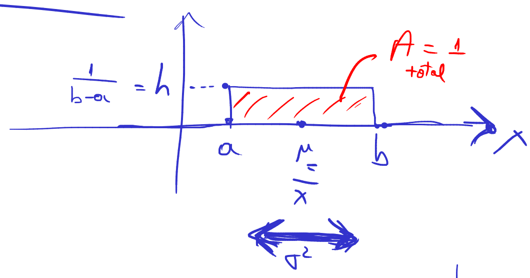
Seminar 2

(1) a) $w_1(x) = U[a,b]$



Same as Sem. 1 :

$$\mu = \overline{x} \stackrel{?}{=} \int_{-\infty}^{\infty} x \cdot w_1(x)\,dx = \int_a^b x \cdot \frac{1}{b-a}\,dx = \frac{1}{b-a}\,\frac{x^2}{2}\Big|_a^b = \frac{1}{\cancel{b-a}} \cdot \left(\frac{b^2-a^2}{2}\right) = \frac{a+b}{2}$$

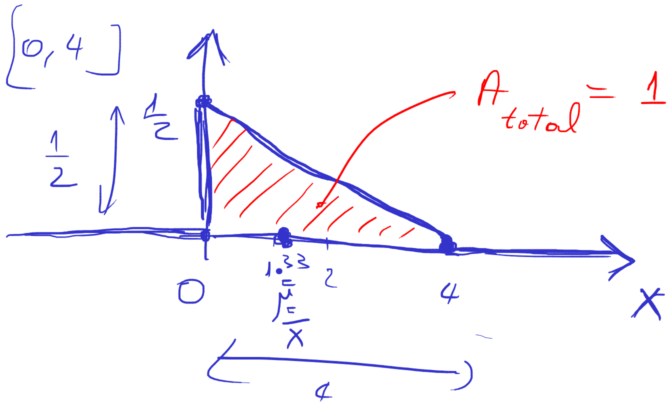
where $b^2-a^2 = \cancel{(b-a)}(b+a)$.

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2 \cdot w_1(x)\,dx = \int_a^b x^2 \cdot \frac{1}{b-a}\,dx = \frac{1}{b-a} \cdot \frac{x^3}{3}\Big|_a^b = \frac{(b^3-a^3)}{3\cancel{(b-a)}} = \frac{a^2+ab+b^2}{3}$$

where $b^3-a^3 = \cancel{(b-a)}(b^2+ab+a^2)$.

$$\sigma^2 = \overline{x^2} - (\overline{x})^2 = \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4} = \frac{4a^2+4ab+4b^2-3a^2-3b^2-6ab}{12} = \frac{a^2+b^2-2ab}{12} = \frac{(a-b)^2}{12} = \frac{(b-a)^2}{12}$$

b). $w_1(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}x, & x \in [0,4] \\ 0, & \text{rest} \end{cases}$



$$\overline{X} = \int_{-\infty}^{\infty} x \cdot w_1(x)\,dx = \int_0^4 x \cdot \left(\frac{1}{2} - \frac{1}{8}x\right)dx = \frac{1}{2}\int_0^4 x\,dx - \frac{1}{8}\int_0^4 x^2\,dx$$

$$= \frac{1}{2} \cdot \frac{x^2}{2}\Big|_0^4 - \frac{1}{8} \cdot \frac{x^3}{3}\Big|_0^4 = \frac{4^2}{4} - \frac{1}{\cancel{24}\,6} \cdot 4^{\cancel{3}\,2}$$

$$= 4 - \frac{16}{6} = 4 - \frac{8}{3} = \frac{4}{3} = 1.33$$

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2 w_1(x) = \int_0^4 x^2\left(\frac{1}{2} - \frac{1}{8}x\right)dx = \frac{1}{2}\int_0^4 x^2\,dx - \frac{1}{8}\int_0^4 x^3\,dx$$

$$= \frac{1}{2}\,\frac{x^3}{3}\Big|_0^4 - \frac{1}{8}\cdot\frac{x^4}{4}\Big|_0^4 = \frac{1}{6}\cdot 4^3 - \frac{1}{\cancel{32}\,2}\cdot 4^{\cancel{4}\,2} =$$

