR decreases
   c. SNR remains the same
7. (5p) Prove that minimizing $I = \int_{-\infty}^{\infty} C(\epsilon)w(\Theta|\mathbf{r})d\Theta$ with a quadratic cost function $C(\epsilon) = \epsilon^2 = (\hat{\Theta} - \Theta)^2$ leads to the formula of the MMSE estimator:

$$\hat{\Theta}_{MMSE} = \int_{-\infty}^{\infty} \Theta w(\Theta|r)d\Theta$$

8. (1p) The **a posteriori** distribution of an unknown parameter Θ is a triangular distribution, as depicted below.

   a. What is the value of the MAP estimator? Explain.
   b. What is the value of the MMSE estimator? Explain.

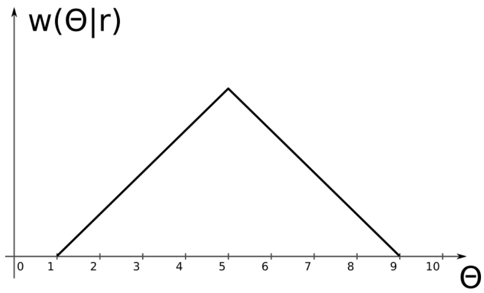

Figure 2:

9. (2p) Consider an estimation algorithm which always produces an estimate $\hat{\Theta}$ which is larger than the true value Θ. Is this estimator biased or unbiased? Justify.