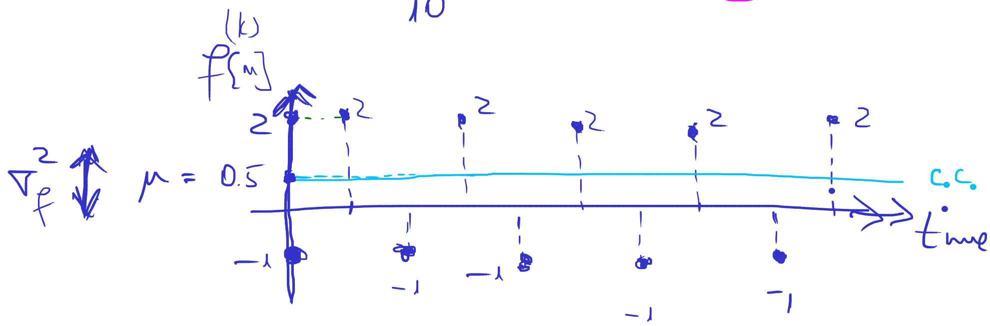
$$= \frac{64}{6} - \frac{16}{2} = \frac{32}{3} - 8 = \frac{32}{3} - \frac{24}{3} = \frac{8}{3}$$

$$\sigma^2 = \overline{x^2} - (\overline{x})^2 = \frac{8}{3} - \left(\frac{4}{3}\right)^2 = \frac{24}{9} - \frac{16}{9} = \frac{8}{9}$$

$$= \overline{(x-\mu)^2}$$

(2) $f^{(k)}[n] = [-1, 2, -1, 2, -1, 2, -1, 2, -1, 2]$

$$\overline{f^{(k)}[n]} = \frac{-1+2-1+2-1+2-1+2-1+2}{10} = 0.5$$



$$\overline{f^{(k)}[n]^2} = \frac{(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2+(-1)^2+2^2}{10} = 2.5$$

$$\sigma_f^2 = \frac{(-1-0.5)^2+(2-0.5)^2+(-1-0.5)^2+\dots}{10} = \frac{1.5^2+1.5^2+\dots}{10} = 1.5^2 = 2.25$$

$$= \overline{f^2[n]} - \left(\overline{f[n]}\right)^2 = 2.5 - 0.5^2 = 2.25$$

$$R_{xx}[\tau] = \overline{x[t] \cdot x[t+\tau]}$$

$$R_{ff}^{(k)}[\tau] = ?$$

$$\begin{array}{cccccccccc}
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
\hline
1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4
\end{array} \longrightarrow 2.5$$

$$R_{ff}^{(k)}[0] = \frac{(-1)(-1) + (2)(2) + (-1)\cdot(-1) + 2\cdot 2 + \dots}{10} = 2.5$$

$$R_{ff}^{\tau}[1] = \frac{(-1)\cdot 2 + (2)\cdot(-1) + \dots + (-1)\cdot 2}{9} = -2$$

periodic

$$\begin{array}{ccccccccccc}
 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & \\
\hline
 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 &
\end{array}$$

x

periodic

9 terms

$$R_{ff}^{\tau}[2] = \frac{(-1)\cdot(-1) + 2\cdot 2 + \dots}{8} = \frac{20}{8} = 2.5$$

$$\begin{array}{cccccccc}
1 & 4 & 1 & 4 & 1 & 4 & 1 & 4
\end{array} \longrightarrow \frac{20}{8}$$

$$R_{ff}[3] =$$

$$-2 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2 \longrightarrow -2$$

⋮

$$\begin{array}{ccccccccccccccccccc}
 & & & & & & & & & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & & & & & & & & & \\
\hline
 & & & & & & & & & -2 & & & & & & & & &
\end{array}$$

$$-2 = R_{ff}[9]$$

$$R_{ff}[-1] = \begin{array}{lccccccccccc} f: & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & \\ f: & & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\ \hline & & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & \end{array} \Longrightarrow -2$$

$$R_{ff}[-k] = R_{ff}[k]$$

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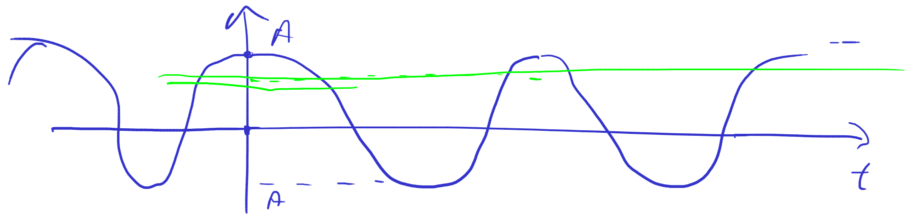
③ $s(t) = \cos(2\pi f t)$



$$c.c. = \overline{s(t)} = \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} \underbrace{\cos(2\pi f t)}_{\Delta(t)}\, dt}_{0} = 0$$

$$P = \overline{s(t)^2} = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

$$P_{ac} = \sigma^2 = \overline{s(t)^2} - \left(\overline{s(t)}\right)^2 = \frac{A^2}{2}$$

$P_a \quad P \quad P_c$